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Mathemathics AA SL

Internal Assessment

“Graphing the IB logo”

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Explanation of the topic and research aim

From the beginning when we were supposed to decide about a topic for our exploration, I knew I wanted to do something unusual. I decided to try to incorporate my interests into mathematics. I am passionate about graphic design, and I take architecture classes, therefore I am not a big fan of explorations that aim to calculate volumes or solve famous mathematical problems. I know that graphic design programs, like Adobe Illustrator, use vectors to create high-resolution graphics. This is the closest I could get to mathematics with my interests, so I decided to check whether instead of using a program, I could graph a logo using mathematical functions learned during Mathematics AA.

During our classes, we have been shown Desmos, an online graphing calculator, which seemed to me like an appropriate tool for this task. During my research, I have seen people online graphing complicated images, like cartoon characters, using functions. So, I decided to learn how to do that and graph. After giving it a thought, an IB logo seemed like a good choice due to the very different types of lines and curves that are in it. I knew some parts of it would be challenging, but I was very eager to try it anyways. I aim to not only check whether I can graph it but also what types and how many functions I will need.

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Predicting the functions to be used and planning the work

I planned to organize my work by dividing the logo into sections and labeling them on a picture

Figure 1 IB logo divided into sections

so that it would be more organized and easier to understand later. The plan can be seen in Figure 1, each section is represented by a color:

Part 1 – the 'arch' of the letter 'b', using quadratic and exponential functions

Part 2 – curves that will require exponential functions

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Part 3 – frame of the logo and the dot in the 'i' using circle equation

Part 4 – circular shape inside the b letter, combining different functions – hard to specify the function types at this point

Part 5 – additional vertical lines that cannot be graphed as functions

After that, I pasted the logo (International Baccalaureate Organisation) into Desmos. Then I resized it, so it lies from -10 to 10 on x axis and from -10 to 10 on y axis, while also turning down the opacity and setting it as the background image. Such even, central positioning of the imagine will help me with calculations. Moreover, in the exploration, I will be using a calculator to help me save time on complicated multiplication and division. All results with multiple decimal places will be rounded to three significant values.

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Part 1 – 'arch' of the b

I decided to experiment with several types of functions for the bottom section of the 'b' letter. I to use quadratic functions for the top bottom and left part, and then eventually fill in the 'corners' of the 'arch' with exponential functions.

The general form of a quadratic function can be written as $y = ax^2 + bx + c$. However, I decided to use the vertex form as it would give me the option of finding the function with the vertex coordinates if I chose to. This form is written as $y = a(x-h)^2 + k$.

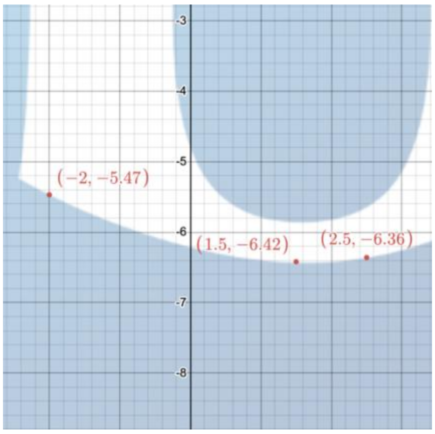

Figure 2 The three chosen points

My first thought was to solve this mathematically. I realized that I haven't done systems of equations with three variables in school, so I had to research it. Fortunately, the method did not turn out to be as complicated as I feared it

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would be. Because this method is long, I will explain it on one example and then just show the outcomes for the other equations. I started by marking three points (as there are three variables), where I wanted my graph to be (Figure 2). The points have values $(-2,-5.47),(1.5,-6.42),(2.5,-6.36)$. I substituted them into the equations of the function and create a system of equations:

$$\begin{cases} -5.47 = a(-2-h)^2 + k \\ -6.42 = a(1.5-h)^2 + k \\ -6.36 = a(2.5-h)^2 + k \end{cases}$$

$$\begin{cases} -5.47 = a(4+4h+h^2) + k \\ -6.42 = a(2.25-3h+h^2) + k \\ -6.36 = a(6.25-5h+h^2) + k \end{cases}$$

$$\begin{cases} -5.47 = 4a + 4ah + ah^2 + k \\ -6.42 = 2.25a - 3ah + ah^2 + k \\ -6.36 = 6.25a - 5ah + ah^2 + k \end{cases}$$

Then I split it into two separate systems of equations:

1. $\begin{cases} -5.47 = 4a + 4ah + ah^2 + k \\ -6.42 = 2.25a - 3ah + ah^2 + k \ /*(-1) \end{cases}$

2. $\begin{cases} -6.42 = 2.25a - 3ah + ah^2 + k \ /*(-1) \\ -6.36 = 6.25a - 5ah + ah^2 + k \end{cases}$

1. $+\begin{cases} -5.47 = 4a + 4ah + ah^2 + k \\ 6.42 = -2.25a + 3ah - ah^2 - k \end{cases}$

2. $+\begin{cases} 6.42 = -2.25a + 3ah - ah^2 - k \\ -6.36 = 6.25a - 5ah + ah^2 + k \end{cases}$

1. $0.95 = 1.75a + 7ah$

2. $0.06 = 4a + 2ah$

Then I created a system of equations again:

$$\begin{cases} 0.95 = 1.75a + 7ah \\ 0.06 = 4a - 2ah \ /*3.5 \end{cases}$$

$$+\begin{cases} 0.95 = 1.75a + 7ah \\ 0.21 = 14a - 7ah \end{cases}$$

$$1.16 = 15.75a \ \ /:15.75$$

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$a \approx 0.0737$

Then I substituted this value into the equation to find h:

$0.95 = 1.75(0.0737) + 7(0.0737)h$

$0.95 = 0.129 + 0.516h$

$0.821 = 0.516h \ /: 0.516$

$h \approx 1.59$

Substituting a and h values to calculate k:

$-5.47 = 4(0.0737) + 4(0.0737)(1.59) + (0.0737)(1.59)^2 + k$

$-5.47 = 0.950 + k$

$k \approx -6.34$

I substituted these values into the function, resulting in $y = 0.0737(x - 1.59)^2 - 6.42$. The outcome can be seen in Figure 3. Now I only needed to read two more points from the graph to set the domain. I settled for D∈x $\{-2.45 \leq x \leq 3.1\}$, as shown in Figure 4.

