Statistical Modelling and Design of Experiments

Assignment II

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1 Introduction

In this assignment, we study the characteristics of marathons with respect to competitiveness, fairness, and safety. To this end, we fashion a model that we can simulate to improve resource allocation and the date for carrying out the Barcelona marathon, taking into account the relevant properties and processes. We identify a group of marathon runners and study the effect of different environmental conditions on their performance with regard to climate change. Lastly, we present a proposal for how the available resources should be allocated and when the race should be scheduled based on our simulation.

2 Problem Description

Conceptual assumptions for optimizing the choice of the month and resource allocation include:

Competitiveness

- Short waiting times
- Sufficient supplies
- Optimal weather conditions to achieve record times. For example, competitive runners prefer the Berlin marathon for its favorable conditions to hit personal records.

Safety

- Adequate distribution of resources to meet runners' medical, hydration, nutrition, and sanitary needs
- Availability of medical support

Comfort

- Reducing waiting time, adverse weather conditions to prioritize runners' comfort

In the following sections, we define our simulation hypothesis. We reason out assumptions in section 4, e.g., based on previous editions of the Barcelona Marathon [2].

2.1 Simplification Hypotheses

- **SH_01** We simulate one block of runners
- **SH_02** There is no block start, meaning all runners start at the same time.

- **SH_03** All runners finish the race.
- **SH_04** The pace at which the runners run is constant throughout the race and normally distributed.
- **SH_05** The needs of runners for certain services (Water, Toilet, etc.) increase linearly over time.
- **SH_06** The time that runners need for services is normally distributed.
- **SH_07** The needs of runners in terms of services are reflected sufficiently accurately.
- **SH_08** Runners use services at a station in random order
- **SH_10** The needs of runners increase at the same rate during both running and queuing.
- **SH_11** Runners are not skipping the queues.
- **SH_12** Once in the queue, runners wait until they can attend the service.
- **SH_13** Besides temperature and humidity, there are no other factors that might affect performance.

2.2 Structural Hypothesis

- **SS_01** Service times for the same resource at different stations are the same.
- **SS_01** The average time and variance for different services is different, e.g. Water takes less time than attending the medical service.
- **SS_02** The amount and layout of the stations as well as the availability and capacity of services are similar to those found in real races.
- **SS_03** The needs of runners in terms of services are reflected sufficiently accurately.
- **SS_04** Capacity at later stations does not affect waiting or running times at previous stations (the queues are not so long that they obstruct the race track significantly)

2.3 Systemic Data Hypotheses

- **SD_01** We obtain the simulated group by the qualifying times, hence, the finish times of previous marathons. We assume that runners perform similarly in future races.

3 System Description

The system we are studying is the Barcelona marathon which first took place in 1978. The 2024 edition [2] was carried out on the 10th of March and about 20.000 runners registered for the race. The participants of a marathon have to cover a distance of 42.195 kilometers to finish the race. The circuit takes the runners through different parts of the city. Throughout this distance, the organizers provide supply stations that offer different services such as water, food, toilets, or medical attention. Typically, several runners can be served at a service point in parallel, e.g., there are multiple helpers who hand out water to runners. However, since the amount of service capacity is limited by cost, service points might get busy. In this case, runners have to stand in a queue until they can take the service and continue the marathon.

4 Model Specification

In this section of the report, we define the entities, their associated properties, and the operations which define the simulation flow. The goal is to model the relevant aspects of the system to enable a meaningful simulation and analysis.

4.1 Station and Service

The race includes k stations which are located at certain kilometers denoted as d_k. Every station hosts a set of services which we define as S_k. Every kind of service s has a given capacity c_s which denotes how many runners can use the service at the same time. Each station has a different amount of capacity which is determined with our experimental setup described in section 6.1.1. Every service point has a queue q_{skt} which denotes the number of runners queuing for this service at station k at the time t. The station arrangements of previous Barcelona marathons [4] serve as a guideline for the configuration. Over the course of the first 25 kilometers, runners arrive at the next station every 5 kilometers, and after that, every 2.5 kilometers. All stations offer at least a WC and Water. Other services are Medical Services and Food. A list of the stations and their respective available services per station can be found in table 1.

