

Heat Transfer

Lumped system assumption

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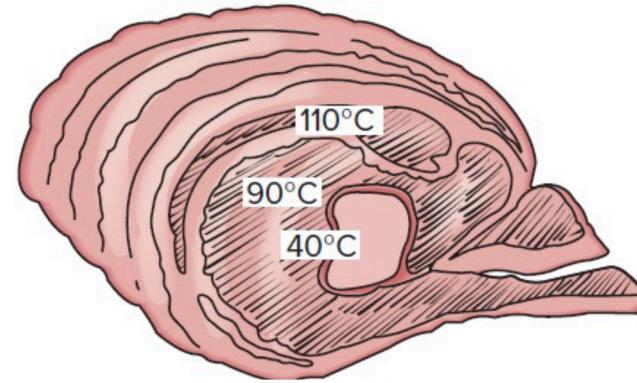
What is the lumped system assumption and why?

- Most heat transfer problems are complex because temperature varies in both with time and space
 - In **lumped systems**, temperature **only varies with time and not space**

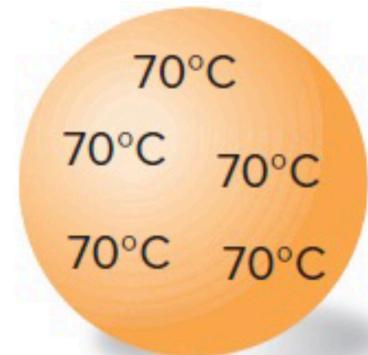
$$T = f(x, y, z, t)$$

$$\longrightarrow T = f(t)$$

- Applying the lumped system assumption **significantly simplifies** the analysis



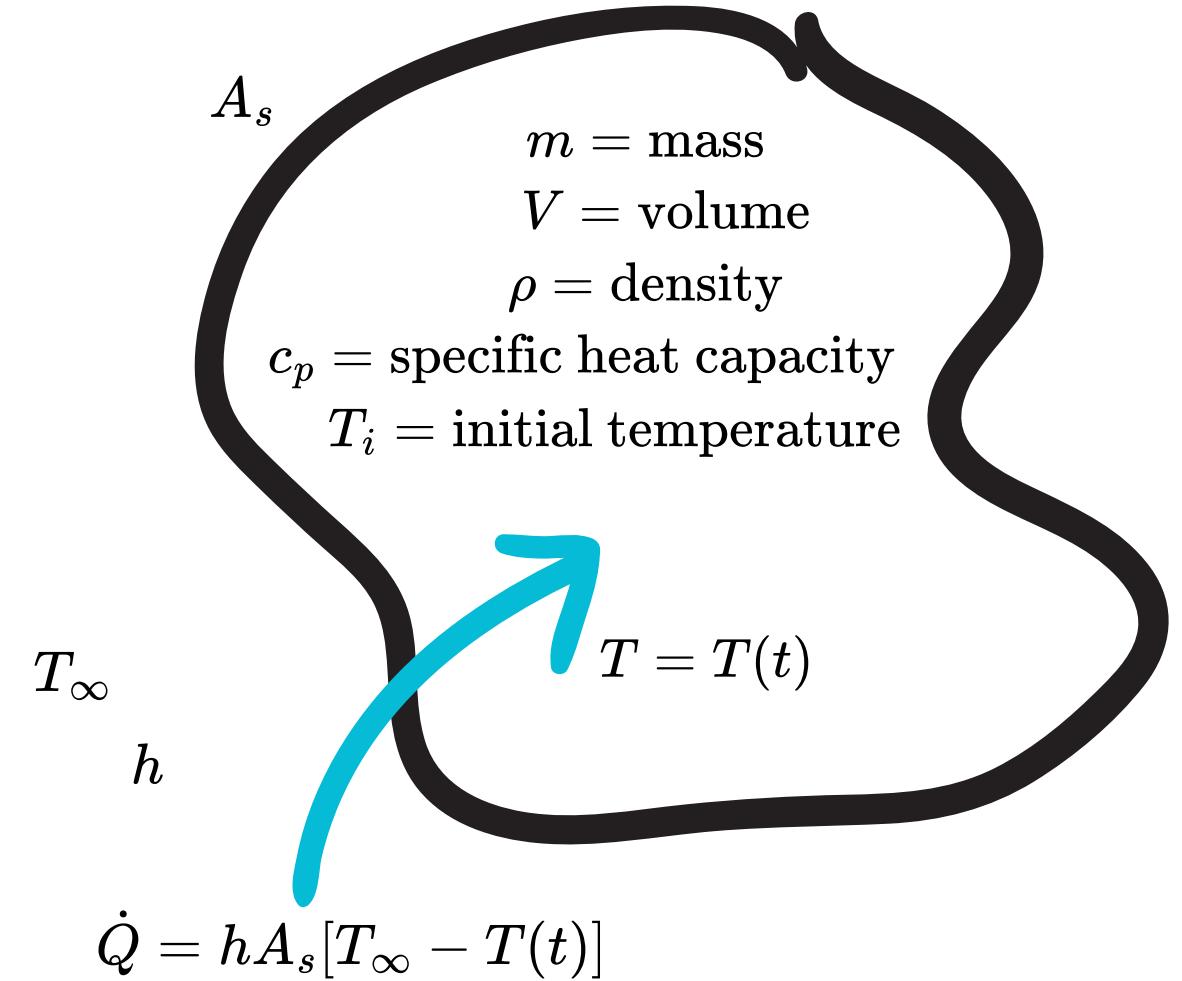
A roast beef (not lumped system)



