Fluid Mechanics
Viscous pipe flows

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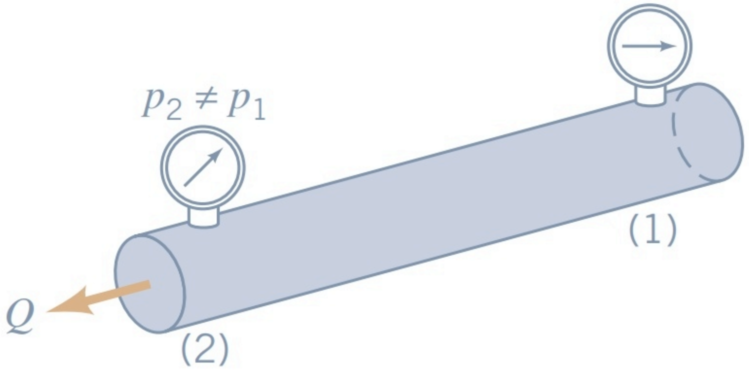
Types of pipe flows

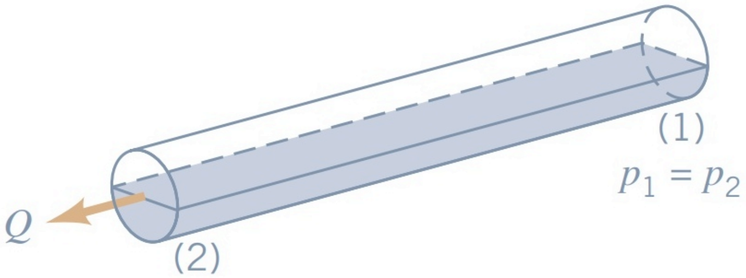
Various pipe flow types exist, each with *distinct* physics

- **Single phase** pipe flows



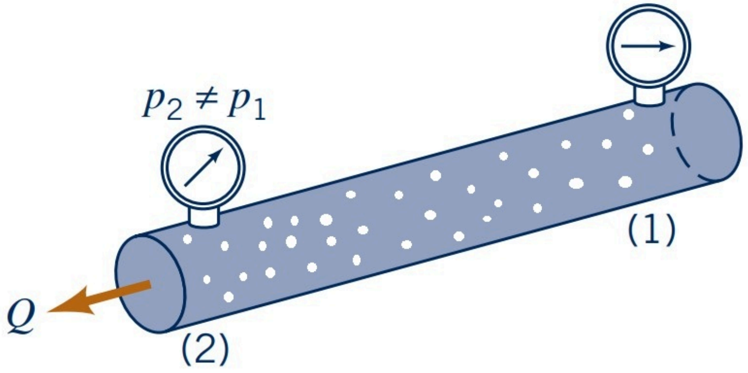
  - Completely filled with liquid (or gas)
  - Pressure difference $(p_2 - p_1)$ drives flow

- **Open channel** pipe flow



  - Partially filled with liquid
  - Gravity drives flow

- **Multiphase** pipe flow



  - Either partially or full of liquid (or gas)
  - Gravity drives flow
  - Complex physics (bubbles, particles, droplets etc)

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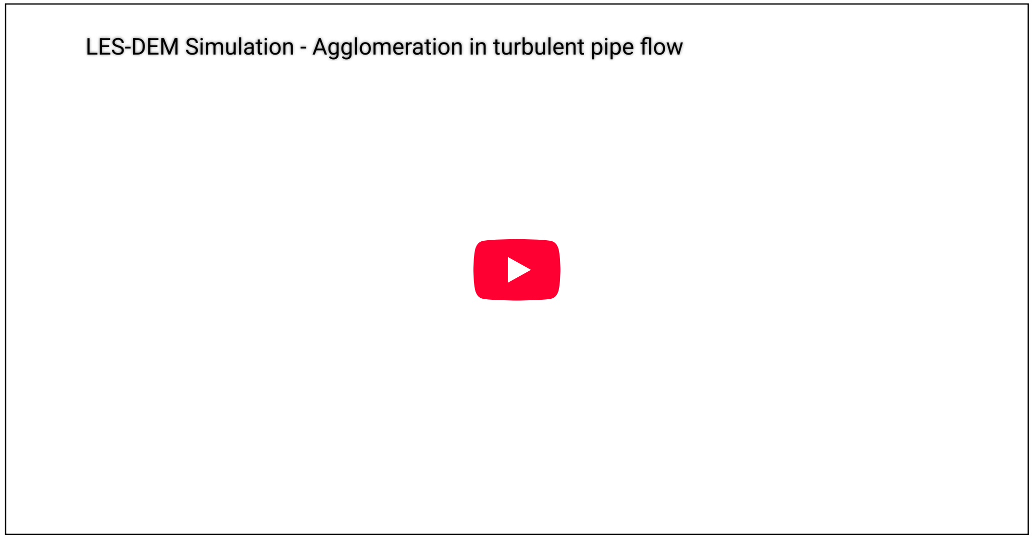
Much research focuses on complex multiphase flows (including own PhD)

