

DEEP LEARNING

HOMEWORK 1

Group 71 members:

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- Vincent Jakl (108529)

Contributions of each member

- Luis Jose De Macedo Guevara (95621):
- Vincent Jakl (108529):

Question 1

1. a)

1. b)

2. a)

2. b)

Question 2

- 1.
2. a)
2. b)
2. c)

Question 3

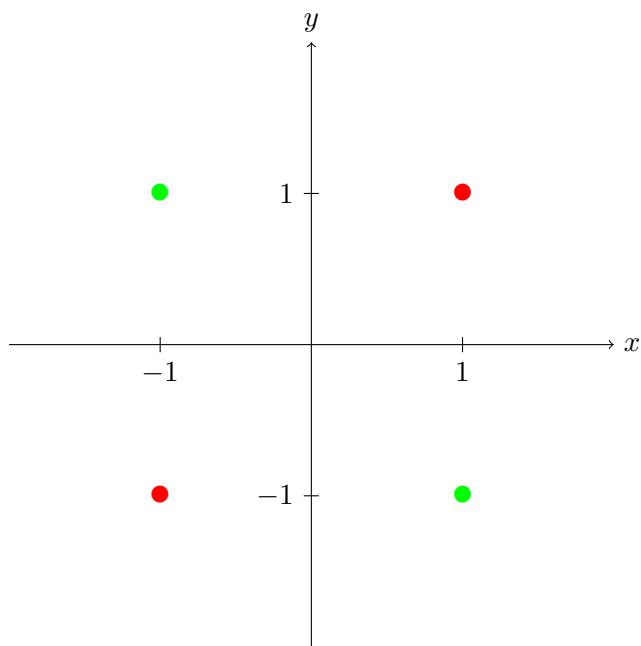
1.

a)

Let

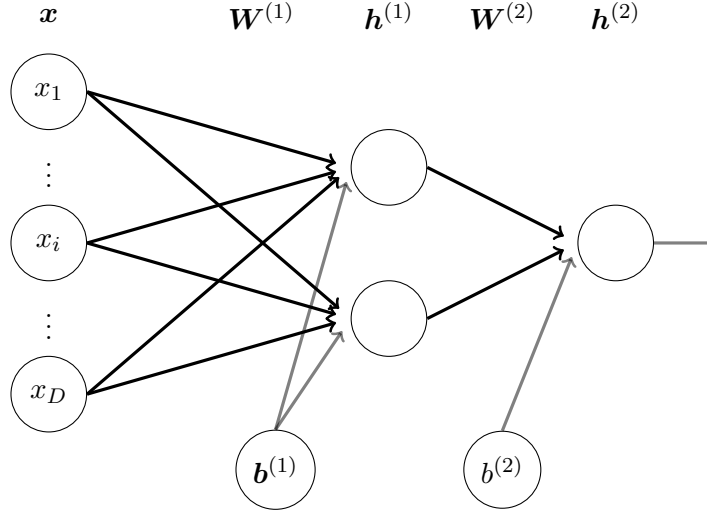
- $D = 2$
- $A = B = 0$
- $\mathbf{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{x}^{(3)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\mathbf{x}^{(4)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$,

We have $f(\mathbf{x}^{(1)}) = -1$, $f(\mathbf{x}^{(2)}) = +1$, $f(\mathbf{x}^{(3)}) = +1$ and $f(\mathbf{x}^{(4)}) = -1$



Which we can see is not linearly separable and therefore a perceptron cannot learn a separating hyperplane.

b)



$$\mathbf{W}^{(1)} = \underbrace{\begin{bmatrix} 1 & 1 & \dots & 1 \\ -1 & -1 & \dots & -1 \end{bmatrix}}_{2 \times D}, \quad \mathbf{b}^{(1)} = \begin{bmatrix} -A \\ B \end{bmatrix}$$

$$\mathbf{W}^{(2)} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad b^{(2)} = -1$$

We have for \mathbf{x} :

$$\begin{aligned} \mathbf{h}^{(2)} &= \text{sign} \left(\mathbf{W}^{(2)} \mathbf{h}^{(1)} + b^{(2)} \right) \\ &= \text{sign} \left(\mathbf{W}^{(2)} \left(\text{sign} \left(\mathbf{W}^{(1)} \mathbf{x} + \mathbf{b}^{(1)} \right) \right) + b^{(2)} \right) \\ &= \text{sign} \left(\begin{bmatrix} 1 & 1 \end{bmatrix} \left(\text{sign} \left(\begin{bmatrix} 1 & 1 & \dots & 1 \\ -1 & -1 & \dots & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix} + \begin{bmatrix} -A \\ B \end{bmatrix} \right) \right) - 1 \right) \\ &= \text{sign} \left(\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \text{sign} \left(\sum_{i=1}^D x_i - A \right) \\ \text{sign} \left(B - \sum_{i=1}^D x_i \right) \end{bmatrix} - 1 \right) \\ &= \text{sign} \left(\text{sign} \left(\sum_{i=1}^D x_i - A \right) + \text{sign} \left(B - \sum_{i=1}^D x_i \right) - 1 \right) \end{aligned}$$

c)