- flip coin 6 times: If heads on ith flip jth round $(1 \le i \le 6)$ $= 12i$ dollars

$(1 \le j \le n)$ otherwise $= -4ij$ dollars

X_{ij} := amount money won for ith flip on jth round

X := amount mony won across all rounds

1) How much \$ won on 2nd round

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ T & T & H & T & H & H \end{array}$$

$$\Rightarrow -4(1)(2) + -4(2)(2) + 12(3) + -4(4)(2) + 12(5) + 12(6)$$

$$= \$112$$

2) Give general formula for expected value of X_{ij}

$$\Rightarrow X_{ij}(s) = \begin{cases} 12i & \text{if } s == \text{heads} \\ -4ij & \text{otherwise} \end{cases}$$

$$\Rightarrow E(X_{ij}) = (1/2)(12i) - (1/2)(4ij)$$

$$E(X_{ij}) = 6i - 2ij$$ // notice if $j = 3$, this would $= 0$

3) Describe random var. X as nested sum of random var. X_{ij}

$$\Rightarrow X = \sum_{j=1}^{n} \sum_{i=1}^{6} X_{ij}$$

4) Give a formula for expected value of this random variable (use linearity of Expectation)

$$\Rightarrow E(X) = E\left(\sum_{j=1}^{n} \sum_{i=1}^{6} X_{ij}\right)$$ // Formula (we can apply linearity of expectation)

// we need to simplify & clean up

$\Rightarrow$ using linearity of expectation

$$\Rightarrow E(X) = \sum_{j=1}^{n} \sum_{i=1}^{6} E(X_{ij})$$

$$= \sum_{j=1}^{n} \sum_{i=1}^{6} 6i - 2ij$$

//We now can apply sum rules

$$\Rightarrow \underbrace{\sum_{j=1}^{n} \sum_{i=1}^{6} 6i}_{①} - \underbrace{\sum_{j=1}^{n} \sum_{i=1}^{6} 2ij}_{②} = 126n - 21n^2 + 21n$$

$$= 126n - 21n^2 - 21n = -21n^2 + 105n$$

① $6 \sum_{j=1}^{n} \sum_{i=1}^{6} i$ $= 1+2+3+4+5+6 = 21$

$$\Rightarrow 6 \cdot 21 \sum_{j=1}^{n} 1 = 6 \cdot 21 \cdot n = 126n$$

② $\sum_{j=1}^{n} \sum_{i=1}^{6} 2ij$

$$= 2 \sum_{j=1}^{n} j \boxed{\sum_{i=1}^{6} i} = 21$$

$$= 2 \cdot 21 \sum_{j=1}^{n} j$$

// Arithmetic series

$$\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$$

$$= \cancel{42} \cdot \frac{n(n+1)}{\cancel{2}}$$

$$= 21n(n+1) = 21n^2 + 21n$$

$$= -21n^2 + 105n$$

//can simplify

$$\Rightarrow 21n(-n+5) = 21n(5-n)$$

5) How many rounds would you want to play
Plug into formula & see what you win
(what n maximizes returns)

$\Rightarrow$ Notice from 2) that if $j = 3$ our formula would equal 0

$\hookrightarrow 6i - 2ij$ // let $j = 3$

$\Rightarrow 6i - 6i = 0$

$\Rightarrow$ This indicates break even, therefore $n = 3$ maximizes gains

6) If you wanted to make game as fair as possible what n would you pick?
(what n makes expected value closest to 0)

$\Rightarrow$ To find what n makes expected value closest to 0; we set formula $= 0$ & solve for n

$\Rightarrow 21n(5-n) = 0$ // Zero factor principle $ab = 0 \Leftrightarrow a = 0$ or $b = 0$

$$\begin{bmatrix} \Rightarrow & n = 0 \\ \Rightarrow & n = 5 \end{bmatrix}$$

& $\dfrac{21n(5-n)}{21n} = \dfrac{0}{21n}$

$\Rightarrow 5 - \underset{+n}{n} = 0 \Rightarrow n = 5$

For $n = 0$ or $n = 5$ creates a fair game