Dermed blir ligningssettet (2)

$$\ddot{\tilde{p}} = \frac{L_1}{J_p} \tilde{V}_d \qquad = f_4$$

$$\ddot{\tilde{e}} = \frac{L_2}{J_e} \cos(\tilde{e} + e^*) + \frac{L_3}{J_e} \tilde{V}_s \cdot \cos(\tilde{p} + p^*) \qquad = f_5$$

$$\ddot{\tilde{\lambda}} = \frac{L_4}{J_e} \tilde{V}_s \cos(\tilde{e} + e^*) \cdot \sin(\tilde{p} + p^*) \qquad = f_6$$

Definerer tilstandsvektor

$$\mathbb{X} = \begin{bmatrix} x_1 = \tilde{p} \\ x_2 = \tilde{e} \\ x_3 = \tilde{\lambda} \\ x_4 = \dot{x}_1 \\ x_5 = \dot{x}_2 \\ x_6 = \dot{x}_3 \end{bmatrix},$$

slik at

$$\dot{\mathbb{X}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} \dot{\tilde{p}} \\ \dot{\tilde{e}} \\ \dot{\tilde{\lambda}} \\ \ddot{\tilde{p}} \\ \ddot{\tilde{e}} \\ \ddot{\tilde{\lambda}} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \end{bmatrix} =$$

$$\mathbb{U} = \begin{bmatrix} \tilde{V}_s + V_s^* \\ \tilde{V}_d + V_d^* \end{bmatrix} = \begin{bmatrix} \tilde{V}_s - \frac{L_2}{L_3} \\ \tilde{V}_d + 0 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$\boxed{P2}$ Linearisere systemet rundt $\boxed{5.\ .2}$

$$\begin{bmatrix} p \\ e \\ \lambda \end{bmatrix} = \begin{bmatrix} p^* \\ e^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{og}$$

$$\begin{bmatrix} \dot{p}^* \\ \dot{e}^* \\ \dot{\lambda}^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \ddot{p}^* \\ \ddot{e}^* \\ \ddot{\lambda}^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

og

$$\begin{bmatrix} V_s \\ V_d \end{bmatrix} = \begin{bmatrix} V_s^* \\ V_d^* \end{bmatrix}$$

Ligningssettet (2) blir da

$$0 = L_1 V_d^* \Rightarrow \boxed{V_d^* = 0}$$

$$0 = L_2 \underbrace{\cos p^*}_{=1} + L_3 V_s^* \underbrace{\cos p^*}_{=1}$$

$$0 = L_2 + L_3 V_s$$

$$\Rightarrow \boxed{V_s^* = -\frac{L_2}{L_3}}.$$

Dermed skal systemet lineariseres rundt

$$\begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} = \begin{bmatrix} V_s + \frac{L_2}{L_3} \\ V_d - 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

og

$$\begin{bmatrix} \tilde{p} \\ \tilde{e} \\ \tilde{\lambda} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} p - p^* \\ e - e^* \\ \lambda - \lambda^* \end{bmatrix} = \begin{bmatrix} p - 0 \\ e - 0 \\ \lambda - 0 \end{bmatrix}$$

Problem 3

5.1.3

2a) stemmer : Modellen viser $\ddot{p} \propto V_d$,

og grafen viser også at

Problem 4

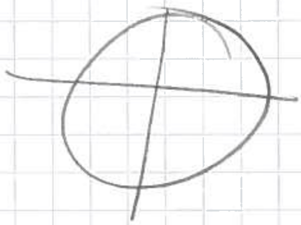
5.1.4

$$\left.\begin{array}{l} K_p = 0 \\ K_e = -3 \end{array}\right\}$$

$\cancel{\frac{\pi\, rad}{180}}$

$$\boxed{\frac{1 rad}{\pi} = \frac{grad^{\circ}}{180}}$$

$\boxed{\frac{\pi}{180}}$ er gale



$V_s = V_s^* = 6{,}3$ volt

$$\underbrace{F_f}_{K_f \cdot V_f} \cdot l_h = 2 m g_p \cdot l_h - F_{g,c} \cdot l_c$$

$= K_f \cdot V_s \cdot l_h =$

$$\Rightarrow \quad K_f = \frac{1}{V_s \cdot l_h}\left(2 m_p g\, l_h - \underbrace{F_{g,c}}_{= m_c \cdot g} \cdot l_c\right) = 0{,}1585$$

$V_s = 6{,}3V$	$m_p = 0{,}72$ kg
$l_h = 0{,}66$ m	$m_c = 1{,}92$ kg
$l_c = 0{,}46$ m	

$$U_d = K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\,\dot{\tilde{p}} \qquad = K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\, s\tilde{p}$$

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\left[K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\, s\tilde{p}\right]$$

