The Ant System: Optimization by a colony of cooperating agents

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Travelling Salesman problem

Formulation:

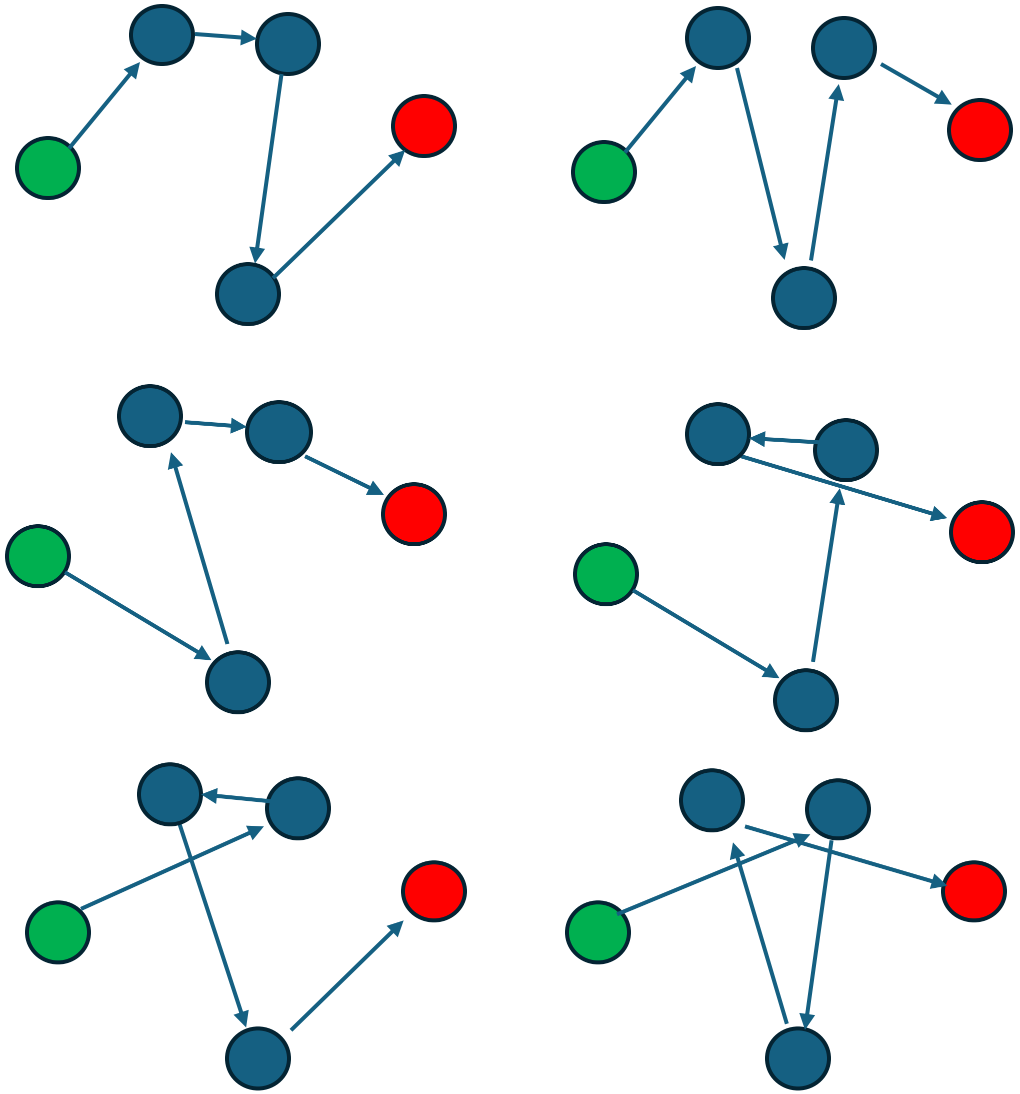
- You have N points with spatial coordinates.
- You have to visit each point exactly once.

Problem statement:

- What is the shortest path?

Implications:

- Delivery (Amazon, Wolt, etc.)

