Faster Weighted and Unweighted Tree Edit Distance and APSP Equivalence

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"Unweighted" (String) Edit Distance: all costs are one

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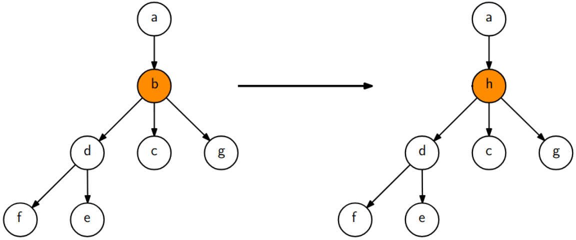

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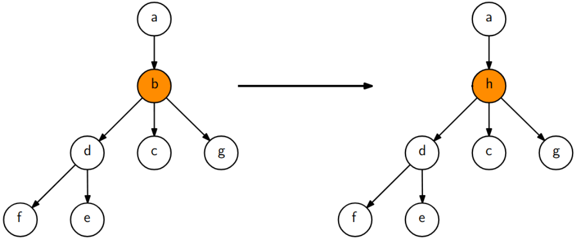

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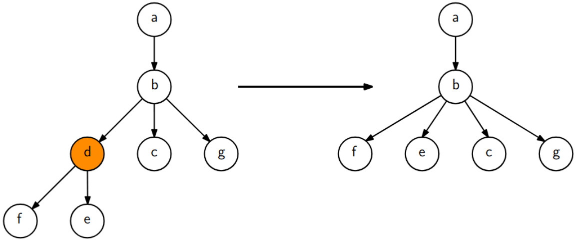

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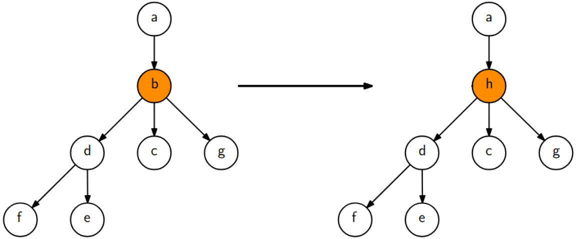

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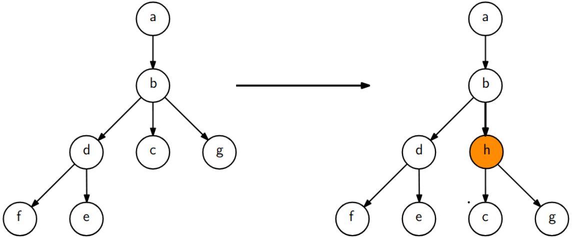

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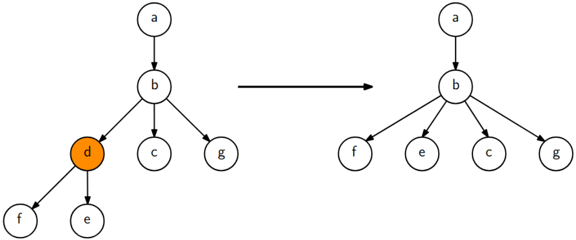

Images from Seddighin, Seddighin 2019

TED reformulated

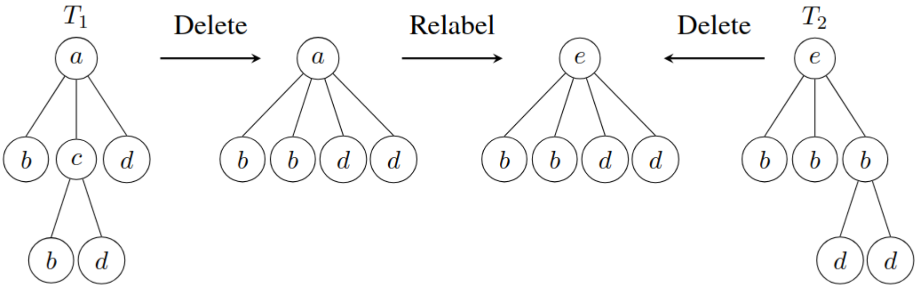

Image from Mao 2022

- Relabel v to v' with cost $\delta(v, v')$.
- Delete v from T_1 with cost $\delta(v, \varepsilon)$.
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Background: Introduced by Selkow in the late 1970s. Applications in computational biology, structured data analysis, image processing, compiler optimization, and more.

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Last three fall within decomposition strategy framework formalized in [Dulucq and Touzet, 2003].
For algorithms within the framework a $\Omega(n^3)$ lower bound exists.

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Input: A weighted and directed graph G.
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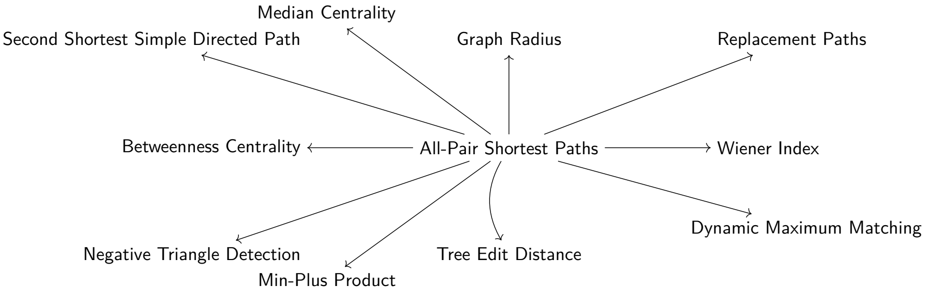

