Fluid Mechanics
Boundary layers

Lecturer: Jakob Hærvig
Slides by Jakob Hærvig (AAU Energy) and Jacob Andersen (AAU Build)

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The beauty of boundary layers

Spatially developing turbulent boundary layer on a flat plate

Why should I care about boundary layers?

Great question — and one that gets surprisingly deep fast.

You should care about **boundary layers** because they govern how fluids (air, water, etc.) actually interact with surfaces — and that interaction affects **drag, heat transfer, lift, pressure loss, and even noise**. Here's a breakdown depending on what you're doing:

🧠 The general idea

The **boundary layer** is the thin region of fluid right next to a surface where viscosity matters — where the fluid "feels" the surface and slows down (for air, it's often just millimeters thick).

Outside that thin layer, the fluid behaves almost as if it were inviscid (frictionless). But inside it, viscosity and shear dominate. That makes the boundary layer the "gateway" through which all momentum, heat, and mass transfer between the surface and the bulk flow must pass.

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⚙️ Why it matters (in practical terms)

1. **Aerodynamics / Hydrodynamics**
   - The state of the boundary layer (laminar vs. turbulent) determines **drag** on airplanes, cars, and ships.
   - It affects **flow separation**, which changes lift and stability — critical for aircraft wings and race car aerodynamics.
   - Example: A golf ball’s dimples trip the boundary layer into turbulence early to delay separation → less pressure drag → flies farther.

2. **Heat transfer**
   - In heat exchangers or electronics cooling, thinner boundary layers mean higher heat transfer rates.
   - Engineers manipulate flow to control boundary layer thickness and turbulence for better thermal performance.

3. **Pipes and ducts**
   - In internal flows, the boundary layers from the walls eventually meet, defining the **fully developed flow** region — key for calculating pressure losses and pumping power.

4. Environmental and biological systems

- Boundary layers over leaves affect gas exchange.
- Ocean boundary layers influence nutrient mixing and sediment transport.
- Atmospheric boundary layers govern weather near the ground.

⚖️ The physics connection

All the “interesting” effects of viscosity — shear stress, friction drag, heat conduction — happen *because* of gradients in the boundary layer. That’s why in fluid dynamics, we often solve for:

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

That derivative (the velocity gradient at the wall) exists because of the boundary layer.

Boundary layer structure

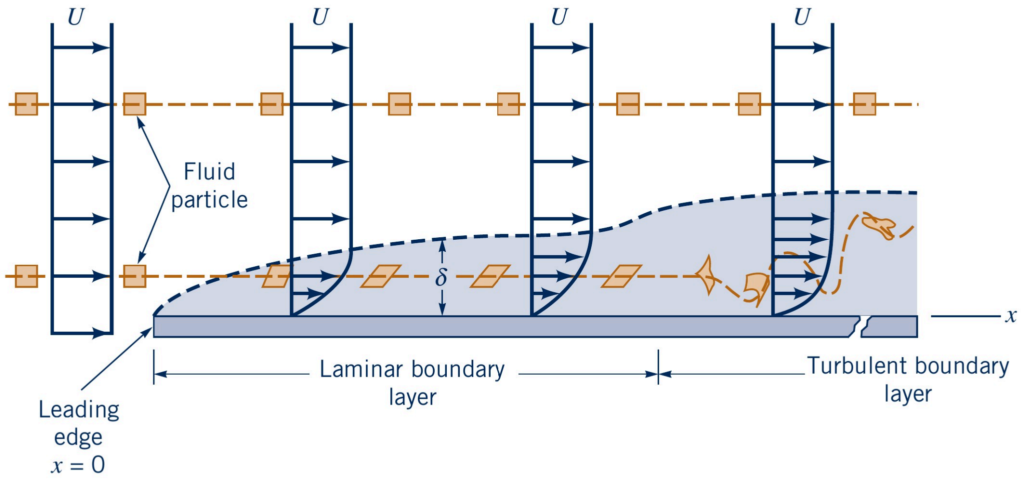
Boundary layer (BL) structure and development for *parallel flow over an infinite flat plate*

