UNIVERSITY OF COPENHAGEN

Extreme Value Theory In Climate Sciences

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Advanced Methods in Applied Statistics

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March 6th 2025

1. What is Extreme Value Theory

-Tail behaviour of PDF

-Areas of Application:

- -Climate Science
- -Finance
- -Biology
- -Many more

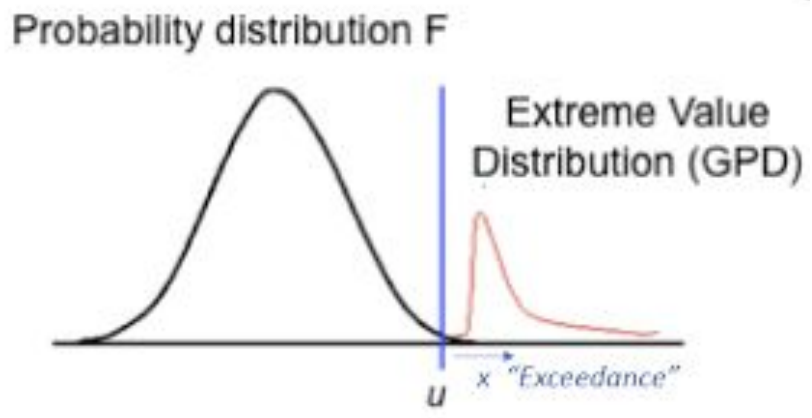


Figure 1: Probability distribution of a gaussian, and extreme value distribution.

1.1 Fischer-Tippet-Gnedenko Theorem

-multiple seqs of i.i.d random variables $(X_1, X_2, \dots X_n)$ with block length **n** drawn from CDF function **F**.

-When $n \rightarrow \infty$ then $M_n = max(X_1, X_2, \dots X_n)$ can also go to infinity. Normalization of max is needed.

-Theorem says: For constants $a_n > 0$ and b_n then if $\frac{M_n - b_n}{a_n}$ converges in F as $n \rightarrow \infty$ to a non degenerate function G(x).

$$\lim_{n \to \infty} P\left(\frac{M_n - b_n}{a_n} \leq x\right) = \lim_{n \to \infty} F^n(a_n x + b_n) = G(x)$$

-Then G(x) is the CDF of a General Extreme Value(GEV) Distribution.

1.2 GEV Distribution

Three parameters :

- **Location** : μ ~ mode (for ξ << 0).
- **Scale** : σ => spread.
- **Shape** : ξ => tail expansion.

PDF :

$$\frac{1}{\sigma}t(x)^{\xi+1}e^{-t(x)}, \tag{2}$$

where

$$t(x) = \begin{cases} 1+\xi\left(\frac{x-\mu}{\sigma}\right)^{-1/\xi} & \text{if } \xi \neq 0, \\ e^{\frac{x-\mu}{\sigma}} & \text{if } \xi = 0. \end{cases}$$