Numerical simulations provide details on the process, which *can be difficult to capture experimentally*.

- Turbulent pipe flow
- Solid particles with $d_p = 10\,\mu\mathrm{m}$
- Particle agglomeration (sticking) occurs due to van der Waals and electrostatic forces

Video: Agglomeration of particles in a turbulent pipe flow



Why look at *viscous pipe flows* ?

For which applications are viscous pipe flows important?

Viscous pipe flows are important in applications where **fluid viscosity significantly affects pressure drop, energy losses, and flow behavior**. Here are the main areas:

🔹 1. Water Supply and Wastewater Systems

- City water distribution networks
- Sewage and stormwater pipelines
- Irrigation systems

👉 Viscous effects determine pumping requirements and pipe sizing.

🔹 2. Oil and Gas Industry

- Crude oil transport through long-distance pipelines (oil can be highly viscous).
- Natural gas pipelines (viscous friction dominates pressure drop).
- Offshore and subsea flowlines.

🔹 3. Chemical and Process Industries

- Transport of viscous fluids like syrups, paints, polymers, and chemicals.
- Cooling and heating systems in reactors (viscosity affects heat transfer too).
- Food industry (chocolate, honey, dairy, sauces).

🔹 4. Energy and Power Systems

- Steam and water circulation in power plants.
- District heating/cooling networks.
- Hydraulic systems in machinery (oil viscosity affects performance).

🔹 5. Biomedical Applications

- Blood flow in arteries, veins, and medical devices (blood is a viscous fluid).
- Drug delivery systems using catheters and microtubes.

🔹 6. Microfluidics & Advanced Tech

- Lab-on-a-chip devices (very small channels where viscosity dominates over inertia).
- Inkjet printing and 3D printing (flow of inks, resins, or melts).