K_1

$$= \frac{L_1}{J_p} K_{pp}(\tilde{p}_c - \tilde{p}) - \frac{L_1}{J_p} K_{pd}\, s\tilde{p} = s^2 \cdot \tilde{p}$$

$$= \frac{L_1}{J_p} K_{pp}\,\tilde{p}_c = \frac{L_1}{J_p} K_{pp}\,\tilde{p} + \frac{L_1}{J_p} K_{pd}\, s\tilde{p} + s^2\,\tilde{p}$$

$$\Rightarrow \quad K_1 K_{pp}\,\tilde{p}_c = \tilde{p}\left[K_1 K_{pp} + K_1 K_{pd} s + s^2\right]$$

$$\Rightarrow \quad \frac{\tilde{p}}{\tilde{p}_c} = \frac{K_1 K_{pp}}{K_1 K_{pp} + K_1 K_{pd} s + s^2}$$

$$= \frac{K_1 K_{pp}}{K_1 (K_{pp} + K_{pd}) + s^2} \quad \Bigg| \; \frac{\frac{1}{K_1 K_{pp}}}{\frac{1}{K_1 K_{pp}}}$$

Problem 3

5.1.3

Fra ligningen (2) er ~~er~~ pitch akselerasjon ~~er~~ ∝ V_d,

$\ddot{e} \propto \cos e$ og $\cos p$. Stemmer ok med

V_d og ~~vinklene til~~ Stemmer dårlig med $\ddot{e}$

og ok med $\ddot{p}$ påvirkning på $\ddot{e}_0$ Aksen

~~til er~~ til yaw er $\propto V_s$, kan endres med

e og stor endring til p ??

Sammenligne med eq 6.

6a: pitch aks er rask, prop med V_d NJA?

6b: aks elevation er prop med V_s: NJA, konstant aks?

6c: prop med pitch? JA

Problem 4

5.1.4

$$\cancel{K_f = \frac{F_f}{V_f} = \frac{F_b}{V_b} \quad F_f - F_b}$$

$$F_f + F_b = K_f V_f + K_f V_b = K_f (\underbrace{V_f + V_b}_{V_s})$$

$$F_v = K_f V_s$$

Likevekt $F_v = F_g$ $\sum M = 0$

$$\Rightarrow F_{g,c} \cdot l_c + F_v \cdot l_h - 2 m g_p \cdot l_h = 0$$

$\Rightarrow$

$$\Rightarrow K_f = \frac{F_{g,c} \cdot l_c}{V_s \cdot l_h}$$

$$\boxed{K_f = \frac{F_{g,c} \cdot l_c}{V_s^* \cdot l_h}}$$

så

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\tilde{V}_d$$

$$\ddot{\tilde{e}} = \frac{L_3}{J_e}\left(\tilde{V}_s - \frac{L_2}{L_3}\right)$$ (fra forelæsning)

$$\ddot{\tilde{\lambda}} = \frac{L_4}{J_e}\tilde{V}_s\,\tilde{p} + \frac{L_4}{J_e}\left(\tilde{V}_s - \frac{L_2}{L_3}\right)$$

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\tilde{V}_d$$

$$\ddot{\tilde{e}} = \frac{L_2}{J_e}\underbrace{\cos 0}_{=1} + \frac{L_3}{J_e}\left(\tilde{V}_s - \frac{L_2}{L_3}\right)\cdot\cos 0$$

$$\ddot{\tilde{\lambda}} = \frac{L_4}{J_e}\left(\tilde{V}_s - \frac{L_2}{L_3}\right)\cdot\underbrace{\cos 0}_{=1}\cdot\tilde{p}$$

$$\Downarrow$$

$$\ddot{\tilde{p}} = \frac{L_1}{J_p}\hat{V}_d$$

$$\ddot{\tilde{e}} = \frac{\cancel{L_2}}{J_e} + \frac{L_3}{J_e}\tilde{V}_s - \frac{\cancel{L_3}\cancel{L_2}}{J_e \cancel{L_3}}$$

$$= \frac{L_3}{J_e}\hat{V}_s$$

$$\ddot{\tilde{\lambda}} = \frac{L_4}{J_e}\left(\tilde{V}_s - \frac{L_2}{L_3}\right)\tilde{p} \;,$$

Dermed er

$$K_1 = \frac{L_1}{J_p} = \frac{l_p \cdot K_f}{J_p}$$

$$K_2 = \frac{L_3}{J_e}$$

$$K_3 = \frac{L_4}{J_e}\left(\cancel{\tilde{V}_s}^{\,0} - \frac{L_2}{L_3}\right) = -\frac{L_4 L_2}{J_\lambda L_3}$$

$$\ddot{\tilde{P}} = K_1 K_{pp} (\tilde{P}_c - \tilde{P}) + K_1 K_{pd} \tilde{P}$$

$K_1 K_{pp} = X_1$

$K_1 K_{pp} = X_2$

$$\ddot{\tilde{P}} = X_1 (\tilde{P}_c - \tilde{P}) - X_2 \tilde{P}$$

$$S^2 \tilde{P}_{(s)} = X_1 \tilde{P}_{c(s)} - X_1 \tilde{P}_{(s)} - X_2 \tilde{P}S$$

