Faster Weighted and Unweighted Tree Edit Distance and APSP Equivalence

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"Unweighted" (String) Edit Distance: all costs are one

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$\Downarrow$ Generalization on Trees

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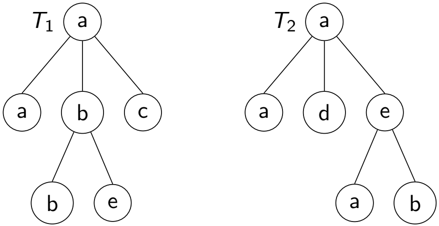

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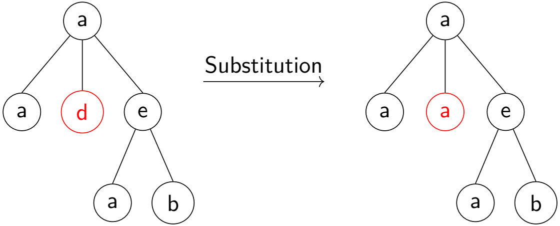

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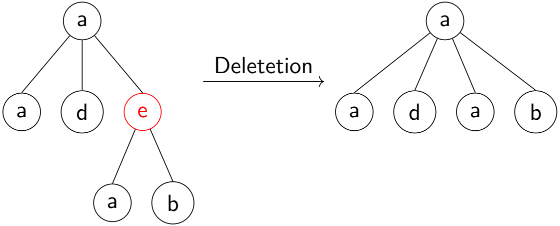

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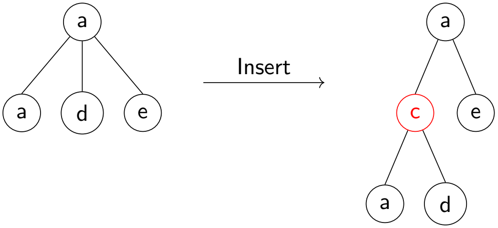

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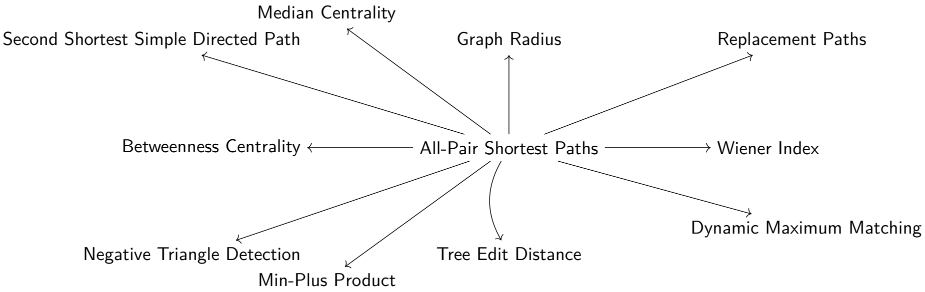

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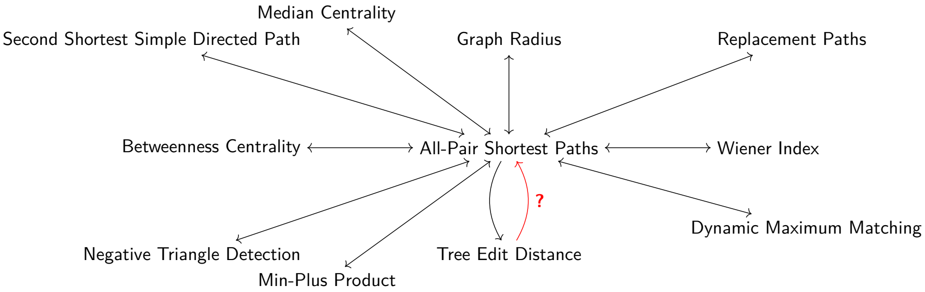