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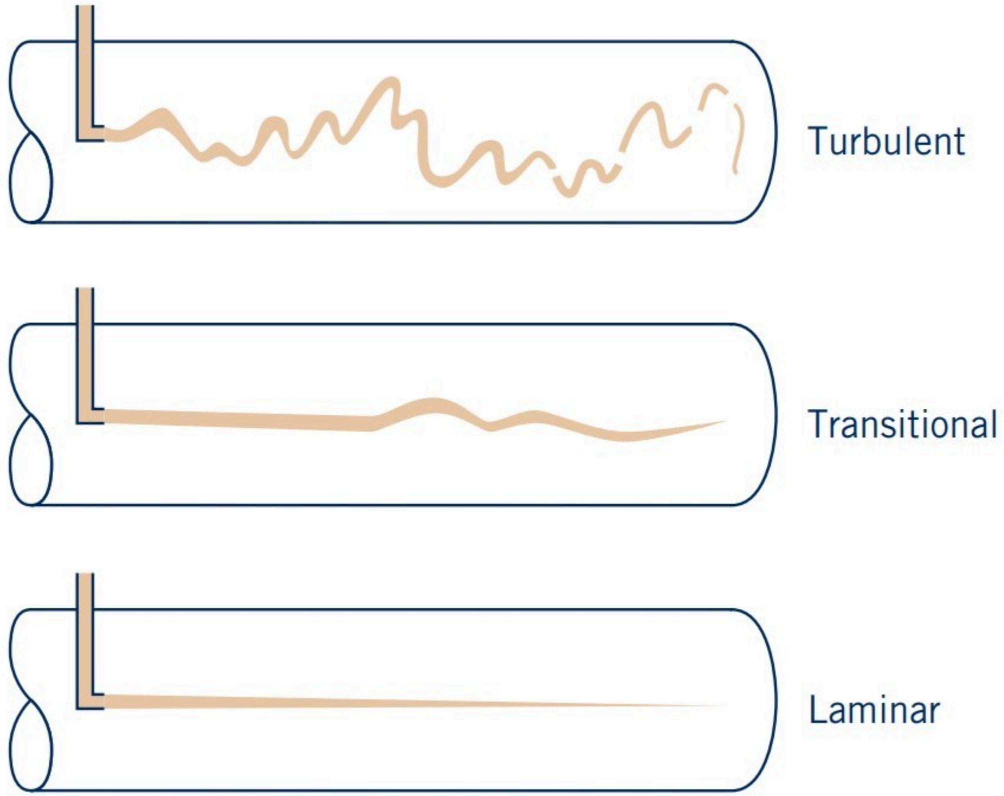
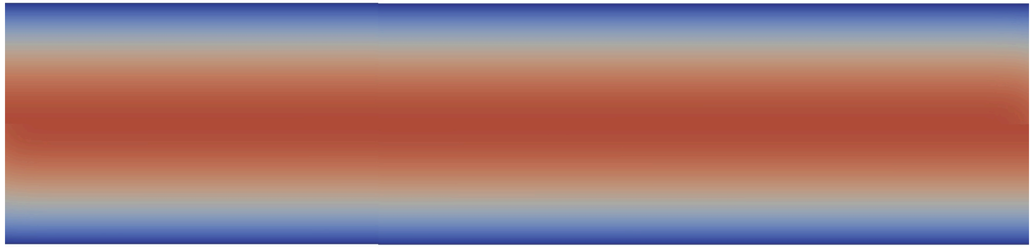
Laminar and turbulent flows





Characterised by different flow regimes:

- *Turbulent flow:* chaotic and irregular *(mixing occurs)*
- *Transitional flow:* between laminar and turbulent
- *Laminar flow:* smooth and orderly *(no mixing)*

Flow regime depends on Reynolds number ($\mathrm{Re}_D = U\rho D/\mu$)

- $\mathrm{Re}_D > 4000$: Turbulent
- $2300 < \mathrm{Re}_D < 4000$: Transitional
- $\mathrm{Re}_D < 2300$: Laminar

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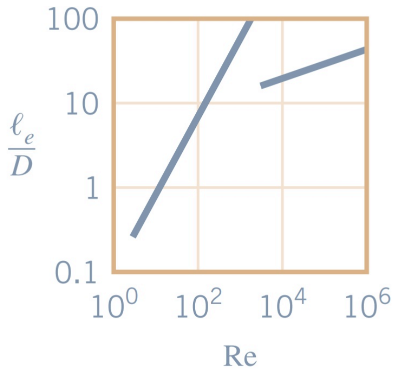
Entrance regions and fully-developed flows

