Alexandroff space as applied to image analysis and Edge Detection

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ملخص

لقد قمنا في هذا البحث بدراسه مفهوم الخل يه المركبه والتي تعني (T_0-Alexandroff space) وكيفيه تطبيقها بوصف البنيه في الصور،حيث نرى ان الطوبولوجيا غير ممكنه الا على المجموعات المنتهيه . ان الخلايا المركبه تقسم الى كتل من الخلايا وسوف نستخدم افكار الخلايا الم ركبه في وصف بنيه الاشياء في الصور ،وكذلك نستخدم بعض الخوارزميات لحساب عدد اويلر و يناقش طريقة مبتكرة وفعالة للكشف عن حافة الصورة من خلال استخدام وصف الطبولوجيا وتطبيق خطوات الخوارزمية للعثور على " الكشف عن الحافة لصورة" والعثور على الآثار النتائج إلى "تحليل الصور" لكل الكتل في كل خطوة.

Abstract

In this paper study the notion of a cell complex (T_0-Alexandroff) which is well known in the topology is applied to describe the structure of images. It is shown that the topology of cell complexes is the only possible topology of finite sets. The process of image segmentation is considered as splitting (in the topological sense) a cell complex into blocks of cells. We are using the notion of a cell complex to describe the structure of an object in an image , also using some algorithm for computing the Euler number and a novel and effective method for image edge detection and polish (rub) is discussed using topological description. Appling steps for Algorithm to find Edge Detection Image and to find the effects of the results in Image Analysis for blocks to each step.

Keywords: Alexandroff space, cell complex , Euler number , Image edge detection

1. Introduction

Topological knowledge plays an important role in computer graphics and image analysis. Images may be represented in computers only as finite sets. We are looking at the Alexandroff topological spaces (T_0-Alexandroff) which are introduced for the first time by P. A. Alexaandroff in 1937 [1]. These topologies satisfy the property that an arbitrary intersection of open sets is open, or equivalently, each element has a minimal neighborhood base .So, any discrete space is Alexandroff, and any finite space is also Alexandroff , that To- Alexandroff Spaces are in one-to-one correspondence with posets , via the relation called (Alexandroff) specialization order $x \le y$ if and only if $x \in \overline{\{y\}}$,that is y belong to smallest open set contains x ,and each one is completely determined by the other. In digital plane as a structure consisting of *heterogeneous elements*, namely, of elements of different dimensions: 0-dimensional points, 1-dimensional line elements, and 2-dimensional area elements [11]. Such a structure is well known in the topology as a cell complex [5]. In section two we are using the notion of a cell complex to describe the structure of an object in an image , also using some algorithm for computing the Euler

number [11] (The number of connected components minus the number of holes). In section 3 we write a Computer program for computing Euler Number, as well as writing an algorithm for working, in addition to a Sequential connected components algorithm using a 4-connectivity. Edge detection is an important field of image processing. It can be used in many applications such as identification of objects in a scene. Edge detection refers to the process of locating sharp discontinuities in an image [2].
The purpose with the description is to quantify a representation of an object. This implies that we instead of talking about areas in the image we can talk about their properties, such as length, curviness, and so on. First segmentation and then representation and description, Image regions can be represented by either the border or the pixels of the region [9]. These can be viewed as external or internal characteristics, respectively. In all representation it is desirable to choose methods that are rotated, scale and translation invariant, To represent and describe information embedded in an image in other forms that are more suitable than the image itself. Edge detection is an important field of image processing. Edges characterize object boundaries and are therefore useful for segmentation, registration, feature extraction, and identification of objects in a scene [4].