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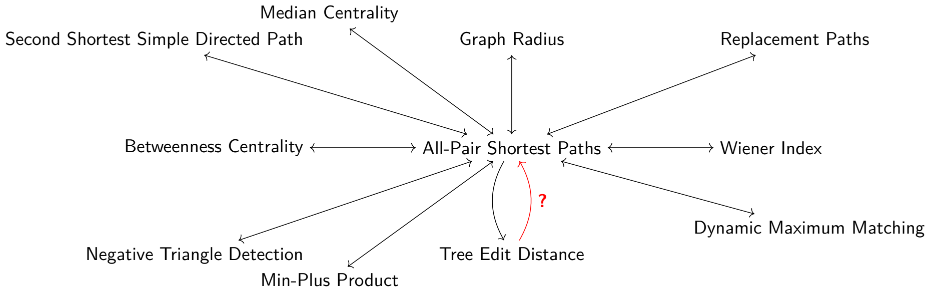

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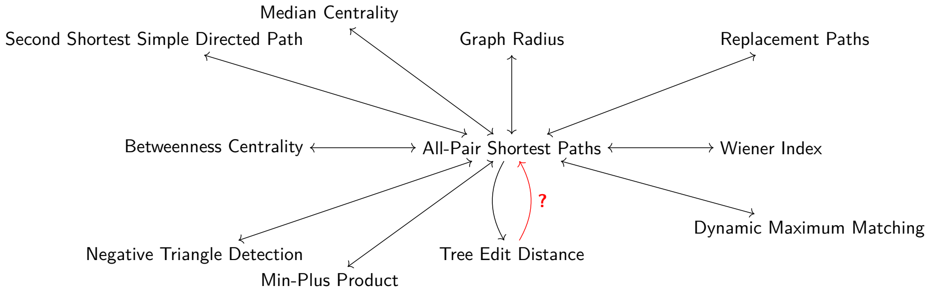
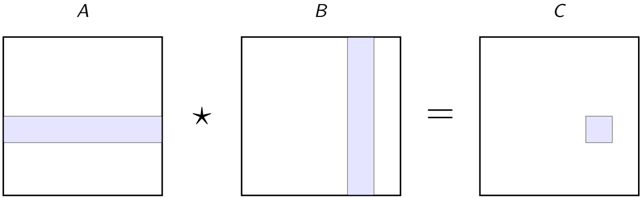


Question 2: is TED equivalent to APSP?

Key component to achieve truly subcubic algorithms for unweighted TED:

Monotone Min-plus Product



$$C_{i,j} = \min_{1 \le k \le n}\{A_{i,k} + B_{k,j}\}$$

- B **is row monotone**:

$$\forall i, j \quad B_{i,j} \le B_{i,j+1}.$$

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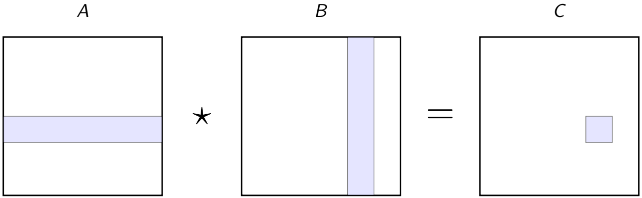

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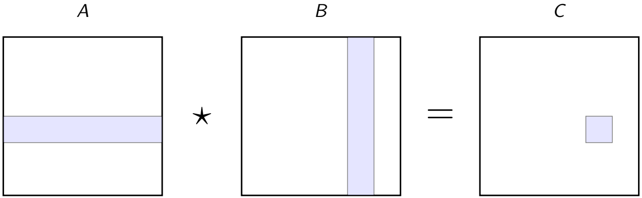

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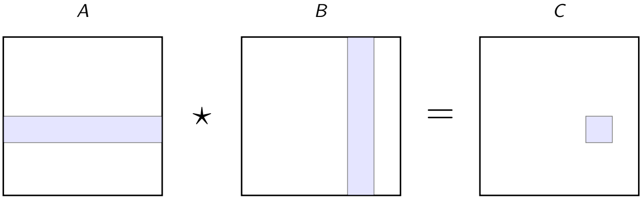

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Question 3: is there a $\mathcal{O}(n^{(\omega+3)/2})$ algorithm for unweighted TED?

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Sketch of the Reduction

Similarity of Strings

Instead of computing the edit distance between two strings $A = a_1 \cdots a_n$, $B = b_1 \cdots b_n$, we compute the similarity between A, B.

m i t a g s e m i n a r

c h r i s t m a s

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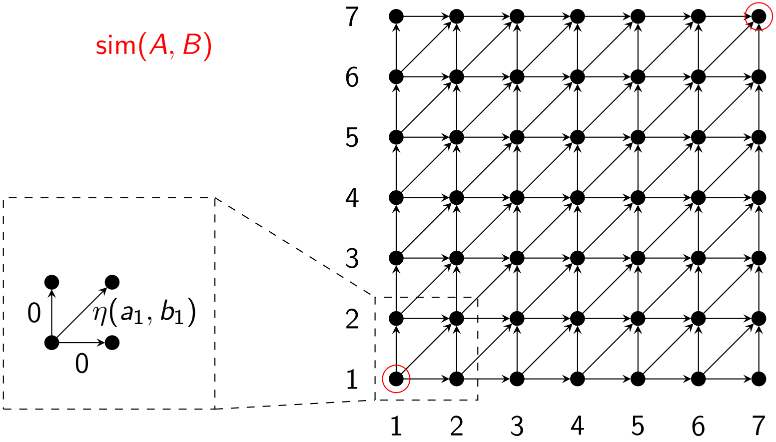
String Alignment Graphs

The *string alignment graph* summarizes the DP scheme computing the similarity.



