

Heat Transfer

# Lumped system assumption

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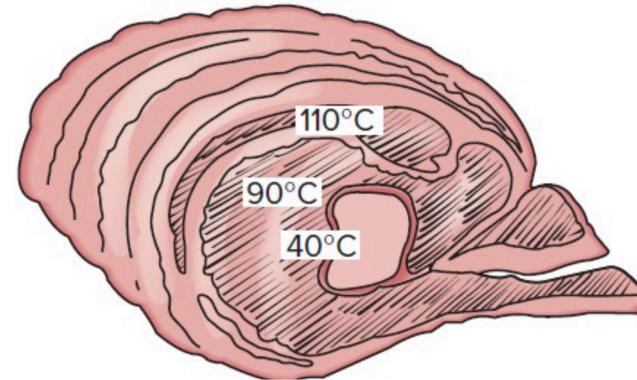
# What is the lumped system assumption and why?

Most heat transfer problems are complex because temperature *varies with both space and time*

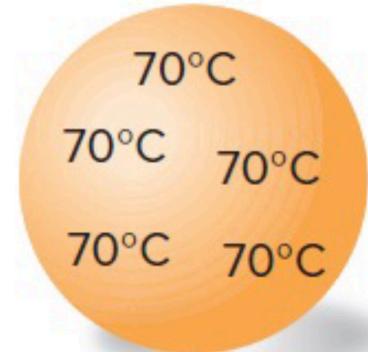
- In **lumped systems**, temperature *only varies with time (not space)*

$$T = f(x, y, z, t) \rightarrow T = f(t)$$

- Significantly *simplifies* the analysis!



A roast beef (not lumped system)

