Predicting Rainfall in Australia

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1. Introduction

Rainfall is a common event that happens in every country. The amount of rain that falls on a given day can depend on several factors. For example, one would expect that if the temperature is high enough, water will evaporate. This evaporated water then cools down when it is far away from the surface. Because of this cooldown, the water transitions from a gaseous phase into a liquid phase, creating clouds containing water. The water subsequently becomes too heavy for the clouds to carry, resulting in rainfall. The temperature depends in its turn on the climate. Take Australia for example. Summers in Australia can become quite warm, with an average temperature of 29 degrees Celsius. Another factor that could influence rainfall is windspeed. With a high wind speed, it is expected that clouds fly by quicker which lowers the rainfall in a certain region. When combined with the dry climate, it is essential to make good predictions about the rainfall so that good precautionary measures can be taken. Therefore, the research question of this report will be: "What are useful predictors of the rainfall in Australia?" I expect that wind speed would be a useful predictor for the reason mentioned before. I also expect that the maximum temperature is a useful predictor of rainfall since Nicholls, Drosdowsky, and Lavery (1997) found a negative correlation between the maximum temperature and rainfall in Australia. Finally, I believe that the minimum temperature will be a useful predictor as well since Nicholls, Drosdowsky, and Lavery (1997) also found a positive correlation between the minimum temperature and rainfall in Australia.

The dataset that will be used to answer the research question is derived from the site Kaggle, which hosts many publicly available datasets that can be used for many projects. The dataset I downloaded from Kaggle contains data about the climate in Australia. Some of the variables used in this dataset are for example the maximum temperature on a given day, the rainfall on that day, and the wind direction on that day. For this project, I will only use the minimum temperature, the maximum temperature, the wind speed, and the amount of rainfall to answer the research question previously addressed. When initially inspecting the dataset, it came to my attention that there are some missing values present in the dataset. Therefore, I decided to impute those missing values to obtain a complete dataset. A complete dataset can potentially provide more information for the question at hand.

To impute the missing values, the MICE algorithm (van Buuren and Groothuis-Oudshoorn (2011)) will be used. This algorithm creates five possible datasets in which the missing data are imputed from the available data. This is done five times, thus creating 25 possible datasets. All these datasets are subsequently pooled to create a final dataset in which the missing values are now replaced by a pooled estimate of the 25 possible datasets. Subsequently, the variables MinTemp, MaxTemp, and WindGustSpeed are centered so that they contain a useful, interpretable zero point when the regression estimates are evaluated later in this project. Finally, I will sample 2000 observations from the original dataset. The reason for this is because the original dataset is quite large, which could lead to memory problems. Additionally, working with a smaller sample generally speeds up the overall project.

2. Estimation

To answer the research question addressed in the previous section, the possible predictors have to be estimated. The estimation will take place using the Bayesian approach, which utilizes prior knowledge about

the parameters of interest and the data at hand. It combines the prior knowledge and the density of the data into a joint posterior distribution. This joint posterior distribution is subsequently used to sample for example 1000 values for the parameters of interest. With these sampled values, Bayesian statistics like the Expected A Posteriori (EAP) and the 95% Central Credible Interval (CCI) can be calculated. The 95% CCI will give me an indication of the relevance of the predictors. As I said earlier, the joint posterior distribution is derived by combining the density of the data and prior knowledge about the parameters of interest. This can be done exactly based on the prior distribution and the density of the data using Bayes' theorem. However, one can also approximate the posterior distribution using Markov-Chain Monte Carlo (MCMC) algorithms, for example, the Gibbs sampler. The Gibbs sampler derives conditional posterior distributions for each of the parameters of interest and uses those to approximate the joint posterior distribution. To derive the conditional posterior distributions for the predictors in this project, I need to specify the density of the data and the prior distributions for each of the predictors. The density of the data can be described with the following regression equation:

