Sep 9

M / Tue / Wed / Thur / Fri

Study (M, Tue) — Lab (Wed) — TUT (Thur) — Problem sets (Fri)

(you may try problems)

Code on tests. → Coding Independent

6 Labs. ~~0~~, 1 2 3 4 5 2WEEK LABs. (25%). Report (2 ppl. &)

2 Exams + 1 Final. (25% each)

⇒ Cumulative Knowledges.

W1

§ 1. Introduction

1.1

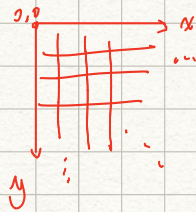
- Digital Image Processing: Manipulation of digital image for enhancement, compression, transmission, information extraction or analysis.

  denoising, enhancement, registration, color mapping, rescaling, feature point detection, illumination reduction, ...

- define 2-D function: $f(x, y)$.

  (x, y): spatial (in-plane) coordinates.

  f: the value @ spatial (x, y) location



- 3D - Images: ① With multiple images of the same scene, one can deduce 3D geometry & pixels in the image could have 3D coords.

  Ex: $f(x, y, z)$, where (x, y, z) rep. spatial coords.

  ② Video: $f(x, y, t)$. (t: time.)

  ③ Color Images have 3 Bands:

  R: $f(x, y, 0)$

  G: $f(x, y, 1)$

  B: $f(x, y, 2)$

  ↳ Multispectral: many bands; in remote sensing: (6~8)

  ↳ Hyperspectral: hundreds of thousands of bands: (image cube)

- Computer Vision.

  SYDE 575 mainly & later

  Scene → Image Processing → Image Analysis → Image Understanding → Information

  - Image Analysis: Content extraction (Ex: segmentation, shape description, boundary detection, mathematical morphology, texture feature extraction, motion estimation.
  - Image Understanding: decision making based on content extraction. (SD 372 & SD 675).

- Why IP difficult?
  - 3D world → 2D plane.
  - Measured intensity is a function of many factors.
  - Interpreting grps of pixels as interesting objects is easy for human but not computers.
  - Loads of data process

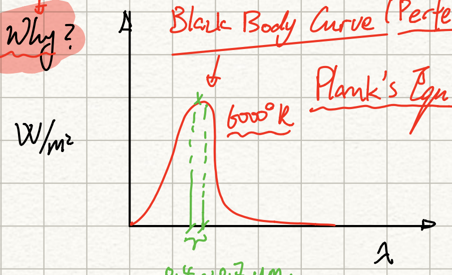
• Local windows vs. global interpretation.

We only see:

• 0.4 ~ 0.7 $\times 10^{-6}$ Electro magnetic range

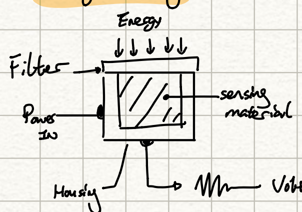
Why?

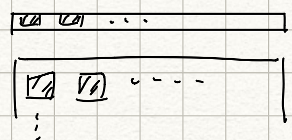
Emits all it absorbs (ex: sun)



(Most powerful / peak wavelength electromagnetic energy).

Image Sensing





↳ **Imaging:** 3D object in world → project to camera plane → discretization ⇒ Form 2D digital images.

└ Formation Model: $f(x,y) = i(x,y)\, r(x,y)$

① source illumination incident on scene

② Amount of illumination reflected by objs in scene. (0 : total absorption ; 1 : total reflection).

↳ Characterized by ① & ②

$$\begin{cases} 0 < i(x,y) < \infty \\ 0 < r(x,y) < 1 \end{cases}$$

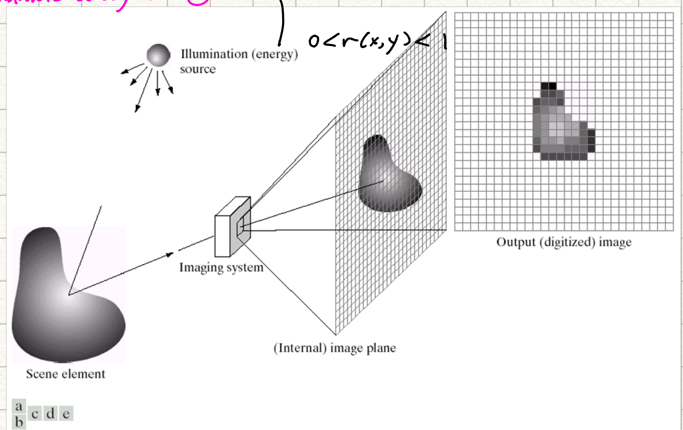
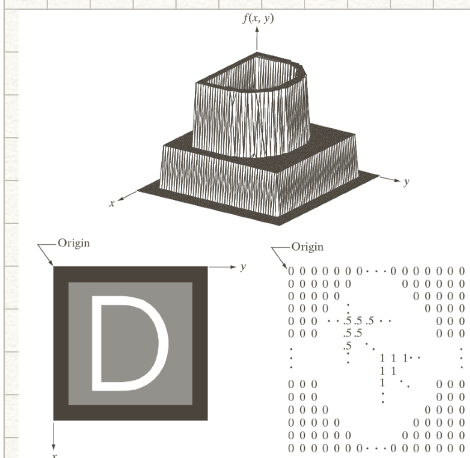
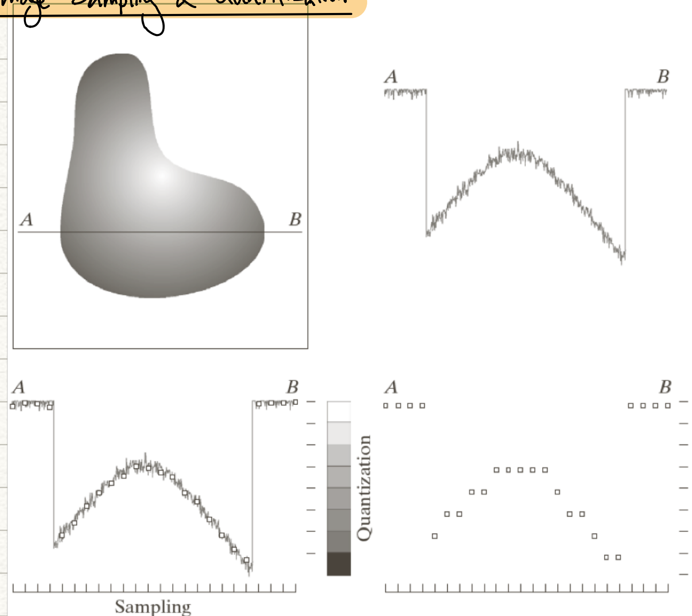


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.



↳ **Image Sampling & Quantization**



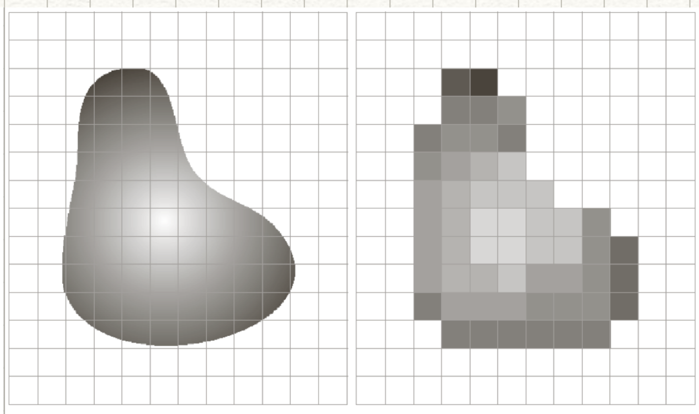
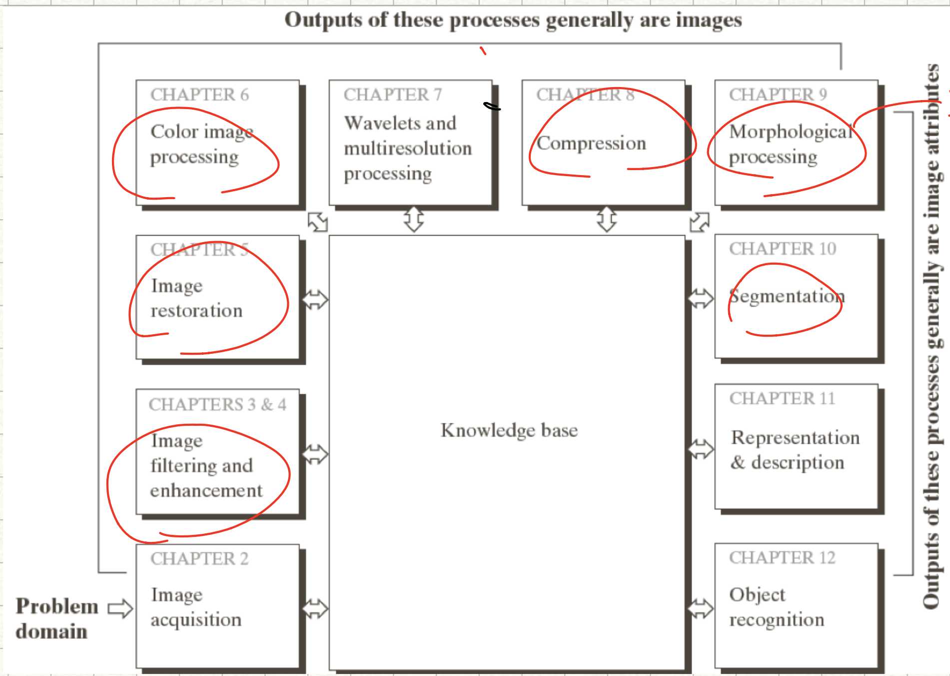


FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Fundamental Steps [This Course]

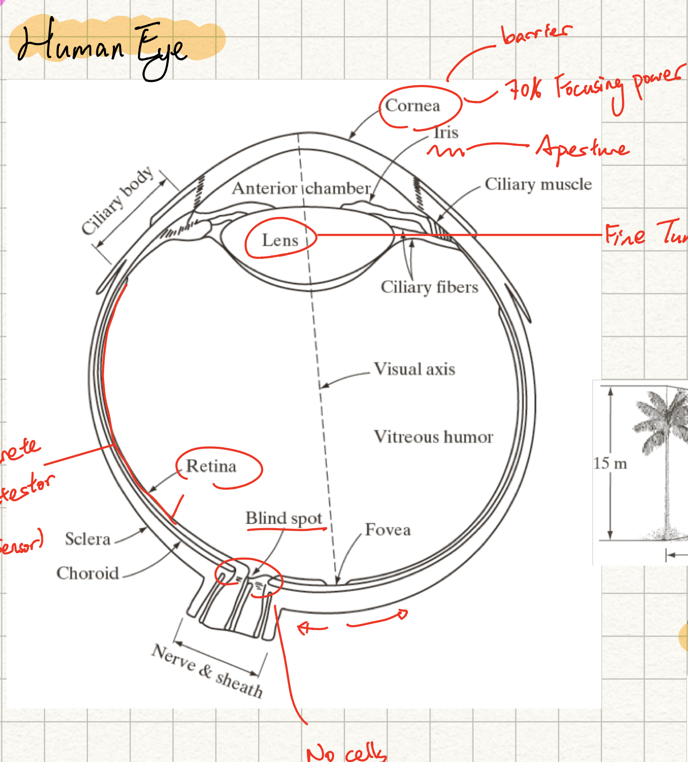
Filtering & Enhancement → Restoration → Color Image Processing →

Compression → Morphological Processing → Segmentation → Neural Networks



1.2 /

Human Eye



→ Brain does the flip

Eye Optics : Flip

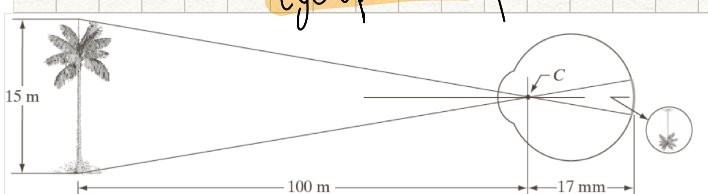
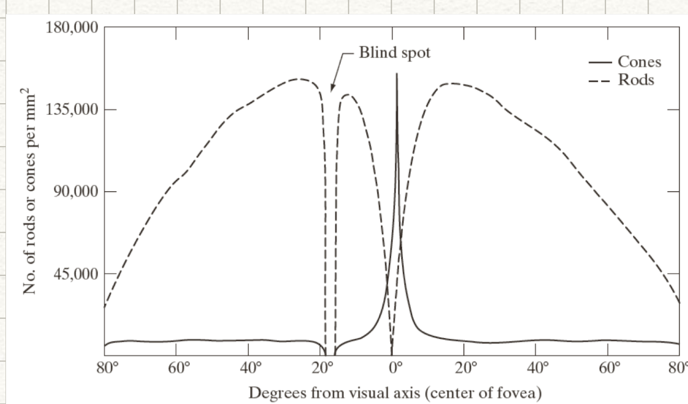


FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point C is the optical center of the lens.

