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Meshfree Methods: A Comprehensive Review of Applications

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The meshfree methods in computational mechanics have been actively proposed and increasingly developed in order to overcome some drawbacks in the conventional numerical methods. Over past three decades meshfree methods have found their way into many different application areas ranging from classical astronomical problems to solid mechanics analysis, fluid flow problems, vibration analysis, heat transfer and optimization to the numerical solution of all kind of (partial) differential equation problems. The present work is an effort to provide a comprehensive review of various Meshfree methods, their classification, underlying methodology, application area along with their advantages and limitations. Key contributions of mesh free techniques to the area of fracture mechanics have been discussed with applications of element free Galerkin method (EFGM) to fracture analysis as primary concern.

Keywords: Meshfree methods; EFGM; fracture mechanics; crack.

1. Introduction

Every phenomenon in nature, physical, chemical or biological, can be easily described in the algebraic, deferential or integral equations and to solve them we use numerical techniques and obtain approximate solutions. The main idea of numerical simulation is to transform a complex practical problem into a simple discrete form of mathematical description, recreate and solve the problem on a computer, and finally reveal the phenomena virtually according to the requirements of the analysts. Engineers and scientists have provided us different numerical techniques such as finite difference method (FDM), finite element method (FEM), boundary element methods, etc. In present scenario, the FEM for the modeling of complex

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problems in applied mechanics and related fields is well established. It is a robust and thoroughly developed technique, but it is not without shortcomings. The reliance of the method on a mesh leads to complications for certain classes of problems. Consider the modeling of large deformation processes; considerable loss in accuracy arises when the elements in the mesh become extremely skewed or compressed. The growth of cracks with arbitrary and complex paths, and the simulation of phase transformations are also difficult. The use of a mesh in modeling these problems creates difficulties in the treatment of discontinuities which do not coincide with the original mesh lines. Computational fracture mechanics is another area which presented a tough challenge to researchers in terms of accuracy in results and capturing stress field oscillations near the crack tip area using FEM.

To overcome these problems meshfree or element-free method has been proposed and achieved remarkable progress in recent years. In MMs approximation is built with the help of nodal points only. The first meshfree method is smooth particle hydrodynamics (SPH) [Gingold and Monaghan (1977); Liu and Liu (2003, 2010)] and it was able to solve problems of fluid dynamics, heat conduction, machining [Jeong *et al.* (2003); Tartakovsky and Meakin (2006)] and solid mechanics [Libersky *et al.* (1993)] with ease. While SPH and their corrected adaptations were strong form based, other methods were developed in the 1990s, based on a weak form. Major utilization of these methods was in solid mechanics. Moës *et al.* [1999] presented a new method for enriching the standard displacement based approximation in framework of partition of unity (PU) concept. Belytschko and Black [1999] proposed the extended finite element method (XFEM) as an extension of standard FEM. This method exploits the effectiveness of basic FE procedure and eliminates the need of remeshing making it simpler in application compared to many meshless techniques. XFEM has been used for the analysis of many variations of fracture problems [Bayesteh and Mohammadi (2011); Shedbale *et al.* (2013); Afshar *et al.* (2015); Pathak *et al.* (2015)]. To remove the instabilities in SPH methods Liu *et al.* [1995] introduced reproducing kernel particle methods (RKPM). Meshless local-Petrov Galerkin (MLPG) [Atluri and Zhu (1998)] method was developed. MLPG was advantageous to other meshless techniques as it required no shadow elements like EFG and no special procedure for integration was needed. For efficient treatment of material discontinuities Tsay *et al.* [1999] proposed numerical manifold method (NMM) which employed the use of physical mesh and a mathematical mesh to dictate the problem. NMM allows destruction and reconstruction of mesh around the crack tip as the crack advances hence claims higher computational efficiency. The cracking particles method (CPM) was introduced by Rabczuk and Belytschko [2004] to model the dynamic crack propagation but it lacked the desired accuracy and was difficult to model in comparison to other meshless techniques. Iso-geometric analysis (IGA) [Nguyen *et al.* (2015)] has recently surfaced as a potential technique in the field of solid mechanics analysis. Bayesteh *et al.* [2015] employed extended iso-geometric analysis (XIGA) for analysis of thermo-elastic fracture of

FGMs. Bhardwaj *et al.* [2015] used NURBS based XIGA for simulation of cracked functionally graded material (FGM) plates by first order shear deformation theory (FSDT) under a variety of loading and boundary conditions.

Of all the major meshfree techniques EFGM has contributed most toward the analysis of fracture problems. The EFG method [Belytschko *et al.* (1994a)] was developed in 1994 and was one of the first meshfree methods that used global weak form as its basic structure. The application of EFGM in the field of fracture mechanics [Belytschko *et al.* (1994b); Brighenti (2005); Singh *et al.* (2010a); Sharma *et al.* (2014a); Jameel and Harmain (2015)] are numerous and it has successfully solved a variety of problems whether under various loadings or conditions immaculately.

The earliest development of EFG method involved the construction of shape function using moving least square (MLS) approximation [Shepard (1968)] and Lagrange's multiplier [Yagawa and Furukawa (2000)] approach for enforcement of boundary conditions [Belytschko *et al.* (1995a); Günther and Liu (1998)]. A problem encountered with MLS methodology was that a set of linear algebraic equations have to be solved at every node at which the primary dependent variable are calculated [Lancaster and Salkauskas (1981)], henceforth the moment matrix has to be inverted at every Gauss point when discrete equations are assembled. This problem was solved by constructing weighted orthogonal basis function [Lu *et al.* (1994); Zhang *et al.* (2008)] for MLS interpolants using Gram Schmidt-orthogonalization process [Björck (1994)]. The MLS shape functions lack Kronecker delta property hence enforcement of boundary conditions is difficult, also the use of Lagrange multiplier method leads to an escalation in number of unknowns which is troublesome for the solver, therefore, to circumvent the use of Lagrange multiplier along with maintaining the satisfaction of essential boundary conditions modified variational principle was used in which Lagrange multipliers were replaced by their physical meaning. The modified variational principle provided a set of banded equations but these equations are not necessarily positive-definite also it was somewhat less accurate than Lagrange multiplier method hence enforcement of boundary conditions was done by penalty method [Gavete *et al.* (2000); Lee and Yoon (2004)] which leads to banded positive-definite equations. Recently radial basis function [Belytschko *et al.* (2004); Xu and Belytschko (2005); Nguyen *et al.* (2014)] in conjugation with MLS approach has been used for the construction of shape function but failed to reach the desired accuracy in results.

In analyzing fracture problems discontinuities can be present in the domain the form of strong or weak discontinuities. Cordes and Moran [1996] in their work presented two methods to deal with material discontinuity, later on modifications were made by introduction of jump function [Ventura *et al.* (2002); Batra *et al.* (2004); Rabczuk *et al.* (2007); Pant *et al.* (2011a); Sharma *et al.* (2014a)] approach and implementing signed distance enrichment functions. Fleming *et al.* [1997] provided the enriched EFG formulations for analysis of fracture problems and Belytschko *et al.* [1996a] developed smoothening techniques for treating cracks and holes in the domain.

In fracture mechanics, the region around the crack tip called the singularity dominated zone is the major area of concern for capturing stress field oscillations. The computational time engineering effort required for such analysis should be minimized to save the overall cost of the project. Hence, to save time, minimize engineering effort and to overcome the inherent flaws of EFGM, EFG was coupled with finite element methods (FEM) [Belytschko *et al.* (1995b); Asadpoure *et al.* (2006)] and fractal finite element methods (FFEM) [Reddy and Rao (2008); Rajesh and Rao (2010)] using ramp function approach. EFG was also coupled with RPIM [Cao *et al.* (2013)] which can also be categorized as a true meshfree method in contrast to coupled FE-EFG approach. This class of hybrid methods acts like a two-edged sword in which the shape functions fulfills the Kronecker delta property along with the smoothness and higher order of continuity of EFGM shape functions.

A complete integration of domain is required for the evaluation of stiffness matrix, displacement matrix and force vector, which corresponds to area integration in two dimensions. A numerical integration scheme such as Gauss quadrature is necessary for computation of stiffness matrix and force vector, for which the subdivision of domain is done. Many integration techniques are proposed in meshless methods over the years [Nguyen *et al.* (2008)], e.g. direct nodal integration, stabilized nodal integration, [Chen *et al.* (2001a)] stress point integration, support based integration. To enhance the accuracy of Gauss quadrature, a sub-triangle technique is used [Ghorashi *et al.* (2011)] as it circumvents the difficulties related to discontinuities are present within a background cell. Sukumar *et al.* [2000] established that a continuous increment in order of Gauss integration will not always improve the integration over a discontinuous element/cell. This numerical difficulty was surmounted by using an approach similar to one projected by Dolbow *et al.* [2000] for extended finite element method (XFEM).

Some major advantages of MMs are (i) selection of basis function is more flexible than FEM, (ii) moving boundary problems such as crack propagation, fluid flow and phase transformation can be treated with ease as there is no need of tedious and time consuming re-meshing procedure [Belytschko *et al.*(1993); Chen *et al.* (1998b); Tsukanov *et al.* (2003)], (iii) large deformation can be handled more robustly as no element distortion is involved due to unavailability of elements, (iv) smooth shape functions are used based on local approximations. The value of the shape function neither equal to one at the node of evaluation nor zero at other nodes. Besides these advantages, MMs have some disadvantages: (i) as MM shape functions lack Kronecker delta property the enforcement of boundary conditions is difficult, (ii) the computation of MM shape functions is difficult thereby causing an overall increase in computational cost.

The contribution of this article aims at providing the researchers an anthology of applications of meshfree techniques with prime focus on analysis of fracture problems. The article will help the future research works in summarizing literature works for further advancement in improvements of meshfree techniques.

2. Meshfree Methods

2.1. *Principle of meshfree methods*

A meshfree method is a method used to establish system algebraic equations for the whole problem domain without the use of a predefined mesh for the domain discretization. It uses a set of points scattered within the problem domain as well as on the boundaries of the domain to represent the problem domain and its boundaries. These set of scattered points are called field nodes or simply nodes.

2.2. *Basic approximations and procedure*

The basic steps in Meshfree formulation are same as FEM except for the formation of shape function and imposition of boundary conditions. The basic approximations for a field variable u in any boundary value problem can be written as

$$u^h(x) = \sum_{i=1}^{n} \emptyset_i(X)u_i = \emptyset^T(X)U_s, \tag{1}$$

where $\emptyset_i$ are the shape functions and the u_i's are the nodal values at particle i located at position x_i and n is the set of nodes included in the local support of domain for which $\emptyset_i(X) \neq 0$. U_s is the vector that collects all the field variables at these nodes. Note, that the above form is identical to an FEM approximation. However, in contrast to FEM, the shape functions in Eq. (1) are only approximants and not interpolants, since $u_i \neq u(X_i)$. This difference can be easily depicted by the flow diagram (Fig. 1), which shows the basic procedure applied for solving any problem, shown below.

2.3. *Classification of meshfree methods*

There are a number of versions of meshfree methods developed so far and since this is in development stage, some new ones will continue to appear in the future. According to the approaches to arrive at the discrete governing equations, they largely fall into three categories. The first category is the meshfree methods based on strong-form formulation, second is based on weak-form formulation and the last one is mixed of both, i.e. based on strong-weak form formulation as shown in Fig. 2. They can also be classified in terms of approximation schemes used during the formulation as shown in Fig. 3.

2.3.1. *Based on strong form formulation*

Many problems in engineering are modeled using partial differential equations (PDE). The set of partial differential equations describing such problems is often referred to as the strong form of the problem.

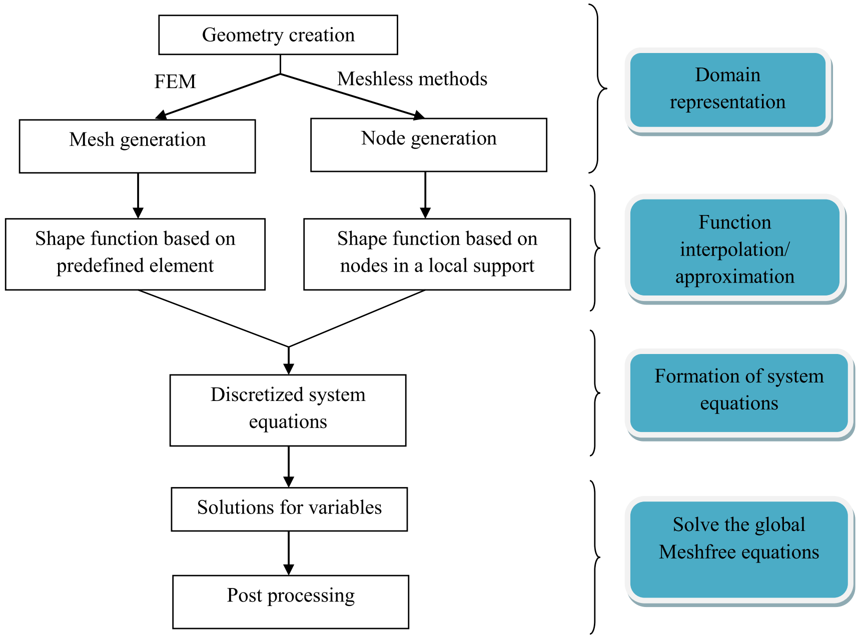


Fig. 1. Meshfree procedure.

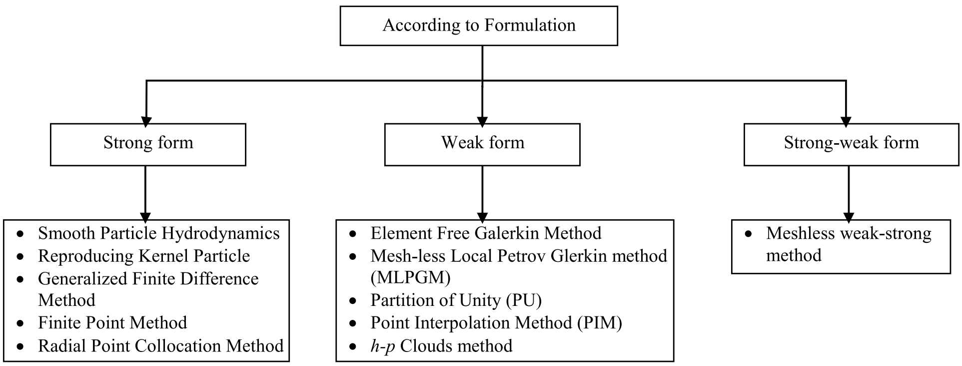


Fig. 2. Classifications according to formulation.