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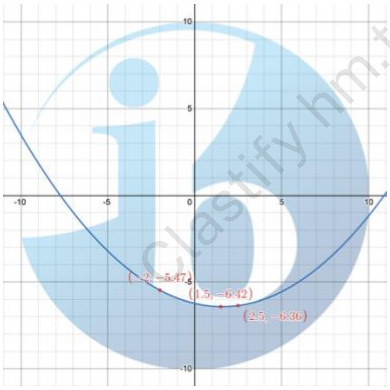

Figure 3 The final equation graphed on the set of axes

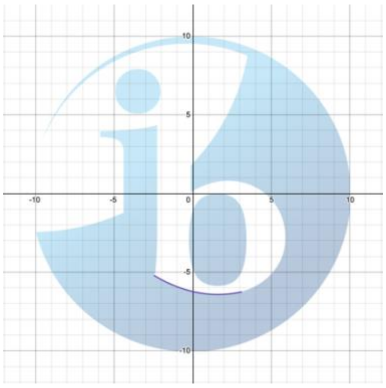

Figure 4 The function after setting the domain

Then I repeated this process for the top part of the b, the results of which can be seen below:

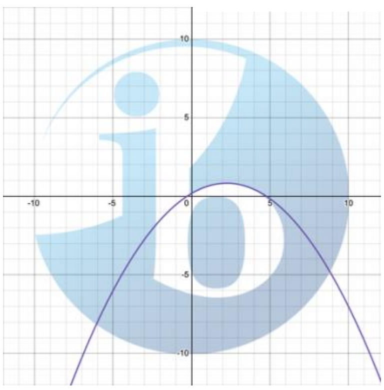

Figure 5 Graph of the equation $y = -0.13(x - 2.25)^2 + 0.83$

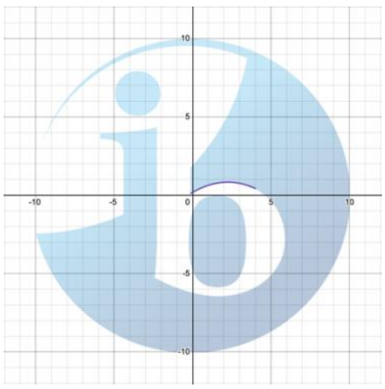

Figure 6 graph of this equation after setting the domain to D∈x {–0.1 ≤ x ≤ –4}

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To fit the right part of the b properly I created an inverse function by swapping x and y, which resulted in $x = a(y - h)^2 + k$. After that, I used the same method again, as shown below (Figure 5 and Figure 6)

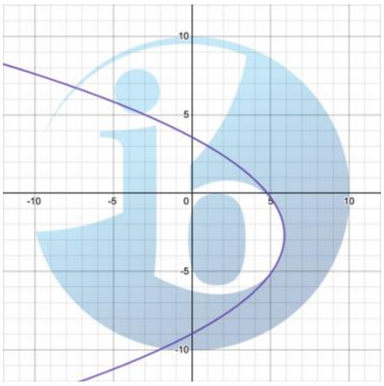

Figure 7 Graph of the equation $x = -0.15(y + 2.7)^2 - 5.9$

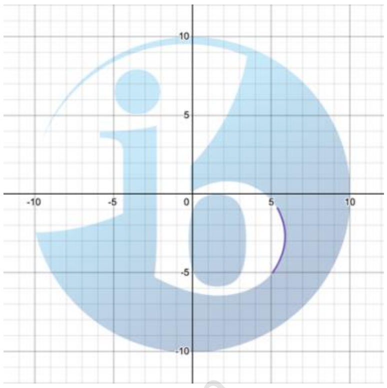

Figure 8 Graph of this equation after setting the range (inverse function) to $R \in y\{-5 \leq y \leq -0.9\}$

The last remaining thing to do was to fill the top-right and bottom-right corners of the 'arc' of the b letter with exponential functions. Because the sections I needed were short, I used the general transformative form

$y = a^{(x-h)} + k$, where

a – vertical stretch; **h** - horizontal translation; **k** - vertical translations

I decided to use this form as it has three variables, so it is like what I have worked with earlier. However, this time I knew the equation would be more complex and lengthier to calculate, and I wanted to implement technology in this exploration as well. Therefore, I decided to find a new method and make use of the technology I was working with. If the equation is inserted into Desmos with variables, the website gives an option to insert different values for these and experiment with the function's shape. Using transformation this way should help me fit the curves accurately.

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After graphing the equation into Desmos I started to experiment with different values of the function. First, when the ‘a’ value was 1 the graph was horizontal, however the value of 2 seemed the right shape. Then I went to settle the translations; by -6.5 to the bottom (6 was too high) and 4 to the right. It was almost good, but I still needed to fit it properly. Increasing the ‘a’ value made the graph steeper; I tried 2.5 which seemed almost perfect and finally decided to 2.6. Then I slightly moved the graph to the right and it was done. The result of the process can be seen in Figure 7. The transformations I used can be described as horizontal stretch with stretch factor 2.3, and translations by the vector $\binom{-2.7}{5.9}$. I was very satisfied with the result. Using transformations seems like a more convenient method, considering how many equations in total I need to do to graph the whole logo. Then, to achieve the function that would fit the top-right corner of the ‘arc’ I simply reflected this equation over the x-axis (by making the ‘a’ value negative) and adjusted the vertical translation (Figure 8). Finally I only needed to establish the domain for both functions, so I did that by looking at the points of intersection between these exponential functions and the previous quadratic ones. The results can be seen below:

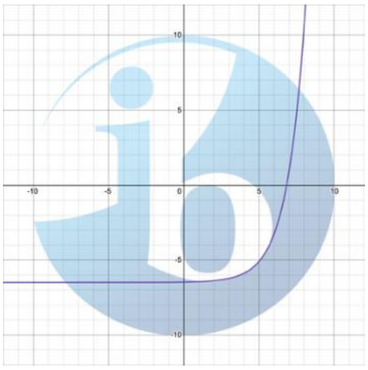

Figure 9 Graph of the equation $y = 2.3^{(x-4.6)} - 6.5$

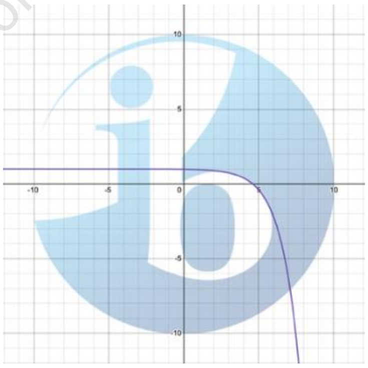

Figure 10 Graph of the equation $y = -2.3^{(x-4.6)} + 1$

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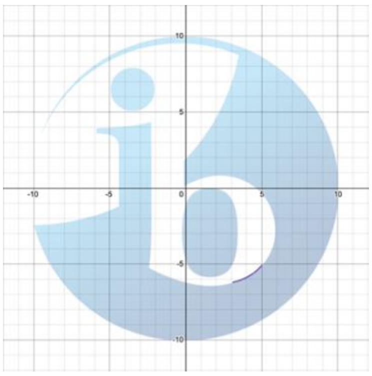

