Dependence of ecosystem equilibrium on predator hunting efficiency in the Wa-Tor Model

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In this experiment, a series of simulations were conducted to investigate the occurrence of equilibrium in the Wa-Tor model ecosystem depending on the hunting efficiency of predators. Measurements were taken for four different simulation area sizes, maintaining the same initial conditions. The dependencies of equilibrium occurrence on hunting efficiency are plotted in Figure 2. For each area size, threshold values were determined, which are the first values of hunting efficiency that caused the appearance of equilibrium. These results are summarized in Table 2.

I. Wa-Tor model

a. Model Description

Wa-Tor is a model simulating a simplified ecosystem of a planet inhabited by only two species: predators and prey (originally sharks and fish [1]). These animals live in a delicate balance, where the population size of one species affects the population size of the other. This dynamic interaction allows the planet to exist in one of three states:

- **Extinction of both species**: when only predators remain on the planet, they start to die slowly due to lack of food, eventually leaving the planet empty.
- **Extinction of predators**: when no predators remain on the planet, prey then have ideal conditions to reproduce and populate the entire planet.
- **Harmony**: when continuous oscillations in species numbers cause cyclical increases and decreases in their populations. Due to the interdependence of species numbers, these changes lead to subsequent oscillations, called an equilibrium state.

The Wa-Tor model can also be interpreted as a projection of a nonlinear system of differential equations (known as **the Lotka-Volterra equations**) onto a discrete, two-dimensional plane. This perspective allows us to write the changes in species numbers as:

$$\begin{cases} \dot{V} = \alpha(a)V(t) - \beta(b)V(t)P(t) \\ \dot{P} = \delta(b,d)V(t)P(t) - \gamma(c)P(t), \end{cases} \tag{1}$$

where $V(t)$ is the prey population, $P(t)$ is the predator population and a, b, c, d are simulation parameters defining the animals' behaviors.
This relationship shows that the population size of one species directly affects the population size of the other.

b. Simulation Parameters

Before starting the simulation, it is crucial to set its parameters. These values significantly influence the simulation process and the obtained results. Properly choosing these parameters can lead to the planet being in a state of harmony or struggling with the extinction of one or both species.
The algorithm used in the study allowed the following parameters to be set:

- **Area size**: the size of the area expressed as the length of a side. The simulation uses periodic boundary conditions.wykorzystuje się periodyczne warunki brzegowe.
- **Initial prey population**: the prey population expressed as a percentage of the simulation area.
- **Initial predator population**: the predator population expressed as a percentage of the simulation area.
- **Prey reproduction** (a): the number of steps after which prey reproduce.
- **Hunting efficiency** (b): the percentage chance of a successful hunt.
- **Predator mortality** (c): the number of steps without a successful hunt after which a predator dies of starvation.
- **Predator reproduction** (d): the number of steps after which predators reproduce.

c. Description of used algorithm

The algorithm consists of two main stages: preparing the simulation and executing its subsequent steps.
The preparation stage involves randomly distributing prey and predators within the ecosystem area.
The simulation steps are discrete time jumps. Each step includes the movement of prey and predators. Prey move to randomly chosen adjacent empty cells (*von Neumann neighborhood* applies). If all cells are occupied, the prey stays in place. After a certain number of steps, prey reproduce, leaving another prey in their cell. Predators also move to randomly chosen adjacent cells, but they prioritize those occupied by prey. If a predator hunts prey (moves to its cell), it has a specified percentage chance of a successful hunt. If the hunt fails or the predator does not hunt, it uses energy for movement. If the used energy equals the critical parameter, the predator dies. A successful hunt resets the energy used by the predator. Additionally, after a certain number of steps, predators reproduce, leaving another predator in their cell.

The program used in the project is available in the form of a repository on GitHub [2].

II. Searching for equilibrium in the simulation

To study the occurrence of equilibrium in the ecosystem depending on predator hunting efficiency, it is necessary to define the concept of equilibrium. Contrary to appearances, this is not an easy task because the time evolution of complex multi-element systems strongly depends on initial conditions. The system is sensitive to parameter changes such as reproduction rate or mortality, and to random events. Even slight perturbations can lead to dramatic changes in ecosystem equilibrium. These changes may not appear in the first oscillation cycles.