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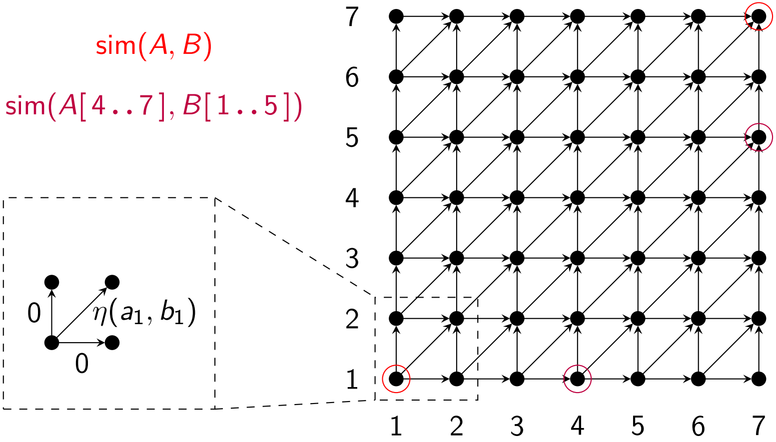
$\mathrm{sim}(A[4\,..\,7], B[1\,..\,5])$



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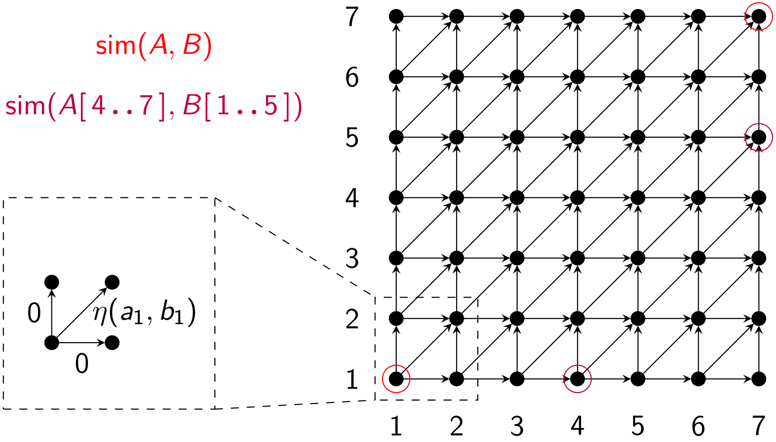

lower-to-right border:

suffix vs prefix

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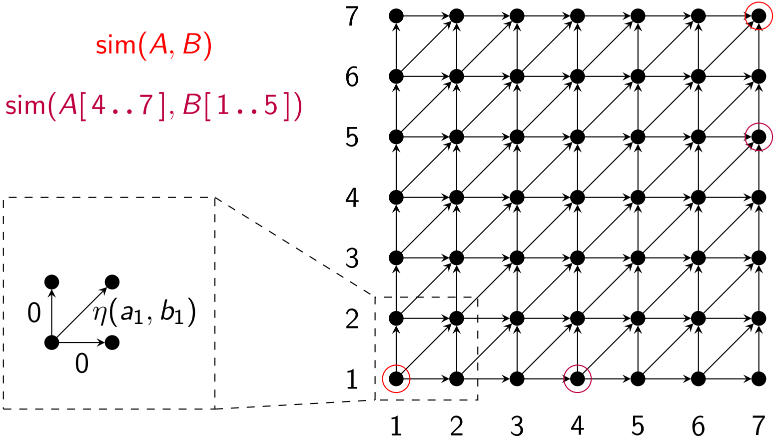

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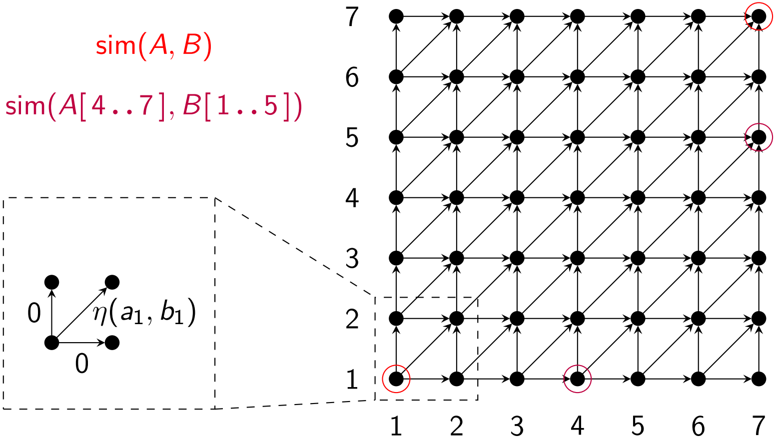

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$\mathcal{O}(n^2)$ to compute all

Bedtime reading: "Semi-local string comparison: algorithmic techniques and applications" by Alexander Tiskin

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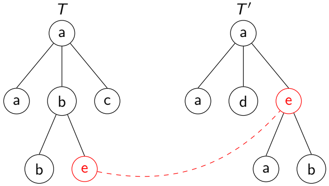
We compute the similarity between T and T'.