Kilometer	Available Services
5	WC, Water, Medical Services
10	WC, Water, Medical Services
15	WC, Water, Medical Services, Food
20	WC, Water, Medical Services, Food
25	WC, Water, Medical Services, Food
27.5	WC, Water
30	WC, Water, Medical Services, Food
32.5	WC, Water
35	WC, Water, Medical Services, Food
37.5	WC, Water, Food
40	WC, Water, Medical Services, Food

Table 1: Service Stations Availability

4.2 Runner

Each runner is running at a constant and individual base pace which is affected by the linear model concerning the climate conditions described in section 6.1.3. At any given time t during the race, each runner has a certain need for a specific service s which we define as n_{st}. Runners start the race with no need for any service, yet, the need for a service is increasing over time by a factor which we denote as x_s. The decrease of a need when using the respective service is denoted as u_s. Hence, we can define the need of a runner arriving at a station k as

$$n_{ks} = n_{k-1s} + x_s(t_k - t_{k-1}) - u_{k-1} \tag{1}$$

When arriving at a station, the runner will engage in the different services with a certain probability p. This probability is determined by the current need n for a service s at station k. The probability function P is defined as

$$P(n_{ks}) = \begin{cases} 1, & \text{if } n_{ks} > 1 \\ n_{ks}, & \text{if } n_{ks} \in [0, 1] \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Hence, the runners use a service with an increasing likelihood as their need gets more urgent. At some point, needs reach a level where a runner cannot ignore them anymore ($n_{ks} \geq 1$). In this case, the runner has to engage in the next possible respective service. On the other hand, a given n_{ks} can also fall below zero after using a service. In this case, the runner will not use this service with certainty. We assume that the time it takes individual runners to finish a service is normally distributed with the parameters shown in table 2.

Service	Average (s)	Deviation (s)
Meal	5	2
Water	5	2
Toilet	40	10
Medical	420	100

Table 2: Average Usage Times and Deviations for the Services. Note that this only includes the pure service time, e.g., the Meal time only includes the distribution of the food.

4.3 Flow Diagram of the Model

The flowchart in figure 1 relates to the structural layout of stations introduced in table 1. To begin with, we generate the simulated runners which will advance through the segments of the race and the respective stations. Depending on the current kilometer, the stations have different services available. It is important to mention that in the actual simulation, runners can take the available services in any order. As described in section 4.2, the probability of runners engaging in a service is based on their needs. Starting from kilometer 25, the density of stations is increased to one station every 2.5 kilometers. However, only every second station is fully-fledged while the others only offer e.g. Water and Food. After leaving the last stop, runners finish the last segment of 2.195 kilometers. Lastly, the simulation terminates.

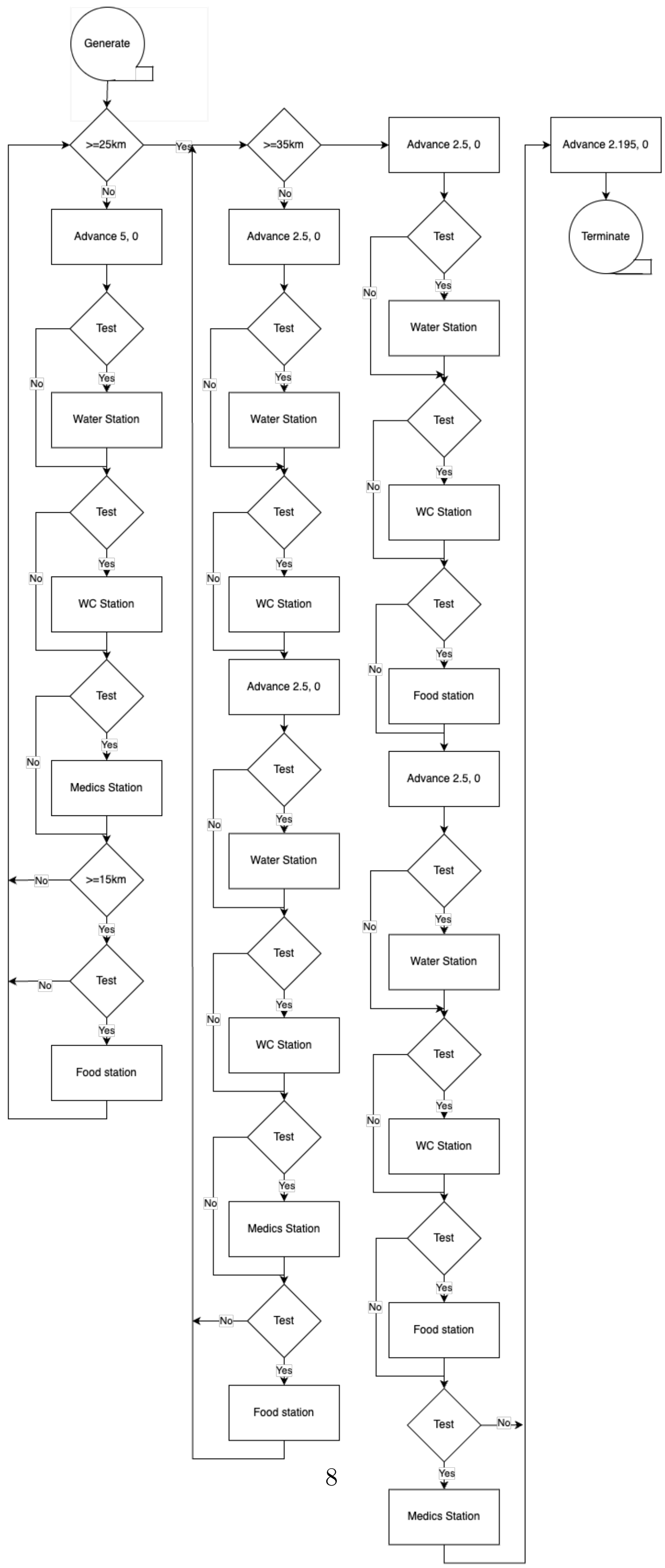

Figure 1: Model Diagram

In figure 2, we zoom into an arbitrary service point. Upon arrival, runners go into the first-in first-out queue until they can seize the desired service or resource. After the associated service time available in table 2, they depart from this service point and hence release the resource.