For obvious reasons, the simulation cannot last indefinitely, so it is necessary to set a certain number of iterations after which it can be concluded that harmony between species numbers has occurred in the ecosystem.

WTo determine the necessary number of iterations, graphs of population changes for a certain number of time steps were created, gradually increasing their number. The parameters of the studied simulation are presented in Table 1. The obtained results were determined as the average of ten runs of the algorithm and are presented in the form of graphs in Figure 1.

The obtained dependencies allowed us to conclude that 2500 iterations are sufficient to determine the occurrence of equilibrium in the system (Figure 1c), which corresponds to about 45 full oscillation cycles. However, this is an arbitrary value established by the experimenter.

III. Studying the dependence of equilibrium occurrence on hunting efficiency

a. Methodology

To study the stability of the simulation concerning hunting efficiency, the parameters presented in Table 1. The study was conducted for area sizes of 25, 50, 75, and 100. Data were collected for hunting efficiencies from 0 to 100 in steps of 1. To ensure the reliability of the results, each simulation step was repeated 100 times, and the average values were then calculated.

Each of the studied runs could end in three ways:

- **2500 iterations**: After 2500 iterations, the algorithm ended the loop and moved to the next step.
- **No prey on the map**: The step in which no prey were left on the map was recorded.
- **No predators on the map**: The step in which no predators were left on the map, resulting in uncontrolled prey population growth and area fill, was recorded.

The collected data were plotted and presented in Figure 2. The average standard deviation of each measurement was also marked on the graphs.

b. Results analysis

The obtained results show that the threshold value for the transition between non-equilibrium and equilibrium states occurs at a hunting efficiency of 50%. Additionally, comparing the graphs, it can be seen that the larger the studied area, the more stable the equilibrium.

e threshold values, i.e., the hunting efficiency at which a sudden increase in simulation length was observed, are presented in Table 2. For an area size of 25, no parameter guaranteeing stability could be found, so the one with the highest average value was chosen.

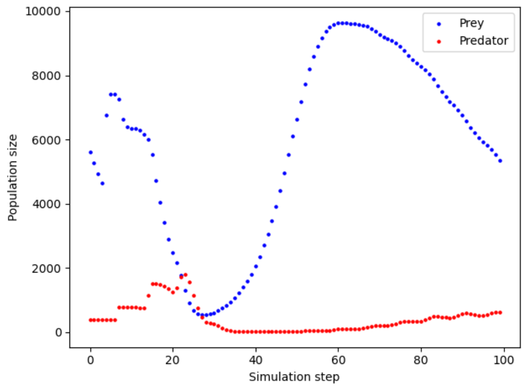
In the case of smaller areas, it can be clearly indicated that too high hunting efficiency also causes instability. This is because when reducing the studied area, it becomes much easier for predators to eat too many prey, ultimately leading to their disappearance. Due to the randomness in the simulation, increasing the area can protect the ecosystem from a situation where none of the predators can "catch up" with the prey before dying. This behavior can be observed in the graphs, with the curve flattening after reaching the threshold value as the area size increases.

The ecosystem could likely be saved, for example, by introducing predator behavior that causes tracking prey - when choosing a path, if there are no prey nearby, the predator would move towards the nearest prey.

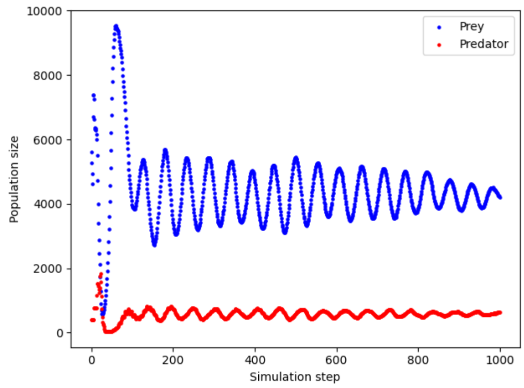
Bibliografia

[1] A. K. Dewdney, *Sharks and fish wage an ecological war on the toridal planet Wa-Tor*. Computer Recreations, 1984.