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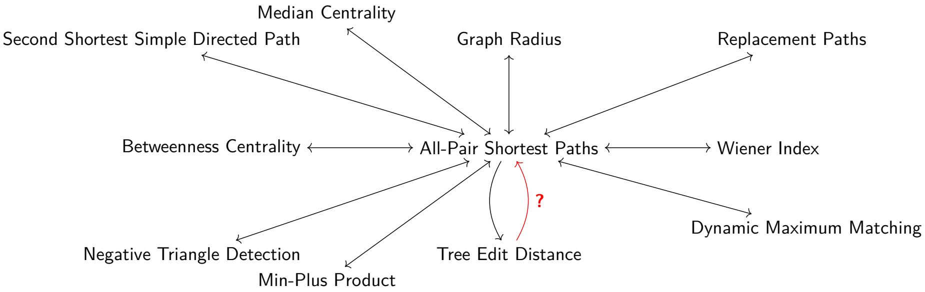

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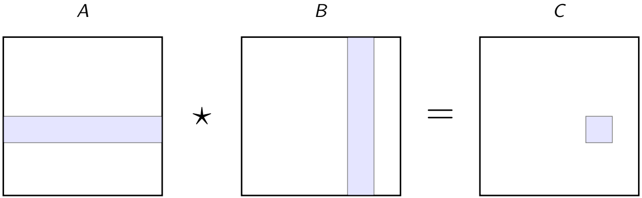
Reducing to Min-Plus Product

Can we reduce TED to computing min-plus product?

Min-plus Product (APSP equivalent)



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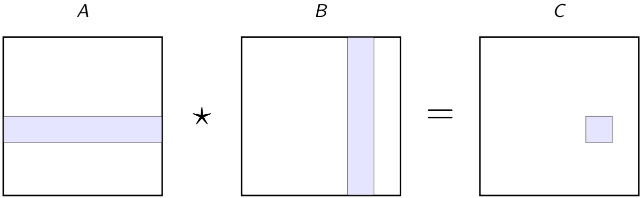

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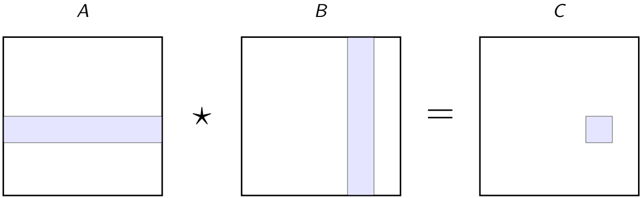

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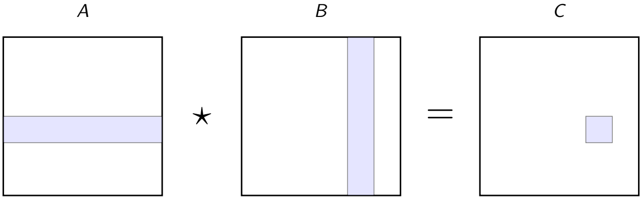

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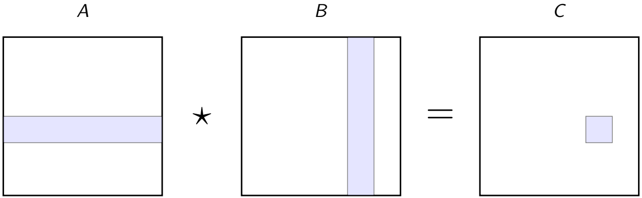

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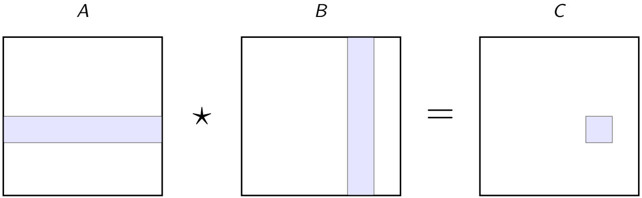

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How to visualize TED to come up with the reduction

Visualization I: Edit Distance as a Matching

We can visualize the least-cost transformation as a matching.