$$rainfall_i = b_0 + b_1 X_{1i} + ... + b_n X_{ni} \tag{1}$$

This means that the density of the data is a normal distribution with as mean $b_0 + b_1 X_{1i} + ... + b_n X_{ni}$ and as variance σ^2. As for the prior, it is typical to use conjugate priors which have the same form as the resulting conditional posterior distribution. For the predictors MaxTemp (centered, from now on called MaxTemp.cen) and MinTemp (centered, from now on called MinTemp.cen), I will use a normal conjugate prior. Since I do not have any historical data that investigated the same question and I am not certain about the relationship between maximum temperature and rainfall, I will use an $X \sim \mathcal{N}(0, 0.0001)$ uninformative prior. The variance of the density of the data is sampled using the normal density and an inverse-gamma prior. I also decided to use an uninformative prior for the conditional posterior of the variance. For the predictor WindGustSpeed (centered, from now on called WindGustSpeed.cen), I decided not to use the same prior as for the predictors MinTemp.cen and MaxTemp.cen. The reason for this is that I am less certain that there is a relationship between wind gust speed and rainfall. Therefore, I use a non-standardized t-distribution as prior distribution to allow for more uncertainty. The non-standardized t-distribution is not a conjugate prior, which means that it will not lead to a closed-form conditional posterior distribution. In such cases, one uses the Metropolis-Hastings (MH) sampler which is described in the next section.

3. Metropolis-Hastings

As described before, I use a non-standardized t-distribution as prior distribution for the variable WindGustSpeed.cen with a normal density of the data. Since the density and the prior distribution are not of the same form, the resulting conditional posterior will not be of a recognizable form. Therefore, I will use a Metropolis-Hastings (MH) sampler to draw samples for the variable WindGustSpeed.cen. Metropolis-Hastings works by deriving functions proportional to the conditional posterior and using those to approximate the joint posterior distribution. The reason that MH is more suitable here than Gibbs is that knowing the distribution up to a proportionality constant is enough to sample from it using MH. Therefore, it does not matter that the final conditional posterior can not be found. In this project, I multiply the t-distribution prior with the normal density and use the resulting proportional function in the MH sampler.

The Metropolis-Hastings sampler works as follows: first, it samples a candidate value from a proposal distribution. The type of proposal distribution depends on the type of MH sampler that is used. In an independent MH sampler, a candidate value is drawn from a proposal distribution with the same parameters in each iteration. The proposal distribution does not depend on the previous value of the parameter of interest, hence the term independent. In a dependent MH sampler, the parameters of the proposal distribution depend on the previous value of the parameter of interest. Therefore, there is more flexibility in the values that can be drawn from it. In this project, I will use a random walk MH sampler with a normal proposal distribution whose mean is the current value of the parameter of interest. Next, a ratio between the candidate value as a function of the proportional function and the current value as a function of the proportional function is

calculated. This ratio is then multiplied with a ratio between the current value of the parameter of interest and the candidate value. The latter ratio is implemented so that the Metropolis-Hastings algorithm does not tend to get stuck at a certain value. When these two ratios are multiplied with each other, the resulting acceptance ratio is compared to a threshold value sampled from a uniform distribution ranging from 0 to 1. If the acceptance ratio is equal to or higher than the threshold value, the candidate value is accepted as the new current value. If the acceptance ratio is lower than the threshold value, the candidate value is rejected and the current value is assigned as the accepted value in that iteration. Since the values of a certain parameter using the MH sampler are based on accepting or rejecting the candidate value, the quality of the proposal distribution can be examined with an acceptance ratio. The quality can also be analyzed by calculating the autocorrelation which will be explained in section 6.

4. Model selection using the DIC

Now that the density of the data and the densities and priors for the predictors have been specified, multiple models can be compared to find the best fitting model to the data in this project. When the best fitting model has been found, I can evaluate the parameter estimates to answer the research question. In this project, I will compare 3 competing models with each other. The first model contains MaxTemp.cen and WindGustSpeed.cen as predictors of Rainfall. The second model tries to predict Rainfall from MinTemp.cen and WindGustSpeed.cen. The third and final model has Rainfall predicted by MaxTemp.cen, MinTemp.cen, and an interaction between these predictors. The reason for the inclusion of the interaction is that [1] found a positive correlation between maximum temperature and minimum temperature. Therefore, I believe the interaction between maximum and minimum temperature to be a possible predictor. The different competing models are compared to each other using the Deviance Information Criterion (DIC). The DIC is based on minimizing the 'loss' or 'deviance,' which is defined as the distance between a model of interest and a saturated model (i.e. a model that fits the data perfectly). Mathematically, the DIC is formulated as follows:

$$DIC = -2logf(y|\theta) + 2pD \tag{2}$$

where $logf(y|\theta)$ is the loglikelihood of the model of interest evaluated at the means of the regression parameters and where pD is defined as the estimated number of 'effective parameters.' The estimated number of effective parameters is calculated with

$$pD = Dbar - Dhat \tag{3}$$

where $Dhat$ is the loglikelihood previously described and where $Dbar$ is the mean of the loglikelihoods of the imposed model evaluated at each set of sampled regression parameters. Using the DIC, the best fitting model can be selected. Usually, a difference in DIC larger than 10 means the model with the higher DIC can be ruled out. A difference between 5 and 10 can also be seen as a substantive difference, thus allowing for the selection of the model with the lower DIC. The first model achieved a DIC of 13345, the second model a DIC of 13340. This means there is a DIC difference of 5, meaning that the second model fits the data substantively better than the first model. Next, the second model is compared to the third model. The third model achieved a DIC of 13288. This means there is a difference of 52 between the third and second model. Since this is much larger than 10, I can safely reject the second model and select the third model as the best-fitting model.

5. Model selection using the Bayes Factor

A good fitting model for the data in this project can also be found by using the Bayes Factor (BF). The Bayes Factor is typically used to test informative hypotheses (e.g. $b_1 > b_2$) against uninformative hypotheses

(e.g. $b_1 = b_2$) or to test informative hypotheses (e.g. $b_1 > b_2$) against their complement (e.g. $b_1 <= b_2$). The BF is calculated by dividing the fit of the specified hypothesis by the complexity of that hypothesis. The complexity of a hypothesis is defined as the proportion of the prior distribution that is in concordance with the specified hypothesis (Hoijtink et al. (2019)). The fit of a hypothesis is defined as the proportion of the posterior distribution that is in concordance with the specified hypothesis (Hoijtink et al. (2019)). The resulting BF value indicates how many more times there is evidence for a specified hypothesis than for its complement or the uninformative hypothesis. Although there are no guidelines for determining when there is enough evidence for a given hypothesis, numbers greater than 1 typically indicate that the specified hypothesis would hold. Using the Bayes Factor, three hypotheses are compared to determine which predictors should be included in the final model. The data for MaxTemp.cen are multiplied by -1 for these comparisons to allow for a better comparison of the effect of MaxTemp.cen with the effects of any other predictors. The hypotheses are evaluated using the bain() package in R.

The first hypothesis, H1, is defined as follows:

$$H_1 : b_1 > b_2$$

where b_1 is the regression coefficient of MaxTemp.cen and b_2 is the regression coefficient of WindGust-Speed.cen. The BF for this hypothesis tested against its complement is 5.44, meaning there is 5.44 times more evidence for H1 than for its complement. Therefore, I would conclude that MaxTemp.cen is a useful predictor to include in the final model.

The second hypothesis, H2, is defined as

$$H_2 : b_3 > b_2$$

where b_3 is the regression coefficient of MinTemp.cen and b_2 still the regression coefficient of WindGust-Speed.cen. The BF for this hypothesis tested against its complement is 6.57, meaning there is 6.57 times more evidence for H2 than for its complement. This leads me to conclude that MinTemp.cen is also a useful predictor for the final model.

The final hypothesis, H3, is defined as

$$H_3 : b_4 > b_1 * b_3$$

where b_4 is the regression coefficient for the interaction between MaxTemp.cen and MinTemp.cen, b_1 the regression coefficient of MaxTemp.cen and b_3 the regression coefficient of MinTemp.cen. The BF for this hypothesis tested against its complement is 34.71, meaning there is 34.71 times for evidence for H3 than for its complement. Therefore, the interaction between MaxTemp.cen and MinTemp.cen should also be included in the final model. Ultimately, all the included parameters using the Bayes Factor lead to the same model as the model selected through the DIC. Therefore, the final model that will be evaluated consists of the predictors MaxTemp.cen, MinTemp.cen and an interaction between these predictors.