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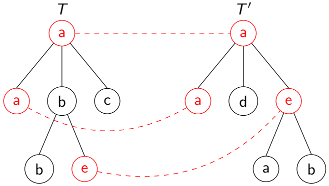

$\eta(v, v') \coloneqq \delta(v, \varepsilon) + \delta(\varepsilon, v') - \delta(v, v')$ "how much I save by substituting v with v'"

$\mathrm{sim}(T, T') =$"maximum weight of similarity matching"

Condition on similarity matching: for any two matched vertices (v, v') and (u, u')

- v is an ancestor of u in T if and only if v' is an ancestor of u' in T',
- v comes before u in the pre-order of T if and only if v' comes before u' in the pre-order of T'.

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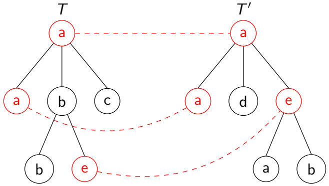

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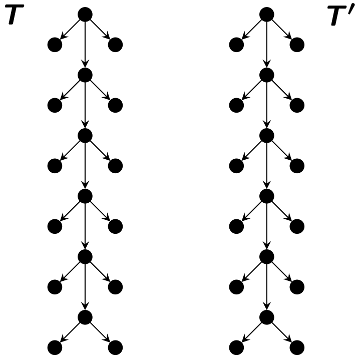
$\mathrm{sim}(T, T') = \sum_{v \in T} \delta(v, \varepsilon) + \sum_{v' \in T'} \delta(\varepsilon, v') - \mathrm{ted}(T, T')$.

TED on Caterpillar Trees I

Let us start by computing the similarity between two caterpillar trees...



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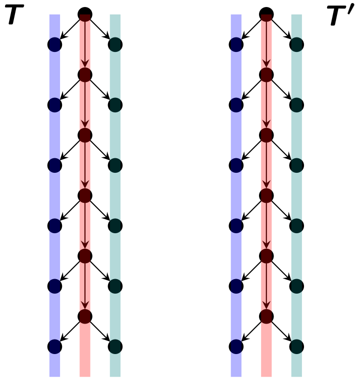


spine nodes

left nodes

right nodes

TED on Caterpillar Trees I

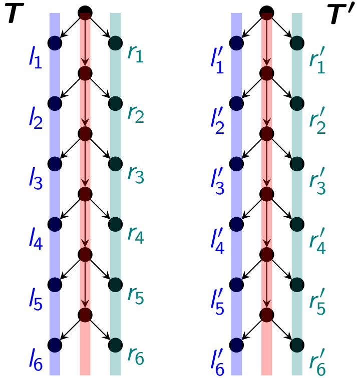
Let us start by computing the similarity between two caterpillar trees...

$L = l_1 l_2 l_3 l_4 l_5 l_6$

$R = r_1 r_2 r_3 r_4 r_5 r_6$

$L' = l'_1 l'_2 l'_3 l'_4 l'_5 l'_6$

$R' = r'_1 r'_2 r'_3 r'_4 r'_5 r'_6$



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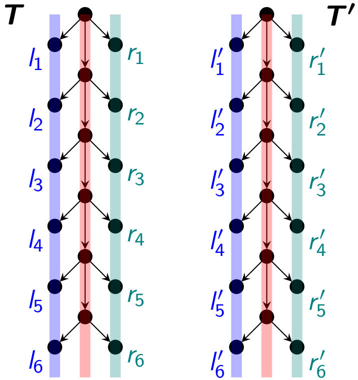


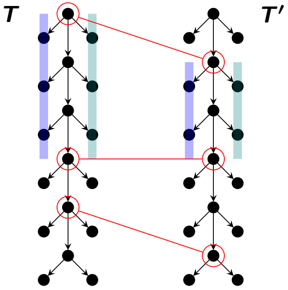
spine nodes

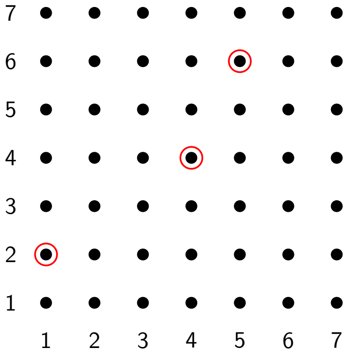
left nodes

right nodes

...with the assumption that spine, left and right nodes of T only match with nodes of their same type (color) in T', respectively.

TED on Caterpillar Trees II





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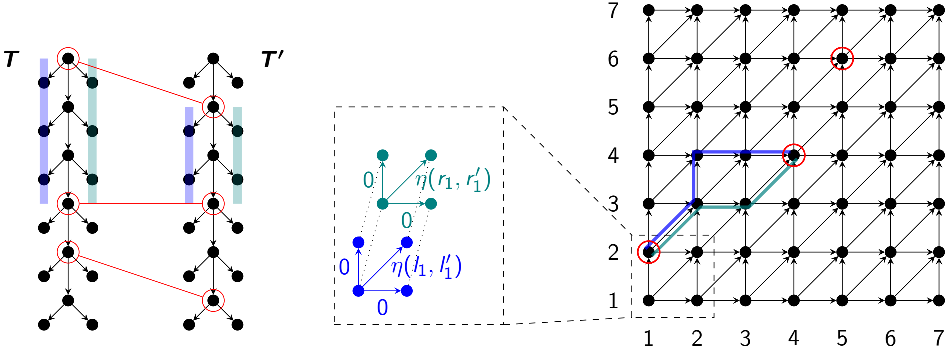

