

Heat Transfer

# Lumped system assumption

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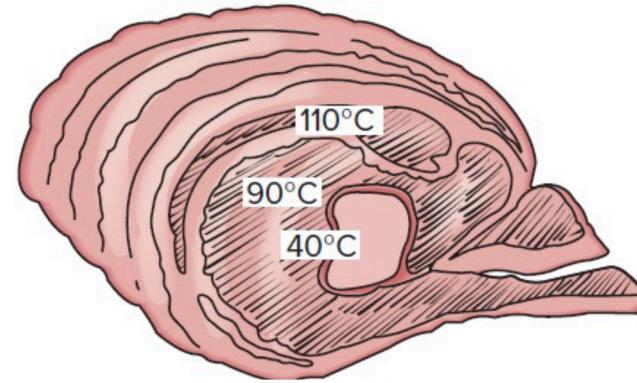
## What is the lumped system assumption and why?

- Most heat transfer problems are complex because temperature varies in both with time and space
  - In **lumped systems**, temperature **only varies with time and not space**

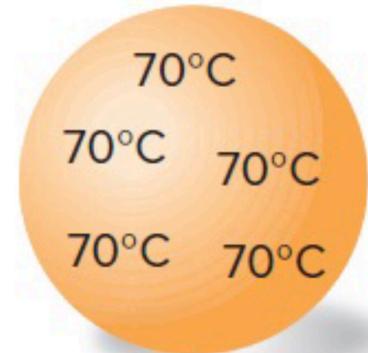
$$T = f(x, y, z, t)$$

$$\longrightarrow T = f(t)$$

- Applying the lumped system assumption **significantly simplifies** the analysis



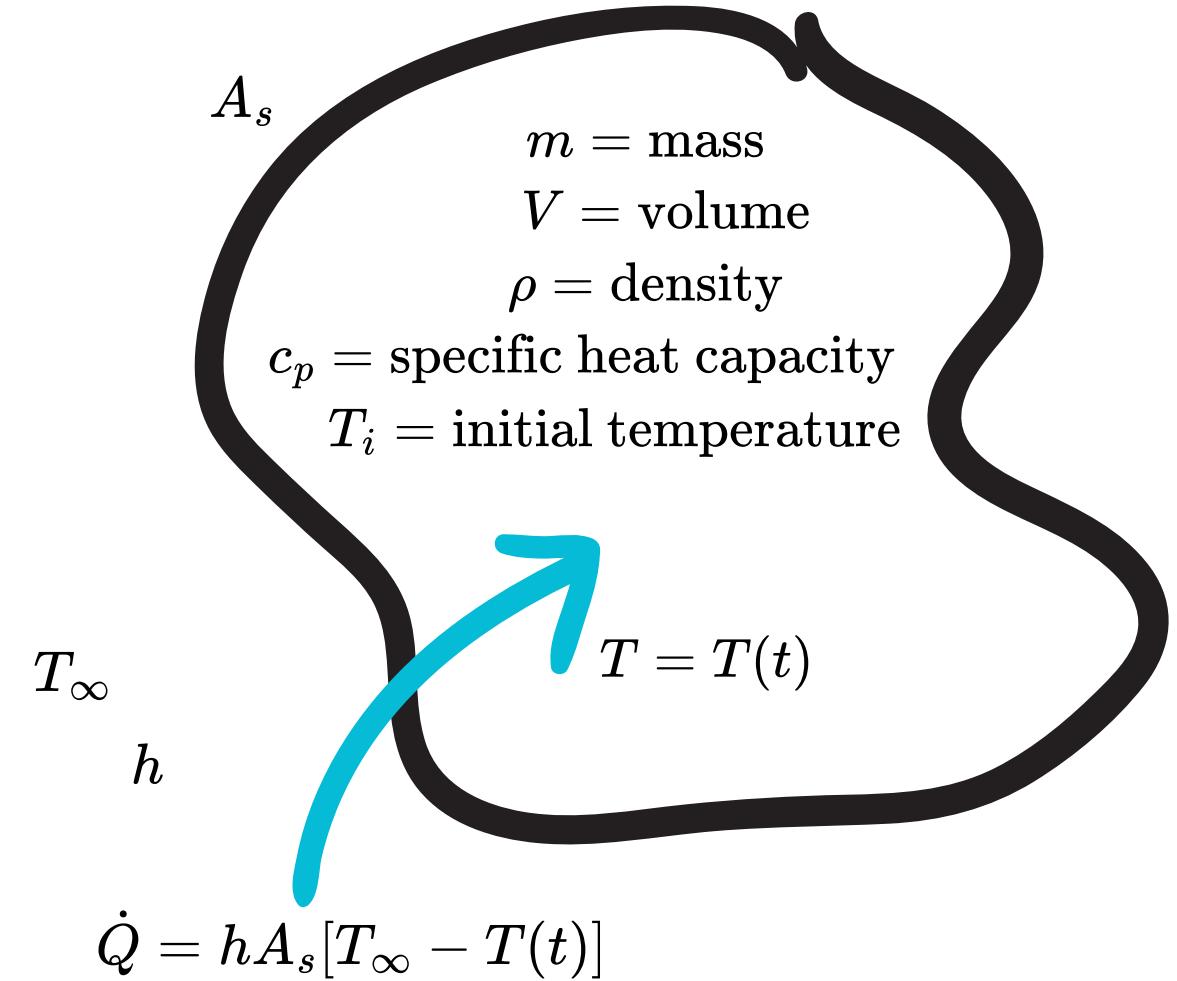
A roast beef (not lumped system)