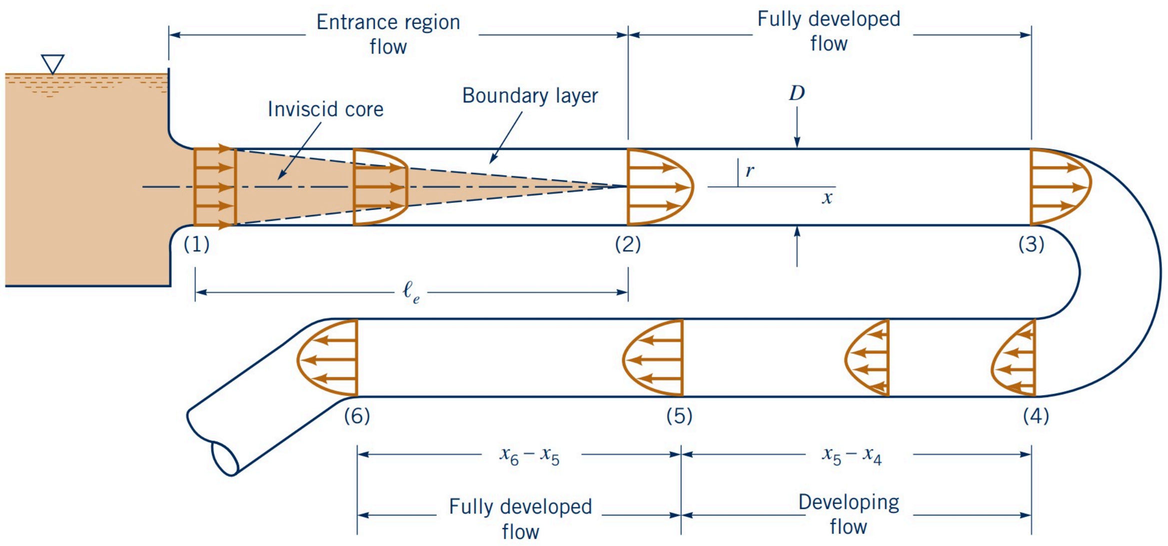
Region close to inlets

- (1): Inviscid (enters with near uniform velocity aka plug flow)
- (2): Reaches fully-developed (boundary layer reached centre)
- (3): Remains fully developed and nothing changes
- (4): Skewed and starts to develop again
- (5): Reaches fully-developed again
- (6): Remains fully-developed and nothing changes

Entrance length l_e depends on Reynolds number:

- Laminar: $l_e/D \approx 0.06\mathrm{Re}_D$
- Turbulent: $l_e/D \approx 4.4\mathrm{Re}_D^{1/6}$





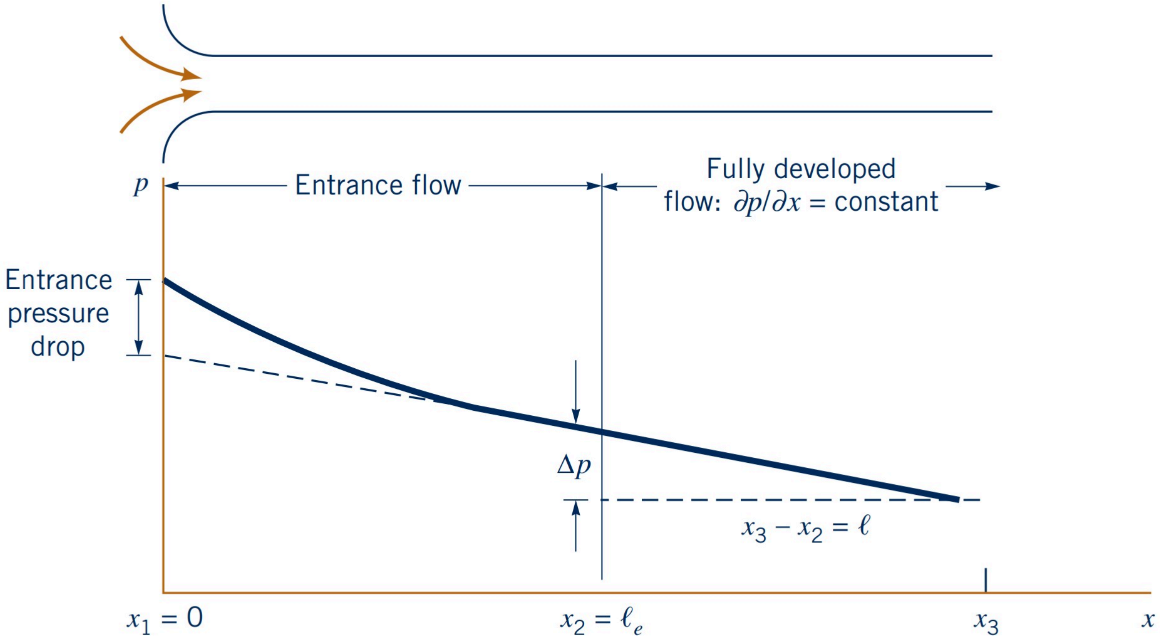
Pressure profile in pipe flows

Pressure varies *in fully-developed laminar flow* ***linearly*** along the pipe

- $\partial p/\partial x = -\Delta p/p < 0$

Pressure drop in pipe flows is balanced by:

- Pressure drop due to friction (viscous effects)
- Pressure drop due to acceleration/deceleration of flow (inertia effects)
- Hydrostatic pressure variation due to elevation changes



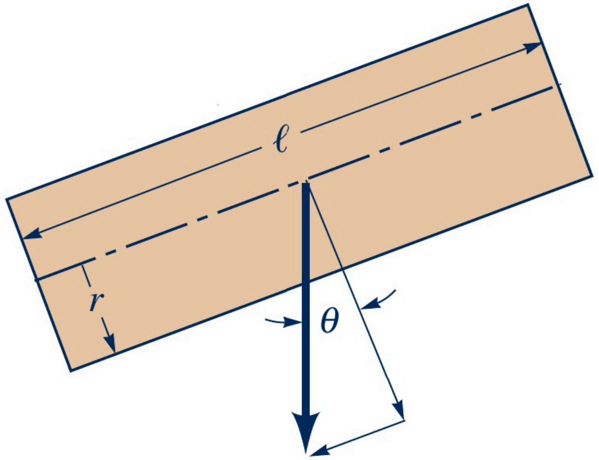
Recall from analytical solutions earlier:

$$Q = \frac{\pi R^4}{8\mu}\left(-\frac{\partial p}{\partial z}\right) = \frac{\pi D^4}{128\mu}\left(-\frac{\partial p}{\partial z}\right) = \frac{\pi D^4 \Delta p}{128\mu l}$$

- *Valid* for laminar, steady, horisontal, fully-developed flow

For non-horisontal pipes:

$$Q = \frac{\pi D^4(\Delta p - \rho g l \sin\theta)}{128\mu l}$$



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Exercise: Laminar Pipe Flow

An oil with a viscosity of $\mu = 0.1\,\mathrm{Pa}\cdot\mathrm{s}$ and density $\rho = 900\,\mathrm{kg/m^3}$ flows in a pipe of diameter $D = 0.020\,\mathrm{m}$.

- *(a)* What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5}\,\mathrm{m^3/s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10\,\mathrm{m}$?
- *(b)* How steep a hill, θ, must the pipe be on to if the oil is to flow through the pipe at the same rate as in part *(a)*, but with $p_1 = p_2$?
- *(c)* For the conditions of part *(b)*, if $p_1 = 200$ kPa, what is the pressure at section $x_3 = 5$ m, where x is measured along the pipe?

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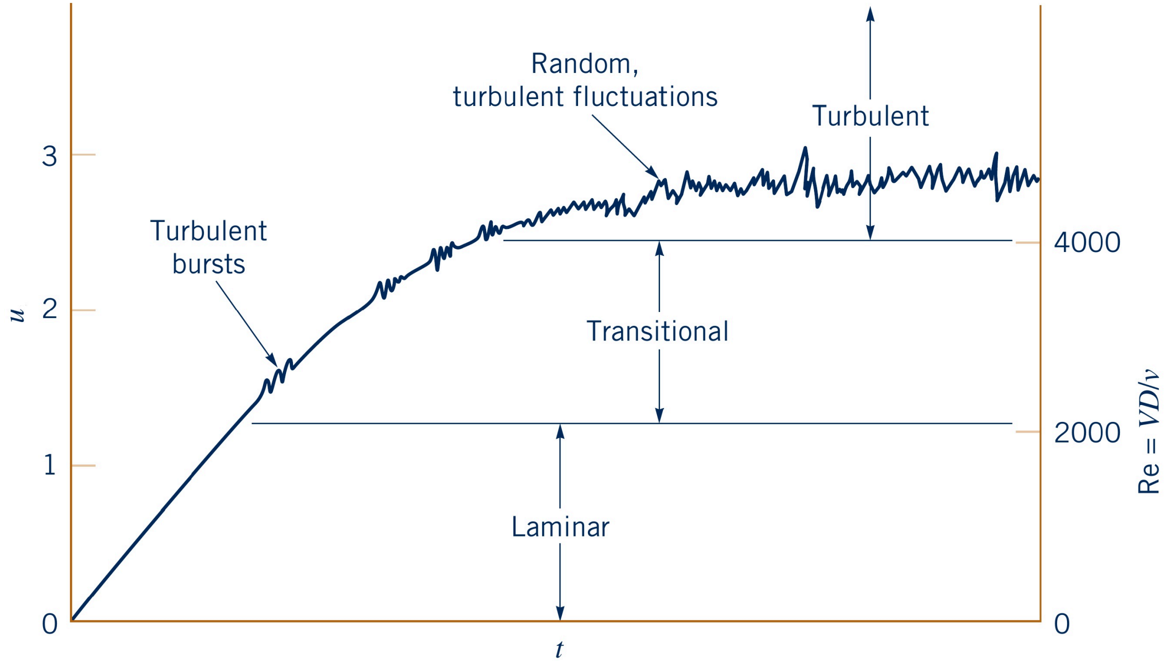
Transition from laminar to turbulent flow