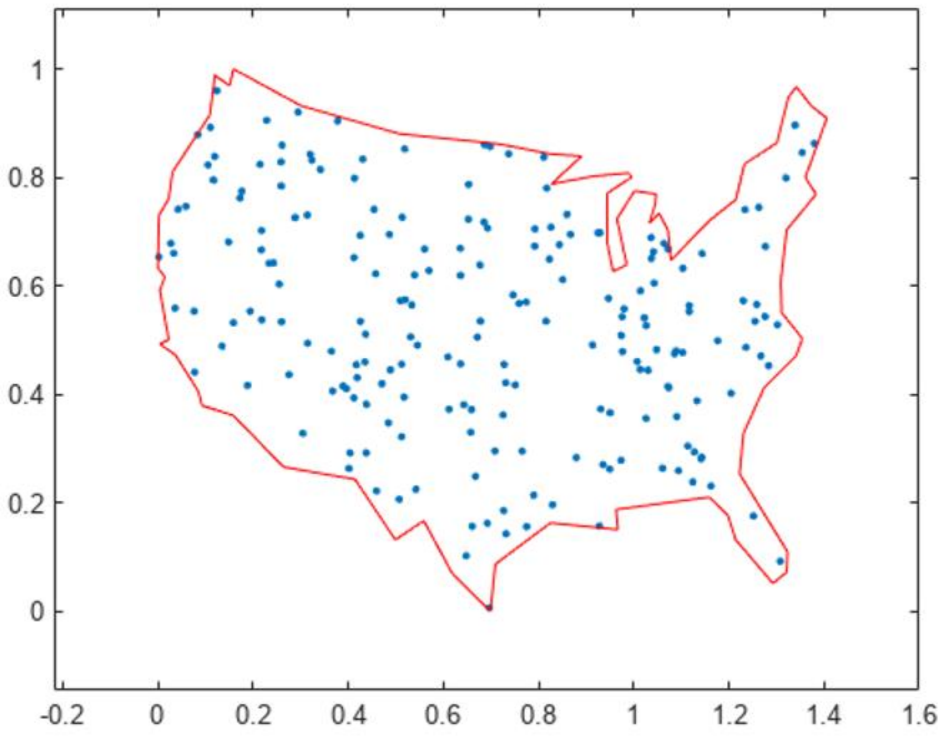


Computational Cost

The computational cost C:

$$C = O(N!)$$

Makes the brute force method computationally infeasible for small systems.



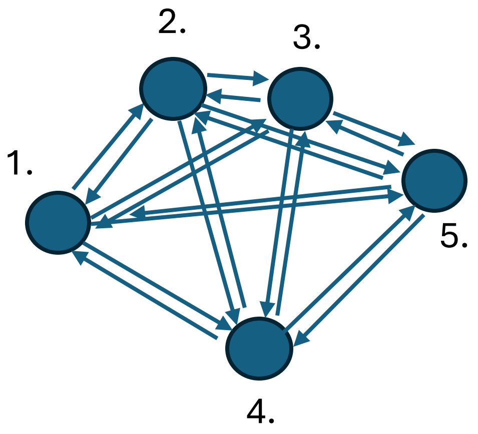
[Traveling Salesman Problem: Solver-Based - MATLAB & Simulink - MathWorks India]

Travelling Salesman Graph Abstraction

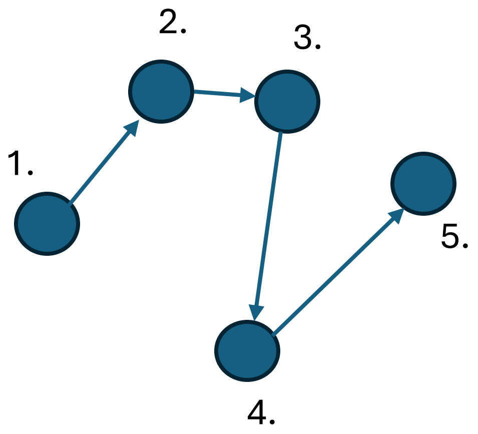
We abstract:

- each coordinate point to a node/vertex.
- each connection from a point to another as a link/edge.

We represent this in an Adjacency Matrix.