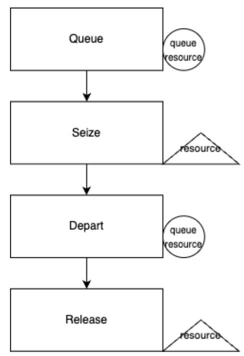

Figure 2: Service Flow Diagram

5 Coding

5.1 Data

In this section, we explain how the relevant data was obtained and how the simulation was implemented.

5.1.1 Obtaining the Simulated Group of Runners

First, we need to select a group of runners to model the marathon finishing time after. It is crucial to ensure the group has normally distributed finishing times to create a realistic and reliable marathon simulation. The normal distribution reflects real-world performance characteristics, avoids bias, and supports valid statistical analysis and predictive modeling. It captures natural variability, handles extremes appropriately, and ensures data integrity and quality.

To achieve this, we used the 2017 Boston Marathon dataset [6] and applied the following selection criteria to ensure a normally distributed group of runners' times. We cannot mix data from the 2015, 2016, and 2017 marathons in the same simulation because the humidity and temperature conditions varied significantly across these years. These environmental factors can substantially impact runners' performances, and mixing data from different years would introduce variability that is not representative of any single year's conditions. To maintain the integrity and realism of the simulation, we chose to use data from a single year with consistent environmental conditions.

Below is the group selection criteria:

1. **Valid Official Time**: Runners must have a valid `Official.Time` which is converted to total seconds for further analysis.

2. **Gender Classification**: Runners must have a valid gender classification in the `M.F` column, which is converted to a factor.

3. **Bib Number Filter**: Runners must have a bib number less than 10000. This criterion ensures the inclusion of a specific subgroup of runners, focusing on elite runners.

4. **Exclusion of Extreme Outliers**: Runners whose total seconds exceed two standard deviations above the mean are excluded to remove extreme outliers and focus on typical performance ranges.

5. **Removal of Boxplot Outliers**: Further outliers identified using a boxplot of total seconds are removed to ensure the remaining data follows a normal distribution.

6. **Normal Distribution of Times**: The selected group of runners must have a normally distributed set of total seconds. We validate the result using the QQ plot that can be seen in figure 4 and plotting run time histogram seen in figure 3.

For the implementation of the selection mechanism refer to `analysis.r` attached in the submission.

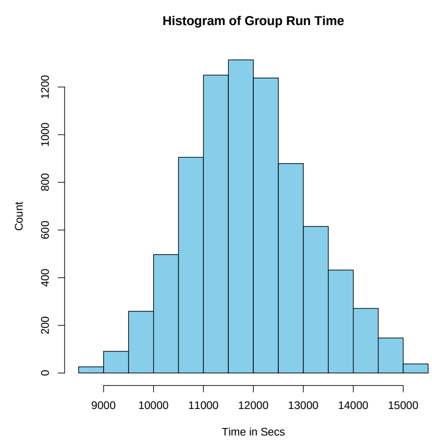

Figure 3: Run time distribution for elite runners.

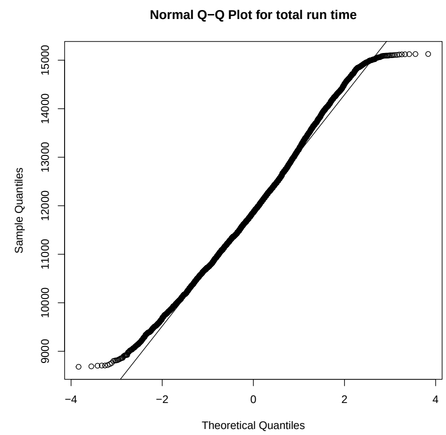

Figure 4: QQ plot of run time for elite runners.