2.3.1.1. A smooth particle hydrodynamics (*SPH*)

SPH method, the oldest meshfree method, developed by Gingold and Monaghan [1977], created a kernel approximation for a single function $u(\boldsymbol{x})$ in a domain Ω by

$$u^h(x) = \int w(x-y,h)u(y)d\Omega_y, \tag{2}$$

where $u^h(x)$ is the approximation, $w(x-y,h)$ a kernel or weight function, and h a measure of the size of the support. The discrete form was obtained by numerical

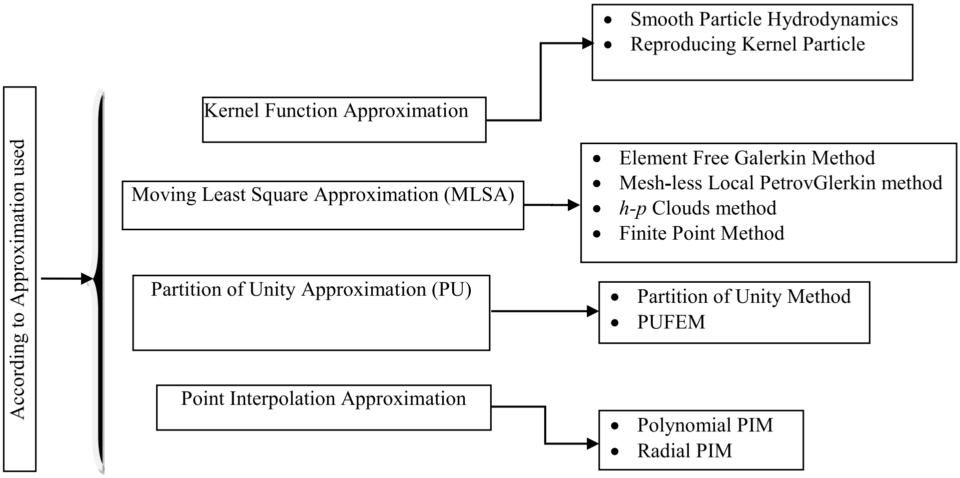


Fig. 3. Classifications according to approximation.

quadrature of the right-hand side in the following type:

$$u^h(x) = \sum_I w(x - x_I)u_I \Delta V_I = \sum_I \phi_I(x)u_I, \tag{3}$$

where ΔV_I is the volume, for 3D, or area, for 2D, or length, for 1D, associated with node I, and $\phi_I(x) = w(x - x_I)\Delta V_I$ the SPH shape function of the approximation.

2.3.1.2. Reproducing kernel particle method (RKPM)

The RKPM by Gosz and Liu [1996] is an improvement of the continuous SPH approximation. In order to increase the order of completeness of the approximation, a correction function $c(x, x - y)$ is introduced into the approximation:

$$(x) = \int c(x, x - y)\phi_\alpha(x - y)u(y)d\Omega_y. \tag{4}$$

The correction function is obtained by imposing the reproducing conditions, i.e., the reproducing equation should exactly reproduce polynomials and can be expressed by a linear combination of polynomial basis functions; α is the dilation parameter of the kernel function $\phi_\alpha(x - y)$.

2.3.1.3. Collocation method

In this, the strong form description of the governing equation and the boundary conditions are used and discretized by collocation techniques as presented by Zhu and Atluri [1998]. Consider a set of n nodes in a domain Ω and boundary Γ. The approximation of field variable is given by Eq. (1) and any of the shape function can be used. The discrete equations are obtained by enforcing the equations on the set of nodes in the domain but do not include the boundary nodes. The equations can be written as

$$\mathcal{L}u^h(x_I) = f(x_I), \quad I \epsilon \Omega - \Gamma \tag{5}$$

$$u(x_I) = \bar{u}(x_I), \quad I \epsilon \Gamma. \tag{6}$$

The above is a set of algebraic equations in the unknowns u_I. $\bar{u}$ represents the prescribed nodal displacement on boundary. The collocation method has two major advantages, namely (i) efficiency in constructing the final system of equations since no integration is required and (ii) shape functions are only evaluated at nodes rather than at integration points as in other methods. The disadvantage is that one must evaluate high-order derivatives of meshfree method shape functions, which are quite burdensome. In addition, two other drawbacks are difficulties in imposing natural boundary conditions and non-symmetric stiffness matrix.

2.3.2. *Based on weak form formulation*

A weak form is a weighted-integral statement of a differential equation in which the differentiation is distributed among the dependent variable and the weight function, and includes the natural boundary conditions of the problem.

2.3.2.1. Element free Galerkin method

One can always write the weighted-integral form of a differential equation, whether the equation is linear or nonlinear. The weak form can be developed if the equations are second order or higher. The method of weighted residuals can be used to approximate the weighted-integral form of any equation. If the trail and test functions are same then the method is better known as EFGM [Belytschko *et al.* (1994c)].

2.3.2.2. Meshfree Petrov–Galerkin method

The trail and test functions in Galerkin methods are given by:

$$\begin{aligned} u^h(x) &= \sum_{I=1}^{N} \emptyset_I(x) u_I; \\ \delta u^h(x) &= \sum_{I=1}^{N} \Psi_I(x) \delta u_I. \end{aligned} \tag{7}$$

If different shape functions are used for the approximation of the test and trial functions, i.e. $\emptyset_I \neq \Psi_I$, then a Meshfree Petrov–Galerkin method [Atluri and Zhu (1998)], is obtained. The advantage over EFGM is that it does not require any background cells for numerical integration. Also no special integration scheme is needed to evaluate the boundary and volume integrals.

2.3.2.3. Point interpolation method (PIM)

The radial basis point interpolation [Liu and Gu (2002)], form is written as

$$u^h(x) = \sum_{i=1}^{n} B_i(r)a_i + \sum_{j=1}^{m} p_j(x)b_j \tag{8a}$$

with the constraint condition

$$\sum_{i=1}^{n} p_{ij}(x)a_i = 0, \quad j = 1 \text{ to } m, \tag{8b}$$

where $B_i(r)$ is the radial basis functions, n is the number of nodes in the neighborhood of x, $p_j(x)$ is monomials in the space coordinates $x^T = (x, y)$, m is the number of polynomial basis functions, coefficients a_i and b_i are interpolation constants. In the radial basis function $B_i(r)$, the variable is only the distance, r, between the interpolation point x and a node x_i.

2.3.3. *Based on weak-strong form formulation*

The key idea of the Meshfree weak-strong method is that in establishing the discretized system equations, both the strong-form and the local weak-form are used for the same problem, but for different groups of nodes that carry different types of equations/conditions. The local weak-form is used for all the nodes that are on or near boundaries with derivative boundary conditions. The strong-form is used for all the other nodes.

The advantages and disadvantages of these classifications are shown in Table 1 below.

Table 1. Advantages and disadvantages of different classifications.

Classification	Advantages	Disadvantages
Strong-form formulation	• Simple to implement • Computationally efficient • No background mesh requirement for integration	• They are unstable • Inaccurate in dealing with Neumann boundary conditions (NBC's)
Weak-form formulation	• Exhibits good stability • Accurate • Capable of imposing NBC's naturally and easily	• Computationally inefficient due to weak form integrations • Background cells are required for integration
Strong-weak form formulation	• NBC's can be imposed accurately using weak form near or on boundary • High efficiency of strong form can be used in inner nodes	

2.3.4. *Based on approximation function schemes*

2.3.4.1. Moving least square approximation (MLSA)

The moving least squares (MLS) approximation was originated by mathematicians working on data fitting and surface construction. Consider an arbitrary point of interest x located in the problem domain. There are n nodes in the support domain of x. The moving least square approximate [Zhu and Atluri (1998)], $u^h(x)$ of $u(x)$ is given as

$$u^h(x) = \sum_{i=1}^{m} p_i(x)a_i(x) \equiv p^T(x)a(x), \tag{9}$$

where $p^T(x)$ is a complete monomial basis and m is the number of basis. For example, in 2D space the basis can be chosen as:

Linear basis: $p^T(x) = \{1, x, y\}, m = 3$.

Quadratic basis: $p^T(x) = \{1, x, y, x^2, xy, y^2\}, m = 6$.

The coefficient vector $a(x)$ is determined by minimizing a weighted discrete norm, defined as

$$\begin{aligned} J &= \sum_I w(x - x_I)[u^h(x, x_I) - u(x_I)]^2 \\ &= \sum_I w(x - x_I)\left[\sum_i p_i(x_I)a_i(x) - u(x_I)\right]^2 \\ &= (Pa - u)^T w(x)(Pa - u), \end{aligned} \tag{10}$$

where $w(x - x_I)$ is a weight function, $P(x_I)$ is the nodal parameter of the field variable at node x_I.

The stationarity of J with respect to $a(x)$ results in following linear equation system:

$$A(x)a(x) = B(x)u. \tag{11}$$

The above equation can be written as

$$a(x) = A^{-1}(x)B(x)u. \tag{12}$$

2.3.4.2. Point interpolation approximation

The point interpolation is a Meshfree interpolation technique that was used by Liu and Gu [2002] and his colleagues to construct shape functions using nodes distributed locally to formulate meshfree weak-form methods. Different from the MLS approximation, PIM uses interpolations to construct shape functions that possess *Kronecker delta function property*. Two different types of PIM formulations using the polynomial basis and the radial function basis (RBF) have been developed.

2.3.4.3. Partition of unity (PU)

The approximation in the Partition of unity meshfree method is given by

$$u^h(x) = \sum_{I=1}^{N} \emptyset_I^0(x) \sum_{j=1}^{l} p_j(x) v_{jI} = \sum_{I}^{N} \emptyset_I^0(x) p^T(x) V_I, \tag{13}$$

where $\emptyset_I^0(x)$ are usually shape functions based on Lagrange polynomials. The coefficients v_{jI} are nodal unknowns. The attractive property of the approximation is that it is the number of terms in polynomial basis which dictates the order of completeness of the approximation. Another useful property of this approximation is that, special enhancement functions, usually a known feature of the sought solution, are easily incorporated into the approximation through this extrinsic basis.

2.3.4.4. hp-clouds approximation

The approximation in the *hp*-clouds method for any point $x \in \Omega$ domain can be written as

$$u^h(x) = \sum_{I}^{N} \emptyset_I(x) \left(u_I + \sum_{j}^{l} p_j(x) v_{jI} \right), \tag{14}$$

where the p_j form the so-called *extrinsic basis* since it contains both high order monomials and enhancement functions as well. *Enhancement functions* or *enrichment functions* are usually introduced into the approximation space to capture special properties such as discontinuities, singularities, boundary layers, or other relevant features of a solution. Different partitions of unity can be used for the standard and enhanced/enriched parts of the approximation as

$$u^h(x) = \sum_{I}^{N} \emptyset_I^k(x) u_I + \sum_{I}^{M} \emptyset_I^m(x) \sum_{j}^{l} p_j(x) v_{jI}, \tag{15}$$

where $\emptyset_I^k(x)$ and $\emptyset_I^m(x)$ are meshfree shape functions of the order of k and m, respectively.

3. Mathematical Approximation Techniques to Solve Fracture Mechanics Problems

Strong discontinuities such as cracks cause a discontinuity in both strain and displacement fields. There are basically six ways to model cracks in EFGM. These methods can be classified under two broad categories first are smoothening techniques based on modification in weight function and then enrichment techniques based on PU concept which involve enrichment of basis function extrinsically or intrinsically.

3.1. *Smoothening techniques*

This section discusses the techniques for modeling non-convex boundaries and strong discontinuities. The smoothness which is a natural property of meshless methods provides approximations of functions and their derivatives which are smooth and have the same continuity as the weight function, on the other hand in cases where a discontinuity is present in geometry or the material, this higher order smoothness causes difficulties which lead to a loss in accuracy. Due to this antecedent smoothness special treatments are given to non-convex boundaries [Belytschko *et al.* (1996b)] which will be elaborated.

3.1.1. *Visibility criterion*

The was the first technique for dealing with strong discontinuities is the visibility criterion [Belytschko *et al.* (1996a); Brighenti (2005)]. In this straightforward approach, the strong discontinuities such as cracks are considered to be opaque and domain of influence is considered as the field of vision at a node. All the nodes that are not truncated by the opaque boundary are not considered in the displacement field approximation. Consider node J in Fig. 4, where the surface of the crack is within its domain of influence and is therefore truncated. This truncation creates a discontinuity in the shape function for node J which will lead to the desired discontinuity in the solution across the crack.

A difficulty with the visibility criterion arises for nodes in close proximity of crack tip. Consider node I in Fig. 4. The field of vision is cut by the crack, leading to a discontinuity along line AC, i.e. the line of the crack. However, the field of vision is also truncated along line AB, which extends into the domain which is an undesired discontinuity.

3.1.2. *Diffraction technique*

Continuous and smooth approximations near nonconvex boundaries can be constructed quite easily by the diffraction technique. The domain of influence is wrapped around nonconvex boundaries similar to the way light diffracts around

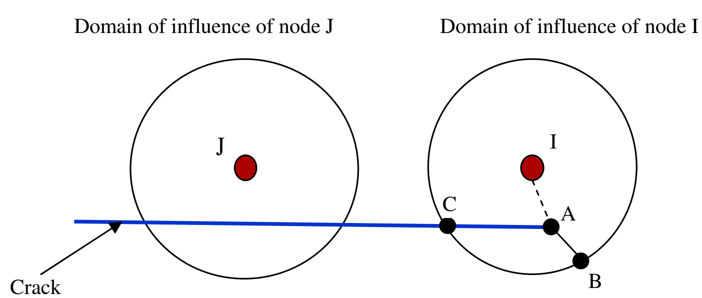


Fig. 4. Domain of influence by visibility criterion near a crack.

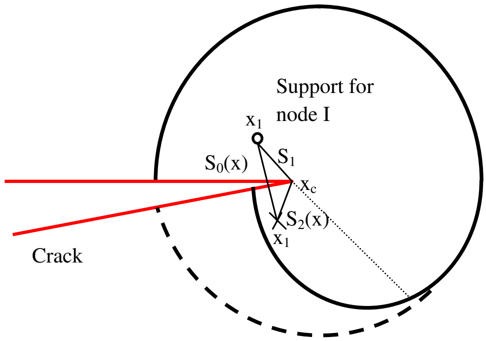


Fig. 5. Diffraction technique for constructing smooth weight functions around non-convex boundaries.

sharp corners as shown in Fig. 5. This technique, which has also been called the wrap-around technique [Organ *et al.* (1996)].

