

Fluid Mechanics

Viscous pipe flows

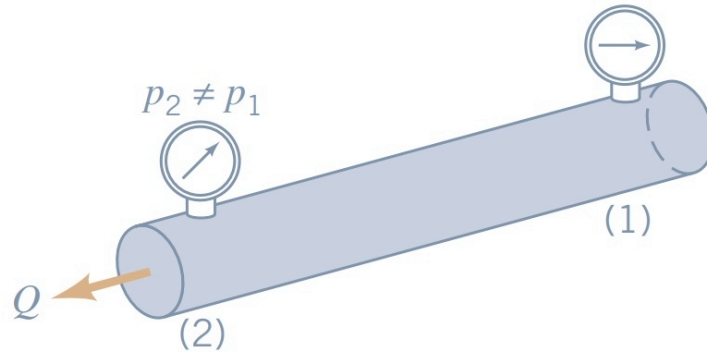
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Types of pipe flows

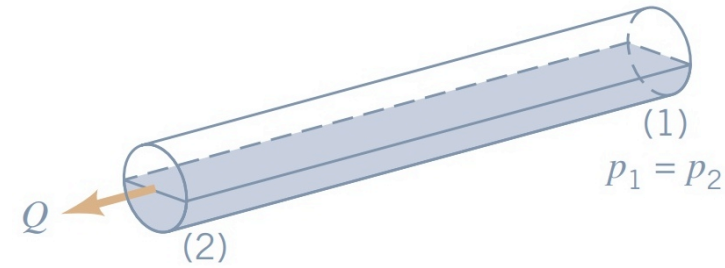
Various pipe flow types exist, each with *distinct* physics

- **Single phase** pipe flows



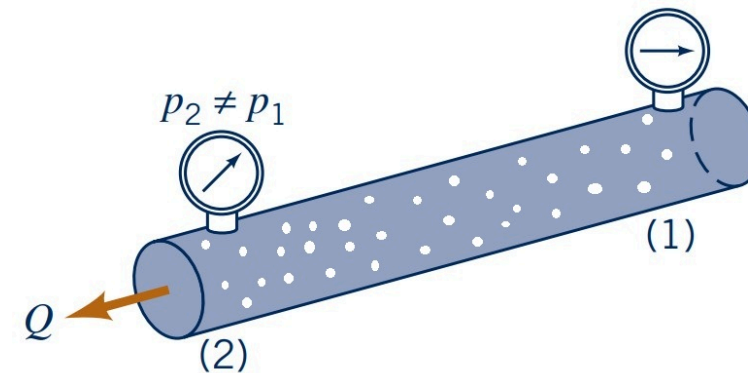
- Completely filled with liquid (or gas)
- Pressure difference ($p_2 - p_1$) drives flow

- **Open channel** pipe flow



- Partially filled with liquid
- Gravity drives flow

- **Multiphase** pipe flow



- Either partially or full of liquid (or gas)
- Gravity drives flow
- Complex physics (bubbles, particles, droplets etc)

Much research focuses on complex multiphase flows (**including own PhD**)

Numerical simulations provide details on the process, which *can be difficult to capture experimentally*.

- Turbulent pipe flow
- Solid particles with $d_p = 10\ \mu\text{m}$
- Particle agglomeration (sticking) occurs due to van der Waals and electrostatic forces

Video: Agglomeration of particles in a turbulent pipe flow

LES-DEM Simulation - Agglomeration in turbulent pipe flow



Why look at *viscous pipe flows* ?

For which applications are viscous pipe flows important?

Viscous pipe flows are important in applications where **fluid viscosity significantly affects pressure drop, energy losses, and flow behavior**. Here are the main areas:

◆ 1. Water Supply and Wastewater Systems

- City water distribution networks
- Sewage and stormwater pipelines
- Irrigation systems
- 👉 Viscous effects determine pumping requirements and pipe sizing.

◆ 2. Oil and Gas Industry

- Crude oil transport through long-distance pipelines (oil can be highly viscous).
- Natural gas pipelines (viscous friction dominates pressure drop).
- Offshore and subsea flowlines.

