

Heat Transfer

Lumped system assumption

Lecturer: Jakob Hærvig

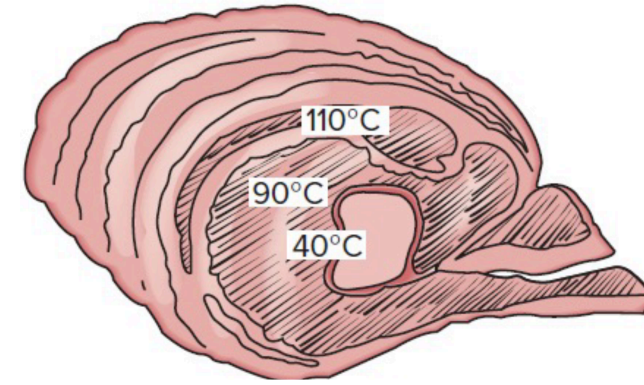
What is the lumped system assumption and why?

Most heat transfer problems are complex because temperature *varies with both space and time*

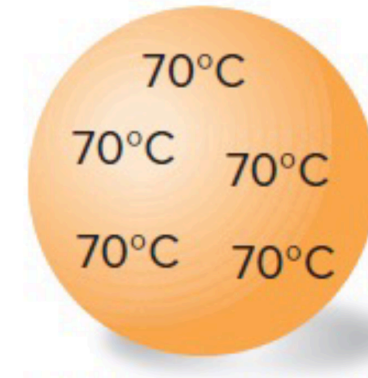
- In **lumped systems**, temperature *only varies with time (not space)*

$$T = f(x, y, z, t) \rightarrow T = f(t)$$

- Significantly *simplifies* the analysis!



A roast beef (not lumped system)

