INDUCTION – MORE EXAMPLES

Ex $$\boxed{\sum_{i=1}^{n} (2i-1) = n^2}$$

<u>Base case ($n=1$)</u> $\sum_{i=1}^{1} (2i-1) = 1 = 1^2$

<u>Induction step</u> Suppose $\sum_{i=1}^{n} (2i-1) = n^2$

Then
$$\begin{aligned}\sum_{i=1}^{n+1} (2i-1) &= \sum_{i=1}^{n} (2i-1) + (2n+1) \\ &\overset{[IH]}{=} n^2 + 2n + 1 \\ &= (n+1)^2\end{aligned}$$

The equality follows by the induction principle.

Ex $$\boxed{n! > 2^n \quad \text{for} \quad n \geq 4}$$

Note that
$$\begin{aligned} 1! &= 1 < 2^1 \\ 2! &= 2 < 2^2 \\ 3! &= 6 < 2^3 \\ 4! &= 24 > 2^4 = 16 \end{aligned}$$

<u>Base case ($n=4$)</u> $4! > 2^4$

<u>Induction step</u> suppose $n! > 2^n$ and $n \geq 4$

Then
$$\begin{aligned}(n+1)! &= (n+1)\, n! \\ &\overset{[IH]}{>} (n+1)\, 2^n \\ &> 2 \cdot 2^n \\ &= 2^{n+1}\end{aligned}$$

The inequality follows by the induction principle