Correlation and CAPM Analysis

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Objective

The aim of this analysis is to examine the relationship between the returns of a high-risk asset (Bitcoin), a relatively safe asset (bonds), and a market benchmark (S&P 500 index). Specifically, this report explores the correlation between these assets and the market and performs a Capital Asset Pricing Model (CAPM) regression on Bitcoin to evaluate its systematic risk (β) and excess returns (α).

Correlation Analysis

The table below presents the correlation matrix for the returns of bonds, Bitcoin, and the S&P 500 index:

	Bond Return	**Bitcoin Return**	**S&P 500 Return**
Bond Return	1.000000	-0.000278	0.242216
Bitcoin Return	-0.000278	1.000000	0.259746
S&P 500 Return	0.242216	0.259746	1.000000

The correlation analysis reveals several key insights:

- **Bitcoin Return and S&P 500 Return**: The correlation between Bitcoin returns and S&P 500 returns is **0.2597**, indicating a weak positive relationship. This suggests that Bitcoin does not strongly follow the general market trends, although they do move together to a limited extent.
- **Bond Return and S&P 500 Return**: The correlation between bond returns and S&P 500 returns is **0.2422**, also showing a weak positive relationship. Bonds, considered safer assets, tend to exhibit less correlation with the market than high-risk assets.
- **Bond Return and Bitcoin Return**: The correlation between bond returns and Bitcoin returns is **-0.00028**, essentially zero, suggesting that there is almost no relationship between the returns of these two vastly different asset classes. This indicates that Bitcoin and bonds behave independently from each other, aligning with traditional expectations of portfolio diversification.

These correlations confirm the expected behaviour that riskier assets, like Bitcoin, have limited alignment with traditional market movements and that bonds, as safe assets, are only moderately correlated with the market index.

CAPM Regression Analysis

The CAPM regression was conducted to evaluate the systematic risk (β) and excess returns (α) for Bitcoin, with the market proxy being the S&P 500 index. The regression results are summarized as follows:

- **Alpha (α): 0.0002**
- **Beta (β): 1.0860**

The equation for the regression model is as follows:

$$R_{Bitcoin} - R_f = 0.0002 + 1.0860 \times (R_{Market} - R_f) + \epsilon$$

Where:

- $R_{Bitcoin}$ is the return on Bitcoin,
- R_{Market} is the return on the S&P 500,
- R_f is the risk-free rate, assumed to be 0 for simplicity,
- ϵ is the idiosyncratic error term.

The results show a **β of 1.0860**, which indicates that Bitcoin is **more volatile than the market**. A β greater than 1 means that Bitcoin exhibits greater sensitivity to market movements, confirming that it is a high-risk asset. In this case, a **1% change in the market's return is associated with an approximately 1.086% change in Bitcoin's return**.

The **α (alpha)** value of **0.0002** suggests that Bitcoin does not offer significant excess returns beyond what is predicted by market movements. According to the CAPM theory, alpha should be zero, as excess returns are typically not expected in an efficient market. However, the small positive alpha observed here is statistically insignificant (p-value = 0.761), meaning that Bitcoin does not provide significant excess returns compared to the market's risk-adjusted return.

The regression model has an **R-squared value of 0.417**, indicating that approximately 41.7% of the variation in Bitcoin's returns can be explained by the market returns (S&P 500). This moderate R-squared value suggests that while the market explains some of Bitcoin's returns, there is a substantial portion of Bitcoin's movements driven by factors unrelated to the overall market.

Interpretation in the Context of CAPM

Under the strict form of CAPM, the alpha should be zero, and the beta provides a measure of systematic risk. In this case:

- The **β** of 1.0860 confirms that Bitcoin is a risky asset, exhibiting higher volatility and a greater risk-return trade-off compared to the market.
- The **α** of 0.0002 is close to zero, indicating that Bitcoin's returns are well-explained by market movements, with no significant excess return beyond what can be expected from its level of risk.

These results are consistent with CAPM theory, which suggests that the risk of an asset is proportional to its expected return. Since Bitcoin has a β greater than 1, it implies that investors can expect higher returns from Bitcoin, but only in exchange for greater risk.

Conclusion

The analysis demonstrates that Bitcoin, as a high-risk asset, has a weak correlation with the overall stock market (S&P 500) and is almost uncorrelated with bonds. The CAPM regression confirms that Bitcoin is significantly more volatile than the market, with a β greater than 1, reinforcing its position as a high-risk, high-return asset. The alpha value, close to zero, indicates no significant excess returns beyond market movements, aligning with the expectations of CAPM. These findings emphasize the importance of understanding the risk-return trade-off when investing in assets like Bitcoin and the value of diversification across asset classes like bonds to manage portfolio risk.