6. Convergence

To arrive at reliable estimates of the regression coefficients, one wants to have sampled as many different values as possible from the approximated joint posterior distribution. This means that one wants the model to have converged so that the model has covered as much from the joint posterior distribution as possible. Convergence can be assessed in four ways and is typically checked using multiple iterations of the sampler which are called chains. The first method is through trace plots, which show for each regression coefficient

the values in the different iterations in each chain. Trace plots can be used to see whether chains are stuck at a local maximum or not, which indicates how much of the joint posterior distribution is covered. When the trace plots show overlapping chains and the chains look stable (i.e. like fat caterpillars), then the model has reached convergence. The second method to check convergence is with autocorrelation plots. Autocorrelation is the correlation between the data of a variable and the same data shifted down one observation. This can be measured at many lags, thus the data can be shifted down by many observations. The resulting autocorrelations can subsequently be plotted to assess convergence. When the model has reached convergence, one would expect the autocorrelation to quickly tend to 0. The autocorrelation at lag 0 will always be 1 since the correlation between a dataset and itself will always be 1. The third method for assessing convergence is with the Gelman-Rubin statistic, which requires at least two chains. It compares the variance within chains to the variance between chains and it should be near 1. Additionally, one can plot the Gelman-Rubin statistic and the within- and between-variability for increasing intervals. The lines for the within- and between-variability should be stable and the line for the Gelman-Rubin statistic should be around 1. The last method for assessing convergence is by calculating the MC error for the different regression coefficients. The MC error for a regression coefficient is calculated as the standard deviation of that coefficient divided by the square root of the number of iterations. Therefore, the MC error decreases as the number of iterations increases. For convergence to be established, the MC error should not be larger than 5% of the standard deviation.

All the plots and convergence statistics can be found in the Appendix.

When checking the convergence, the trace plots of all the coefficients show overlapping chains that resemble fat caterpillars. The autocorrelation plots also indicate convergence, but they show a decreasing slope that tends to 0 after around 10 lags. However, since the corresponding trace plots indicate convergence, I believe that the autocorrelation should not be a problem. The Gelman-Rubin plots overall also show convergence for each of the coefficients. The between-variability for b_0, b_1, b_2, and the variance exhibits instability between the different iterations. However, the Gelman-Rubin statistic for these parameters is stable around 1, so I believe that convergence has been reached nonetheless. Finally, the MC errors also indicate convergence. I have used 10000 iterations in my Gibbs sampler, meaning that each standard deviation is divided by 100. Therefore, every MC error will automatically be 1% of the corresponding standard error.

Based on the information above, I conclude that my model has converged and that I have reached reliable estimates for the regression coefficients. These will be interpreted in the next section.

7. Posterior Predictive Check

Before the results from the Gibbs sampler can be interpreted, one of the multiple linear regression assumptions has to be checked. More specifically, I will assess whether the residuals of the observed data are normally distributed using the posterior predictive p-value. To calculate this p-value, I first need to create simulated datasets. For $t = 1,\ldots,18000$ I will sample 2000 values, let's call them y^t, from a normal distribution with as mean $b_0^t + b_1^t X_1 + b_2^t X_2 + b_3^t X_3$ and as standard deviation the square root of $\sigma^{2,t}$. Next, for $t = 1,\ldots,18000$, a dataset with simulated residuals sim^t is created using $y^t - b_0^t - b_1^t X_1 - b_2^t X_2 - b_3^t X_3$. At the same time, for $t = 1,\ldots,18000$, a dataset with observed residuals obs^t is created using $y - b_0^t - b_1^t X_1 - b_2^t X_2 - b_3^t X_3$. After the residuals are created, a test statistic is calculated from each simulated dataset sim^t and each observed dataset obs^t. For this project, I have chosen to assess the normality of residuals based on the proportion of residuals in the tails of the distribution of each simulated and observed residual dataset. Typically, in a standard normal distribution, you would expect that 5% of the observations lie in the tails, thus 2 or more standard deviations away from the mean. Therefore, the test statistic for each simulated and observed dataset that will be calculated in this project is the proportion of observations that are 2 or more standard deviations away from the mean. When this proportion is lower than or equal to 0.05 for a certain dataset, which is the proportion of observations in the tails of the standard normal distribution, then this dataset will receive a 1. Otherwise, it will receive a 0. Finally, the posterior predictive p-value is calculated as the proportion of simulated statistics greater than or equal to the observed statistics. I received a p-value of 0.93. This means that the residuals are more normally distributed than expected. Thus, I can assume that the assumption of normally distributed residuals is met.