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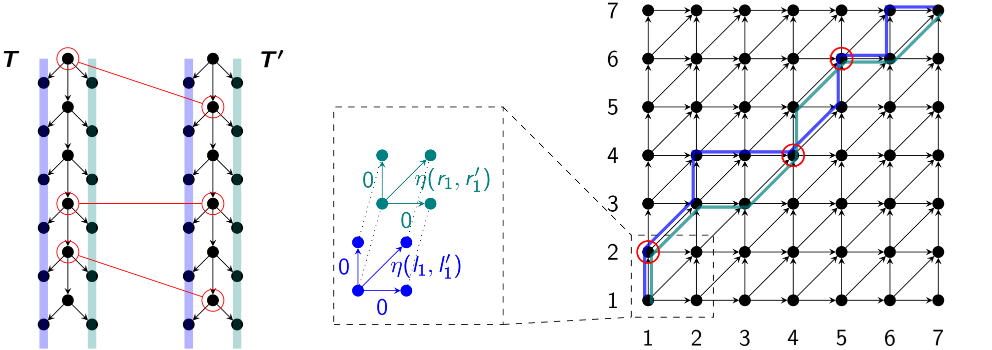

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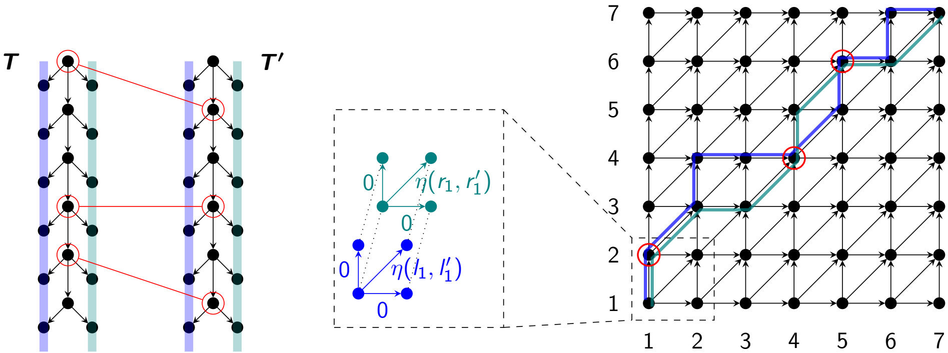
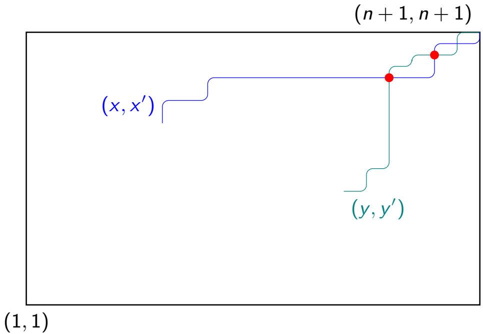


sim(T, T') equals to the maximum achievable sum of:

1. the weight of a path from $(1, 1)$ to $(n+1, n+1)$ in the alignment graph of sim(L, L');
2. the weight of a path from $(1, 1)$ to $(n+1, n+1)$ in the alignment graph of sim(R, R'); and
3. values $\eta(c_i, c'_{i'})$ for (i, i') where the two paths intersect (each c_i and $c'_{i'}$ appears at most once).

A Divide and Conquer Scheme for TED on Caterpillar Trees I



$\mathrm{sim}((x, x'), (y, y'))$ equals to the maximum achievable sum of:

1. the weight of a path from (x, x') to $(n+1, n+1)$ in the alignment graph of $\mathrm{sim}(L, L')$;
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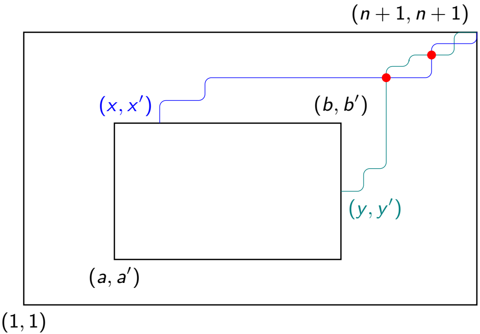

Divide et Conquer Scheme

Input:

- The lower-left corner (a, a') and upper-right corner (b, b') of a rectangle.

Output:

sim$((x, x'), (y, y'))$ equals to the maximum achievable sum of:

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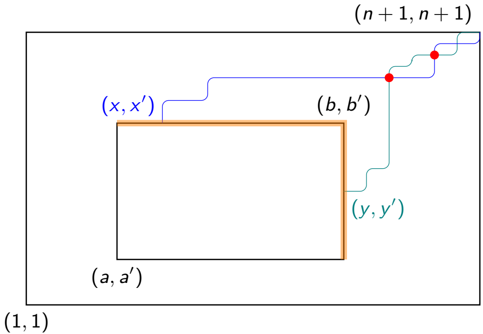

Divide et Conquer Scheme

Input:

- The lower-left corner (a, a') and upper-right corner (b, b') of a rectangle.
- $\mathrm{sim}((x, x'), (y, y'))$ $\forall (x, x'), (y, y') \in ([\,a \,..\, b\,] \times \{b'\}) \cup (\{b\} \times [\,a' \,..\, b'\,])$.