`i n t r o s e m i n a r`

`o` is substituted with `r`

`c h r i s t m a s`

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`m` is left untouched

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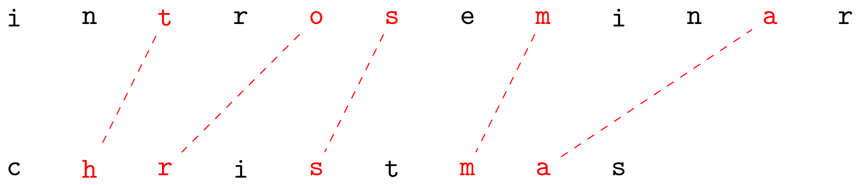

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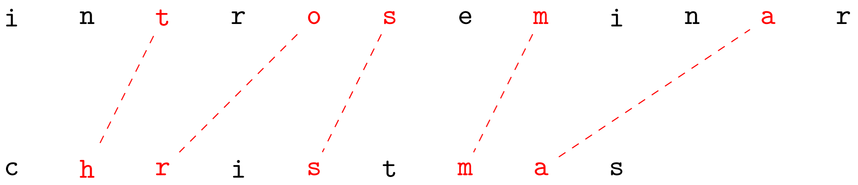
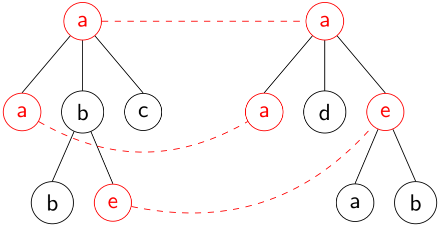
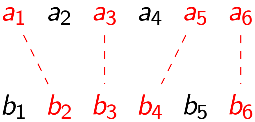
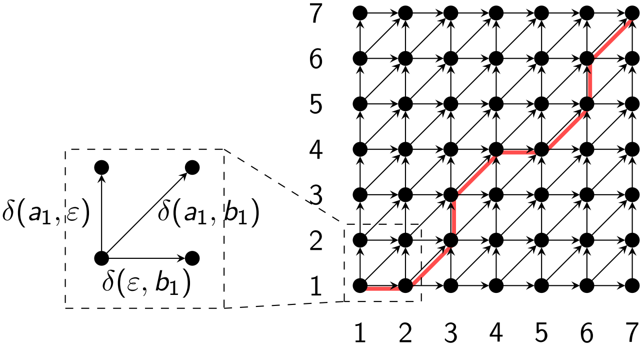




Visualization II: Edit Distance as a Path

The *string alignment graph* summarizes the DP scheme computing the edit distance.





Note: All border-to-border distances can be computed in $\mathcal{O}(n^2)$ time

TED on Caterpillar Trees I

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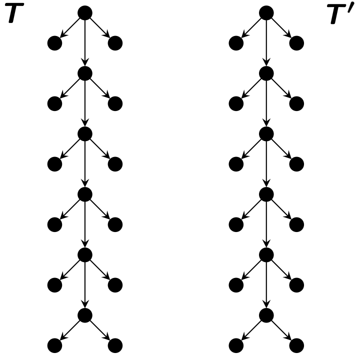

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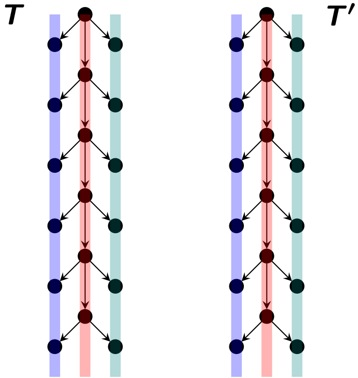

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right nodes

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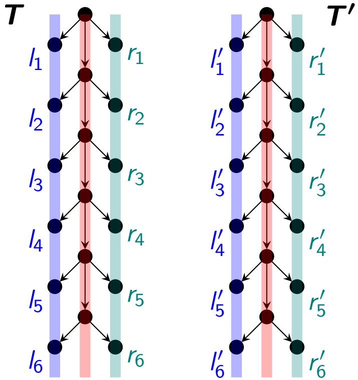

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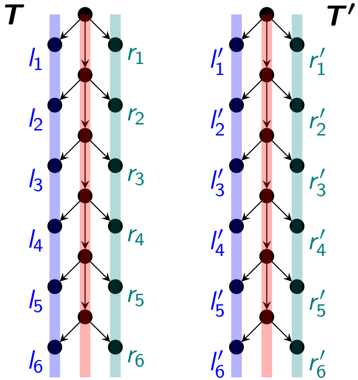

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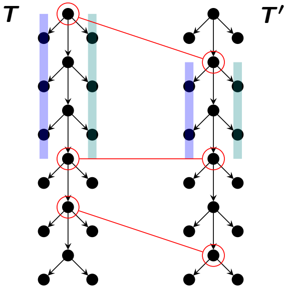
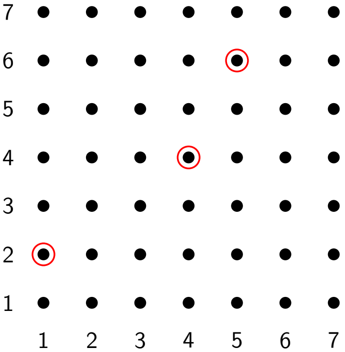
...with the assumption that spine, left and right nodes of T only match with nodes of their same type (color) in T', respectively.

