Ant Colony Optimization Review: a robust nature based optimizer for graph based problems

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(Dated: March 1, 2024)

I. INTRODUCTION

Nature based optimization methods have been increasing in popularity over the last couple of decades, as the growing computational capacity in recent history has provided these nature based optimization techniques with a realization pathway. Consequently, a few state-of-the-art nature based optimization methods have emerged, - e.g. Genetic Algorithm, Particle Swarm Optimization, and Ant Colony Optimization (ACO). The ACO algorithm, introduced as the Ant System in 1995 by Marco Dorigo *et al.*, has been of particular importance, as it has been widely used as the minimizer for the travelling salesman problem, revealing itself in transportation networks, amazon algorithms, and many more [1]. Moreover, this algorithm has been generalized to solve computationally complex graph-based problems, of which some include: (i) routing [1, 2], (ii) assignment [2], (iii) scheduling [3], (iv) subset [4], and (v) others - e.g. protein folding, Bayesian networks, and many more [5–8].

Intuitively, the ACO algorithm is focused around imitating the behaviour of ants, specifically, how the collective system of ants (agents) use pheromones to navigate the variability of the solution space to find the shortest path. The simplicity of the ACO, reveals the inherent collective intelligence of 'ants' to navigate non-trivial and complex loss spaces.

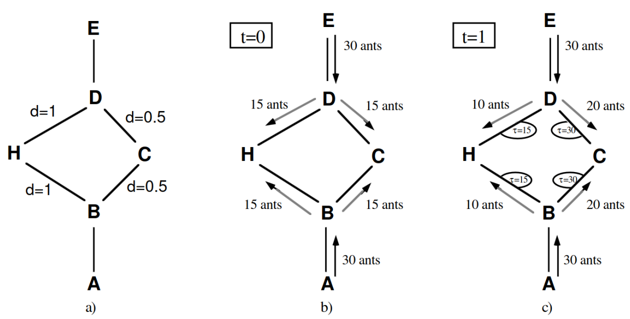

FIG. 1. showcases an intuitive figure for the ACO, with (a) a simple graph where a shortest distance between A-E is desired, (b) the initialization of the ants, and (c) how the pheromones and transition probabilities will evolve after $t = 1$.

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II. ANT COLONY OPTIMIZATION FORMULATION

The ACO leverages on the collective intelligence of ants to find the shortest path. By abstracting the travelling salesman problem as a directed graph, the shortest path can be found by simulating the behaviour of ants in this abstract loss space. Specifically, whenever an ant travels from one vertex in the graph to another, it leaves pheromones along that edge inversely proportional to the length of the edge d_{ij}. Therefore, we consider N vertices in a graph $\mathcal{N}$, then $d \in \mathbb{R}^{N\times N}$ represents the adjacency matrix of $\mathcal{N}$, where the weight of each edge d_{ij} is the euclidean distance between those two vertices. Now, the amount of pheromone dropped on the trail $\Delta\tau_{ij}^k$ by ant k can be formalized as:

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{d_{ij}}, \text{if ant } k \text{ traverses edge } i,j \text{ between } t \text{ and } t+n \\ 0, \text{otherwise} \end{cases} \tag{1}$$

where, Q is a constant. Now, the pheromones τ_{ij} on the edge i, j is updated at every time step $t \to t+n$ given κ ants,

$$\tau_{ij}(t+n) = \rho\tau_{ij}(t) + \sum_{k=1}^{\kappa} \Delta\tau_{ij}^k, \tag{2}$$

where $\rho \in [0, 1]$ is an evaporation factor of the pheromones. Now, the *visibility* η_{ij} of vertex j from vertex i is defined as the reciprocol of the distance between vertices i and j,

$$\eta_{ij} = \frac{1}{d_{ij}}. \tag{3}$$

Now, we turn the ants into random walkers in the graph $\mathcal{N}$, where the transition probability $p_i^k(t)$ of an ant k on vertex i at time t is defined as a Markov Chain,

$$p_{ij}^k(t) = \begin{cases} \frac{[\tau_{ij}]^\alpha[\eta_{ij}]^\beta}{\sum_{k\in \text{allowed}_k}[\tau_{ik}]^\alpha[\eta_{ik}]^\beta}, \text{if } j \in \text{allowed}_k \\ 0, \text{otherwise} \end{cases} \tag{4}$$

where, allowed_k are the traversable vertices from vertex i of ant k, and α and β are the scaling of the pheromones and visibility factors. Trivially, a large α will correspond to the ants having a high probability to follow the dominant pheromones, and small α will make their walks more random with respect to the pheromones. Similarly, a large β will make the ants preferentially walk to closer cities, and a small β will make the ants traverse the edge regardless of its distance.