Distribution of Rods & Cones



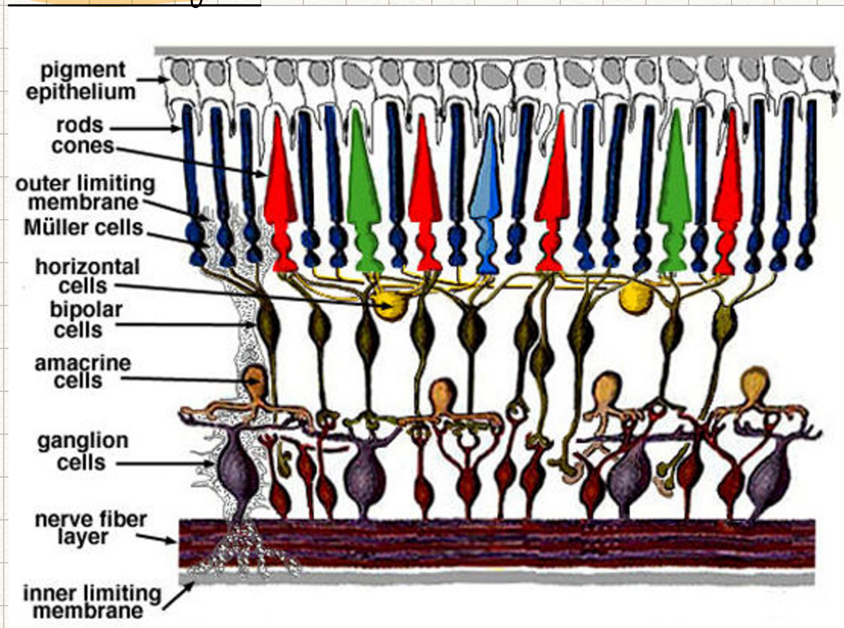
Rods : + 75 ~ 150 million.
+ sensitive to low level of illumination
+ Dim-light vision (scotopic)
+ Monochrome vision
+ Low visual resolution (shares nerve ends)

Cones : + 6 ~ 7 million

+ Sensitive to high levels of illumination
+ Color
+ Bright light / day-time vision (photopic)
+ High visual resolution (has own nerve end).

Retinal Layers

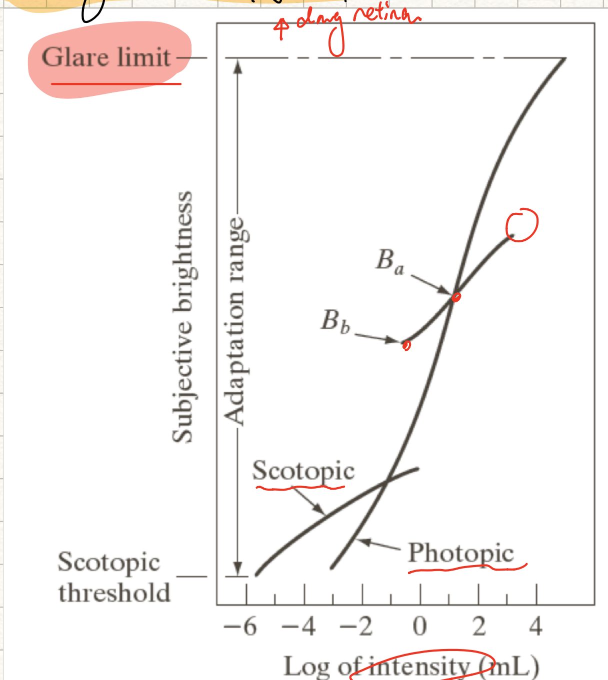
99% Reduction



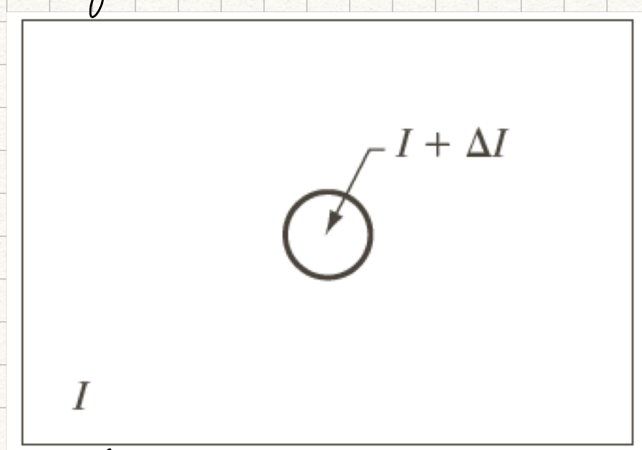
(Compression happened).

Brightness Adaptation

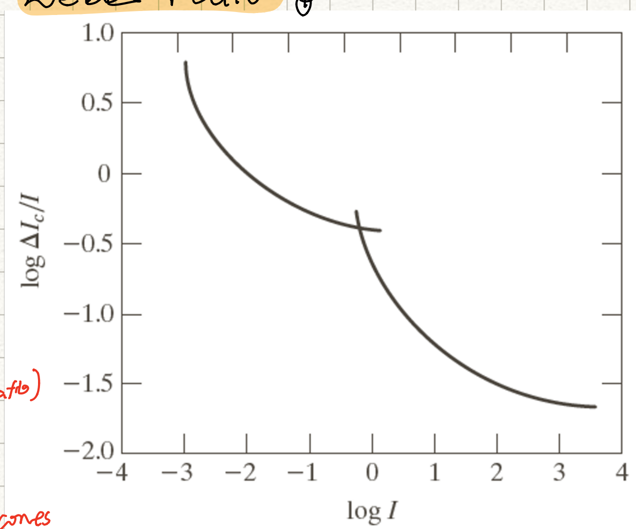
4 along retina



Brightness Discrimination



Weber ratio



- Brightness discrimination poor (large Weber Ratio) at low levels of illumination.
- Brightness discrimination improves (small Weber Ratio) at high levels of illumination.
- Reflects fact that dim-light vision carried out by rods, while bright-light vision carried out by cones

Human Vision System:

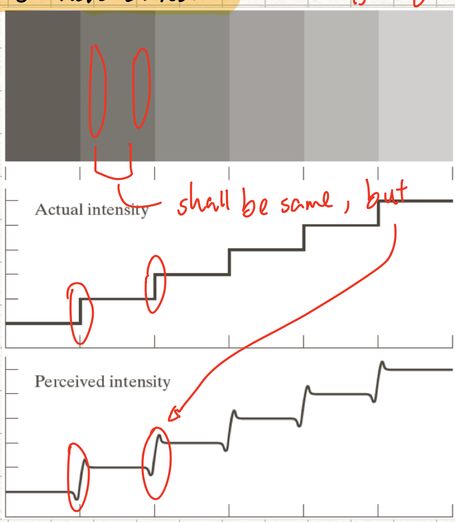
is less sensitive to changes in chrominance (color) than luminance.

original

1/4 Color information

Human Eye cannot tell diff.

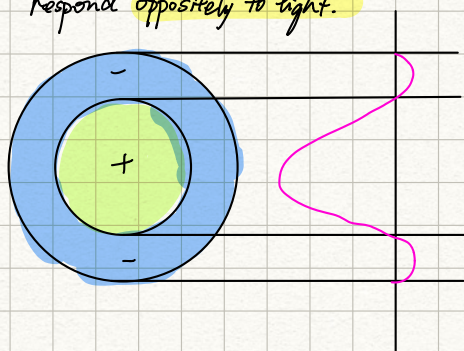
Mach Band → Intensify edges,



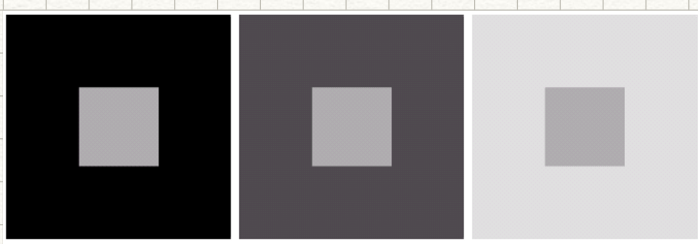
shall be same, but

⇒ Lateral Inhibition

- Receptive field of retina organized for contrast detection (discontinuities in distribution of light)
- Centre (lateral excitation) & surround regions (lateral inhibition) respond oppositely to light.

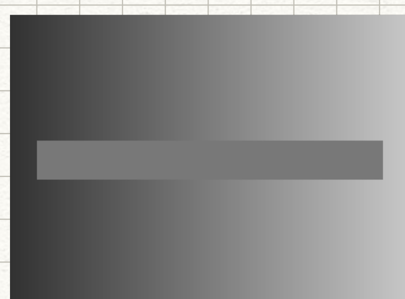


Simultaneous Contrast



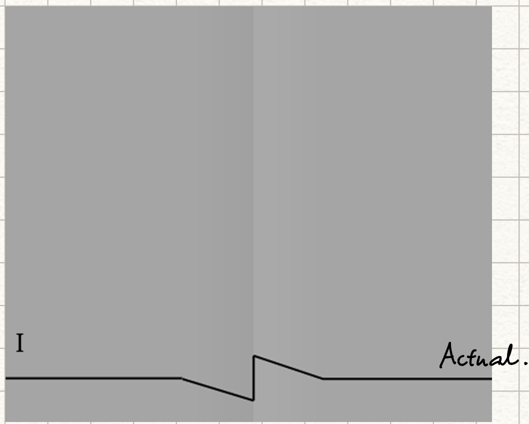
a b c

FIGURE 2.8 Examples of simultaneous contrast. All the inner squares have the same intensity, but they appear progressively darker as the background becomes lighter.



- inner cube appears to be darker by our perception.
- Perception changes, but reality is not.

Cornsweet Effect (Perception of Large Area)



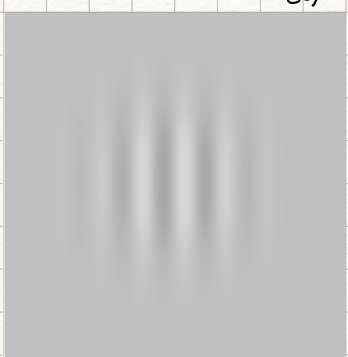
Spatial Contrast Sensitivity

- Size of retina's perceptive field controls spatial-frequency sensitivity
- Small receptive fields:
  - Sensitive to high spatial frequencies
  - Responsible for fine detail
- Large receptive fields:
  - Sensitive to low spatial frequencies.
  - Responsible for coarse detail.