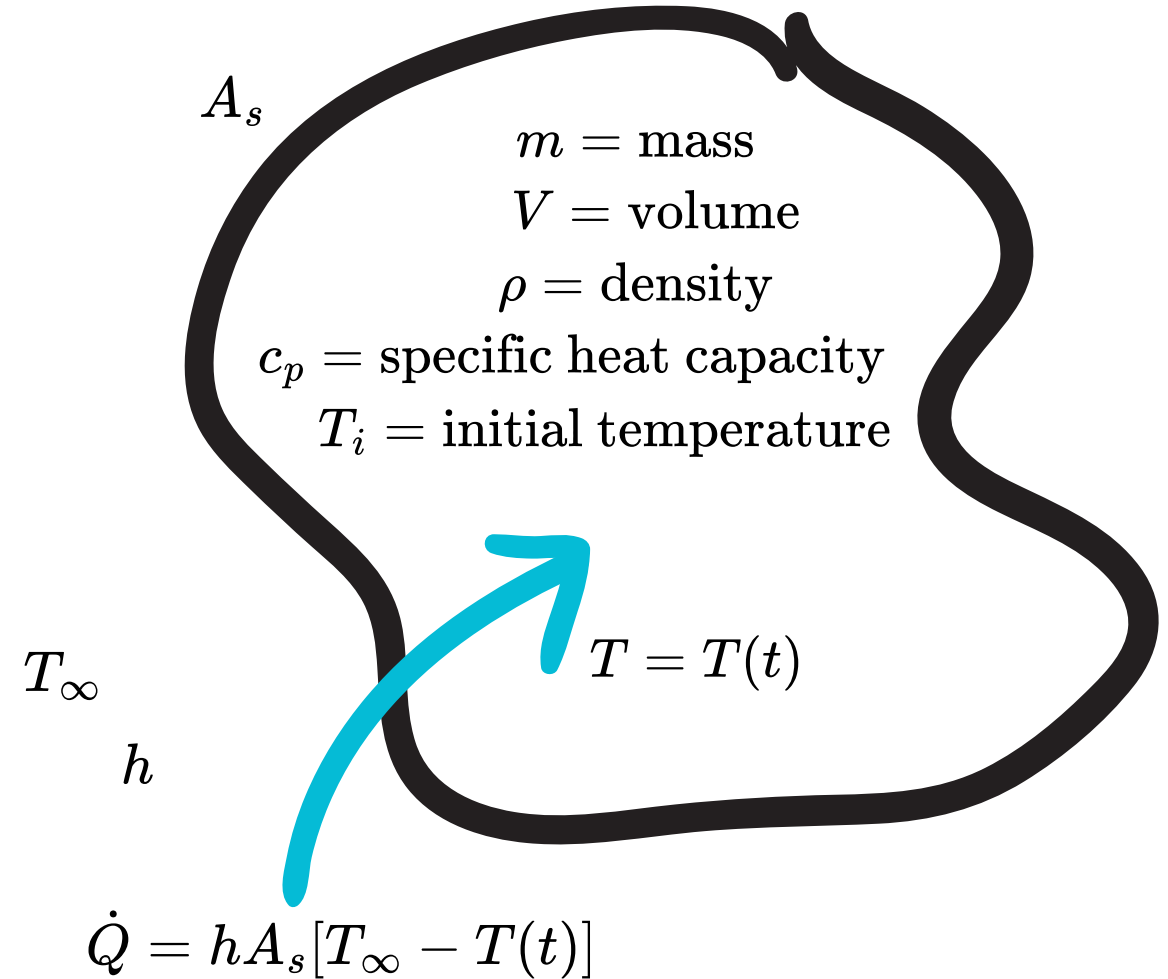


A copper ball (lumped system)

Starting point of lumped system analysis

Consider a body of arbitrary shape with

- At time $t = 0$ the body is exposed to convective heat transfer from the outside:
 - Surrounding temperature T_∞
 - Heat transfer coefficient h
 - Surface area A_s



Derivation of the lumped system equation