Figure 11 Graph of the equation $y = 2.3^{(x-4.6)} - 6.5$ *with the domain* $D \in x\{3.1 \leq x \leq 5\}$

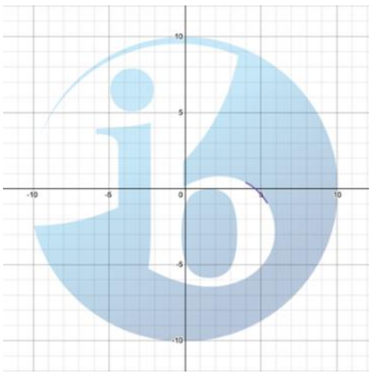

Figure 12 Graph of the equation $y = -2.3^{(x-4.6)} + 1$ *with the domain* $D \in x\{4 \leq x \leq 5.4\}$

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Having found all the necessary functions for the ‘arc’ of the b, I was able to put them all together, as shown on the Figure 10. I changed the color of every function so that I could see how well they fit together.

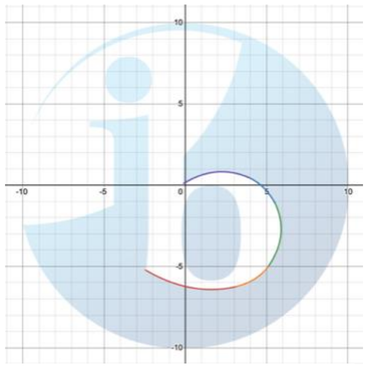

Figure 13 The complete ‘arch’ of the b graphed with functions

Part 2 - Exponential functions

From my experience so far, I realized that the method was a good one, however I needed one more variable to be able to manipulate both the horizontal and vertical stretch. Therefore, I decided to create the general transformative form by inserting variables

$y = a^{b(x-c)} + d$, where letters represent following

a – vertical stretch; **b** - horizontal stretch; **c** - horizontal translation; **d** - vertical translations.

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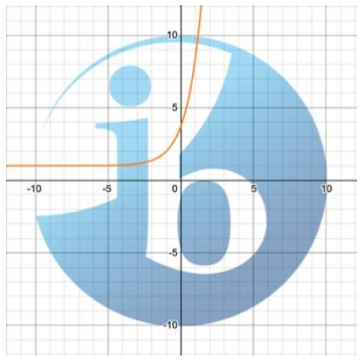

Figure 14 Graph of the exponential function

Having four variables would allow me to have bigger flexibility in transformations and get the best results. I first drew the general function onto the grid (Figure 11). Then, started experimenting to fit the functions onto the lines of the logo.

The way I have found the equations of these functions was by understanding how changing the values of certain transformations affect the graph and then adjusting them accordingly. First, I changed the horizontal and vertical translations to move it closer to the part I wanted to graph. These values I have evaluated based on the graph, vertical translation downwards by 2.5, and horizontal translation around -5. Then for the horizontal stretch, any value of a smaller than 1 would result in reflection over y-axis, and a value of 1 – a horizontal line. The higher the ‘a’ value – the steeper the graph has become. Because this fragment of the logo was not as steep, I visually estimated the value to be around 3. Now the value of ‘b’ had similar effects (except the

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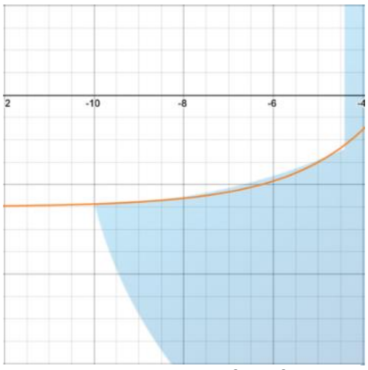

Figure 15 The graph of the function with estimated transformations

reflection happened for negative values) but seemed to adjust the graph more precisely. Because the value of 'a' made the graph quite steep I needed the b to ease out the curve, meaning I needed a value less than 1 (otherwise it would just enhance it) so I estimated it for 0.5. The 'first version' of the graph can be seen in Figure 15. It can be seen that the estimation was obviously not accurate enough, yet already pretty close. Therefore, slightly adjusting each value was necessary. Having in mind the properties, both horizontal and vertical stretch values were decreased, while both the horizontal and vertical translation values were slightly increased. Figure 16 shows graph of the equation after vertical stretch with stretch factor 2.8, horizontal stretch – with stretch factor 0.38 and translation by the vector $\binom{-5.3}{-2.6}$. The result is the equation:

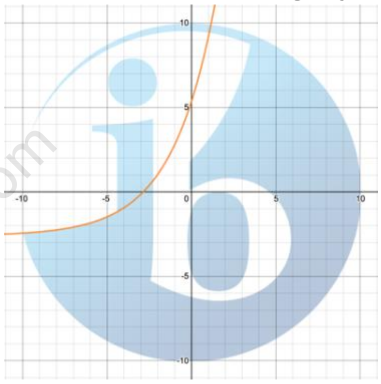

Figure 16 Graph of the exponential equations after adjusting transformations

$$y = 2.8^{0.38(x-5.3)} - 2.6$$

After getting this result, I used the same method, for which the results are shown below with the descriptions of applied transformations for each:

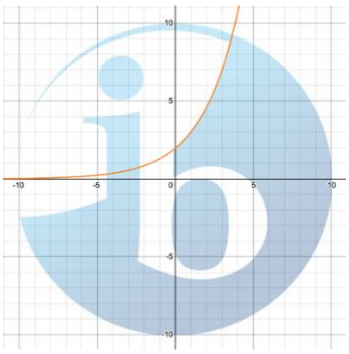

Figure 17 Graph of the equation after:

vertical stretch with stretch factor 0.56,

horizontal stretch with stretch factor –0.73

translation by the vector $\binom{1.6}{0}$

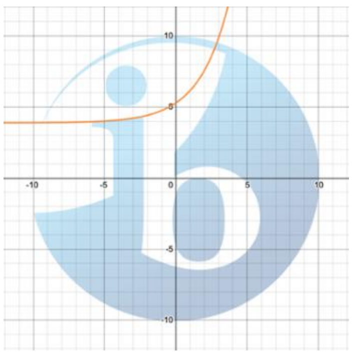

Figure 18 Graph of the equation after:

vertical stretch with stretch factor 10,

horizontal stretch with stretch factor 0.21

translation by the vector $\binom{0.7}{3.88}$

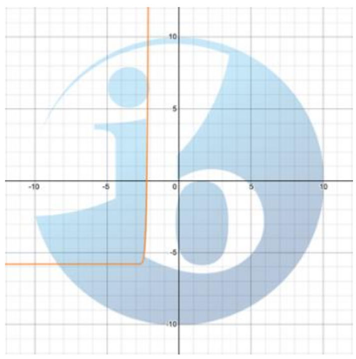

Figure 19 Graph of the equation after:

vertical stretch with stretch factor 4.2,

horizontal stretch with stretch factor 8.7

translation by the vector $\binom{2.38}{-5.8}$

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Now that I had found the equations that fit as close to the lines in the logo as possible, I had to set the domain so that they only appeared in the part that I needed them to. Placing the logo on the grid in the center allowed me to read the points from the grid. The results are shown below with final equations and the established domain for each:

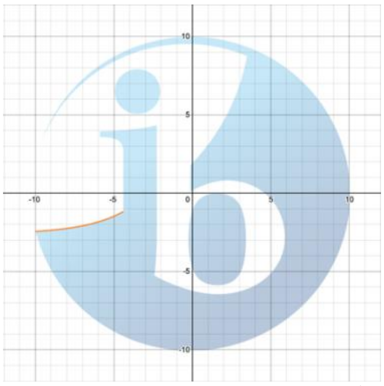

Figure 20 The graph of the equation $y = 2.8^{0.38(x+5.3)} - 2.6$ *with the domain* $D \in x\ \{-9.99 \le x \le -4.42\}$

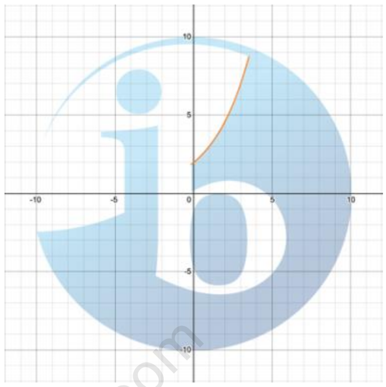

Figure 21 The graph of the equation $y = 0.56^{-0.73(x+1.6)}$ *with the domain* $D \in x\ \{-0.16 \le x \le 3.52\}$

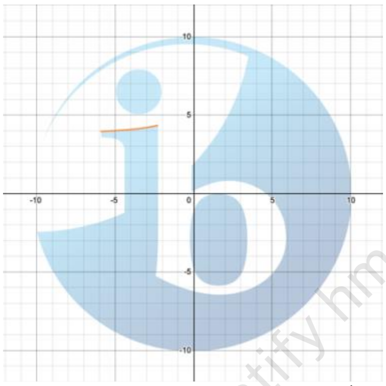

Figure 22 The graph of the equation $y = 10^{0.21(x+0.7)} + 3.88$ *with the domain* $D \in x\ \{-5.9 \le x \le -2.3\}$

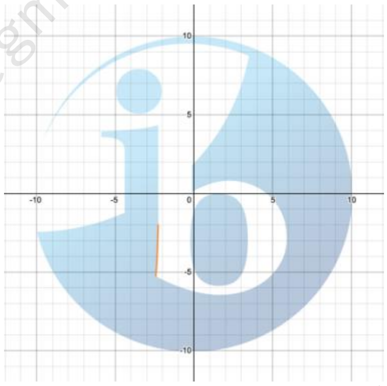

Figure 23 The graph of the equation $y = 4.2^{8.7(x+2.38)} - 5.8$ *with the domain* $D \in x\ \{-5.24 \le x \le -2\}$

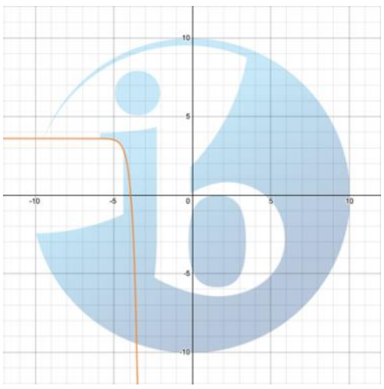

Figure 24 The last exponential function

After these 4 functions were done, I had one part left where I planned an exponential function. By reflecting an exponential function over the x-axis and applying transformations as described earlier I managed to fit it as shown on the Figure 24, which resulted in an equation

$$y = -1.33^{12(x+4.3)} - 3.6$$

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Although the graph looks fitted enough, after drawing the vertical line that would connect to the function, I realized the exponential function cannot be connected to the vertical line smoothly (Figure 25) so I had to look for another solution.

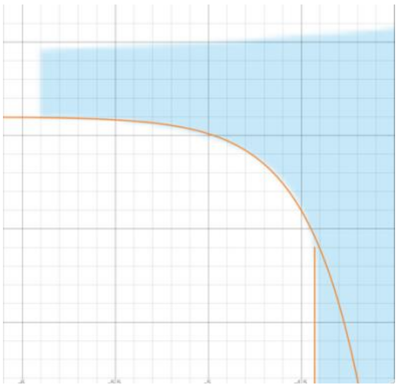

Figure 25 The disconnection of the function and the vertical line

I decided to try a logarithmic function which has an equation $y = \log_a(x)$. I added variables to the equation, creating the general transformative form:

$$y = \log_a\big(b(x+h)\big) + k$$

where certain variables represent

b – horizontal stretch; h – horizontal translation; k – vertical translation

Again, I experimented with the transformations. This one took me the longest so far as I had to establish the 'a' value. Small values where not steep enough so I began increasing them by 50 and checking if the curve will fit. I settled for 300 at the beginning and then adjusted it to 304. Using reflection over the y-axis, horizontal stretch (stretch factor 3.9) and translation by the vector $\binom{4.4}{3.3}$ resulted in the equation:

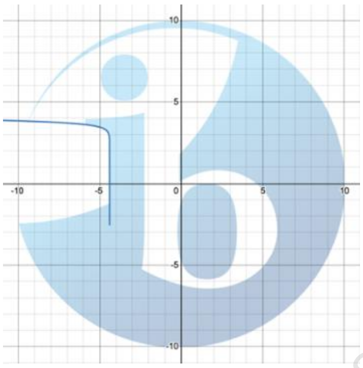

Figure 26 Graph of the logarithmic function

$$y = \log_{304}\big(-3.9(x+4.4)\big) + 3.3$$

I was very satisfied with the result, until I zoomed in and realized the curve does not fit the lines on the logo. I probably could have left it that way, however it would not be corresponding with my research aim: an illustrating program would not be so inaccurate, and so couldn't I.

