Numerical Computations - Ex 2, Oct 22, 2020

5. Implement Hermite interpolation using divided differences: find $p \in P^{2n+1}$ such that

$$p(x_i) = f(x_i)$$
 and $p'(x_i) = f'(x_i)$ for $i = 0 \dots n$

Test your implementation for $f(x) = \sin(x)$, and $x_0 = 0, x_1 = \pi/2, x_2 = \pi$.

You can start from the notebook NewtonInterpolation.ipynb provided in the lecture.

- 6. Compare the interpolation errors (convergence plots) of polynomial interpolation and spline interplation with classical cubic splines for the functions
 - $f(x) = \sqrt{x}$ on [0, 1]
 - $f(x) = \arctan(10x)$ on [-1, 1]

You can start from the notebook *splines.ipynb* using the interpolate module from scipy. Try to find good knot distributions.

7. Find a good approximation to

$$\sum_{j=1}^{\infty} \frac{6}{j^2}$$

by extrapolation. Hint: define the function

$$f: \{1/n : n \in \mathbb{N}\} \to \mathbb{R} : x \mapsto \sum_{i=1}^{1/x} \frac{6}{j^2},$$

apply polynomial interpolation with knots $x_i = 2^{-i}$, and evaluate the interpolation polynomial at x = 0.

8. Let

$$I_n: C([a,b]) \rightarrow C([a,b])$$

 $f \mapsto \sum_{i=0}^n f(x_i)l_i(\cdot)$

be the Lagrange interpolation operator. Prove that its operator norm defined by

$$||I_n||_{\infty} := \sup_{f \in C([a,b]), f \neq 0} \frac{||I_n f||_{\infty}}{||f||_{\infty}}$$

evaluates to

$$||I_n||_{\infty} = \sup_{x \in [a,b]} \sum_{i=0}^n |l_i(x)|.$$

Hint: part of the work was already done in class