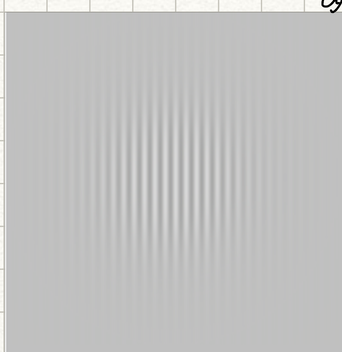
Sine Wave Grating

(cpd)

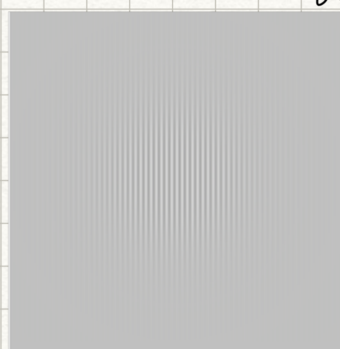
8 cycles/deg

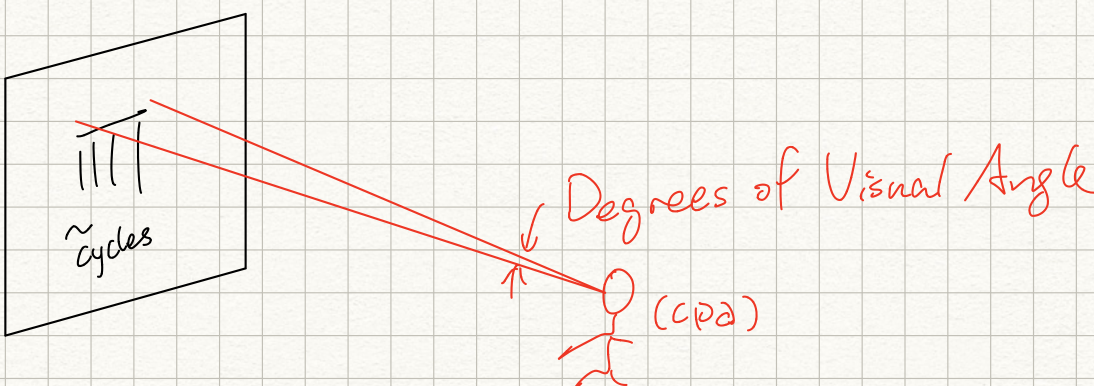


32 cycles/deg



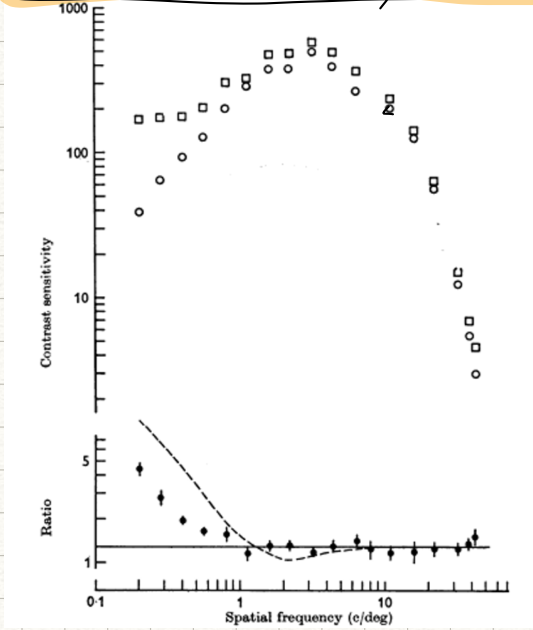
64 cycles/deg





Closer / outter: cycles per degree changes.

Contrast Sensitivity Function



Cells sensitive / tune differently
depends on frequency & orientation

Effects of Spatial Contrast Sensitivity

- Human vision good at spotting small brightness differences in low to medium frequencies
- Human vision poor @ judging brightness diff in regions with high freq information.
- Human vision more sensitive to noise in low to medium frequency regions than high frequency regions

Primary Visual Cortex (V1)

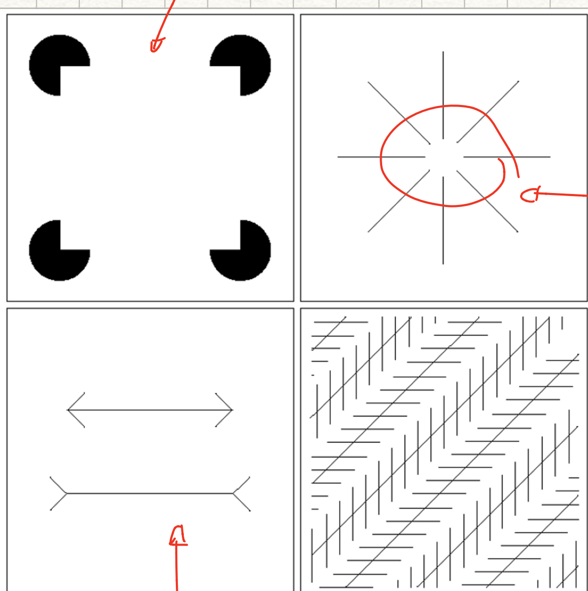
- Earliest & largest cortical visual area
- Majority of all visual information enters cortex through V1
- V1 neurons considered simplest of all neurons in visual cortex
- V1 creates a "saliency map" that identifies the important parts of the scene to guide positioning of the eyes (gaze)
- Early V1 neurons tuned to low-level visual characteristics such as orientations & spatial frequencies.
- Often modeled using Gabor functions.

Optical Illusions

perceive square

a b
c d

FIGURE 2.9 Some well-known optical illusions.

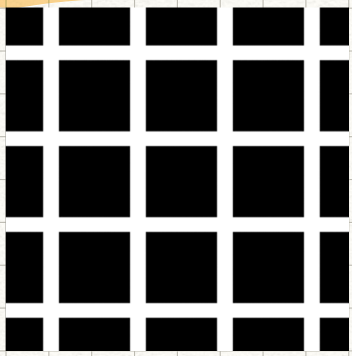


perceive as circle

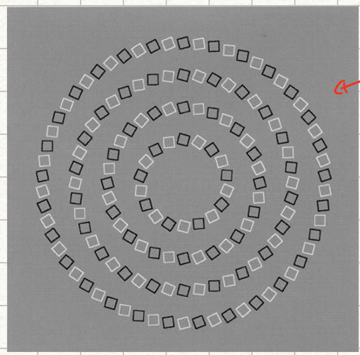
perceive unparalleled.
(Optical Distortion)

perceive lower longer

Hermann Grid



spiral illusions



- HVS picks up on the edges of the boxes.
- assumes continuity, forming a spiral shape

↳ Why do we care?

- Image quality is highly subjective & greatly dependent on the way our vision sys works.
- By taking into account the psychovisual characteristics of the human vision systems, we can:
  - Design Image Processing algorithms that make images looks better perceptually.
  - Design Image Compression algorithms that look almost as good as the original while storing much less information.
  - Design information extraction algorithms that provides good representation of the image content

Human Vision vs. Computer Vision.

Human Vision

- Easily processes 3D & video
- Excellent visual interpretation
- Success under range of lighting conditions.
- Not well understood.

Computer Vision

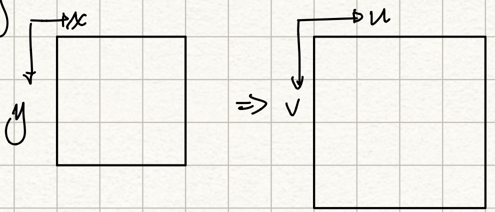
- Difficult to get 3D info.
- Noise. (Due to sensor/env.)
- Static Viewpoints
- Low spatial Resolution.
- Well understood algo, limited and currently improving tremendously.

W2

§ 2 – Point Operations

§ 2.1.1 Image Zooming / Resampling

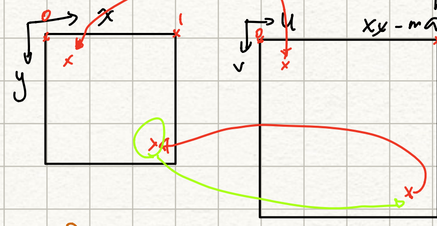
- **Resampling**: when resizing an image
- **Resizing**:
  - change # pixels by ↑/↓ # of rows/columns.
- **Zooming**:



Three Approaches

① Nearest Neighbour

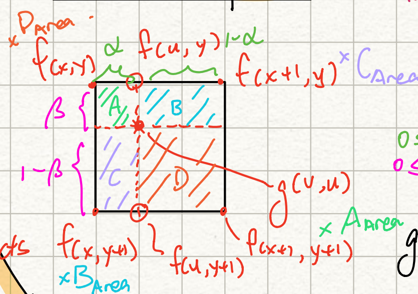
{ ① map uv pixel to xy coord
{ ② find the nearest neighbor to the mapped coordinate in xy-map, copy value to selected uv pixel



Pro: Quick, Easy to implement, Retains orig. value (important for calibrated data)

Con: Blocking Effect: (ex: if image is rotated, a straight horizontal edge will have a "staircase effect")

② Bilinear Interpolation



$0 \le \alpha \le 1$
$0 \le \beta \le 1$

$g(u,v) = (1-\beta)(1-\alpha) f(x,y) + (1-\beta)\alpha\, f(x+1,y) + \beta(1-\alpha) f(x,y+1) + \beta\alpha\, f(x+1,y+1)$

③ Cubic Interpolation

Summary

① **Nearest Neighbor Interpolation**
- Look for closest pix in orig. image.
- Fast but causes undesirable checkboard effects

② **Bilinear Interpolation**
- Determines pix value based on four nearest neighbours.
  - Perform linear interpolation in x direction.
  - Perform linear interpolation in y direction based on results of interpolation from x-direction
- Does not suffer from checkerboard / staircase effect but can result in a blurred appearance.

③ **Bicubic Interpolation**
+ Determines pixel value based on sixteen nearest neighbours (4×4)
  - ↳ perform cubic spline interpolation in x-dir.
  - ↳ ______ in y-dir. based on results of interpolation from x-dir.
+ Does not suffer from checkerboard effect
+ Preserves fine details better than bilinear interpolation.

§2.1.2 Image Quality Metrics

① Mean Square Error:

$$\boxed{MSE = \underbrace{\frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}}_{\text{mean}} \| \overbrace{f(i,j) - g(i,j)}^{\text{diff}} \|^{\overset{\text{square}}{2}}}$$

- Pro:
  - Easy to understand
  - Easy to implement
  - commonly used
- Con:
  - May not accurately reflect image quality
  - Ex: If there are shifts in image, MSE will not be zero

(Improved): SSIM (Structural Similarity Index Measurement) by Zhou Wang (ECE).

② Peak Signal-to Noise Ratio:

$$\boxed{NSE = 10 \log_{10}\left(\frac{MAX_f^2}{MSE}\right) = 20 \log_{10}\left(\frac{MAX_f}{\sqrt{MSE}}\right)}$$

Ex: MAX → 8 bit data, set to 255

§2.1.3 Point Noise Models

Ⓐ Simple Image Noise Degradation Models

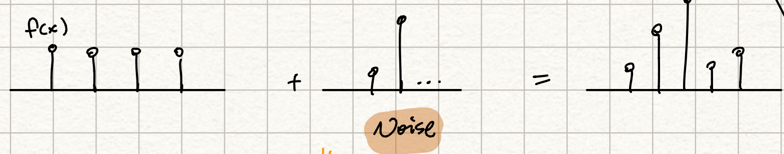
Ⓑ Additive Noise Model: → $\overset{\text{output}}{g(x,y)} = \overset{\text{input}}{f(x,y)} + \overset{\text{noise}}{h(x,y)}$

Ⓒ Multiplicative Noise Model: → $g(x,y) = f(x,y)\, h(x,y)$

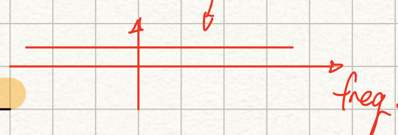
Ⓑ Additive Noise Model

(Note: great for ultrasonic sensor)

$f(x,y)$ → $\boxed{\text{random noise}}$ → $g(x,y)$



typically:
① - zero mean
② - signal & noise independent → sig changes cause no change in noise
③ - Noise is white

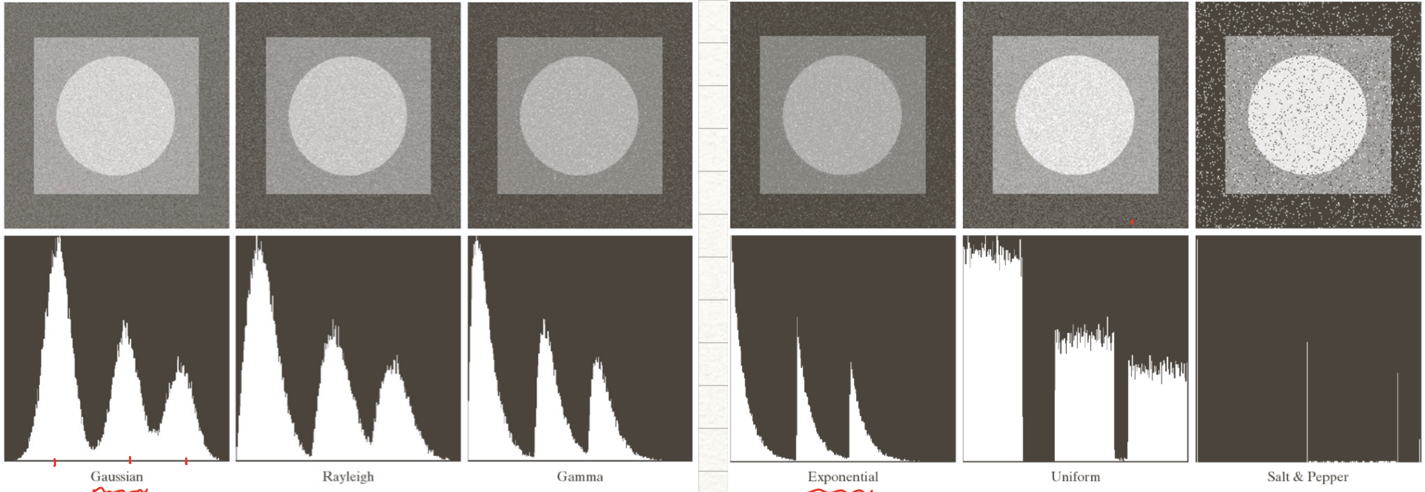


Noise Models

- Principal src of digital image noise arise during img acquisition and/or transmission
  - Thermal Noise
  - Shot Noise
  - Corruption during transmission over network
- In Simple Noise Models, noise is assumed to be Independent & White
- See section §5.2 of Gonzalez & Woods

Some Point Noise Models

ORIGIN



↖ statistical noise distribution.