Energy transferred to body during dt :

$$\dot{Q} = hA_s(T_\infty - T)dt = mc_p dT = \rho V c_p dT$$

Because T_∞ is constant, we may expand dT :

$$\dot{Q} = hA_s(T_\infty - T)dt = \rho V c_p d(T - T_\infty)$$

..Re-arranging:

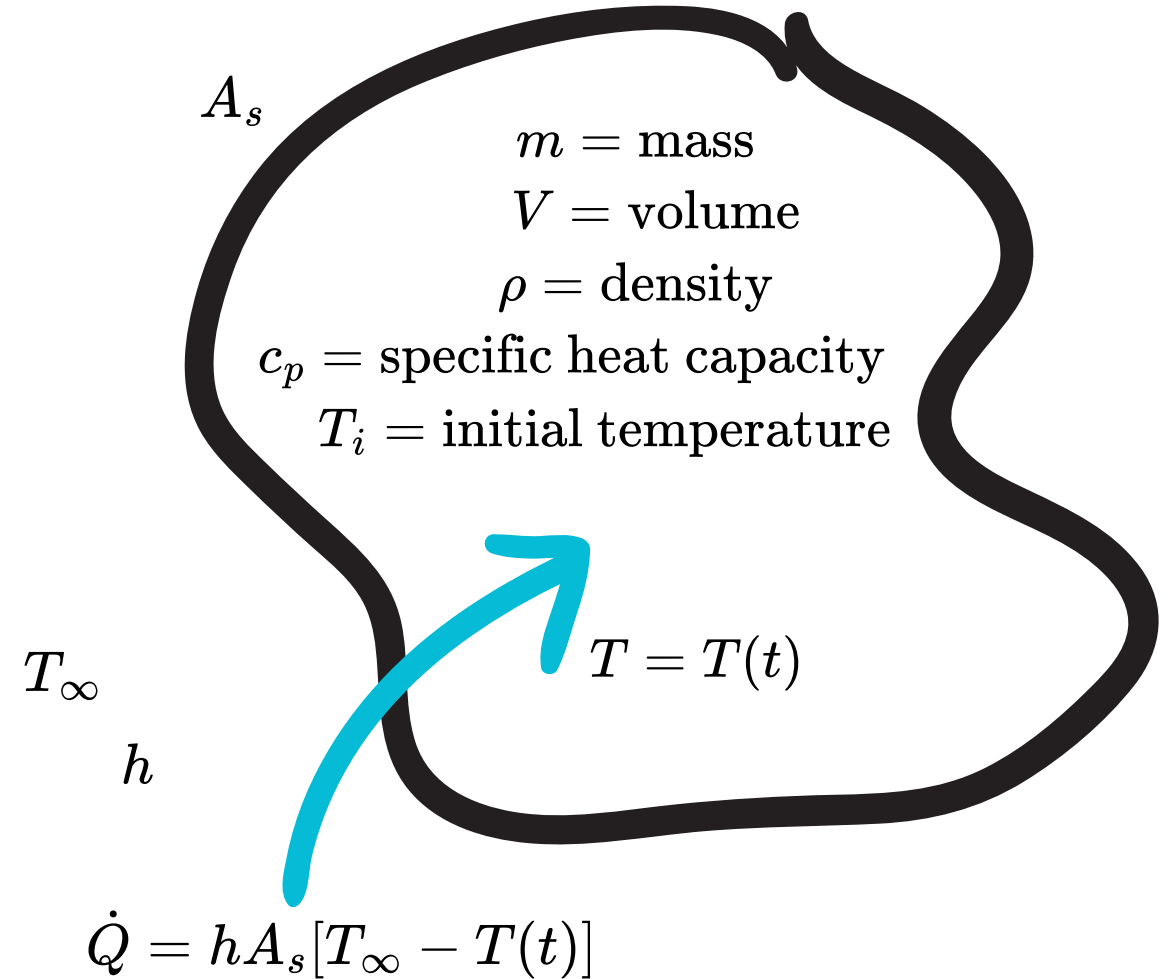
$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p} dt$$

