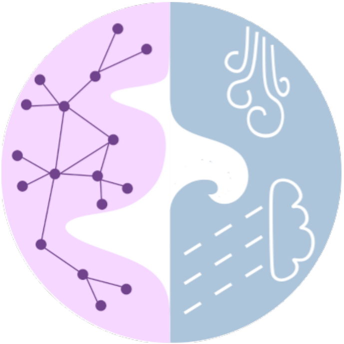
Introduction to Neural Networks



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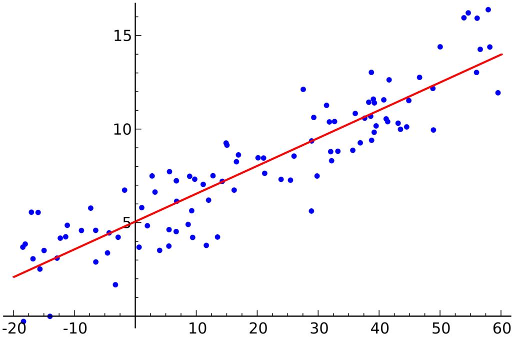
Linear Regression

As always ... we start with **linear regression**:

$$\hat{y}(\mathbf{w}, x) = \sum_{j=1}^{M} w_j x_j + w_0$$



Linear Regression

Fitting a linear model to observations $\{y(x_1), \dots, y(x_N)\}$ by

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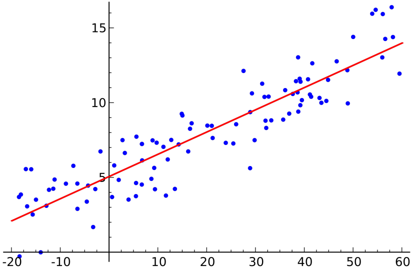

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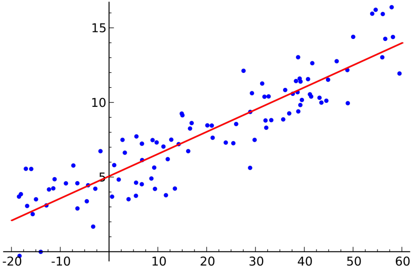
2. Pick loss function, e.g. MSE
$$L(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} \left(\hat{y}\left(x_n, \mathbf{w}\right) - y_o(x_n)\right)^2$$



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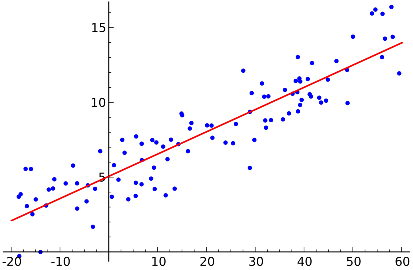
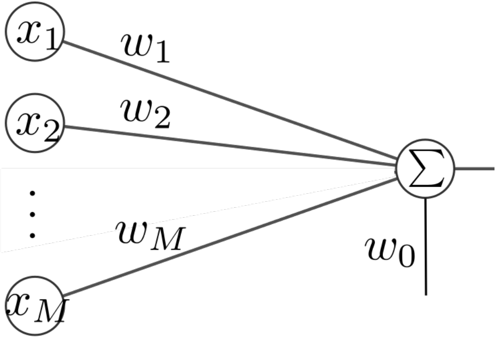
3. Minimize loss function wrt. to parameters $\mathbf{w}$,
$$\arg\min_{\mathbf{w}} L(\mathbf{w}).$$
For linear regression an analytic solution exists:
$$\frac{\partial L}{\partial \mathbf{w}} = \frac{\partial L}{\partial \hat{y}(x,\mathbf{w})} \frac{\partial \hat{y}(x,\mathbf{w})}{\partial \mathbf{w}} \overset{!}{=} 0$$



Linear Regression

Linear regression can be illustrated as a graph:

$$\hat{y}(\mathbf{w}, x) = \sum_{j=1}^{M} w_j x_j + w_0$$

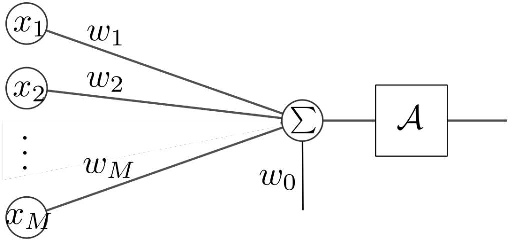


Neural Perceptron

Building block of Fully Connected Neural Networks (FCNN)

- Instead of linear combination add non-linear function

$$y = \mathcal{A}\left(\sum_j x_j \cdot w_j + w_0\right)$$



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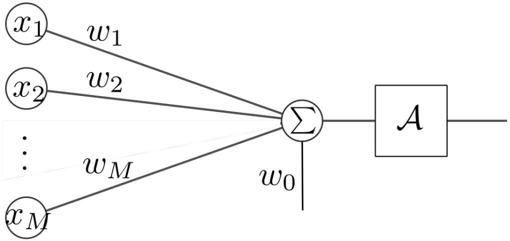
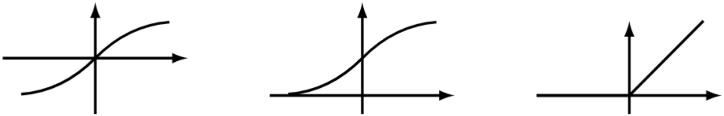