◆ 3. Chemical and Process Industries

- Transport of viscous fluids like syrups, paints, polymers, and chemicals.
- Cooling and heating systems in reactors (viscosity affects heat transfer too).
- Food industry (chocolate, honey, dairy, sauces).

◆ 4. Energy and Power Systems

- Steam and water circulation in power plants.
- District heating/cooling networks.
- Hydraulic systems in machinery (oil viscosity affects performance).

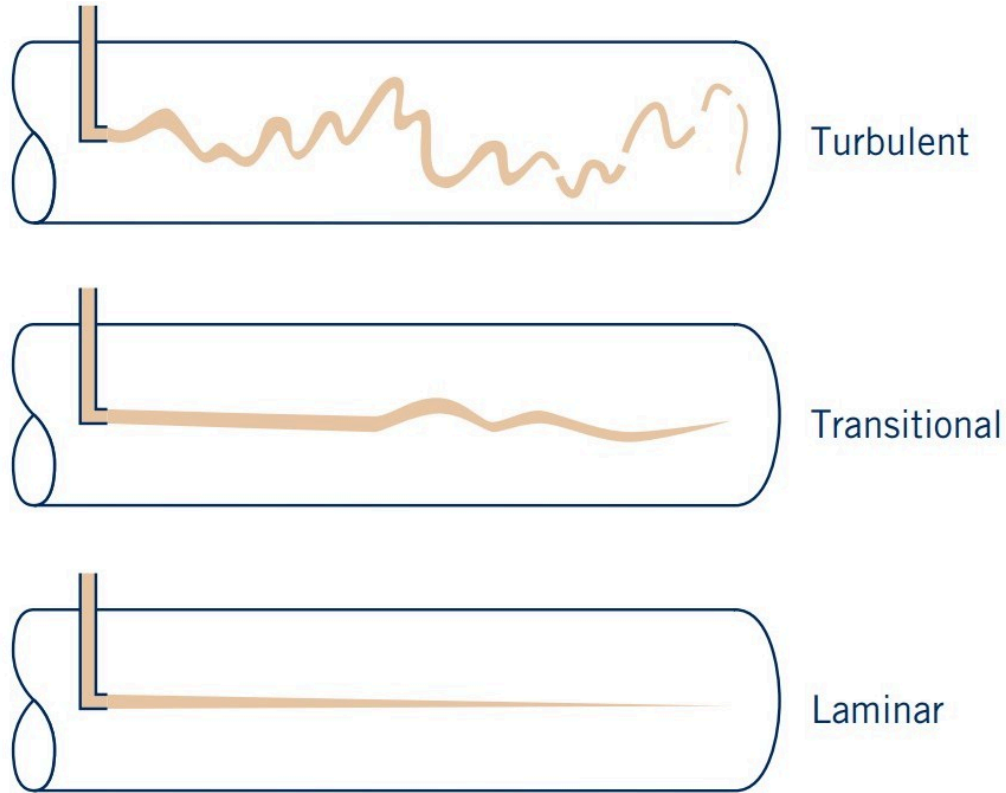
◆ 5. Biomedical Applications

- Blood flow in arteries, veins, and medical devices (blood is a viscous fluid).
- Drug delivery systems using catheters and microtubes.

◆ 6. Microfluidics & Advanced Tech

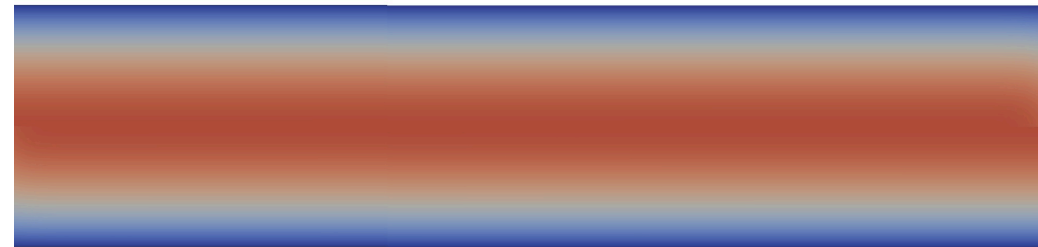
- Lab-on-a-chip devices (very small channels where viscosity dominates over inertia).
- Inkjet printing and 3D printing (flow of inks, resins, or melts).