A copper ball (lumped system)

Starting point of lumped system analysis

- Consider a body of arbitrary shape with:
 - Mass m
 - Volume V
 - Density ρ
 - Specific heat capacity c_p
 - Initial internal temperature T_i
- At time $t = 0$ the body is exposed to convective heat transfer from the outside:
 - Surrounding temperature T_∞
 - Heat transfer coefficient h
 - Surface area A_s



Derivation of the lumped system equation

- Energy transferred to body during dt :

$$\begin{aligned} hA_s(T_\infty - T)dt &= mc_p dT \\ &= \rho V c_p dT \end{aligned}$$

- Because T_∞ is constant, we may expand dT :

$$\begin{aligned} hA_s(T_\infty - T)dt &= -hA_s(T - T_\infty)dt \\ &= \rho V c_p d(T - T_\infty) \end{aligned}$$

- Re-arranging:

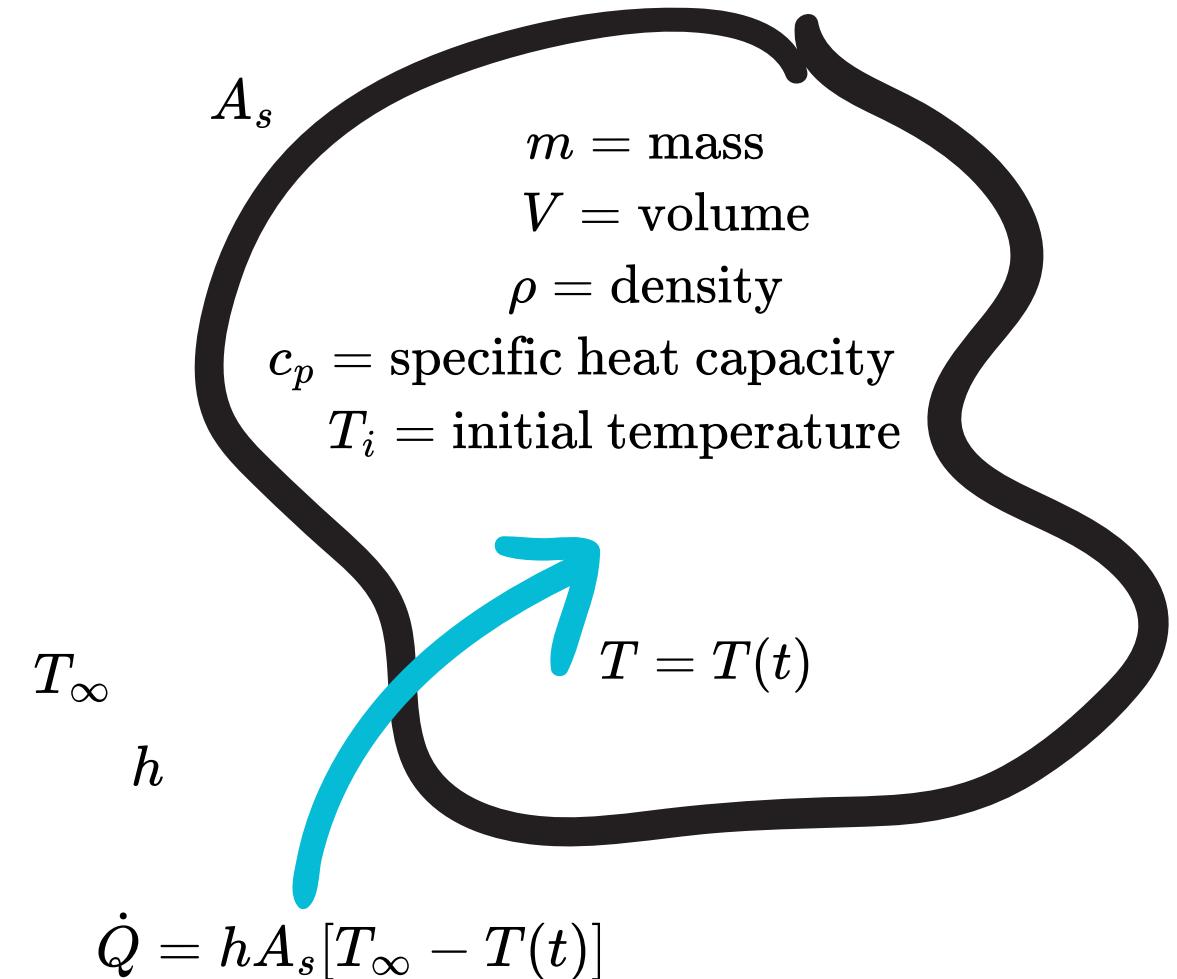
$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p} dt$$