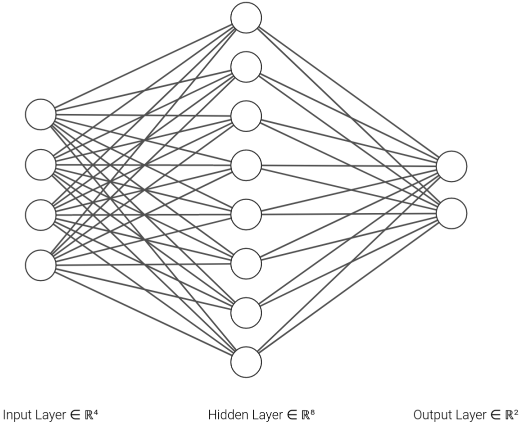
- *f* called the activation function
- Common activation functions:

tanh(x)   sigmoid(x) = $\frac{1}{1+e^{-x}}$ relu(x) = max(0,x)



Multilayer Perceptron

Combining perceptrons into Neural Network.

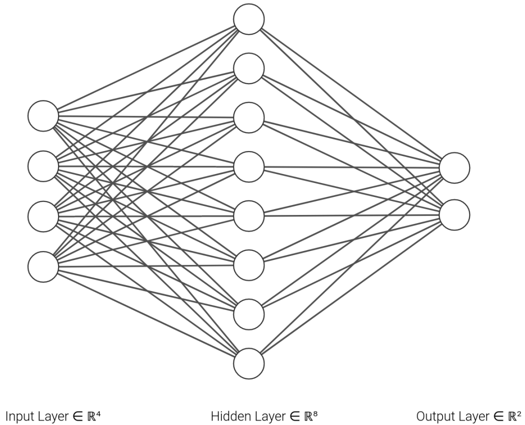


Multilayer Perceptron

Combining perceptrons into Neural Network.

Gain by depths:

- Each layer is a processing step
- Multiple processing step allows to approx any function
- Metaphor: NN and computing



Training a Neural Network

1. Define NN $\hat{y} = f(\mathbf{w}, x)$

2. Pick loss function, e.g. MSE $L(\mathbf{w}) = \frac{1}{N}\sum_{n=1}^{N}(\hat{y}(x_n, \mathbf{w}) - y_o(x_n))^2$

3. Minimize loss function wrt. to parameters **w**, $\arg\min_{\mathbf{w}} L(\mathbf{w})$

   How do we adjust the weights without analytic solution?
   - Gradient descent
   - Back-propagation

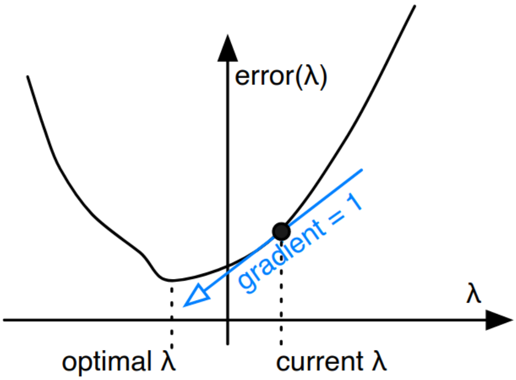
Gradient descent:

- Error is a function of the weights

$$L(\mathbf{w}) \propto \left|\hat{y}\,(x_n, \mathbf{w}) - y_o(x_n)\right|$$

- Compute gradients to move towards error minimum

$$\mathbf{w_{i+1}} = \mathbf{w_i} - \alpha \cdot \frac{\partial L}{\partial \mathbf{w_i}}$$



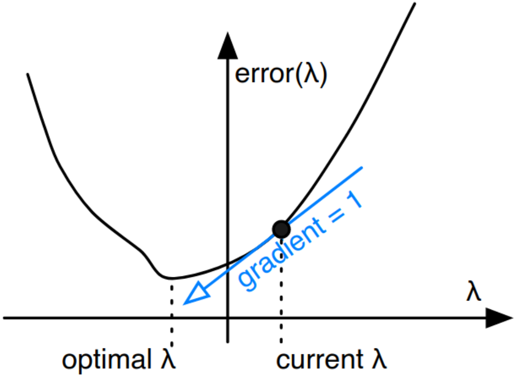
Gradient descent and back-propagation

Gradient descent:

- Error is a function of the weights
  $$L(\mathbf{w}) \propto \left|\hat{y}\,(x_n,\mathbf{w}) - y_o(x_n)\right|$$
- Compute gradients to move towards error minimum
  $$\mathbf{w_{i+1}} = \mathbf{w_i} - \alpha \cdot \frac{\partial L}{\partial \mathbf{w_i}}$$

Back-propagation:

- Adjust weights of last layer first
- Propagate error back to each previous layer
- Adjust weights of hidden layers



Summary

- Neural perceptron: Linear model with non-linear function
- MLPs are simply stacked perceptrons
- Theoretically: MLPs can approximate any function

Hands-on!

jupyter

python™

https://github.com/jakob-schloer/tutorials

Reminder:

$$y\,(h, \mathbf{w}) = \mathcal{A}(\underbrace{\sum_j h_j \cdot w_j}_{s}), \qquad L(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} \left(y(x_n, \mathbf{w}) - y_o(x_n)\right)^2$$

1. Derivative of error of last layer with regard to one weight w_k
$$\frac{dL}{dw_k} = \frac{dL}{dy}\frac{dy}{ds}\frac{ds}{dw_k} = (y - y_o)y' h_k$$
2. Hidden layer error
$$\delta = (y - y_o)y'$$
3. Update weights $\qquad w_k^{(i+1)} = w_k^{(i)} - \alpha\,\delta\,h_k$