TED on Caterpillar Trees II





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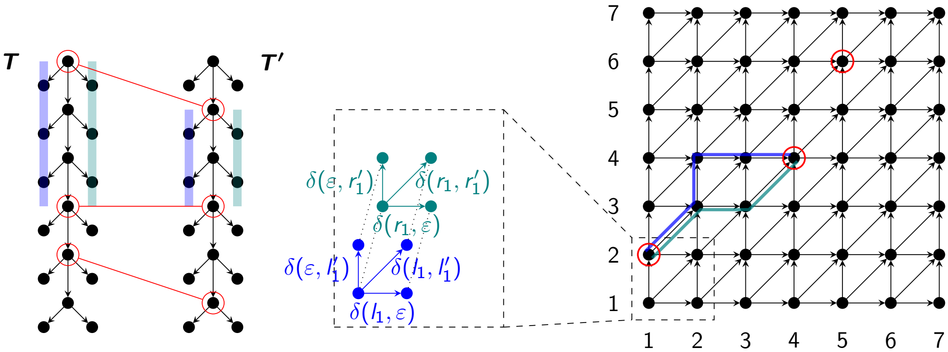
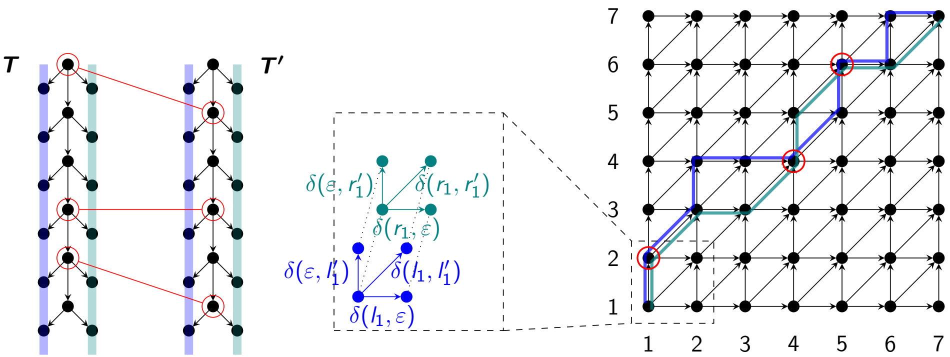
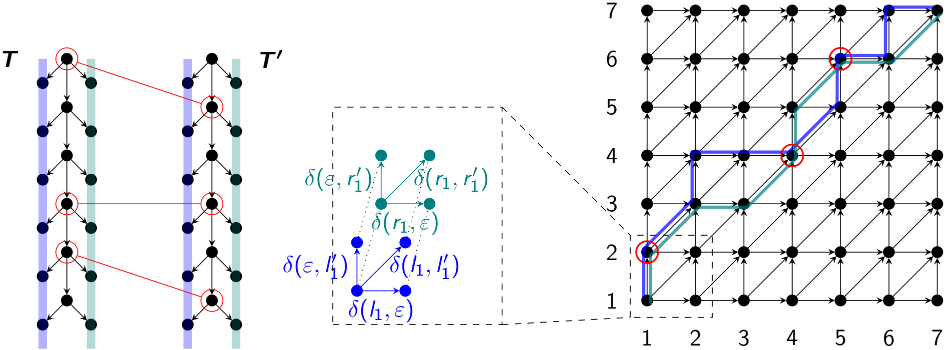


TED on Caterpillar Trees II



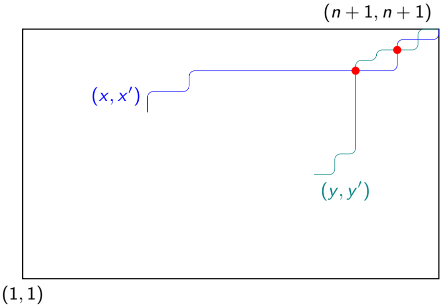
TED on Caterpillar Trees II



ted(T, T') is a pair of **interlaced paths** that minimize three terms:

1. corner-to-corner path in the alignment graph of ed(L, L');
2. corner-to-corner path in the alignment graph of ed(R, R'); and
3. a set of spine-to-spine matchings where the two paths intersect.