2. Cell Complexes and Alexandroff Spaces

The topology of cell complex is a branch of the general topology was founded by the famous Riemann in the 19th century. It considers structures consisting of elements of different dimensions called *cells* [5]. The cells may be interpreted as faces of a polyhedron or simply considered as abstract elements of a set **.**
The base notions and some important Theorems predominantly based on the publications of V.A. Kovalevsky, but also of other authors**.**

Definition (2.1) [5] *An abstract cell complex is a structure* $(X, \leq, \dim)$*,* where
$(X, \leq)$ is a post (partially ordered set, that is, $\leq$ is a binary reflexive transitive and anti-symmetric relation on the set X), and $\dim : X \to N$ is a function such that $x \leq y$ implies that $\dim(x) \leq \dim(y)$, for any $x \in X$. The elements of X are called cells, and, for $x, y \in X$, if $\dim(x) = k$, x is named k-cell. The dimension of $(X, \leq, \dim)$ is defined by $\sup\{\dim(x) : x \in X\}$. If $X = (X, \leq, \dim)$ is an abstract cell complex, then a sub complex M= (M, ,dim M)
of X is entirely determined by the subset $M \subseteq X$, by defining $\leq_M$ as the restriction of $\leq$ onto $M \times M$, and dim M as the restriction of the function dim onto M.

Examples (2.2) Cell complex of a d-dimension or d-cell , d=1,2,3 are shown in figure(1)

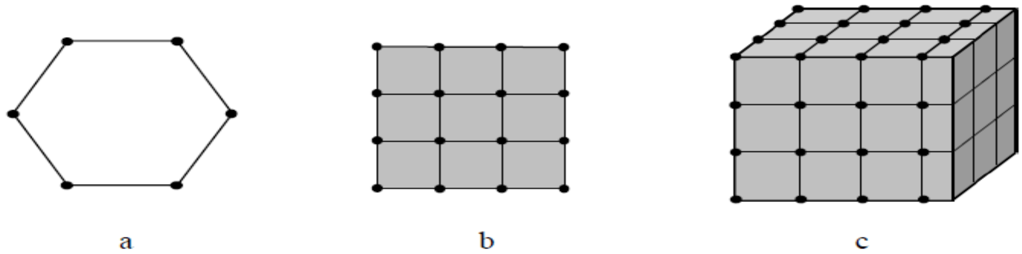


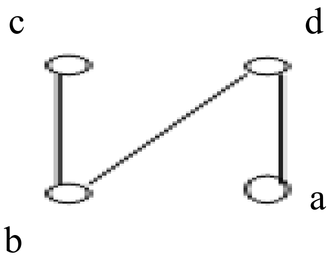
Fig (1)

Thus it is possible to speak of a sub complex $E' \subset E$ while understanding the sub complex $C' = (E', B', \dim')$. All sub complexes of C may be regarded as subsets of E and There the 0-cells are called *points*, 1-cells are called *cracks*, 2-cell is called *pixels* and 3-cells are called *voxels* .

Theorem (2.3) [5] Every To-finite topological space is isomorphic to an abstract cell complex.

Definition (2.4)[5] A sub complex *S* of *C* is called *open* in *C* if for every element *e′* of *S* all elements of *C* which are bounded by *e′* are also contained in *S*.

Example (2.5) Let S= {a, d} is open in the cell complex of figure (2) .But S*= {a, b, d} is not open in it since c $\in$SN (b) but c $\notin$ S*.



Definition (2.6)[5] The open subset *St*(*e′*, *C*) consisting of *e′* and all elements of *C* which are bounded by *e′* is called the *open star* of *e′* in *C*. It is exactly the same as the smallest neighborhood SN(*e′*) of *e′* in *C*.

The notion of an open star is of great importance since open stars are the simplest open subsets in a complex. All other open subsets are unions of some open stars.

Remark (2.7) The family of open star is a base of a T_0-Alexandroff topology on X. Conversely, for a given (X, T ($\leq$)) a T_0-Alexandroff space is defined by $x \leq y \Leftrightarrow y \in SN(x) \Leftrightarrow y \in st(x)$.Therefore cell complex is equivalently, T_0-Alexandroff space .

Definition (2.8)[5]The boundary of a subset $S \subset C$ relative to C is the subset Fr(S,C) of all elements e' of C such that any open neighborhood of e' contains elements both of S and of its complement C \ S.