Now, the ACO is defined, and κ ants will be initialized on a graph $\mathcal{N}$ with adjacency matrix d, and after some time $t >> 0$, the ants will be almost unanimously agreeing upon a traversal of graph $\mathcal{N}$, which, will correspond to the ants solution of the shortest path. From Fig. 1 we notice that even after a single time step, the majority of the ants are walking in the shortest path direction, which is trivially due to the low complexity of the toy problem.

III. RESULTS

The ACO has had particular success due to three important factors: (i) the *versatility* of the algorithm, (ii) the *robustness* of the approach, and (iii) its *population* based approach, making it ready for parallel implementations. Moreover, the relatively low computational cost of the algorithm, along with its ability to converge to the best known solutions is unparalleled. In Fig. 2, the famous CCA0 travelling salesman problem of 10 vertices is showcased, where, the optimal solution is found within $t = 100$ time steps. This algorithm hasn't only had success for smaller systems, in fact, it has been tested on many of the iconic test problems with great results, as seen in Table. I. This table was generated by letting all of the different heuristic algorithms run for an hour on a given problem, and reporting the best solution. Here, we notice that the Ant System, and Ant System with non deterministic hill climbing (synonyms for ACO) produce very good results, always being within 5% of the best known solution. More impressively, the AS with non-deterministic hill climbing produced the best known result in all the test problems, except for the Nugent (30) case.

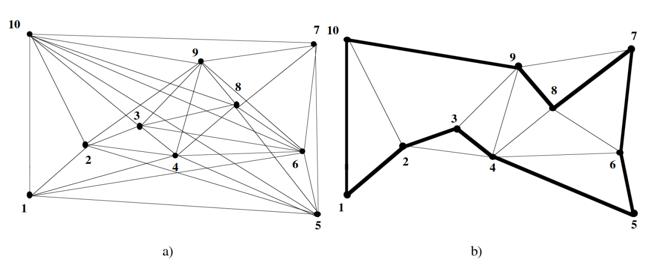

FIG. 2. Evolution of the pheromones on the trail of the CCA0 problem. Panel (a) represents the initial distribution of pheromones, and (b) represents the distribution of pheromones along the trials after $t = 100$ time steps.

Interestingly, the set of parameters was constant for (α, β, κ) for all of these different problems, which, could be a potential reason for the imperfect result for Nugent (30), as, some of these parameters could potentially scale with the complexity or size of the problem - i.e. having lower α and β parameters to search more of the solution space before converging. To further support this attribution, we present Fig. 3, which, showcases the rapid convergence of the ants to a given solution, thus, rendering the majority of the iterations in the hour long simulation necessary to produce Table. I unnecessary. Finally, Fig. 3 reveals that indeed the use of pheromones increases the performance of the algorithm, verifying the 'null-hypothesis' that the ants use pheromones for their collective intelligence.

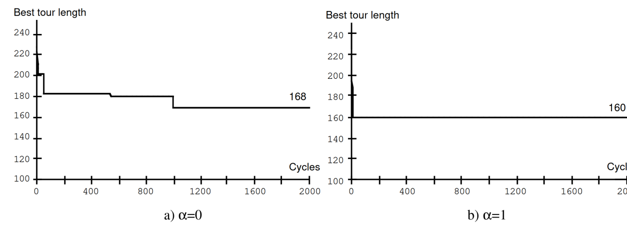

FIG. 3. Synergy: communication among ants ($\alpha > 0$) improves ACO performance - (a) shows $\alpha = 0$ and (b) shows $\alpha = 1$ for the CCA0 problem.

IV. CONCLUSION

This paper provides a novel nature-based algorithm to solve the travelling salesman problem, that is highly efficient, parallelizeable, robust, and versatile. Moreover, this algorithm has been applied to various graph-based problems with great results. That being said, the paper provides proof that the pheromones are necessary to efficiently search the solution space, however, there is not much insight into what the ideal ACO parameters are, and how they scale with the size or complexity of the problem. We hypothesize that the more complex and larger the system gets i.e. - the solution space has more local minima that are potentially more disparate, the lower should the values for the α and β parameters be, in order to increase the thoroughness of the search of the ants in their abstract loss space - i.e. avoid getting stuck in local minima.

	Nugent (15)	Nugent (20)	Nugent (30)	Elshafei (19)	Krarup (30)
Best known	**1150**	**2570**	**6124**	**17212548**	**88900**
Ant System (AS)	**1150**	2598	6232	18122850	92490
AS with non deterministic hill climbing	**1150**	**2570**	6128	**17212548**	**88900**
Simulated Annealing	**1150**	**2570**	6128	17937024	89800
Tabu Search	**1150**	**2570**	**6124**	**17212548**	90090
Genetic Algorithm	1160	2688	6784	17640548	108830
Evolution Strategy	1168	2654	6308	19600212	97880
Sampling & Clustering	**1150**	**2570**	6154	**17212548**	**88900**

TABLE I. Comparison of the ACO with other heuristic approaches. Results are averaged over five runs. Best known restults are in bold.

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