Happens *gradually* between $Re \approx 2100$ and $Re \approx 4000$

- Small *bursts* of turbulence start at $\mathrm{Re} \approx 2100$
- Bursts grow in *size* and *frequency* with increasing Re
- Increased mixing and momentum transfer occurs
- Higher *heat transfer rates* and *pressure drop*

Velocity field $\boldsymbol{V}$ *changes* from being 1-D to 3-D

- *Laminar:* $\boldsymbol{V} = v_x \cdot \hat{\mathbf{i}}$
- *Turbulent:* $\boldsymbol{V} = v_x \cdot \hat{\mathbf{i}} + v_y \cdot \hat{\mathbf{j}} + v_z \cdot \hat{\mathbf{k}}$



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Decomposing a turbulent flow

Consider a turbulent flow with instantaneous velocity $u(t)$

- *Time-averaged* velocity component $\overline{u}$

$$\overline{u}(x,y,z) = \frac{1}{T}\int_{t_0}^{T+t_0} u(x,y,z,t)\,\mathrm{d}t$$

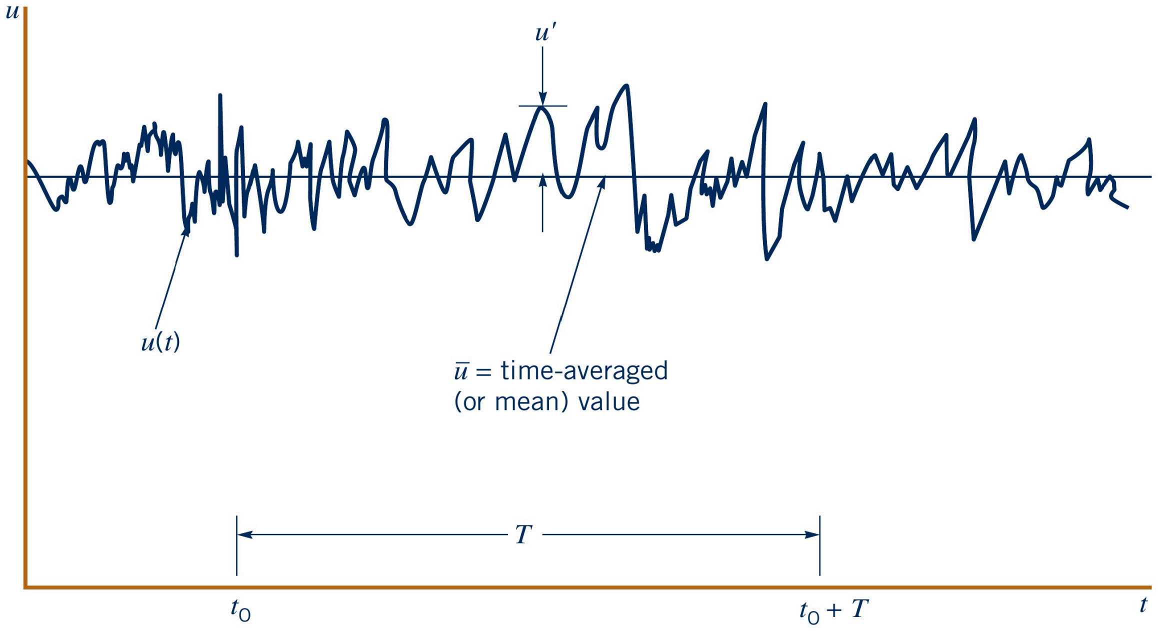
  - T is the *averaging time period* (should be long enough to capture the relevant flow structures)