$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

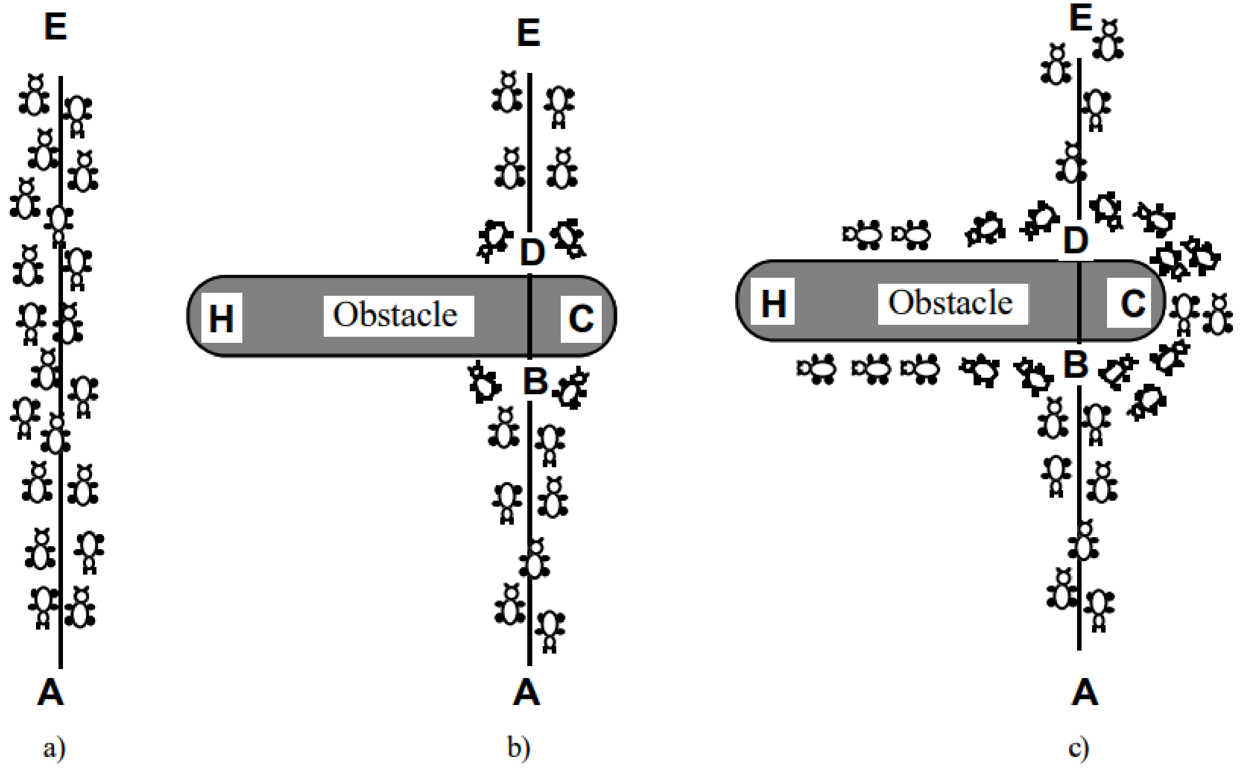


$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Ant Colony Optimization Motivation

Assumptions:

- The artificial ants will have some memory.
- The artificial ants will not be completely blind.
- The artificial ants live in a system where time is discrete.



