Counterfactuals and Directed acyclic graphs in epidemiology

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- We write the causal effect we're interested in as $\mathrm{E}[Y^{a=1} - Y^{a=0}]$

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- We can then use DAGs to explore the causal relationship between variables using our observational data

Directed acyclic graphs (DAGs)

george davey smith
@mendel_random

Following

Epidemiologists rejoice! "'the task of selecting an appropriate set of covariates to control for confounding has been reduced to a simple "roadblocks" puzzle manageable by a simple algorithm" arxiv.org/abs/1801.04016 and

Comments on: The tale wagged by the DAG | International ...
I am grateful to the editors for the opportunity to comment on Nancy Krieger and George Davey Smith's article, 'The tale wagged by the DAG', which appeared in t
academic.oup.com

This is true! But the DAG has to be correctly specified which is the hard (impossible) part.

Rules of DAGs

- Show conditioning on a node with a box

$$
\begin{tikzcd}
U \arrow[d] \arrow[drr] & & \\
A \arrow[rr] & & Y
\end{tikzcd}
$$

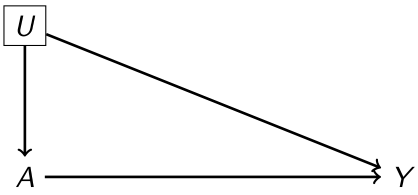
Rules of DAGs

- Show conditioning on a node with a box
- The effect of A on Y is confounded because\
  $E[Y|A=1] - E[Y|A=0] \neq E[Y^{a=1}] - E[Y^{a=0}]$

$$\begin{tikzcd}
U \arrow[d] \arrow[drr] & & \\
A \arrow[rr] & & Y
\end{tikzcd}$$

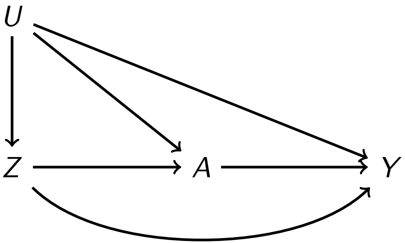
Rules of DAGs

- Show conditioning on a node with a box



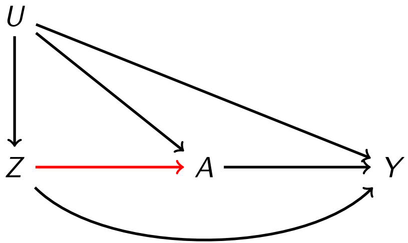
Showing the IV assumptions in a DAG

- This DAG makes the least amount of assumptions about the observed data. Every node is connected to every other



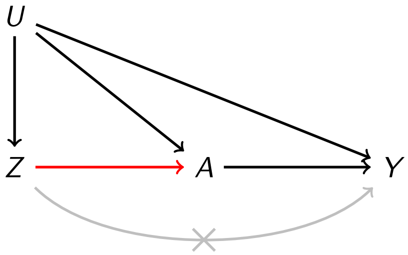
Showing the IV assumptions in a DAG

- The relevance assumption is represented by the red arrow, showing that there is a relationship between Z and A



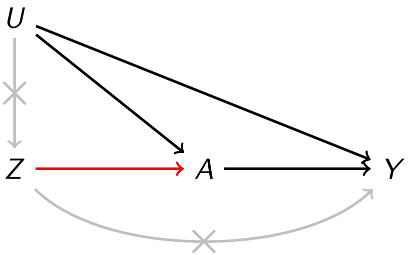
Showing the IV assumptions in a DAG

- The exclusion restriction (ER) assumption assumes that the direct edge between Z and A is not present



Showing the IV assumptions in a DAG

- The exchangeability assumption assumes that Z is not confounded with Y



Showing the IV assumptions in a DAG

- With our three assumptions, we end up with our classic IV DAG

$$
\begin{tikzcd}
 & U \arrow[d] \arrow[dr] & \\
Z \arrow[r] & A \arrow[r] & Y
\end{tikzcd}
$$

Showing the IV assumptions in a DAG

- With our three assumptions, we end up with our classic IV DAG
- Note: monontonicity and homogeneity cannot be drawn on a DAG because they are parametric concepts

$$
\begin{tikzcd}
 & U \arrow[d] \arrow[dr] & \\
Z \arrow[r, red] & A \arrow[r] & Y
\end{tikzcd}
$$

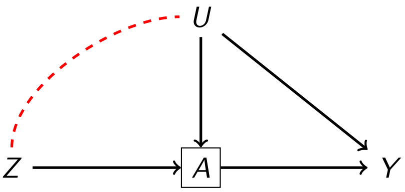
Conditioning on exposure in IV analyses

- Sometimes people like to stratify or restrict on exposure

$$\begin{tikzcd}
 & U \arrow[d] \arrow[dr] & \\
Z \arrow[r] & \boxed{A} \arrow[r] & Y
\end{tikzcd}$$