5.1.2 Humidity, Temperature and Wind Speed in Barcelona

Climate data, including temperature and precipitation, was obtained from Servei Meteorològic de Catalunya of the Observatori Fabra[3]. We calculated monthly averages for 2017-2019. As for the wind data, we first obtained hourly wind data from Kaggle [7] and then calculated monthly averages.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp.	8.80	10.00	12.53	14.10	17.02	22.33	24.95	25.18	21.25	18.07	12.23	10.30
Prec.	9.1	9.5	12.3	14.1	16.6	21.9	25.2	25.4	21.8	17.9	12.1	10.9
Wind	3.48	3.49	3.63	3.43	3.20	2.99	3.05	2.99	2.96	3.15	3.37	3.37

Table 3: Monthly Temperature in Celsius degrees, Precipitation in mm, Wind speed in m/s for 2017-2019

5.2 Implementation

Our implementation of the experiment was done using the Python framework SimPy. Like GPSS it is a process-based discrete-event simulation framework. We chose it because of its capacity to directly measure events and run the analysis in the same Python notebook.

The software design is based on 2 classes.

- The MonitoredResource which is a generic queue, measures relevant information such as queue sizes as well as waiting times at any given moment.
- The Runner class which simulates the runner and measures related events like the needs when reaching a station as well as finishing times

The experiment was evaluated using a function called run_simulation_with_capacity which allows specifying the mean running time of the marathon for the normally distributed group of runners, the usage times, need functions as well as capacities of the resource queues at each station.

This setup allowed us to run the different experiments described in the following section.

6 Definition of the Experimental Framework

In the experiment, we employ the previously described model to simulate and study how different allocations of the fixed amount of available stations and their respective services and capacities affect the flow of the race. Given the mean running time obtained from the linear model, we aim to find the best configuration of parallel resource queues for each service point. As a target metric, we aim to allocate the available service capacity such that the maximum waiting time is between 7 and 15 seconds. The maximum is chosen because 15 is still an acceptable waiting time. The lower bound was chosen to ensure that we avoid overprovisioning the resources.

6.1 Experimental Setup

6.1.1 Obtaining Resource Configuration

Identifying and tuning the relevant factors for efficient resource allocation can be done as a binary search for each station from start to finish. As mentioned before, we assume that for each service, we have a capped amount of parallel slots for serving them which can be arbitrarily distributed across stations. In the binary search, each node represents a simulation and the associated queuing times. If the maximum waiting time at a given service point exceeds the acceptable upper bound, we consider this service point *underprovisioned* and we branch into a new simulation that allocates an additional service slot to the respective station. Likewise, if the present allocation resulted in a maximum waiting time below the lower bond which we consider acceptable, we remove a slot from the service pool and put it back into the available pool of resources. This way, we can avoid *overprovisioning* which might result in very high throughput at some stations, yet globally there might be more queuing. Running a fully-fledged simulation with this approach takes approximately 20 minutes and yields good results which are described in section 8.

6.1.2 Analyzing the Variance

One potential drawback of the previously described approach is, that it is sensitive to the variance of of the simulations. Thus it cannot be used to obtain the exact resources required. Rather it gives an estimate for how many resources should be allocated. Hence, in the second step of our experiment, we ran the simulation 50 times given the obtained resource configuration in the previous step. This was done to analyze how sensitive our simulation is to randomness. For this step, we focus on the maximum waiting time across all stations by service.

6.1.3 Linear Model

Finally, given the resource configuration obtained using the procedure described in section 6.1.1, we change the mean running time according to the linear model for climate conditions. The group of runners for which we perform our analysis are considered the best runners in the race. For this group, we applied the linear model described by Knechtle et al. [5]. The linear model consists of the three categorical variables humidity, temperature, and wind, each of which is marked as either "medium" or "high". Given that there are only 8 possible combinations, we ran the linear model for each of them. Table 4 shows the linear parameters applied to the running speed. Because the 2017 marathon had medium conditions, we used its speed and subtracted the medium condition linear parameters from it to obtain the constant parameter for our linear model.

	temperature	humidity	wind speed
medium	2.4	4.8	90.6
high	112.2	33.6	81

Table 4: Linear parameters of finishing time in seconds

6.2 Interactions

In our model, the only elements that can interact are the service queues and the runners. Long queues at earlier stations spread out the running time. This in turn will lead to fewer queues at later stations.

7 Model Validation

Validating the model is crucial because we need to ensure that our simulation represents the real-world system to an extent where it can produce meaningful insights. To this end, we propose different methods for validation in this section.

7.1 Data Validation

Data validation is a critical step to ensure that the data used in our marathon simulation model is accurate and correctly defined. This involves validating normality, data sources, and support structures to maintain the data's integrity. Data validation tests the Systemic Data assumptions.

7.1.1 Data Selection

First, we must ensure that the data aligns with the modeling goals. For the marathon simulation, this involves validating that the finishing times of the selected group of runners follow a normal distribution. Data selection is also described in section 5.1.1. We have chosen elite runners from the 2017 Boston Marathon, specifically the top 10,000 runners. We then removed outliers by excluding those with running times outside of two standard deviations from the mean. Additional outliers identified in a box plot were also removed. To confirm normality, we used a QQ plot (figure 4) and visually inspected the histogram (figure 3). Although we couldn't use the Shapiro-Wilk test due to our large sample size of approximately 8,000 runners, which exceeds the test limit of 5,000, we can confidently confirm that the data is normally distributed.