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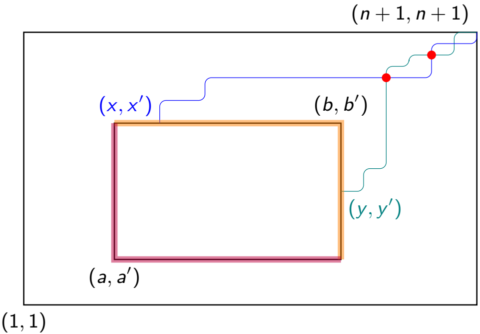

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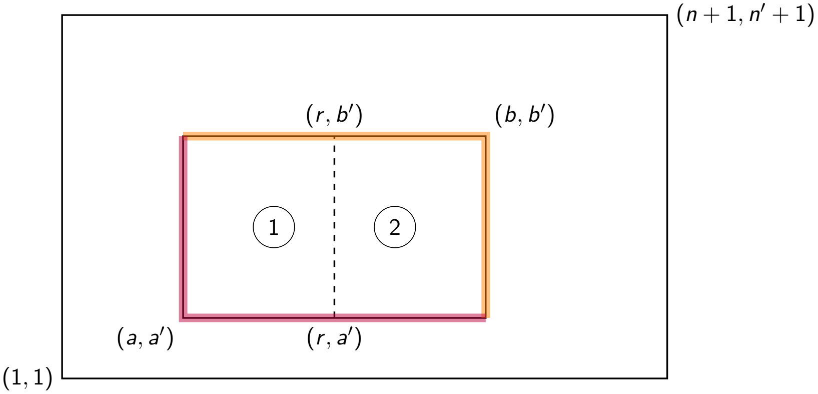

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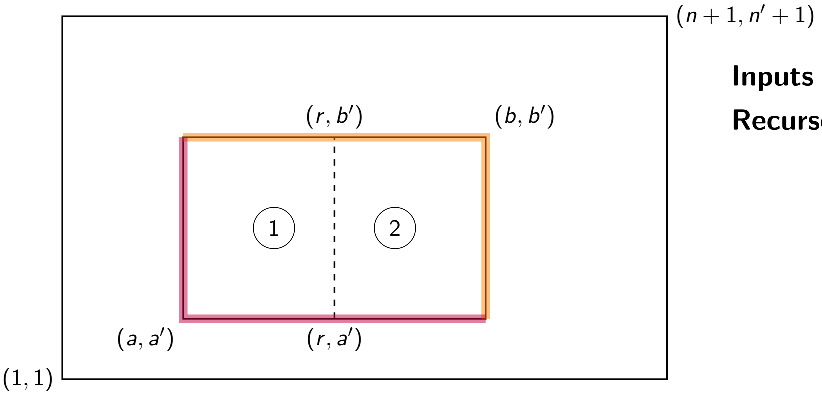

Inputs of subrectangle 2. ✔

Recurse on subrectangle 2. ✔

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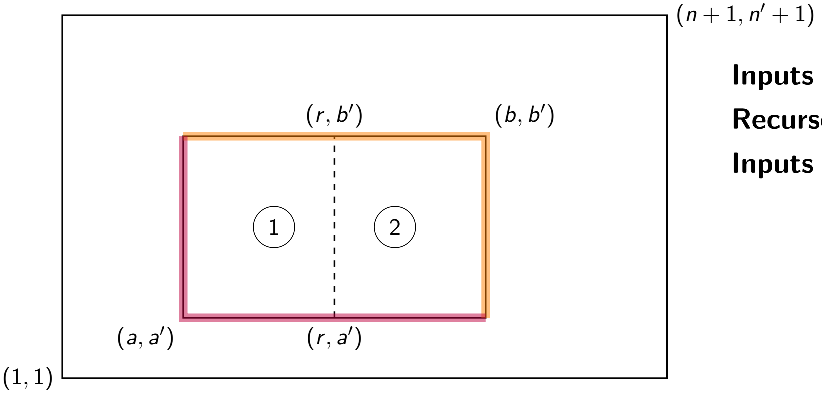

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Inputs of subrectangle 1.

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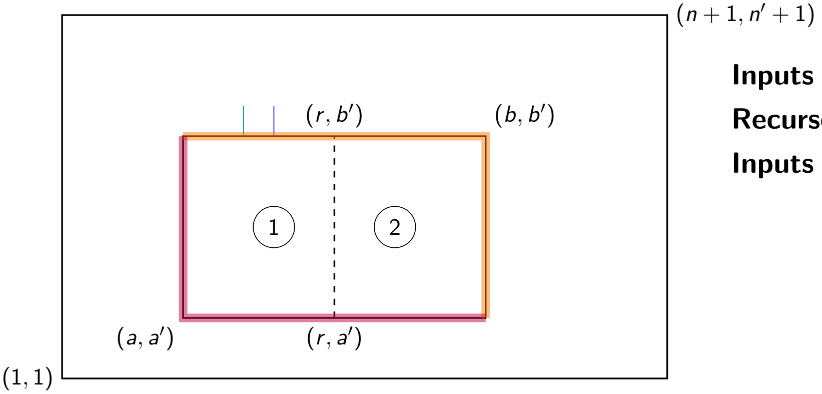