Consider Fig. 5, where a line between the node x_I, and a sampling point x intersects a crack and the tip is within the domain of influence of the node. The weight function distance d_I, is described here by

$$d_I = \left(\frac{s_1 + s_2(x)}{s_0(x)}\right)^{\lambda} s_0(x), \tag{16}$$

where

$$s_1 = \|x_I - x_c\|, \quad s_2(x) = \|x - x_c\|, \quad s_0(x) = \|x - x_I\|. \tag{17}$$

And x_I is the node, x is the sampling point, and x_c is the crack tip. The parameter λ is used to adjust the distance of the support on the opposite side of the crack. It was found that $\lambda = 1, 2$ perform well.

The spatial derivatives of the weight function are computed using the chain rule:

$$\frac{dw}{dx_i} = \frac{\partial w}{\partial d_I}\frac{\partial d_I}{\partial x_i}. \tag{18}$$

Since $\partial w/\partial d_I$ is unchanged, all that is necessary are expressions for $\partial d_I/\partial x_i$

$$\frac{\partial d_I}{\partial x_i} = \lambda\left(\frac{s_1 + s_2}{s_0}\right)^{\lambda - 1}\frac{\partial s_2}{\partial x_i} + (1 - \lambda)\left(\frac{s_1 + s_2}{s_0}\right)^{\lambda}\frac{\partial s_0}{\partial x_i}, \tag{19}$$

where

$$\frac{\partial s_0}{\partial x_i} = \frac{x_i - x_{Ii}}{s_0}, \quad \frac{\partial s_2}{\partial x_i} = \frac{x_i - x_{ci}}{s_2}. \tag{20}$$

The diffraction technique works well for general nonconvex boundaries as well. The tangent point between the node and the nonconvex boundary is used as the wrap-around point $\mathbf{x}_c$.

3.1.3. *Transparency technique*

Another technique for constructing continuous approximations considered as a substitute to diffraction technique is the transparency technique [Organ *et al.* (1996)]. The basic concept of this technique is to bestow the crack tip with a varying measure of transparency such that it is completely transparent at the tip and becomes completely opaque a short distance behind the tip. By doing this, the abrupt truncation of field of vision of node close to crack tip does not take place, but rather diminishes smoothly to zero a short distance behind the tip of the crack.

When a ray passes between a node x_I and a sampling point x, and crosses the crack as shown in Fig. 6, the distance parameter d_I in the weight function is modified (lengthened) by the following:

$$d_I(\mathbf{x}) = \mathbf{s}_0(\mathbf{x}) + d_{mI}\left(\frac{s_c(\mathbf{x})}{\overline{s_c}}\right)^{\lambda} \lambda \geq 2, \tag{21}$$

where $s_0(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_I\|, d_{mI}$ is the radius of support for node I, and $s_c(\mathbf{x})$ is the intersection distance behind the crack.

The parameter s_c sets the distance behind the crack tip at which complete opacity occurs:

$$s_c = kh, \tag{22}$$

where h is the nodal spacing and k is a constant, usually $0 < k < 1$.

The spatial derivatives of the distance parameter, d_I obtained by chain rule, are

$$\frac{\partial d_I}{\partial x_i} = \frac{\partial s_0}{\partial x_i} + \lambda d_{mI}\frac{s_c^{\lambda-1}}{s_c^{-\lambda}}\frac{\partial s_c}{\partial x_i}, \tag{23}$$

where $\frac{\partial s_0}{\partial x_i} = \frac{x_i - x_{Ii}}{s_0}$, $\frac{\partial s_c}{\partial x_1} = \frac{x_b - x_c}{s_c} = \cos\theta$, $\frac{\partial s_c}{\partial x_1} = \frac{y_b - y_c}{s_c} = -\sin\theta$, θ is the angle between the crack and x-axis and x_b is the intersection point behind the crack tip.

One drawback of the transparency technique is that it does not work well when nodes are placed too close to the crack surface. Note the trough which appears in the shape function ahead of the crack. This trough appears because although the

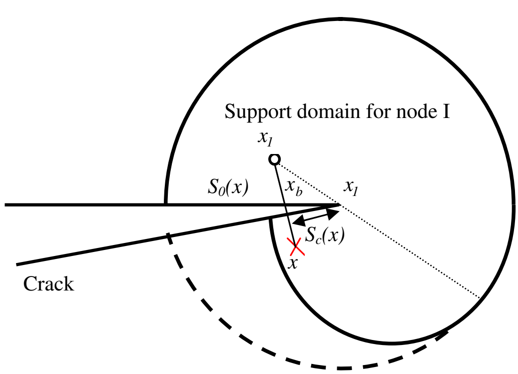


Fig. 6. Transparency technique for computing smooth weight functions.

crack tip is transparent for this node, the change in the degree of transparency with respect to the change in angle is very sharp. To circumvent this problem in the transparency technique, a restriction has been placed on the position of the nodes. All nodes should be placed so that the normal distance from the node to the crack surface is greater than roughly $1/4h$, where h is the nodal spacing.

3.2. *Enrichment techniques*

The isoparametric finite elements can be enriched by the inclusion of near crack tip fields in trial functions to capture stress field oscillations or singular stress fields in finite elements. This method can provide the stress intensity factor directly as a part of solution. However, the results of elements enriched by singular fields are dependent on their size, and fail to exhibit uniform convergence. The other difficulty associated with these elements is that their implementation becomes intricate because the stiffness matrix and force vector have to be expanded to compensate for the extra unknowns.

3.2.1. *Extrinsic enrichment*

In extrinsic enrichment of a meshfree approximation, a function closely related to the solution is added to the polynomial expansion of MLS approximation. For example, in linear elastic fracture mechanics, the near tip asymptotic field or its constituents can be added. The approximation takes the form

$$\mathbf{u}_\alpha^h(x) = \mathbf{p}^T(x)\mathbf{a}_\alpha(x) + \sum_{j=1}^{n_c} k_1^j Q_{1\alpha}^j(x) + k_2^j Q_{2\alpha}^j(x), \tag{24}$$

where $\mathbf{u}_\alpha^h(\mathbf{x})$ denotes the approximation for $\mathbf{u}_\alpha(\mathbf{x})$, $\mathbf{P}(\mathbf{x})$ is the standard polynomial basis defined earlier, n_c is the number of cracks in the model, $a_\alpha(x)$ are the coefficients of the polynomial basis; k_1^j and k_2^j are global unknowns associated with crack. Lower case Greek subscripts have a range of 2 and refer to Cartesian components. The functions $Q_{1\alpha}(x)$ and $Q_{2\alpha}(x)$ stand for the near-tip displacement field.

The coefficients $\mathbf{a}_\alpha(\mathbf{x})$, are functions of the spatial coordinates and are determined by the MLS methodology. However, additional terms arise from the inclusion of the near-tip field and so the MLS formulation will be again derived here in the interest of completeness. A weighted, discrete L_2 norm is written

$$L = \sum_{I=1}^{n} w(\mathbf{x} - \mathbf{x}_I) \left[\mathbf{p}^T(\mathbf{x}_I)\mathbf{a}_\alpha(\mathbf{x}) + \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}_I) + k_2^j Q_{2\alpha}^j(\mathbf{x}_I)] - u_{I\alpha} \right]^2, \tag{25}$$

where n is the number of points in the neighborhood of x for which the weight function $w(\mathbf{x} - \mathbf{x}_I)$ is non-zero, and $u_{I\alpha}$ is the component of the nodal value at x_I.

The stationarity of L with respect to $\mathbf{a}_\alpha(\mathbf{x})$ leads to

$$\mathbf{A}(\mathbf{x})\mathbf{a}_\alpha(\mathbf{x}) = \sum_{I=1}^{n} \mathbf{C}_I(\mathbf{x}) \left\{ u_{I\alpha} - \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}_I) + k_2^j Q_{2\alpha}^j(\mathbf{x}_I)] \right\}, \tag{26}$$

where

$$\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w(\mathbf{x} - \mathbf{x}_I)\mathbf{P}(\mathbf{x}_I)\mathbf{P}^T(\mathbf{x}_I), \tag{27}$$

$$B_I(\mathbf{x}) = w(\mathbf{x} - \mathbf{x}_I)\mathbf{P}(\mathbf{x}_I). \tag{28}$$

It should be noted that k_1^j and k_2^j are global parameters in this technique and they are considered fixed in the process of obtaining the parameters $\mathbf{a}_\alpha$ for the local fit. Solving Eq. (26) for $\mathbf{a}(\mathbf{x})$ gives

$$\mathbf{a}_\alpha(\mathbf{x}) = \sum_{I=1}^{n} \mathbf{A}^{-1}(\mathbf{x})B_I(\mathbf{x}) \left\{ u_{I\alpha} - \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}_I) + k_2^j Q_{2\alpha}^j(\mathbf{x}_I)] \right\}. \tag{29}$$

Expressing in terms of the nodal parameter $u_{I\alpha}$ and the enriched field parameters k_1^j and k_2^j yields

$$\mathbf{u}_\alpha^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_I(\mathbf{x}) \left\{ u_{I\alpha} - \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}_I) + k_2^j Q_{2\alpha}^j(\mathbf{x}_I)] \right\} + \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}) + k_2^j Q_{2\alpha}^j(\mathbf{x})], \tag{30}$$

$$\mathbf{u}_\alpha^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_I(\mathbf{x}) u_{I\alpha} + \sum_{j=1}^{n_c} k_1^j \left[Q_{1\alpha}^j(\mathbf{x}) - \sum_{I=1}^{n} \Phi_I(\mathbf{x}) Q_{1\alpha}^j(\mathbf{x}_I) \right] + \sum_{j=1}^{n_c} k_2^j \left[Q_{2\alpha}^j(\mathbf{x}) - \sum_{I=1}^{n} \Phi_I(\mathbf{x}) Q_{2\alpha}^j(\mathbf{x}_I) \right], \tag{31}$$

where the shape function, $\Phi_I(\mathbf{x})$, is defined as

$$\Phi_I(\mathbf{x}) = \mathbf{P}^T(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})B_I(\mathbf{x}). \tag{32}$$

These shape functions are capable of representing the smooth part of the solution. Equation (31) will be written as

$$\mathbf{u}_\alpha^h(\mathbf{x}) = \sum_{I=1}^{n} \Phi_I(\mathbf{x}) \tilde{u}_{I\alpha} + \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}) + k_2^j Q_{2\alpha}^j(\mathbf{x})], \tag{33}$$

where the modified nodal coefficients, $\tilde{u}_{I\alpha}$, are

$$\tilde{u}_{I\alpha} = u_{I\alpha} - \sum_{j=1}^{n_c} [k_1^j Q_{1\alpha}^j(\mathbf{x}_I) + k_2^j Q_{2\alpha}^j(\mathbf{x}_I)]. \tag{34}$$

3.2.2. *Intrinsic enrichment*

Meshless approximations can be intrinsically enriched by including a special function in the basis. For example, in fracture mechanics, one can include the asymptotic near-tip displacement field, or an important ingredient such as $\sqrt{r}$. The choice of functions depends on the coarse mesh accuracy desired. For higher accuracy, include the full asymptotic field, while for higher speed at some cost of accuracy, only the $\sqrt{r}$ function can be included in the basis. In full intrinsic enrichment of EFG approximations for fracture problems, the entire near-tip asymptotic displacement field is included in the basis. Following some trigonometric manipulation, it can be shown that all the functions (linear, quadratic etc.) are spanned by the basis:

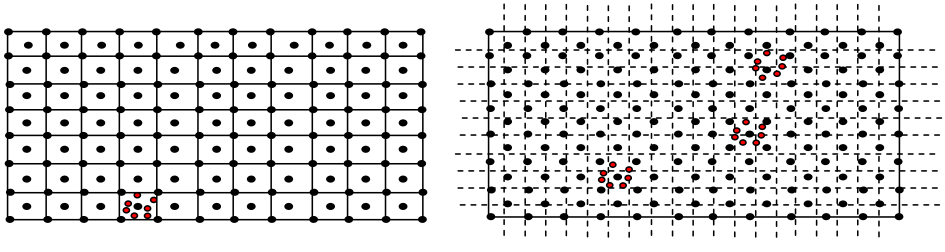
$$\mathbf{P}^T(\mathbf{x}) = \left[1, x, y, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right]. \tag{35}$$

(The linear terms are not related to the near-tip fields and are represented through the linear completeness of the EFG approximant).

In contrast to the extrinsic techniques, this technique involves no additional unknowns. However, because of the increased size of the basis, an additional computational effort is required to invert the moment matrix $A(x)$. For multiple cracks, four additional terms would have to be added to the basis for each crack.

4. Numerical Integration

A complete integration of domain is required for the evaluation of stiffness matrix (K), displacement matrix (u) and force vector (f) in, which corresponds to area integration in two dimensions. A numerical integration scheme such as Gauss quadrature is necessary for computation of stiffness matrix and force vector, for which the subdivision of domain is done. Many integration techniques are proposed in meshless methods over the years [Nguyen *et al.* (2008)], e.g. direct nodal integration, stabilized nodal integration, [Chen *et al.* (2001)] stress point integration, support based integration but in the case of fracture analysis using EFGM two types of subdivision techniques as shown in Fig. 7 are mostly used for the purpose of integration.



(a) Element quadrature (b) Cell quadrature

Fig. 7. Integration techniques of EFGM.

Element quadrature as shown in Fig. 7(a) uses a finite element mesh generator to create a cell structure which matches with the problem domain. The second integration technique, which is often called cell quadrature, uses background cells, which is independent of the problem domain as shown in Fig. 7(b). During integration over the problem domain, a particular quadrature point is checked whether it lies inside the domain or not.

To enhance the accuracy of Gauss quadrature, a sub-triangle technique is used [Ghorashi *et al.* (2011)] as it circumvents the difficulties related to discontinuities are present within a background cell. Sukumar *et al.* [2000] established that a continuous increment in order of Gauss integration will not always improve the integration over a discontinuous element/cell. This numerical difficulty was surmounted by using an approach similar to one projected by Dolbow *et al.* [2000] for extended finite element method (XFEM). According to this technique, any background cell which intersects with a crack is subdivided at both sides into sub-triangles whose edges are adapted to crack faces. It is imperative to note that, while triangulation of the crack tip element considerably improves the accuracy of integration by increasing the order of Gauss quadrature, it also shuns the numerical complications of singular fields at the crack tip because none of the Gauss points are placed on the position of the crack tip.

5. Review of Applications of Meshfree Method in Engineering

The papers dealing with Meshfree methods are summarized in Table 2. In this paper, each paper is described in terms of work done, structure/problem type, theory used, method/approximation algorithm used and remarks.