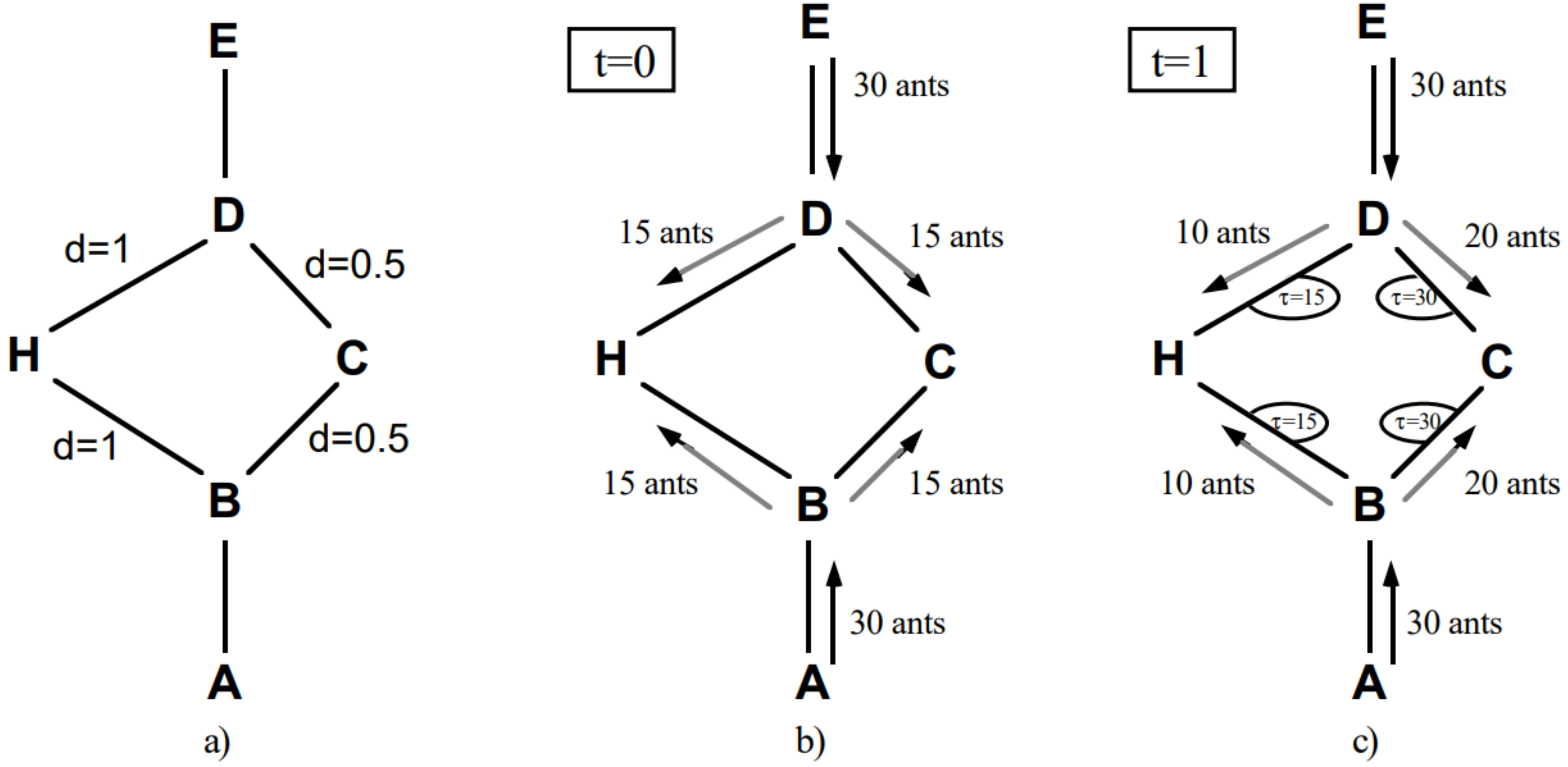
How do the ants find the shortest path?

a. Ants want to go from A to E.

b. At t=0, ants have no information, and will walk randomly.

c. At t=1, more ants will walk towards C, as more pheromone has been placed.

This process continues until all ants follow the same path, and then, the algorithm has converged.



Mathematical Formulation

1. Let,

$$d_{ij}=[(x_i-x_j)^2 + (y_i-y_j)^2]^{1/2})$$

i.e. the adjacency matrix of the fully connected graph with edges representing the distance from the connected points.

2. Let,

$$\tau_{ij}(t+n)=\rho\cdot\tau_{ij}(t)+\Delta\tau_{ij}$$

i.e. the intensity of pheromones on edge (i,j) at time t. Where ρ, is the evaporation rate of the pheromones.