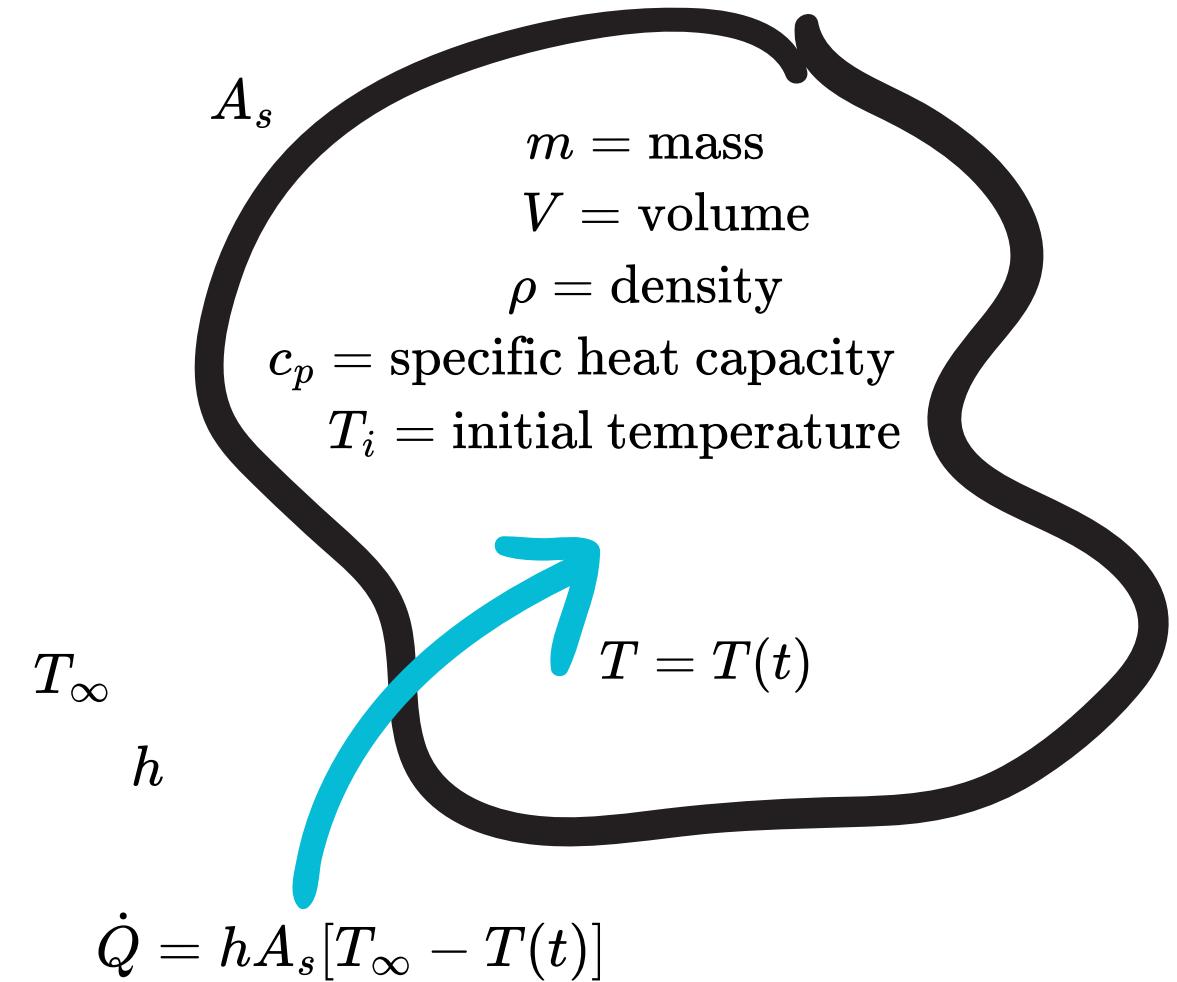


A copper ball (lumped system)

## Starting point of lumped system analysis

Consider a body of arbitrary shape with

- At time  $t = 0$  the body is exposed to convective heat transfer from the outside:
  - Surrounding temperature  $T_\infty$
  - Heat transfer coefficient  $h$
  - Surface area  $A_s$



## Derivation of the lumped system equation

Energy transferred to body during  $dt$ :

$$\dot{Q} = hA_s(T_\infty - T)dt = mc_p dT = \rho V c_p dT$$

Because  $T_\infty$  is constant, we may expand  $dT$ :

$$\dot{Q} = hA_s(T_\infty - T)dt = \rho V c_p d(T - T_\infty)$$

..Re-arranging:

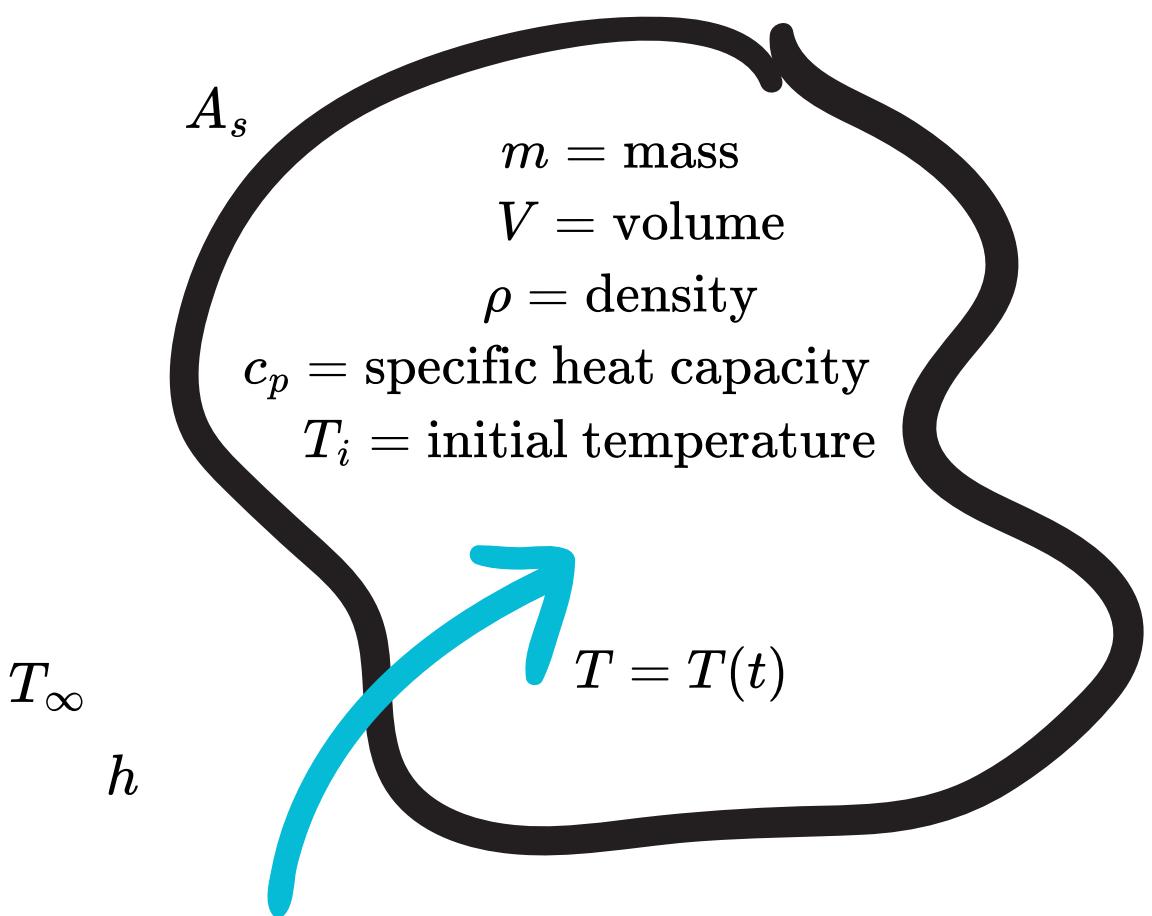
$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p} dt$$