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

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§ 2.2 Histograms.

§ 2.2.1 Intro.

we understand.

① Enhancement vs. Restoration

↳ qualitative approach to improve the perceived appearance of an image.

↳ Model-based approach to improve the statistics of an image &, hopefully, improve perceived appearance.

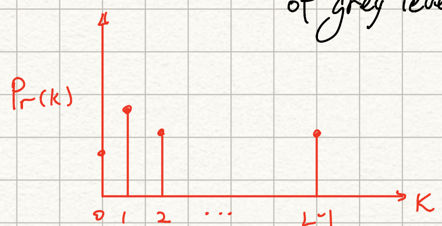
② Point Processing

- the simplest of image enhancement techniques.
- based only on intensity of individual pixels.
- Given $g(x,y)$ as the output pixel, $f(x,y)$ as the input pixel, and T as some transformation:

Based on

$$\boxed{g(x,y) = T[f(x,y)]}$$

②-1 Histograms : simply the distribution of grey level in an image



K: intensity
L: # of intensities
n_k: # of pixels per K
M, N: # rows / cols of img

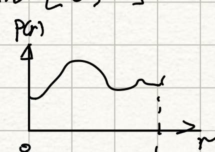
$$P_r(k) = \frac{n_k}{MN}, \quad k = 0, 1, \dots, L-1$$

$P_r(k) \triangleq$ probability mass func. (Distribution).

Discrete ⇒ Continuous Rep:

- For a distribution bound in $[0,1]$

$$\boxed{\int_0^1 P_r(r)\,dr = 1}$$



pdf (Probability Density Function)

≜ CDF : Cumulative Distribution Function.

2-1-1 Mean & Variance

Mean: $\boxed{\mu_r = \int_0^1 r \, p_r(r) \, dr}$ $\qquad$ $\mu_r = \sum_{k=0}^{L-1} k \cdot p_r(k)$

Variance: $\boxed{\sigma_r^2 = \int_0^1 (r-\mu)^2 \, p_r(r) \, dr}$ $\qquad$ $\sigma_r^2 = \sum_{k=0}^{L-1} (k-\mu_r)^2 \, p_r(k)$

Continuous $\qquad$ Discrete

2-1-2 Contrast

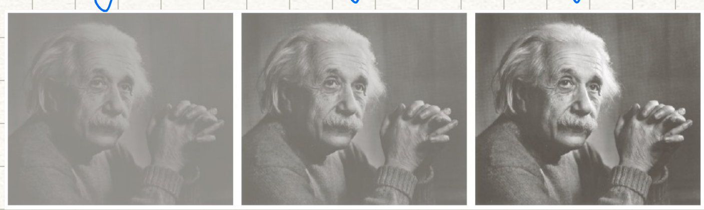
- Distribution of grey levels in an image
- Rep. by Histogram
- Types:

Flat (Strong Visual Rep.)

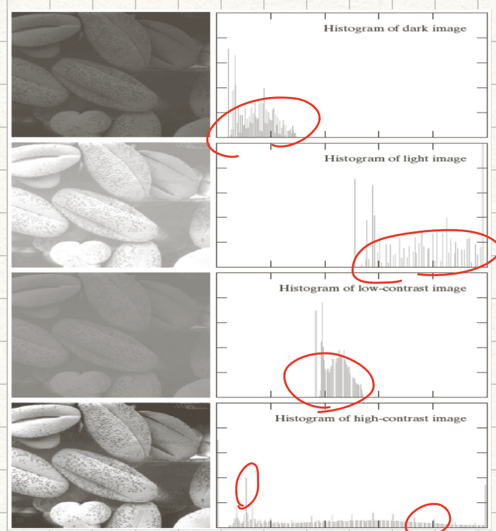
Low $\qquad$ High $\qquad$ Best ?

not much distinguish. $\qquad$ Lots Dark $\qquad$ Lots Brightness $\qquad$ Ideal

Eye Contrast.



The best.



§2.2.2 Histogram Transformation

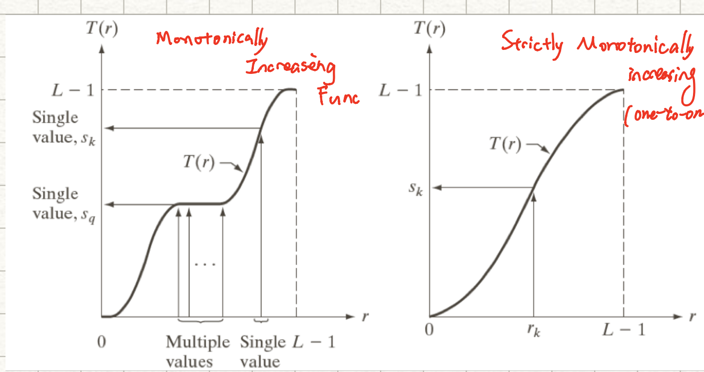
- Conditions:

a) Transformation should be monotonically increasing or ~ decreasing

↳ Ensure no artifacts based on reversals of intensity

b) Range of output shall be the same as range of input.

Monotonically Increasing Func $\qquad$ Strictly Monotonically increasing (one-to-one)



• Impact of Transformation

↳ consider impact of point operation on a histogram PDF.

↳ Look at a Linear transformation

↳ What happens to the PDF params after transformation?

① Linear Trans.

$s = ar + b = T(r)$

Expectation ↳ mean of s

$E[s] = \mu_s = E[ar + b] = E[ar] + E[b] = aE[r] + b$

$= a\mu_r + b$

Variance

↳ $Var[s] = \sigma_s^2 = E[(s - \mu_s)^2]$

$= E[(ar + b - a\mu_r - b)^2]$

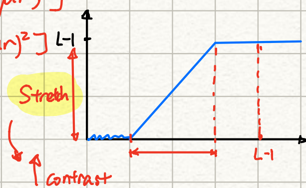
$= E[a^2 (r - \mu_r)^2]$

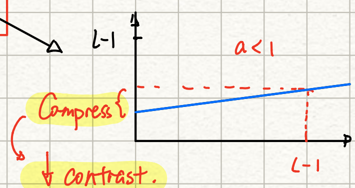
$= a^2 E[(r - \mu_r)^2]$

$= a^2 \sigma_r^2$

⇓

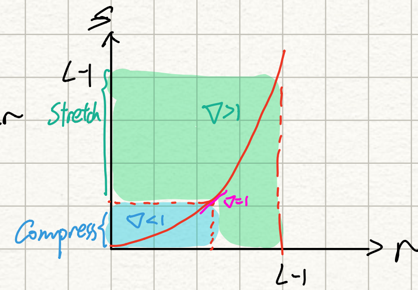
$$\begin{cases} a > 1, & \uparrow \text{contrast} \\ a < 1, & \downarrow \text{contrast} \end{cases}$$





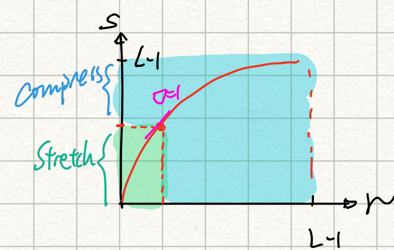
② Quadratic Trans.

$s = ar^2$, $\nabla = \frac{ds}{dr} = 2ar$



③ Square Root Trans

$s = a\sqrt{r}$, slope $= \frac{ds}{dr} = \frac{a}{2\sqrt{r}}$

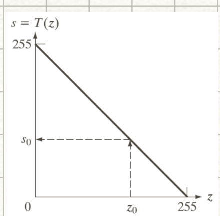


slope $= 1 = \frac{a}{2\sqrt{r}}$

⇓

$r = \frac{a^2}{4}$

Ex : Negative Intensity Linear Trans. Funct.



Result :

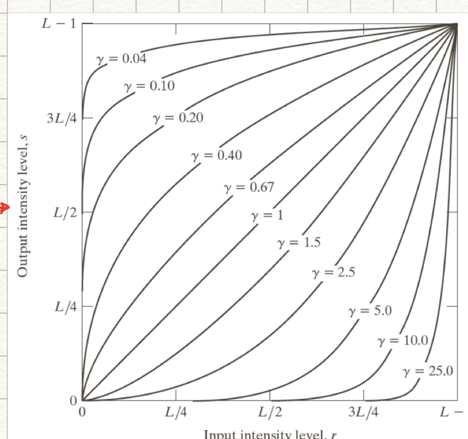
• Gamma Corrections

• Various devices used for image acquisition & display respond based on power law.

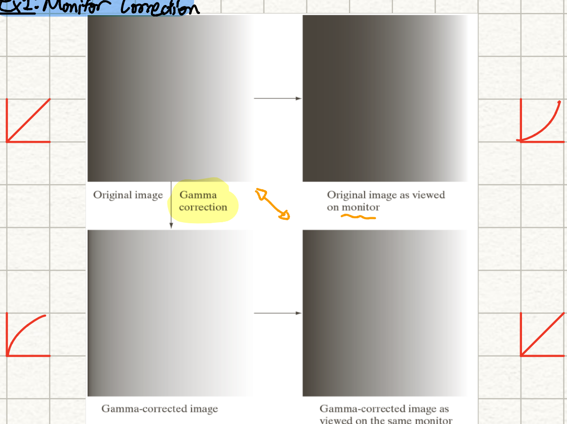
• Process used to correct power law response phenomena is called gamma correction.

$$\boxed{s = r^{\gamma}}$$

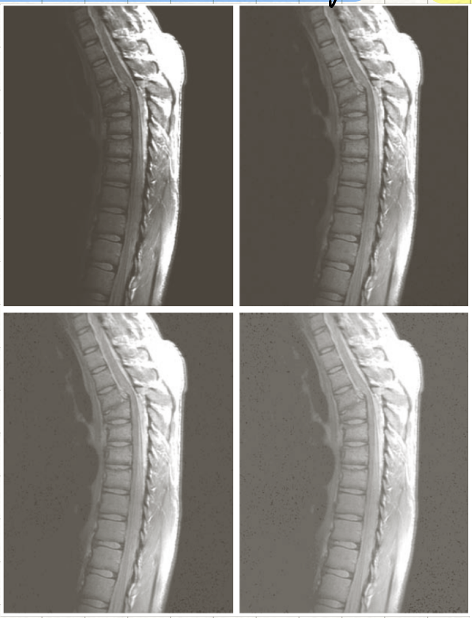
GAMMA CURVE



Ex1: Monitor Correction



Ex 2| Stretch range & Compress Hi h Ra ge ⇒ To Provide More details Visually.



a b
c d

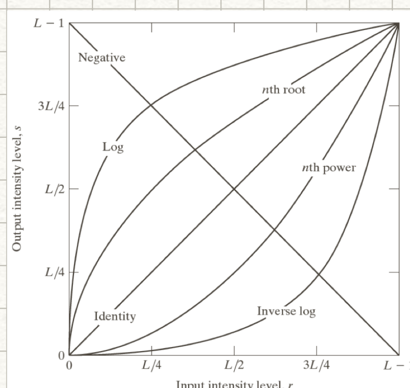
FIGURE 3.8 (a) Magnetic resonance image (MRI) of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4$, and 0.3, respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

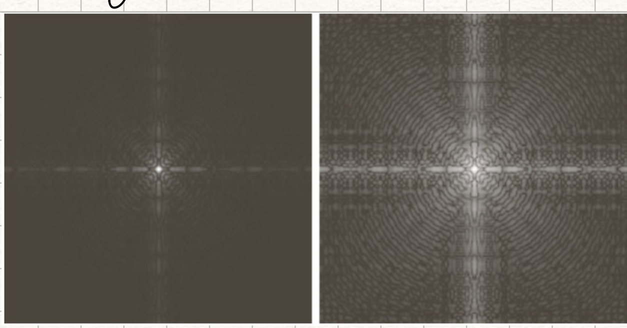
- Store img orig.
- correct @ viewing on fly

a b
c d

FIGURE 3.9 (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0$, and 5.0, respectively. (Original image for this example courtesy of NASA.)