- $u(y) = 0$ at the surface (*no-slip condition*)
- $u(y) = U$ far away from surface (*free stream condition*)

Convenient to *define Reynolds number* as $\mathrm{Re}_x = xU/\nu$

- Laminar BL: $\mathrm{Re}_x < 5 \times 10^5$:
- Turbulent BL: $\mathrm{Re}_x > 5 \times 10^5$:

Critical Reynolds number varies with *surface roughness* and *upstream turbulence*



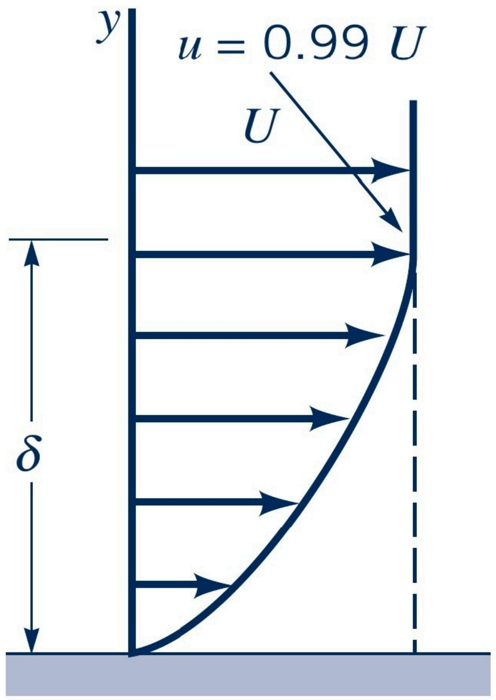
Boundary layer thickness: *Standard*

Flat plate exposed to flow with *free stream velocity*, U

- Boundary layer *thickness* δ defined as:

$$\delta = y \qquad \text{where} \qquad u(y) = 0.99U$$

Note: The 99% criterion is kind of arbitrary



Boundary layer thickness: *Displacement* and *momentum thicknesses*

Flat plate exposed to flow with *free stream velocity*, U

- Boundary layer *displacement thickness δ^** defined as:

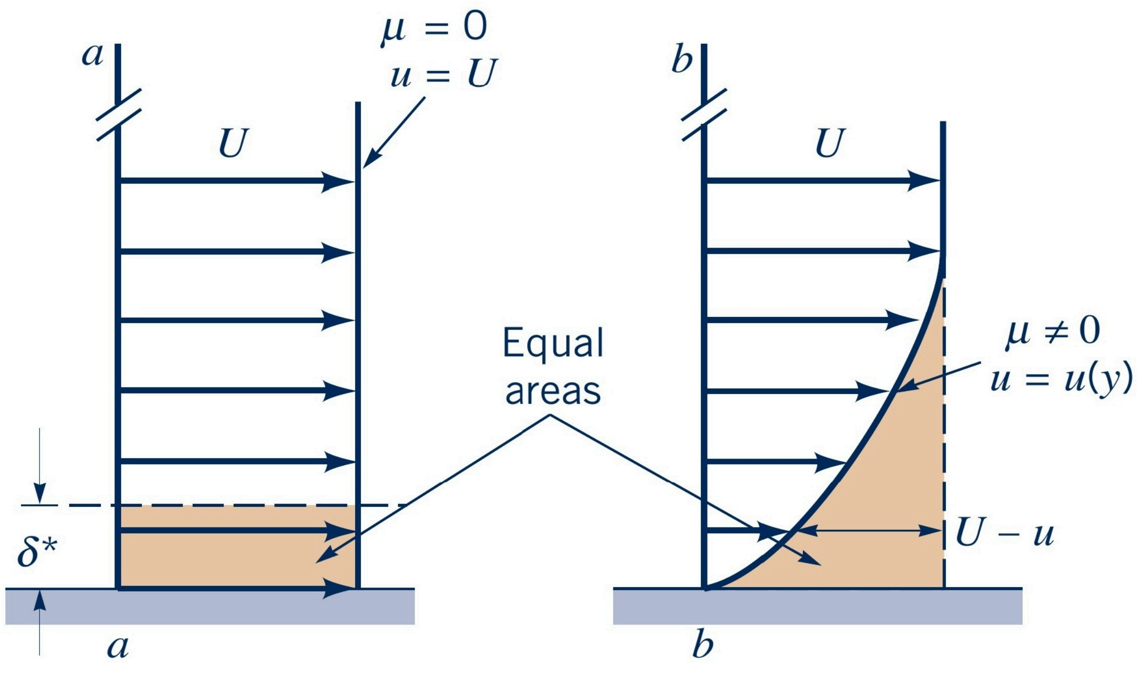
$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) \mathrm{d}y$$

