A8-Assignment-CH5.1

Problem 11 - Obtain an upper bound on the absolute error when we compute $\int_0^6 \sin(x^2), dx$ by means of the composite trapezoid rule using 101 equally spaced points.

1. Number of subintervals and step size:
    - 101 points
    - Gives us 100 subintervals ($n = 100$).
    - So, our step size will be
        - Step size $h = \frac{6-0}{100}$
        - $= 0.06$.
2. Error formula for composite trapezoid rule:
    - Absolute error bound: $|E| \leq \frac{1}{12}(b-a)h^2 \max_{\xi \in [a,b]} |f''(\xi)|$.
3. Second derivative of $f(x) = \sin(x^2)$:
    - $f''(x) = 2\cos(x^2) - 4x^2 \sin(x^2)$.
    - Max of $|f''(x)|$ on $[0, 6]$:
        - Approximate $\max |f''(x)| \leq 2 + 4(6)^2 = 146$.
4. Compute the error bound:
    - $|E| \leq \frac{1}{12} \times 6 \times (0.06)^2 \times 146 = 0.2628$.

Aaand the answer is 0.2628

Problem 17 - Compute two approximate values for $\int_1^2 \frac{dx}{x^2}$ using $h = \frac{1}{2}$ with the composite trapezoid rule.

1. First, lets get the step size and subintervals:
    - $h = \frac{1}{2}$
    - partition points will be: 1, 1.5, 2.
2. Composite trapezoid rule formula:
    - $T = \frac{h}{2}[f(1) + 2f(1.5) + f(2)]$.

- Lets evaluate $f(x) = \frac{1}{x^2}$ at partition points (using scientific calculator):
    - $f(1) = 1$
    - $f(1.5) = \frac{4}{9}$
    - $f(2) = \frac{1}{4}$
- Compute the approximation:
    - $T = \frac{1/2}{2}\left[1 + 2\left(\frac{4}{9}\right) + \frac{1}{4}\right]$
    - $= \frac{77}{144} \approx 0.5347$.

Problem 18 - Consider $\int_1^2 \frac{dx}{x^3}$. What is the result of using the composite trapezoid rule with the partition points - 1, $\frac{3}{2}$, and 2?

1. Partition points:
    - Subintervals: $[1, \frac{3}{2}]$ and $[\frac{3}{2}, 2]$.
2. Composite trapezoid rule formula:
    - $T = \frac{h}{2}\left[f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right]$, where $h = \frac{1}{2}$.
3. **Evaluate $f(x) = \frac{1}{x^3}$ at partition points:
4. Still using the scientific calculator since it makes it faster**
    - $f(1) = 1$
    - $f\left(\frac{3}{2}\right) = \frac{8}{27}$
    - $f(2) = \frac{1}{8}$
5. Compute the approximation:
    - $T = \frac{1/2}{2}\left[1 + 2\left(\frac{8}{27}\right) + \frac{1}{8}\right]$
    - $= \frac{371}{864} \approx 0.4294.$