Now, integrate from  $t = 0$  where  $T = T_i$  to  $t$  at which  $T = T(t)$ :

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$

Exponential on both sides gives **lumped system equation**:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t}$$



$$\dot{Q} = hA_s[T_\infty - T(t)]$$

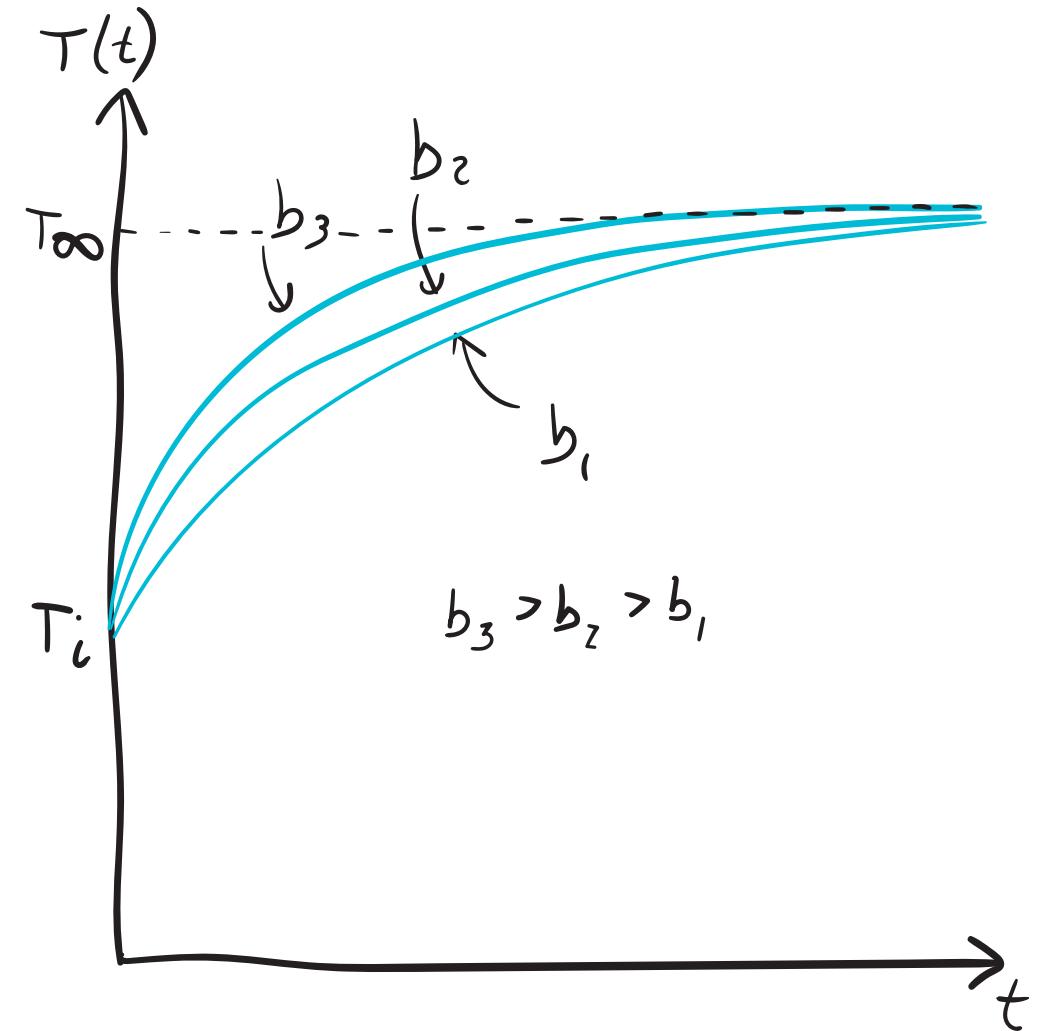
## The **time constant** for lumped systems