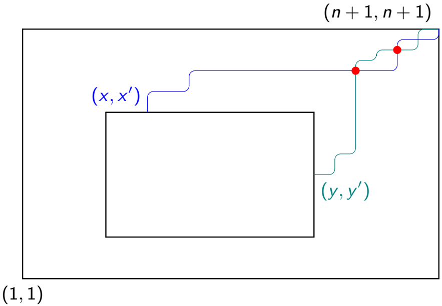
A Divide and Conquer Scheme for TED on Caterpillar Trees



val$((x, x'), (y, y'))$ is a pair of **interlaced paths** that minimize three terms:

1. (x, x')-to-corner path in the alignment graph of ed(L, L');
2. (y, y')-to-corner in the alignment graph of ed(R, R'); and
3. a set of spine-to-spine matchings where the two paths intersect.

A Divide and Conquer Scheme for TED on Caterpillar Trees



Divide and Conquer Scheme

Input:

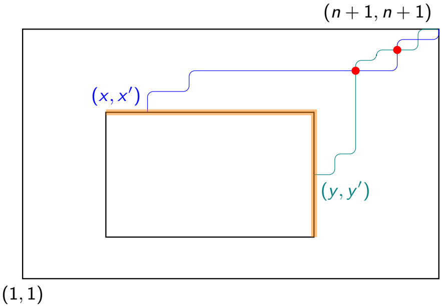
- A rectangle in the grid.

Output:

val$((x, x'), (y, y'))$ is a pair of **interlaced paths** that minimize three terms:

1. (x, x')-to-corner path in the alignment graph of ed(L, L');
2. (y, y')-to-corner in the alignment graph of ed(R, R'); and
3. a set of spine-to-spine matchings where the two paths intersect.

A Divide and Conquer Scheme for TED on Caterpillar Trees



Divide and Conquer Scheme

Input:

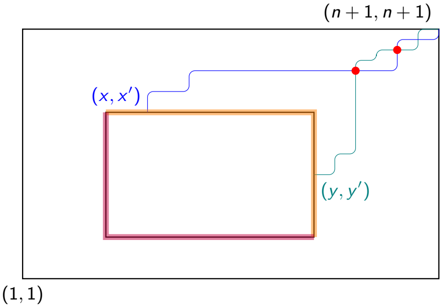
- A rectangle in the grid.
- $\mathrm{val}((x, x'), (y, y'))$ w/ $(x, x'), (y, y')$ on upper-right border.

Output:

$\mathrm{val}((x, x'), (y, y'))$ is a pair of **interlaced paths** that minimize three terms:

1. (x, x')-to-corner path in the alignment graph of $\mathrm{ed}(L, L')$;
2. (y, y')-to-corner in the alignment graph of $\mathrm{ed}(R, R')$; and
3. a set of spine-to-spine matchings where the two paths intersect.

A Divide and Conquer Scheme for TED on Caterpillar Trees



Divide and Conquer Scheme

Input:

- A rectangle in the grid.
- $\mathrm{val}((x, x'), (y, y'))$ w/ $(x, x'), (y, y')$ on upper-right border.

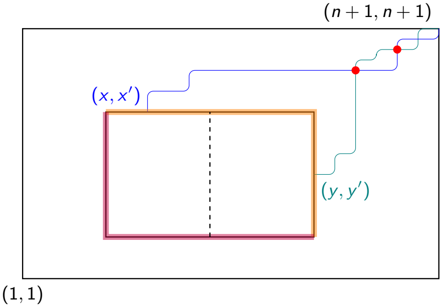
Output:

- $\mathrm{val}((x, x'), (y, y'))$ w/ $(x, x'), (y, y')$ on lower-left border.