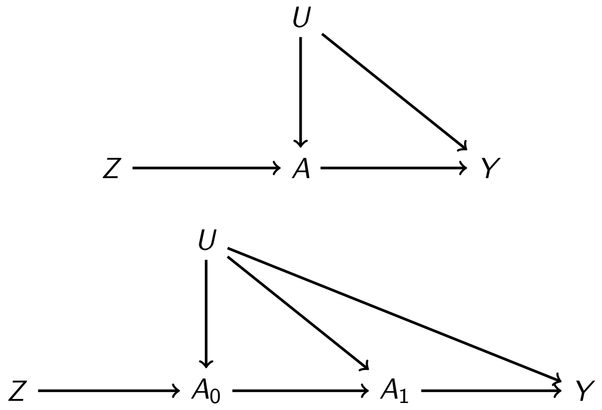
Conditioning on exposure in IV analyses

- But we create a new association between Z and U therefore violating the exchangeability assumption



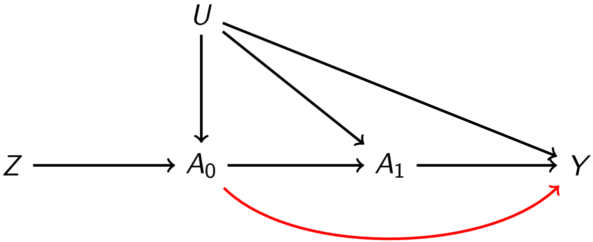
Using DAGs to think about IV longitudinally

- We can depict a time-varying exposure by splitting the A node into separate nodes for each time point



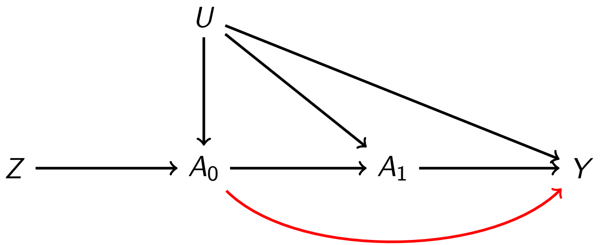
Using DAGs to think about IV longitudinally

- We have assumed that A_0 does not have a direct effect on Y



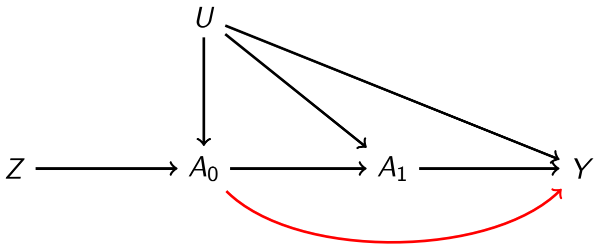
Using DAGs to think about IV longitudinally

- We have assumed that A_0 does not have a direct effect on Y
- If A_0 does affect Y then we can no longer estimate the effect of A_1 ony Y



Using DAGs to think about IV longitudinally

- We have assumed that A_0 does not have a direct effect on Y
- If A_0 does affect Y then we can no longer estimate the effect of A_1 ony Y
- This is because, although the ER holds for A as a whole, it does not hold for A_1



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- We have assumed that A_0 does not have a direct effect on Y
- If A_0 does affect Y then we can no longer estimate the effect of A_1 ony Y
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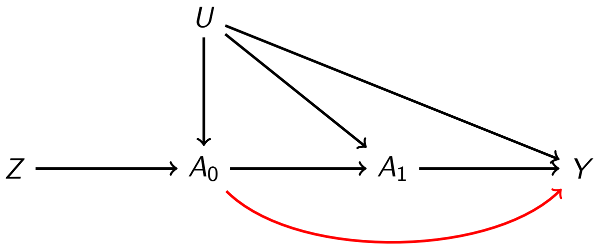
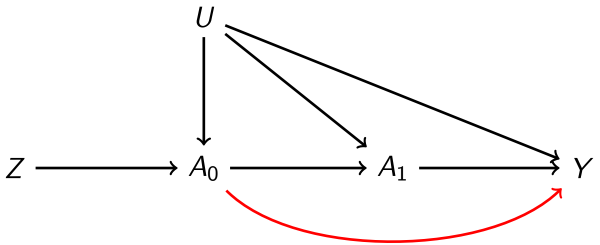
- So if we measure A_0 we're alright?



