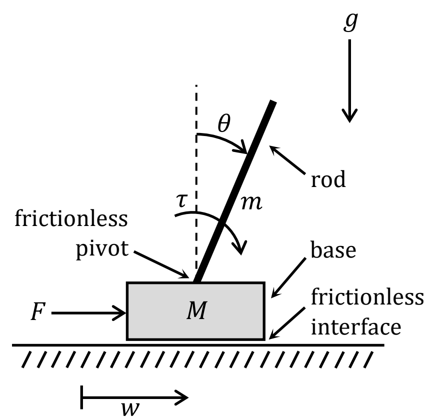
4.10 PROJECT EXERCISE P5

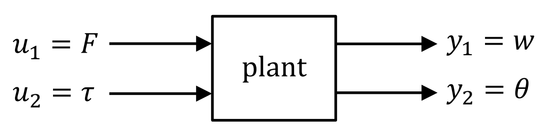
Problem P5: Decentralized MIMO control of the aiming system

So far in the project we've looked at SISO control of the one-rod aiming system and, in the context of performance limitations, SISO control of the two-rod aiming system. We now consider the use of multiple control signals to control the one-rod aiming system.

As shown at the right, we introduce a force input applied to the base, in addition to the torque input applied to the rod. Now that we have two control channels, we can expect to be able to control two different signals, so let's make the obvious choice: we'll try to control the base position in addition to the rod angle.



[Note that this is different from the inverted pendulum setup considered on page 211 since the regular inverted pendulum has only one control signal (i.e., the force applied to the base), and our goal there is to control the base position while keeping the pendulum balanced at $\theta = 0$. Here we are trying to get independent control of the base position and the rod angle. In general, if we have p actuators then we can independently control p things.]



Key variables and parameters are summarized below:

Signal	Unit	Interpretation
$F(t)$	N	Force applied to base (control signal u_1)
$\tau(t)$	$\mathrm{N} \cdot \mathrm{m}$	Torque applied to rod (control signal u_2)
$w(t)$	m	Position of the base (plant output y_1)
$\theta(t)$	rad	Rod angle (plant output y_2)

Parameter	Value	Interpretation
M	1 kg	Mass of the base
m	0.5 kg	Mass of the rod
L	1.0 m	Length of the rod
g	9.8 $\mathrm{m/s^2}$	Acceleration due to gravity

With the additional control channel, the equations of motion in (P1) (see page 99) are modified as follows:

$$\frac{3}{2}\ddot{w} + \frac{1}{4}\left[\ddot{\theta}\cos(\theta) - \left(\dot{\theta}\right)^2 \sin(\theta)\right] = F$$
$$\frac{1}{6}\ddot{\theta} + \frac{1}{4}[\ddot{w}\cos(\theta) - g\sin(\theta)\ ] = \tau.$$

Linearize the above equations about the "up" equilibrium operating point (where $\theta = 0$ and w is arbitrary but fixed) to yield the following transfer function matrix, where the inputs and outputs are ordered as indicated on the previous page:

$$P_3(s) = \begin{bmatrix} \dfrac{0.8889(s+3.834)(s-3.834)}{s^2(s-4.427)(s+4.427)} & \dfrac{-1.333s^2}{s^2(s-4.427)(s+4.427)} \\ \dfrac{-1.333}{(s-4.427)(s+4.427)} & \dfrac{8}{(s-4.427)(s+4.427)} \end{bmatrix}. \qquad \text{...(P7)}$$

As expected, the (2,2) entry matches what we obtained in (P2) on page 99. Also note the unstable pole-zero cancellation in the (1,2) entry: this is not a meaningless mathematical artifact, but it arises because of physical reasons (essentially because the torque has limited ability to control the base position). However, this cancellation won't cause serious problems.

Let's now try to control (P7). There are several reasonable ways forward, but I suggest the following one since it results in two relatively easy SISO design problems. The approach is a modified version of the decentralized control approach of Section 4.2 (page 197).

(a) First, note that the unstable pole-zero cancellation in the (1,2) entry of $P_3(s)$ implies that we should not try to control the base position using just the torque input because the resulting feedback loop will necessarily be unstable. (Of course, it would be awkward to want to control the base via the torque input anyways!) On the other hand, there are no mathematical reasons why we cannot try the following:

- Control the base position using the force input
- Control the rod angle using the torque input

... (P8)

Matching up the inputs and outputs this way is also appealing from a physical perspective since, loosely speaking, the force input directly influences the base position and the torque input directly influences the rod angle. Can you justify the choice (P8) to be reasonable by inspection of the MIMO Bode plot of $P_3(s)$?