$\boxed{5.2.1}$?

$$(S^2 + X_2 S + X_1) \tilde{P}_{(s)} = X_1 \tilde{P}_{c(s)}$$

$$\frac{\tilde{P}(s)}{\tilde{P}_c(s)} = \frac{X_1}{S^2 + X_2 S + X_1} = \frac{K_1 K_{pp}}{S^2 + K_1 K_{pd} S + K_1 K_{pp}}$$

$$\frac{1}{\cancel{S(S + K_1 K_{pd}) + K_1 K_{pp}}}$$

$$\frac{1}{\frac{S^2}{K_1 K_{pp}} + \frac{K_{pd}}{K_{pp}} S + 1}$$

$\omega_n^2 = K_1 K_{pp}$

$\frac{2\zeta}{\omega_n} = \frac{K_{pd}}{K_{pp}}$

$$\frac{1}{\left(\frac{S}{\omega_n}\right)^2 + \left(\frac{K_{pd}}{K_{pp}}\right) S + 1}$$

$\zeta = 1$

$\frac{2}{\omega_n} = \frac{K_{pd}}{K_{pp}}$

$\omega_n^2 = K_1 K_{pp}$

$$\tilde{\dot{\gamma}}_d = K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\dot{\tilde{p}}$$

$$K_{pp} = \frac{K_1}{\omega_n^2}$$

$$K_{pd} = \frac{2\zeta}{\omega_n} K_{pp}$$

$$\tilde{p}_c = K_{rp}(\tilde{\dot{\lambda}}_c - \tilde{\dot{\lambda}})$$

$\tilde{e}_c = \text{ref-value} = 0$

$$\tilde{V}_d = K_{pp}(\tilde{P}_c - \tilde{P}) - K_{pd}\dot{\tilde{P}} \qquad \boxed{5.2.1}$$

$$\ddot{\tilde{P}} = K_1 \tilde{V}_d = \ddot{\tilde{P}} = K_1 K_{pp}(\tilde{P}_c - \tilde{P}) - K_{pd}\dot{\tilde{P}}$$

$$S^2\tilde{P} = K_1 K_{pp}(\tilde{P}_c - \tilde{P}) - K_1 K_{pd} S\tilde{P}$$

$$S^2\tilde{P} = K_1(K_{pp}\tilde{P}_c - K_{pp}\tilde{P} - K_{pd}S\tilde{P})$$

$$S^2 \cancel{P} \quad \tilde{P}(S^2 + K_{pp}K_1 + K_{pd}K_1 S \cancel{P}) = K_1 K_{pp}\tilde{P}_c$$

$$\frac{\tilde{P}}{\tilde{P}_c} = \frac{(K_1 K_{pp})}{(1)S^2 + (K_{pd}K_1)S + (K_{pp}K_1)}$$

$$\mathcal{L}(P(t)) = P(s)$$

$$\omega_n^2 = K_1 K_{pp}$$

$$\zeta$$

$$\frac{\tilde{p}(s)}{\tilde{p}_c(s)} = \frac{1}{\left(\frac{s}{\sqrt{K_1 K_{pp}}}\right)^2 + \left(\frac{K_{pd}}{K_{pp}}\right)s + 1}$$

$$\boxed{\omega_n^2 = K_1 K_{pp} \qquad \frac{2\zeta}{\omega_n} = \frac{K_{pd}}{K_{pp}}}$$

setter $\zeta = 1$ for krit. demp.

$$\omega_n^2 = K_1 K_{pp} \qquad \frac{2}{\omega_n} = \frac{K_{pd}}{K_{pp}}$$

$\omega_n =$

$\boxed{5.2.2 \; ?}$

$$k_1\left(c\tilde{p}_c + d\mathring{\tilde{e}} - g\tilde{p}\right) = 0$$

$$k_2\left(a\tilde{p}_c + b\mathring{\tilde{e}} - f\mathring{\tilde{e}}\right) = 0$$

$$\longrightarrow p\,\mathcal{B} = \begin{bmatrix} 0 & f \\ g & 0 \end{bmatrix}$$

$$c\tilde{p}_c + d\mathring{\tilde{e}}_c - g\tilde{p} = 0 \qquad (1)$$

$$a\tilde{p}_c + b\mathring{\tilde{e}}_c - f\mathring{\tilde{e}} = 0 \qquad (2)$$

$$\left(b\mathring{\tilde{e}}_c - f\mathring{\tilde{e}}\right) - a\tilde{p}_c$$

$$\tilde{p}_c\,(c-g) = -d\mathring{\tilde{e}}_c$$

$$\mathring{\tilde{e}}_c\,(b-f) = -a\tilde{p}_c$$

$$\tilde{p}_c = \frac{d\mathring{\tilde{e}}_c}{g-c} \qquad \mathring{\tilde{e}}_c = \frac{a\tilde{p}_c}{f-b}$$