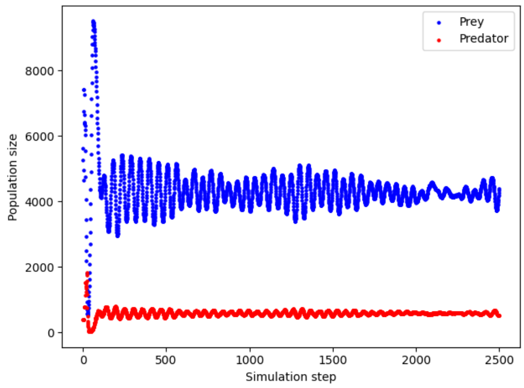
[2] J. Rogala, *Wa-Tor*. [Online]. Available: https://github.com/jakrog01/Wa-Tor



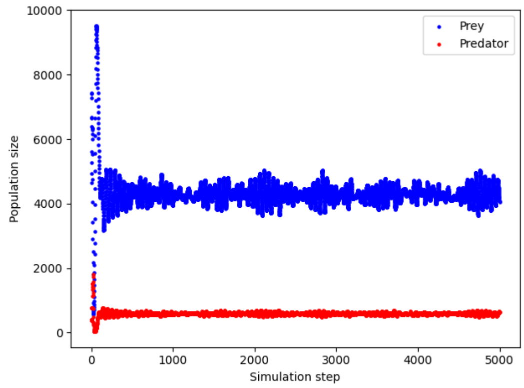
(a) Changes in populations for 100 iterations



(b) Changes in populations for 1000 iterations

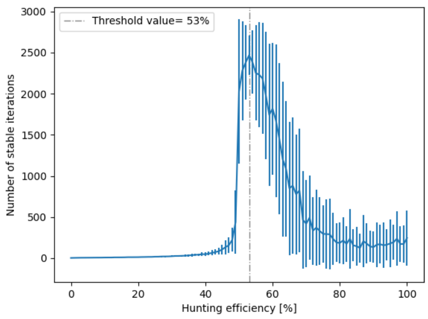


(c) Changes in populations for 2500 iterations

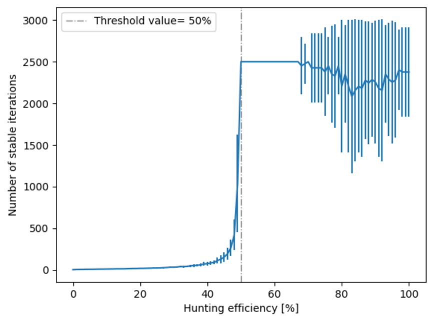


(d) Changes in populations for 5000 iterations

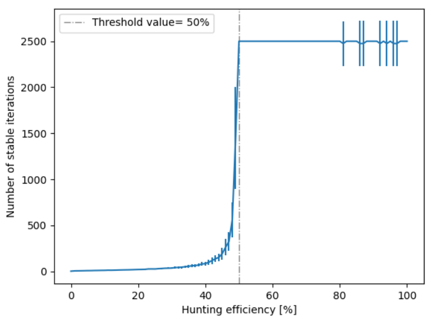
Rys. 1: Changes in population size for different numbers of algorithm iterations



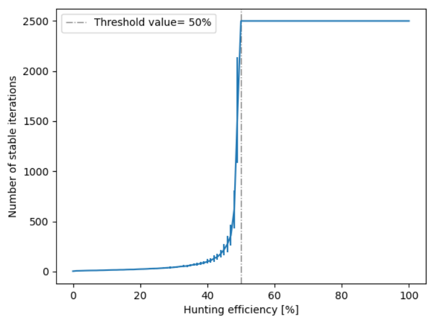
(a) Dependence of the simulation length on hunting efficiency for an area of size 25



(b) Dependence of the simulation length on hunting efficiency for an area of size 50



(c) Dependence of the simulation length on hunting efficiency for an area of size 75



(d) Dependence of the simulation length on hunting efficiency for an area of size 100

Rys. 2: The dependence of obtaining balance in the simulation on the hunting efficiency of various sizes of the study area

TABELA 1: SIMULATION PARAMETERS USED TO STUDY THE EQUILIBRIUM STATE

Area size	100
Initial prey population [%]	60
Initial predator population [%]	4
a [steps]	5
b [%]	100
c [steps]	3
d [steps]	7

TABELA 2: THRESHOLD VALUES OF HUNTING EFFICIENCY FOR WHICH EQUILIBRIUM OCCURRED

Area size	25	50	75	100
Threshold values [%]	53	50	50	50