

$$\begin{aligned}
 4) \int \sin^3 x \cos^2 x \, dx &= \int \sin x \cdot \sin^2 x \cdot \cos^2 x \, dx = \\
 &= \int (1 - \cos^2 x) \cdot \cos^2 x \cdot \sin x \, dx = \begin{cases} t = \cos x \\ \sin x \, dx = -dt \end{cases} \\
 &= \int (1 - t^2) t^2 (-dt) = -\int t^2 \, dt + \int t^4 \, dt = -\frac{1}{3} \cos^3 x + \frac{1}{5} \cos^5 x
 \end{aligned}$$

$$\begin{aligned}
 8) \int \sin^3 x \cos 5x \, dx &= \int \frac{1}{2} (\sin(2x) + \sin(8x)) \, dx = \\
 &= \frac{1}{2} \int \sin 2x \, dx + \frac{1}{2} \int \sin 8x \, dx = -\frac{1}{4} \cos 2x - \frac{1}{16} \cos 8x \\
 I_1 = \int \sin 2x \, dx &= \begin{cases} t = 2x \\ dt = 2 \, dx \end{cases} = \frac{1}{2} \int \sin t \, dt = -\frac{1}{2} \cos t = -\frac{1}{2} \cos 2x \\
 I_2 = \int \sin 8x \, dx &= \begin{cases} t = 8x \\ dt = 8 \, dx \end{cases} = \frac{1}{8} \int \sin t \, dt = -\frac{1}{8} \cos t = -\frac{1}{8} \cos 8x
 \end{aligned}$$

$$\text{Ans. } -\frac{1}{4} \cos 2x - \frac{1}{16} \cos 8x$$

$$15) \int \frac{dx}{4 + \sin^2 x} = \left\{ \begin{array}{l} \sin^2 x = \frac{t^2}{1+t^2} \\ dx = \frac{dt}{1+t^2} \end{array} \right\} =$$

$$= \int \frac{\frac{dt}{1+t^2}}{4 + \frac{t^2}{1+t^2}} = \int \frac{dt}{\frac{4+5t^2}{1+t^2}} \cdot (1+t^2) = \int \frac{dt}{4+5t^2} =$$

$$= \int \frac{dt}{4 \left(1 + \left(\frac{\sqrt{5}}{2} t \right)^2 \right)} = \left\{ \begin{array}{l} k = \frac{\sqrt{5}}{2} \\ dk = \frac{\sqrt{5}}{2} dt \end{array} \right\} =$$

$$= 4 \int \frac{dk}{1+k^2} \cdot \frac{2}{\sqrt{5}} = \frac{8}{\sqrt{5}} \operatorname{arctg} \frac{\sqrt{5}}{2} \operatorname{tg} \frac{x}{2}$$

$$16) \int \frac{\cos^3 x}{\sin^5 x} dx = \int \frac{\cos x (1 - \sin^2 x)}{\sin^5 x} dx = \left\{ \begin{array}{l} t = \sin x \\ dt = dx \cdot \sin x \end{array} \right\} =$$

$$= \int \frac{(1-t^2)}{t^3} dt = \int \frac{1}{t^3} dt - \int \frac{t^2}{t^3} dt = -\frac{1}{2} (\sin x)^{-2} + \frac{1}{2} (\sin x)^{-2}$$