

# IDMA 2025: Summary of Course Contents

## The Basic Computer Model

RAM model (memory consisting of big array of fixed-size words)

Unit time cost for (essentially) all simple operations that can be performed on a single word

## Asymptotic analysis

Big-oh notation: Meaning and use

Complexity analysis for algorithms (algorithms covered in the course + simple algorithms in exercises)

## Basic Data Structures

Stacks

Queues

Lists (implemented as arrays and as singly or doubly linked lists)

Heaps (min heaps and max heaps)

Priority queues

## Sorting and Searching

Linear search

Binary search

Insertion sort

Merge sort

Heap sort

## Basic Arithmetic and Set Theory

Quotient, remainder, mod function, divisibility

Prime numbers and prime factorization

Greatest common divisor (GCD), Euclidean algorithm

Least common multiple (LCM)

Representation of numbers in different bases (decimal, binary, octal, hexadecimal)

Sets, subsets, power sets, intersection, union, complement, Venn diagrams

Inclusion-exclusion principle

Different functions: polynomials, exponentials, logarithms

Sequences and sums and ways to specify them (explicit, recursive)

Connections to asymptotic analysis

Sums of n first numbers, n first squares, n first cubes; geometric sums

## Logic, Mathematical Proofs, and Proof Techniques

Propositional logic, connectives, truth tables, satisfiability, tautologies, contradictions

Predicate logic, universal and existential quantifiers

Direct proofs

Proofs by contraposition

Proofs by contradiction

Mathematical induction (and reasoning with invariants)

## Combinatorics and Probability

Multiplication principle

How to choose  $k$  elements out of  $n$  with and without repetition and caring or not caring about order

Sequences, permutations, multisets, sets

Binomial coefficients ("n choose  $k$ ")

Combinatorial proofs (show that two sets have the same size by exhibiting a bijection)

Pigeonhole principle

Basic probability theory (for uniform distribution, with outcomes equally likely, this is just counting)

## Matrices and Linear Algebra

Matrices, square matrices, diagonal matrices, and identity matrices

Addition and multiplication of matrices

Boolean multiplication

Matrix transpose and symmetric matrices

## Relations

Cartesian product of sets

Relations as subsets of Cartesian products

Domain, range, R-relative sets, in-degree and out-degree

Representation of relations as matrices or directed graphs

Relations  $R^n$  and  $R^\infty$

Functions: total and partial, surjective/onto, injective/one-to-one, bijective

Reflexive and irreflexive relations

Symmetric, asymmetric, and antisymmetric relations

Transitive relations

Equivalence relations (reflexive, symmetric, transitive) and partitions

Complement and inverse relations

Reflexive, symmetric, and transitive closure

Partial orders (reflexive, antisymmetric, transitive) and partially ordered sets (posets)

Hasse diagrams

Lattices, greatest lower bound, least upper bound

## Graph Theory

Graph representation: adjacency matrix, adjacency lists; graphs as binary relations

Directed and undirected graphs; weighted graphs

Neighbours, vertex degree (in-degree, out-degree), paths/walks, simple paths

Connected (undirected) graphs

Graph isomorphism

Trees: root, leaves, children, parents, siblings, ancestors, descendants

Spanning trees of (undirected) graphs

Forests

Graph traversal: Breadth-first search and depth-first search

Topological sort

Strongly connected components

Minimum spanning trees, Prim and Kruskal

Single-source shortest path, Dijkstra