A copper ball (lumped system)

## Starting point of lumped system analysis

- Consider a body of arbitrary shape with:
  - Mass  $m$
  - Volume  $V$
  - Density  $\rho$
  - Specific heat capacity  $c_p$
  - Initial internal temperature  $T_i$
- At time  $t = 0$  the body is exposed to convective heat transfer from the outside:
  - Surrounding temperature  $T_\infty$
  - Heat transfer coefficient  $h$
  - Surface area  $A_s$



## Derivation of the lumped system equation

- Energy transferred to body during  $dt$ :

$$\begin{aligned} hA_s(T_\infty - T)dt &= mc_p dT \\ &= \rho V c_p dT \end{aligned}$$

- Because  $T_\infty$  is constant, we may expand  $dT$ :

$$\begin{aligned} hA_s(T_\infty - T)dt &= -hA_s(T - T_\infty)dt \\ &= \rho V c_p d(T - T_\infty) \end{aligned}$$

- Re-arranging:

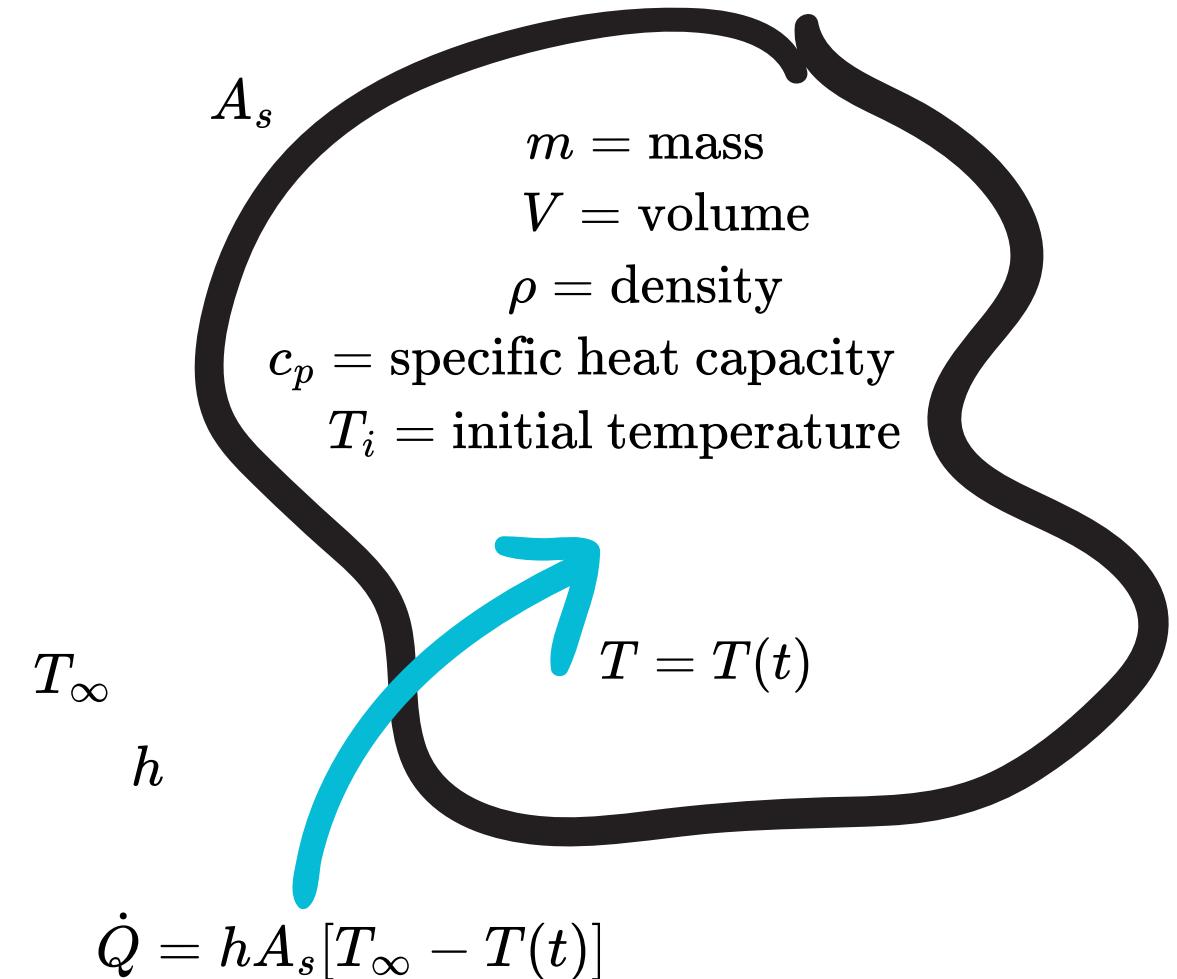
$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p} dt$$

