Deep Learning

Homework 1

Group 71 members:

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Contributions of each member

- Luis Jose De Macedo Guevara (95621):
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Question 1

1. a)

The single perceptron was not the best choice for this task. As can be seen in the figure below, the perceptron was not able to fit to the high dimensional data also with no sign of improvement. With the 20 epochs, it ended up with a 0.3422 test accuracy.

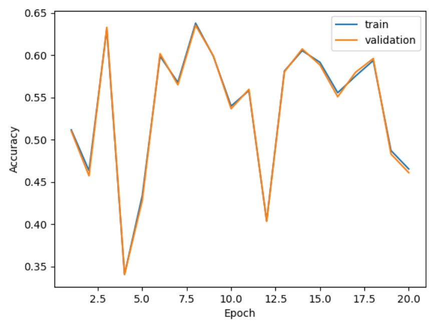


Figure 1: Single perceptron

1. b)

2. a)

2. b)

Question 2

1.

2. a)

2. b)

2. c)

Question 3

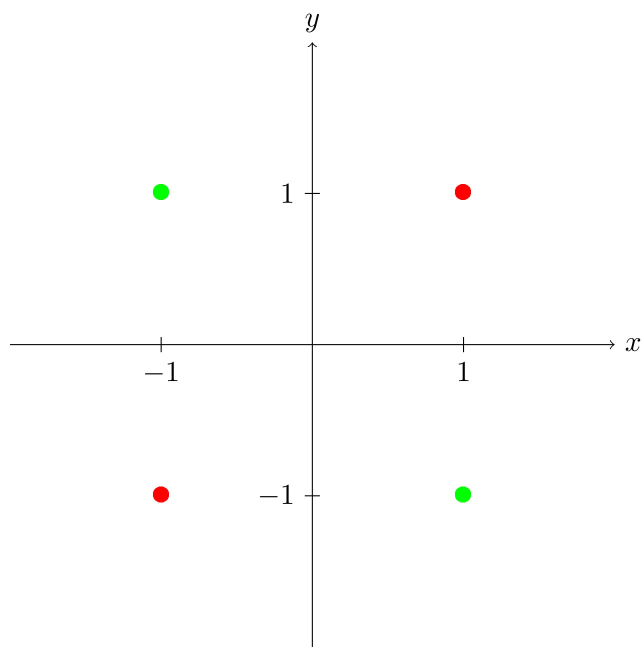
1.

a)

Let

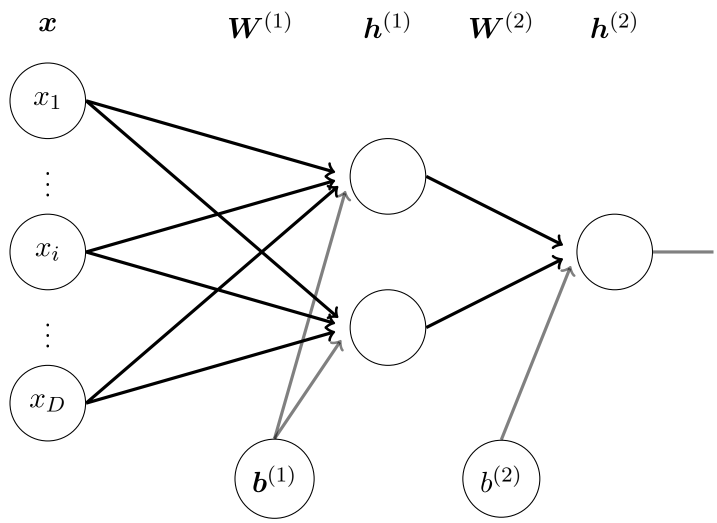
- $D = 2$
- $A = B = 0$
- $\boldsymbol{x}^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\boldsymbol{x}^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\boldsymbol{x}^{(3)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ and $\boldsymbol{x}^{(4)} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$,

We have $f(\boldsymbol{x}^{(1)}) = -1$, $f(\boldsymbol{x}^{(2)}) = +1$, $f(\boldsymbol{x}^{(3)}) = +1$ and $f(\boldsymbol{x}^{(4)}) = -1$



Which we can see is not linearly separable and therefore a perceptron cannot learn a separating hyperplane.

b)