3. Let,

$$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

i.e. the per unit length amount of pheromone on edge (i,j), be the per unit length sum of all of the pheromones added on edge (i,j) by all 'm' ants, stated mathematically, where Q constant:

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{Q}{L_k} & \text{if k - th ant uses edge } (i, j) \text{ in its tour (between time t and t + n)} \\ 0 & \text{otherwise} \end{cases}$$

Mathematical Formulation

Finally, let

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}]^{\beta}}{\sum\limits_{k \in allowed_k} [\tau_{ik}(t)]^{\alpha} \cdot [\eta_{ik}]^{\beta}} & \text{if } j \in allowed_k \\ 0 & \text{otherwise} \end{cases}$$

where $p_{ij}^{k}(t)$ is the transition probability of ant k at time t, to go from node i to node j (Markov Chain), and η_{ij} is the visibility of node j from node i – i.e. the reciprocal of the distance $1/d_{ij}$.

The effect of pheromones: null-hypothesis

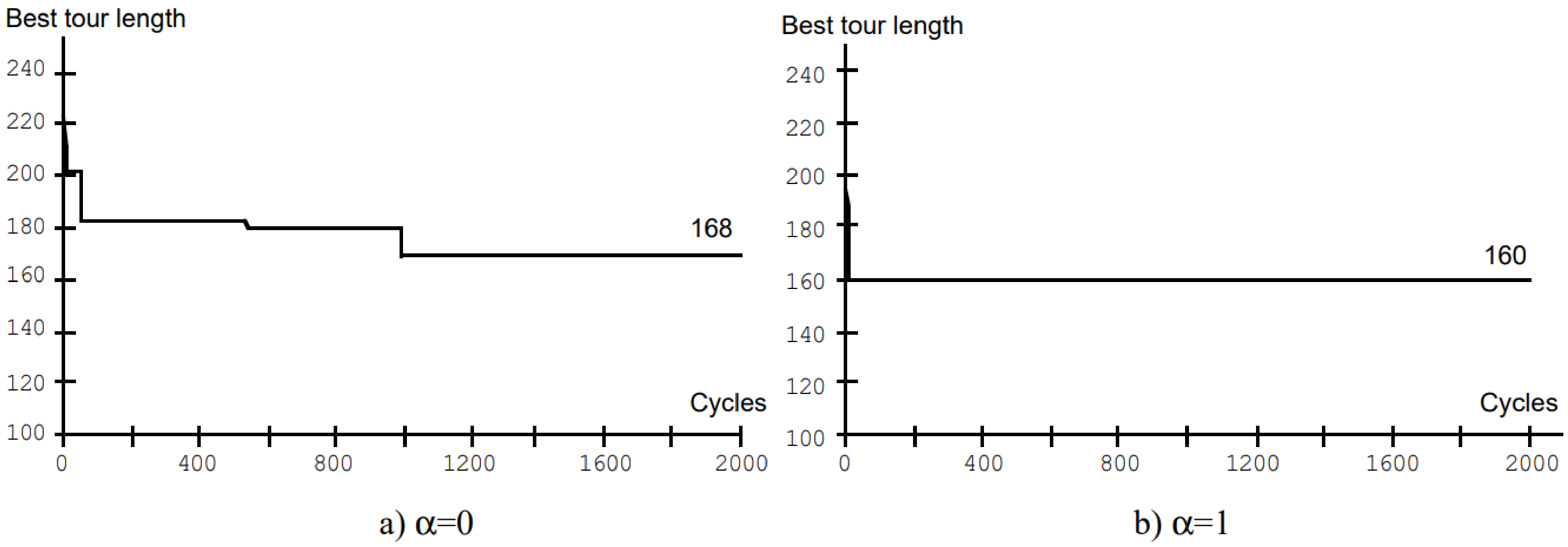


Fig. 12. Synergy: Communication among ants (α>0) improves performance. In (a) α=0, in (b) α=1.