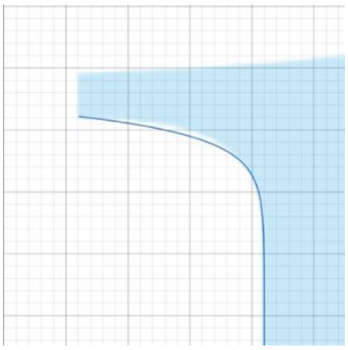

Figure 27 The inaccuracy of the logarithmic function

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I decided to come back to this part after I have done other parts to decide if I want to do it differently. After learning the ellipses, I was able to graph this part using a combination of exponential function, an ellipse, and a horizontal line. The process for exponential function has already been explain, and the rest is later in the text, therefore I will only show the results here, in Figure 28. I have found the points of intersection, which helped me state the domains and range.

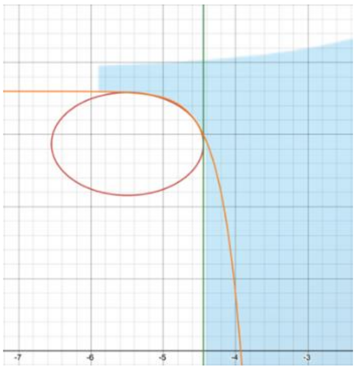

Figure 28 The combination of three equations graphed on one plot

The equations of all these are as following:

$y = -1.34^{12(x+4.3)} + 3.6 \rightarrow$ exponential function, with the domain D∈ $\{-1.2 \leq x \leq 2.88\}$

$\frac{(x+5.498)^2}{1.05^2} + \frac{(y-2.87)^2}{0.714^2} = 1 \rightarrow$ equation of the ellipse, with the domain D∈ $\{-5.5 \leq x\}$ and range R∈ $\{2.88 \leq y\}$

$x = -4.445 \rightarrow$ horizontal line, with range R∈ $\{-1.2 \leq y \leq 2.88\}$

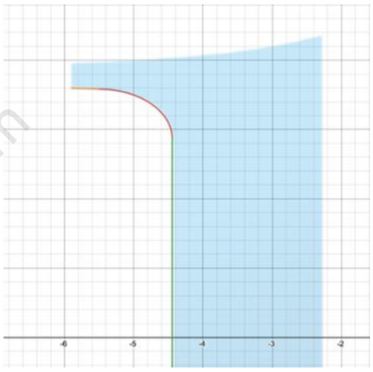

Figure 29 The equations after their domains and ranges were set

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Part 3 – Frame of the logo and the dot in the 'i'

From the beginning of planning, it was clear to me I would have to use circles to graph this logo. Because I am on the Mathemathics SL, we did not cover this topic and I had to research it myself. I found the equation of the circle which is written as:

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h, k) is the center of the circle, and r is the radius.

Having that knowledge, I substituted the h and k for zeroes and r for 10 (due to the positioning of the image on the grid) creating the following equation:

$$(x-0)^2 + (y-0)^2 = 10^2$$

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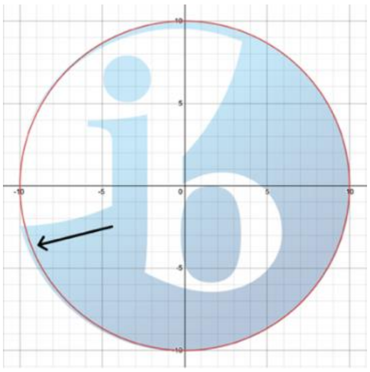

Figure 30 Circle equation graphed onto the logo

At that point, I realized that the logo is not a perfect circle, and it was my mistake in assuming it is. It can be mostly seen on part on the left from the y-axis especially the bottom left quarter of the logo. Because of the wrong estimation I made, I had to find another equation to graph the 'frame' of the logo. Since it came out wider than the estimated circle, I decided to research an equation of an ellipse. The equation of the ellipse is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

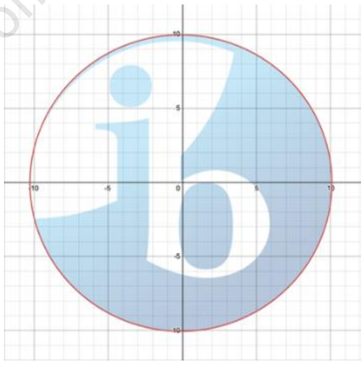

Figure 31 The ellipse equation graphed onto the logo

Where (h, k) is the center of the ellipse, a – half of the distance between the two vertexes (longer axis of the ellipse), and b – half of the distance between the semi-vertexes (shorter axis of the ellipse). I substituted (h, k) for (0,0) and b for 10 again (the height of the ellipse – 20 divided by 2), and tried to stretch the ellipse by manipulating the values of a. When a was equal to 10, the shape looked identical to the one circle. Increasing the values of a seemed to stretch the graph horizontally and decreasing – compress it. I experimented with different values and managed to fit the ellipse onto the logo, setting a to 15 and adjusting the horizontal and vertical translations slightly. The equation of the ellipse I drew is

$$\frac{(x+0.1)^2}{10.15^2} + \frac{(y+0.1)^2}{10^2} = 1$$

The IB logo though is not a full ellipse. Because the missing part is only in one of the quarters, I had to find a way to set the domain and range and create the shape I need. So, I duplicated the function which allowed me to set the domain separately for the top and bottom half. The outcome can be seen below:

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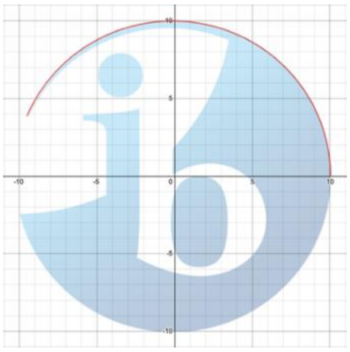

Figure 32 The top part of the ellipse equation with D∈{–9.5 ≤ x} and R∈{0 ≤ y}

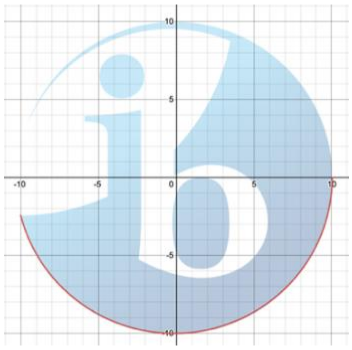

Figure 33 The bottom part of the ellipse equation with D∈{–10 ≤ x} and R∈{y ≤ 0}

Next, I used the equation I calculated above and applied translation by changing the values of h and k to fit the remaining part of the frame.

the previous equation:

$$\frac{(x+0.1)^2}{10.15^2} + \frac{(y+0.1)^2}{10^2} = 1$$

equation after adjusting the translations:

$$\frac{(x+0.3)^2}{10.15^2} + \frac{(y+0.5)^2}{10^2} = 1$$

Then I stated the domain to D∈ {–9.5 ≤ x ≤ 3.48} and range to R∈ {0 ≤ y}, resulting in the graph shown in Figure 34.