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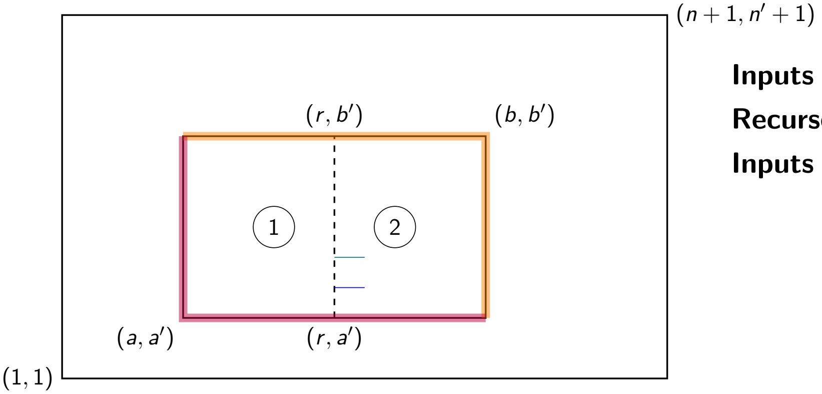

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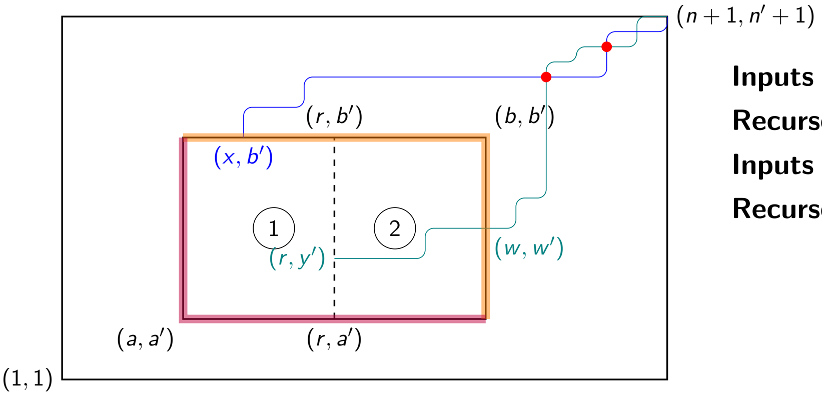

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Recurse on subrectangle 2. ✔

Inputs of subrectangle 1. ✔

Recurse on subrectangle 1. ✔

$$\mathrm{sim}((x, b'), (r, y')) = \max_{(w,w')} \left\{ \mathrm{sim}(R[\, r \mathinner{..} w \,], R'[\, y' \mathinner{..} w' \,]) + \mathrm{sim}((x, b'), (w, w')) \right\}.$$

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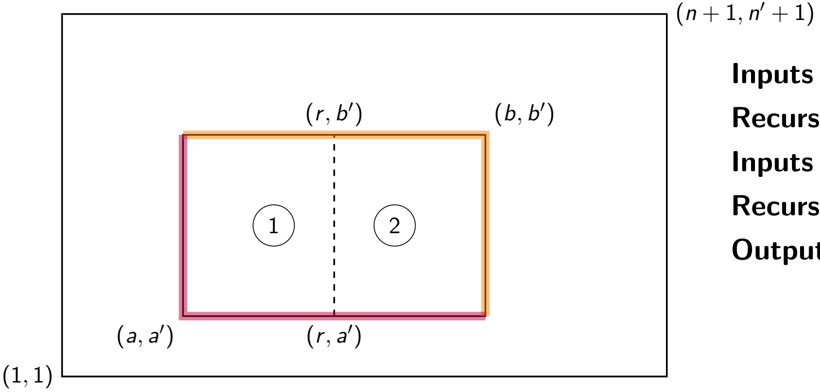


Inputs of subrectangle 2. ✔

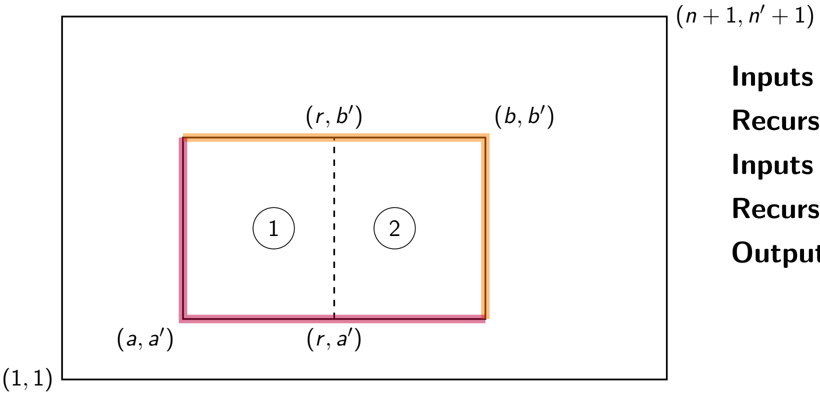
Recurse on subrectangle 2. ✔

Inputs of subrectangle 1.✔

Recurse on subrectangle 1. ✔

Outputs of big rectangle.

A Divide and Conquer Scheme for TED on Caterpillar Trees II



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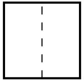
Outputs of big rectangle. ✔

A Divide and Conquer Scheme for TED on Caterpillar Trees III

Since we can handle vertical “cuts”, we can also handle horizontal “cuts”.



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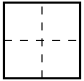

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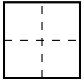

We obtain the recurrence

$$T(n) = 4T(n/2) + \mathcal{O}(T_{\text{APSP}}).$$

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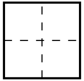

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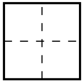

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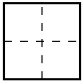

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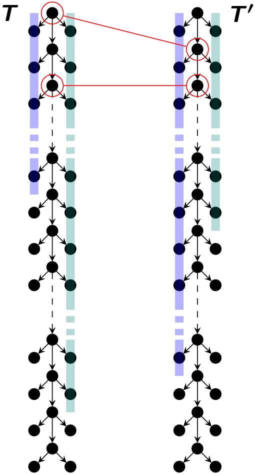
I cheated a bit... in the scheme I need to remember whether spines nodes are already mapped.