7.1.2 Data Expiry

Next, we need to ensure that the data expiration constraints are valid. We used data from the 2017 Boston Marathon and historical climate data in our modeling.

The marathon data was scraped from the official race website [1]. Given the prestige and competitiveness of the Boston Marathon, we can trust the accuracy of this data. Since the event has already occurred, no updates are needed.

For the weather data, we used information from the Servei Meteorològic de Catalunya of the Observatori Fabra [3], which provides official weather records. This data is regularly updated and maintained. Although we are only using historical data, this ensures that the data remains accurate and relevant.

7.2 Experimental Validation

Experimental validation assesses the adequacy of the experimental procedures used to obtain results from a simulation model. As simulation models are executed within an experimental framework, it is crucial to validate the design and execution of these experiments. For this validation type, we focused on verifying the number of replications for each scenario in the experiment is sufficient and obtained correctly.

In our implementation, we analyzed queue times for a given resource allocation by running the same experiment 50 times. We then analyzed the maximum queue time, the maximum median queue time, and the overall average. The results are presented in Tables 7, 9, 6, and 8. This approach provides a more accurate representation of wait times compared to running the simulation only once, as it balances the results across 50 trials and reduces the influence of random, non-representative outcomes. The analysis conducted in section 8 has shown that our resource allocation logic is robust, with experimental validation confirming the stability and reliability of our model. The probability of the maximum waiting time falling outside three times the standard deviation from the mean is less than 0.1%, indicating low sensitivity to randomness and high confidence in our simulation results.

7.3 Operational Validation

We need to test the operational validity of the model to verify that it behaves as expected and produces accurate outputs when subjected to real-world scenarios. To achieve this, we can perform cross-validation with real-world data. In our case, we simulated runners' paces from the 2017 Boston Marathon and used station allocations from the Barcelona Marathon. By comparing the wait times from the Barcelona Marathon to our simulated results, we can evaluate the model's accuracy. It is essential to correctly input the month of the race and the humidity conditions to ensure the comparison is valid. This process helps us validate how well our model performs in real-world scenarios, ensuring its reliability and robustness.

8 Results

8.1 Resource Allocation

Table 5 shows the resource allocations obtained by the binary search. We see that the most provisioned resource is toilets, due to the large amount of time that a runner takes to use them. Except for medical facilities, we see a trend that in later stations of the race, we need fewer resources for each of the services because the runners are more spread out.

	meal	water	toilet	medical
0	N/A	104	167	4
1	N/A	53	133	7
2	36	35	79	5
3	18	27	38	6
4	10	22	17	6
5	N/A	20	26	N/A
6	9	18	34	5
7	N/A	16	32	N/A
8	8	13	31	9
9	4	10	27	N/A
10	5	11	18	8

Table 5: Obtained resource allocations

resource	max queue	max median queue	average overall
meal	13.19	0.00	0.65
medical	206.08	0.00	7.29
toilet	31.56	2.50	2.20
water	18.08	0.76	1.36

Table 6: Medians over all runs for collected statistic by resource

resource	max queue	max median queue	average overall
meal	18.19	0.00	0.97
medical	573.32	267.30	56.93
toilet	61.55	15.22	3.94
water	38.82	2.23	2.04

Table 7: Maximum of collected statistic by resource for 50 repeated runs

resource	max queue	max median queue	average overall
meal	2.25	0.00	0.11
medical	131.46	50.12	12.63
toilet	9.09	3.59	0.62
water	6.15	0.61	0.28

Table 8: Standard deviation of statistics by resource for 50 repeated runs

resource	max queue	max median queue	average overall
meal	13.29	0.00	0.67
medical	213.10	16.34	10.80
toilet	32.64	3.63	2.26
water	19.11	0.79	1.39

Table 9: Means of statistics by resource for 50 repeated runs

Table 6 shows the medians of the collected stats over the 50 runs. We can see that the median max queuing time is higher than our targeted 15 seconds for all resources except meals. This is due to the fact this max queue is actually the maximum of the maximum queuing time over all of the 11 stations. Under both, the maximum shown in table 7 and the medians over all runs, medical seems to be the most volatile, breaking far over the maximum waiting time that we want to see in a race. It is, however, in the order of the 420 seconds of mean treatment time for this service. This gives an indicator that perhaps it is prudent to slightly overprovision this resource to ensure the fairness of the race.

Furthermore, if we look at the standard deviations shown in table 8, we can see relatively low values for all of them. A Shapiro-Wilk test of the max queuing time confirmed that the max-queuing time is normally distributed except in the case of water with $p = 0.002$. We thus conclude that our resource allocations are not very sensitive to randomness and quite stable, with the probability of the maximum waiting time being outside of the range of 3 times the standard deviation from the mean being less than 0.1%.

