Assignment 3 - Jakob Austrheim

onsdag 31. januar 2024 14:52

1)

- The set of unobserved variables (X_t) in the umbrella world is whether it's raining or not (Defined as R_t in the book)
- The set of observable variables (E_t) is whether the security guards director brings an umbrella to work or not. (Defined as U_t in the book)

The umbrella world as an HHM

Dynamic model

- A model where the probability distribution for whether it's currently raining or not depends on previous values. To solve this probability one has to make a Markov assumption that the current probability only depends on a given amount of previous values. Assuming a first order Markov process for the umbrella problem the matrix will look like:

R_t-1 \ R_t	Rain	No rain
Rain	0.7	0.3
No rain	0.3	0.7

Observation model

- A model where the probability distribution depends on the U_t representing the current state of R_t (P(U_t|R_t)). As a matrix this will look like:

U_t \ R_t	Rain	No rain
Umbrella	0.9	0.2
No umbrella	0.2	0.9

- There are several assumptions encoded in this model. One assumes that a first order Markov model is sufficient to give an accurate probability of rain. In this case this is a reasonable assumption since the probability of the director carrying a umbrella wont depend much on whether it rained the previous days or not.

  The time size of the timesteps in this problem is a day. This means that using our model it either rains the entire day, or it doesn't. This is not realistic, and one should ideally have shorter timesteps to calculate the probability more accurately. However this particular problem enforces restrictions on the timesteps.

2)

- See implementation of the forward operation in the attached python script. Calculated the normalized probability of rain given the evidence that the umbrella was used on both day one and day two to be [0.8833, 0.1166]
- The forward messages up to day 5, for the sequence given in the task, are:

Evidence	Normalized forward message
Umbrella	[0.81818182 0.18181818]
Umbrella	[0.88335704 0.11664296]

No umbrella	[0.29518943 0.70481057]
Umbrella	[0.763759 0.236241]
Umbrella	[0.87352896 0.12647104]