- Other Point Transformation) →
  - Exponential : $s = e^{(\beta r)} - 1$
  - Log (base e) : $s = \alpha \ln(r+1)$





a b

FIGURE 3.5 (a) Fourier spectrum. (b) Result of applying the log transformation in Eq. (3.2-2) with $c = 1$.

§2.2.3 Histogram Equalization

- High-contrast images are often desirable from a visual perspective
- High-contrast images have histograms where components cover a wide dynamic range
- Distribution of pixels are close to a uniform distribution.

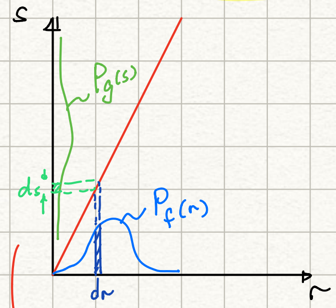
- Intuitively, Low-Contrast Img can be enhanced by transforming its pixel distribution into a uniform distribution to achive improved contrast.

Motivation:

- Let $P_f(r)$ & $P_g(s)$ denote the PDF of r & s
- For $s = T(r)$,

$$\boxed{P_g(s)\,ds = P_f(r)\,dr}$$

Total Area Stay Const. B/A Equalization.



(Area = Area)

Pixels will not disapper

$$P_g(s) = P_f(r)\,\frac{dr}{ds}\bigg|_{r=T^{-1}(s)}$$

Ex: $s = ar+b$

$r = \frac{s-a}{b}$

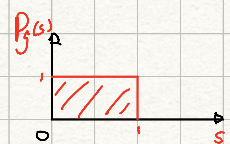
$\frac{ds}{dr} = a.$

$P_g(s) = \frac{1}{a} P_f\left(\frac{s-b}{a}\right)$

Derivation

- We want to transform the input img such that the pixel distribution is uniform.

$P_g(s) = 1 \quad , 0 \le s \le 1$



$\therefore P_g(s) = 1 = P_f(r)\,\frac{dr}{ds}$

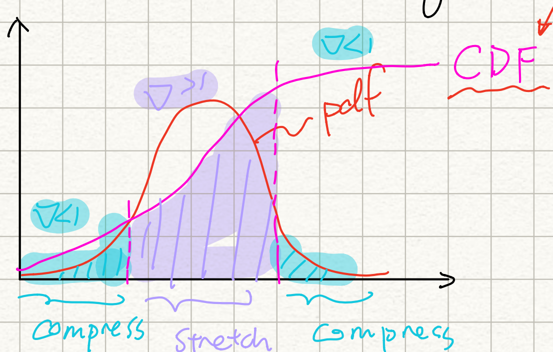
$\therefore ds = P_f(r)\,dr$

$\therefore T(r) = s = \underbrace{\int_0^r P_f(x)\,dx}$

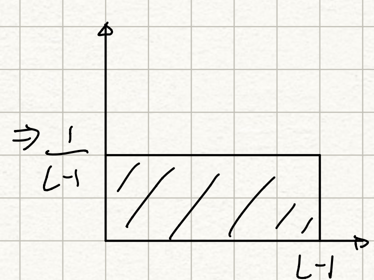
CDF of that histogram.

Qualitively, What's happening?

Apply CDF → stretch / compress needed to force histogram to be flat based on slope.



Goal



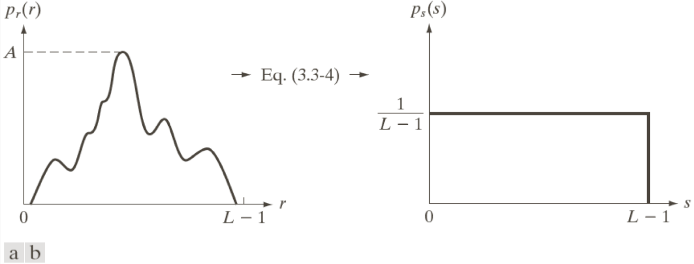


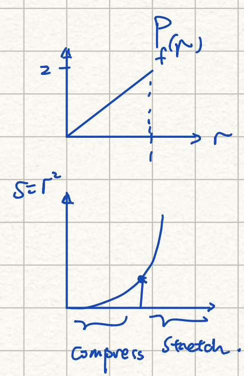
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram EQ - Cont. Ex

$p_r(r) = 2r, \ 0 \le r \le 1$

$s = T(r) = \int_0^r 2x \, dx$

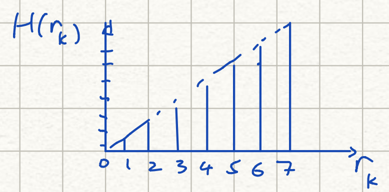
$= x^2 \Big|_0^r = r^2$



Discrete Eqn. Ex.

$$S_k = T(r_k) = (L-1) \sum_{j=0}^{k} P_r(r_j)$$

Ex. $H(k) = 2k, \ 0 \le k \le 7 \ (L=8)$

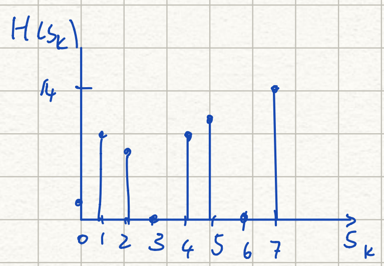


(Total # pix = 56)

r_k	0	1	2	3	4	5	6	7
$P_r(r_k)$	0	2/56	4/56	6/56	8/56	10/56	12/56	14/56
cdf	0	2/56	6/56	12/56	20/56	30/56	42/56	1
$(L-1)$ cdf	0	14/56	42/56	84/56	140/56	210/56	294/56	7
(S_k) round	0	0	1	1	2	4	5	7
output	0	4	8	∅	10	12	∅	14
	+2	+6						
	2	10						

int. only pix.

TEST on test.



→ Not flat, because its discrete.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

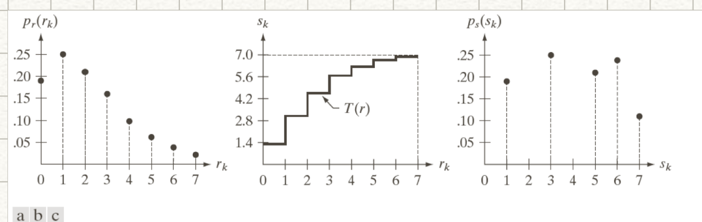


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Ex Result

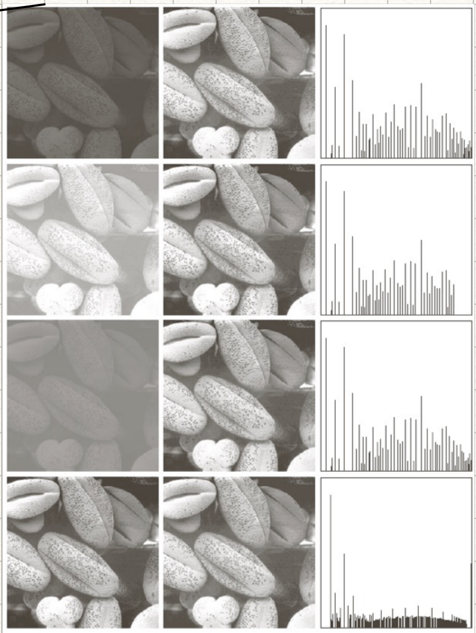


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

Local Histogram Equalization

- Global approach good for overall contrast enhancement.
- However, there may be cases where its necessary to enhance details over small areas in image.
- Soln : perform histogram equalization over a small neighbourhood.

§ 3 – Spatial Operation

§ 3.1 Linear Sys.

§ 3.1.1 Linear Sys & Convolution

$f(m,n) \longrightarrow \boxed{T[f(m,n)]} \longrightarrow g(m,n)$.

m, n | discrete variable

output

'T': Linear Transformation

: can be a point operation, but, more robustly, works on a neighborhood of pixs.

$g(m,n) = T[f(m-k, n-\ell)]$

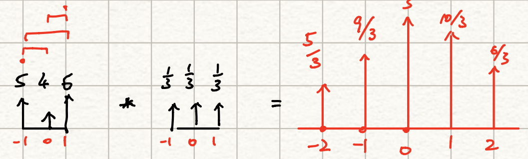
$\{k, \ell\}$ defines local Neighbourhood.

↳ Discrete Convolution:

$$g[n] = f[n] * h[n] = \sum_{k=-\infty}^{\infty} f[k] \cdot h[n-k]$$

output   input   impulse Response.

Ex



↳ Discrete Convolution in 2D

$$g[m,n] = f[m,n] * \underbrace{h[m,n]} = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} f[i,j] \cdot h[m-i, n-j]$$

{ 1D: "impulse response"
2D: "point spread function"
(spatial filter).

§ 3.1.2 Properties of Sys

① **Linearity**: sys is linear when
$a_1 f_1(m,n) + a_2 f_2(m,n) \to a_1 g_1(m,n) + a_2 g_2(m,n)$

- **Additivity** : $f_1(m,n) + f_2(m,n) \to g_1(m,n) + g_2(m,n)$
- **Homogenity**: $af(m,n) \to ag(m,n)$ ← mathematically, linear
  But reality for 8 bit image, the max pix val is capped.

② **Shift Invariance**: A shift input causes shift in output

If $T[f(m,n)] = g(m,n)$
$\Rightarrow T[f(m-m_0, n-n_0)] = g(m-m_0, n-n_0)$

③ **Memory(less)**:
Sys is memorless if its output is dependent on the input @ same location
(ex: Histogram Equalization is memoryless).

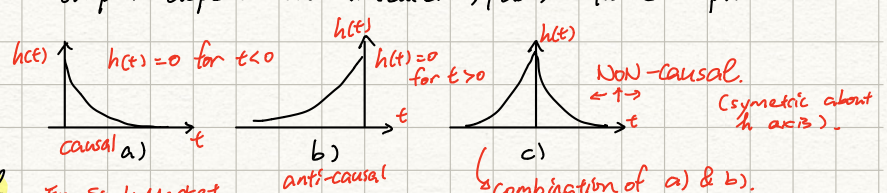
④ **Invertibility**:
Ability to determine input uniquely from the output.

$$f \to \boxed{h} \to g \to \boxed{h_0} \to f.$$

$f * \underline{\delta} = f$.
$\therefore\ h * h_0 = \delta$.
↑ we can fully recover the image if we know exact inverse of "h" as h_0

By system itself.

⑤ **Causality**:
- a) **Causal** sys: output depends only on current & past inputs (can't anticipate)
- b) **Anti-Causal** sys: output depends only on current & future inputs
- c) **Non-Causal** sys: output depends on all (current, past, & future) inputs
- Impulse Response Plot:



⑥ **Stability**
- Bounded Inputs → Bounded Outputs   Ex: Stock Market.
- To achieve stability,
  sum/integral of sys impulse response must be finite.
  Ex: $\sum |h(m,n)| < \infty$

BIG ①

§ 3.2 Spatial Smoothing

§ 3.2.1 Multi-Image Averaging

Noise Reduction

so if take ∞ images & avg them ⇒ Noise free image

- Assume noise is additive & gaussian distributed with zero mean.

  $n \sim N(0, \sigma)$ (σ: deviation)

  ← Gaussian Distribution.

- Suppose we take the avg of q # of Noise samples $(n_1, n_2, \dots, n_q)$ @ a point in the image.

  $$\boxed{m = \frac{1}{q}\sum_{k=i}^{q} n_k}$$

  ↓

- Apply concept to each pixel in the whole img.