Under the topology of cell complexes, there exists a unique definition of the boundary, there is no need to distinguish between the inside and outside boundary as it was the case with the neighborhood graphs. We may notion this assertion when supposing a 0-cell to be an end point on the boundary of a subset S and considering all possible combinations of the membership in S of the four pixels bounded by this 0-cell.

Definitions (2.9) [5]

(1) A sequence of elements of a subset S of a complex C beginning with e' and finishing at e" is called a path in S from e' to e" if for every two elements which are adjacent in the sequence one of them is bounding the other.

(2) A subset S is called connected if for any two elements e' and e" of S there exists a path in S from e' to e".

(3) Two sub complexes S1 and S2 of a complex C is called incident to each other if they do not intersect and there are two elements e1$\in$S1 and e2$\in$S2 such that one of them bounds the other.

That is two element x, y (X are called incident if $x \leq y$ or $y \leq x$.

3. Topological Descriptor

Topology is the study of properties of an object that is not affected by deformation, Topology is the study of properties of a figure that are unaffected by any deformation, as long as there is no tearing or joining of the figure (sometimes these are called rubber-sheet distortions).

Definition (3.1) [9,11] The Euler number is defined as the number of connected components minus the number of holes.

Topological properties are useful for global descriptions of regions in the image plane. Simply defined. Topology is the study of properties of a figure that are unaffected by any deformation. As long as there is no tearing or joining of the figure (sometimes these are called rubber-sheet distortions) [6]. If a topological descriptor is defined by the number of holes in the region , this property obviously will not be affected by a stretching or rotation transformation. [6, 9].

$$E = C - H \tag{1}$$

In general, the number of holes will change if the region is torn or folded note that, as stretching affects distance, topological properties do not depend on the notation of distance or any properties implicitly based on the concept of a distance measure. [6] .
The Euler Number function in Matlab called bweuler of ban- of binary image

eul = bweuler(BW,n) (2)

The Euler Number function return value eul is a scalar whose value is the total number of objects in the image minus the total number of holes in those objects. The argument n can have a value of either 4 or 8, where 4 specifies 4-connected objects and 8 specifies 8-connected objects; if the argument is omitted, it defaults to 8. bweuler computes the Euler number by considering patterns of convexity and concavity in local 2-by-2 neighborhoods [4, 7].

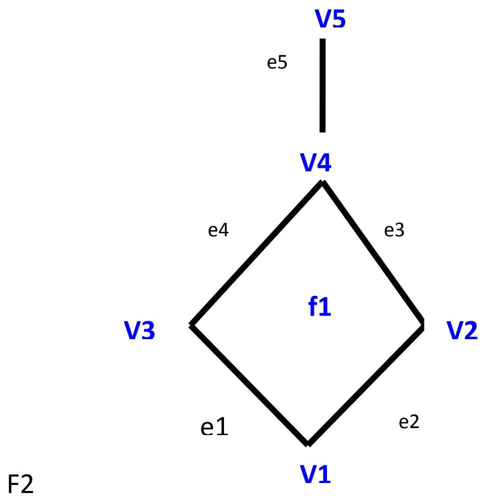
Regions represented by straight-line segments (referred to as polygonal networks) have a particularly simple interpretation in terms of Euler number. Classifying interior regions of such a network in to faces and holes often is important. Denoting the number of vertices by V , the number of edges by Q ,and the number of faces by F gives the following relationship, called the Euler formula:

$$\mathbf{V-Q+F=C-H} \tag{3}$$

The Euler number is also Topological property. The regions shown in fig. 3 for example, cell complex with 5 vertices, 5 edges, and 2 faces : 5-5+2=2

Which, in view of Eq. (3), is equal to the Euler number:

$$\mathbf{V - Q + F = C - H = E} \tag{4}$$



Fig(3)

4- Algorithm for work

1-start
2-read image from database for images
3-Find information for image
4-convert image to Gray scale image
5-find the Number of Convexities, Number of Concavities and Euler Number
6-Find Edge detection for image
7-Scan all pixels for image
8-Extended (expansion) the image
9-Fill all holes in Image
10-A number of blocks in the image from the image
11-Find all objects that intersect with the image borders to clarify confined within only image objects
12-polish (rub) image, Body image twice weaken with any masek regulating element
13-if you have another image to process then go to (2)
14-End