Q: How to drop the assumption that spines, left and right nodes map only nodes of the same type?

TED on Caterpillar Trees: Dropping the Assumption



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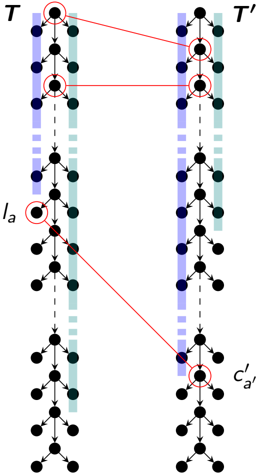

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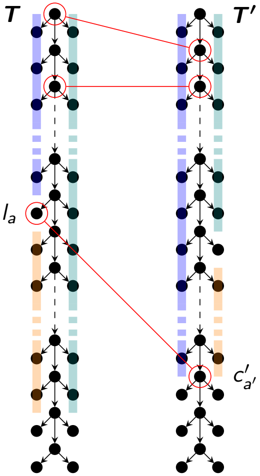

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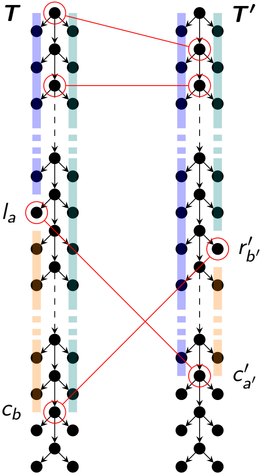

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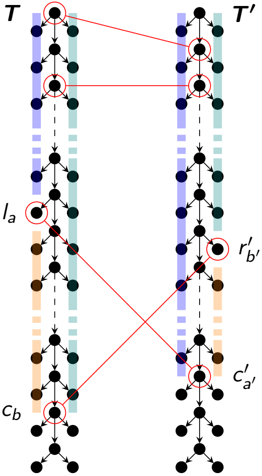

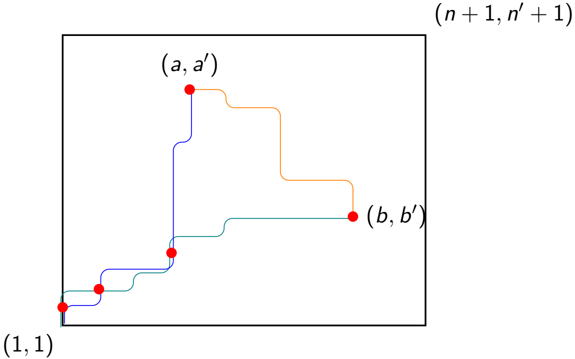

From Caterpillar Trees to Arbitrary Trees

We generalize TED on caterpillar trees to Spine Edit Distance.

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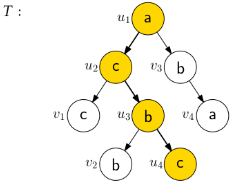
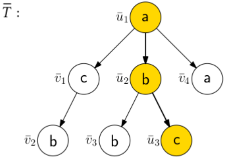
Input: trees T, T', root-to-leaf paths $S \subseteq T, S' \subseteq T'$, and $\mathrm{sim}(\mathrm{sub}(v), \mathrm{sub}(v')) \;\; \forall (v, v') \in (T \times T') \setminus (S \times S')$.





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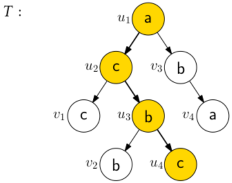

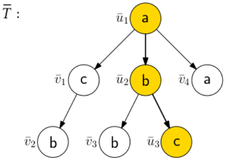

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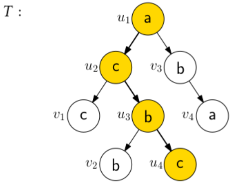

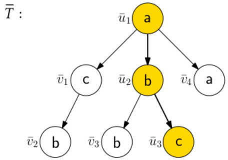

Output: $\mathrm{sim}(\mathrm{sub}(v), \mathrm{sub}(v'))\ \ \forall (v, v') \in S \times S'$.

Why?

- There are still spine, left, and right nodes.
- The underlying paths are not in string alignment graphs anymore but in *forest alignment graphs*.

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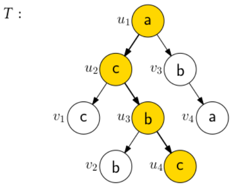
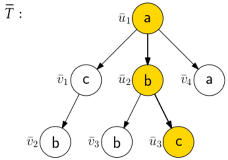
Input: trees T, T', root-to-leaf paths $S \subseteq T, S' \subseteq T'$, and $\mathrm{sim}(\mathrm{sub}(v), \mathrm{sub}(v')) \;\; \forall (v, v') \in (T \times T') \setminus (S \times S')$.





Output: $\mathrm{sim}(\mathrm{sub}(v), \mathrm{sub}(v')) \;\; \forall (v, v') \in S \times S'$.

Why?

- There are still spine, left, and right nodes.
- The underlying paths are not in string alignment graphs anymore but in *forest alignment graphs*.

We also prove that TED on arbitrary trees is fine-grained equivalent to Spine Edit Distance.

Thanks!