Heat Transfer

Lumped system assumption

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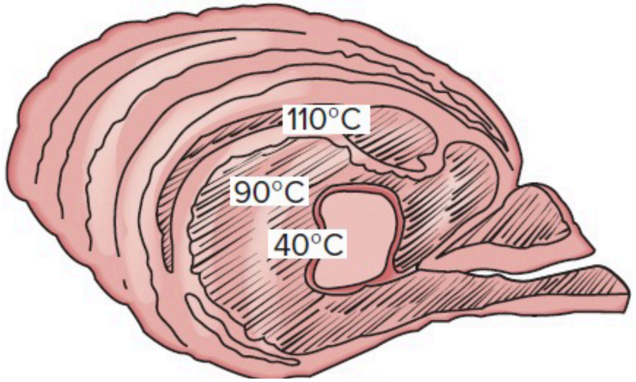
What is the lumped system assumption and why?

- Most heat transfer problems are complex because temperature varies in both with time and space
  - In **lumped systems**, temperature **only varies with time and not space**

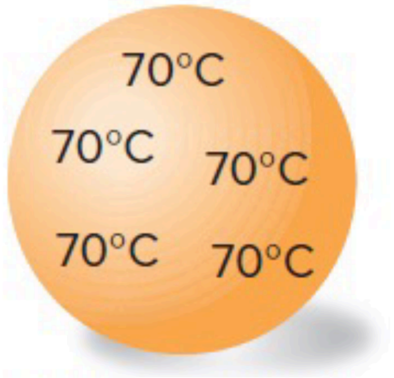
$$T = f(x, y, z, t)$$

$$\longrightarrow T = f(t)$$

- Applying the lumped system assumption **significantly simplifies** the analysis



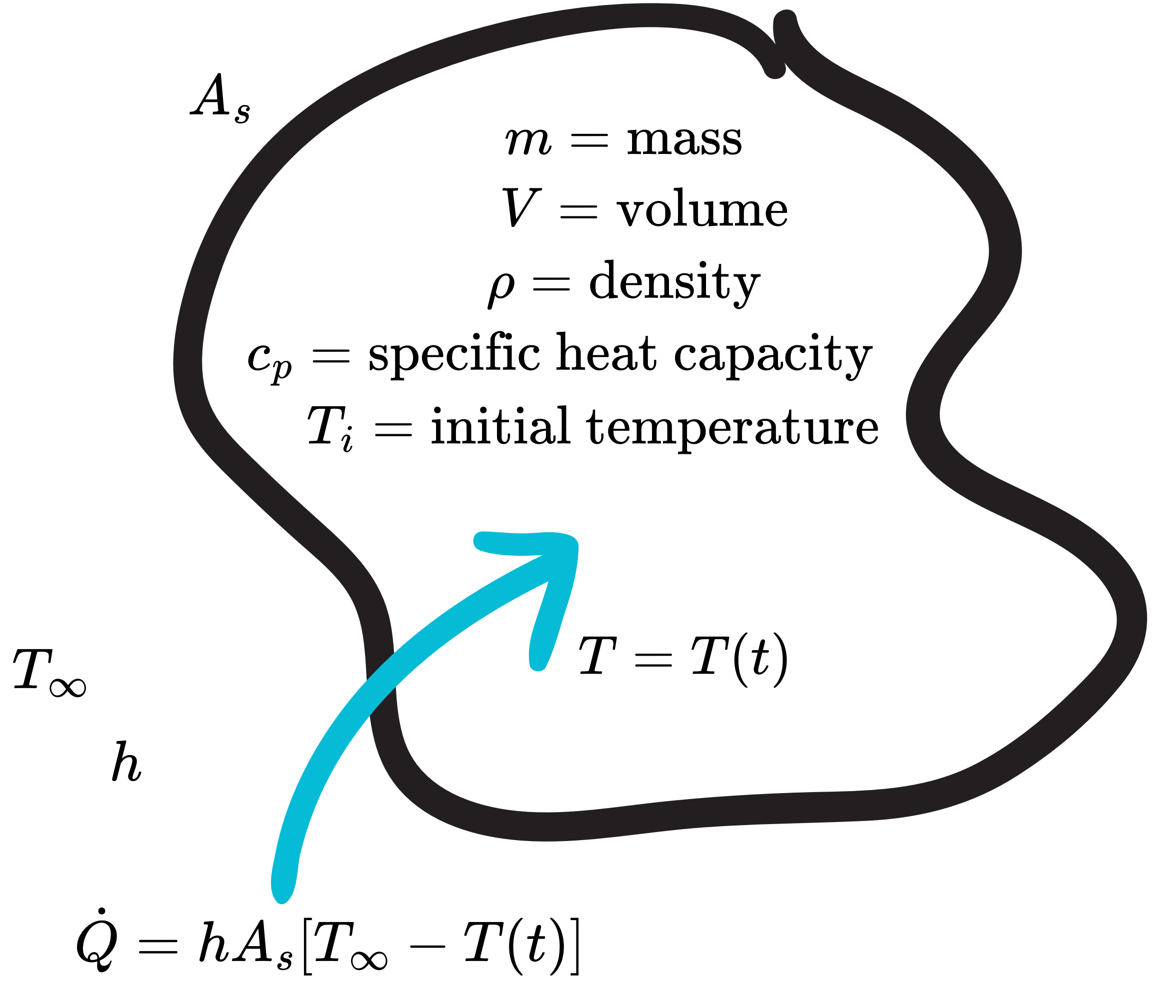
A roast beef (not lumped system)