Ant Colony Optimization Results

Best parameter set	Average result	Best result
α=1, β=5, ρ=0.99	426.740	424.635
α=1, β=5, ρ=0.99	427.315	426.255
α=1, β=5, ρ=0.5	424.250	423.741

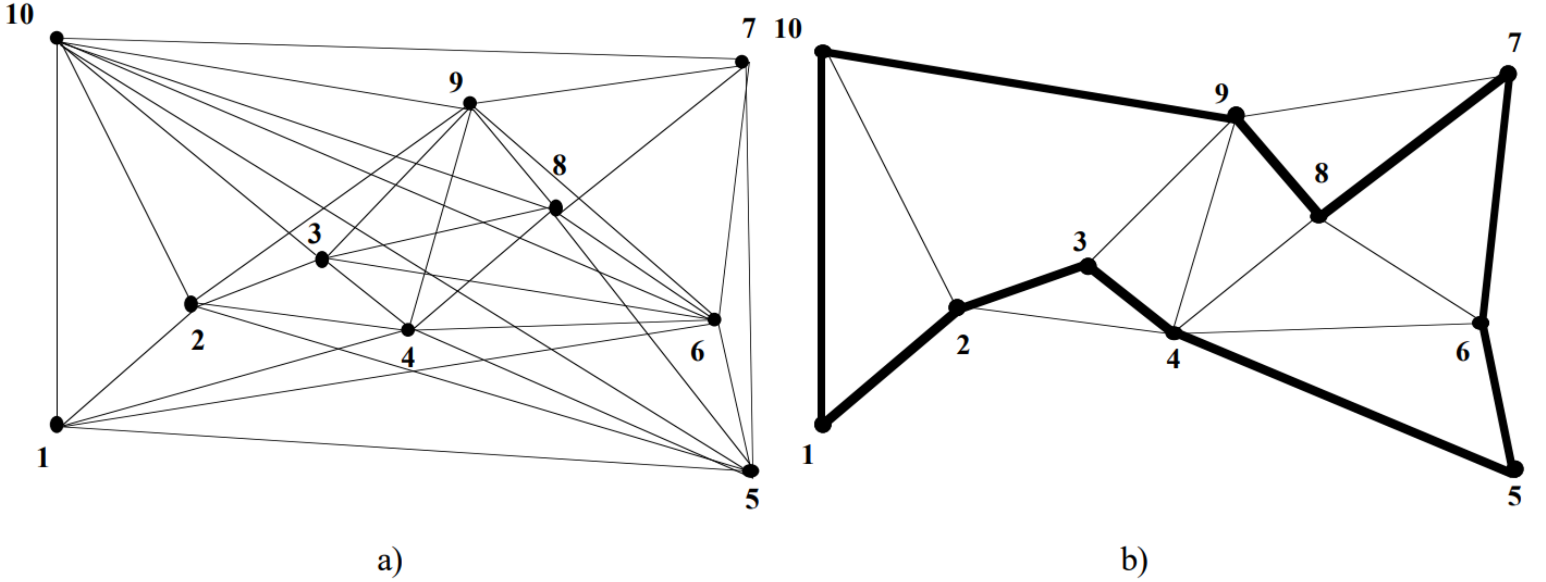


Fig. 6. Evolution of trail distribution for the CCA0 problem.
a) Trail distribution at the beginning of search.
b) Trail distribution after 100 cycles.

Oliver-30

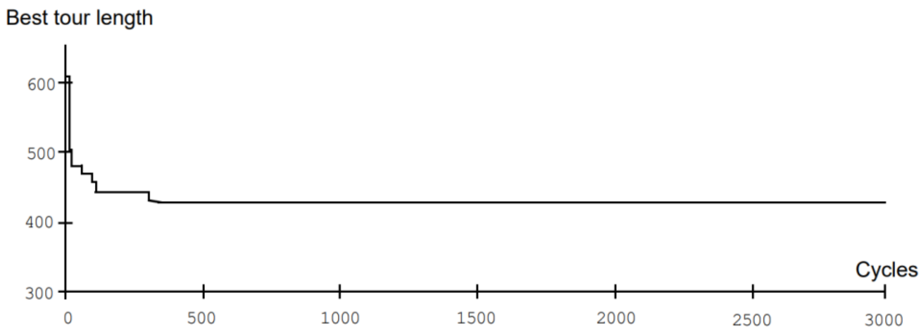


Fig. 3. Evolution of best tour length (Oliver30). Typical run.