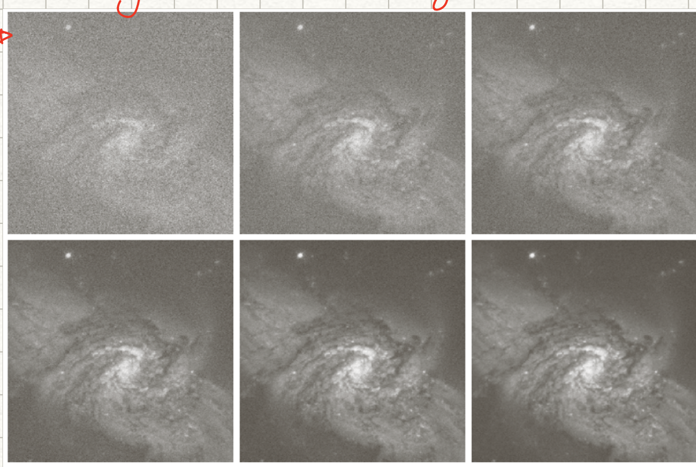
  ↓

- Given an ∞ # of noise samples, the avg approaches the mean of the distribution (= 0)

$$g(x,y) = \frac{1}{q}\sum_{k=1}^{q} f(x,y) + \frac{1}{q}\sum_{k=1}^{q} n_k(x,y)$$

$$\boxed{\text{As } q \to \infty, \Rightarrow \frac{1}{q}\sum_{k=1}^{q} n_k(x,y) \to 0,\ g(x,y) \to f(x,y)}$$

Take infinity in q ⇒ Noisy free, original scene.



a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

→ Clarity ↑ Noise ↓

§ 3.2.2 Spatial Filter - Linear

LINEAR

- Image avging takes advantage of info. redundancy from the individual image ⇒ reduce noise
- Not always possible to acquire so many imgs.
- Imgs may not be perfectly due to spatial misalignment
- Alternative: Take advantage of info. redundancy from diff pixels within the same img ⇒ reduce noise

Convolution

$$g[m,n] = f[m,n] * \underbrace{h[m,n]}_{\text{spatial filter}} = \underbrace{\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}}_{\text{infinity}} f[i,j] \cdot h[m-i, n-j]$$

Simplified: (Kernel)

- $W(x,y)$ as a 2D convolution mask

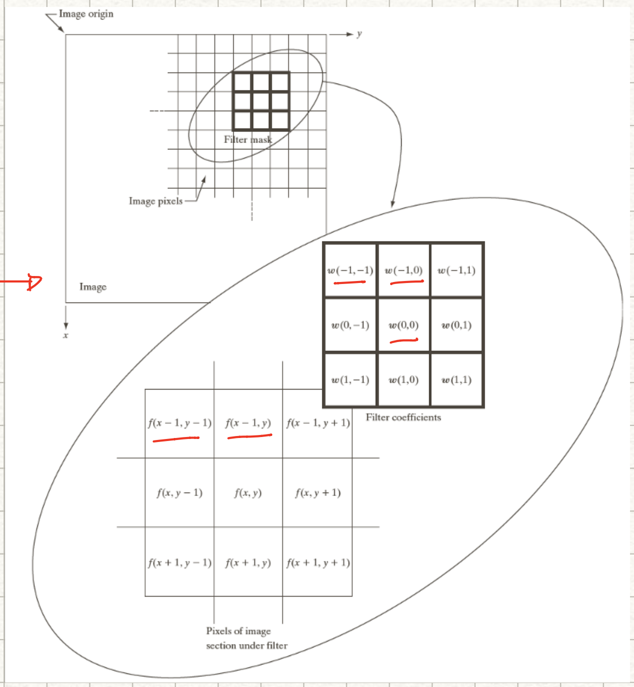
Ex)

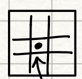
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

3×3 mask

- Spatial Filtering of an img f with 2D convolution using symmetrical mask 'w' of size $(m \times n)$ can be expressed as:

$$\underbrace{g(x,y)}_{\text{output.}} = \sum_{s=-(m-1)/2}^{(m-1)/2} \sum_{t=-(n-1)/2}^{(n-1)/2} \underbrace{w(s,t)}_{\text{convolution mask.}} \cdot \underbrace{f(x+s,\,y+t)}_{\text{input}}$$





kernel for convolution

Back to central pixel of the kernel

Linear

Local average

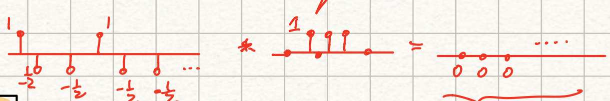
- Removes noises But also Blurs

destroying edges.

Properties:

- Linear (✓) ← Additive & Homogenity
- Shift Invariant (✓)
- Memory (✓) ← need mask
- Causal (×) ← Non-causal
- Stable (✓) ← $\sum h(m,n) = \text{const.}$
- Invertible (×)

↳ A bit not clear, clear in freq domain.



if its all zeros, ⇒ cannot recover.

Weighted Avg Filter

- simple avg of neighboring pixs ⇒ over-smoothing

⇓

- sol^n: instead of weighting equally, assign higher weights to pixs that are closer to the input pixel.

$\frac{1}{16} \times$

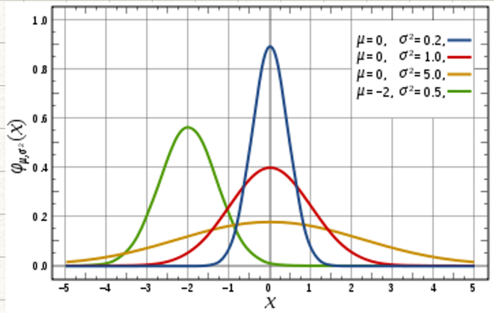
1	2	1
2	4	2
1	2	1

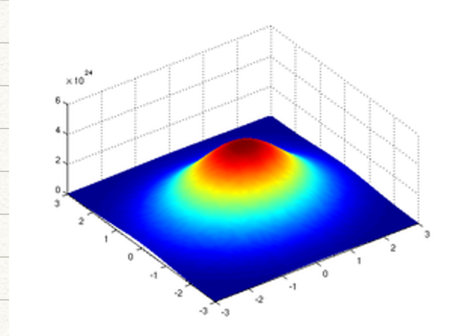
Ex)

Noisy Average Weighted Average

Gaussian Filter

(Typical Weighted Avg).





1D

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\left[-\frac{1}{2}\left(\frac{x}{\sigma_x}\right)^2\right]}}$$

2D

$$\boxed{f_{(x,y)} = \frac{1}{2\pi\sigma_x\sigma_y}\, e^{\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2}+\frac{y^2}{\sigma_y^2}\right)\right]}}$$

$\begin{cases} \sigma \uparrow \Rightarrow \text{wider, shorter} \\ \sigma \downarrow \Rightarrow \text{narrower, taller} \end{cases}$

Characteristics of (Local Avg.)

a) $h(m,n) \geq 0 \quad \forall (m,n)$
b) $\sum h(m,n) = 1$ (DC Gain is normally 1) ← (Don't change grey scale)
c) Non-causal
d) Typically odd-dimensions (practical)
e) Typically even symmetry

§3.2.3 Spatial Filter – Non-linear → $a_1 f_1(m,n) + a_2 f_2(m,n) \neq a g_1(m,n) + a_2 g_2(m,n)$

NONLINEAR

Order-Statistic Filter

Best Known Ex:

Median Filter
- provides good noise reduction for certain types of noise (ex: impulse noise)
- Considerably less blurring than weighted avging filter.
- Forces a pix to be like its neighbors.

Steps:
- ① Order pixs within an area
- ② Replace value of center pixel with median value ($\frac{1}{2}$ of all pixs have intensities $\geq$ median value).

(salt & pepper noise)/(impulse noise).

Ex

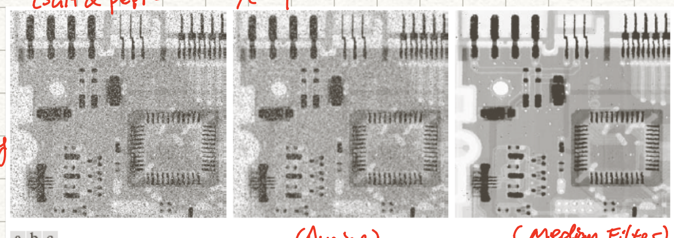
255	10	9
10	253	10
8	10	10

⇒

255	10	9
10	10	10
8	10	10

median → prevents avging

8	9	10	10	10	10	10	255	255



(Avging) (Median Filter)

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

S-statistic Filter
- Another method to avoid impact of outliers on smoothing is the S-statistic Filter
- only use those values within a region that are within a certain range relative to the mean.
- Done by calc the μ & σ of the local region & determining a local avg based only on pixs within a certain # of σ

Recap:

Spatial Smoothing Filter

- Weight of each pix in the neighborhood covered by the filter depends on the proximity of the pix to the center pix being filtered.

⇓

Problem: oversmoothing of edges & other fine detail (with ideal / Gaussian Linear Filters)

mass dependent

⇓

Alt. Soln: Use ⇓

'Range Smoothing Filter'

- dependent on img content.
- Weight of each pix in the neighborhood covered by the filter depends on the similarity of the pix's intensity value to that of the center pix being filtered.
  - The closer the pix's intensity val is to that of center pixel, the higher the weight.
- Problem: simply remaps intensity values.
  - No notion of space
  - Poor Noise Reduction Performance, (Not really useful by itself).

⇓

Bilateral Filtering

reduces small img details while preserving large edge details. ⇒ if Multiple times?

⇓

- More & More smaller details get smoothed out
- Large edges remain well-preserved

⇓

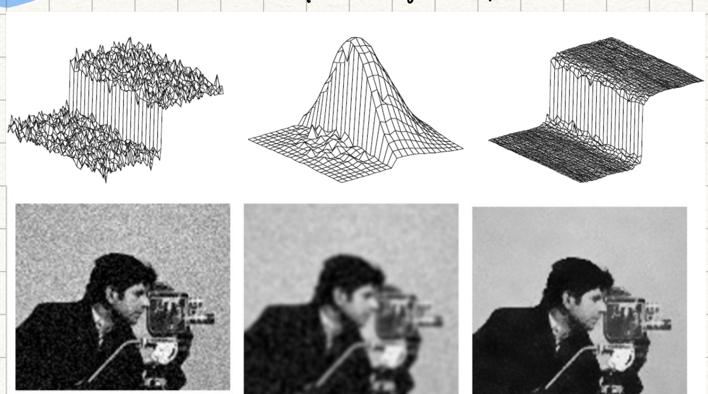
Cartoon Effects

⇓

- Combine "Spatial Smoothing" + "Range Smoothing"
  - Good Noise Reduction
  - Preserve good edge & details.
- Goal: For regions with
  - Low Dynamic Range → Smoothing (Noise Reduc).
  - High Dynamic Range → Preserve Details.
- Result: Non-linear;

$$h_{bf} = \underbrace{h(|f(x,y) - f(x_c, y_c)|)}_{\text{Range smoothing Filter}} \cdot \underbrace{h(|x - x_c|, |y - y_c|)}_{\text{spatial smoothing}}$$

- Ex) Gaussian Model for Range & Spatial



noisy | Gaussian spatial filter | Bilateral filter

§3.2.4 Implementation of 2D separable Filters

- Faster way to implement a 2D separable filter is to use 2× 1D arrays.

Ex)

1	2	3	4
1	1	1	1
1	2	2	3
1	1	1	1

"Image" $*$ $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$ (Smoothing Mask) $\Rightarrow \frac{1}{3}\begin{bmatrix}1&1&1\end{bmatrix} * \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$

- **Computational Complexity**
  - Assume $M \times N$ img & $m \times n$ filter mask
  - 2D mask: # of multiplications: $M \times N \times m \times n$
  - 2× 1D mask: # of multiplications: $M \times N \times (m+n)$ — Huge Savings.
  - ✱ But 1D implementation requires storing an additional img in memory.

§3-3 : Spatial Operations — Edges (1st order deriv.)

- **Why edges are important?**
  - ⇒ Psychovisually, most important characteristic that HVS identifies
  - ⇒ HVS loves Edges. (Not Absolute Graylevels)
  - ⇒ Dificult to distinguish 2 similar grey lvls, unless side-by-side with a boundary separating.
  - ⇒ Phase v.s. Magnitude:
    - reconstruct an img with only
      - Phase ⇒ Structure Is Retained ⇒ to interpret the img
      - Magnitude ⇒ An Unintelligible img is produced (Lab#3) ⇒ Garbage!