Laminar and turbulent flows



Characterised by different flow regimes:

- *Turbulent flow*: chaotic and irregular (*mixing occurs*)
- *Transitional flow*: between laminar and turbulent
- *Laminar flow*: smooth and orderly (*no mixing*)



Flow regime depends on Reynolds number ($Re_D = U\rho D/\mu$)

- $Re_D > 4000$: Turbulent
- $2300 < Re_D < 4000$: Transitional
- $Re_D < 2300$: Laminar

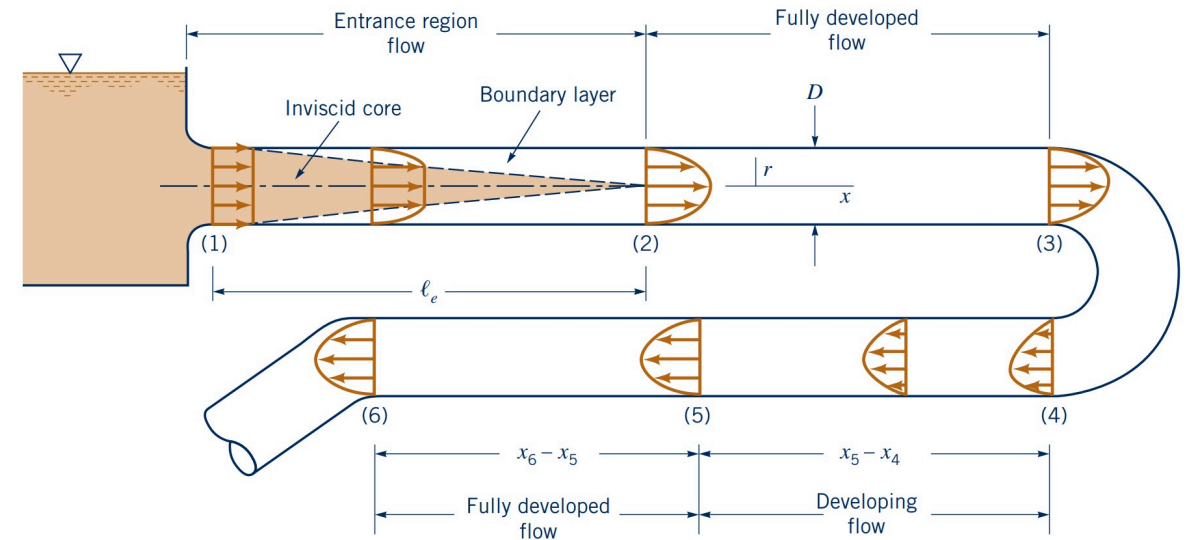
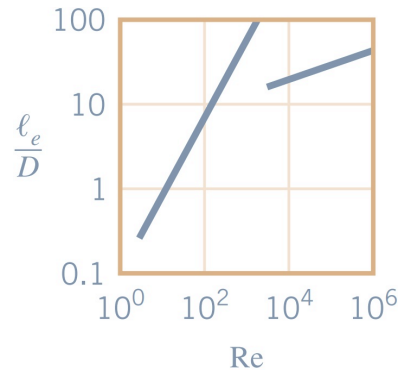
Entrance regions and fully-developed flows

Region close to inlets

- **(1)**: Inviscid (enters with near uniform velocity aka plug flow)
- **(2)**: Reaches fully-developed (boundary layer reached centre)
- **(3)**: Remains fully developed and nothing changes
- **(4)**: Skewed and starts to develop again
- **(5)**: Reaches fully-developed again
- **(6)**: Remains fully-developed and nothing changes

Entrance length l_e depends on Reynolds number:

- Laminar: $l_e/D \approx 0.06\text{Re}_D$
- Turbulent: $l_e/D \approx 4.4\text{Re}_D^{1/6}$



Pressure profile in pipe flows

Pressure varies *in fully-developed laminar flow linearly* along the pipe

- $\partial p / \partial x = -\Delta p / l < 0$

Pressure drop in pipe flows is balanced by:

- Pressure drop due to friction (viscous effects)
- Pressure drop due to acceleration/deceleration of flow (inertia effects)
- Hydrostatic pressure variation due to elevation changes

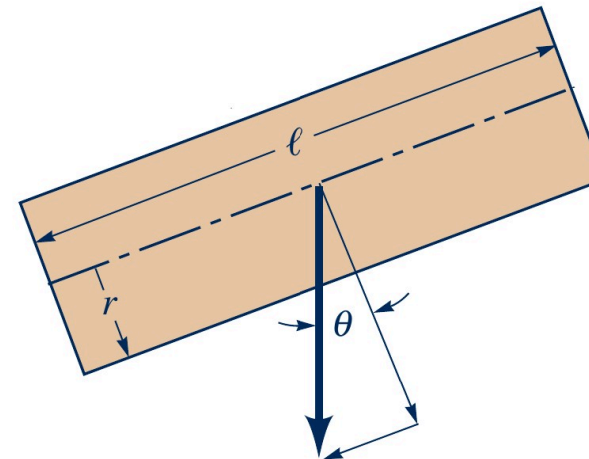
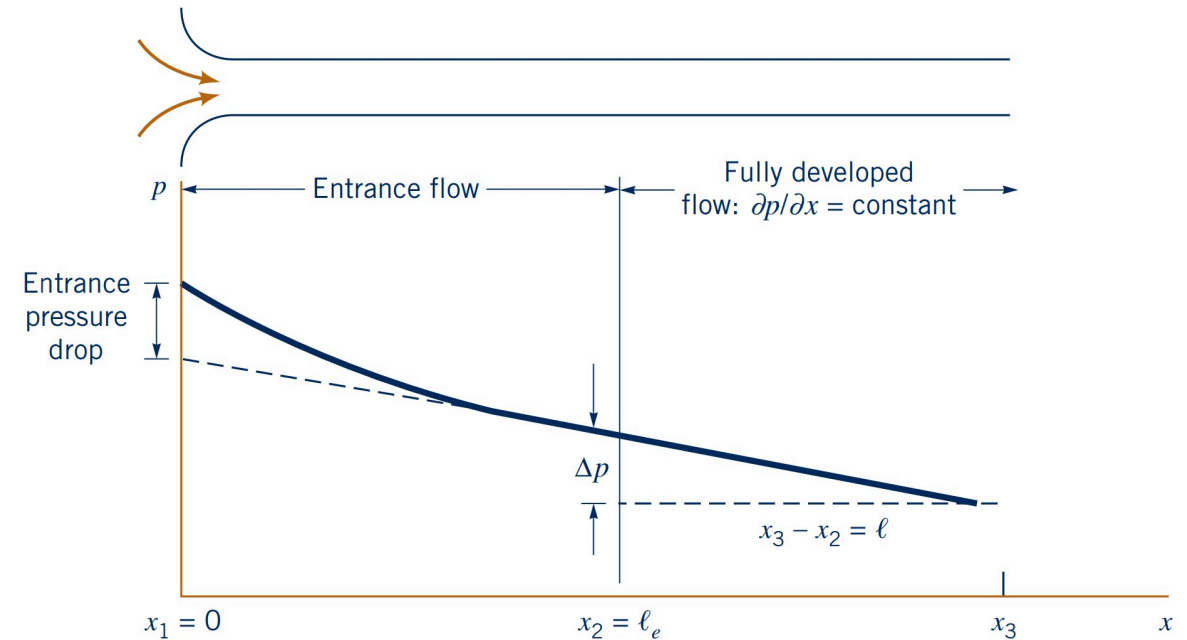
Recall from analytical solutions earlier:

$$Q = \frac{\pi R^4}{8\mu} \left(-\frac{\partial p}{\partial z} \right) = \frac{\pi D^4}{128\mu} \left(-\frac{\partial p}{\partial z} \right) = \frac{\pi D^4 \Delta p}{128\mu l}$$

- *Valid* for laminar, steady, horizontal, fully-developed flow

For non-horizontal pipes:

$$Q = \frac{\pi D^4 (\Delta p - \rho g l \sin \theta)}{128\mu l}$$



Exercise: Laminar Pipe Flow

An oil with a viscosity of $\mu = 0.1 \text{ Pa} \cdot \text{s}$ and density $\rho = 900 \text{ kg/m}^3$ flows in a pipe of diameter $D = 0.020 \text{ m}$.

