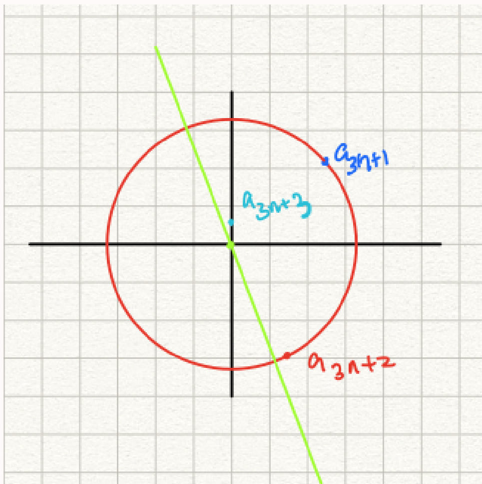
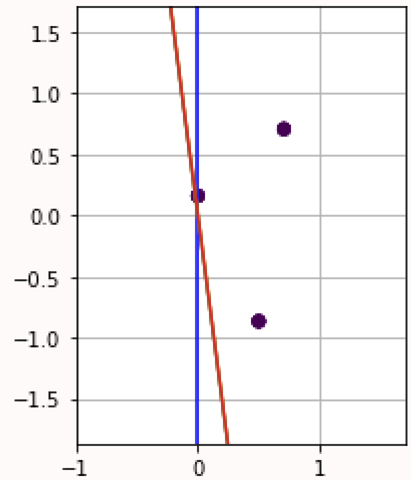
**Proof 2.1: Property I - Strictly Linearly Separable**

Since all labels are +1, and all data points are asymmetric, hence the data $\mathscr{D}$ is strictly linear separable with $b = 0$.

And the algorithm shall only make the first mistake in the ordinary perceptron with $b = 0$, simply updating $w$ and $b$ to $w = (1/\sqrt{2}, 1/\sqrt{2}), b = 0$, and then halt update. Hence, there exists $w = (1/\sqrt{2}, 1/\sqrt{2}), b = 0$ such that $\forall i, \mathbf{a}_i^T w \geq s \geq 0$. As a result, we conclude the dataset is truly strictly linearly separable.

As shown in the graph below, there would exist some hyperplane, where all points on one side of the hyperplane through the origin with $\gamma > 0, b = 0$, that



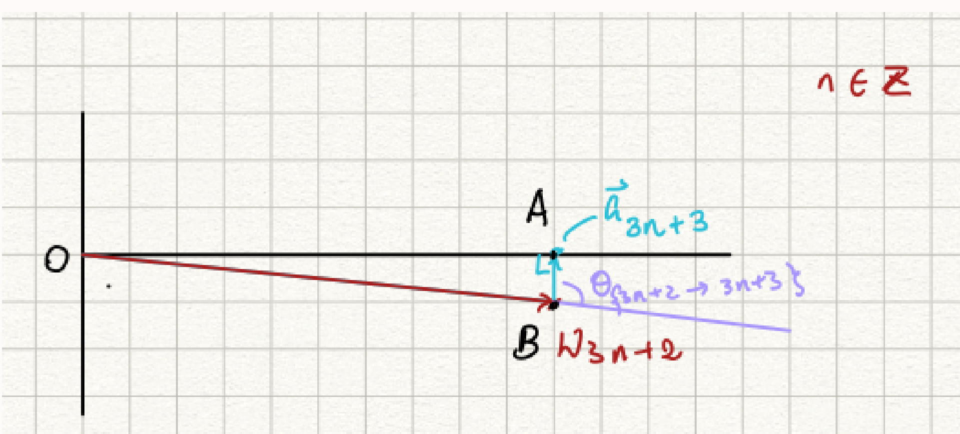

**Proof 2.2: Property II - max $||x_i||_2 \leq 1$**

$$\because \quad \|\mathbf{a}_{3n+1}\|_2 = 1 \quad \|\mathbf{a}_{3n+2}\|_2 = 1 \quad \|\mathbf{a}_{3n+3}\|_2 = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \quad \forall n \in \mathbb{Z} \tag{30}$$

$$\therefore \quad \|x_i\|_2 = \|a_i\|_2 \leq 1, \forall i \in \mathbb{Z} \tag{31}$$

**Proof 2.3: Property III: infinite number of mistakes**

We can guarantee the angle formed between every third point and prior weight vector would be larger or equal than 90 degree. As the figure below shown, the angle $\theta w_{3n+2} \to \mathbf{a}_{3n+3} \geq \frac{\pi}{2}, \forall n \in \mathbb{Z}$.