Now, integrate from $t = 0$ where $T = T_i$ to t at which $T = T(t)$:

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$

Exponential on both sides gives **lumped system equation**:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t}$$



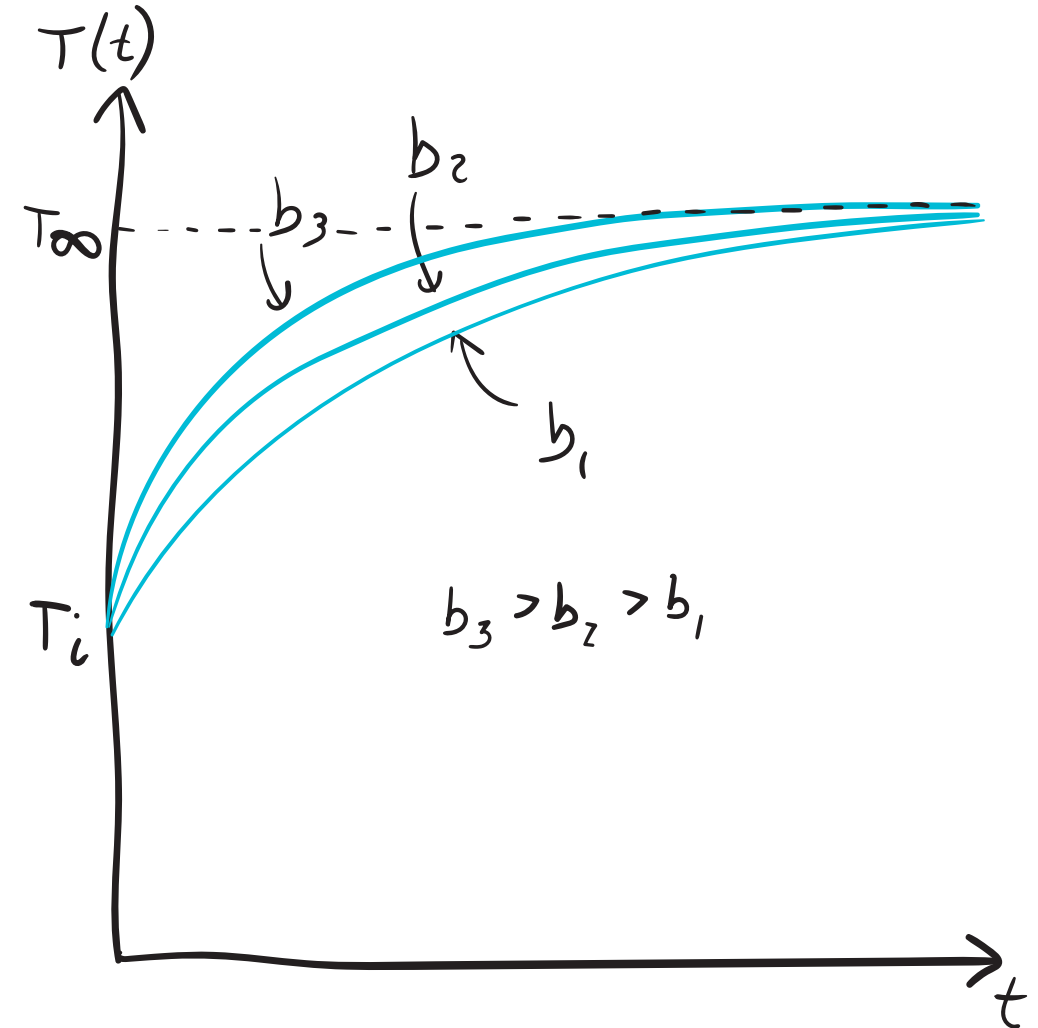
The *time constant* for lumped systems

Introducing $b = hA_s/(\rho V c_p)$, we obtain:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t} = e^{-bt}$$

The *time constant* ($1/b$) describes the rate at which the system approaches the surrounding temperature T_∞

- *Large b* : Temperature approaches T_∞ quickly
- *Small b* : Temperature approaches T_∞ slowly



Validity of lumped system assumption

We define a *Biot number*

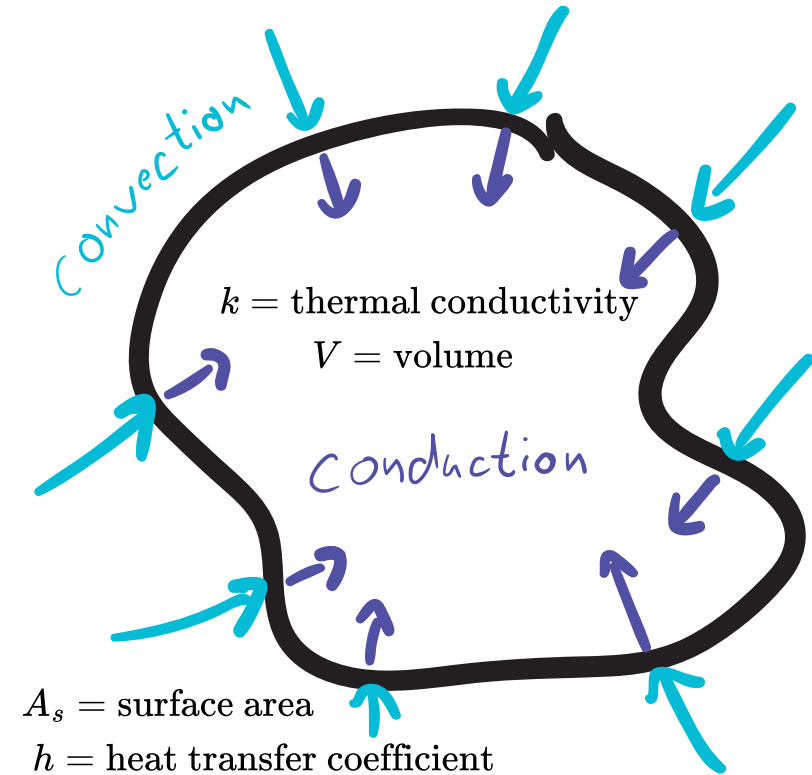
$$Bi = hL_c/k$$

- with the characteristic length $L_c = V/A_s$

If $Bi = 0$: lumped system assumption is **exact**

If $Bi \leq 0.1$: lumped system assumption is **reasonable accurate**

If $Bi > 0.1$: lumped system assumption is **inaccurate**



Exercise: Fast temperature measurements with thermocouples

The temperature of a gas stream is to be measured by a thermocouple. The thermocouple junction can be approximated as a 1 mm diameter sphere. The junction properties are $k = 35 \text{ W/m}\cdot\text{K}$, $\rho = 8500 \text{ kg/m}^3$ and $c_p = 320 \text{ J/kg}\cdot\text{K}$.

How quickly can the thermocouple respond to changes in temperature when the convective heat transfer coefficient between the junction and the gas is $h = 210 \text{ W/m}^2\cdot\text{K}$?