6. Review of Existing Literature on Application of Meshfree Methods to Fracture Mechanics

The papers dealing with application of Meshfree methods to fracture mechanics are summarized in Table 3. In the review, each paper is described in terms of method used, structure/problem discussed in the article, any modifications or noteworthy specifications mentioned in technique and conclusion or remarks. Since major part of contribution of meshfree techniques used for analysis of fracture problems is achieved by EFGM, the articles in Table 3 also cover all the major modifications in EFGM over the years for analysis of fracture.

7. Conclusion

This paper reviews widely used meshfree methods and their applications in the analysis of various engineering problems. It is found that meshfree methods are able to solve more accurately and efficiently than FEM, even overcoming the shortcomings of FEM especially in case of discontinuities and large deformation problems.

Table 2. Summary of research papers dealing with meshfree methods.

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Nayroles *et al.* [1992]	A modified FEM is developed using diffuse approximation to overcome the limitations of regular FEM	Beam	Saint-Venant's beam theory	Diffuse Approximation	Provides better gradients of unknown variables than FEM and FEM re-meshing was avoided by fitting polynomials into nodal values via least square approximations
Belytschko *et al.* [1994a]	New meshfree method called Element Free Galerkin (EFG) is developed	Beam, hole in a infinite plate and crack	Timoshenko beam theory, Hencky–Mindlin plate theory	It is combination of EFGM and MLSM	No need for mesh. The dependent variables and its derivatives are continuous in entire domain with higher accuracy
Belytschko *et al.* [1995b]	Finite element and EFG methods are coupled together to improve efficiency	Cantilever beam, wave propagation in rod and fracture	Timoshenko beam theory	Standard bilinear shape functions and MLSM	Computationally efficient over full mesh less method
Krysl and Belytschko [1995]	Structural analysis of thin plates is carried out by the EFGM	Kirchhoff plates	Kirchhoff's Theory	EFGM using MLSM	Lagrange multiplier is used for essential boundary condition (EBC). Shows higher accuracy
Gosz and Liu [1996]	Introduced new ways to enforce EBC in the reproducing kernel particle method	Helmholtz equation and cantilever beam	Helmholtz equations and Timoshenko beam theory	Reproducing Kernel Particle Method (RKPM)	Convergence rate for forcing the window function to zero were not good.
Liu *et al.* [1996]	Advances in multiple scale kernel particle methods are discussed with reference to Elastic contact problem approximation	Rubber ring and thin biconvex airfoil	Large deformation theory with Mooney-Rivilin rubber	RKPM	Accurate mesh free algorithm and superior convergence

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Yagawa and Yamada [1996]	A new mesh-less finite element method is proposed called Free Mesh method	2D steady heat conduction problem	The Poisson's theory	Free Mesh method	Does not require connectivity information between elements
Krongauz and Belytschko [1997]	A Petrov–Galerkin Diffuse Element Method (PG DEM) is proposed and compared to EFGM	Plate with hole and cantilever beam	Hencky–Mindlin plate theory and Timoshenko beam theory	Petrov–Galerkin method with shepard approximation	Formulation does not pass patch test and converges very slowly.
Sukumar *et al.* [1997]	An EFGM is proposed for 3-dimensional fracture mechanics	Crack in an infinite body	Mode 1 crack problem	EFGM with MLSA	Provides accurate stress intensity factor
Liu *et al.* [1997]	Reproducing kernel particle method for multiresolution analysis is addressed	Crash analysis, plane wave scattering, rubber ring and plate	Large deformation theory with Mooney–Rivilin rubber	RKPM	Accuracy is enhanced. Ability to perform hp-like adaptive refinement without a mesh
Chen *et al.* [1998a]	A Lagrangian reproducing kernel particle method for metal forming analysis is presented	Sheet metal forming, ring compression and upsetting simulation	Lagrangian formulation	Lagrangian reproducing kernel method	Stable during large deformation. Does not require readjustment during contact computation.
Onate and Idelsohn [1998]	A mesh-free finite point method is developed for adjective-diffusive transport and fluid flow problems	Fluid flow	Navier–Stokes equations	Finite point method with least square approximation Coupled with collocation point	Effective method for compressible and incompressible fluid flow problems

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Zhu *et al.* [1998]	A meshfree local boundary integral equation (LBIE) method is presented for solving nonlinear problems	Cubic solution with mixed and essential boundary conditions	Linear potential theory	Local boundary integral equation using moving least square method	High convergence rates and accuracy of unknown variables and its derivatives Symmetric matrix is banded
Zhu and Atluri [1998]	New techniques are discussed for enforcing the essential boundary conditions in the EFGM	Plate with circular hole, beam	Hencky–Mindlin plate theory and Timoshenko beam theory	EFGM with modified collocation and penalty method	Better than direct collocation, penalty method yields banded, symmetric and positive definite system matrix
Atluri and Zhu [1998]	A new Meshfree Local Petrov–Galerkin (MLPG) approach is presented in computational mechanics	Potential flow around cylinder	Navier–Stokes equations	Local symmetric weak form with MLSA	No background mesh is required.
Chung and Belytschko [1998]	Error estimation is carried out in the EFG method	Bar, cantilever Beam, plate with circular hole and crack	Hencky–Mindlin plate theory, Timoshenko beam theory and mode 1 crack	EFGM	Local and global error estimates are provided which can be used in adaptive analysis of EFG problems.
Xu and Saigal [1998]	EFG study is presented for steady quasi-static crack growth in plane strain tension in elastic-plastic materials	Crack in elastic-perfectly plastic material	Mode 1 crack theory	EFGM	Steady and steady quasi-static growth of a mode 1 crack was studied.

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Donning and Liu [1998]	Various Meshfree methods are presented for shear-deformable beams and plates	Straight beam, curbed beam, plates	Mindlin–Reissner theory	Displacement based Galerkin method	Completely eliminates shear and membrane locking in beams and plates Error estimation is simplified
Atluri *et al.* [1999b]	A critical assessment of the truly Meshfree Local Petrov–Galerkin (MLPG), and Local boundary integral equation (LBIE) methods is presented	Cantilever beam and plate with circular hole	Hencky–Mindlin plate theory and Timoshenko beam theory	MLPG & LBIE	Shows good accuracy with MLS, Shepard and PU interpolations.
Aluru [1999]	The RKPM is presented for meshfree analysis of microelectromechanical systems	Both end Fixed beam, cantilever beam and electromechanic pressure sensor	Euler-Bernoulli beam theory, thin plate theory	RKPM	Gives results better than FEM
Chen *et al.* [1999a]	Improvement technique is proposed for tensile instability in smoothed particle hydrodynamics	Bar	—	Corrected smooth particle hydrodynamics	Shortcomings of the tensile instability and boundary deficiency in standard SPH apparently do not appear.
Atluri *et al.* [1999a]	Analysis of thin beams, using the meshfree local Petrov–Galerkin method, with generalized moving least squares interpolations is proposed	Thin beam	Euler beam theory	MLPGM with generalized MLSA	Displacement and slope boundary conditions are imposed at same point with more accurate results

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Chen *et al.* [1999b]	A new technique called corrective smoothed particle method for linear elastodynamics is discussed	Square plate and beam	Euler beam theory	Corrected smooth particle method (CSPM)	The results of displacement based CSPM than stress based CSPM.
Dolbow and Belytschko [1999]	Numerical integration of the Galerkin weak form in meshfree methods	Integration error in Poison's equation	Gauss quadrature	EFGM with MLSA	For large changes in nodal spacing severe errors may result from the quadrature with large background cells
Saigal and Barry [2000]	A slices based EFG formulation is presented	Cantilever beam and clamped plate	Timoshenko beam theory and thin plate theory	EFGM with MLSA and method of slices	Nodal distribution is employed easily.
Liu and Gu [2000a]	Coupling of EFG and hybrid boundary element methods is presented	Cantilever beam and plate with hole	Timoshenko beam theory and Hencky–Mindlin plate theory	Coupled EFG and HBEM	Less computation cost, EBC are easily imposed and shape functions have higher order of continuity
Breitkopf *et al.* [2000]	A new approach is proposed which is extension of generalized FDM	Poison's equation and cantilever beam	Timoshenko beam theory	Diffuse collocation method	Double grid approach is proposed resulting in smaller DOI and sparsity of global matrix.
Liu and Gu [2000b]	A coupled MLPG method which combines with finite element and boundary element approaches are proposed	Cantilever beam, hole in an infinite plate and internal pressurized hollow cylinder	Timoshenko beam theory and Hencky–Mindlin plate theory	Coupling of MLPG with FEM or BEM	Reduced computational cost, imposition of EBC's is easier

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Chen and Wang [2000]	Transformation method and boundary singular kernel method were employed in meshfree computation of contact problems.	Rubber ring under compression, Cylindrical punch, Rubber door seal compression, Metal upsetting forging process	—	RKPM	Transformation method was modified so that the mathematical complexity associated with it can be reduced. The boundary singular kernel method required no co-ordinated transformation therefore reduces computational effort
Zhang *et al.* [2000]	Proposed Meshfree methods based on collocation with radial basis functions	Poison's eq. cantilever beam and plate with hole	Timoshenko beam theory and Hencky–Mindlin plate theory	Collocation with radial basis functions	Improved accuracy than direct collocation. Leads to full coefficient matrix
Kim *et al.* [2000]	Meshfree shape design sensitivity analysis and optimization for contact problem with friction are presented	Door seal	Frictional interface law	RKPM	Shape optimization of the frictional contact problem can be carried out effectively
Li *et al.* [2000]	A meshfree method is proposed for large deformation of thin shell structures	shells	3D continuum direct approach	RKPM	Eliminates volumetric and shear locking. Captures gradients in thickness direction
Chen *et al.* [2000a]	Improvements in meshfree methods are presented for incompressible finite elasticity boundary value problems	Inflation of plane strain tube, rubber ring compression and engine mount	Incompressible boundary value problem	RKPM	Higher accuracy is achieved, volumetric locking is absent

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Sladek *et al.* [2000]	A truly meshfree method local boundary integral equation (LBIE) is proposed and implemented for linear elasticity	Cantilever beam and plate with circular hole	Timoshenko beam theory and Hencky–Mindlin plate theory	Local boundary integral equation with MLSA	High accuracy of numerical integration is achieved without special scheme, no derivatives of shape functions are required in constructing system matrix
Atluri and Zhu [2000]	MLPG approach is presented for solving problems in elasto-statics	Cantilever beam and plate with circular and elliptical hole	Timoshenko beam theory and Hencky–Mindlin plate theory	Local Petrov–Galerkin method with MLSA	No smoothing technique is required to compute stresses and strains
Lee *et al.* [2000]	A two scale meshfree method is presented for the adaptivity of 3D stress concentration problems	Beam, L-shaped plate and plate with circular hole	Timoshenko beam theory and Hencky–Mindlin plate theory	RKPM	High stress regions can be detected without posteriori estimation.
Chen *et al.* [2001b]	A corrective smoothed particle method is presented for transient elastoplastic dynamics	Forced vibration of beam type material	Euler–Bernoulli Theory	Corrective smooth particle method	Conditions of nodal completeness and integrablity are satisfied
Gu and Liu [2001]	MLPG method is presented for free and forced vibration analyses for solids	Cantilever beam, shear wall with four openings	Initial/boundary value problem	Meshfree local Petrov–Galerkin method	Easily implemented and very flexible for free and forced vibration analysis.

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Ohs and Aluru [2001]	Meshfree analysis is presented for piezoelectric devices	Shear deformation of piezoelectric strip and piezoelectric bimorphs	Single/multi layered problem	Meshfree point collocation method	Lesser time for numerical computation is required in point collocation method
Liew *et al.* [2002a]	Analysis of laminated composite beams and plates with piezoelectric patches using the element-free Galerkin method is presented	Cantilever piezoelectric bimorph beam and plate	Hencky–Mindlin plate theory	EFGM	Analyzes accurately the shape control of laminated beams and plates
Liu and Gu [2002]	Comparison between two meshfree local point interpolation methods for structural analysis is presented	Cantilever beam	Timoshenko beam theory	LPIM	Efficiencies of LPIM and MLPG is same
Liew *et al.* [2002b]	Meshfree method is proposed for modeling of human proximal femur treatment of nonconvex boundaries and stress analysis	Femur stress analysis	Euler beam theory	RKPM	Good for biomechanic problems
Rao and Rahman [2002]	Reliability analysis for cracked structure is carried out	Crack in rectangular plate	Mode 1 and mixed mode crack theory	EFGM	Accurate compared to Monte-Carlo simulation

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Liu *et al.* [2003]	A radial point interpolation method is proposed for simulation of two-dimensional piezoelectric structures	Piezoelectric bimorph beam, eigenvalue analysis of piezoelectric transducer	Euler Bernoulli beam theory	RPIM	Possess delta property Easy to implement, flexible and stable for static and dynamic analysis for piezoelectric structures
Wu and Liu [2003]	A meshfree formulation of local radial point interpolation method (LRPIM) is proposed for incompressible flow simulation	Natural convection in square cavity and concentric annulus	Vorticity-stream function formulation	Local radial PIM	Accurate than finite difference. Expensive when no. of nodes is high
Liu and Gu [2004]	A meshfree weak–strong (MWS) form method is proposed for 2D solids	Cantilever beam and hole in infinite plate	Timoshenko beam theory and Hencky–Mindlin plate theory	Point collocation method with MLSA at boundaries	More accurate and stable than strong form. More efficient than weak form methods
Sladek and Sladek [2003]	A meshfree method is proposed for large deflection of plates	Square plate simply supported	Decoupled Berger equations	Boundary integral equations with MLSA	Greater efficiency
Ng *et al.* [2003]	A novel true meshfree numerical technique, hybrid meshfree–differential order-reduction (hM-DOR), is proposed for the deformation control of circular plates integrated with piezoelectric sensors/actuators	Circular plate	Hencky–Mindlin plate theory	Hybrid meshfree-differential order-reduction method	Directly discretize the overlapping boundary conditions

