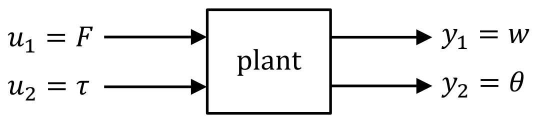
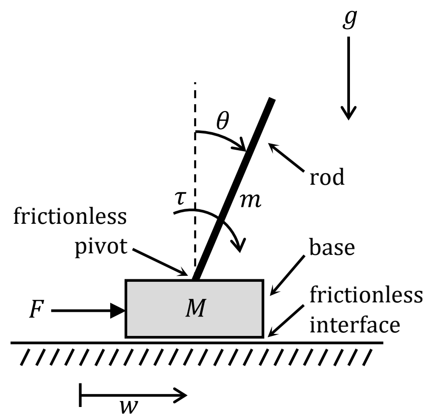
5.12 PROJECT EXERCISE P6

Problem P6: Use of state-space methods to control the MIMO aiming system

In this problem we use state-space methods to control both the one-rod aiming system and the two-rod aiming system. I realize there's not much time to do this problem, but it is actually one of the easiest, so give it a shot!

(a) Consider again the MIMO one-rod aiming system dealt with in Problem P5:





Signal	Unit	Interpretation
$F(t)$	N	Force applied to base (control signal u_1)
$\tau(t)$	$\mathrm{N \cdot m}$	Torque applied to rod (control signal u_2)
$w(t)$	m	Position of the base (plant output y_1)
$\theta(t)$	rad	Rod angle (plant output y_2)

Parameter	Value	Interpretation
M	1 kg	Mass of the base
m	0.5 kg	Mass of the rod
L	1.0 m	Length of the rod
g	$9.8\ \mathrm{m/s^2}$	Acceleration due to gravity

Back in Problem P5 (page 234) we used transfer function methods to control the system. Our approach there, typical of transfer function methods, required some clever approximations and original thinking. In contrast, state-space methods are mechanical and systematic, and we'll see that controlling the above system is not a big deal. All we need is a state-space model for the linearized system. One such model is given on the next page…

$$\left.\begin{aligned}\dot{x} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -3.2667 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 19.600 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0.8889 & -1.333 \\ 0 & 0 \\ -1.333 & 8.000 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u, \end{aligned}\right\} \qquad \text{... (P10)}$$

where

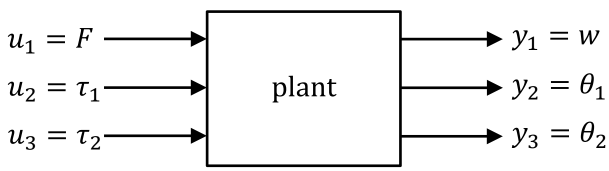
$$x = \begin{bmatrix} w \\ \dot{w} \\ \theta \\ \dot{\theta} \end{bmatrix}, \qquad u = \begin{bmatrix} F \\ \tau \end{bmatrix}, \qquad y = \begin{bmatrix} w \\ \theta \end{bmatrix}.$$

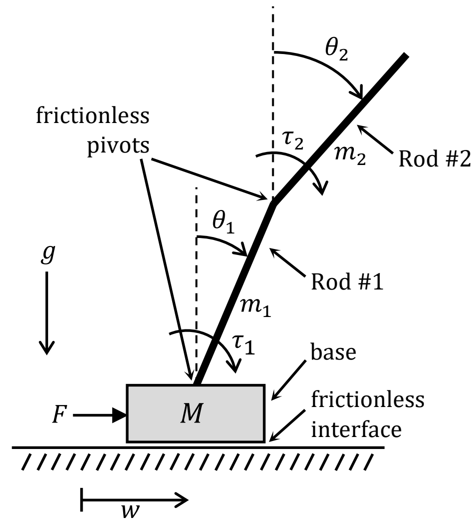
(i) Use the Matlab command “zpk” to show that the transfer function matrix for state-space model (P10) equals that given in (P7) back on page 235.

(ii) Assume the state is measurable. Use the *plain state-feedback control with integral action* control method to design a controller for the system (P10):

- Confirm that the augmented system is controllable.
- Place all the poles at $s = -20$. [Hint: The “place” command doesn’t like repeated poles, so place the poles at different locations that are very close to $s = -20$.]
- Look at the closed-loop step response. Verify that the response agrees with the response that you would expect from a system whose poles are all at $s = -20$.
- [Optional] Use “single_pend_fancy_sim.m” to visualize how your MIMO system behaves.
- [Optional] Place the poles further from the origin to see how the response speeds up. Mathematically, there is no limit to how fast you can make the system behave, but there are reasons such a scheme will eventually fail. Can you think of at least two such reasons?

