

# Teorija grafov - Zapiski predavanj

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# 1 Introduction

A graph is defined as  $G = (V, E)$ .  $n = |V|$  is the number of vertices,  $m = |E|$  is the number of edges. We also denote them as  $V(G), n(G), E(G), m(G)$ .

## 2 Independence, matching, covers

**Definition.** The set of vertices  $S \subseteq V$  is an **independent set** if  $G(S)$  contains no edges. (No two vertices in the independent set are adjacent)

The independence number  $\alpha(G)$  is the size of the maximum independent set.

**Definition.** The set of vertices  $T \subseteq V$  is a **vertex cover** if  $\forall e \in E T \cap e \neq \emptyset$ . (All edges have at least one endpoint in the vertex cover)

The vertex cover number  $\beta(G)$  is the size of the minimum vertex cover.

**Definition.** A **matching** is a set of edges  $M \subseteq E$  such that  $\forall e, f \in M e \neq f \implies e \cap f = \emptyset$ . (No two edges share a vertex)

The matching number  $\alpha'(G)$  is the size of the maximum matching.

**Definition.** An **edge cover** is a set of edges  $C \subseteq E$  such that  $\forall v \in V \exists e \in C v \in e$ . (All vertices are covered by at least one edge from  $C$ )

The edge cover number  $\beta'(G)$  is the size of the minimum edge cover. Some graphs have no edge covers, for example graphs with isolated vertices.

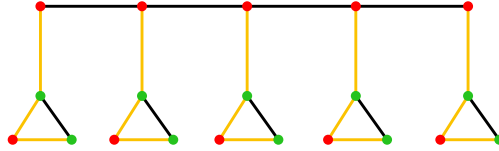


Figure 1:  $G$  from example

**Example.**  $\alpha(G) = 8$

$h(G) = 20$

$\beta(G) = 12 \rightarrow$  complement of vertex set

$\alpha'(G) = 10$  maximum for  $\alpha'$  is  $\frac{h(G)}{2}$

$\beta'(G) = 10$

Observations

**Theorem.**  $\alpha(G) + \beta(G) = |V|$