Prewitt Edge Detector Filter creates an image where edges (sharp changes in grey level values) are shown. Only a 3x3 filter size can be used with this filter [6]. This filter uses two 3x3 templates to calculate the Prewitt gradient value as shown below:
Templates:

Table (1): Prewitt Edge Detector Filter information

Horizontal Filter		
1	1	1
0	0	0
-1	-1	-1

Vertical Filter		
-1	0	1
-1	0	1
-1	0	1

3x3 filter window		
A1	A2	A3
A4	A5	A6
A7	A8	A9

X = -1*A1 + 1*A3 - 1*A4 + 1*A6 - 1*A7 + 1*A9

Y = 1*A1 + 1*A2 + 1*A3 - 1*A7 - 1*A8 - 1*A9
Where A1 .. A9 are grey levels of each pixel in the filter window.
Note that the actual formula uses the horizontal and vertical components into the final form
Pixel = SQRT ((X*X) + (Y*Y)) (5)
Where X = (A1+A2+A3-A7-A8-A9) and Y = (A3+A6+A9A1-A4-A7)
But for performance reasons we approximate the result and leave the final formula out [12]

5. Experimental Results

The proposed approach is tested using the 20 color images table (1) show some of these images . Table (2) shows Convert Images to gray Images. Table (3) finds image edge detection using Prewitt to detect edges. When we use general arithmetic to mean filtering technique to clear the noise, the output image is not at all a pleasant one. However, in this proposed arithmetic mean filtering technique, the output will be a visually pleasing one.
Table (4) show extended (expansion) the images for repeat all pixels which values between 0 to 90. Table(6) fill all holes in Images for all pixels no zero values .The topology descriptors for image not change for all these process table(5) show the all blocks for Image name(B). And Table (6) finds polish (rub) image, Body image twice weaken.
While removing the noisy pixel we should preserve the details of edge information as well as spatial resolution. The proposed filtering algorithm meets these conditions without any negligence. Using the new filtering technique, the edges of the object are detected by using the similarity criteria. In future, they can be upgraded them according to achieve better performance.

6. Conclusions

In this paper we have developed a new technique for edge detection which is better than the existing filtering techniques for salt and pepper noise. The visual examples and associated tables show that the proposed methods are better than the traditional median-type filters in the aspects of the noise removal, edge and fine details preservation, as well as minimal signal distortion. The fact that topological descriptors can be derived from the boundary surface representation suggests for Image may be suited for providing topological matching constraints for use in higher vision reasoning figure (4) show the Image analysis for each step. In the line drawing interpretation, despite much activity directed towards (geometrically) cataloguing junctions of both polyhedral and curved objects results on real edge data have been disappointing. The lack of robustness of these schemes to improperly classified and missing junctions remains a major source of frustration.

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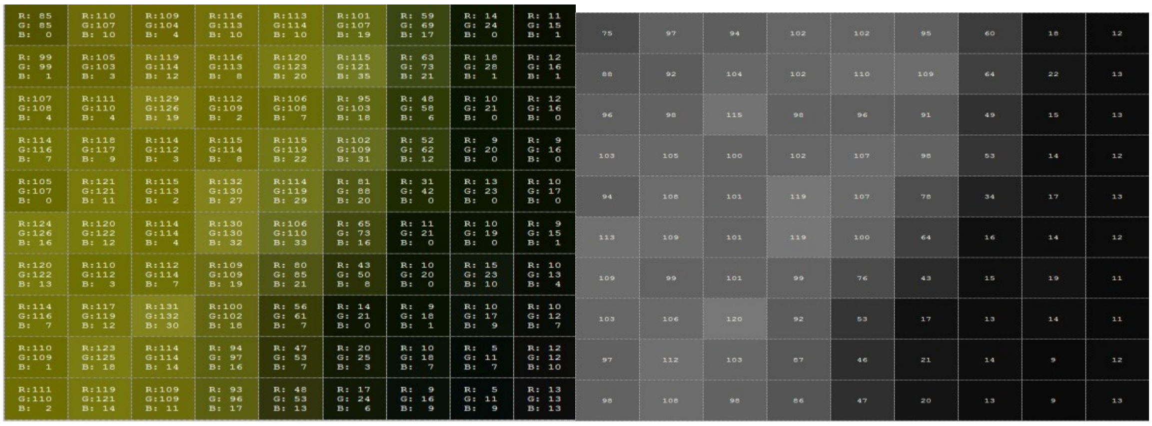