A copper ball (lumped system)

Starting point of lumped system analysis

- Consider a body of arbitrary shape with:
  - Mass m
  - Volume V
  - Density ρ
  - Specific heat capacity c_p
  - Initial internal temperature T_i
- At time $t = 0$ the body is exposed to convective heat transfer from the outside:
  - Surrounding temperature T_∞
  - Heat transfer coefficient h
  - Surface area A_s



Derivation of the lumped system equation

- Energy transferred to body during $\mathrm{d}t$:

$$hA_s(T_\infty - T)\mathrm{d}t = mc_p\mathrm{d}T$$

$$= \rho V c_p \mathrm{d}T$$

- Because T_∞ is constant, we may expand $\mathrm{d}T$:

$$hA_s(T_\infty - T)\mathrm{d}t$$

$$= -hA_s(T - T_\infty)\mathrm{d}t$$

$$= \rho V c_p \mathrm{d}(T - T_\infty)$$

- ..Re-arranging:

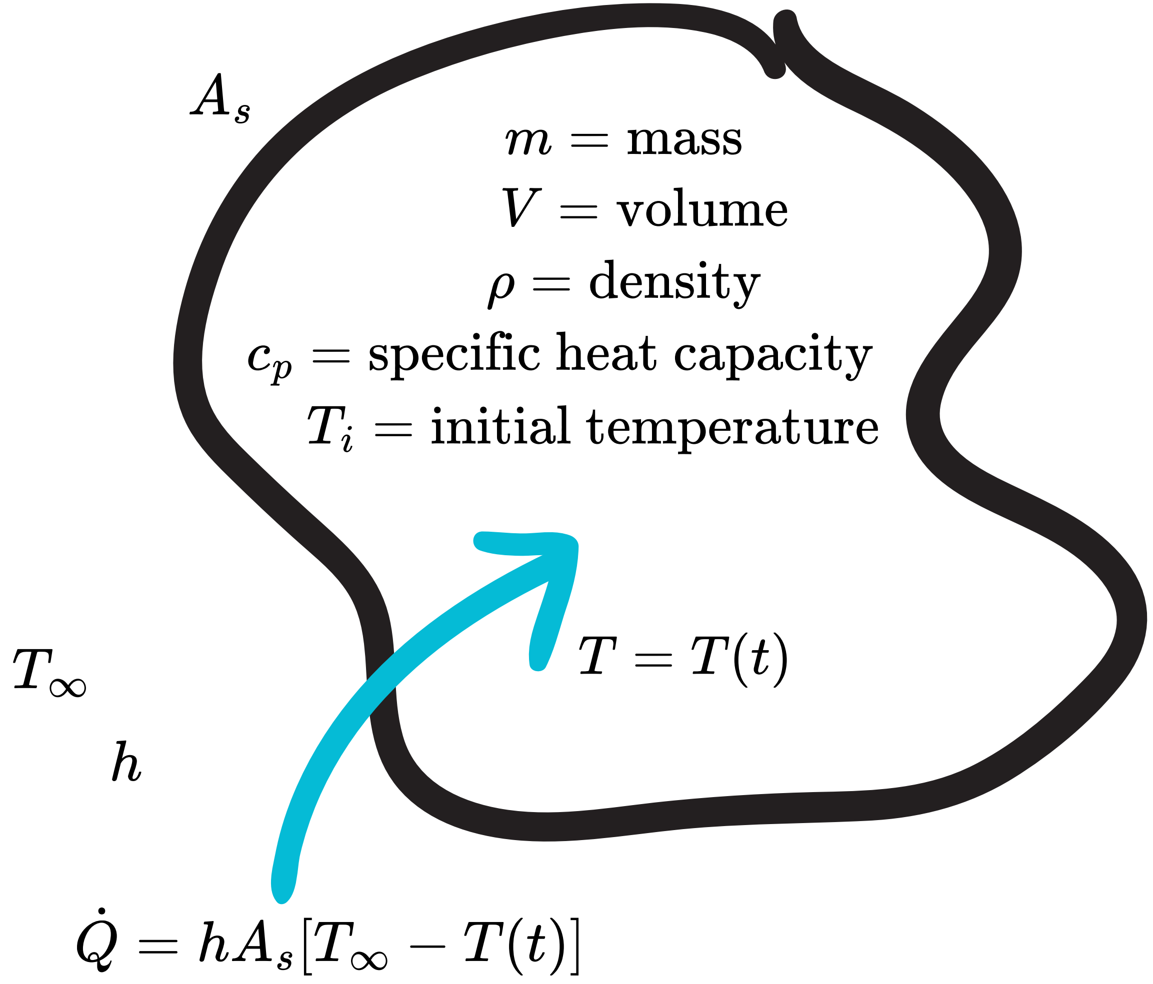
$$\frac{\mathrm{d}(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\mathrm{d}t$$