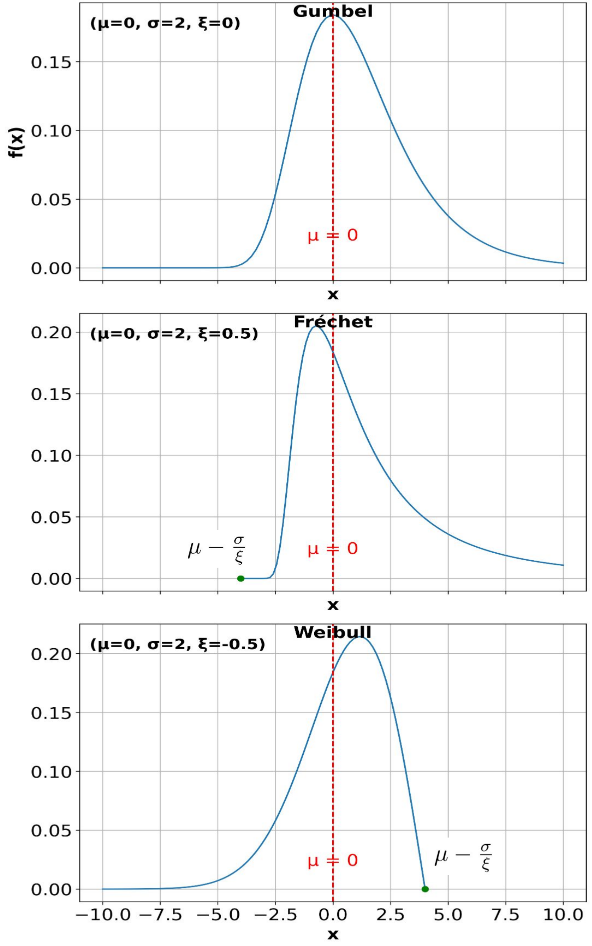


Figure 2: The three families of a GEV distribution

1.3 Return level

-The return level x_r is the value in the GEV distribution that is on average exceeded once per **r** years(because our block length **n** is one year).

-Probability of exceeding x_r a given year given by: $P(x_r < X) = \frac{1}{r}$

- Example r= 50: *1/50 change to exceed* x_r *a given year, you expect to exceed* x_r *once every 50 years.*

-Probability of not exceeding x_r is $P(X \leq x_r) = 1 - 1/r$

-This is the same as looking at the CDF of the GEV: $G(x_r) = 1 - 1/r.$

2. Application to Extreme Temperatures

Article of reference: *Estimating changes in temperature extremes from millennial-scale climate simulations using generalized extreme value (GEV) distributions* by Huang et al. (2016).

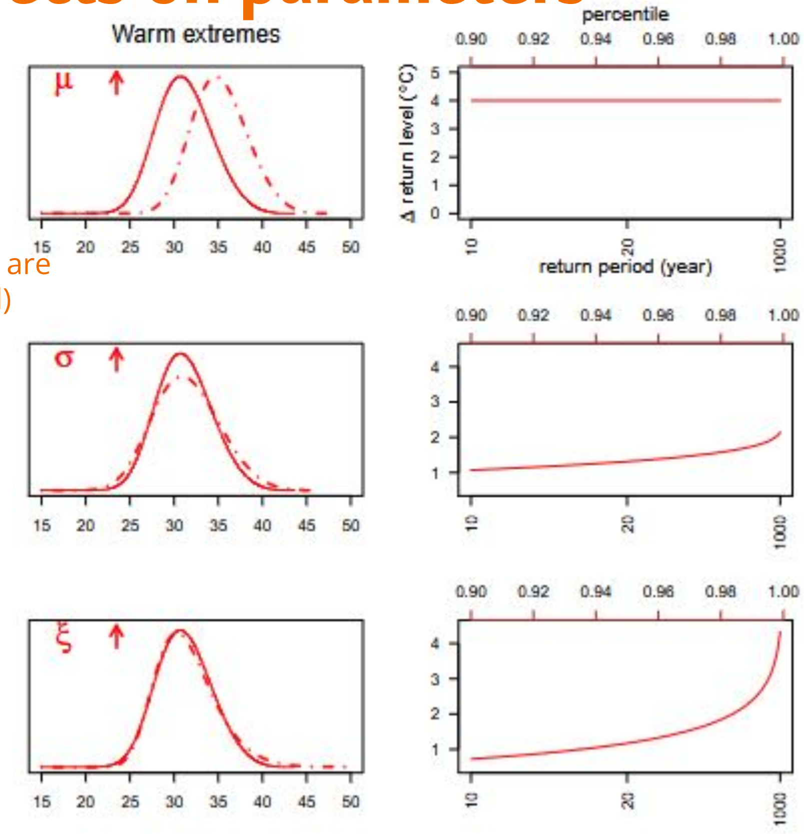
How the parameters (μ,σ,ξ) change in a changing climate (GHG concentrations scenarios) and how it affects the GEV distribution and the return level ?

Relevant assumption:

- **Stationary** condition.
- Extreme = **annual maxima** of daily maximum temperatures and **annual minima** of daily minimum temperatures.