8. Interpretation of estimates and intervals

Since the Posterior Predictive Check indicated that the residuals are normally distributed, I can investigate the estimates of the regression parameters. To answer the research question, I will look at the Expected A Posteriori (EAP) estimate and the 95% Central Credible Interval (CCI). The values for the EAP estimate and the CCI are displayed below.

Table 1: EAP estimates and CCI intervals for the regression parameters

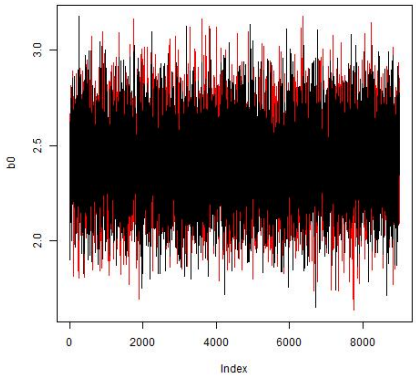
Parameter	EAP (SE)	95% CCI Lower Bound	95% CCI Upper Bound
Intercept	2.42 (0.0046)	2.02	2.83
MaxTemp.cen	-0.34 (0.0008)	-0.41	-0.27
MinTemp.cen	0.39 (0.0009)	0.32	0.47
MaxTemp.cen*MinTemp.cen	-0.01 (0.0001)	-0.01	0.001

The intercept (b_0 = 2.42, SE = 0.0046, CCI = [2.02; 2.83]) denotes that in a place with an average temperature of 0 degrees Celsius, the expected rainfall on one day is 2.42 ml. The predictor MaxTemp.cen (b_1 = -0.34, SE = 0.0008, CCI = [-0.41; -0.27]) expresses that for every degree Celsius above the average temperature in a place, the expected rainfall on one day in that place is expected to decrease by 0.34 ml. The predictor MinTemp.cen (b_2 = 0.39, SE = 0.0009, CCI = [0.32; 0.47]) expresses that for every degree above the average minimum temperature in a place, the expected rainfall on one day in that place is expected to increase by 0.39 ml. The interaction between the predictors (b_3 = -0.01, SE = 0.0001, CCI = [-0.01; 0.001]) indicates that for every degree above the average minimum temperature in a place, the effect of the maximum temperature on the rainfall in that place decreases by 0.01 ml. Based on the CCI's of the predictors, I can conclude that only the interaction can not be distinguished from 0. Therefore, only the maximum temperature and the minimum temperature are reasonable predictors of the rainfall in a certain place. This is also the answer to the research question proposed in the introduction.

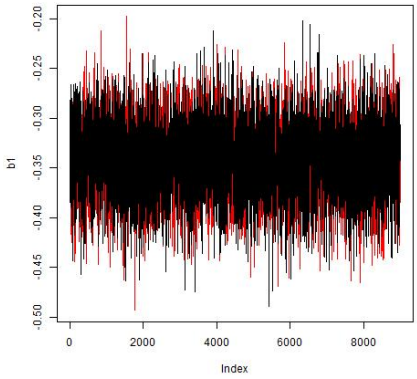
All the code used to produce the results in this manuscript can be found on this GitHub repository.

