IDMA 2026

– Ugeseddel 4 –

General Plan

This week will be dedicated to **combinatorics** as discussed in Chapter 3 of KBR. In combinatorics we consider questions like:

- In how many different ways can one draw a hand of 5 cards from a set of 52 different cards?
- What happens to the number if the order of the cards matter?

One particularly important combinatorial principle that will come up is the **pigeonhole principle**, which simply says that if you put strictly more than n objects (pigeons) in n containers (pigeonholes), then there will be more than one object in at least one of the containers. Although this is completely, blindingly obvious, it can be a surprisingly useful fact.

When talking about combinatorics it is also natural to touch briefly on **probability theory**, so that we can answer questions like

- What is the probability that two of the cards that are drawn are spades?

We will see that it is essential to differentiate between combinatoric questions where the elements are ordered and questions where the order does not matter.

Just as a side note, an important application of probabilities in computer science is when we go beyond worst-case analysis to perform *average-case running time analysis*, where one looks at how efficient an algorithm is on average instead of focusing on the worst-case performance. Another application is in *randomized algorithms*, which can make random choices during execution to find the answer faster (but might also commit errors with small probability). Such things are beyond the scope of this course, however, so you will have to wait until your next course on algorithms for this.

Reading Instructions

- Our lectures will cover KBR Sections 3.1–3.4.
- Section 3.5 is not included in the course requirements, but is *definitely* useful to read if you feel that you have already mastered the rest of the material.

Exercises

1. Here are two exercises to understand the proof we did in class of how many different multisets of size r can be chosen from a universe of size n.

   (a) Do the translation from multisets and coloured boxes for the multiset $[2, 3, 5, 5, 8, 8]$ chosen from the universe $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Then take the row of coloured boxes that you get and translate them back to a multiset, just to check that you get back the multiset with which you started.

   (This example also illustrates an interesting corner case in the proof, namely when there are copies of the last element in the universe in the multiset. It is hard to find time to talk properly about this during the lecture, but the details are ironed out in the lecture notes that are posted on Absalon.)

   (b) With the same parameters $n = 8$ and $r = 6$, consider the row of boxes where all prime numbers between 1 and 13 are coloured red. Which multiset does this correspond to? Take this multiset and run the colouring translation to check that you get the same coloured boxes back.

2. To test your basic understanding, solve KBR exercises 3.1.1, 3.1.4, 3.1.8, 3.1.9

3. For some more interesting problems, solve KBR exercises 3.1.22 and 3.1.31.

4. For some more interesting problems, solve KBR exercises 3.1.33 and 3.1.34.

5. For a mix of combinatorial problems, solve KBR exercises 3.2.1, 3.2.6, 3.2.16, 3.2.25, 3.2.33.

6. For a mix of combinatorial problems, solve KBR exercises 3.3.6, 3.3.23

7. To work on probability theory, solve KBR exercises 3.4.12, 3.4.20, 3.4.33, 3.4.34.

8. For a challenging probability problem, solve KBR exercise KBR 3.4.36 (part (c) is [**]).

9. Continue the discussion from the lecture about poker by calculating and comparing the associated probabilities that we did not already compute in class. The hierarchy is as follows:

   (1) Straight flush
   (2) Four of a kind
   (3) Full house
   (4) Flush
   (5) Straight
   (6) Three of a kind
   (7) Two pairs
   (8) A pair