- **Basic Principle**
  - Avging (Blurring) is analogous to interpolation
  - ∴ ⇒ Logically, Edge Detection accomplished by Differentiation
  - ⇓ Both derivative in x, y dire.

PRINCIPLE

- **First Derivative**
  - Magnitude of Gradient.
  - Continuous✱ ⇒ Discrete needs $(\Delta x, \Delta y)$
  - $\nabla f = \begin{bmatrix}\frac{\partial f}{\partial x}\\ \frac{\partial f}{\partial y}\end{bmatrix} \Rightarrow \boxed{|\nabla f| = \left[\frac{\partial f}{\partial x}^2 + \frac{\partial f}{\partial y}^2\right]^{\frac{1}{2}}}$

Methods : (Discrete)

- Simple: 1^{st} difference, Roberts, Prewitt, Sobel
- sophisticated: Canny

a) Forward First Difference

First Difference

shift by 1 & subtract the original

- $\Delta x = f(m+1, n) - f(m, n)$ ⇒ x-dir ?

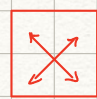
  OR: $[1 \ -1]$ (as an impulse response)

- $\Delta y = f(m, n+1) - f(m, n)$ ⇒ y-dir ?

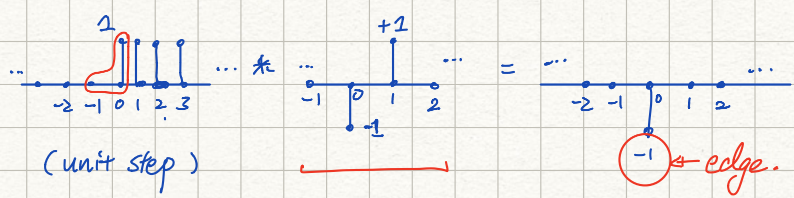
  OR: $[1 \ -1]'$ (as an impulse response)

Roberts Cross Operator

- $|\Delta f(m,n)| = |f(m,n) - f(m+1, n-1)| + |f(m, n-1) - f(m+1, n)|$



Ex/



(unit step)

-1 ← edge.

Σ weights = Φ
DC gain is zero

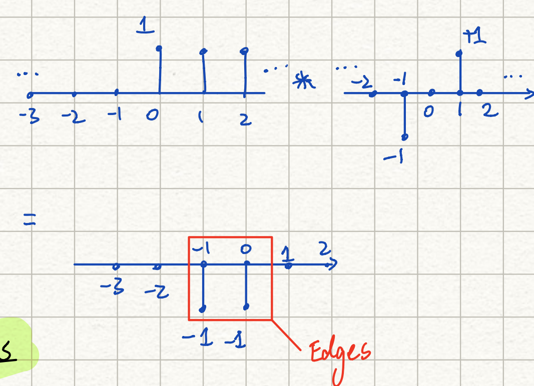
Comments:

- 1^{st} diff. operators are not effective
- very sensitive to noise because of small spatial extent

⇓

Sol^n: Use a larger mask.

Ex/ 1-D sobel Operator.



Gradient Mask

intensity values

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Roberts - Cross

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Horizontal   Vertical Edges

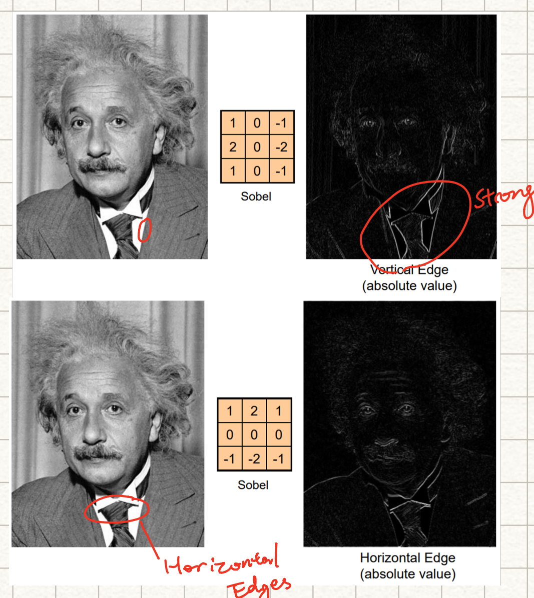
Sobel Operators

a
b c
d e

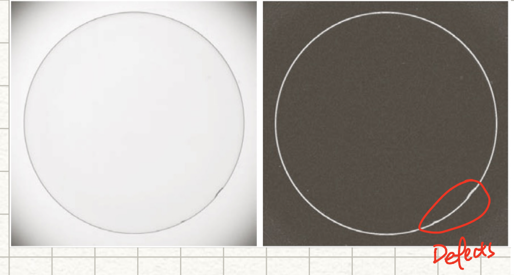
FIGURE 3.41 A 3 × 3 region of an image (the zs are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

0 DC gain

Results



Sobel Operator



a b
FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

Edge Detector Characteristics

a) Zero weights required ⇒ Zero gain
b) Zero DC gain
c) Non-causal
d) Typically odd Dimension
e) ...... } Symmetry

Canny Edge Detector

- specified 3 issues that an edge detector must address:

3 CRITERIA

① Error Rate : Edge detector must respond ONLY to Edges & detect them ALL
② Localization : Distance btwn detected edges & true edges must be minimized
③ Response : Do not identify multiple edge pixels where only a single edge exists

- **Assumption:**
  - Step edge with point Gaussian noise

⇓

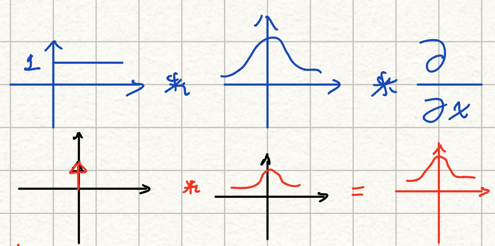
Goal: Tried to derive single filter to optimize edge detection based on 3 criteria for given edge model.

⇓

Outcome:
- too complex to be solved analytically!
- A reasonable sol^n is :

A derivative of a Gaussian:

Step * Gaussian * edge detector =



- might be noisy
- But provide a good localization of the edge

- **Steps:**

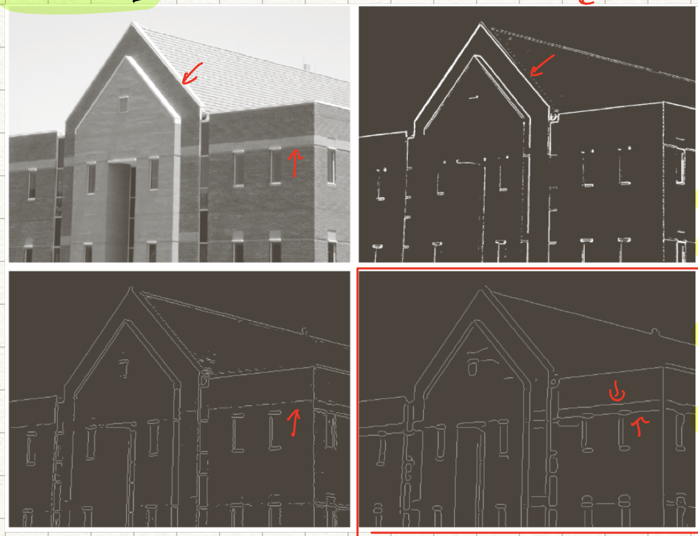
① Convolve image with derivative of a Gaussian

② Non-maximum Suppression :
- Thins edge boundary to 1-pixel thick
- Threshold based on direction of gradient
- Magnitude of gradient @ edge pixel should be greater than magnitude of gradients on each side of edge

③ Hysteresis :
- Two threshold T_h & T_l
- Any gradient $> T_h$ → An edge
- Iteratively, any pixel connect to T_h with gradient $> T_l$ → An edge

Results

← Threshold Gradient



a b
c d

FIGURE 10.25 (a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) Thresholded gradient of smoothed image. (c) Image obtained using the Marr-Hildreth algorithm. (d) Image obtained using the Canny algorithm. Note the significant improvement of the Canny image compared to the other two.

Marr-Hildreth

Canny (significant improvements)

(13:04:33) §3-3 Edge Enhancement (2nd order deriv)

- **Goal:** Highlight / Enhance details in images.
- **Application:**
  - Photo Enhancement
  - Medical Img Visualization
  - Industrial Defect Detection
  - ...
- **Two Methods:**
  - 1- Laplacian
  - Unsharp masking
- **1st vs. 2nd Deriv.**
  - FWD dif.: $f_1'(n) = f(n+1) - f(n)$
  - BWD dif.: $f_2'(n) = f(n) - f(n-1)$
  - 2nd dif.: $\boxed{f''(n) = f_1'(n) - f_2'(n) = f(n+1) - 2f(n) + f(n-1)}$
    - Impulse Response: output of impulse to the sys.

      $f \rightarrow \boxed{h} \rightarrow g = f''$

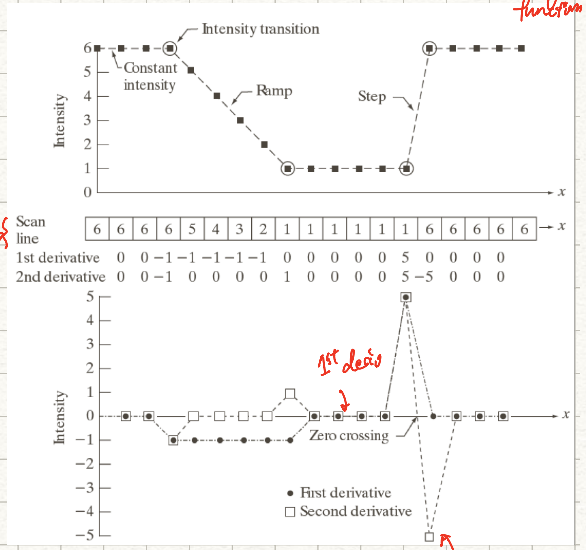
      $\delta \rightarrow \boxed{h} \rightarrow h$

      (impulse) impulse response

      $h(n) = \delta(n+1) - 2\delta(n) + \delta(n-1)$

PRINCIPLE

Illustration of 1st & 2nd derivatives of a 1-D Digital function



Intensity profile of an img

1st deriv

2nd deriv.

- **Comments:**
  - 2nd-order deriv. have stronger response to fine details. (ex: Thin Lines & Points)
  - 2nd-order deriv. have zero response to ramps.
  - 1st-order deriv. have stronger response to step changes
  - 2nd-order deriv. produce double response @ step changes

Laplacian

- 2D (2nd order) Continuous

$$\boxed{\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}}$$

- Discrete ↓

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

- Laplacian Filter

includes diagonal terms

(a)

0	1	0
1	−4	1
0	1	0

(b)

1	1	1
1	−8	1
1	1	1

(c)

0	−1	0
−1	4	−1
0	−1	0

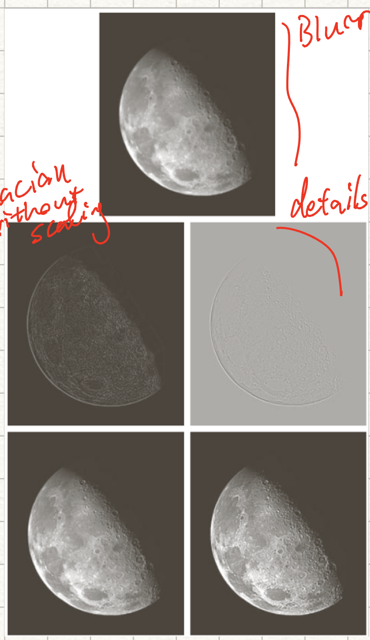
(d)

−1	−1	−1
−1	8	−1
−1	−1	−1

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.

2 freq. in practice

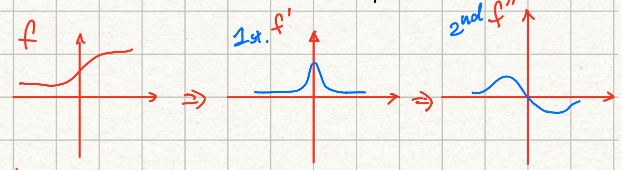


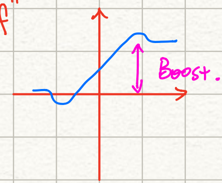
a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

Ex) How can 2nd deriv. (Laplacian) be used?