Appendix

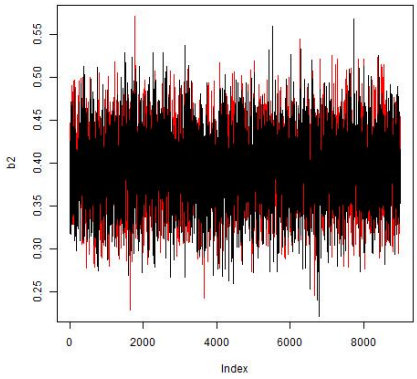
Appendix A: Traceplots



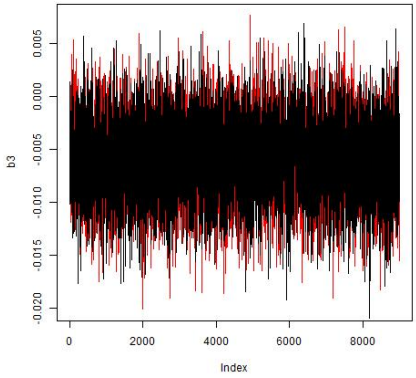
(a) Traceplot for the intercept



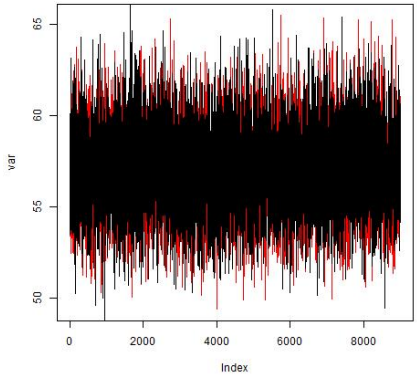
(b) Traceplot for b_1



(c) Traceplot for b_2



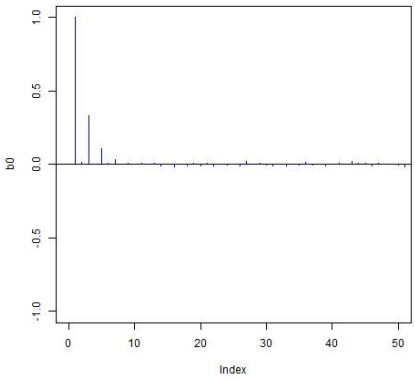
(d) Traceplot for b_3



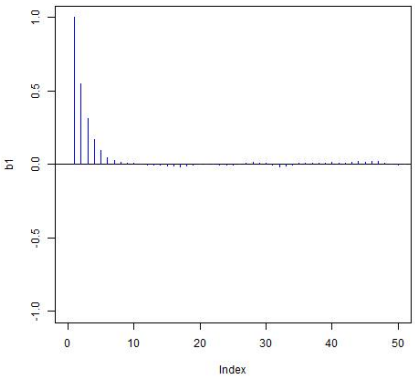
(e) Traceplot for the variance

Figure 1: Traceplots for the regression parameters

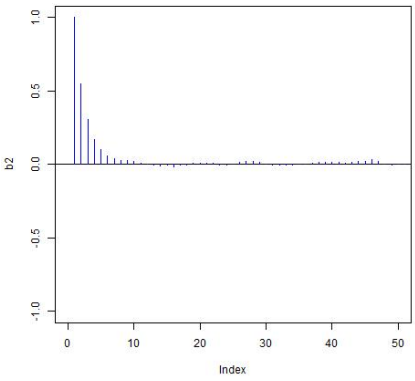
Appendix B: Autocorrelation plots



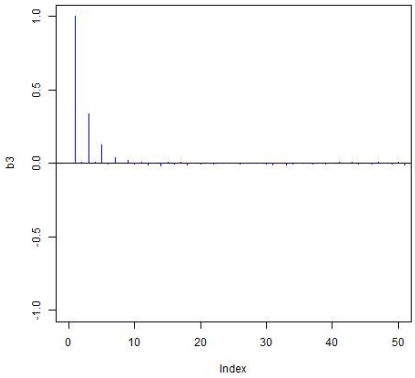
(a) Autocorrelation plot for the intercept



