

MATH 373 Project 1 (100 points)

Due on 10:50 AM of Feb 14, 2025

**Please submit your report, source codes, and environment which
TA can run your codes**

Here we consider two numerical differentiation formulas,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \quad (1)$$

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \quad (2)$$

Study and compare the two formulas for $f(x) = \sin x$ and $x = 1$ as $h \rightarrow 0$.

- 1) Find truncation error bounds for (1) and (2).
- 2) Find rounding error bounds for (1) and (2).
- 3) On the two graphs for (1) and (2), plot truncation error bound, rounding error bound and total error using a log-scale; the axes in the plot should be $\log_{10} |\text{error}|$ versus $\log_{10} h$ as $h = 10^{-k}$, $k = 1, \dots, 16$.
- 4) Discuss the optimal values h and the relations between errors.
- 5) Compare (1) and (2) for your conclusion.