ECE 457 B: COMPUTATIONAL INTELLIGENCE

ASSIGNMENT #3
(Fuzzy Logic Module)

Due Date: Please upload your solutions in the Dropbox folder "Assignment#3" by 4pm on April 13, 2021

NOTE: To help with the adequate marking of your assignments, please follow these rules carefully:

1. All work is expected to be carried out **individually.**
2. The first page of the assignment must have the name and ID of the student
3. The items required must be numbered, labelled, and in numerical order.
4. The solutions should be typewritten. All graphs should be computer generated. You can use existing libraries in Matlab, Python or others (but state the software used and attach a copy of your program code).
5. All pages must be numbered sequentially
6. Show your steps and state any additional assumption you make.
7. Presentation of your results, the organization of your copy, and completion level count for **5% of the total mark.**
8. **The minimum score for the average of all assignments is required to be 50%. Invalid submission or no submission leads to INC in the course.**
9. All copies will be checked by Turnitin similarity software

Problem 1. (20 marks) Draw the membership function $\mu_A(x) = e^{-\lambda(x-a)^n}$ for $\lambda = 2$, $n = 2$, and $a = 3$ for the support set $S = [0,6]$. On the drawn curve, highlight the shaded areas that represent the following fuzziness measures given by M_1, M_2 and M_3:

(a) $M_1 = \int_S f(x)dx$ where $f(x) = \begin{cases} \mu_A(x) & \text{for } \mu_A(x) \le 0.5 \\ 1-\mu_A(x) & \text{for } \mu_A(x) > 0.5 \end{cases}$

(b) $M_2 = \int_S \left| \mu_A(x) - \mu_{A_{1/2}}(x) \right| dx$

where $\mu_{A_{1/2}}$ is the $\alpha-$cut of $\mu_A(x)$ for $\alpha = 1/2$

(c) $M_3 = \int_S \left| \mu_A(x) - \mu_{\overline{A}}(x) \right| dx$

where $\overline{A}$ is the complement of the fuzzy set A. Evaluate numerically, the values of M_1, M_2 and M_3 for the given membership function.

i. Establish relationships between M_1, M_2 and M_3.

ii. Indicate how these measures can be used to represent the degree of fuzziness of a membership function.

Problem 2. (20 marks)

Consider an *n*-nary (higher dimension version of *bi*-nary) fuzzy relation $R(x, y, z, ...)$, which has *n* independent variables (coordinate axes), $x, y, z, ...$.

a) How many different projections are possible for this relation, in various subspaces? Explain how you arrived at this number.

b) Consider a discrete ternary ($n = 3$) fuzzy relation (rule base) given by the following "pages" of binary (x_i, y_j) relation along the z axis:

$$R_{z_1}(x_i, y_j) = \begin{bmatrix} 0.5 & 0.4 & 0.2 \\ 0.3 & 0.8 & 0.5 \\ 0.1 & 0.6 & 0.6 \end{bmatrix}; \quad R_{z_2}(x_i, y_j) = \begin{bmatrix} 0.6 & 0.5 & 0.3 \\ 0.4 & 1.0 & 0.6 \\ 0.2 & 0.6 & 0.8 \end{bmatrix};$$

$$R_{z_3}(x_i, y_j) = \begin{bmatrix} 0.4 & 0.3 & 0.1 \\ 0.2 & 0.7 & 0.3 \\ 0.0 & 0.5 & 0.5 \end{bmatrix}$$

Determine all possible projections of this relation

Problem 3. (20 marks) Suppose that the state of "fast speed" of a machine is denoted by the fuzzy set *F* with membership function $\mu_F(v)$. Then the state of "very fast speed", where the *linguistic hedge* "very" has been incorporated, may be represented by μ_F (*v*-

v_o) with $v_o > 0$. Also, the state "presumably fast speed", where the linguistic hedge "presumably" has been incorporated, may be represented by $\mu_F^{\,2}(v)$.

(a) Discuss the appropriateness of the use of these membership functions to represent the respective linguistic hedges.

(b) In particular, if

$$F = \{\frac{0.1}{10}, \frac{0.3}{20}, \frac{0.6}{30}, \frac{0.8}{40}, \frac{1.0}{50}, \frac{0.7}{60}, \frac{0.5}{70}, \frac{0.3}{80}, \frac{0.1}{90}\}$$

in the discrete universe $V = \{0, 10, 20, \ldots, 190, 200\}$ rev/s and $v_o = 50$ rev/s, Determine the membership functions of "very fast speed" and "presumably fast speed". Display both membership functions over the discrete Universe V.

Problem 4. (5 marks) Show that $max[0, x + y - 1]$ is a t-norm. Also, determine the corresponding t-conorm (i.e., s-norm)..