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Dai *et al.* [2004]	A meshfree radial point interpolation method is presented for analysis of functionally graded material (FGM) plates	Square plate simply supported	Hencky–Mindlin plate theory	RPIM	Shape functions possess delta function property. Good convergence rate and accuracy
Gu and Liu [2004]	A meshfree weak-strong (MWS) is presented for time dependent problems	1-D truss member and cantilever beam	Timoshenko beam theory	Point collocation method with MLSA at boundaries	Efficient, accurate and stable
Lam *et al.* [2004]	A novel meshfree approach — Local Kriging (LoKriging) method with two-dimensional structural analysis is presented	Cantilever beam and infinite circular plate with circular hole	Timoshenko beam theory and Hencky–Mindlin plate theory	LoKriging method	Possess delta property Good accuracy
Raju *et al.* [2004]	A radial basis function approach in the meshfree local Petrov–Galerkin method is presented for Euler–Bernoulli beam problems	Cantilever beam	Euler-Bernoulli beam theory	MLPG with Radial basis functions	Better results than MLPG with MLSA
Kitipornchai *et al.* [2005]	A boundary element-free method (BEFM) is presented for three-dimensional elasticity problems	Cube and hollow sphere with inner pressure	Classical elasticity theory	Boundary element free method	Combination of BEM and improved MLSA Possess greater computational efficiency

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Liu *et al.* [2005]	A meshfree radial point interpolation method (RPIM) is presented for three-dimensional solids	3D cantilever beam	Timoshenko beam theory	RPIM using Radial basis function	Reproducing property and shows good convergence and accuracy
Sellountos and Polyzos [2005]	A meshfree local boundary integral equation method is presented for solving transient elastodynamic problems	Rectangle under a uniform step load and flexural load	Navier–Cauchy partial differential equation	Local boundary/ volume integral method	Accurate and stable results are obtained where abrupt changes in displacement and traction occur
Deeks and Augarde [2005]	A meshfree local Petrov–Galerkin scaled boundary method is presented	Plate with circular and square hole	Hencky–Mindlin plate theory	Scaled boundary method with MLPG	Increased continuity and smoothness No stress recovery process is needed
Pan *et al.* [2005]	Meshfree Galerkin least-squares method is presented	Cantilever beam and plate with circular hole	Euler–Bernoulli beam theory	Meshfree Galerkin least-square method	Better accuracy and more economical
Wang and Chen [2006]	A locking-free meshfree curved beam formulation with the stabilized conforming nodal integration is presented	Clamped free curved beam, pinched ring and straight beam	Mindlin–Reissner theory	Locking free meshfree method	Free from shear and membrane locking whereas Gauss Integration solution has severe locking.
Liu *et al.* [2006]	A mesh-free minimum length method is presented for 2D problems	Cantilever beam and plate with circular hole	Timoshenko beam theory and Hencky–Mindlin plate theory	Minimum length method	Better results for problem with steep gradients

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Liew *et al.* [2006]	Buckling analysis of corrugated plates is presented using a mesh-free Galerkin method based on the first-order shear deformation theory	Corrugated plates	Hencky–Mindlin plate theory	Meshfree Galerkin method	Solution for corrugated plates is obtained by analyzing them as orthotropic plates and has shown good agreement with available literature.
Sladek *et al.* [2006]	Meshfree local Petrov–Galerkin method is presented for continuously nonhomogeneous linear viscoelastic solids	Viscoelastic strip and hollow cylinder	Stehfest's inversion method	MLPG	Less computational effort than FEM
Guo *et al.* [2006]	Analysis of piezoelectric ceramic multilayer actuators is presented based on an electro-mechanical coupled meshfree method	Piezoelectric plate with hole	Hencky–Mindlin plate theory	EFGM	Yields accurate near-tip stress field
Rabczuk and Zi [2007]	A meshfree method is presented based on the local partition of unity for cohesive cracks	Crack in beam	Lemaitre damage model and Johnson-Cook model	Extended EFGM	Accuracy and smoothening of crack propagation
Rabczuk *et al.* [2007]	A 3D meshfree method is presented for continuous multiple-crack initiation, propagation and junction in statics and dynamics	Penny shaped crack in finite cube, flyer plate impact and beam	Lemaitre damage model and Johnson–Cook model	Extended EFGM	Avoid locking and inaccuracies

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Zou *et al.* [2007]	A Truly meshfree method is presented based on partition of unity quadrature for shape optimization of continua	Shape optimization of fillet and a portal frame	Linear elastic solid problem	Meshfree method based on partition of unity	Higher convergence rates and better optimization efficiencies obtained as compared with classical RKPM method
Lee and Shuai [2007]	An automatic adaptive refinement procedure is presented for the reproducing kernel particle method. Part I Stress recovery and *a posteriori* error estimation	Timoshinko beam, plate with hole and domain near crack tip	Timoshenko beam theory and Hencky–Mindlin plate theory	Adaptive refinement procedure using RKPM	More accurate and converges at higher rate than original RKPM
Lee and Shuai [2007]	An automatic adaptive refinement procedure is presented for the reproducing kernel particle method. Part II Adaptive refinement	Timoshinko beam, plate with hole and domain near crack tip	Timoshenko beam theory and Hencky–Mindlin plate theory	Adaptive refinement procedure using RKPM	More accurate than RKPM
Wang *et al.* [2007]	Analysis of Microelectromechanical Systems (MEMS) devices by the meshfree point weighted least-squares method is presented	Fixed-fixed microswitch, Cantilever micro switch and the micro tweezer	Euler-Bernoulli beam theory	Point weighted least square method	The boundary conditions can be easily enforced; and the final coefficient matrix is symmetric compared to collocation method
Zi *et al.* [2007]	Extended meshfree methods without branch enrichment for cohesive cracks is proposed	Beam, cracks in dams, crack branching and john & shah's beam	Lemaitre damage model and cohesive crack model	Extended EFGM	Branch enrichment is removed from the discontinuous displacement field

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Peng and Kitipornchai [2007]	Free vibration analysis of folded plate structures by the FSDT Mesh-free Method is presented	Cantilever folded plate and Cantilever folded plate stiffened by two stiffeners	Hencky–Mindlin plate theory	Meshfree Galerkin method	Shows good accuracy and rates of convergence
Sladek *et al.* [2007]	Heat Conduction analysis of 3D axisymmetric and anisotropic FGM bodies by meshfree Local Petrov–Galerkin Method is proposed	FGM (finite full and hollow cylinder)	Boundary value problem	MLPGM	No special integration technique is required
Wan *et al.* [2007]	Meshfree point collocation method with intrinsic enrichment for interface problems are presented	1D rod	Interface problem	Point collocation method	Accurately captures sharp jumps in the derivative fields. Higher order wedge function provides additional accuracy
Xiong *et al.* [2007]	A study is carried out on background cells during the analysis of bulk forming processes by the Reproducing Kernel Particle Method	Friction less upsetting of cylinder, headings of cylindrical billets	Bulk metal forming process	RKPM	Triangular background shells are capable of handling large deformations without remeshing
Rossi and Alves [2006]	A study is carried out on the Analysis of an EFG method under large deformations and volumetric locking	Necking of circular bar, stretching problem, etc.	F-bar method	Modified EFGM (combination of EFG & PU)	Introduction of F-bar approach reduces the volumetric locking

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Khoei *et al.* [2007]	Reproducing Kernel Particle Method is presented for plasticity of pressure-sensitive material with reference to powder forming process	Cutting tool component, rotational flanged components	Powder compaction process	RKPM	Adaptive remeshing technique can be effectively replaced by RKPM with efficient and accurate results
Batra and Zhang [2007]	Search algorithm, and simulation of elastodynamic crack propagation by modified smoothed particle hydrodynamics (MSPH) method are presented	Dynamic stress intensity factor, crack propagation	Mode 1 crack theory	Modified smooth particle hydrodynamics	Saves 10% of CPU time.
Singh *et al.* [2007]	Thermal analysis of CNT-Based Nano-Composites by EFG method is presented	Carbon nano-tube based composite structure	Steady state heat conduction eq.	EFGM	Due to accuracy and capability of handling complicated geometries, the EFGM is extended to predict the thermal properties of CNT-composites
Batra *et al.* [2008]	A study is carried out for free and forced vibrations of a segmented bar by a Meshfree Local Petrov–Galerkin (MLPG) Formulation	Free and forced vibrations of segmented bar	Wave propagation problem	MLPGM	Higher accuracy, convergence rates and computation time achieved
Balachandran *et al.* [2008]	Meshfree Galerkin method is proposed based on natural neighbors and conformal mapping	Block under uniform tension	Natural neighbor concept	EFGM with natural neighbor concept	Shows good estimates of stress strain field.

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Zhao *et al.* [2009]	A mesh-free method is presented for analysis of the thermal and mechanical buckling of functionally graded cylindrical shell panels	Functionally graded ceramic metal cylindrical shell	1st order shear deformation shell theory and Sanders kinematic eq.	Element free *kp*-ritz method based on FSDT	Eliminates membrane and shear locking in thin shells
Vavourakis [2009]	A meshfree local boundary integral equation method for two-dimensional steady elliptic problems is proposed	Heat transfer in hollow cylinder Plate in uniaxial plane stress, cantilever beam	Steady state heat conduction and Timoshenko beam	LBIE	Essential boundary conditions can be imposed directly on nodal values. Stiffness matrix is banded
Zhang *et al.* [2009]	Analyzing three-dimensional potential problems with the improved Element-Free Galerkin method	Poison's eq. and Laplace eq. with dirichlet BC on a cube	Potential equations	Improved EFGM with Improved MLSA	Achieved greater computational efficiency and precision, system can be solved without taking inverse matrix
Boroomand *et al.* [2009]	The generalized finite point method is proposed	Timoshenko cantilever beam Plane stress elasticity on a square domain, etc.	Timoshenko beam theory	Generalized finite point method	The use Heaviside step functions as the weights leads to simple boundary integral equations which can be evaluated explicitly without the need for numerical integration.
Cheng and Liew [2009]	The reproducing kernel particle method is presented for two-dimensional unsteady heat conduction problems	2D heat conduction problem	Unsteady heat conduction eq.	Reproducing kernel particle method	Results obtained are accurate and efficient

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Sadeghirad *et al.* [2009]	A numerical approach is proposed based on the meshfree collocation method in elastodynamics	Cantilever beam, rectangular strip with Heaviside tension load	Navier's equations of dynamic equilibrium for plane stress and strain state	Collocation method with modified equilibrium on line method (ELM)	ELM is used for NBC satisfaction.
Wang *et al.* [2010]	A meshfree collocation method is proposed based on the differential reproducing kernel interpolation	Static analysis of bars, 2D potential problem, cantilever beam	Potential equations Timoshenko beam theory	Differential reproducing kernel method	Satisfy Kronecker Kronecker delta property and derivatives of reproducing kernel approximants are less time consuming
Erkmen and Bradford [2010]	Elimination of slip-locking in composite beam-column analysis by using the element-free Galerkin method is presented	Simply supported and overhanging beam analysis	Euler–Bernoulli's Theory	Coupled EFG and FEM	Allows direct assembly of stiffness matrix and direct application of BC's
Wang and Lin [2010]	Free vibration analysis is carried out for thin plates using Hermite Reproducing Kernel Galerkin Meshfree method with sub-domain stabilized conforming integration	Thin plate and beam, clamped square plate, etc.	Kirchhoff's hypothesis for thin plates	Hermite reproducing kernel Galerkin meshfree formulation	Reduced support size of kernel function, still HRK shape function satisfies the consistency condition

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Ahmadi and Aghdam [2010]	A study is carried out on micromechanics of fibrous composites subjected to combined shear and thermal loading using a truly meshfree method	Composite systems	Modified generalized plane strain model	Representative volume element method	Provided highly accurate results with lesser no. of nodes with lesser computational time.
Soric and Jarak [2010]	Mixed meshfree formulation is proposed for analysis of shell-like structures	Clamped thin square plate, cylindrical shell subjected to uniform line load	Reissner–Mindlin theory	MLPGM	Shear locking is completely suppressed in mixed formulation. Better accuracy and convergence rate than FEM
Quak *et al.* [2011]	A comparative study is carried out on the performance of meshfree approximations and their integration	Plate with hole, distortion analysis and tapered bar	Timoshenko beam theory and Hencky–Mindlin plate theory	MLS, local maximum entropy function and linear triangular interpolation	Offers better accuracy than liner triangular interpolation
Zhang and Li [2011]	A mixed finite element and mesh-free method is proposed using linear complementarity theory for gradient plasticity	1D Tensile bar Square panel in plane stress	Lexico–Lekme method	Coupled FE and Meshfree method	No need to derive non-local consistent tangent elasto-plastic matrix.

Table 2. (*Continued*)

Source	Work done	Structure/ Problem type	Theory/ Problem	Method/ Algorithm used	Remark
Loukopoulos *et al.* [2011]	Localized meshfree point collocation method is proposed for time-dependent magneto-hydrodynamics flow through pipes under a variety of wall conductivity conditions	Rectangular duct with insulating and conducting wall and under the influence of an oblique magnetic field	Navier–Stokes and Maxwell equations	Local meshfree point collocation method	Simple and accurate results are obtained as compared to Local Boundary Integral Element and MLPG method
Paola *et al.* [2011]	De Saint Venant flexure-torsion problem handled by Line Element-less method is presented	Beam	Saint Venant beam theory	Line element less method	Robust but more accurate
Zhao and Liew [2011]	Free vibration analysis of functionally graded conical shell panels is carried out by a meshfree method	Functionally graded conical shell panel	Hencky–Mindlin plate theory and	Meshfree *kp*-ritz method	Bending and shear stiffness are separately treated to eliminate the shear locking
Bui *et al.* [2011]	An efficient meshfree method for analysis of 2D Piezoelectric structure is proposed	Piezoelectric strip in shear deformation and bending, etc.	Timoshenko beam theory and Hencky–Mindlin plate theory	Moving Kriging interpolation	Possess Kronecker delta property.

Table 3. Summary of research papers dealing with analysis of fracture problems using meshfree methods.