$$= \frac{a}{(f-b)} \cdot \frac{d\tilde{e}_c}{(g-c)} \quad \Big| \cdot \frac{1}{\tilde{e}_c}$$

$$1 = \frac{a}{(f-b)} \cdot \frac{d}{(g-c)} = \frac{ad}{(f-b)(g-c)}$$

$$(g-c) = \frac{ad}{(f-b)}$$

$$\tilde{p}_c = \frac{a\tilde{p}_c}{f-b} \cdot \frac{d\tilde{e}_c}{ad}\,(f-b)$$

$$\begin{bmatrix} V_s \\ V_c \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \tilde{p}_c \\ \tilde{e}_c \end{bmatrix} - \begin{bmatrix} \cancel{C} \end{bmatrix} \begin{matrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot \\ - & \cdot & \cdot \\ \cdot & \cdot \; - & \cdot \end{bmatrix}$$

$\tilde{P}_c = K_{RP}\left(s\tilde{\lambda}_c^{(s)} - s\tilde{\lambda}^{(s)}\right)$ $\boxed{5.2.2}$ ou!

$\dfrac{\tilde{P}_c}{s\,K_{rp}} = \dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}$

$s\dot{\tilde{\lambda}} = K_{rp}(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}})$

$\cancel{s\dot{\tilde{\lambda}} = K}$

$\tilde{P}_c = K_{PR}\left(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}\right)$

$\dot{\tilde{\lambda}} = K_3\tilde{P}$

$s\dot{\tilde{\lambda}} = K_3K_{PR}\left(\dot{\tilde{\lambda}}_c - \dot{\tilde{\lambda}}\right)$

$(s + K_3K_{PR})\dot{\tilde{\lambda}} = K_3K_{Pr}\,\dot{\tilde{\lambda}}_c$

$\dfrac{\dot{\tilde{\lambda}}}{\dot{\tilde{\lambda}}_c} = \dfrac{K_3K_{PR}}{s + K_3K_{Pr}}$ $\dfrac{e}{s+e}$ $e = K_3K_{pr}$

$c\tilde{p}_c + d\tilde{e} - g\tilde{p} = 0$ $\boxed{c = g}$

$\tilde{p}_c - \tilde{p} + d\tilde{e} = 0$ $\boxed{5.2.2}$

$\tilde{p}_c = \tilde{p} + d\tilde{e}$ $\boxed{d = 0}$ $\infty =$

$\rightarrow \tilde{p}_c = \tilde{p}$

$a\tilde{p}_c + b\tilde{e}_c - f\tilde{e} = 0$ $\cancel{B}$ $b = f$

$\cancel{a\tilde{p}_c} + b(\tilde{e}_c - \cancel{f}\tilde{e}) = 0$ $a = 0$

$\boxed{\tilde{e} = \tilde{e}}$

$c\tilde{p}_c + d\tilde{e} - c\tilde{p} = 0$

$c(\underbrace{\tilde{p}_c - \tilde{p}}_{=0}) = -d\tilde{e} = 0$ med $d \neq 0$ $p_c - p = 0 \Rightarrow \boxed{p_c = p}$

$a\tilde{p}_c + b\tilde{e}_c - \mathring{e} \cdot f = 0$ $\boxed{f = b}$

$b(\tilde{e}_c - \tilde{e}) = -d\tilde{p}_c = 0$ $b = 0 \vee e_c = \tilde{e}$

$u = \ell r - u_x$

$$\tilde{V}_s = a\tilde{p}_c + b\dot{\tilde{e}}_c - 3\dot{\tilde{e}}$$

$$\tilde{V}_d = c\tilde{p}_c + d\dot{\tilde{e}}_c - \tilde{p} - 10\dot{\tilde{p}}$$

8

$$\dot{x} = Ax + Bu = 0 \qquad x$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & k_1 \\ k_2 & 0 \end{bmatrix}}_{B} \left(\begin{bmatrix} a & b \\ c & D \end{bmatrix} \begin{bmatrix} p_c \\ \cdot \end{bmatrix} - \begin{bmatrix} 0 & 0 & f \\ g & h & \end{bmatrix} \right)$$

$$0 = \begin{bmatrix} \dot{\tilde{p}} \\ 0 \\ 0 \end{bmatrix} + B\left(\begin{bmatrix} a\tilde{p}_c + b\dot{\tilde{e}}_c \\ c\tilde{p}_c + d\dot{\tilde{e}}_c \end{bmatrix} - \begin{bmatrix} \dot{\tilde{e}} \cdot f \\ g \cdot \tilde{p} + h \cdot \dot{\tilde{p}} \end{bmatrix} \right)$$