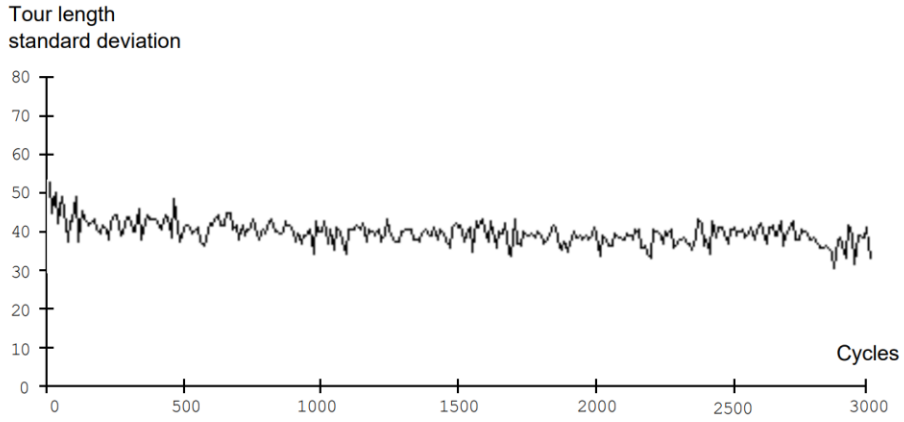


Fig. 4. Evolution of the standard deviation of the population's tour lengths (Oliver30). Typical run.

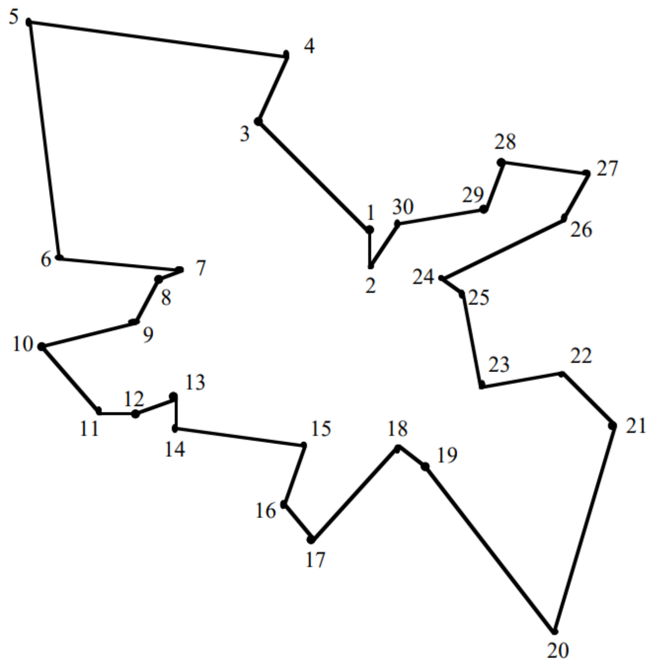


Fig. 9. The best tour obtained with 342 cycles of the *ant-cycle* algorithm for the Oliver30 problem (α=1, β=5, ρ=0.5, Q=100), real length = 423.741, integer length = 420.

Conclusion

Table V. Comparison of the AS with other heuristic approaches. Results are averaged over five runs. Best known results are in bold.

	Nugent (15)	Nugent (20)	Nugent (30)	Elshafei (19)	Krarup (30)
Best known	**1150**	**2570**	**6124**	**17212548**	**88900**
Ant System (AS)	**1150**	2598	6232	18122850	92490
AS with non deterministic hill climbing	**1150**	**2570**	6128	**17212548**	**88900**
Simulated Annealing	**1150**	**2570**	6128	17937024	89800
Tabu Search	**1150**	**2570**	**6124**	**17212548**	90090
Genetic Algorithm	1160	2688	6784	17640548	108830
Evolution Strategy	1168	2654	6308	19600212	97880
Sampling & Clustering	**1150**	**2570**	6154	**17212548**	**88900**

- Relatively low computational cost
- Consistent results and state-of-the-art results.
- Generalizeable algorithm – can be applied to:
  - Studies of social animal behaviour.
  - Research in "natural heuristic algorithms".
  - Stochastic Optimization