A6-Assignment-Ch4.1

Section 4.1: 6, 7.b, and 10.a due

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6. Find the polynomial p of least degree that takes these values:

- $p(0) = 2$
- $p(2) = 4$
- $p(3) = -4$
- $p(5) = 82$
  Use divided differences to get the correct polynomial. It is *not* necessary to write the polynomial in the standard form $a_0 + a_1 x + a_2 x^2 + \cdots$.

Theory Recap

- Divided differences provide a way to recursively compute the coefficients of the interpolation polynomial.
- The Newton form of the interpolation polynomial is:
  - $p(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots$

Working Steps

- Step 1. List the data points:
  - $x_0 = 0,\ f(x_0) = 2$
  - $x_1 = 2,\ f(x_1) = 4$
  - $x_2 = 3,\ f(x_2) = -4$
  - $x_3 = 5,\ f(x_3) = 82$
- Step 2. Calc the first divided differences:
  - $f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{4-2}{2-0} = \frac{2}{2} = 1$
  - $f[x_1, x_2] = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{-4-4}{3-2} = \frac{-8}{1} = -8$
  - $f[x_2, x_3] = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = \frac{82-(-4)}{5-3} = \frac{86}{2} = 43$
- Step 3. Calc the second divided differences:

- $f[x_0, x_1, x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{-8-1}{3-0} = \frac{-9}{3} = -3$
- $f[x_1, x_2, x_3] = \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1} = \frac{43-(-8)}{5-2} = \frac{51}{3} = 17$

- Step 4. Calc the third divided difference:
  - $f[x_0, x_1, x_2, x_3] = \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0} = \frac{17-(-3)}{5-0} = \frac{20}{5} = 4$
- Step 5. Write the Newton interpolation polynomial:
  - The general form is:
    - $p(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x$
  - Substituting the computed values:
    - $p(x) = 2 + 1\,(x - 0) - 3\,(x - 0)(x - 2) + 4\,(x - 0)(x - 2)(x - 3)$
- Final Answer for Problem 6:
  - $p(x) = 2 + x - 3(x)(x - 2) + 4(x)(x - 2)(x - 3)$

Divided Difference Table

x	$f[,]$	$f[,,,]$	$f[,,,,,]$	$f[,,,,,,,]$
0	2			
2	4	1		
3	-4	-8	-3	
5	82	43	17	4

7.b.

Complete the following divided-difference tables, and use them to obtain polynomials of degree 3 that interpolate the function values indicated:

x	$f[\,]$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,,\,]$
-1	2			
1	-4			
3	46	53.5		
4	99.5			

Write the final polynomials in a form most efficient for computing.

Theory Recap

- As before, the Newton form is:
  - $p(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0$
- Each level of divided differences is computed using:
  - $f[x_i, x_{i+1}, \ldots, x_{i+k}] = \frac{f[x_{i+1},\ldots,x_{i+k}] - f[x_i,\ldots,x_{i+k-1}]}{x_{i+k} - x_i}$

Working Steps

- **Step 1. List the data points:**
  - $x_0 = -1$, \quad $f(x_0) = 2$
  - $x_1 = 1$, \quad $f(x_1) = -4$
  - $x_2 = 3$, \quad $f(x_2) = 46$
  - $x_3 = 4$, \quad $f(x_3) = 99.5$
- **Step 2. Calc the first divided differences:**
  - $f[x_0, x_1] = \frac{-4-2}{1-(-1)} = \frac{-6}{2} = -3$
  - $f[x_1, x_2] = \frac{46-(-4)}{3-1} = \frac{50}{2} = 25$
  - $f[x_2, x_3] = \frac{99.5-46}{4-3} = \frac{53.5}{1} = 53.5$
    - (Note: The table already gives $f[x_2, x_3] = 53.5$)
- **Step 3. Calc the second divided differences:**
  - $f[x_0, x_1, x_2] = \frac{25-(-3)}{3-(-1)} = \frac{28}{4} = 7$
  - $f[x_1, x_2, x_3] = \frac{53.5-25}{4-1} = \frac{28.5}{3} = 9.5$
- **Step 4. Calc the third divided difference:**
  - $f[x_0, x_1, x_2, x_3] = \frac{9.5-7}{4-(-1)} = \frac{2.5}{5} = 0.5$
- **Step 5. Write the Newton interpolation polynomial:**
  - Using $x_0 = -1$, the polynomial is:
    - $p(x) = f[x_0] + f[x_0, x_1](x - (-1)) + f[x_0, x_1, x_2](x - (-1))(x - 1) + f[x_0, x_1, x_2, x$
  - Substitute the values:
    - $p(x) = 2 - 3\,(x + 1) + 7\,(x + 1)(x - 1) + 0.5\,(x + 1)(x - 1)(x - 3)$
- **Final Answer for Problem 7 (b):**
  - $p(x) = 2 - 3(x + 1) + 7(x + 1)(x - 1) + 0.5(x + 1)(x - 1)(x - 3)$

Completed Divided Difference Table

x	$f[,]$	$f[,,,]$	$f[,,,,,]$	$f[,,,,,,,]$
-1	2	-3	7	0.5
1	-4	25	9.5	
3	46	53.5		
4	99.5			

10.a. Construct Newton's interpolation polynomial for the data shown.

x	0	2	3	4
y	7	11	28	63

Theory Recap

- The Newton interpolation polynomial is built as:
  - $p(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + f[x_0, x_1, x_2, x_3](x - x_0$
- This method uses the computed divided differences to progressively build the polynomial.

Working Steps

- Step 1. List the data points:
  - $x_0 = 0,\ f(x_0) = 7$
  - $x_1 = 2,\ f(x_1) = 11$
  - $x_2 = 3,\ f(x_2) = 28$
  - $x_3 = 4,\ f(x_3) = 63$
- Step 2. Calc the first divided differences:
  - $f[x_0, x_1] = \frac{11-7}{2-0} = \frac{4}{2} = 2$
  - $f[x_1, x_2] = \frac{28-11}{3-2} = \frac{17}{1} = 17$
  - $f[x_2, x_3] = \frac{63-28}{4-3} = \frac{35}{1} = 35$
- Step 3. Calc the second divided differences:

- $f[x_0, x_1, x_2] = \frac{17-2}{3-0} = \frac{15}{3} = 5$
- $f[x_1, x_2, x_3] = \frac{35-17}{4-2} = \frac{18}{2} = 9$

- Step 4. Calc the third divided difference:
  - $f[x_0, x_1, x_2, x_3] = \frac{9-5}{4-0} = \frac{4}{4} = 1$
- Step 5. Write the Newton interpolation polynomial:
  - Since $x_0 = 0$, the polynomial is:
    - $p(x) = f(x_0) + f[x_0, x_1](x-0) + f[x_0, x_1, x_2](x-0)(x-2) + f[x_0, x_1, x_2, x_3](x-$
  - Substitute the values:
    - $p(x) = 7 + 2x + 5x(x-2) + 1 \cdot x(x-2)(x-3)$
- Final Answer for Problem 10 (a):
  - $p(x) = 7 + 2x + 5x(x-2) + x(x-2)(x-3)$

Divided Difference Table

x	$f[,]$	$f[,,,]$	$f[,,,,,]$	$f[,,,,,,,]$
0	7	2	5	1
2	11	17	9	
3	28	35		
4	63			

Newton Interpolating Polynomial

- $p(x) = 7 + 2x + 5x(x-2) + x(x-2)(x-3)$