A5-Assignment-CH3.3

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2. If we use the secant method on $f(x) = x^3 - 2x + 2$ starting with $x_0 = 0$ and $x_1 = 1$, what is x_2?

1. First, we'll calculate $f(x_0)$ and $f(x_1)$:
   - $f(0) \quad = 0^3 - 2(0) + 2 \quad = 0 - 0 + 2 \quad = 2$
   - $f(1) = 1^3 - 2(1) + 2 = 1$
2. And i'll use the secant formula:
   $X_2 = x_1 - f(x_1) \cdot \frac{x_1 - x_0}{f(x_1) - f(x_0)} = 1 - 1 \cdot \frac{1-0}{1-2} = 1 + 1 = 2$

> ✓ Success
>
> $x_2 = 2$

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11. Show that if the iterates in Newton's method converge to a point r for which $f'(r) \neq 0$, then $f(r) = 0$. Establish the same assertion for the secant method.

Hint: In the latter, the Mean-Value Theorem of Differential Calculus is useful. This is the case $n = 0$ in Taylor's Theorem.

Newton's Method:

- Assume $x_n \to r$. The iteration is:
  $X_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- Taking the limit $n \to \infty$:
  $R = r - \frac{f(r)}{f'(r)} \implies 0 = -\frac{f(r)}{f'(r)} \implies f(r) = 0$

Secant Method:

- Assume $x_n \to r$. The iteration is:
  $X_{n+1} = x_n - f(x_n) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$
- By the Mean Value Theorem, $f(x_n) - f(x_{n-1}) = f'(c)(x_n - x_{n-1})$ for some c.
- As $n \to \infty$, $c \to r$, so:
  $X_{n+1} = x_n - \frac{f(x_n)}{f'(r)}$
- Taking the limit $n \to \infty$:
  $R = r - \frac{f(r)}{f'(r)} \implies f(r) = 0$

> ✓ Success
>
> For both methods, if $x_n \to r$ and $f'(r) \neq 0$, then $f(r) = 0$.

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13 b. Test the sequence $x_n = 2^{-n}$ for different types of convergence (i.e., linear, super linear, or quadratic), where $n = 1, 2, 3, \ldots.$

Linear Convergence:

- Lemme check if $|x_{n+1}| \leq C|x_n|$ for $0 < C < 1$:
- $|x_{n+1}| = 2^{-(n+1)}$
- $= \frac{1}{2} \cdot 2^{-n}$
- $= \frac{1}{2}|x_n|$
- Well, $C = \frac{1}{2}$, so convergence is linear.
  Super linear
- We need $\frac{|x_{n+1}|}{|x_n|} \to 0.$
- But $\frac{|x_{n+1}|}{|x_n|} = \frac{1}{2}$ (constant)
- So, not superlinear

Quadratic

$|x_{n+1}| \leq C|x_n|^2$

- $\frac{|x_{n+1}|}{|x_n|^2}$

- $= \frac{2^{-(n+1)}}{(2^{-n})^2}$
- $= 2^{n-1} \to \infty$
- It is not bounded, so it is not super linear

✓ **Success**

The sequence $x_n = 2^{-n}$ converges linearly to 0.
But it isn't super linear or quadratic