- Now, integrating from $t = 0$ where $T = T_i$ to any time t at which $T = T(t)$ we get:

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$

- Exponential on both sides gives **lumped system equation**:

$$\frac{T(t) - T_\infty}{T - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t}$$



The time constant for lumped systems

- Introducing $b = hA_s/(\rho V c_p)$, we obtain:



$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t} = e^{-bt}$$

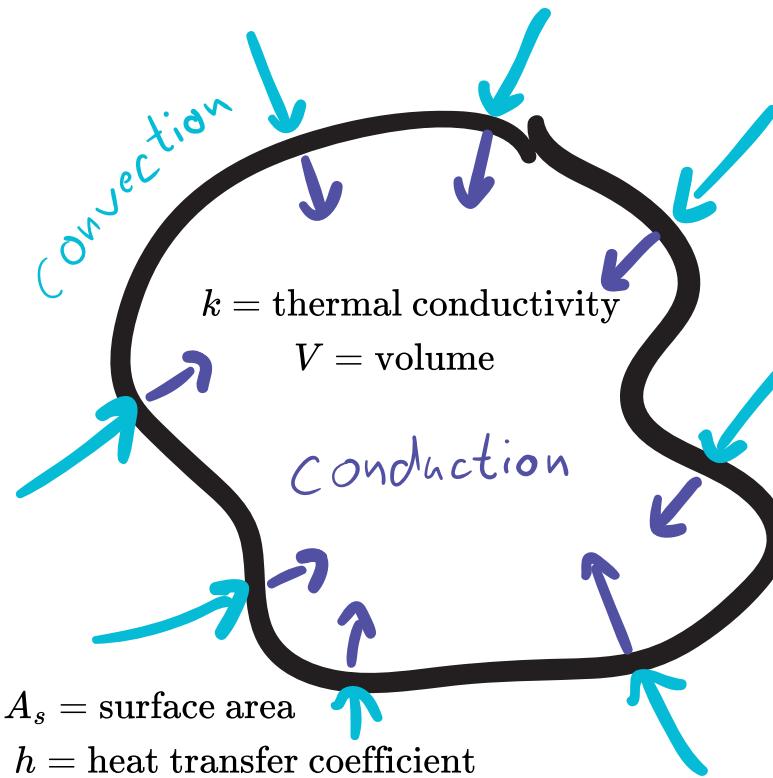
- The time constant ($1/b$) describes the rate at which the system approaches the surrounding temperature T_∞
 - Large b value**, system approaches surrounding temperature **quickly**
 - Small b value**, system approaches surrounding temperature **slowly**

Validity of lumped system assumption

- We define a **Biot number**:

$$Bi = hL_c/k$$

- with the characteristic length $L_c = V / A_s$
- If $Bi = 0$: lumped system assumption is **exact**
- If $Bi \leq 0.1$: lumped system assumption is **reasonable accurate**
- If $Bi > 0.1$: lumped system assumption is **inaccurate**



Example

Course material - Lumped system analysis - Example 1

