Jakub Piotrowczyk | 252885 | mod(85,4) = 3 | MUD-LAB03

$$4\dddot{x} + 13\ddot{x} + 3x = 2u$$

1° dla $u(t)=1$, $\dot{x}(0)=0$, $x(0)=2$

Rozwiązanie swobodne:

$$4\ddot{x}_s + 13\dot{x}_s + 3x_s = 0$$

$$x_s = Ae^{zt}$$

$$\dot{x}_s = zAe^{zt}$$

$$\ddot{x}_s = z^2Ae^{zt}$$

$$4z^2Ae^{zt} + 13zAe^{zt} + 3Ae^{zt} = 0 /:Ae^{zt}$$

$$4z^2 + 13z + 3 = 0$$

$$\Delta = 169 - 48 = 121$$

$$\sqrt{\Delta} = 11$$

$$\begin{cases} z_1 = \frac{-13+11}{2\cdot4} = \frac{-2}{8} = -\frac{1}{4} \\ z_2 = \frac{-13-11}{2\cdot4} = \frac{-24}{8} = -3 \end{cases}$$

$$\begin{cases} x_{s1} = A_1e^{-\frac{1}{4}t} \\ x_{s2} = A_2e^{-3t} \end{cases}$$

$$x_s = x_{s1} + x_{s2}$$

$$x_s = A_1e^{-\frac{1}{4}t} + A_2e^{-3t}$$

Rozwiązanie wymuszone:

$$4\ddot{x}_w + 13\dot{x}_w + 3x_w = 2u = 2\cdot1 = 2$$

$$2;\ 2' = 0$$

$$x_w = C_1\cdot2 + C_2\cdot0 = 2C_1$$

$$\dot{x}_w = 0$$

$$\ddot{x}_w = 0$$

$$4\cdot0 + 13\cdot0 + 3\cdot2C_1 = 2$$

$$C_1 = \frac{1}{3}$$

$$x_w = 2C_1 = 2\cdot\frac{1}{3} = \frac{2}{3}$$

Rozwiązanie ogólne:

$$x = x_w + x_s = A_1e^{-\frac{1}{4}t} + A_2e^{-3t} + \frac{2}{3}$$

Rozwiązanie szczególne:

$$\dot{x} = -\frac{1}{4}A_1e^{-\frac{1}{4}t} - 3A_2e^{-3t} + 0$$

$$\dot{x} = -\frac{1}{4}A_1e^{-\frac{1}{4}t} - 3A_2e^{-3t}$$

$$\begin{cases} x(0) = 2 = A_1e^0 + A_2e^0 + \frac{2}{3} \\ \dot{x}(0) = 0 = -\frac{1}{4}A_1e^0 - 3A_2e^0 \end{cases}$$

$$\begin{cases} 2 = A_1 + A_2 + \frac{2}{3} \Rightarrow \frac{4}{3} = A_1 + A_2^{*1} / \cdot 3 \\ 0 = -\frac{1}{4}A_1 - 3A_2 \end{cases}$$

$$\begin{cases} 4 = 3A_1 + 3A_2 \\ 0 = -\frac{1}{4}A_1 - 3A_2 \end{cases} \Big| +$$

$$4 = \frac{11}{4}A_1$$

$$16 = 11A_1$$

$$A_1 = \frac{16}{11}$$

$$*_1\ \frac{4}{3} = A_1 + A_2$$

$$\frac{4}{3} = \frac{16}{11} + A_2$$

$$A_2 = \frac{44}{33} - \frac{16}{11}$$

$$A_2 = \frac{44}{33} - \frac{48}{33}$$

$$A_2 = \frac{-4}{33}$$

$$\boxed{x(t) = \frac{16}{11}e^{-\frac{1}{4}t} - \frac{4}{33}e^{-3t} + \frac{2}{3}}$$