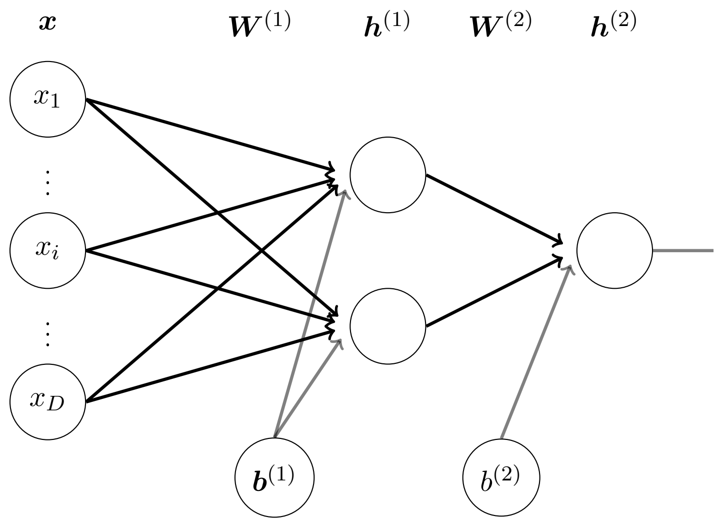
$$\boldsymbol{W}^{(1)} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -1 & -1 & \cdots & -1 \end{bmatrix}}_{2\times D}, \qquad \boldsymbol{b}^{(1)} = \begin{bmatrix} -A \\ B \end{bmatrix}$$

$$\boldsymbol{W}^{(2)} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \qquad b^{(2)} = -1$$

We have for $\boldsymbol{x}$:

$$\begin{aligned}
\boldsymbol{h}^{(2)} &= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\boldsymbol{h}^{(1)} + b^{(2)}\right) \\
&= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\left(\operatorname{sign}\left(\boldsymbol{W}^{(1)}\boldsymbol{x} + \boldsymbol{b}^{(1)}\right)\right) + b^{(2)}\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} 1 & 1 \end{bmatrix}\left(\operatorname{sign}\left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ -1 & -1 & \cdots & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix} + \begin{bmatrix} -A \\ B \end{bmatrix}\right)\right) - 1\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} \operatorname{sign}\left(\sum_{i=1}^{D} x_i - A\right) \\ \operatorname{sign}\left(B - \sum_{i=1}^{D} x_i\right) \end{bmatrix} - 1\right) \\
&= \operatorname{sign}\left(\operatorname{sign}\left(\sum_{i=1}^{D} x_i - A\right) + \operatorname{sign}\left(B - \sum_{i=1}^{D} x_i\right) - 1\right)
\end{aligned}$$

c)



$$\boldsymbol{W}^{(1)} = \underbrace{\begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 1 & \cdots & 1 \end{bmatrix}}_{2\times D}, \qquad \boldsymbol{b}^{(1)} = \begin{bmatrix} A \\ -B \end{bmatrix}$$

$$\boldsymbol{W}^{(2)} = \begin{bmatrix} -1 & -1 \end{bmatrix}, \qquad b^{(2)} = 0$$

We have for $\boldsymbol{x}$:

$$\begin{aligned}
\boldsymbol{h}^{(2)} &= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\boldsymbol{h}^{(1)} + b^{(2)}\right) \\
&= \operatorname{sign}\left(\boldsymbol{W}^{(2)}\left(\operatorname{ReLU}\left(\boldsymbol{W}^{(1)}\boldsymbol{x} + \boldsymbol{b}^{(1)}\right)\right) + b^{(2)}\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} -1 & -1 \end{bmatrix}\left(\operatorname{ReLU}\left(\begin{bmatrix} -1 & -1 & \cdots & -1 \\ 1 & 1 & \cdots & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix} + \begin{bmatrix} A \\ -B \end{bmatrix}\right)\right)\right) \\
&= \operatorname{sign}\left(\begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} \operatorname{ReLU}\left(A - \sum_{i=1}^{D} x_i\right) \\ \operatorname{ReLU}\left(\sum_{i=1}^{D} x_i - B\right) \end{bmatrix}\right) \\
&= \operatorname{sign}\left(-\operatorname{ReLU}\left(A - \sum_{i=1}^{D} x_i\right) - \operatorname{ReLU}\left(\sum_{i=1}^{D} x_i - B\right)\right)
\end{aligned}$$