$$= \begin{bmatrix} \dot{\tilde{p}} \\ 0 \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & k_1 \\ k_2 & 0 \end{bmatrix}}_{B} \begin{bmatrix} a\tilde{p}_c + b\dot{\tilde{e}}_c - \dot{\tilde{e}}f \\ c\tilde{p}_c + d\dot{\tilde{e}}_c - g\cdot\tilde{p} - h\cdot\dot{\tilde{p}} \end{bmatrix}$$

$$0 = \begin{bmatrix} \dot{\tilde{p}} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ k_1(c\tilde{p} + d\dot{\tilde{e}}_c - g\tilde{p} - h\dot{\tilde{p}}) \\ k_2(a\tilde{p} + b\dot{\tilde{e}}_c - \dot{\tilde{\imath}}_f) \end{bmatrix}$$

$$\boxed{\dot{\tilde{p}} = 0} \;!$$

$\boxed{5.3.1}$

$$Bu = B \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix}$$

$$\begin{array}{cc|c} & & \tilde{V}_s \\ & & V_d \\ \hline 0 & \frac{1}{K_r} & \end{array}$$

$$Bu = \begin{bmatrix} 0 & -\frac{1}{K_{pd}} \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix}$$

$$\begin{bmatrix} \tilde{\dot{p}} \\ \tilde{\ddot{p}} \\ \tilde{\ddot{\theta}} \end{bmatrix} = Ax + Bu = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \tilde{p} \\ \tilde{\dot{p}} \\ \tilde{\dot{\theta}} \end{bmatrix}} + \underbrace{\begin{bmatrix} 0 & -1/K_{pd} \\ 0 & K_1 \\ K_2 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix}}_{u}$$

$$C = \begin{bmatrix} B & AB & (A^2B) \end{bmatrix} \in [\ 3 \times 3$$

$$Y = Cx + Du$$

$$= \begin{bmatrix} 0 & -1/K_{pd} \\ & \\ & \end{bmatrix}$$

$$\boxed{\tilde{\dot{p}} = \tilde{\dot{p}}}$$

$a = a$

$\boxed{1 = 1}$

$\dot{x} = Ax + BU$

$A = \begin{bmatrix} \tilde{p} \\ \tilde{p} \\ \tilde{e} \end{bmatrix}$

$\begin{bmatrix} & 2 & \\ & & 3 \end{bmatrix}$

$\dot{x} = A \begin{bmatrix} \\ K_1 \tilde{V}_d \\ K_2 V_s \end{bmatrix} + BU$

$\begin{bmatrix} \dot{p} \\ \\ \\ \end{bmatrix} = A \begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \end{bmatrix} + B \begin{bmatrix} 0 \\ \tilde{V}_s \\ V_d \end{bmatrix}$

$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$B = \begin{bmatrix} [illegible] & 1/k_{rd} & 0 & 0 \\ 0 & & K_1 & 0 \\ 0 & & 0 & K_2 \end{bmatrix} \begin{bmatrix} \\ \end{bmatrix}$

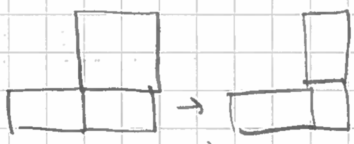
$$\begin{bmatrix}\dot{\tilde p}\\ \ddot{\tilde p}\\ \ddot{\tilde e}\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\tilde p\\ \dot{\tilde p}\\ \dot{\tilde e}\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}0 & 0\\ 0 & K_1\\ K_2 & 0\end{bmatrix}}_{B}\underbrace{\begin{bmatrix}\tilde V_s\\ \tilde V_d\end{bmatrix}}_{u}$$

Farsley

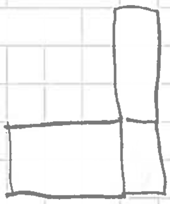
$$Q = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \Rightarrow$$

$$\boxed{5.3.2}$$

$$R = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \Rightarrow u^T R u \equiv$$



$$K = \overset{3\times 2}{\cancel{}} \quad : -Kx \equiv$$



$= u$

$$\boxed{u = Pr - Kx}$$

P här vara 3×2 $\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix}\tilde p_c\\ \dot{\tilde e}_c\end{bmatrix}$

$$u = \begin{bmatrix}\tilde V_s\\ \tilde V_d\end{bmatrix} = \underbrace{\begin{bmatrix}a & b\\ c & d\end{bmatrix}}_{P}\begin{bmatrix}\tilde p_c\\ \dot{\tilde e}_c\end{bmatrix} - \underbrace{\begin{bmatrix}0 & 0 & f\\ g & h & 0\end{bmatrix}}_{K}\underbrace{\begin{bmatrix}\tilde p\\ \dot{\tilde p}\\ \dot{\tilde e}\end{bmatrix}}_{x}$$