8.2 Weather conditions

humidity	wind	temperature	mean finish time	mean expected finish time
medium	medium	medium	12100	11934
medium	high	medium	12090	11924
medium	medium	high	12198	12034
medium	high	high	12209	12043
high	medium	medium	12143	11975
high	high	medium	12102	11938
high	medium	high	12249	12079
high	high	high	12216	12050

Table 10: Finish times with queuing vs expected finish times without queuing or using resources

Table 10 shows the mean finishing times based on weather conditions for 1 simulation run under these conditions. In comparison to the overall running time, the effect on weather conditions is quite small. When looking at the resource usage statistics, we don't see values far from the mean in terms of standard deviation compared to our repeated baseline run.

This shows that weather conditions do not play a big role in allocating resources to the services at stations. Therefore the recommendation would be to pick a time of year when the humidity and temperature are medium and the wind speed is high to have the optimal conditions for runners to run fast.

8.3 Digging Deeper into the needs

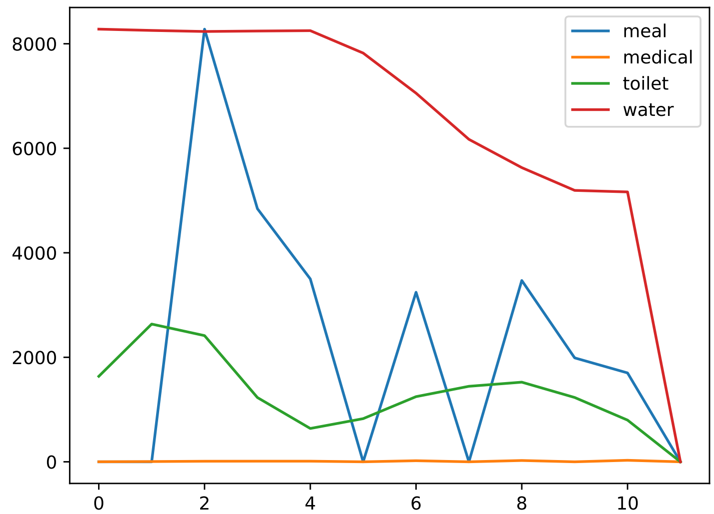

Figure 5: Usage by station

In this section, we analyze the usage and need characteristics of one single baseline race run. Figure 5 shows how many runners use each service at every station. We can clearly see that at the first few stations, all runners use the water service. This only drops at the 6th station where the distance between stations reduces from 5km to 2.5km. Under our model of hydration needs, this indicates that more water stations would need to be placed at the beginning of the race.

Similarly, the runners all need to eat by km10 under our assumptions, thus indicating that it would be useful to add food to at least station 1 but potentially also station 0.

Medical and toilet usage is far below all of the runners per station, so under our model, we conclude that it is well spaced out.

These assumptions are further validated by the median needs shown in figure 6. We can see that the median need for water and meal rises above 1 indicating a problem given our assumptions of needs.

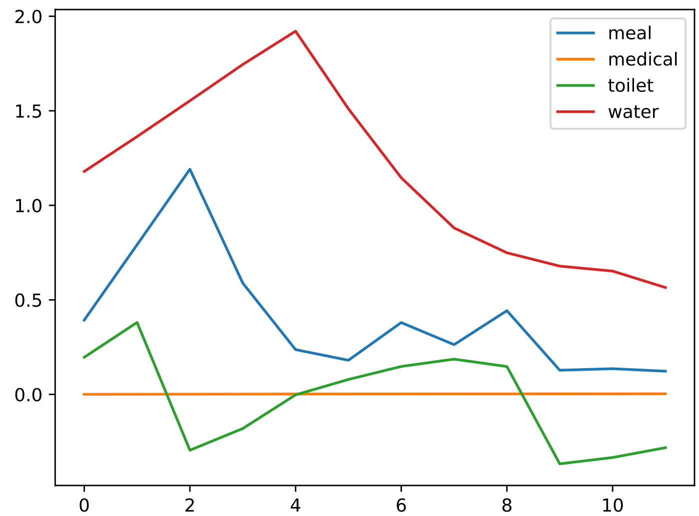

Figure 6: Median Needs

Figure 7 shows the median waiting time per station. We can see that at the third station, there is a significant median waiting time for toilet usage. For all other stations and resources, the median waiting time is close to 0. This shows that we could potentially provision more toilets at that station.