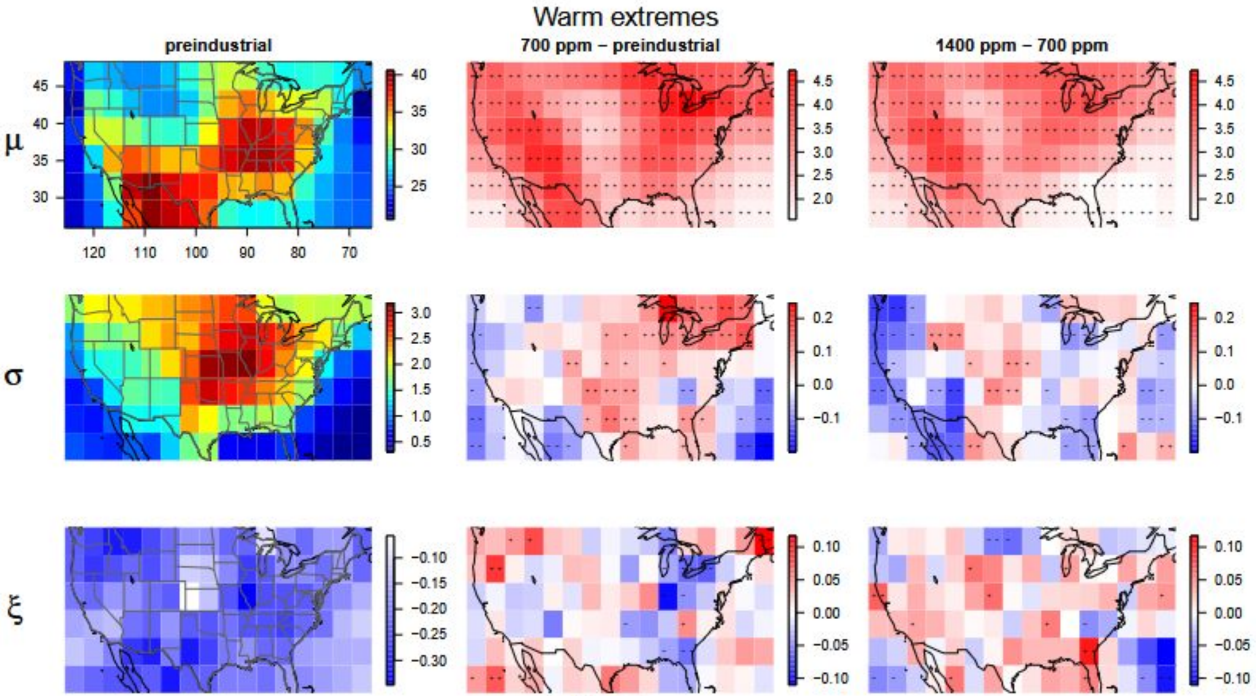
2. 1. Effects on parameters

Weibull GEV fitting
(High temperatures are physically restricted)



Huand et. al (2016)

Figure 3 : Illustration of the effect on return levels of changing individual GEV parameters. We show consequences for warm extremes. Columns 1 and 3 show GEV distributions for baseline (solid) and future (dashed) climates. Columns 2 and 4 show resulting changes in return levels for different return periods. Note that 1000-year periods are on the right for warm and left for cold extremes, to conform with percentiles. Location and shape parameters used here are chosen as representative of our model results, with larger effects in cold extremes than warm extremes, while shape parameter are identical in both cases. Top row: changing the location parameter shifts return levels uniformly across return periods. Middle row: increasing the scale parameter produces effects dependent on return period. All return levels increase (decrease for cold extremes), but more so for longer return periods. Bottom row: increasing the shape parameter produces dramatic increases in return levels at very long return periods.