$\mathrm{val}((x, x'), (y, y'))$ is a pair of **interlaced paths** that minimize three terms:

1. (x, x')-to-corner path in the alignment graph of $\mathrm{ed}(L, L')$;
2. (y, y')-to-corner in the alignment graph of $\mathrm{ed}(R, R')$; and
3. a set of spine-to-spine matchings where the two paths intersect.

A Divide and Conquer Scheme for TED on Caterpillar Trees



Divide and Conquer Scheme

Input:

- A rectangle in the grid.
- $\mathrm{val}((x,x'),(y,y'))$ w/ $(x,x'),(y,y')$ on upper-right border.

Output:

- $\mathrm{val}((x,x'),(y,y'))$ w/ $(x,x'),(y,y')$ on lower-left border.

$\mathrm{val}((x,x'),(y,y'))$ is a pair of **interlaced paths** that minimize three terms:

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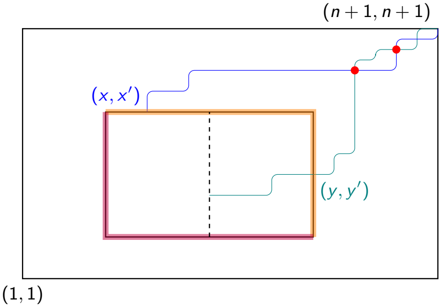
A Divide and Conquer Scheme for TED on Caterpillar Trees



Divide and Conquer Scheme

Input:

- A rectangle in the grid.
- $\mathrm{val}((x, x'), (y, y'))$ w/ $(x, x'), (y, y')$ on upper-right border.

Output:

- $\mathrm{val}((x, x'), (y, y'))$ w/ $(x, x'), (y, y')$ on lower-left border.

$\mathrm{val}((x, x'), (y, y'))$ is a pair of **interlaced paths** that minimize three terms:

1. (x, x')-to-corner path in the alignment graph of $\mathrm{ed}(L, L')$;
2. (y, y')-to-corner in the alignment graph of $\mathrm{ed}(R, R')$; and
3. a set of spine-to-spine matchings where the two paths intersect.

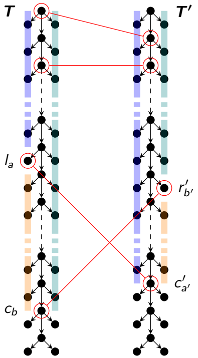
Extending to General Trees

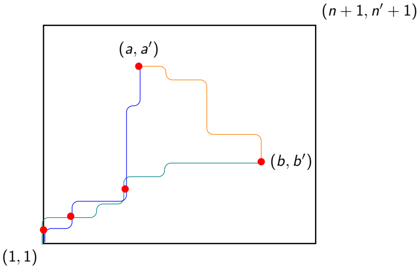
Steps needed to extend to general trees:

Extending to General Trees

Steps needed to extend to general trees:

1. Drop the assumption on caterpillars that left matches w/ left, right w/ right and spine w/ spine.





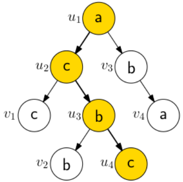
Extending to General Trees

Steps needed to extend to general trees:

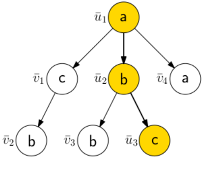
1. Drop the assumption on caterpillars that left matches w/ left, right w/ right and spine w/ spine.
2. Generalize TED on caterpillar to spine edit distance on general trees.

Input: trees T, T', root-to-leaf paths $S \subseteq T, S' \subseteq T'$, and $\text{ted}(\text{sub}(v), \text{sub}(v')) \;\; \forall (v, v') \in (T \times T') \setminus (S \times S')$.

T :



$\bar{T}$:



Images from **[BGHS19]**

Output: $\text{ted}(\text{sub}(v), \text{sub}(v')) \;\; \forall (v, v') \in S \times S'$.

Extending to General Trees

Steps needed to extend to general trees:

1. Drop the assumption on caterpillars that left matches w/ left, right w/ right and spine w/ spine.
2. Generalize TED on caterpillar to spine edit distance on general trees.
3. Devise algorithm computing border-to-border distances in forest alignment graphs in APSP time.

Thanks!