- Now, integrating from  $t = 0$  where  $T = T_i$  to any time  $t$  at which  $T = T(t)$  we get:

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p} t$$

- Exponential on both sides gives **lumped system equation**:

$$\frac{T(t) - T_\infty}{T - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t}$$



$$\dot{Q} = hA_s[T_\infty - T(t)]$$

## The time constant for lumped systems

- Introducing  $b = hA_s/(\rho V c_p)$ , we obtain:



$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c_p} t} = e^{-bt}$$

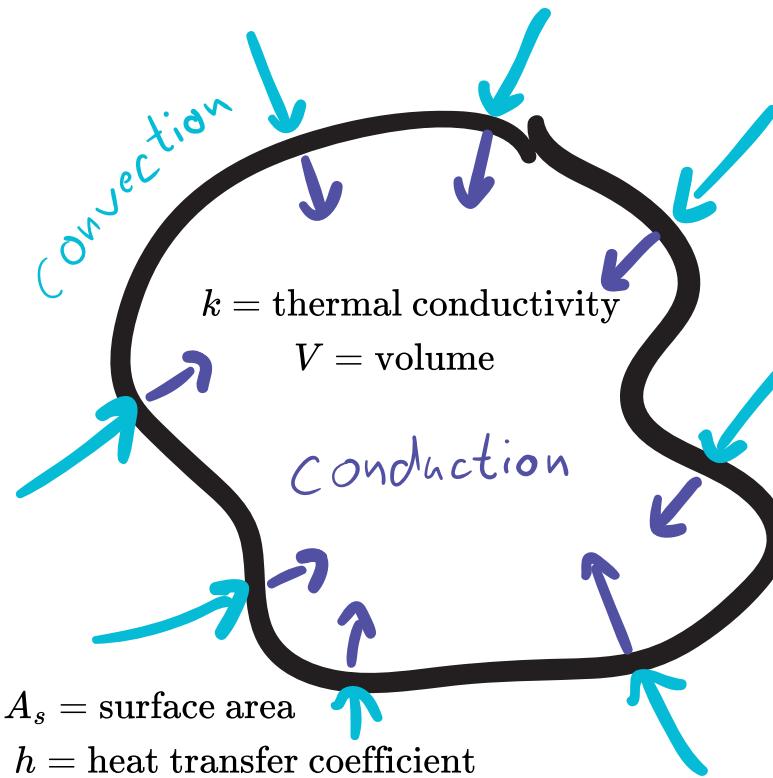
- The time constant ( $1/b$ ) describes the rate at which the system approaches the surrounding temperature  $T_\infty$ 
  - Large  $b$  value**, system approaches surrounding temperature **quickly**
  - Small  $b$  value**, system approaches surrounding temperature **slowly**

## Validity of lumped system assumption

- We define a **Biot number**:

$$Bi = hL_c/k$$

- with the characteristic length  $L_c = V / A_s$
- If  $Bi = 0$ : lumped system assumption is **exact**
- If  $Bi \leq 0.1$ : lumped system assumption is **reasonable accurate**
- If  $Bi > 0.1$ : lumped system assumption is **inaccurate**



# Example

Course material - Lumped system analysis - Example 1