(b) Now reconsider the two-rod aiming system originally introduced in Problem P4(b) (page 176). Back then we had just a single input, but since state-space design techniques can handle MIMO situations with almost no additional effort, let's introduce two other inputs, and we can then use the three inputs to control three outputs, as shown at the right and below:





The signals and parameters are summarized in the following table:

Signal	**Unit**	**Interpretation**
$F(t)$	N	Force applied to base (control signal #1)
$\tau_1(t)$	N·m	Torque applied to Rod #1 (control signal #2)
$\tau_2(t)$	N·m	Torque applied to Rod #2 (control signal #3)
$w(t)$	m	Position of the base (plant output #1)
$\theta_1(t)$	rad	Rod #1 angle (plant output #2)
$\theta_2(t)$	rad	Rod #2 angle (plant output #3)

Parameter	**Value**	**Interpretation**
M	1 kg	Mass of the base
m_1	0.25 kg	Mass of Rod #1
m_2	0.25 kg	Mass of Rod #2
L_1	0.5 m	Length of Rod #1
L_2	0.5 m	Length of Rod #2
g	9.8 m/s^2	Acceleration due to gravity

To account for the new inputs, the messy nonlinear equations from Problem P4(b) get modified as follows:

$$\frac{3}{2}\ddot{w} - \frac{3}{16}\dot{\theta}_1^2\sin(\theta_1) + \frac{3}{16}\ddot{\theta}_1\cos(\theta_1) - \frac{1}{16}\dot{\theta}_2^2\sin(\theta_2) + \frac{1}{16}\ddot{\theta}_2\cos(\theta_2) = F$$

$$\begin{aligned}\frac{1}{12}\ddot{\theta}_1 + \frac{3}{16}\ddot{w}\cos(\theta_1) + \frac{1}{32}\ddot{\theta}_2\cos(\theta_1-\theta_2) - \frac{1}{32}\big(\dot{\theta}_1-\dot{\theta}_2\big)\dot{\theta}_2\sin(\theta_1-\theta_2)\\ - \frac{3}{16}g\sin(\theta_1) + \frac{1}{32}\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) = \tau_1\end{aligned}$$

$$\begin{aligned}\frac{1}{48}\ddot{\theta}_2 + \frac{1}{16}\ddot{w}\cos(\theta_2) + \frac{1}{32}\ddot{\theta}_1\cos(\theta_1-\theta_2) - \frac{1}{32}\big(\dot{\theta}_1-\dot{\theta}_2\big)\dot{\theta}_1\sin(\theta_1-\theta_2)\\ - \frac{1}{16}g\sin(\theta_2) - \frac{1}{32}\dot{\theta}_1\dot{\theta}_2\sin(\theta_1-\theta_2) = \tau_2.\end{aligned}$$

Linearization of these equations about the operating point corresponding to where both rods are pointing straight up (i.e., when $\theta_1 = \theta_2 = 0$ and for any value of w) yields the following state-space realization:

$$\left.\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -4.41 & 0 & 0.49 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 61.74 & 0 & -26.46 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -79.38 & 0 & 67.62 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0.9333 & -2.40 & 0.80 \\ 0 & 0 & 0 \\ -2.40 & 33.60 & -43.20 \\ 0 & 0 & 0 \\ 0.80 & -43.20 & 110.40 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} u,
\end{aligned}\right\} \quad \ldots \text{(P11)}$$

where

$$x = \begin{bmatrix} w \\ \dot{w} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix}, \qquad u = \begin{bmatrix} F \\ \tau_1 \\ \tau_2 \end{bmatrix}, \qquad y = \begin{bmatrix} w \\ \theta_1 \\ \theta_2 \end{bmatrix}.$$

(i) Use Matlab to find the transfer function matrix for system (P11). Imagine trying to use transfer function methods to control this system. It is possible, but what a horrible mess! I do not recommend pursuing transfer function methods.

(ii) Assume the state is measurable. Use the *plain state-feedback control with integral action* control method to design a controller for system (P11).

- Confirm that the augmented system is controllable.
- Place all the poles at $s = -20$ and look at the step response. Verify that the response agrees with the response that you would expect from a system whose poles are all at $s = -20$.

(iii) Suppose now that the state is not measurable, so an observer is needed to estimate the state. Construct a controller for system (P11) based on the *observer-based state-feedback control with integral action* technique.

- Confirm that the state-space realization (P11) is observable.
- Place the state-feedback poles at $s = -20$ and place the poles of the observer error dynamics at any reasonable location.
- Look at the closed-loop step response and choose initial conditions in your simulation to clearly demonstrate the "locking on" effect of the observer. [Hint: Use "lsim" instead of "step" to handle non-zero initial conditions.]
- [Optional] Use "double_pend_fancy_sim.m" which can be downloaded from LEARN to visualize how your MIMO system behaves, both with and without the observer.