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Raveendra and Banerjee [1991]	Boundary element method (BEM)	• Centre crack under plane stress and strain • Edge crack under plane stress and strain	• Boundary element method with special near tip crack elements to capture stress fields.	• The stress intensity factors were found to be dependent on material properties.
Belytschko *et al.* [1994a]	EFGM	• Patch test. • Load on cantilever • Heat conduction. • Edge crack.	• Moving least square approximation for construction of shape function. • Lagrange's multiplier to enforce boundary conditions.	• The method scored over DEM and FEM in many ways as the absence of elements avoided volumetric locking and higher convergence was achieved.
Belytschko *et al.* [1994b]	EFGM	• Edge crack • Cracks emanating from circular hole	• Moving least square approximation for construction of shape function. • Lagrange's multiplier to enforce boundary conditions.	• While boundary element method (BEM) can also avoid meshing EFGM has many advantages. It is comparatively easy to move dense arrangement of nodes around the crack tip to capture singularity.
Lu *et al.* [1994]	EFGM	• Patch test • Beam • Hole in an infinite plate • Edge crack	• Orthogonal basis function for moving least square interpolant • Gram-Schmidt orthogonalization to diagonalize shape function matrix • Modified variational principle (MVP) to enforce boundary conditions in place of Lagrange's multipliers (LM).	• This form is slightly less accurate than previous version although it reduces engineering effort to invert the shape function matrix in every step. The numerical results have better convergence than FEM and BEM

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Belytschko *et al.* [1995a]	EFGM	• Hole in an infinite plate • Edge crack • Cracks emanating from circular hole	• Modified variational principle (MVP) to enforce boundary conditions in place of Lagrange's multipliers (LM)	• Radial mesh provides better accuracy for this method around the crack tip. • The method is more domain dependent than FEM. • Domain integrals used for calculations of stress intensity factors (SIF) are not path independent.
Belytschko *et al.* [1995b]	Coupled EFGM-FEM	• Problems on elastostatics: cantilever beam • Problems on elastodynamics wave propagation and dynamic fracture	• Ramp function for combining finite and boundary elements is used.	• This method produced very accurate displacement results the EFG approximation was used near the crack tip only to reduce computational costs.
Liu *et al.* [1995]	RKPM	• Convergence study • 2D analytical problems	• Modifications in previous meshfree techniques like SPH by introduction of correction function to meet the reproducing conditions	• The correction function removes the instabilities in SPH methods. • The independence in selection of smooth correction function and the window function results in the solution as well as its derivatives to be continuous throughout the entire domain of problem.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Cordes and Moran [1996]	EFGM	• 1D bi-material rod • Inclusion in an infinite plate	• Modifications were made to define material discontinuity in terms of domain of influence in homogeneous and inhomogeneous material. • To perform integration of domain 4-node quadrilateral elements were used to increase accuracy.	• This method produced very accurate displacement results however oscillations about the exact solution were inherent while taking derivatives of displacement.
Melenk *et al.* [1996]	Partition of unity finite element method (PUFEM)	• 1D and 2D model problems.	• The PUFEM constructs a global conforming finite element space out of a set of given local approximation spaces, Henceforth, the PUFEM separates the issues of interelement continuity and local approximability.	• The results were more smooth and accurate compared to standard finite element method.
Fleming *et al.* [1997]	Enriched EFGM	• Near tip crack field • Shear edge crack • Double cantilever beam (DCB) • Crack growth from fillet	• Enriched trial functions (Extrinsic enrichment) • Enriched basis (Intrinsic enrichment)	• Method 1 can be used for multiple cracks with little expense • Method 2 is easy to program • Both methods offer significant reduction in number of unknowns required to obtain accurate solution by meshfree method.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Mukherjee and Mukherjee [1997]	EFGM	• Patch test • Dirichlet problem • Mixed problem • Neumann problem	• Modified weighted norm in conjugation with fluxes as Lagrange multipliers is used to modify the imposition of boundary conditions. • h-refinement of the EFG is also presented which enhances the number of integration cells while maintaining the ratio of nodes to cells small and roughly the same	• The new method produced good results in comparison to previous modifications proposed by researchers.
Nusier and Newaz [1998]	Virtual crack extension method and FEM	• Cylindrical specimen • Stepped disc specimen	• FEM to calculate j-integral • Virtual crack extension method to calculate G.	• The use of temperature dependent properties cause variations in values of J and G when compared to the use of constant properties. • Edge delaminations in stepped disc grow due to mode-II conditions under pure thermal load. • cylindrical specimen with circumferential crack has mixed mode conditions
Dolbow and Belytschko [1998]	EFGM	• 2D problems	• The paper provides the basics about the programming of EFGM using MATLAB.	• A comprehensive explanation to the programing of EFG method is provided. • The comparison of EFG with FEM in regard to accuracy, computational time and versatility has been discussed. • EFGM is appreciably accurate but the computational time taken is more.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Tsay *et al.* [1999]	NMM	• Infinite plate with circular hole • Mixed mode stress intensity factors • Rectangular plate with edge crack • Simply supported beam with edge central crack	• Use of dual cover systems i.e. mathematical cover system and physical cover system. • Use of physical mesh and a mathematical mesh to dictate the problem. • Adaptive finite covers approximation to construct meshes.	• The method overcame the difficulties associated with EFGM in dealing with discontinuous deformation of structures. • Crack growth is accompanied by destruction and construction of meshes causing increased computational efficiency.
Moës *et al.* [1999]	Partition of unity (PU) enrichment	• Robustness analysis • Shear edge crack • Crack growth • Plate with angled center crack	• The standard displacement based approximation is enriched near a crack by incorporating both discontinuous fields and near tip asymptotic fields.	• Cracks is treated as a completely different entity from the mesh and very accurate SIFs can be computed even with coarse meshes.
Belytschko and Fleming [1999]	EFGM	• Infinite plate with a hole • Near-tip crack problem • Plate with a hole and two cracks • Compression loaded cracks	• Comparison on smoothing and enrichment techniques is made for different problems.	• In smoothing technique visibility criterion is best for cracks and see through technique is best for hole. • Intrinsic enrichment provides better results for studying crack problems with comparison to extrinsic enrichment.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Rao and Rahman [2000]	EFGM	• Stationary crack under mode-1 • Near tip mode-1 stress field • Stationary crack under mixed mode • Propagating crack under mixed mode	• Enforcement of essential boundary conditions by transformation method and a new weight function based on student's t distribution. • To avoid discontinuities in shape function use of diffraction method	• This method eliminates the problems with Lagrange's multiplier method like loss of Kronecker delta property. The computational results were verified with experimental results and are in good agreement.
Chen *et al.* [2000b]	J-integral and EFGM	• Edge crack	• The normal J-integral for homogeneous material is modified for particular domain and is validated by EFGM	• The work analyzed the influence of non-homogeneity on the standard J-integral and defines a modified J-integral which is path independent even for FGMs and also enables to calculate energy release rate at crack tip.
Belytschko *et al.* [2000]	EFGM	• Double cantilever beam • Crack growth from fillet • Beam under 3 point bending	• The MLS shape function is enriched by using jump function and branch function	• This method for representation of discontinuity is particularly effective at crack tips. The method can be easily extended for branching and intersecting cracks.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Shields [2001]	FEM	• Plate with notches subjected to bi-axial tension	• FEM analysis using global and local crack growth criterions.	• The major impact of this research is the discovery of easy-to-use realistic models for structures with multiple cracks. Computer algorithms provide generic tools for the fracture mechanics of structures. In particular, the tools rigorously account for the effects of all cracks. This computer analysis opens up new approaches for finding crack growth and life cycles before failure occurs
Stolarska *et al.* [2001]	XFEM and LSM	• Crack growth in a square mesh subjected to tensile load • Edge crack • Crack growth from a fillet • Center crack	• Level set method was used to model location of crack and XFEM is used to capture stress and displacement fields for determining rate of crack growth.	• LSM updates the crack tip at each iteration. The geometry of crack is easily represented by two zero level sets that are orthogonal to each other at the crack tip. The combined process with XFEM produces very accurate results.
Rao and Rahman [2001]	Coupled EFGM-FEM	• Stationary crack under mode-1 • Stationary crack under mixed mode • Propagating crack under mixed mode	• EFGM was used to model nodes near the crack tip and FEM was used to model nodes in rest of region.	• The SIF calculated by coupled method are in good agreement with all EFGM and FEM. • The L/L_{EFGM} domain ratio of 0.5 gives best convergence and saves CPU time.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Chang-chun *et al.* [2002]	J-integral and Coupled EFGM-FEM	• Edge crack	• The normal J-integral for homogeneous material is modified for particular domain and is validated by EFGM	• The J-integral was extended for dynamic fracture by taking into account the non-homogeneity property. Numerical results show that extended integral varies with time and crack propagation.
Chiou *et al.* [2002]	NMM and Virtual crack extension method.	• Crack propagation on concrete structure using fictitious crack model [Hillerborg *et al.* (1976)]	• The use of interface element is eliminated as force due to cohesive normal stress in fracture process zone can be directly applied to fictitious crack surface.	• The method removed the difficulties associated with conventional FEM.
Li *et al.* [2002]	RKPM	• Simulation of dynamic shear band propagation and failure mode transition	• Thermo-elasto-viscoplastic constitutive model is used to carry out simulations	• The simulations of various procedures were replicated for the first time for various problems in engineering and some key observations were reported.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Rao and Rahman [2003]	EFGM Interaction integral (II) to calculate SIFs.	• Edge cracked plate under mode-1 • Three point bend specimen under mode-1 • Composite strip under mode-1 • Slanted crack in a plate under mixed mode • Plate with interior inclined crack under mixed mode.	• Modifications were made to interaction integral approach for calculating SIFs in FGMs	• The newly developed interaction integrals show good agreement with analytical and other previously proposed methods for calculation of SIF. The new IIs can be coupled together with FEM as well for numerical evaluation of SIF.
Ventura *et al.* [2003]	Extended FEM (X-FEM)	• DCB specimen • crack hole interaction	• Crack geometry is described by three-tuple for cracks in two dimensions and the level set function is updated by simple geometric formulas.	• The method can handle arbitrary crack geometries that are independent of mesh.
Budyn *et al.* [2004]	Extended FEM (X-FEM)	• Multiple crack growth for homogeneous and inhomogeneous material	• Higher order elements are used i.e. quadratic for standard displacement field and linear for enrichment.	• SIF and energy release rates were calculated and compared with standard analytical solutions. The results were in good agreement.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Kargarnovin *et al.* [2004]	Elasto-Plastic Element Free Galerkin Method (EP-EFGM)	• Uniform elasto-plastic tensile stretch • Capturing mode-1 of elasto-plastic crack tip plane stress singularity	• Standard EFG procedure was combined with incremental plasticity to obtain results in elasto-plastic region	• Although this method produced satisfactory results for analysis of crack tips in elasto-plastic region the computational costs involved are high compared to FE and EFG methods. • The method seeks some other requirements such as solution of highly non-linear system of equations which increases engineering effort as well.
Rabczuk and Belytschko [2004]	CPM	• The notched concrete beam of Arrea and Ingraffea • Four-point-bending with two notches • Crack branching • Fragmentation of a cylinder under internal pressure	• Crack is represented by set of discrete particles and segments rather than a continuous line.	• More complex to model • Accuracy is low compared to other meshfree methods. • Good to model dynamic crack branching.
Lee and Yoon [2004]	Enhanced EFGM	• Patch test for mode-1 and mode-2 problems • Mode-1 crack growth • Shear edge crack • Slanted crack growth	• For modeling discontinuities a discontinuity function is used. • Essential boundary conditions are enforced with penalty method in conjunction with FEM.	• The enhancement function enabled the enhanced EFG to capture sharp stress field near the crack tip. • This enhancement or enrichment is less complex compared to previous enrichment stated in literature.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Fan *et al.* [2004]	Partition of unity finite element method (PUFEM)	• Cracked plates under mode-1 and mode-2 traction • Inclined cracks with uniaxial tension	• Nodal enrichment based approximation is considered in comparison to element based approximation in FEM	• The results show that even with coarse mesh and higher order polynomial and more terms of asymptotic function are able to yield very accurate results.
Duflot and Nguyen-Dang [2004]	EFGM	• Single centered crack • Single edge crack • Single centered angled crack • Single edge angled crack • Two internal non-collinear cracks	• Nodes around the crack tip possess special weight functions similar to trigonometric functions used in intrinsic enrichment.	• The results were in good agreement to the previous literature with better computational efficiency.
Yan [2004]	BEM	• Centre and inclined crack in infinite plate in tension • A crack emanating from a triangular hole in an infinite plate under tension • A pair of symmetric cracks emanating from a square hole in an infinite plate under tension	• Constant displacement discontinuity method • Crack tip displacement discontinuity elements	• Numerical examples are included to show that the method is very efficient and accurate for calculating stress intensity factors of plane elasticity crack problems. Specifically, the numerical results of stress intensity factors of cracks emanating from a triangular or square hole in an infinite plate subjected to internal pressure are given.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Dai *et al.* [2005]	EFGM	• Static analysis under mechanical and thermal load.	• Penalty method was used to enforce boundary conditions.	• The work proves that EFGM can be successfully applied to FGMs under various types of loads.
Brighenti [2005]	EFGM	• Thick plate with an edge crack under tension • Finite thin plate under tension with a central slant crack • Penny shaped crack in a cube under remote tension	• Penalty method was used to enforce boundary conditions. • Exponential weight function was used in conjunction with visibility criterion.	• Mode-1 and Mode-2 SIFs were calculated and compared with analytical results and they were in agreement.
Ching and Yen [2005]	MLPG	• An FG link bar under a unit axial tension load. • A pressurized hollow FG cylinder. • An FG beam loaded by an exponentially change in temperature through the thickness. • An FG beam subjected to thermoelastic deformation of the cylindrical bending	• Penalty method to enforce boundary conditions.	• The work extended the domain of MLPG method to analysis of FGMs.
Li *et al.* [2005]	NMM	• Infinite plate with a hole • Near-tip crack problem • Crack growth	• Intrinsic enrichment • Extrinsic enrichment	• Overcome difficulties in the conventional meshfree methods for problems with a discontinuous domain