$$\tilde V_s = a\tilde p_c + b\dot{\tilde e}_c - 3\dot{\tilde e}$$

$$\tilde V_d = c\tilde p_c + d\dot{\tilde e}_c - \tilde p - 10\dot{\tilde p}$$

$$x = \begin{bmatrix} \tilde{p} \\ \mathring{\tilde{p}} \\ \mathring{\tilde{e}} \end{bmatrix}, \quad \mathring{x} = \begin{bmatrix} \mathring{\tilde{p}} \\ \ddot{\tilde{p}} \\ \ddot{\tilde{e}} \end{bmatrix} =$$

$$\mathring{\tilde{p}} = -\frac{\hat{V}d}{K_{pd}}$$

$$\ddot{\tilde{p}} = K_1 \cdot \tilde{V}_d$$

$$\ddot{\tilde{e}} = K_2 \cdot \tilde{V}_s$$

$$\hat{V}d = K_{pp}(\tilde{p}_c - \tilde{p}) - K_{pd}\mathring{\tilde{p}}$$

$$K_{pd}\mathring{\tilde{p}} = -\tilde{V}d + K_{pp}(\tilde{p}_c - \tilde{p})$$

$$\mathring{\tilde{p}} = \frac{K_{pp}(\tilde{p}_c - \tilde{p}) - \hat{V}d}{K_{pd}}$$

$$= \frac{K_{pp}\,\tilde{p}_c - K_{pp}\cdot\tilde{p} - Vd}{K_{pd}} = \frac{K_{pp}\cdot\tilde{p}_c - K_{pp}\cdot\tilde{p}}{K_{pd}} - \frac{Vd}{K_{pd}}$$

$$\cancel{= \frac{K_{pp}\cdot p - K_{pp}\cdot\tilde{p} - Vd}{K_{pd}}}$$ dus

$$\cancel{= \tilde{p}(K_{pp} -}$$

$$\tilde{p}_c = K_{rp}(\mathring{\tilde{\lambda}}_c - \mathring{\tilde{\lambda}}) \qquad \mathring{\tilde{p}} = -\frac{\tilde{V}d}{K_{pd}}$$

$$\underbrace{\begin{bmatrix} a & & & & \\ & b & & 0 & \\ & & c & & \\ & 0 & & d & \\ & & & & e \end{bmatrix}}_{Q} \qquad \underbrace{\begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \\ \gamma \\ \zeta \end{bmatrix}}_{x} \begin{array}{l} \text{pitch} \\ \text{pitch rate} \\ \text{elev. rate} \\ \tilde{p}_c - \hat{p} \quad (\text{diff pitch}) \\ \dot{\hat{e}}_c - \dot{\hat{e}} \quad (\text{diff elevation}) \end{array}$$

$$\underbrace{\begin{bmatrix} f & 0 \\ 0 & g \end{bmatrix}}_{R} \qquad \underbrace{\begin{bmatrix} \tilde{V}_s \\ \\ \tilde{V}_d \end{bmatrix}}_{u} \begin{array}{l} \sim \text{Lift} \\ \\ \sim \text{pitch} \end{array}$$

$\boxed{5.3.3}$

$$\begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \ddot{\tilde{e}} \\ \dot{\gamma} \\ \dot{\xi} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \dot{\tilde{e}} \\ \gamma \\ \zeta \end{bmatrix}}_{x}$$

$$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} V_s \\ V_d \end{bmatrix} \qquad Y = Cx$$

$$\boxed{A \quad + Bu} = Ax + B(\underbrace{Pr \;- Kx}_{u})$$

$$\vec{r} = \begin{bmatrix} \tilde{p}_c \\ \dot{\tilde{e}}_c \end{bmatrix}$$

$$\cancel{Bu} \quad Bu = B \begin{bmatrix} \hat{V}_s \\ \hat{V}_d \end{bmatrix} \qquad Ax + Bu = Fr$$

$$\cancel{u = Pr - Kx}$$

$$F_r = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}}_{F} \begin{bmatrix} \tilde{p}_c \\ \dot{\tilde{e}}_c \end{bmatrix}$$

$$\begin{bmatrix} \dot{X} \\ \dot{X}_a \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}$$

$$\begin{bmatrix} \varphi & \\ & 0 \end{bmatrix}$$

$$y = Cx$$

$$u = \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix} = \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} \tilde{p}_c \\ \tilde{\dot{e}}_c \end{bmatrix}}_{\vec{r}} - \underbrace{\begin{bmatrix} 0 & 0 & f \\ g & h & 0 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} \tilde{p} \\ \tilde{\dot{p}} \\ \tilde{\dot{e}} \end{bmatrix}}_{x}$$

$$\tilde{V}_s = a\tilde{p}_c + b\tilde{\dot{e}}_c - f\tilde{\dot{e}}$$

$a=0,\ d=0$

$$\tilde{V}_d = c\tilde{p}_c + d\tilde{\dot{e}}_c - g\tilde{p} - h\tilde{\dot{p}}$$

$$\begin{bmatrix} 0 & g \\ f & 0 \end{bmatrix}$$

$$\dot{x} = Ax + Bu = 0$$

$$\Rightarrow \tilde{V}_s = f\tilde{\dot{e}}_c - f\tilde{\dot{e}} = f(\tilde{\dot{e}}_c - \tilde{\dot{e}})$$