Introducing  $b = hA_s/(\rho V c_p)$ , we obtain:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t} = e^{-bt}$$

The **time constant ( $1/b$ )** describes the rate at which the system approaches the surrounding temperature  $T_\infty$

- **Large  $b$ :** Temperature approaches  $T_\infty$  quickly
- **Small  $b$ :** Temperature approaches  $T_\infty$  slowly



# Validity of lumped system assumption

We define a *Biot number*

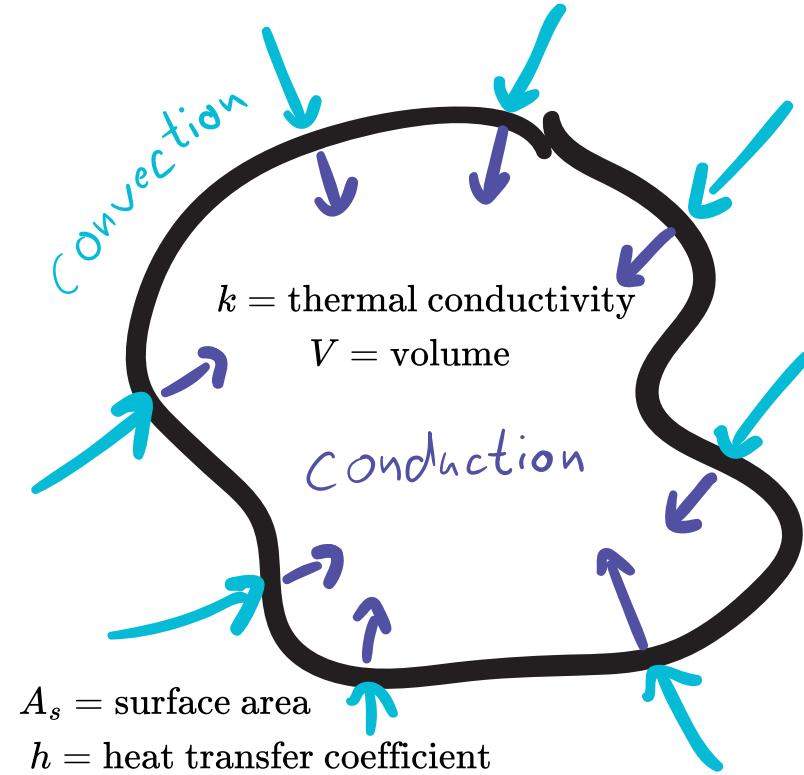
$$Bi = hL_c/k$$

- with the characteristic length  $L_c = V/A_s$

If  $Bi = 0$ : lumped system assumption is **exact**

If  $Bi \leq 0.1$ : lumped system assumption is **reasonable accurate**

If  $Bi > 0.1$ : lumped system assumption is **inaccurate**



## Exercise: Fast temperature measurements with thermocouples

The temperature of a gas stream is to be measured by a thermocouple.

The thermocouple junction can be approximated as a 1 mm diameter sphere. The junction properties are  $k = 35 \text{ W/m}\cdot\text{K}$ ,  $\rho = 8500 \text{ kg/m}^3$  and  $c_p = 320 \text{ J/kg}\cdot\text{K}$ .

How quickly can the thermocouple respond to changes in temperature when the convective heat transfer coefficient between the junction and the gas is  $h = 210 \text{ W/m}^2\cdot\text{K}$ ?