  - **Think as**: Distance the wall should be moved outward *to maintain the same volume flow rate for an inviscid flow*

- Boundary layer *momentum thickness θ* defined as:

$$\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) \mathrm{d}y$$

  - **Think as**: Distance the wall should be moved outward *to maintain the same momentum flow rate for an inviscid flow*



$$\delta^* b U = \int_0^\infty (U - u)\, b\mathrm{d}y$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) \mathrm{d}y$$

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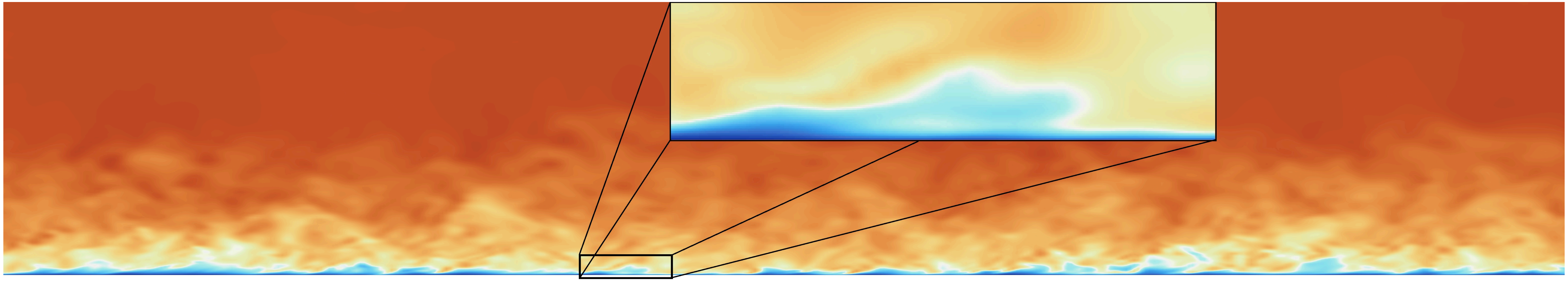
Turbulent boundary layer regions

Divided into *three regions* away from the wall

1. *Viscous sublayer*: Laminar part, viscous effects
2. *Buffer layer*: Transitional part, both viscous and inertial effects
3. *Log-law layer*: Turbulent part, inertial effects dominant