- (a) What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5} \text{ m}^3/\text{s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10 \text{ m}$?
- (b) How steep a hill, θ , must the pipe be on to if the oil is to flow through the pipe at the same rate as in part (a), but with $p_1 = p_2$?
- (c) For the conditions of part (b), if $p_1 = 200 \text{ kPa}$, what is the pressure at section $x_3 = 5 \text{ m}$, where x is measured along the pipe?

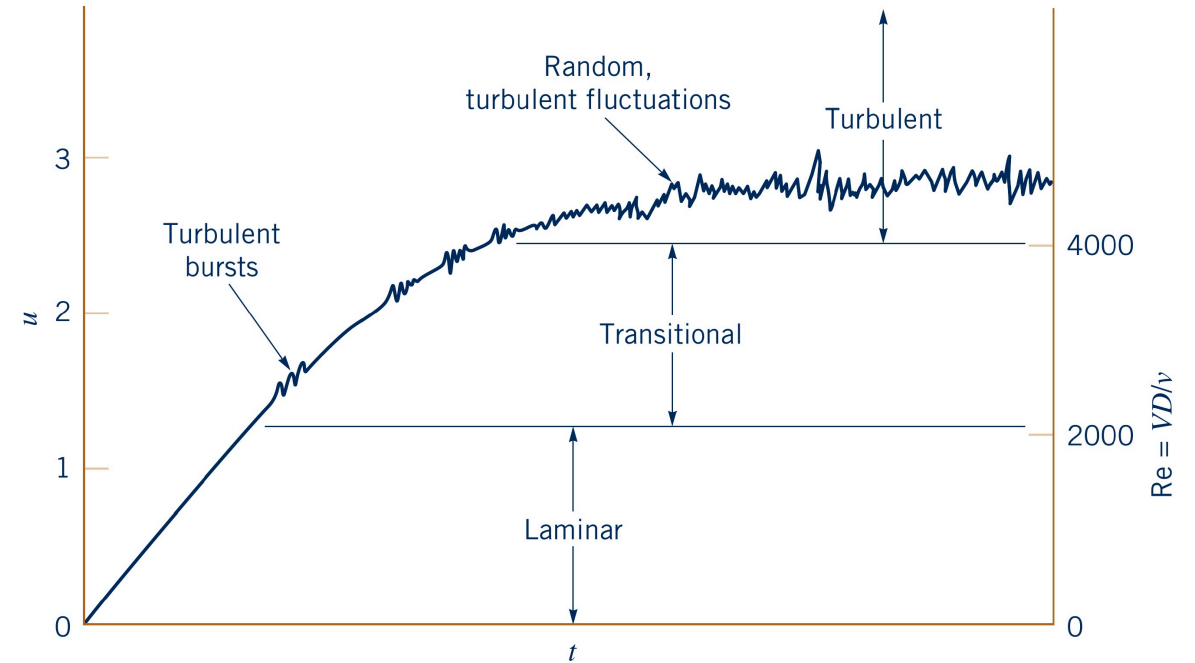
Transition from laminar to turbulent flow

Happens *gradually* between $Re \approx 2100$ and $Re \approx 4000$

- Small *bursts* of turbulence start at $Re \approx 2100$
- Bursts grow in *size* and *frequency* with increasing Re
- Increased mixing and momentum transfer occurs
- Higher *heat transfer rates* and *pressure drop*

Velocity field \mathbf{V} *changes* from being 1-D to 3-D

- *Laminar*: $\mathbf{V} = v_x \cdot \hat{\mathbf{i}}$
- *Turbulent*: $\mathbf{V} = v_x \cdot \hat{\mathbf{i}} + v_y \cdot \hat{\mathbf{j}} + v_z \cdot \hat{\mathbf{k}}$