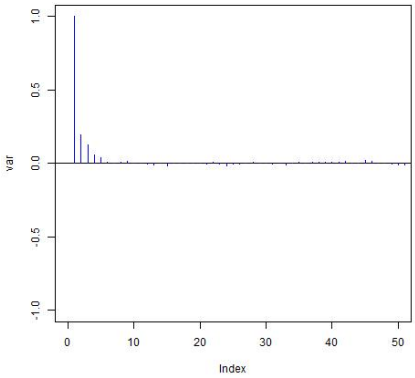
(b) Autocorrelation plot for b_1



(c) Autocorrelation plot for b_2



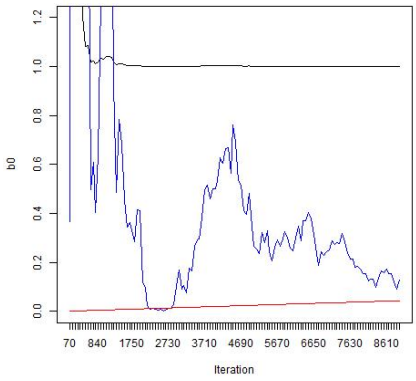
(d) Autocorrelation plot for b_3



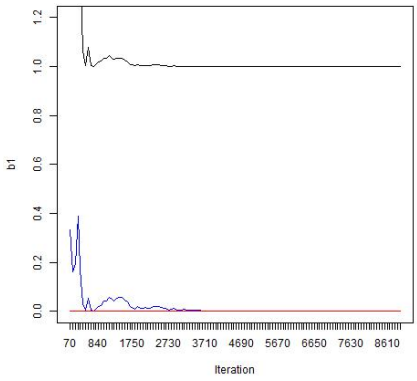
(e) Autocorrelation plot for the variance

Figure 2: Autocorrelation plots for the regression parameters

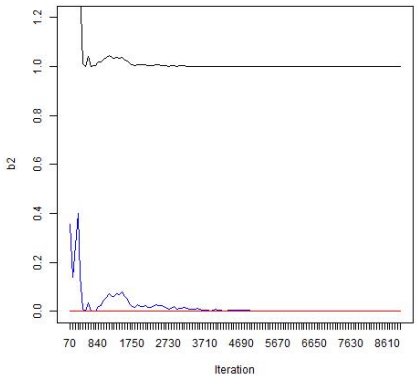
Appendix C: Gelman-Rubin plots



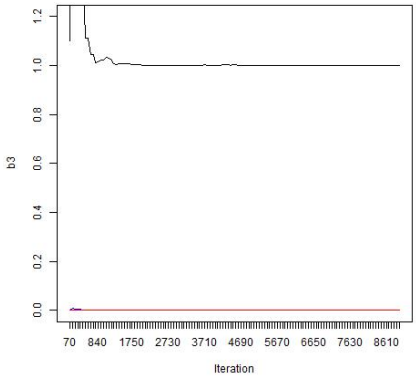
(a) Gelman-Rubin plot for the intercept



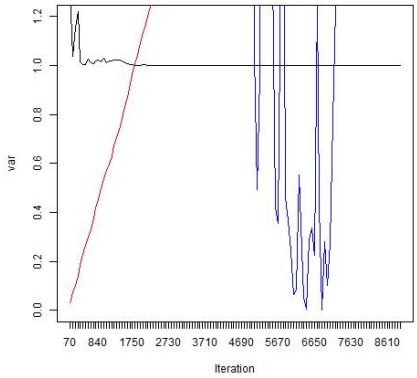
(b) Gelman-Rubin plot for b_1



(c) Gelman-Rubin plot for b_2



(d) Gelman-Rubin plot for b_3



(e) Gelman-Rubin plot for the variance

Figure 3: Gelman-Rubin plots for the regression parameters

Appendix D: MCMC Errors

Table 2: MCMC Errors for each of the regression parameters

Parameter	SD	MC Error	MC Error/SD
b_0	0.207	0.0021	0.01
b_1	0.035	0.0003	0.01
b_2	0.039	0.0004	0.01
b_3	0.004	0.0000	0.01
var	2.229	0.0223	0.01

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