$g\ f\ C$

$$\tilde{V}_d = g\tilde{p}_c - g\tilde{p} - h\tilde{\dot{p}}$$

$$= g(\tilde{p}_c - \tilde{p}) - h\tilde{\dot{p}}$$

$$\boxed{u = \underbrace{\left[(BK-A)^{-1} \cdot B\right]^{-1}}_{E} \cdot \underbrace{\vec{r}}_{r} - Kx}$$

$C' = C$

$$\dot{x} = Ax + Bu$$

$$0 = Ax + Bu$$

$$0 = Ax + B(Pr - Kx)$$

$$0 = (A - BK)x + BPr$$

$$Cx = C((KB-A))BPr$$

$$x = \begin{bmatrix} p \\ \dot{p} \\ \dot{e} \end{bmatrix} \neq r = \begin{bmatrix} p \\ \dot{e} \end{bmatrix}$$

$$\cancel{y = C(KB-A)^{-1}BPr}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$I = C(KB-A)^{-1}BP$$

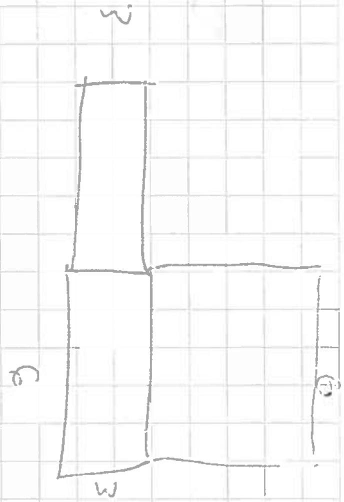
$$\boxed{P = \left[C(KB-A)^{-1}B\right]^{-1}}$$

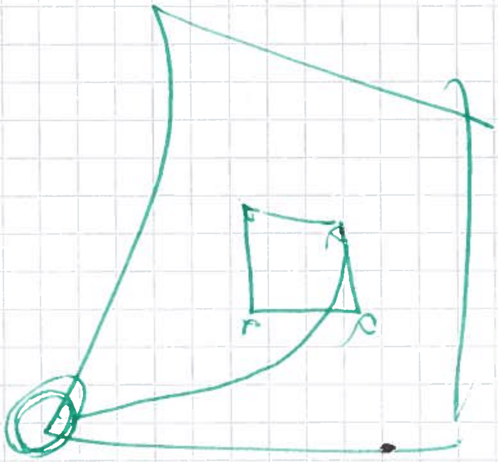
$\boxed{4}$ Problem 1

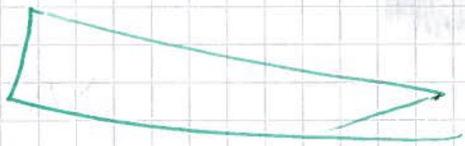
$$
\begin{bmatrix} \dot{\tilde p} \\ \dot{\tilde e} \\ \dot{\tilde \lambda} \\ \ddot{\tilde p} \\ \ddot{\tilde e} \\ \ddot{\tilde \lambda} \end{bmatrix}
=
\underbrace{\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\text{[redacted]} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}}_{=A}
\underbrace{\begin{bmatrix} \tilde p \\ \tilde e \\ \tilde \lambda \\ \dot{\tilde p} \\ \dot{\tilde e} \\ \dot{\tilde \lambda} \end{bmatrix}}_{x}
+
$$

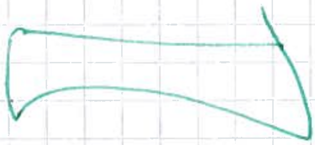
$V_s^* \frac{L_4}{J_{\lambda}} = K_3$

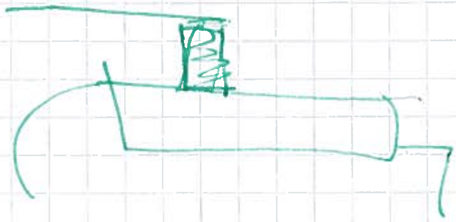
$$
+ \underbrace{\begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & K_1 \frac{L_1}{J_p} \\
K_2 \frac{L_3}{J_e} & 0 \\
\cancel{\frac{L_4}{J_\lambda}}\ 0 & 0
\end{bmatrix}}_{B}
\underbrace{\begin{bmatrix} \tilde V_s \quad \cancel{-\frac{L_2}{L_3}} \\ \tilde V_d \end{bmatrix}}_{u}
$$