Decomposing a turbulent flow

Consider a turbulent flow with instantaneous velocity $u(t)$

- *Time-averaged* velocity component \bar{u}

$$\bar{u}(x, y, z) = \frac{1}{T} \int_{t_0}^{T+t_0} u(x, y, z, t) dt$$

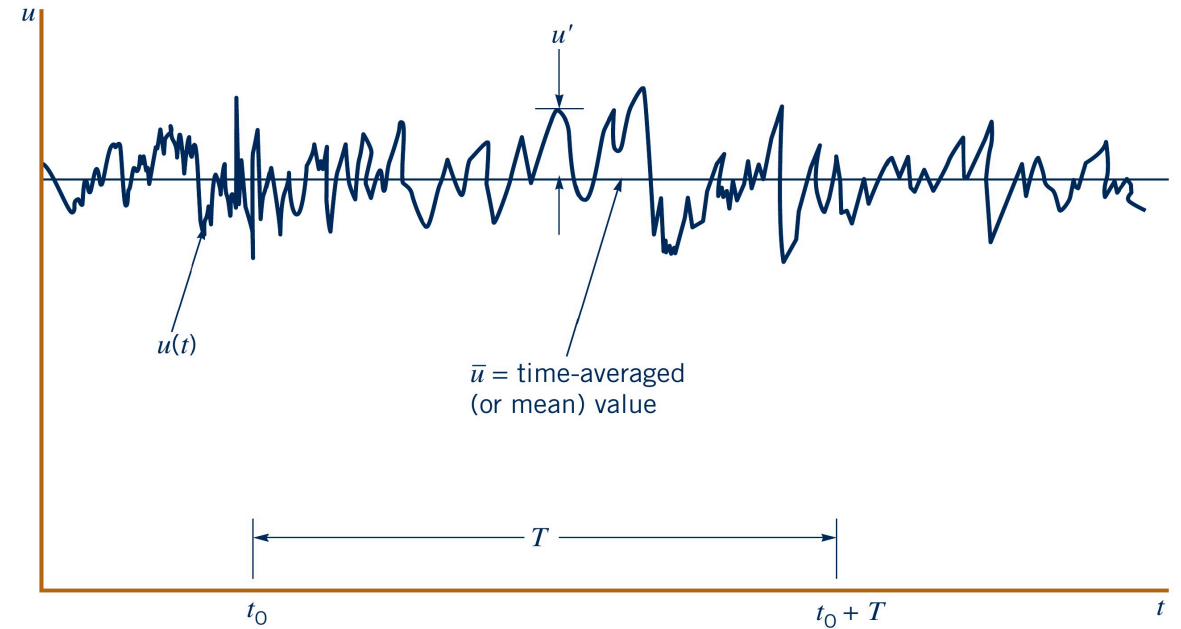
- T is the *averaging time period* (should be long enough to capture the relevant flow structures)

- *Instantaneous* velocity can then be decomposed as:

$$u(x, y, z, t) = \bar{u}(x, y, z) + u'(x, y, z, t)$$

- *Fluctuating* velocity component u'

$$u'(x, y, z, t) = u(x, y, z, t) - \bar{u}(x, y, z)$$



Describing turbulence

Relevant to quantify strength of fluctuations

- Clearly, *time-averaging* fluctuations $\overline{u'}$ does not make sense

$$\overline{u'}(x, y, z) = \frac{1}{T} \int_{t_0}^{T+t_0} u'(x, y, z, t) dt = 0$$

- What if we take *square* fluctuations and then *time-average*?

$$\overline{(u')^2}(x, y, z) = \frac{1}{T} \int_{t_0}^{T+t_0} u'(x, y, z, t)^2 dt > 0$$

- To get "back" to same order of magnitude, we now take *the square root of* fluctuations

$$u_{\text{rms}}(x, y, z) = \sqrt{\overline{(u')^2}(x, y, z)}$$

- This is called the *root-mean-square (RMS)* of the fluctuations
- Often normalised by the time-averaged velocity to get a *relative measure*, aka *turbulence intensity*

$$I(x, y, z) = \frac{u_{\text{rms}}(x, y, z)}{\overline{u}(x, y, z)}$$