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Asadpoure and Mohammadi [2007]	XFEM	• Plate with crack parallel to material axis orthotropy • Edge crack with several orientation of axes of orthotropy • Single edged notched tensile specimen with crack inclination • Central slanted crack. • Inclined center crack in discs subjected to point loads.	• Heaviside and near tip functions are utilized in framework of partition of unity for modeling discontinuities.	• The enrichment functions developed can be applied to any case of orthotropic media in contrast to previous methods.
Zhang *et al.* [2008]	Interpolating moving least squares (IMLS) and EGF or IEFG	• Perforated plate under distributed load. • Rectangular plate with crack under distributed load • Rectangular plate with slant edge crack	• Orthogonal basis function for moving least square interpolant • Gram–Schmidt orthogonalization to diagonalize shape function matrix	• The IMLS approximation has greater computational efficiency than MLS approximation and saves computational time. • For 2D crack problems enriched function with improved EFG is used and results obtained are in good agreement with literature.
KC and Kim [2008]	Finite element method (FEM)	• An edge crack in a plate • A crack in functionally graded thermal barrier coating	• In this work mixed mode stress intensity factors and the non-singular T-stress in FGMs under steady state thermal loads are evaluated by means of interaction integral in conjunction with the 2D and 3D FE analysis.	• Various numerical examples are presented to verify the accuracy and performance of present method. The FEM results showed very good agreement with reference results.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Gu and Zhang [2008]	Coupled FE-EFG method	• Cantilever beam • Near tip crack field • Two edge cracked plates loaded in tension • Shear edge crack • Crack in a complex shaped plate	• Near crack tip fields are simulated using EFG approximations and rest geometry is simulated using FE approximation. • Legrange's multiplier method was used in transition zone. • Relay model was used for construction of meshfree approximations.	• The coupled method has good convergence and lower computational costs. • The SIF calculated by coupled method are in good agreement with all EFGM and FEM.
Duflot [2008]	XFEM	• Edge crack under constant flux • Square plate with a center crack • Rectangular plate with an inclined crack • Adiabatic arc-shaped crack • Crack propagation example • Three-dimensional edge crack under constant flux	• The analysis of cracks under adiabatic and isothermal conditions is presented. • The method to implement cracks and boundary conditions under thermo-elastic loading is presented.	• The work extended the domain of XFEM in modeling thermo-elastic fracture analysis.
Nguyen *et al.* [2008]	EFGM, RKPM, SPH	• Timoshenko beam • Infinite plate with center crack • Infinite plate with center inclusion • Quasi-static crack propagation	• The article discusses major aspects of mathematical formulations used in meshless methods over the years. • Major technique modifications in construction of weight functions, boundary condition applications are discussed.	• The paper provides a very comprehensible review of meshfree techniques and their coding.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Netuzhylov [2008]	Interpolating moving least squares (IMLS)	• Comparison with traditional methods to check the convergence of the method	• Singular weight functions were used to obtain solution of PDEs	• There was a problem in obtaining the inverse of singular matrix which was removed by regularization technique. • The method is much straightforward and saves computational time.
Rao and Kuna [2008]	FEM	• Finite horizontal crack in infinite medium • Finite slant crack in infinite medium	• Three new methods for calculating SIFs are proposed • Constitutive tensor formulation • Incompatibility formulation • Nonequilibrium formulation	• Accuracy of the predicted intensity factors using the interaction integrals based on three formulations is invest gated by comparing with those obtained by using displacement extrapolation method by means of two examples. Very stable results of intensity factors are obtained regardless of the type of the auxiliary field. • The interaction integral based on constant constitutive tensor formulation requires the derivatives of the actual stress and electrical displacement fields, which in turn requires second order derivatives of finite element shape functions.
Dong *et al.* [2009]	EFGM	• Tensile shear beam • Nooru–Mohamed test • 3-point bending	• Crack modeling is done by representing crack by a set of cohesive segments • Rankine criterion was used to generate crack.	• The method produced good results for fracture analysis of quasi-brittle materials.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Wen and Aliabadi [2009]	EFGM	• Rectangular plate with a slant edge crack under static loads • Rectangular plate with an inclined edge crack under a dynamic load	• Stiffness matrix is derived by the variational principle of potential energy. • The EFGM is presented in the Laplace transformed domain with enriched RBF interpolation. • The solution in the time domain is evaluated by the Durbin inversion method.	• Computational efficiency is decreased using enriched RBF.
Koohkan *et al.* [2009]	Local Petrov–Galerkin method	• Plate under mode-1 and mixed mode conditions.	• Local sub-domain and support domain techniques used to model more nodes are considered in the direction of material variation and extra nodes are located near the crack tip of the FGM body to obtain an accurate meshfree model.	• Results are compared with the available solutions in the literature which shows a good agreement.
Li *et al.* [2009]	Coupled MLPG and FEM	• Two collinear cracks in a magneto–electro-elastic plate • An edge crack in a magneto–electro-elastic plate • A penny-shaped crack in a magneto–electro-elastic cylinder	• Hybrid shape functions used at interface of MLPG and FEM domains.	• Lower computational cost compared to MLPG • The use of volumetric elements for axisymmetric problems provides small number of integral terms in comparison with the corresponding equations derived by the planar elements.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Singh *et al.* [2010b]	EFGM	• Two edge cracks on same side • Collinear edge cracks in opposite directions • Three parallel edge cracks	• The intrinsic enrichment criterion was modified and was based on normalized radius between evaluation points and crack tip.	• Results obtained were more accurate than diffraction criterion and presence of other cracks show a shielding effect over stress field of main crack. • The nature and amount of interaction depends on spatial and angular orientation of crack.
Rabczuk *et al.* [2010]	CPM	• Various cases of static and dynamic problems.	• The additional unknowns employed earlier (Rabczuk and Belytschko 2004) in variational formulation are replaced by modeling the crack segment as a set of two separate particles lying on associated cohesive crack segments. • Visibility criterion to represent crack.	• The modification achieved by removing additional degrees of freedom leads to increase in computational efficiency.
Pant *et al.* [2010]	EFGM	• Edge crack under constant flux • Square plate with center crack • Rectangular plate with inclined center crack • Bi-material body with edge interface crack • Bi-material body with central interface crack • Bi-material Brazilian disc with central interface cracks	• Jump function was used to model bi-material discontinuity • Introduction to thermal interaction integral • Adiabatic and isothermal crack modeling shown	• Comparison of temperature profiles and heat flux discontinuity due to presence of crack in a body for both adiabatic and isothermal cases is shown. • The normalized SIFs are calculated and compared with previous literature and were in agreement.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Sladek *et al.* [2010]	MLPG	• A central crack in a finite homogeneous strip • Edge crack in a finite strip under a thermal shock	• Houbolt finite-difference scheme for solution of system of ordinary differential equations	• Compared to the conventional BEM, the present method requires no fundamental solutions and all integrands in the present formulation are regular.
Rajesh and Rao [2010]	Coupled EFGM and fractal finite element method.	• Single crack problem • Multiple crack problems	• Fractal FEM is used near the crack tip and EFGM is used for rest of domain	• Numerical examples based on all four orthotropic cases are presented to illustrate the proposed coupled EFGM-FFEM method by calculating SIFs and T-stress. The convergence was checked against Q8 and L9 fractal mesh configurations
An *et al.* [2011]	NMM	• One-dimensional bimaterial bar problem • Circular inclusion in an infinite plate under uniaxial tension • Multiple circular inclusions/holes in a finite plate	• Introduction of customized physical covers for weak discontinuities with use of jump functions	• Eliminates the cumbersome process previously used in NMM for treating material discontinuities as the compatibility conditions are automatically satisfied without additional constraints
Wen and Aliabadi [2011]	EFGM	• Rectangular sheet with an edge slant crack under uniform and bending loads • Crack growth modeling • Parametric study	• Shape functions are constructed by radial basis function (RBF) and moving least square (MLS) approximation. • SIF are computed by boundary integrals	• Crack growth path and SIFs computed are in agreement with previous literature although MLS has better accuracy then RBF but with proper selection of free parameters RBF can be more stable.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Gu *et al.* [2011]	Enriched RPIM	• Mode I single edge-cracked plate. • Mode I double edge-cracked plate. • Mode-II edge-cracked plate • Cracks in a complex shaped plate	• Radial basis function enriched with trigonometric basis function similar to intrinsic enrichment procedure used in EFGM.	• Better accuracy and performance compared to conventional radial basis approach.
Pant *et al.* [2011a]	EFGM	• Edge crack bi-material plate under tension • Center crack bi-material plate under tension	• Material discontinuity i.e. weak discontinuity is treated with jump function while the interface crack i.e. strong discontinuity is treated with intrinsic enrichment criterion	• The discontinuity in strain field is observed due to change in material property i.e. young's moduli at the interface. • The normalized SIFs obtained are in agreement with pre-defined values in literature.
Singh *et al.* [2011]	EFGM	• Cracks in convex domains • Cracks in non-convex domains	• Partial domain enrichment is done to capture stress oscillations in non-convex domains.	• The results of PDE are compared with FDE for both convex and non-convex boundaries and PDE works better than FDE solutions.
Pant *et al.* [2011b]	EFGM	• Study of cracks under thermal and mechanical loads.	• Moving least square approximation for construction of shape function. • Lagrange's multiplier to enforce boundary conditions. • Diffraction criterion.	• Thermal loads have qualitatively same effect as the mechanical load but the severity of stress field near the crack tip due to crack interaction effect is different.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Simpson and Trevelyan [2011]	Enriched Boundary Element Method (EBEM)	• Cracks under mode-1 and mixed mode loadings	• Enrichment through partition of unity is applied to BEM.	• The results are compared with dual Boundary element Method. The EBEM provides better accuracy than DBEM at the cost of small increase in computational effort due to additional DOFs.
Ghorashi *et al.* [2011]	Orthotropic enriched EFGM	• Infinite isotropic tensile plate with center crack • Finite isotropic edge crack plate • Finite square orthotropic plate with a central crack • Finite rectangular orthotropic plate with an edge crack under tension • Edge crack in a cantilever orthotropic plate under shear stress • Finite rectangular orthotropic plate with a central slanted crack • An inclined central crack in an orthotropic disk subjected to point loads	• For increasing the solution accuracy, recently developed orthotropic enrichment functions used in the XFEM are adopted along with a sub-triangle technique for enhancing the Gauss quadrature accuracy near the crack.	• Several isotropic and orthotropic problems with central and edge cracks have been solved by the proposed method. Results of mixed-mode stress intensity factors (SIFs) and J-integrals have been compared with the reference results and proved the accuracy, robustness and efficiency of the proposed orthotropic enriched EFG.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Yu and Liu [2011]	Generalized finite element method (GFEM) and XFEM	• Mode-1 crack in the infinite plate • Edge crack plate under tension • Edge crack plate under shear	• GFEM improves the general accuracy of FEM by introducing generalized degrees of freedom and re-interpolating nodal degrees of freedom	• Enrichment of XFEM function with GFEM is presented and improvement in accuracy of results is seen despite of increase in computational costs.
Liew *et al.* [2011]	EFGM and RKPM	• Crack problems • Vibration analysis • Solids and structures problems • Nonlinear analysis	• This article is a review article comparing EFGM and RKPM with other meshfree techniques.	• ----------------------------------
Moosavi *et al.* [2012]	Orthogonal meshfree finite volume method (OMFVM)	• Plate with a middle edge crack subjected to end shear • Plate with a central inclined crack under traction • Plate with a central star crack under traction	• An orthogonal weighted basis function is used to construct shape function so there is no problem of singularity in this new form	• The OMFVM unities the major advantages of meshfree methods and finite volume method in one single scheme. • Orthogonal moving least square approximation instead of the moving least square, this method does not have any singularity or ill-conditioning in calculation of shape function.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Sladek *et al.* [2012]	MLPG	• Central interface crack in a finite strip under a pure mechanical load	• Use of 4th order spline-type weight function in MLS approach. • Simplified procedure to deal with each domain and interface nodes separately is presented.	• Interface can significantly increase both the stress and the electrical displacement intensity factors with respect to the corresponding homogeneous case. • Growth of an interface crack in piezoelectric bimaterials is initiated at a lower load than a crack in a homogeneous counterpart. • The effect of permeable and impermeable crack-face boundary conditions on the intensity factors in a piezoelectric biomaterial can be opposite to that for a crack in a homogeneous piezoelectric medium.
Sharma *et al.* [2012]	EFGM	• Edge crack problem under mode-1 and mode-2 loading • Crack modeling using extrinsic PU enrichment	• In extrinsic PU enrichment the approximation augmented by enrichment function added extrinsically to EFG approximations.	• The comparison of modeling techniques with extrinsic PU enrichment criterion is found to be more appealing owning to its simplicity accuracy and convergence. In general for accuracy PU > intrinsic > smoothening.
Pathak *et al.* [2012]	EFGM and XFEM	• Interfacial edge crack under various loading • Interfacial center crack under various loading • Study of interaction in presence of minor crack.	• Material discontinuity at interface is modeled by signed distance enrichment function • Heaviside function is used to model strong discontinuity.	• Discontinuity is strain field obtained due to change in material property. • Minor crack has significant effect on SIFs of major center crack and this effect is more in strongly inhomogeneous materials

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Pant *et al.* [2013]	EFGM	• Kinked crack modeling • Quasi-static crack growth	• Modification in basic intrinsic enrichment criterion is made by making changes in angular position of evaluation point.	• The proposed method of modeling kinked cracks was put to test by modeling for quasi-static fracture where crack tracking is required at each step and showed good results in agreement with results obtained from other methods.
Shi *et al.* [2013]	Extended meshfree method based on partition of unity	• Parallel cracks on two boundaries • Double crack on one boundary	• Embedding jump and singularity field items near the crack tip into meshfree approximation function. • Enhanced test and trial functions.	• This method is effective in preventing discontinuity problems introduced by the visualization method.
An *et al.* [2013]	Numerical manifold method	• A center crack in a finite bimaterial plate • A notched bimaterial four-point bending beam • Crack interaction in interface center crack	• Introduction of two new physical covers for treatment of strong and weak discontinuities i.e. weak-discontinuous physical covers and interface-singular physical covers are introduced to represent those completely intersected by material interfaces and partially intersected by interface cracks	• The modifications extended the capability of NMM in handling strong and weak discontinuities simultaneously in one domain.
Bouhala *et al.* [2013]	XFEM	• Crack terminating at bi-material interface	• SIF is calculated by body force method (BFM) • Enrichment functions are created and tested via XFEM.	• The XFEM solution was improved with the use of singular enrichment functions compared to non-enrichment solution.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Chamat *et al.* [2013]	ABAQUS numerical simulation and singular integral equation method	• Study of crack propagation • Fatigue testing • Analysis of coating	• ABAQUS numerical simulation and singular integral equation method used for studying crack behavior.	• First method: the crack effect on the material was investigated. It was found that the energy release rate decreases as soon as the crack approaches the interface and increases when crossing it. • Second method: the penetration and deflection crack lengths were assumed to be equal and the ratio between the energies of penetration and deflection was computed in the elastic case. The results showed that the probability for a crack to deflect was very high. • Third method: the volumetric approach has been used to evaluate the ratio between the deflection and penetration energy release rates in the elastoplastic case. The same behavior as in the case of the second method has been observed. • A satisfactory agreement between the experimental and modeling results was obtained. Indeed, 98% of pictures taken with SEM showed that the cracks are either stopped or deflected at the interface and only 2% showed a crack penetration through the interface.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Bayesteh and Mohammadi [2013]	XFEM	• Plate with inclined edge and center cracks. • Four point bending specimen	• Orthotropic enrichment functions are used for crack tip fields • Maximum circumferential stress criterion	• Orthotropic XFEM needs far fewer DOFs than conventional FEM to achieve the same level of • Accuracy. In addition, high convergence rates are obtained by this method. Three cases of tip enrichment (Isotropic enrichment, orthotropic enrichment and no enrichment) have been compared for orthotropic FGM problems and shown that for the same mesh configuration and contour integral, orthotropic enrichments yield far accurate results. Also, simulation of crack propagation in an isotropic problem showed good agreement with the experimental results
Cao *et al.* [2013]	Combined EFG and RPIM	• Cantilever beam • Plate with central hole • Poisson's equation	• The coupling between EFG and RPIM is obtained via collocation method.	• It is easy to implement essential boundary conditions via this method as RPIM shape functions posses Kronecker delta property.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Bhattacharya *et al.* [2013]	XFEM	• FGM plate with edge crack	• M-integral for SIF calculation	• The fatigue life of aluminium alloy, FGM and equivalent composite is found minimum when the discontinuities are simultaneously present in the domain. The minor cracks have the least effect on the fatigue lives of the materials whereas effect of holes is quite significant. In case of FGM, if a crack is present on the ceramic rich side, the fatigue life becomes quite small as compared to alloy rich side.
Sharma *et al.* [2013]	XFEM	• Edge cracked plate with sharp cracks holes and inclusions	• Level sets for circular holes and Heaviside shape function for modeling crack.	• It was observed that presence of all inhomogeneities cause and increase in SIF of the crack. • L2 error norm is more in FEM compared to XFEM
Gope *et al.* [2014]	Photoelastic bench and FEM	• 2D plate with multiple cracks	• Photoelastic and Finite element method	• The study was carried out for SIF, J-integral, von mises stress distribution and the results predicted by this approach matches very well with the literature