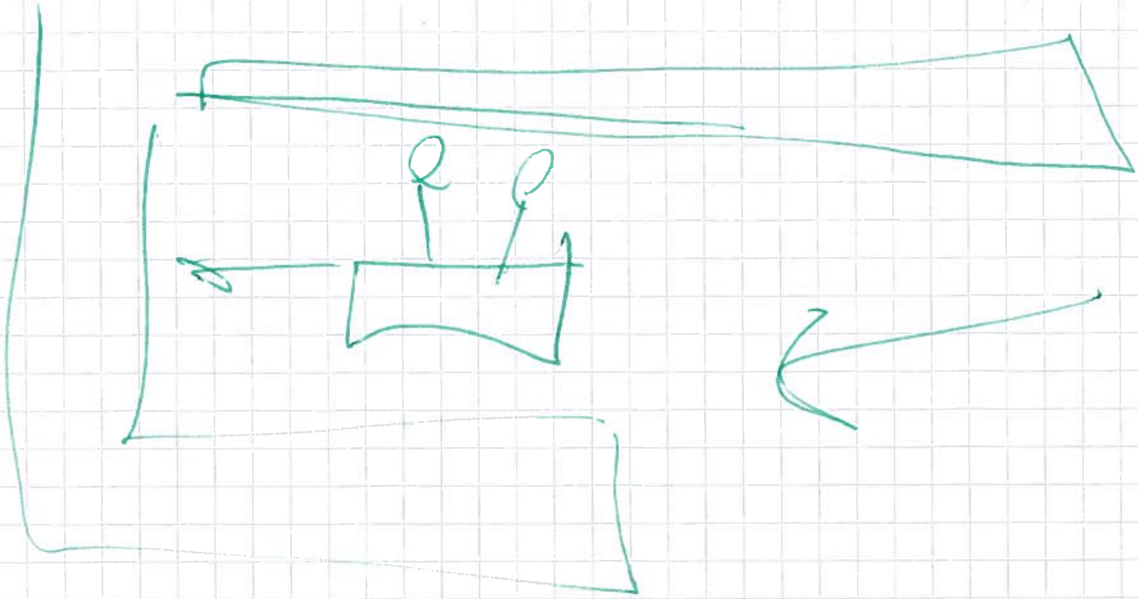












5.4.1

Pinv(A) gir pseudoinverse

$$
\begin{bmatrix} \dot{\tilde{p}} \\ \ddot{\tilde{p}} \\ \dot{\tilde{e}} \\ \ddot{\tilde{e}} \\ \dot{\tilde{\lambda}} \\ \ddot{\tilde{\lambda}} \end{bmatrix}
=
\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ K_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} \tilde{p} \\ \dot{\tilde{p}} \\ \tilde{e} \\ \dot{\tilde{e}} \\ \tilde{\lambda} \\ \dot{\tilde{\lambda}} \end{bmatrix}
+
\begin{bmatrix} 0 & 0 \\ 0 & K_1 \\ 0 & 0 \\ K_2 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} \tilde{V}_s \\ \tilde{V}_d \end{bmatrix}
$$

$$\dot{x} = A x + B u$$

$$y = Cx$$

$$
\begin{bmatrix} \tilde{p} \\ \tilde{e} \\ \tilde{\lambda} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}
\begin{bmatrix} x \end{bmatrix}
$$

5.4.2

Find observability matrix $\mathcal{O}$ to determine if it's observabl

$$
\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & & 0 \\
0 & 0 & & 1 & & 0 \\
0 & 0 & & & & 1 \\
0 & 0 & & & & \\
0 & 0 & & & & \\
K_3 & 0 & & & & \\
0 & 0 & & & & \\
0 & 0 & & & & \\
0 & K_3 & & & & \\
0 & 0 & & & & \\
0 & 0 & & & & \\
0 & 0 & & & & \\
 & 0 & & & & \\
 & 0 & & & & \\
 & 0 & & & &
\end{bmatrix}
= \cancel{\text{obsv}}(A, C)
$$

$\text{Rank}(\cancel{\text{Obsv}(A,C)}\,\mathcal{O}) = 6$

Full rank

$$\dot{\hat{x}} = A\hat{x} + Bu + L(Cx - C\hat{x}) \quad ?$$

$$= A\hat{x} + Bu + LC(x - \hat{x}) \quad 0$$

$$L = \text{place}(A, B, \text{poles})$$

$$\dot{e} = \dot{\tilde{x}} - \dot{\hat{x}}$$

$$= A\hat{x} + \cancel{Bu} + LC(x - \hat{x}) - Ax - \cancel{Bu}$$

$$= A\hat{x} - Ax + LC(x - \hat{x}) \qquad = A(e) - eLC$$

$$= e(-LC + A)$$

Poles of $\; =$

$$\det(\lambda I - A + LC) = 0$$

$$L = \text{place}(A, C, q) \qquad q = \begin{bmatrix} -20 \\ -40 \\ -60 \\ -80 \\ -100 \\ -120 \end{bmatrix}$$