IDMA 2025

– Ugeseddel 7 –

General Plan

This week we will continue talking about graphs and graph algorithms. After rounding off the discussion of the graph traversal algorithms (and applications) that we began the previous week, we will discuss minimum spanning trees as in CLRS Chapter 21 and then make a detour to learn about the useful heap data structure in CLRS Chapter 6 (this will be useful in computing shortest paths in graphs next week).

As already mentioned, graphs are a truly foundational topic in computer science. They can be used to model all kinds of real-world problems, and efficient graph algorithms therefore have applications that are too numerous to list. Since this is an introductory course in discrete math and algorithms, though, we do not really have time to get into any serious applications at this point (you will see them later during your studies), but will focus on covering some of the basic algorithms used to manipulate these objects.

Reading Instructions

As usual, I will do my best to try to cover as much as possible of the most important material in class, but it is very important that you also read the textbook, which contains additional material.

- CLRS Chapter 21 about minimum spanning trees (all of it).
- CLRS introduction to Part II (especially if you want to get a bit of an overview why we are so interested in sorting data).
- CLRS Chapter 6 about heaps (all of it).

Exercises

Some of the exercises below were listed also last week, but I am mentioning them here as well to give an overview of good exercises concerning graphs.

CLRS Chapter 20: Elementary Graph Algorithms

1. Draw some moderately sized directed graphs and make sure that you can run DFS and BFS on them.

2. Run topological sort on the same algorithms. Check that the algorithm works precisely when your directed graphs are acyclic.

3. Use the algorithm in Sec 20.5 to compute strongly connected components of some of your directed graphs.

4. CLRS Section 20.1 exercises 20.1-1, 20.1-3, 20.1-7.

5. CLRS Section 20.2 exercises 20.2-1, 20.2-2, 20.2-4, 20.2-5, 20.2-7

6. CLRS Section 20.3 exercises 20.3-2, 20.3-11, 20.3-12

7. CLRS Section 20.4 exercise 20.4-1

8. CLRS exercise 20-3.

CLRS Chapter 21: Minimum spanning trees

1. Draw some moderately sized undirected graphs with edge weights (making sure to consider also graphs with several edges of the same weight) and run Prim's algorithm starting from different vertices. Can you find graphs where different starting points yield different MSTs?

2. Draw some moderately sized undirected graphs with edge weights (making sure to consider also graphs with several edges of the same weight) and run Kruskal's algorithm. Can you find graphs for which you get different MSTs depending on which order edges of the same weight happen to be sorted?

3. CLRS Section 21.1 exercises 21.1-1, 21.1-3–21.1-7, and 21.1-9

4. CLRS Section 21.2 exercises 21.2-1 and 21.2-2

5. [*] CLRS exercise 21-4

CLRS Chapter 6: Heaps

1. Take some moderately sized arrays filled with numbers (or comparable keys of your choice) and build min-heaps and max-heaps from them.
2. CLRS Section 6.1 exercises 6.1-1–6.1-7
3. CLRS Section 6.2 exercises 6.2-1 and 6.2-3–6.2-6
4. CLRS Section 6.3 exercises 6.3-1–6.3-3
5. CLRS Section 6.4 exercises 6.4-1–6.4-4
6. CLRS Section 6.5 exercises 6.5-1, 6.5-2, 6.5-5, and 6.5-7–6.5-9
7. CLRS exercise 6-1