

Fluid Mechanics

# Short introduction to Navier-Stokes equations

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# Any experience with the Navier-Stokes Equations?

# Richard Feynman on Turbulence

*“Turbulence is the most important unsolved problem of classical physics.” - Eames & Flor (2011)*

*“There is a physical problem that is common to many fields, that is very old, and that has not been solved. It is not the problem of finding new fundamental particles, but something left over from a long time ago—over a hundred years. Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of circulating or turbulent fluids.” - Feynman et al. (1965)*

# A Millennium Problem

The Clay Mathematics Institute has identified the existence and smoothness of solutions to the Navier-Stokes equations as one of the seven Millennium Prize Problems.

- *Sketch of the Problem*

Prove or give a counter-example that in 3-D, solutions to the incompressible Navier-Stokes equations always exist and are smooth for all time.

- *Prize*

\$1,000,000 for a correct solution.



## Millennium Prize Problems

The **Millennium Prize Problems** are seven well-known complex [mathematical](#) problems selected by the [Clay Mathematics Institute](#) in 2000. The Clay Institute has pledged a US \$1 million prize for the first correct solution to each problem.

The Clay Mathematics Institute officially designated the title **Millennium Problem** for the seven unsolved mathematical problems, the [Birch and Swinnerton-Dyer conjecture](#), [Hodge conjecture](#), [Navier–Stokes existence and smoothness](#), [P versus NP problem](#), [Riemann hypothesis](#), [Yang–Mills existence and mass gap](#), and the [Poincaré conjecture](#) at the Millennium Meeting held on May 24, 2000. Thus, on the official website of the Clay Mathematics Institute, these seven problems are officially called the **Millennium Problems**.

To date, the only Millennium Prize problem to have been solved is the [Poincaré conjecture](#). The Clay Institute awarded the monetary prize to Russian mathematician [Grigori Perelman](#) in 2010. However, he declined the award as it was not also offered to [Richard S. Hamilton](#), upon whose work Perelman built.<sup>[1]</sup>

## Overview

The Clay Institute was inspired by a set of twenty-three problems organized by the mathematician [David Hilbert](#) in 1900 which were highly influential in driving the progress of mathematics in the twentieth century.<sup>[2]</sup> The seven selected problems span a number of mathematical fields, namely [algebraic geometry](#), [arithmetic geometry](#), [geometric topology](#), [mathematical physics](#), [number theory](#), [partial differential equations](#), and [theoretical computer science](#). Unlike Hilbert's problems, the problems selected by the Clay Institute were already renowned among professional mathematicians, with many actively working towards their resolution.<sup>[3]</sup>

The seven problems were officially announced by [John Tate](#) and [Michael Atiyah](#) during a ceremony held on May 24, 2000 (at the amphithéâtre [Marguerite de Navarre](#)) in the [Collège de France](#) in [Paris](#).<sup>[4]</sup>

[Grigori Perelman](#), who had begun work on the [Poincaré conjecture](#) in the 1990s, released his proof in 2002 and 2003. His refusal of the Clay Institute's monetary prize in 2010 was widely covered in the media. The other six Millennium Prize Problems remain unsolved, despite a large number of unsatisfactory proofs by both amateur and professional mathematicians.

[Andrew Wiles](#), as part of the Clay Institute's scientific advisory board, hoped that the choice of [US\\$1 million](#) prize money would popularize, among general audiences, both the selected problems as well as the "excitement of mathematical endeavor".<sup>[5]</sup> Another board member, [Fields medalist](#) [Alain Connes](#), hoped that the publicity around the unsolved problems would help to combat the "wrong idea" among the public that mathematics would be "overtaken by computers".<sup>[6]</sup>

Some mathematicians have been more critical. [Anatoly Vershik](#) characterized their monetary prize as "show business" representing the "worst manifestations of present-day mass culture", and thought that there are more meaningful ways to invest in public appreciation of mathematics.<sup>[7]</sup> He viewed the superficial media treatments of Perelman and his work,

# The Navier-Stokes Equations

*x*-direction:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x$$

*y*-direction:

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y$$

*z*-direction:

$$\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z$$

Continuity equation (*incompressible flow*):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

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