- Now, intergrating from $t = 0$ where $T = T_i$ to any time t at which $T = T(t)$ we get:

$$\ln\frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}t$$

- Exponential on both sides gives **lumped system equation**:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \mathrm{e}^{-\frac{hA_s}{\rho V c_p}t}$$



The time constant for lumped systems

- Introducing $b = hA_s/(\rho V c_p)$, we obtain:

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = \mathrm{e}^{-\frac{hA_s}{\rho V c_p}t} = \mathrm{e}^{-bt}$$

- The time constant $(1/b)$ describes the rate at which the system approaches the surrounding temperature T_∞
  - **Large b value**, system approaches surrounding temperature **quickly**
  - **Small b value**, system approaches surrounding temperature **slowly**

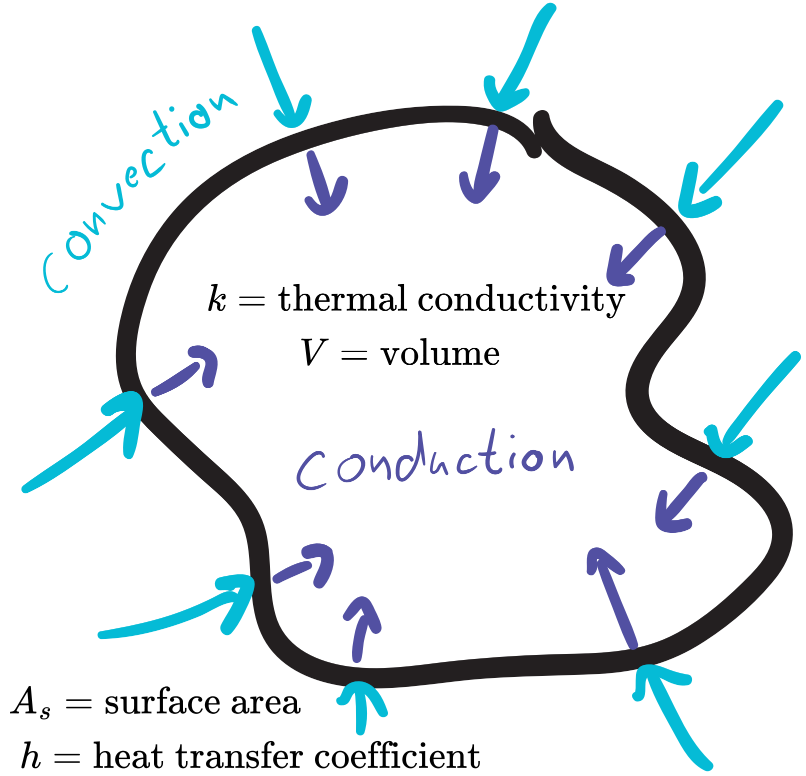
Validity of lumped system assumption

- We define a **Biot number:**

$$\mathrm{Bi} = hL_c/k$$

  - with the characteristic length $L_c = V/A_s$

- If $\mathrm{Bi} = 0$: lumped system assumption is **exact**
- If $\mathrm{Bi} \leq 0.1$: lumped system assumption is **resonable accurate**
- If $\mathrm{Bi} > 0.1$: lumped system assumption is **inaccurate**



Example

Course material - Lumped system analysis - Example 1