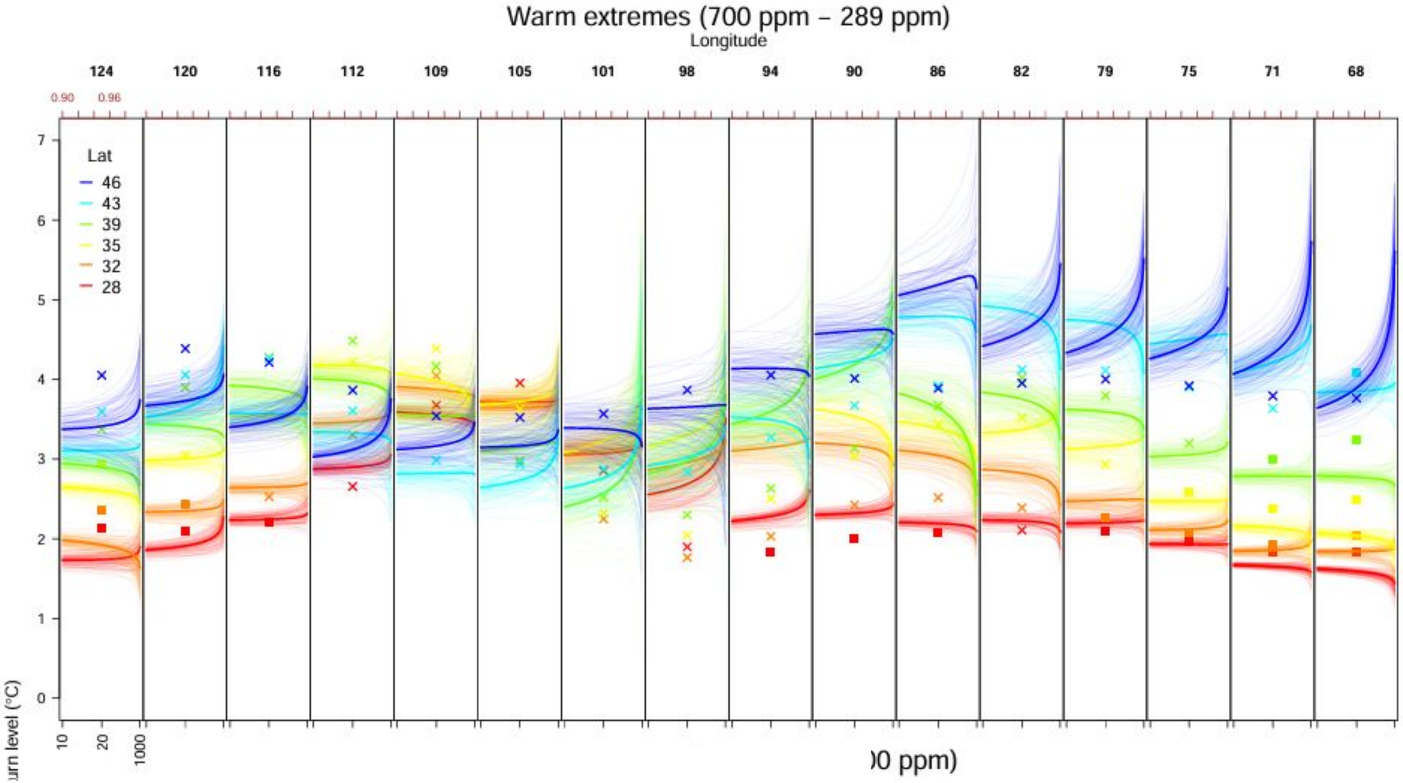


Huand et. al (2016)

<u>Figure 4 :</u> The estimated CCSM3 GEV parameters and their changes in possible future climate conditions. Left: fitted GEV parameters (location μ, scale σ, and shape ξ) for annual extremes for the baseline model run at pre-industrial CO2 concentration. Top 3 panels are for warm extremes and bottom are for cold extremes. Negative ξ is expected for temperature distributions. Middle: changes in parameters (Δμ, Δ log σ, Δξ) after a warming of global mean temperature by 3.4 ◦C (by raising atmospheric CO2 to 700 ppm). Right: changes in parameters after an additional 2.7 ◦C warming (by raising CO2 from 700 to 1400 ppm). The symbols ++ or −− mean the bootstrapped p-value for testing whether the parameter is different in the scenarios is < 0.02; + or − mean the p-value is between 0.02 and 0.10.

2.2 Effects on return level

<u>Figure 5 :</u> Estimated changes in warm extremes return levels for 700 ppm CO2 vs. 289 ppm CO2 climate conditions. Panel shows results from the 6 grid cells in a longitude band, while each color represents a latitude band. Estimated changes in return levels are plotted as solid lines in their corresponding colors. Estimates obtained from block bootstrapped samples by resampling years are plotted as lighter and thinner curves to form envelopes representing the associated uncertainties. The corresponding summertime mean changes for inland and ocean grid cells are marked by cross and filled box symbols, respectively.



Huand et. al (2016)

Conclusion - Considerations

- What is an extreme ?

  Peak over threshold, Centiles.

- Quid of multivariate extremes ?

  As a climate is not defined by only one variable, what is an extreme climate ?

- Stationary conditions assumption

Thanks for listening
Questions ?