Deep Learning

Homework 1

Group 71 members:

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Contributions of each member

- Luis Jose De Macedo Guevara (95621):
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Question 1

1. a)

1. b)

2. a)

2. b)

Question 2

1.

2. a)

2. b)

2. c)

Question 3

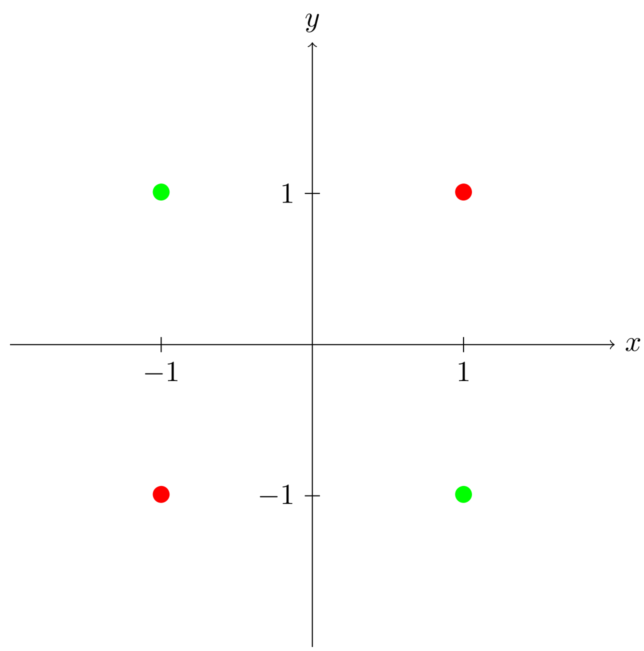
1.

a)

Let

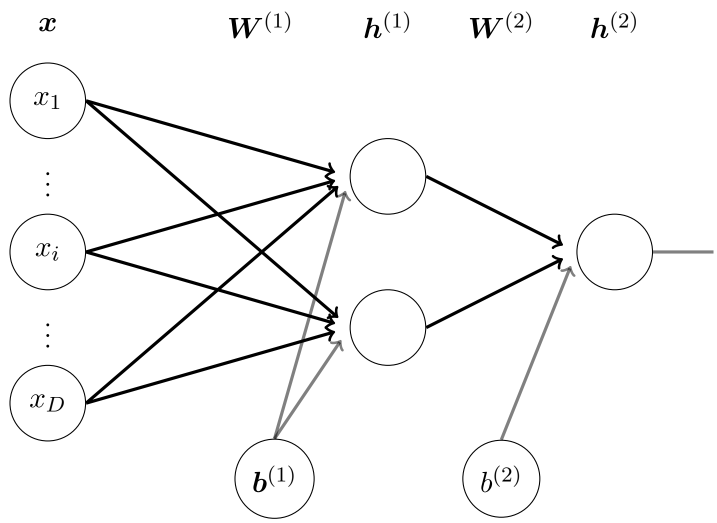
- $D = 2$
- $A = B = 0$
- $\boldsymbol{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\boldsymbol{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\boldsymbol{x}^{(3)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\boldsymbol{x}^{(4)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$,

We have $f(\boldsymbol{x}^{(1)}) = -1$, $f(\boldsymbol{x}^{(2)}) = +1$, $f(\boldsymbol{x}^{(3)}) = +1$ and $f(\boldsymbol{x}^{(4)}) = -1$



Which we can see is not linearly separable and therefore a perceptron cannot learn a separating hyperplane.

b)



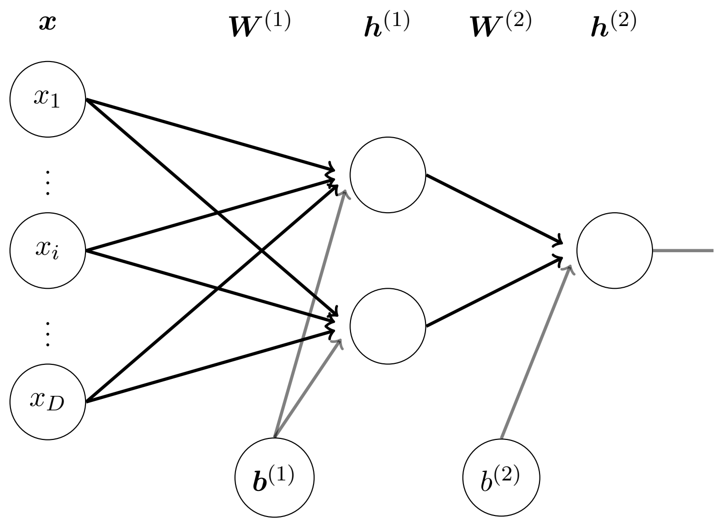
$$\boldsymbol{W}^{(1)} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -1 & -1 & \cdots & -1 \end{bmatrix}}_{2\times D}, \qquad \boldsymbol{b}^{(1)} = \begin{bmatrix} -A \\ B \end{bmatrix}$$

$$\boldsymbol{W}^{(2)} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad b^{(2)} = -1$$

We have for $\boldsymbol{x}$:

$$\begin{aligned}
\boldsymbol{h}^{(2)} &= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\boldsymbol{h}^{(1)} + b^{(2)}\right) \\
&= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\left(\operatorname{sign}\left(\boldsymbol{W}^{(1)}\boldsymbol{x} + \boldsymbol{b}^{(1)}\right)\right) + b^{(2)}\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} 1 & 1 \end{bmatrix}\left(\operatorname{sign}\left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -1 & -1 & \cdots & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix} + \begin{bmatrix} -A \\ B \end{bmatrix}\right)\right) - 1\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} \operatorname{sign}\left(\sum_{i=1}^{D} x_i - A\right) \\ \operatorname{sign}\left(B - \sum_{i=1}^{D} x_i\right) \end{bmatrix} - 1\right) \\
&= \operatorname{sign}\left(\operatorname{sign}\left(\sum_{i=1}^{D} x_i - A\right) + \operatorname{sign}\left(B - \sum_{i=1}^{D} x_i\right) - 1\right)
\end{aligned}$$

c)



$$\boldsymbol{W}^{(1)} = \underbrace{\begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 1 & \cdots & 1 \end{bmatrix}}_{2\times D}, \qquad \boldsymbol{b}^{(1)} = \begin{bmatrix} A \\ -B \end{bmatrix}$$

$$\boldsymbol{W}^{(2)} = \begin{bmatrix} -1 & -1 \end{bmatrix}, \qquad b^{(2)} = 0$$

We have for $\boldsymbol{x}$:

$$\begin{aligned}
\boldsymbol{h}^{(2)} &= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\boldsymbol{h}^{(1)} + b^{(2)}\right) \\
&= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\left(\operatorname{ReLU}\left(\boldsymbol{W}^{(1)}\boldsymbol{x} + \boldsymbol{b}^{(1)}\right)\right) + b^{(2)}\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} -1 & -1 \end{bmatrix}\left(\operatorname{ReLU}\left(\begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 1 & \cdots & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix} + \begin{bmatrix} A \\ -B \end{bmatrix}\right)\right)\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} \operatorname{ReLU}\left(A - \sum_{i=1}^{D} x_i\right) \\ \operatorname{ReLU}\left(\sum_{i=1}^{D} x_i - B\right) \end{bmatrix}\right) \\
&= \operatorname{sign}\left(-\operatorname{ReLU}\left(A - \sum_{i=1}^{D} x_i\right) - \operatorname{ReLU}\left(\sum_{i=1}^{D} x_i - B\right)\right)
\end{aligned}$$