Buck-Boost converter

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# Introduction

Diagram, schematic

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Figure 1 Buck-Boost Converter

The use of a Buck-Boost converter is the simplest way to control the batter charging current for WPT systems. Under dynamic conditions the power supplied to the secondary side is expected to fluctuate; The use and design on a Buck-Boost converter is intended to manage these fluctuations to provide a more constant supply to vehicle batteries, in the interest of improving their lifespan as a result.

# Transfer Functions

First the system is analyzed mathematically to give equations for parameter design and transfer functions for each mode are found, later used to design for ripple current and PI controller values based on damping factor and response time.

## Buck mode

During Buck mode , where .

When S1 is on

|  |  |  |
| --- | --- | --- |
|  |  | (1a) |
|  |  | (1b) |

When S1 is off

|  |  |  |
| --- | --- | --- |
|  |  | (2a) |
|  |  | (2b) |
|  |  | (3) |
|  |  | (4) |

Where D is the duty cycle,

Current ripple on (Use to define ),

|  |  |  |
| --- | --- | --- |
|  |  | (5) |
| Confirming :    Figure 2 Confirming for Buck mode. | | |

Calculating average value over one period (T), from equations (1) and (2):

|  |  |  |
| --- | --- | --- |
|  |  | (6a) |
|  |  | (6b) |

Rearranging and simplifying (6) yields:

|  |  |  |
| --- | --- | --- |
|  |  | (7a) |
|  |  | (7b) |

Where D is the duty cycle , T is the period of the PWM signal and

Small signal modelling

|  |  |  |
| --- | --- | --- |
|  |  | (8a) |
|  |  | (8b) |

Assuming is constant and Laplace transform (8):

|  |  |  |
| --- | --- | --- |
|  |  | (9a) |
|  |  | (9b) |

Substituting yields:

|  |  |  |
| --- | --- | --- |
|  |  | (10) |
| The transfer function is in the general form for second order systems Making .  Chart, line chart  Description automatically generated  Figure 3 Comparing Buck-mode transfer function with simulation results for a step change D. | | |

## Boost mode

During Buck mode , where .

When S2 is on

|  |  |  |
| --- | --- | --- |
|  |  | (11a) |
|  |  | (11b) |

When S2 is off

|  |  |  |
| --- | --- | --- |
|  |  | (12a) |
|  |  | (12b) |
|  |  | (13) |
|  |  | (14) |

Current ripple on (Use to define ),

|  |  |  |
| --- | --- | --- |
|  |  | (15) |
|  |  |  |
| Confirming | | |
| Combining (11) and (12) | | |
|  |  | (16a) |
|  |  | (16b) |

Small signal modelling, assuming is constant and Laplace transform:

|  |  |  |
| --- | --- | --- |
|  |  | (17a) |
|  |  | (17b) |

Substituting yields:

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

As the transfer function depends on the duty cycle (D), it is confirmed at different values of D:

A picture containing graphical user interface

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Figure 4 Comparing Boost-mode transfer function with simulation results for a step change D.

## Buck-Boost mode

Assuming S1=S2

When S1 is turned on

|  |  |  |
| --- | --- | --- |
|  |  | (19a) |
|  |  | (19b) |

When S2 is turned off

|  |  |  |
| --- | --- | --- |
|  |  | (20a) |
|  |  | (20b) |
|  |  | (21) |
|  |  | (22) |

Where

Current Ripple on

|  |  |  |
| --- | --- | --- |
|  |  | (23) |

Confirming

A picture containing shape

Description automatically generated

|  |  |
| --- | --- |
|  | (24a) |
|  | (24b) |

Small-signal (assuming is constant)

|  |  |  |
| --- | --- | --- |
|  |  | (25a) |
|  |  | (25b) |
|  | S domain |  |
|  |  | (26a) |
|  |  | (26b) |
| Substitute |  | (27) |
|  |  | (28) |
|  |  | (29) |

Graphical user interface, chart

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## Summary of transfer functions

All operation modes are second order systems for which the general form is

Table 1 Summary of design equations for all modes

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Mode |  |  |  |  |  |
| Buck (Buck) |  |  |  |  |  |
| Boost |  |  |  |  |  |
| Buck-Boost |  |  |  |  |  |

# Parameter design

Designing for an acceptable , use equations from Table 1 to find . Then

The transfer function for all operation modes is in second order for which the general form is:

|  |  |  |
| --- | --- | --- |
|  |  | (30) |

And subsequently the response time:

|  |  |  |
| --- | --- | --- |
|  |  | (31) |

Designing for a desired time response and damping factor:

|  |  |  |
| --- | --- | --- |
|  |  | (32) |
|  |  | (33) |

Knowing the required natural frequency from (33) we can find finding the value of and subsequently .

Table 2 Finding Cb and Rb

|  |  |  |
| --- | --- | --- |
| Mode |  |  |
| Buck |  |  |
| Boost |  |  |
| Buck-Boost |  |  |

# PI-controller Design

**General Approach:**

For a PI controller, the transfer function is denoted as , where Kp and Ki are the proportional and integral gains respectively. For a known system (Gp) in a close loop set up, the overall transfer function is denoted at:

|  |  |  |
| --- | --- | --- |
|  |  | (34) |

Where is the controller transfer function and is the system transfer function, found in the previous step.

Substituting into (34) will give the close loop transfer function where the denominator (or pole position) is related to the damping ratio and natural frequency or response time.

## Buck -Mode

To simplify equation, take where the coefficients a,b and c are known from the previous stage (based on RLC and input voltage values). Following the process shown previously, the current transfer function can be found as: .

|  |  |
| --- | --- |
|  | (35) |

Where for . As the transfer function in (35) is a 3rd order system, we expect 3 poles, 2 symmetrical and another real pole at a value of . We can therefore use the denominator from (35) to achieve poles at desired values of .

|  |  |  |
| --- | --- | --- |
|  |  | (36) |
|  |  |  |
| Equating coefficients | | |
|  |  | (37) |
|  |  | (38) |
|  |  | (39) |

Designing for a desired , can be found for ~1.5% settling criteria:

|  |  |  |
| --- | --- | --- |
|  |  | (40) |

From here the PI values can be found by:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

Confirming this design process:

Chart, diagram

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Figure 5 Theoretical and simulation results for Buck-mode PI-controller design.

## Boost Mode

Transfer function for the current can be derived as:

|  |  |  |
| --- | --- | --- |
|  |  | (41) |

Following the same design process from eq. (35-40), we get:

|  |  |  |
| --- | --- | --- |
|  |  | (42) |
|  |  | (43) |
|  |  | (44) |
| For a settling criteria of ~1.5%: | | |
|  |  |  |
|  |  |  |
|  |  |  |

Confirming with simulation:

Chart, line chart

Description automatically generated

Figure 6 Theoretical and simulation results for Boost-mode PI-controller design.

## Buck-Boost mode

|  |
| --- |
|  |