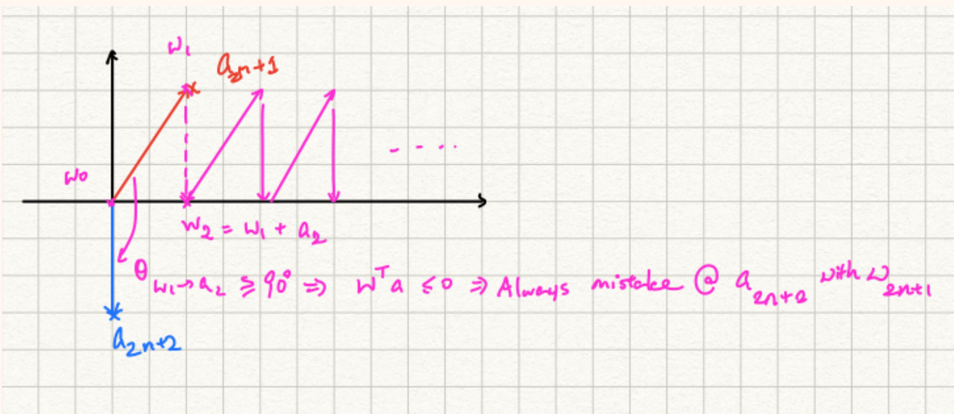
**Interesting Conclusion**

Interestingly to note, for given any repeated set of points, as long as we ensure the last point $\mathbf{a}_k$ will make $w_k = w_{k-1} + \mathbf{a}_k$ back to x-axis and form an 90 degree or larger angle with with prior weight $w_{k-1}$.

For a 2 point series case in 2D, it is much simpler, simply any arbitrary first point in quadrant 1, and a vector on negative y-axis that will bring the first point back to x-axis:

**Remark 2.2: Generalized Two-point Solution of an infinite sequence of points**

$$(\mathbf{x}_i, y_i) = \begin{cases} (\mathbf{x}_{2n+1}, y_{2n+1}) = ((x_{2n+1,1}, x_{2n+1,2}), 1) \\ (\mathbf{x}_{2n+2}, y_{2n+2}) = ((0, -x_{2n+1,2}), 1) \end{cases} \qquad x_{2n+1,1} \in \mathbb{R}^+, x_{2n+1,2} \in \mathbb{R}, n \in \mathbb{Z} \tag{32}$$

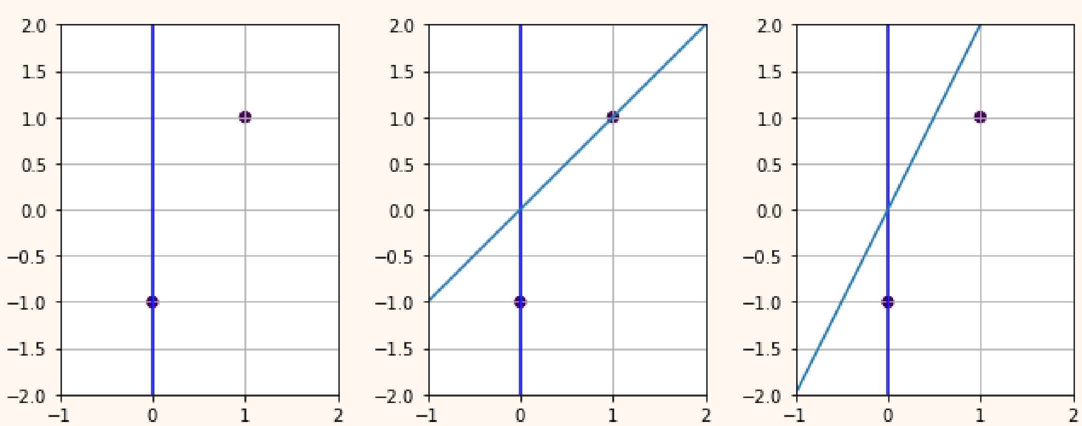



For example:

**Remark 2.3: Two-point Example of an infinite sequence of points**

$$(\mathbf{x}_i, y_i) = \begin{cases} ((1,1),1) & i = 2n+1 \\ ((0,-1),1) & i = 2n+2 \end{cases}, \quad n \in \mathbb{Z} \tag{33}$$

It is strictly linearly separable with $b = 0$:



**Q.E.D.**