APPLICATION





Edge Enhancement

↓ blur ↑ contrast

$$g(n) = f(n) - f''(n)$$
$$= f(n) - f(n-1) + 2f(n) - f(n+1)$$
$$= \underline{-f(n-1) + 3f(n) - f(n+1)}$$

shift, → x3 →, shift back

$$\boxed{h(n) = -\delta(n-1) + 3\delta(n) - \delta(n+1)}$$

As Mask

1D: $[-1 \quad 3 \quad -1]$ $\Sigma = 1$ gain of 1

2D: $g(m,n) = f(m,n) - f''(m,n)$

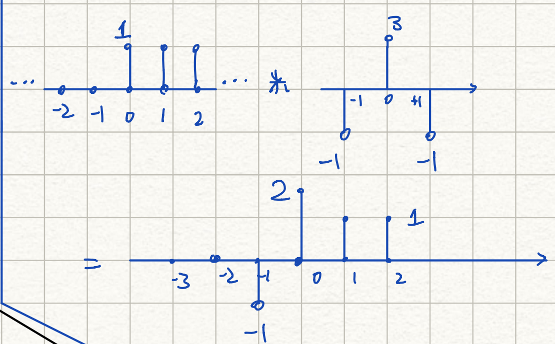
$$h(m,n) = 1 - \nabla^2$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

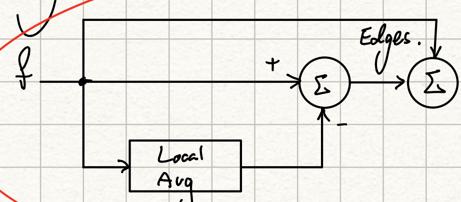
DC gain of 1

Step Response (1D)



Unsharp Masking

[Sys. Diagrams]



$$g = f + f - f_{smoothed}.$$
$$= 2f - f_{smoothed}.$$

[Masks] Consider 3x3 smoothing mask:

2-D: $h(m,n) = 2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \underbrace{\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{Normalized}}$

$$= \frac{1}{9}\begin{bmatrix} -1 & -1 & -1 \\ -1 & 17 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

1-D: $h(n) = [0\ 2\ 0] - \frac{1}{3}[1\ 1\ 1]$

$$= [-\tfrac{1}{3}\quad \tfrac{5}{3}\quad -\tfrac{1}{3}]$$

[Generalized]

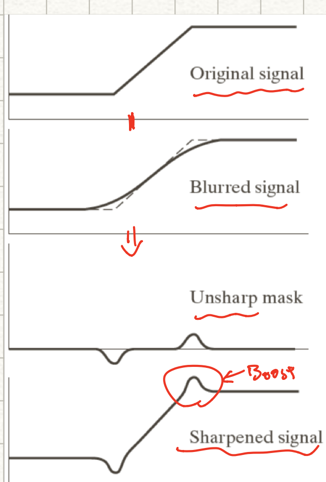
- used in publishing
- Subtract Blurred version of img from the img itself ⇒ produce sharp img:

$\boxed{g_{edge}(x,y) = f(x,y) - \mu(x,y)}$ Edge Map.

$g(x,y) = f(x,y) + k\, g_{edge}(x,y)$

↑ output ↑ input

$\ast \begin{cases} k=1 \rightarrow \text{unsharp masking} \\ k>1 \rightarrow \text{High Boost Filtering} \end{cases}$



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Unsharp Mask

Unsharp Masking

HighBoost

a
b
c
d
e

FIGURE 3.40 (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

Edge Enhancement Filter Characteristics

a) Even Symmetry
b) Negative Weights (Surrounding)
c) Typically DC gain of 1

Retina in HVS.

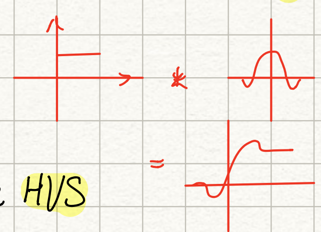
Filters mimic the "Lateral Inhibition"

⇓

These aspects characterize "Lateral Inhibition"

└ Neighbouring inputs inhibit response of a pixel.

└ Well known as a model of neural interaction in the HVS



§3.5.3 Exact Blur Compensation

→ Determine Blur Model (Deconvolution)

- Assume sharpening filter is known (ex: 1-Laplacian)
  - ⇒ What Blur Model is exactly compensated by the sharpening filter?

$$s(n) \longrightarrow \boxed{\begin{matrix}\text{Blur}\\ h(n)\end{matrix}} \longrightarrow \boxed{\begin{matrix}\text{Enhancement}\\ h_1(n)\end{matrix}} \xrightarrow{\text{⇒ sharpening}} g(n) = s(n)$$

- Assume $h_1(n) = -\delta(n-1) + 3\delta(n) - \delta(n+1)$ $\quad$ $(1-\nabla^2)$ $\underset{\text{minus}}{1}$-Laplacian (Enhancement Operator)
- For $g(n) = s(n)$, $h(n) * h_1(n) = \delta(n)$
- $h(n) * h_1(n) = h(n) * [-\delta(n-1) + 3\delta(n) - \delta(n+1)]$

$$\boxed{\delta(n) = -h(n-1) + 3h(n) - h(n+1)}$$

LDE : Linear Difference Eqn

⇒ How to solve $h(n)$?

↳ Assume & plug-in :)

- Try $h(n) = Az^n$, where $z = re^{j\Omega}$ is a complex #. (Assume Complex Sol^n)

$\therefore -Az^{n-1} + 3Az^n - Az^{n+1} = \emptyset \quad , n \neq \emptyset$

$\Rightarrow Az^n[-z^{-1} + 3 - z^1] = \emptyset$

$\Rightarrow z^2 - 3z + 1 = \emptyset \Rightarrow z = \frac{3 \pm \sqrt{5}}{2}$ ≈ 0.38
≈ -2.6

$\therefore$ For $z = \frac{3-\sqrt{5}}{2}$

⇒ $h(n) = A\left(\frac{3-\sqrt{5}}{2}\right)^n$

$\because$ for $n>0$, $\left(\frac{3-\sqrt{5}}{2}\right)^n$ will decay with $\uparrow n$

$\therefore h(n) = A\left(\frac{3-\sqrt{5}}{2}\right)^{-n}$ for $n<0$

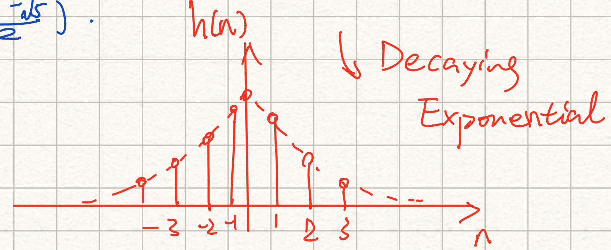
At $n=0$, $\delta(0) = -h(-1) + 3h(0) - h(1)$

$1 = -A\left(\frac{3-\sqrt{5}}{2}\right) + 3A - A\left(\frac{3-\sqrt{5}}{2}\right)$.

$1 = A(3 - 3 + \sqrt{5})$

$A = 1/\sqrt{5}$

$\therefore h(n) = \frac{1}{\sqrt{5}}\left(\frac{3-\sqrt{5}}{2}\right)^{|n|}, \forall n$



Infinity extend filter
- Finite extend filter
= Impulse Response

→ General Model (Z-Domain)

$$s(n) \longrightarrow \boxed{\begin{matrix}\text{Blur}\\ h(n)\end{matrix}} \longrightarrow \boxed{\begin{matrix}\text{Sharpening}\\ h_1(n)\end{matrix}} \longrightarrow g(n) = s(n)$$

$$h(n) * h_1(n) = \delta(n)$$

z-Domain

$$H(z) \cdot H_1(z) = 1 \Rightarrow H(z) = H^{-1}(z)$$

- Assume $h(n) = k a^{|n|}$ for $|n| < 1$ (Decaying Exponential)

↳ else grow exponential $\Rightarrow$ unstable.

① What should 'k' be set to?

↳ $\sum_{n=-\infty}^{\infty} k a^{|n|} = 1$ for DC gain to be 1

Break, Discontinuity @ origin.

$$K\left[\sum_{n=-\infty}^{0} a^{-n} + \sum_{n=0}^{\infty} a^{n} - 1\right] = 1$$

Recall $\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$, for $|\alpha| < 1$.

⇓ symmetry (repeated 0)

$$K\left[2\sum_{n=0}^{\infty} a^{n} - 1\right] = 1$$

⇓

$$K\left[\frac{2}{1-a} - 1\right] = 1$$

$$\Rightarrow K = \frac{1-a}{1+a}$$

$$\therefore \boxed{h(n) = \left(\frac{1-a}{1+a}\right) a^{|n|}}$$

② What is $h_1[n]$?

a) Find z-trans of $h(n) = H(z)$

b) Determine $H^{-1}(z) = H_1(z)$

c) Take inverse transformation of $H_1(z)$

a)

$$H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$$

$$= \left(\frac{1-a}{1+a}\right)\left[\sum_{n=-\infty}^{0} a^{-n} z^{-n} + \sum_{n=0}^{\infty} a^{n} z^{n} - 1\right]$$

⇓ switch order

$$= \left(\frac{1-a}{1+a}\right)\left[\sum_{n=0}^{\infty} a^{n} z^{n} + \sum_{n=0}^{\infty} a^{n} z^{n} - 1\right]$$

$$= \left(\frac{1-a}{1+a}\right)\left[\frac{1}{1-az} + \frac{1}{1-az^{-1}} - 1\right]$$

$$= \left(\frac{1-a}{1+a}\right)\left[\frac{1-\cancel{az^{-1}} \cancel{+1} - \cancel{az} \cancel{-1} + \cancel{az} + \cancel{az^{-1}} - a^{2}}{(1-az)(1-az^{-1})}\right]$$

$$= \left(\frac{1-a}{\cancel{1+a}}\right)\left[\frac{(1-a)\cancel{(1+a)}}{1+a^{2}-az^{-1}-az}\right]$$

$$= \frac{(1-a)^{2}}{1+a^{2}-az^{-1}-az}$$

b) $\cdot\ H_1(z) = H^{-1}(z) = \frac{1+a^2}{(1-a)^2} - \frac{a}{(1-a)^2} z^{-1} - \frac{a}{(1-a)^2} z$

z^{-1}

$$\boxed{h_1(n) = \frac{1+a^2}{(1-a)^2}\delta(n) - \frac{a}{(1-a)^2}\delta(n-1) - \frac{a}{(1-a)^2}\delta(n+1)}$$

Compensate Infinity Filter with Finite Filter

↓

Corresponding to difference eqn:

$$g(n) = \frac{1+a^2}{(1-a)^2} f(n) - \frac{a}{(1-a)^2} f(n-1) - \frac{a}{(1-a)^2} f(n+1)$$

$\therefore$ Exponential Blur Model $h(n) = \left(\frac{1-a}{1+a}\right) a^{|n|}$, For $|a|<1$

Has an inverse $h_1(n)$ that perfectly reconstructs the original image.

Ex) a) For $[-1\ 3\ -1]$

↳ $a = \frac{3-\sqrt{5}}{2} \approx 0.38 \Rightarrow h(0) = \frac{1-a}{1+a} \approx 0.45$

b) For $[-1/3\ \ 5/3\ \ -\frac{1}{3}]$

$-\frac{1}{3} = \frac{-a}{(1-a)^2} \Rightarrow a \approx 0.2$ or $\cancel{4.8}$

↓

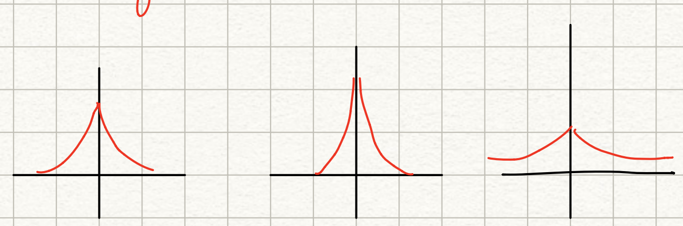
$h(0) = \frac{1-0.2}{1+0.2} \approx 0.67.$

c) $[-2\ 5\ -2]$

$-2 = \frac{-a}{(1-a)^2} \Rightarrow a \approx \frac{1}{2}$ or $\cancel{2}$ $|a|<1$!!!

↳ $h(0) = \frac{1}{3}$

Larger "a" is → modify how it decays.



a) (Blur)

b) (least Blur) large delay

c) Slow decay (Most Blur)

MIDTERM 1