Using DAGs to think about IV longitudinally

- We have also assumed that Z does not have a direct effect on A_1



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- If Z does affect A_1 then we can no longer estimate the effect of A_0 ony Y

$$
\begin{tikzcd}
 & U \arrow[d] \arrow[dr] \arrow[drr] & & \\
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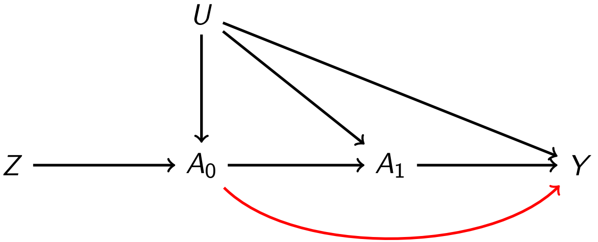

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- If Z does affect A_1 then we can no longer estimate the effect of A_0 ony Y
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- All this to illustrate that, in contexts such as Mendelian randomization, important thought must be given to how exposures change longitudinally

$$\begin{tikzcd}
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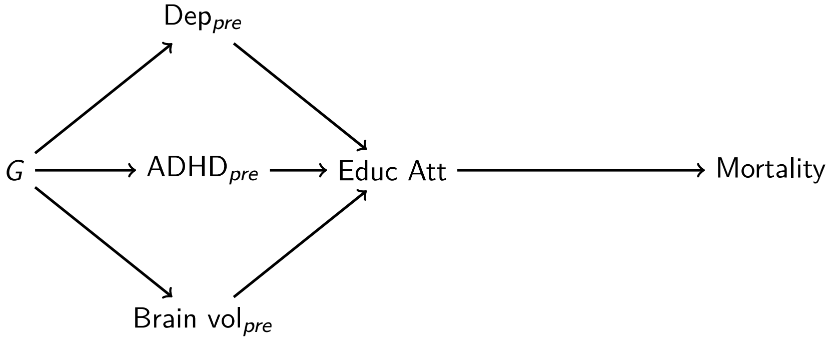
Z \arrow[r] & A_0 \arrow[r] \arrow[rr, bend right, red] & A_1 \arrow[r] & Y
\end{tikzcd}$$

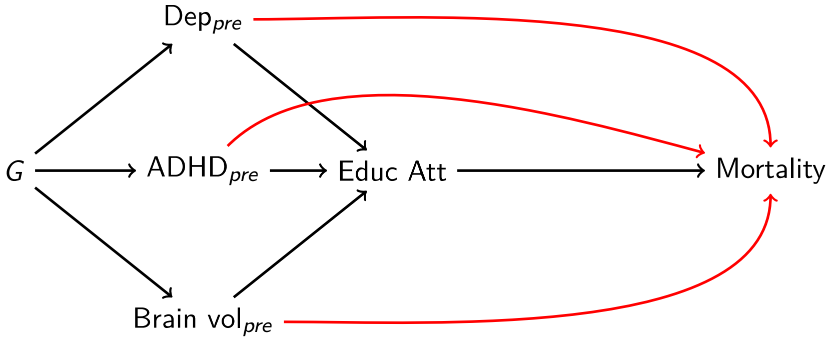
Using DAGs to think about Mendelian randomization

- DAG for a MR study of the effect of educational attainment on mortality



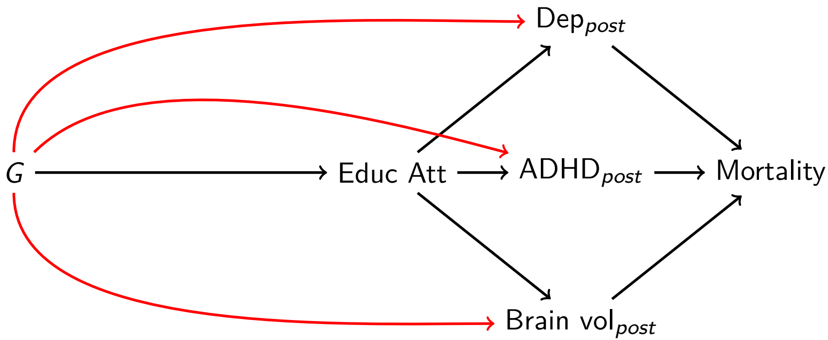
Using DAGs to think about Mendelian randomization

- But what if the intermediates between G and education attainment have a direct effect on mortality?



Using DAGs to think about Mendelian randomization

- Or what if G has a direct affcect on the intermediates after educational attainment



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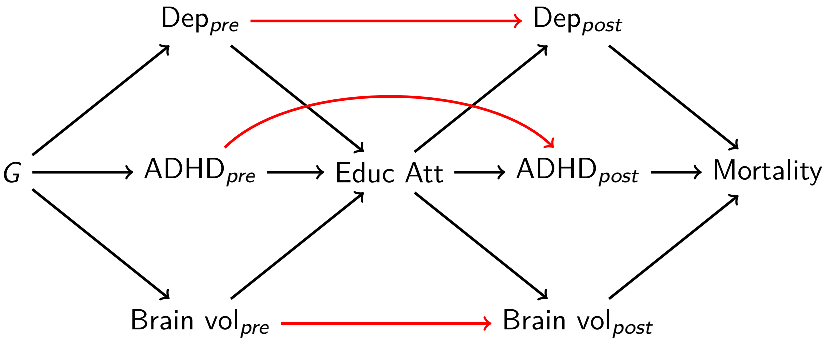

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