(b) Rather than just following Section 4.2 blindly and ignoring the off-diagonal terms within $P_3(s)$, which would leave us with a nasty-looking (1,1) entry to deal with, we can make a different type of approximation. Specifically:

- From the perspective of the force input, imagine that the entire base-rod system is one fixed mass (i.e., pretend that the pivot is welded so the rod cannot rotate).
- From the perspective of the torque input, imagine that the position of the base is fixed, so that the only effect of the torque input is to rotate the rod.

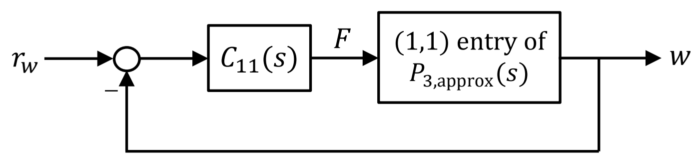
Use these two ideas to come up with an approximation to the plant $P_3(s)$ that has the following form:

$$P_{3,\text{approx}} = \begin{bmatrix} ? & 0 \\ 0 & ? \end{bmatrix}. \qquad \text{...(P9)}$$

Hints:

- To minimize confusion, use the same ordering of inputs and outputs that we used for $P_3(s)$.
- Use "$F = ma$" to model the (1,1) term.
- Use the angular version of "$F = ma$" to model the (2,2) term. The moment of inertia of the rod about its pivot point is $J = mL^2/3$. Don't forget gravity.
- You should find that each of the diagonal entries in (P9) has two poles and no zeros.

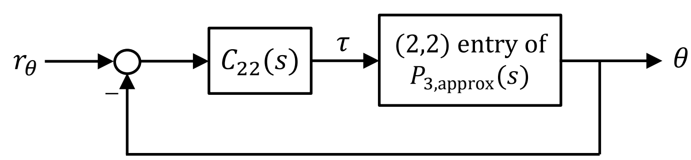
(c) Design a 1-DOF SISO controller, $C_{11}(s)$, for the (1,1) entry of $P_{3,\text{approx}}(s)$ so that the resulting SISO feedback loop, shown at the right, satisfies the following specifications:



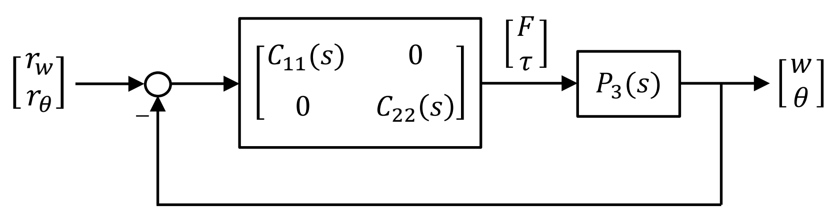
- the closed-loop system is stable,
- perfect steady-state tracking is attained for step reference signals, and
- the phase margin is at least 50°.

Show explicitly that the three specifications are satisfied.

(d) Repeat part (c) for the (2,2) entry of $P_{3,\text{approx}}(s)$. Denote this SISO controller by $C_{22}(s)$.



(e) Now analyze what happens when your two SISO controllers are connected to the actual plant $P_3(s)$ (i.e., (P7) on page 235) as follows:



Note that the MIMO controller is decentralized.

Specifically, answer the following questions:

- Assuming the SISO loops from parts (c) and (d) are stable, do you necessarily expect the MIMO closed-loop system to be stable? Explain your reasoning, and determine whether or not the MIMO system is stable.
- Assuming the MIMO closed-loop system is stable, is there a guarantee of perfect steady-state tracking for step references? Explain your reasoning, and use Matlab to determine the actual steady-state performance.
- Assuming the MIMO closed-loop system is stable, is there a guarantee that the step response transients will be identical to the step response transients of the individual SISO feedback loops from parts (c) and (d)? Explain your reasoning and use Matlab to determine the actual transient performance.

(f) [Optional] Use “single_pend_fancy_sim.m” to visualize how your MIMO system behaves.