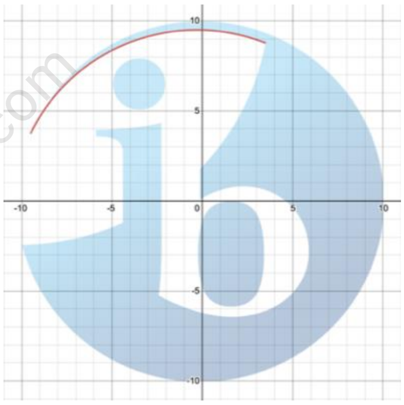

Figure 34 The adjusted ellipse equation after stating the domain and range

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Although I did not find the equation of the circle useful before, I still had the dot above the ‘i’ left. I graphed an equation of the circle again and checked the center of the circle on the plot to be (–3.5, 6.5). I found its radius by subtraction of the lowest y–value from the highest y–value on the circle and dividing it by 2:

$$r = \frac{7.93-5.07}{2} = 1.43$$

Then I substituted the values into the equation

$$(x+3.5)^2 + (y-6.5)^2 = 1.43^2$$

The result has been shown in Figure 35.

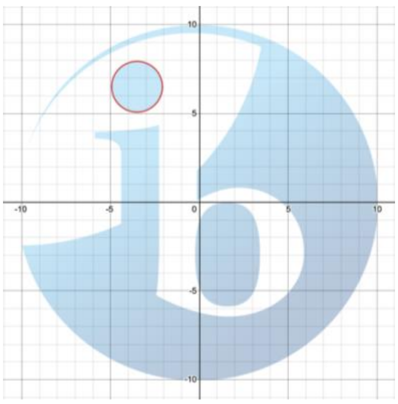

Figure 35 Graph of the circle equation

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Part 4 –Circular shape inside the b letter

The last function that I needed to find were the ones needed to draw the shape inside of the letter ‘b’. I tried to fit an ellipse into it (Figure 36), but unfortunately, it was not a good fit and I had to combine few functions together, as before. I decided to use the skills I have already learned and try to draw this part with two quadratic functions and two ellipses. I created an inverse quadratic function, $x = a(y - h)^2 + k$, and experiment with transformations. Because I was able to estimate my vertex from the graph, I applied translation by the vector $\begin{pmatrix}-0.25\\-2.9\end{pmatrix}$. Then only the ‘a’ value needed to be changed. Any value over 1 was very steep, so I tried 0.1. It seemed almost fitting, so I settled for stretch factor 0.07. To produce the second equation, I simply reflected it over the y-axis and applied a horizontal translation to the right.

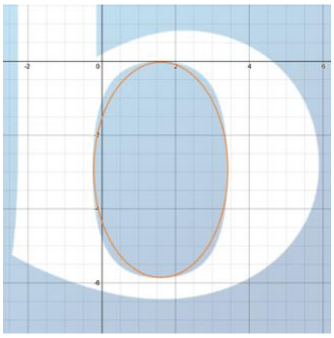

Figure 36 An ellipse drawn onto the b letter

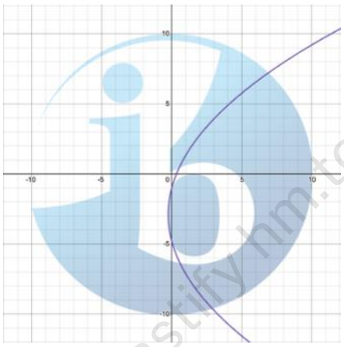

Figure 37 Graph of the equation $y = 0.07(x + 2.9)^2 - 0.25$

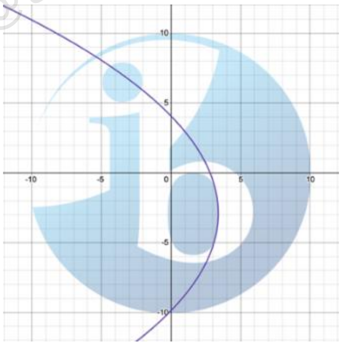

Figure 38 Graph of the equation $y = -0.07(x + 2.9)^2 - 3.4$

Then I created the top part by drawing the equation of the ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Now that I have worked with ellipses, I used the same method of finding the values by fitting it in the online graphing calculator. I have found the h value from the graph and estimated the h value (which I later adjusted). Then, I settled the values of a and b and corrected the translation of the ellipse by the vector $\begin{pmatrix}1.58\\-1.5\end{pmatrix}$, resulting in the equation:

$$\frac{(x - 1.58)^2}{1.65^2} + \frac{(y + 1.5)^2}{1.5^2} = 1$$

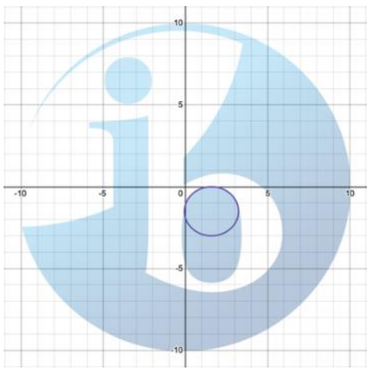

Figure 39 Graph of the found ellipse drawn onto the set of axes

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To create the second one, I simply duplicated this ellipse and applied a vertical translation downwards. The adjusted equation:

$$\frac{(x-1.58)^2}{1.65^2}+\frac{(y+4.36)^2}{1.5^2}=1$$

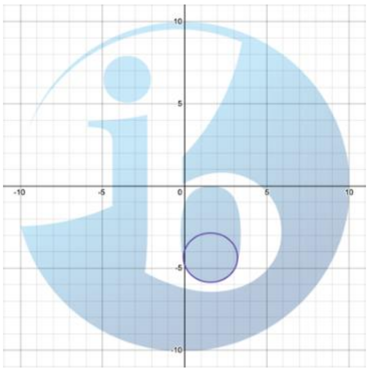

Figure 40 The second ellipse after translation downwards

After finding all the equations I stated domains or ranges for all of them, based on where they intersect, resulting in:

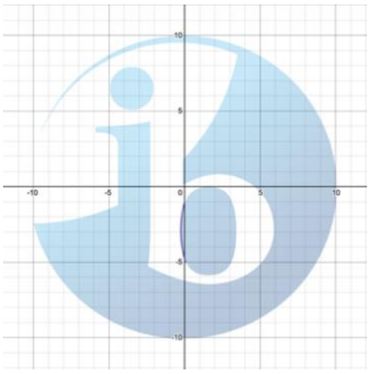

Figure 41 The inverse quadratic function $x = 0.07(y + 2.9)^2 - 0.25$ *with range* $R\in\{-5 \le y \le -1\}$

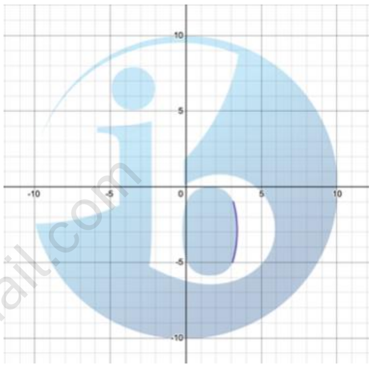

Figure 42 The inverse quadratic function $x = 0.07(y + 2.9)^2 - 0.25$ *with range* $R\in\{-5 \le y \le -1\}$

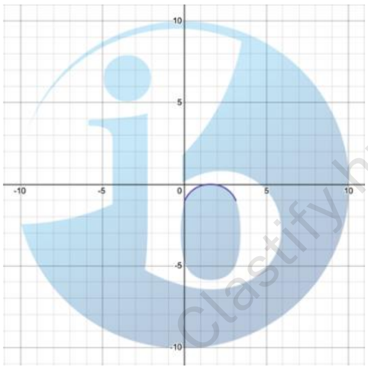

Figure 43 The graph of the ellipse equation $\frac{(x-1.58)^2}{1.65^2}+\frac{(y+1.5)^2}{1.5^2}=1$ *with range* $R\in\{-1\le y\}$

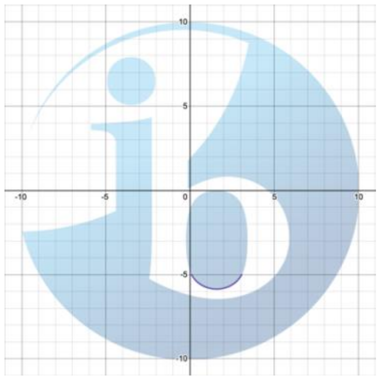

Figure 44 The graph of the ellipse equation $\frac{(x-1.58)^2}{1.65^2}+\frac{(y+4.36)^2}{1.5^2}=1$ *with range* $R\in\{y\le -5\}$

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Part 5 - Vertical lines

One challenge I came across, as early as in the phase of planning my work, were the vertical lines in the IB logo. If these were horizontal lines, a linear function could be used. But from the understanding how functions are created, it is known that a function cannot be vertical. Therefore, I decided to draw those 3 vertical segments using vertical lines in the form of x = a

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(where a is the value on the x-axis equal to the position of a certain vertical line) I have created vertical lines on the graph to match the lines on the logo. Then I stated the range for all of them (by looking at the points where they need to intersect with the other functions) so there only the parts that are needed stay on the picture:

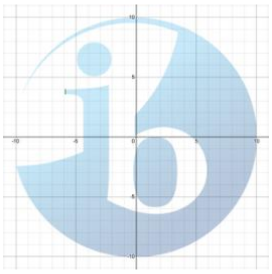

Figure 45 Vertical line $x = -5.9$ with range $R\in\{3.59 \le y \le 3.97\}$

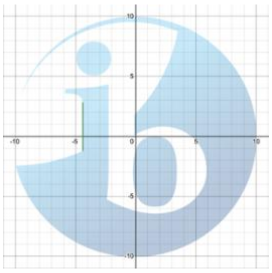

Figure 46 Vertical line $x = -4.4$ with range $R\in\{-1.2 \le y \le 2.8\}$

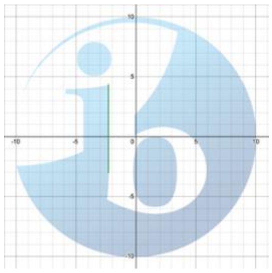

Figure 47 Vertical line $x = -2.3$ with range $R\in\{-3 \le y \le 4.34\}$

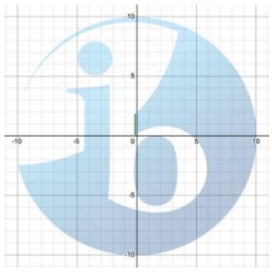

Figure 48 Vertical line $x = -0.15$ with range $R\in\{0.1 \le y \le 1.8\}$

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<u>Final logo</u>

Finally, I inserted all the equations onto one graph. I needed to readjust some of the domains and ranges so the lines would not overlap. The result can be seen in Figure 49. For better effect, I deleted the background so that the logo made up of functions can be seen more clearly.

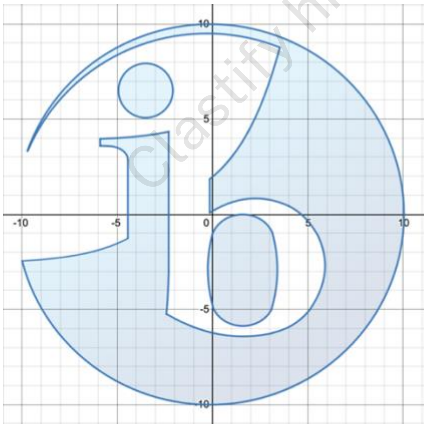

Figure 49 All equations graphed onto the IB logo

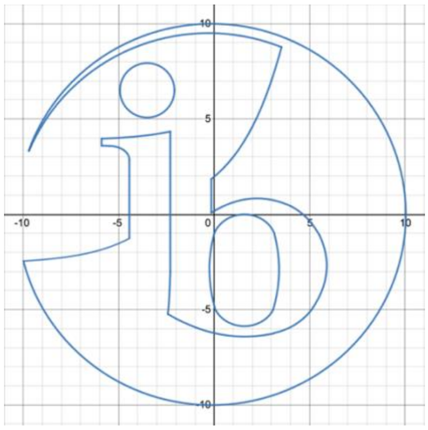

Figure 50 All equations graphed after the background was removed

The IB logo is defined by the following functions:

'Arch' of the b:

$y = 0.0737(x - 1.59)^2 - 6.42$ for D$\in\{-2.45 \le x \le 3.1\}$

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$y = -0.13(x - 2.25)^2 + 0.83$ for D∈ $\{-0.1 \leq x \leq -4\}$

$x = -0.15(y + 2.7)^2 - 5.9$ for R∈ $\{-5 \leq y \leq -0.9\}$

$y = 2.3^{(x-4.6)} - 6.5$ for D∈x $\{3.1 \leq x \leq 5\}$

$y = -2.3^{(x-4.6)} + 1$ for D∈x $\{4 \leq x \leq 5.4\}$

Exponential functions (with ellipse equation and a vertical line):

$y = 2.8^{0.38(x+5.3)} - 2.6$ for D∈ $\{-9.99 \leq x \leq -4.42\}$

$y = 0.56^{-0.73(x+1.6)}$ for D∈ $\{-0.16 \leq x \leq 3.52\}$

$y = 10^{0.21(x+0.7)} + 3.88$ for D∈ $\{-5.9 \leq x \leq -2.3\}$

$y = 4.2^{8.7(x+2.38)} - 5.8$ for D∈ $\{-5.24 \leq x \leq -2\}$

$y = -1.34^{12(x+4.3)} + 3.6$ for D∈ $\{-1.2 \leq x \leq 2.88\}$

$\frac{(x+5.498)^2}{1.05^2} + \frac{(y-2.87)^2}{0.714^2} = 1$ for D∈ $\{-5.5 \leq x\}$ and R∈ $\{2.88 \leq y\}$

$x = -4.445$ for R∈ $\{-1.2 \leq y \leq 2.88\}$

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Frame of the logo and the dot in 'i':

$\frac{(x+0.1)^2}{10.15^2} + \frac{(y+0.1)^2}{10^2} = 1$ for D∈ $\{-9.5 \leq x\}$ and R∈ $\{0 \leq y\}$

$\frac{(x+0.1)^2}{10.15^2} + \frac{(y+0.1)^2}{10^2} = 1$ for D∈ $\{-10 \leq x\}$ and R∈ $\{y \leq 0\}$

$\frac{(x+0.3)^2}{10.15^2} + \frac{(y+0.5)^2}{10^2} = 1$ for D∈ $\{-9.5 \leq x \leq 3.48\}$ and range to R∈ $\{0 \leq y\}$

$(x + 3.5)^2 + (y - 6.5)^2 = 1.43^2$

Circular shape inside the b letter:

$y = 0.07(x + 2.9)^2 - 0.25$ for R∈ $\{-5 \leq y \leq -1\}$

$y = -0.07(x + 2.9)^2 - 3.4$ for R∈ $\{-5 \leq y \leq -1\}$

$\frac{(x-1.58)^2}{1.65^2} + \frac{(y+1.5)^2}{1.5^2} = 1$ for R∈ $\{-1 \leq y\}$

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$\frac{(x-1.58)^2}{1.65^2}+\frac{(y+4.36)^2}{1.5^2}=1$ for R∈ {y ≤ –5}

Horizontal lines:

x = –5.9 for R∈ {3.59 ≤ y ≤ 3.97}

x = –4.4 for R∈ {–1.2 ≤ y ≤ 2.8}

x = –2.3 for R∈ {–3 ≤ y ≤ 4.34}

x = –0.15 for R∈ {0.1 ≤ y ≤ 1.8}

Evaluation

All things considered, I believe my exploration has been successful. When I first started my work, I doubted if my goal was not too difficult or extensive to solve within such page number restrictions. However, not only did I manage to graph the IB logo but also to learn new topics and use both typical mathematical methods as well as make use of the technology available to students like me. I believed my aim has been reached, and I can now affirm that it took 13 functions, 6 circle and ellipse equations as well as 5 vertical lines to graph the whole IB logo. During the process, I had to overcome some hardships and use problem-solving skills by changing my initial plan of the functions and adapting it to how it needed to be done. Although in fact there are parts of the logo that I have created with the functions I planned, there are some – especially the ellipse equation – that I did not expect to use. If there is any constructive criticism that I could give to this exploration it's that, maybe, some parts could be done with less but more fitted functions (ones that I haven't learned about yet).

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Running into different dilemmas while creating those functions and equations made me realize how much work the graphic design programs do for us. I cannot imagine how long would a person need to spend to design and create such a relatively simplistic logo from scratch, let

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alone some more complicated ones. Technology can be used in mathematics to aid progress and help achieve more than it was possible without it. Having tested out two methods in the paper I would like to reflect on them a little. When starting out, I did not intend to rely on technology as much as I finally did. However, it was only when I tried both, I realized how much more convenient it was. The method of calculating the equations from given points does have its advantages, it is very accurate and relatively easy. However, considering the downsides – it is repetitive and does not require a precise understanding of what the calculated values mean. It was a good way of starting the exploration by calculating some functions with it, however, it would not be convenient to continue it throughout the whole work. The second method, using transformations and the graphing calculator, helped me understand the dynamics of transformations on functions better. As an advantage, this one also allows for relatively quicker results, which is important with the number of equations I needed in this case. A prominent disadvantage could be the trial and error it sometimes takes to find specific values, however, that issue decreased after getting used to this method. I believe it was beneficial to use both methods, as it allowed me to compare them and highlight the importance of incorporating different mediums in work.

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<u>Application</u>

The completed leaves an open door for many further options. Firstly, since the IB logo can only be downloaded from the internet as a ".png" image format, with some skill my work could be transferred to a graphic design program and serve as a graphic itself. It would allow for resizing and editing and could be useful for creating high-resolution graphics, for example, huge posters. Secondly, I have seen the so-called "Desmos art" being colored on the inside. With a little research and effort, it would probably be possible to color the inside of the logo blue. Then, I believe the colored area could be calculated allowing for real-life use. Having the area, it would be possible to estimate, for example, how much ink is needed to print such a logo, as mentioned

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in the first application idea. Moreover, it would be possible to resize the logo accordingly to create a mural, for example in an IB school. Then, the graphic created from the logo would serve as a project and the area could be used to calculate the amount of wall paint (therefore price) of such a project. In conclusion, these are just ideas that came to my mind during the exploration, but I am sure the work can be used in other ways as well.

Bibliography

International Baccalaureate Organisation. *International Baccalaureate Logo*. https://www.ibo.org/digital-toolkit/logos-and-programme-models/. Accessed 11 Mar. 2022.

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