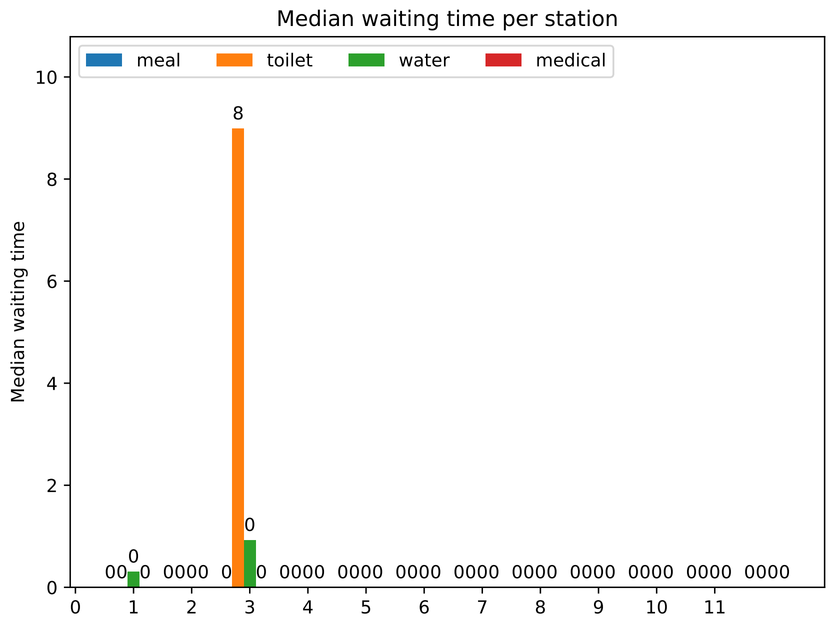

Figure 7: Median waiting time per station in seconds

With respect to the maximum waiting times, medical services are the most critical as shown in figure 8. At several stations there is a queue, showing that we need to ensure more provisioning for these stations if we wish to treat patients immediately.

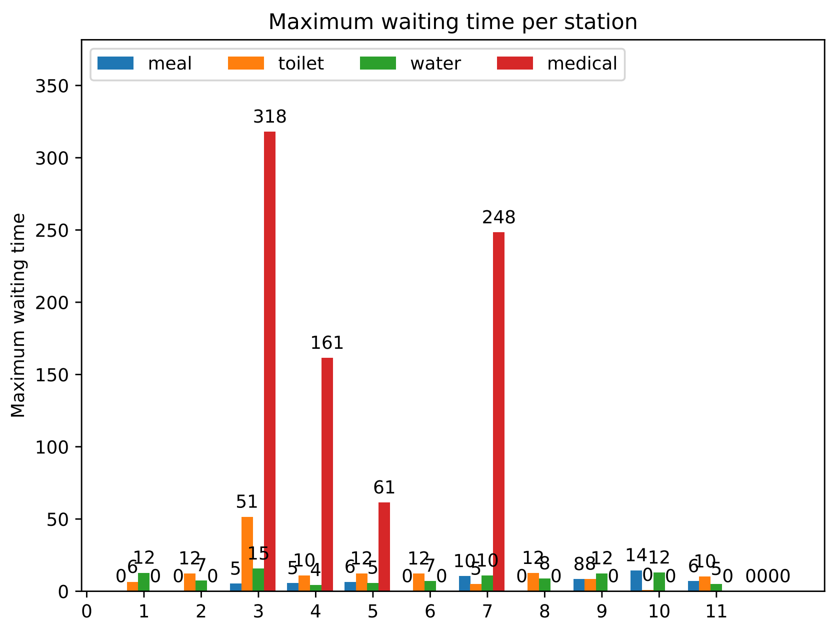

Figure 8: Maximum waiting time per station in seconds

Except for medical services which are subject to large variances due to small queue sizes and large service times, only the maximum waiting time for toilets at the third station was above the threshold of 15 seconds that was set as a target metric. This indicates that our provisioning scheme using the binary search is generally quite accurate in assigning the right amount of resources.

Lastly, we can compare the performance of the resource allocation obtained with the binary search method to a baseline allocation with uniformly distributed service slots. In figure 9 we can see that the median queuing time is considerably higher which indicates that the first stations are quite underprovisioned in terms of water, toilets, and meals. Especially the long queues at station 3 make sense since runners need to start their nutrition early in the race. However, if we look at the maximum queuing time displayed in 10, we see no queuing for medical services. In general, a poor allocation can affect the finishing times of runners quite significantly, as we can see in the histograms if figure 12 and 11.