Problem 5. (10 marks)

Consider the membership function $\mu_A(x) = e^{-\lambda|x-a|^n}$, for a fuzzy set A.
Interpret the meaning of the parameters a, λ and n. In particular, discuss how (1) fuzziness and (2) a fuzzy adjective or fuzzy modifier such as "very" or "somewhat" of a fuzzy state may be represented using these parameters.

Problem 6. (20 marks)
Consider the experimental setup of an inverted pendulum shown in Figure 1.

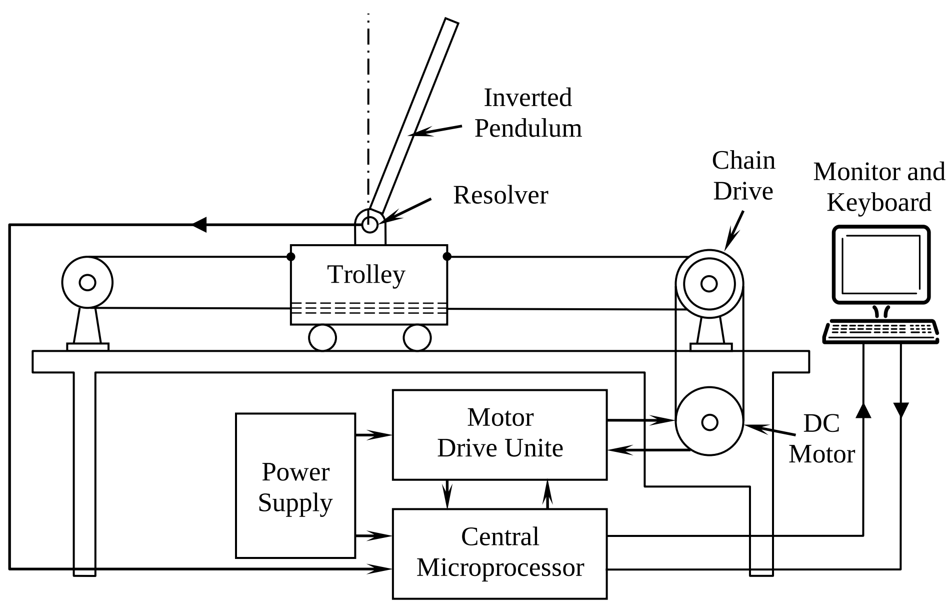

Figure 1. A computer-controlled inverted pendulum.

Suppose that direct fuzzy logic control is used to keep the inverted pendulum upright. The process measurements are the angular position, about the vertical (*ANG*) and the angular velocity (*VEL*) of the pendulum. The control action (*CNT*) is the current of the motor driving the positioning trolley. The variable *ANG* takes two fuzzy states: positive large (*PL*) and negative large (*NL*). Their memberships are defined in the support set [-30°, 30°] and are trapezoidal. Specifically,

μ_{PL}	= 0	for *ANG* = [-30°, -10°]
	= linear [0,1.0]	for *ANG* = [-10°, 20°]
	= 1.0	for *ANG* = [20°, 30°]
μ_{NL}	= 1.0	for *ANG* = [-30°,- 20°]
	= linear [1.0, 0]	for *ANG* = [-20°, 10°]
	= 0	for *ANG* = [10°, 30°]

The variable *VEL* takes two fuzzy states *PL and NL,* which are quite similarly defined in the support set [-60°/s, 60°/s]. The control inference *CNT* can take three fuzzy states: Positive large (*PL*), no change (*NC*), and negative large (*NL*). Their membership functions are defined in the support set [-3A, 3A] and are either trapezoidal or triangular. Specifically,

μ_{PL}	= 0	for *CNT* = [-3A, 0]
	= linear [0,1.0]	for *CNT* = [0, 2A]
	= 1.0	for *CNT* = [2A, 3A]
μ_{NC}	= 0	for *CNT* = [-3A, -2A]
	= linear [0, 1.0]	for *CNT* = [-2A, 0]
	= linear [1.0,0]	for *CNT* = [0, 2A]
	= 0	for *CNT* = [2A, 3A]
μ_{NL}	= 1.0	for *CNT* = [-3A, -2A]
	= linear [1.0,0]	for *CNT* = [-2A, 0]
	= 0	for *CNT* = [0, 3A]

The following four fuzzy rules are used in control:

	If	*ANG*	is	*PL*	and	*VEL*	is	*PL*	then	*CNT*	is	*NL*
else	if	*ANG*	is	*PL*	and	*VEL*	is	*NL*	then	*CNT*	is	*NC*
else	if	*ANG*	is	*NL*	and	*VEL*	is	*PL*	then	*CNT*	is	*NC*
else	if	*ANG*	is	*NL*	and	*VEL*	is	*NL*	then	*CT*	is	*PL*
end	if.											

(a) Sketch the four rules in a membership diagram for the purpose of making control inferences using individual rule-based inference.

(b) If the process measurements of *ANG*=5° and *VEL*=15°/s are made, indicate on your sketch the corresponding control inference.