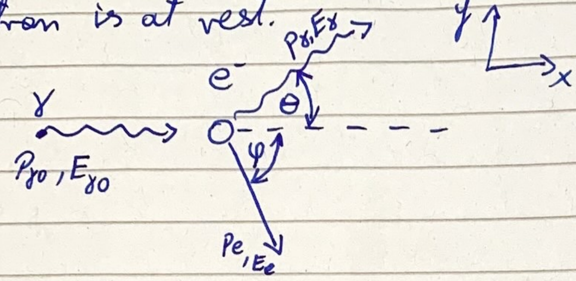
1) Derivation of eq. 1 - describing Compton effect

- electron is at rest.



initial photon momentum : $p_{\gamma 0} = \frac{E_{\gamma 0}}{c}$ initial photon Energy : $E_{\gamma 0}$

electron rest mass energy : $m_e c^2$

1° Energy conservation :

$$E_{\gamma 0} + m_e c^2 = E_\gamma + \underbrace{E_e}_{}$$ → energy of outgoing electron

$$= E_\gamma + \sqrt{m_e^2 c^4 + p_e^2 c^2} \quad \Rightarrow \quad E_\gamma = E_{\gamma 0} + m_e c^2 - \sqrt{m_e^2 c^4 + p_e^2 c^2}$$

$$\Rightarrow (E_{\gamma 0} - E_\gamma + m_e c^2)^2 = m_e^2 c^4 + p_e^2 c^2$$

$$\vec{P}_{\gamma 0} + \underbrace{\vec{P}_{e 0}}_{=0} = \vec{P}_\gamma + \vec{P}_e \quad \Rightarrow$$

X axis :

$$P_{\gamma x} + P_{ex} = p_{\gamma 0}$$

y axis:

$$P_{\gamma y} + P_{ey} = 0 \qquad P_{\gamma y} = -P_{ey}$$

$$P_{\gamma x} = \cos(\theta) \cdot P_\gamma \quad \rightarrow \quad P_{ex} =$$

$$(-\vec{P}_\gamma + \vec{P}_{\gamma 0})^2 = \vec{P}_e^{\,2} \quad \Rightarrow \quad P_\gamma^2 + P_{\gamma 0}^2 - 2 P_\gamma P_{\gamma 0} \cos\theta = p_e^2$$

/ multiply by c^2

use $p_\gamma c = E_\gamma$

$$(*) \quad E_\gamma^2 + E_{\gamma 0}^2 - 2 E_\gamma E_{\gamma 0} \cos\theta = p_e^2 c^2$$

plug (*) into

$$(E_{\gamma 0} - E_\gamma + m_e c^2)^2 = m_e^2 c^4 + E_\gamma^2 + E_{\gamma 0}^2 - 2E_\gamma E_{\gamma 0}\cos\theta$$

$$\cancel{E_{\gamma 0}^2} + \cancel{E_\gamma^2} + \cancel{m_e^2 c^4} + \cancel{2}E_{\gamma 0} m_e c^2 - \cancel{2}E_\gamma m_e c^2 - \cancel{2}E_{\gamma 0} E_\gamma = \cancel{E_\gamma^2} + \cancel{E_{\gamma 0}^2} - \cancel{2}E_\gamma E_{\gamma 0}\cos\theta + \cancel{m_e^2 c^4}$$

$$m_e c^2 (E_{\gamma 0} - E_\gamma) - E_{\gamma 0} E_\gamma = -E_{\gamma 0} E_\gamma \cos(\theta)$$

$$m_e c^2 (E_{\gamma 0} - E_\gamma) = (1-\cos(\theta)) E_{\gamma 0} E_\gamma$$ // devide by $(1-\cos(\theta))E_{\gamma 0}$

$$m_e c^2 E_{\gamma 0} = E_\gamma (m_e c^2 + (1-\cos\theta) E_{\gamma 0})$$ // $m_e c^2$

$$E_{\gamma 0} = E_\gamma \left(1 + \frac{E_{\gamma 0}}{m_e c^2}(1-\cos(\theta))\right)$$

$$E_\gamma(\theta) = \frac{E_{\gamma 0}}{1 + \frac{E_{\gamma 0}}{m_e c^2}(1-\cos(\theta))}$$