

Mundtlig eksamen for Introduktion til diskret matematik og algoritmer (IDMA) Eksamens 3

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June 2025

Combinatorics

A deck of card has

- **4 suits:** ♠, ♦, ♣, ♠
- **13 ranks:** 2–10, jack, queen, king, and ace

A **poker hand** contains 5 cards. Match the three poker hands below with the correct probabilities of drawing them!

(1) **Four of a kind:** 4 of the cards have the same rank

(2) **Full house:** 3 cards of one rank, 2 cards of another rank

(3) **Flush** (including **straight flush**): All cards are of the same suit

$$(a) \frac{4 \cdot \binom{13}{5}}{\binom{52}{5}}$$

$$(b) \frac{13 \cdot \binom{4}{3} \cdot 12 \cdot \binom{4}{2}}{\binom{52}{5}}$$

$$(c) \frac{13 \cdot 48}{\binom{52}{5}}$$

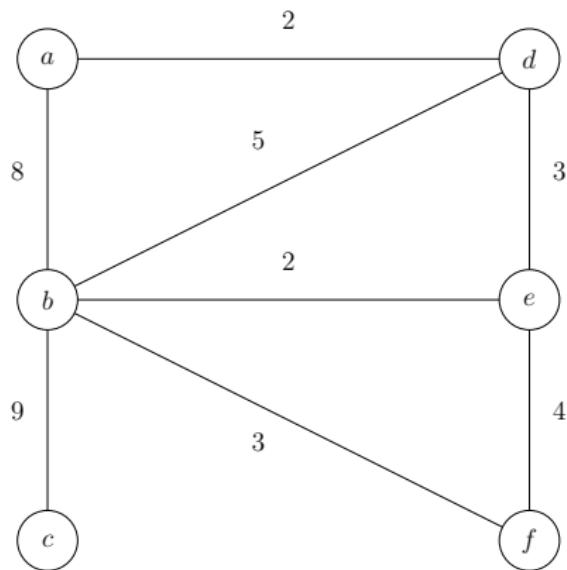
$$(d) \frac{\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 44}{\binom{52}{5}}$$

Graph Algorithms

Run the algorithms below on the graph to the right and explain what spanning trees they will produce:

- ① Depth-first search
- ② Dijkstra's algorithm

(Algorithms that need a starting vertex begin with a ; all neighbour lists are sorted in alphabetical order.)



Algorithm Analysis

Below, A and B are arrays indexed from 1 to n containing numbers

```
for i := 1 upto n {  
    B[i] := 0  
    for j := 1 upto i {  
        B[i] := B[i] + A[j]  
    }  
}
```

- ➊ Explain in plain language what the algorithm does
- ➋ Provide an asymptotic analysis of the running time
- ➌ Can you improve the code to run faster while retaining the same functionality?

Propositional Logic

Decide for each of the propositional logic formulas below whether it is a tautology, contradiction, or neither

① $(p \rightarrow q) \rightarrow (q \rightarrow p)$

② $(p \wedge q) \vee (p \rightarrow \neg q)$

Relations

Let R be the relation on $A = \{e_1, e_2, e_3, e_4, e_5\}$ represented by

$$M_R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

(where element e_i corresponds to row and column i).

Can you combine **transitive closure**, **symmetric closure**, **reflexive closure**, and/or **inverse** to obtain the relations below? How?

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$M_3 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Sorting

Explain how **merge sort** will sort the list

[4, 3, 5, 2, 6, 1]