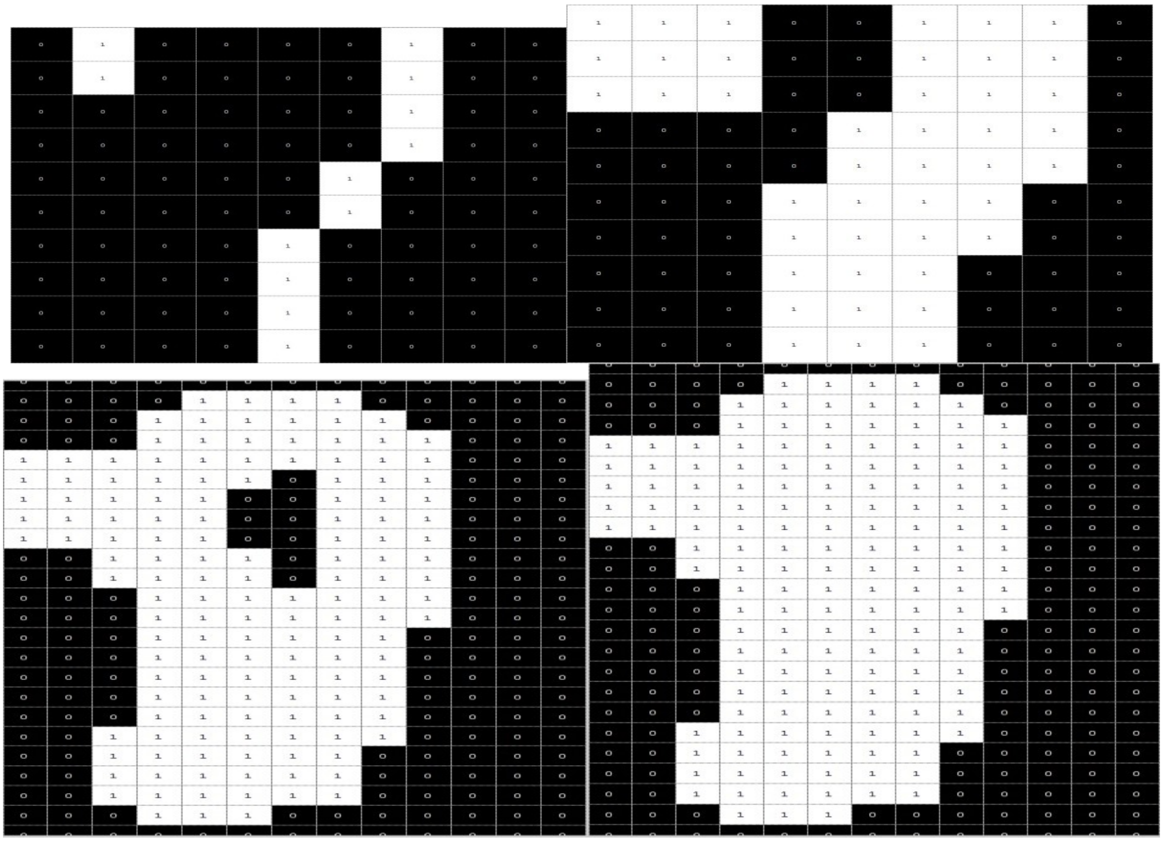

Fig. (4) : Image analysis results for each step