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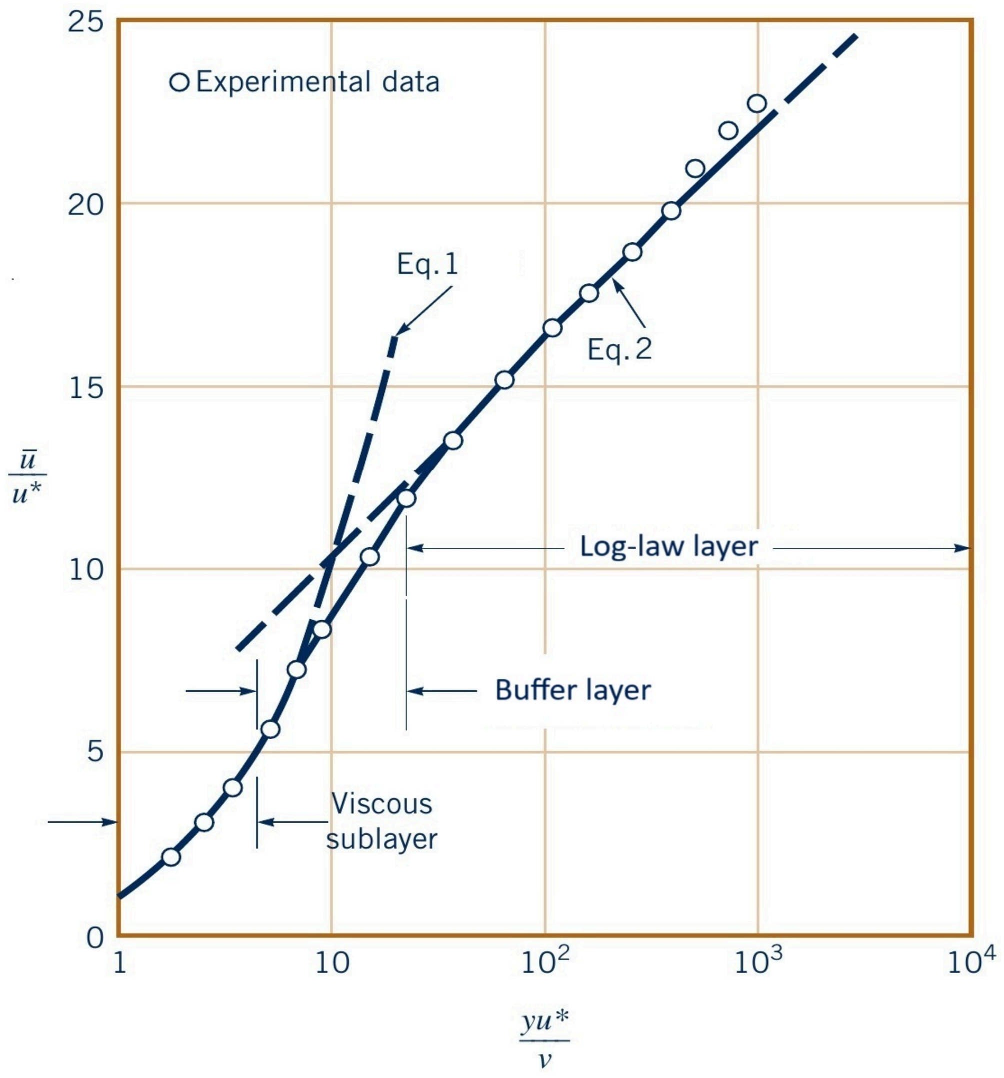
Turbulent boundary layer velocity profile

Based on dimensional analysis, we obtain *two dimensionless groups*:

- *Dimensionless velocity*: $u^+ = \overline{u}/u^*$
- *Dimensionless position*: $y^+ = yu^*/\nu$

  - Time-averaged velocity, $\overline{u}$
  - Friction velocity, $u^* = \sqrt{\frac{\tau_w}{\rho}}$ where $\tau_w = \mu \frac{\mathrm{d}\overline{u}}{\mathrm{d}y}|_{y=0}$
  - Distance from wall, y



Velocity profiles in *log-log plot*

- *Viscous sublayer*: Linear profile, $u^+ = y^+$ (**Eq. 1**)
  - Valid for $y^+ < 5$
- *Log-law layer*: Logarithmic profile, $u^+ = \frac{1}{2.5} \ln y^+ + 5.0$ (**Eq. 2**)
  - Valid for $y^+ > 30$

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Laminar boundary layer velocity profile

Boundary layer thickness δ (Blasius solution)

$$\delta = \frac{5x}{\sqrt{\mathrm{Re}_x}}$$

Boundary layer displacement thickness δ^* (Blasius solution)

$$\delta^* = \frac{1.721x}{\sqrt{\mathrm{Re}_x}}$$

Boundary layer momentum thickness θ (Blasius solution)

$$\theta = \frac{0.664x}{\sqrt{\mathrm{Re}_x}}$$

Note: Boundary layer thickness decreases with increasing Re_x:

$\delta \to 0$ as $\mathrm{Re}_x \to \infty$

Looking for an *analytical solution*..

x-direction:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

y-direction:

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \right)$$

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Boundary conditions:

$u = v = 0$ at $y = 0$

$u = U$ at $y \to \infty$