- *Instantaneous* velocity can then be decomposed as:

$$u(x,y,z,t) = \overline{u}(x,y,z) + u'(x,y,z,t)$$

- *Fluctuating* velocity component u'

$$u'(x,y,z,t) = u(x,y,z,t) - \overline{u}(x,y,z)$$



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Describing turbulence

Relevant to quantify strength of fluctuations

- Clearly, *time-averaging* fluctuations $\overline{u'}$ does not make sense

$$\overline{u'}(x,y,z) = \frac{1}{T}\int_{t_0}^{T+t_0} u'(x,y,z,t)\,\mathrm{d}t = 0$$

- What if we take *square* fluctuations and then *time-average*?

$$\overline{(u')^2}(x,y,z) = \frac{1}{T}\int_{t_0}^{T+t_0} u'(x,y,z,t)^2\,\mathrm{d}t > 0$$

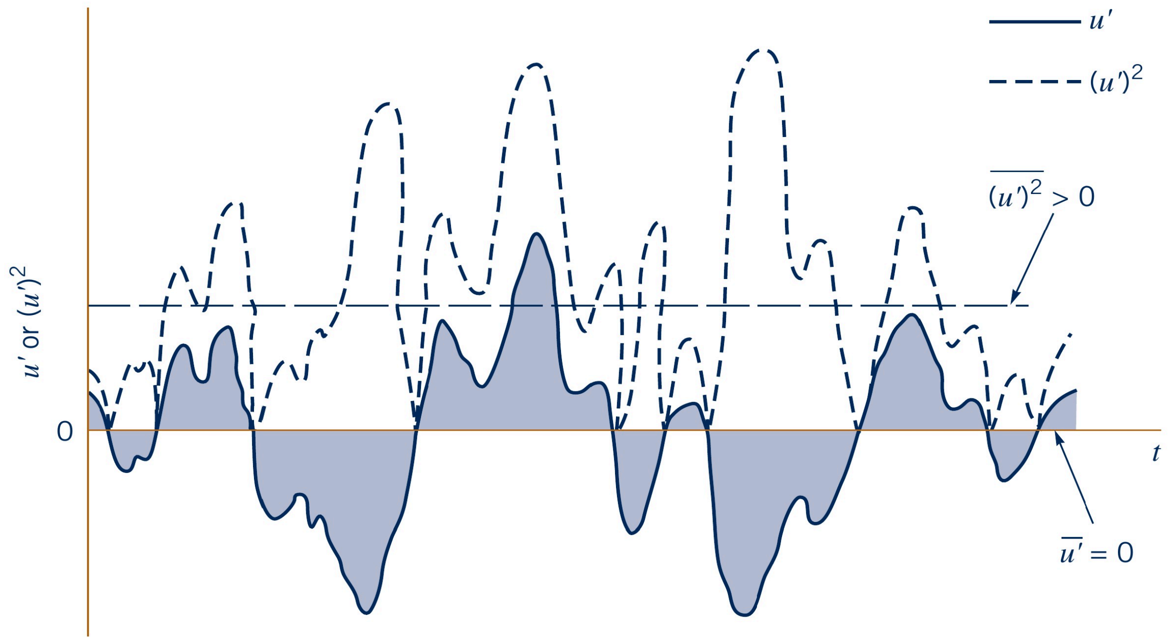
- To get "back" to same order of magnitude, we now take *the square root of* fluctuations

$$u_{\mathrm{rms}}(x,y,z) = \sqrt{\overline{(u')^2}(x,y,z)}$$

- This is called the *root-mean-square (RMS)* of the fluctuations

- Often normalised by the time-averaged velocity to get a *relative measure*, aka *turbulence intensity*

$$I(x,y,z) = \frac{u_{\mathrm{rms}}(x,y,z)}{\overline{u}(x,y,z)}$$



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