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Kumar *et al.* [2014]	Coupled FE-EFG	• Crack growth modeling in CT specimen using J–R curve. • Crack growth modeling in triple point bend specimen using J–R curve. • Crack growth modeling in bi-material triple point bend specimen using J–R curve.	• PU based extrinsic enrichment technique is used for EFG procedure. • Ramp function is used in transition zone of FE and EFG	• The simulations by the proposed method show that variation of load with crack mouth opening displacement (CMOD) predicted by this approach matches very well with the literature. • Holes have a more severe effect in front on crack tip for CT specimen as compared to back of crack tip due to reduction of plastic zone.
Nasri *et al.* [2014b]	ABAQUS numerical simulation	• Study of crack propagation of crack at different orientations	• The first part of work deals with crack normal to interface and second part deals with crack terminating at interface at different orientations	• It was found that the normalized SIF decreases and then increases rapidly once crack crosses the interface.
Cheng *et al.* [2014]	Interpolating moving least squares (IMLS) and EGF or IEFG	• Cantilever beam • Plate with a hole	• Interpolating moving least squares is used to obtain weight functions. • Increment tangent stiffness matrix method for elastoplasticity problems.	• The IMLS methodology used to obtain shape function possess the Kronecker delta property and the results obtained via this method are in agreement with the previous literature.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Sharma *et al.* [2014a]	EFGM	• Interfacial edge crack under tension • Interfacial center crack under tension	• Jump function is used to model material discontinuity	• The results obtained for normalized SIFs are in agreement with literature.
Pathak *et al.* [2014]	XFEM	• Interfacial center crack	• Crack surface is modeled by Heaviside function and material interface is modeled by level set function. • Fatigue crack growth has been modeled using modified kinking criterion	• The simulations shows that the crack gradually kinks towards softer material.
Namakian *et al.* [2014]	EFGM with moving least square reproducing kernel method (MLSRKM)	• Single center crack • Single edge crack • Slanted edge crack • Slanted center crack • Shear edge crack	• The enriched particles associated with the EWF are added to crack tip which previously occupied by only the particle with ordinary weight functions.	• Two types of EWFs are constructed one fully enriched and other partially enriched and their prowess is tested in many cases and these enriched functions provide higher workability than standard extrinsic enrichment techniques.
Muthu *et al.* [2014]	Modified crack closure integral (MCCI)	• Edge crack in finite plate. • Angled crack centrally located in finite plate • Mode-1 crack face pressure loading • Thermal loading.	• The closure nodal forces at the crack tip and at nodes ahead of it are multiplied with opening displacements at the corresponding nodes behind the tip to obtain strain energy release rates (SERR).	• This technique for extraction of SIFs has advantages over other techniques when it comes to nodal density independency, influence of order of gauss quadrature, effect of domain of influence and local refinement.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Nguyen *et al.* [2014]	Extended mesh free Galerkin radial point interpolation method (X-RPIM)	• Edge crack plate under tensile loading • An edge crack plate under a uniform shear loading. • Crack growth from fillet • Crack growth in perforated panel with circular hole.	• Crack topology is treated by aid of level set function and jump functions are used to model crack tip while step function is used to model displacement discontinuity around the crack faces. • Shape functions are constructed via RPIM.	• The RPIM methodology used to obtain shape function possess the Kronecker delta property and the results obtained via this method are in agreement with the previous literature.
Singh *et al.* [2014]	EFGM	• Homogeneous material under thermal and mechanical load • Bi-material under thermal and mechanical load.	• Moving least square approximation for construction of shape function. • Lagrange's multiplier to enforce boundary conditions. • Paris fatigue crack growth law has been used for life estimation of various problems	• Thermo-elastic fracture problem was decoupled into thermal and elastic problem and extrinsic enrichment technique was used and the results obtained via this method are in agreement with the previous literature.
Nasri *et al.* [2014a]	Combined XFEM and FEM	• Study and comparison of cracks in mono material and bi-material Zn/Al, Zn/steel and Zinc	• Standard XFEM procedure used to study interaction between cracks.	• The study was carried out to check the effect of substrate rigidity on crack behavior and it was found that more the substrate is stiff more the cracks move away from each other.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Ren *et al.* [2014]	IEFG	• Dirichlet and Neumann problems	• Error estimation in MLS approximation by using consistency conditions.	• The error is proportional to the radius of the weight functions, and is inversely proportional to the nodes number when $d_{\max}$ is fixed. • The error also depends on the bound of the norm of derivatives of shape functions.
Kumar *et al.* [2014]	Combined EFGM and FEM	• Finite size plate with an edge crack • Finite size plate with an edge crack and multiple holes • Finite size plate with an edge crack and multiple inclusions • Finite size plate with an edge crack and multiple minor cracks • Finite size plate with an edge crack, multiple holes, inclusions and minor cracks	• Defects are modeled in 20% of region while remaining domain is modeled with properties of equivalent homogeneous material • Homogenization is done to calculate material properties of complex materials by selection of proper representative volume element (RVE).	• The results obtained by proposed multigrid coupled approach are found in good agreement with those obtained by analytical and/or meshfree method. • The presence of holes, inclusions and minor cracks significantly affect the fatigue life of the plate. • The minor cracks have least effect on the fatigue life of the plate whereas the holes provide maximum damage to the plate. • 5% holes reduce the fatigue life of the plate nearly by 18% whereas 5% inclusions reduce the fatigue life of the plate nearly by 9%.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Pant and Sharma [2014]	EFGM	• Bi-material beam with vertical interface • Bi-material beam with horizontal interface	• Three approaches have been used to compare the efficiency of modeling techniques namely Domain portioning, Lagrange's multiplier and Jump function approach.	• On comparison of results it was found that jump function approach gives best results for both vertical and horizontal interface problems.
Zhang and Ma [2014]	NMM	• A plate with an edge crack • A plate with a slanted crack • Three-point bending specimen with an edge crack • A plate with a multi-branched crack	• Representing FGM domain by various physical and mathematical covers formed. • Use of manifold elements formed by combination of physical and mathematical covers.	• The proposed method of analysis using dual covers provided more accurate results.
Živojinović *et al.* [2014]	XFEM using ABAQUS	• Friction stir welded joint made of aluminum alloy	• Different welding zones are modeled in ABAQUS and further analysis was made	• During its (stable) growth, the crack remains within the base material. As it gets closer to the FSW joint (HAZ), considerable crack growth leading to structure failure starts to occur, before the crack can reach the HAZ. • During the propagation of the crack through the structure, change of its direction can be noticed. Therefore, a combination of crack opening and shearing occurs during its growth, which leads to deforming of the structure as a whole. This phenomenon is related to shear stresses appearing in the structure, and the two additional fracture modes are quantified by corresponding stress intensity factors K_{II} and K_{III}.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Sharma *et al.* [2014a]	XFEM	• Pipe bend with axial part through crack at extrados and intrados • Pipe bend with circumferential part through crack at extrados and intrados • Straight pipe with circumferential part through crack under internal pressure • Straight pipe with circumferential elliptical part through crack under opening moment	• 3D XFEM formulation is done. The integration of enriched elements is achieved by dividing them into several tetrahedrons above and below surface.	• XFEM works better than FEM and it was found that axial part through crack located at intrados is more severe than extrados. • Severity of circumferential crack is not location dependent. • Axial crack is more severe than circumferential crack.
Singh *et al.* [2014]	XFEM	• Double edge crack under mechanical load • Double adiabatic edge crack under thermal load	• Unknown temperature field was obtained by solving heat conduction equation and it was then used as load input for elastic problem to get displacement and stress field.	• SIF of main crack remains the same with no major change • SIF of auxiliary crack varies with angel of orientation of crack.
Peng *et al.* [2015]	Transformation toughening theory and Eshelby inclusion method	• Mode-1 crack with semicircular and rectangular hole inclusion studied	• Transformation toughening theory and Eshelby inclusion method	• The change of SIFs due to stiffness and shapes of inclusion is studied.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Jameel and Harmain [2015]	EFGM and LSM	• Inclined edge crack • Center crack in a rectangular plate • Effect of Bi-material interface on Fatigue crack Growth • Effect of Holes on Fatigue Crack Growth	• EFGM has been applied in conjugation with level set methods to model crack propagation under fatigue load.	• This work enhances the applicability of EFGM to simulate crack propagation under fatigue loading with much efficiency when compared to previous counterparts.
Miao *et al.* [2015]	Hybrid displacement discontinuity method	• Two collinear square hole cracks	• Hybrid displacement discontinuity method and generalization of Bueckner's principle used to study interaction of two collinear square hole cracks	• Square hole has shielding effect on crack emanating from hole
Lee *et al.* [2015]	Particle difference method	• Stationary mixed mode edge crack • Growing mode I edge crack. • Mixed mode crack propagation of an edge-cracked plate	• Eliminates the mesh dependency by constructing strong form. • The higher order derivative approximation can be obtained without using mesh or grid.	• Works well with lower number of nodes compared to EFGM • There is no need for numerical integration by avoiding weak formulation makes the simulation much faster.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Pant and Bhattacharya [2016]	EFGM and XFEM	• Plate with an edge crack. • Plate with a center crack • Edge crack under mode II loading • Center crack under mode II loading	• Anew criterion to model the crack geometry by modifications in angular position of Gauss point. • Convergence analysis provides insight about the suitable parameters for simulation process.	• The modification extends the prowess of EFGM in modeling Quasi-static crack growth in FGMs.
Garg and Pant [2016]	EFGM	• Edge crack plate subjected to Mode-I mechanical loading • Edge crack plate subjected to thermal loading • Crack in functionally graded Thermal Barrier Coating (TBC)	• Intrinsic enrichment done for EFGM • Modified interaction integral for thermal fracture of FGMs. • The temperature field obtained by solving the heat transfer problem is then employed as input for the mechanical problem to determine the displacement and stress fields	• The modification extends the domain of EFGM in modeling thermo-elastic fracture in FGMs.
Khosravifard *et al.* [2016]	EFGM, RPIM and Background Decomposition Method (BDM)	• Single-edge-crack in a rectangular domain under uniform tension • Rectangle with an inclined edge crack • Crack propagation from a fillet in a structural member • Crack propagation in a three-point bending specimen with circular holes	• Intrinsic enrichment done for EFGM • No enrichment strategy adopted in RPIM hence refined nodal distribution used around the crack tip. • BDM is used for evaluation of domain integrals of the meshfree method	• The use of BDM enhances the capability of meshfree methods in handling oscillations in stress fields efficiently and produces more accurate results. • BDM helps in uniform distribution of nodal points and even without enrichment procedures the accurate results can be obtained by fewer nodes compared to other meshfree techniques.

Table 3. (*Continued*)

Source	Mesh-less technique used	Problems discussed	Technique specifications	Conclusions/Remarks
Li *et al.* [2016]	Finite block method	• Circumferentially loaded disk. • Rectangular plate with cracks under tensile load • Rectangular sheet with central crack under dynamic load	• Use of laplace transform method and the Durbin's inverse method for dynamic problems. • Polygonal or circular core is used at the crack tip to obtain SIFs and the T-stress in the Williams' stress expansion	• The method provided good results in comparison to previous literature.
Tan and Jiao [2017]	Combined BEM and NMM	• Dirichlet problems • Neumann problem	• Boundary integral formulation and the finite cover approximation	• Compared with the finite element method and the numerical manifold method, the new approach has the well-known dimensionality of the BEM; compared with the conventional BEM, it can perform the p-adaptive analysis conveniently without adding intermediate nodes in elements.
Garg and Pant [2017]	Optimized EFGM (OEFG)	• Adiabatic and Isothermal cracks (Singe Edge and Multiple) in FGMs	• Modified algorithm to select nodes in influence domain. • Use of Taguchi's Optimization to select best parameters for EFG simulation.	• The method removed the dilemma about selection of scaling parameter. • The optimized method provided approximately 80% reduction in computational time required to perform an EFG simulation.

Various meshfree techniques have been discussed in this article with major focus on analysis of fracture problems. All the developments in various meshfree techniques like PU based techniques, NMM, CPM, RKPM, MLPG, etc. have been developed to compensate the shortcomings of their counterparts.

EFGM has contributed most towards the analysis of fracture problems and it is easy to model when compared to other techniques. EFGM is easy to model and most of the results produced by EFG method match the desired accuracy. The shortcomings in EFGM can be removed by coupling it with other However, there are still some challenges remaining. Few of them are as:

(1) The greatest challenges appear to be developing the speed and robustness in meshfree method. It is still an expedient task to construct an efficient and effective method to construct meshfree shape function, which should satisfy the consistency and compatibility conditions and probably possess the delta function property.
(2) In some meshfree methods background cells are used for numerical integration whereas in some no background cells are used but in these cases accuracy and stability decreases. So there is need of efficient algorithm and stability techniques.
(3) Development of a robust commercial software package using Meshfree methods or at least incorporation of Meshfree method in existing FEM software to treat special problems is also an imminent task for researchers and engineers.
(4) An engineering application such as analysis of piezolaminated composite structures including the pyroelectric effect.

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