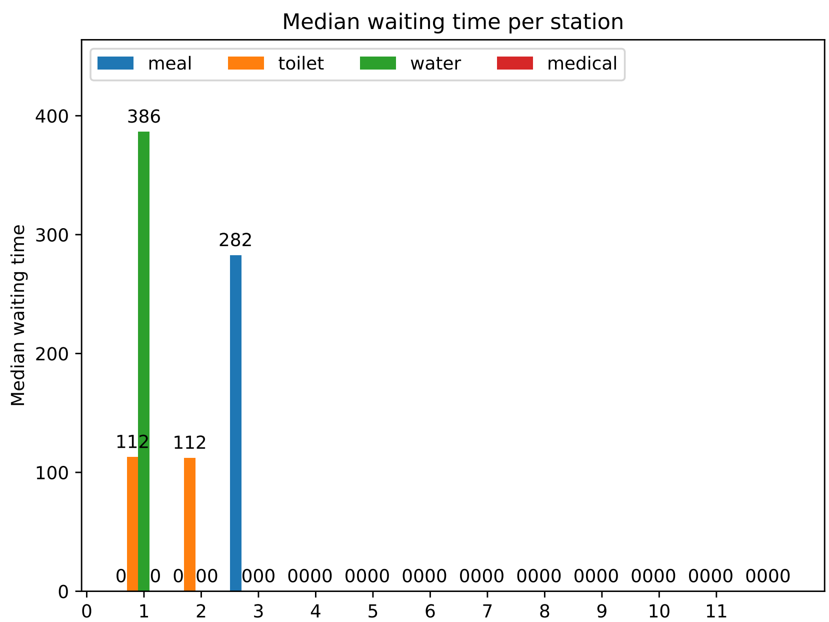

Figure 9: Median Queue time in seconds when allocation the service points uniformly.

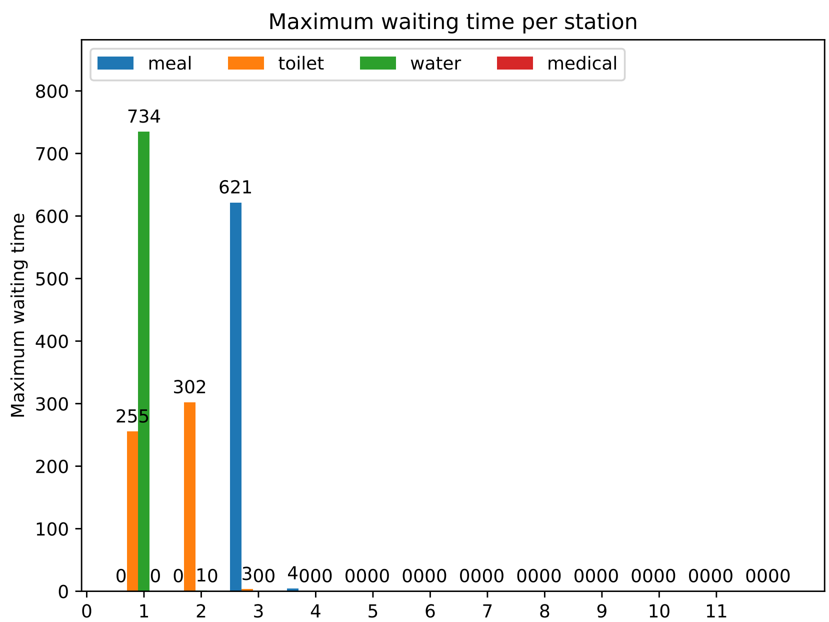

Figure 10: Maximum Queue time in seconds when allocation the service points uniformly.

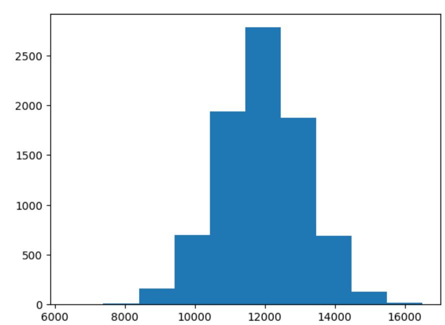

Figure 11: Expected finish time seconds when using the baseline allocation.

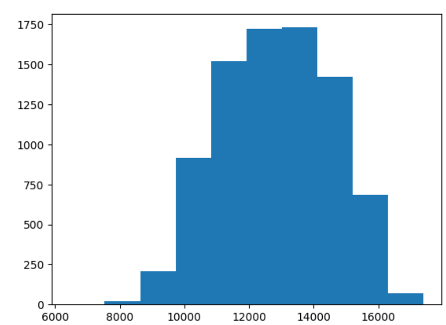

Figure 12: Actual finish time in seconds when using the baseline allocation.

9 Conclusions

In this project, we described the relevant entities and their behavior of the Barcelona marathon formally. We reasoned our assumptions and simplifications based on domain knowledge and external sources. Based on the developed model, we fashioned a process-based discrete-event simulation to study how different parameters such as resource availability and climate change affect the competitiveness and the safety of the race. We implemented a binary search scheme that finds an efficient allocation of resources. We found that services with longer usage times were subject to more sensitivity to maximum waiting time variations. Under our assumptions of needs, the current configuration of the Barcelona marathon has too sparsely distributed water and food stations at the beginning of the race. The most needed resource is toilets due to the high need factor and usage times. Additionally, we found that moving the race to date medium temperature and high wind conditions will improve the competitiveness of the run in terms of the mean finish times of the runners.

Future Work

The binary search gave a good estimation of how many resources would be needed at each station. With some further fine-tuning, exact numbers could be obtained. Additionally, the model does not take into account the potential obstruction that queues can cause as well as the flows of runners to get to the service stations. Lastly, one could analyze the required resources before the start of the race and at the end, e.g. in the finish line area.

References

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