

# Business Analytics & Machine Learning

## Tutorial sheet 11: Convex Optimization

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### Exercise T11.1 *Convex function*

You are given the following function:

$$f(x, y) = a \exp(3x) + \frac{b}{2}xy + y^2$$

Determine all parameters  $a, b \in \mathbb{R}$  such that  $f$  is convex.

### Exercise T11.2 *Operations preserve convexity*

You are given the following convex functions  $g_1(x), g_2(x)$ . Prove that the following functions are also convex functions:

- $h_1(x) = g_1(Ax + b)$  where  $A$  is a matrix and  $b$  is a vector.
- $h_2(x) = C_1g_1(x) + C_2g_2(x)$ , where  $C_1$  and  $C_2$  are nonnegative constant.
- $h_3(x) = \max\{g_1(x), g_2(x)\}$ .

### Exercise T11.3 *Gradient descent*

You are given the following function:

$$f(x, y) = 2x^2 + 0.5y^2 - 3x - y - 2xy + 5$$

With starting point  $z^{(1)} = (0, 0)$ , conduct two steps of the gradient descent algorithm. Choose the step size  $\alpha$  using line search.