

# Face Recognition From Single - image Per Person - A survey

## One Sample Per Person Problem

Given a stored database of faces, the goal is to identify a person from the database later in time in any different and unpredictable poses, lighting, etc. from just one image.

### Advantages

1 → easy to collect samples  
↳ Smart cards, Passport, Driver's license. etc.

2 → Save Storage Cost

3 → Save Computational Cost.

Objective:- How to improve performance against extremely small sample size.



# Face Recognition Methods

## Holistic methods

→ Identify face using whole face image as input.

## Local methods

→ use local facial features for recognition.

## Hybrid methods

→ Use both → local & holistic features for recognition.



## Holistic methods

- PCA
- Fisherface
- $(PC)^2A$
- 2DPCA
- NoBe model
- Enlarge training set
  - New representation
    - SVD + Perturbation
  - Generate novel views
    - Parallel Deformation
- Dis-advantages of Holistic methods



PCA

Preprocessing

- Convert each image to same width & Height
- Take training set of  $M$  images each with dimension  $\rightarrow M \times M$
- Create Matrix  $A$

each column is a Vector representation of  $N \times M$  image matrix

$$A = \left\{ \begin{bmatrix} \uparrow & \uparrow & \dots & \uparrow \\ im-1 & im-2 & \dots & im-M \\ \downarrow & \downarrow & \dots & \downarrow \end{bmatrix} \right.$$

$\underbrace{\hspace{10em}}_M$

$$\underline{A \rightarrow (N^2 \times M)}$$



→ Find mean face by taking average of each row.

$$N^2 \left\{ \begin{array}{l} \left[ \begin{array}{l} \leftarrow \frac{im_1(i,j) + im_2(i,j) \dots}{M} \\ \vdots \\ \leftarrow \frac{im_1(N \times N) + im_2(N \times N) \dots}{M} \end{array} \right] \\ \underbrace{\hspace{1cm}}_I \end{array} \right.$$

→ Subtract the mean Vector from each  $N^2 \times 1$  Vector → so data become normalized → Zero mean

→ Find Covariance Matrix

$$AA^T \rightarrow N^2 \times N^2$$

↳ find eigenvectors →

$$\frac{\text{total } N^2}{=} \rightarrow \frac{\text{tolarge}}{=}$$



\*

$$ATA \rightarrow (M \times 1)$$

So,

$$A^T A v_i = \mu_i v_i$$

mutiply both side by A

$$AATAV_i = u_i AV_i$$

$$C(AV_i) = (\mu_i A) V_i$$

So  $AV_i \rightarrow$  eigen vectors of  $C$

$$\frac{A \cdot V_i}{-} \rightarrow (N^2 \times M) \times (M \times 1) \rightarrow N^2 \times 1$$

So by taking eigen vectors  $V_1, V_2, \dots, V_M$  in lower space we can get the eigen vectors of  $A^T A$

Higher space.  $\rightarrow V_i = \sum_{k=1}^M A_{ik} V_{1k}$   $\leftarrow (N^2 \times M) \times (M \times 1)$   
 $L = 1, 2, \dots, M$   $(N^2 \times 1)$



# Recognition

→ accurate reconstruction of face is not needed.

total eigenvectors required for recognition  $\ll M$

original paper  $\rightarrow M=16$  then  $M'=7 \leftarrow$  top seven eigenvectors corresponding to highest eigenvalues

→ each training image

↳ convert

$tr-1$	$\rightarrow$	$tr_1$ - mean face
$\vdots$	$\rightarrow$	$tr_2$ - mean face
$tr \rightarrow m$		$tr_m$ - mean face.

→ Compute weight vector for each training image.

$$w_{11} = [e.g. v-1]^T [tr-1]$$

$$w_{12} = [e.g. v-2]^T [tr-1]$$

$$w_{1m} = [e.g. v-M]^T [tr-m]$$



→ represent each image with weight Vectors

$$Tr_1 \rightarrow [W_{11} \ W_{12} \ \dots \ W_{1M}]$$

$$\vdots$$
$$Tr_M \rightarrow [W_{M1}, W_{M2} \ \dots \ W_{MM}]$$

→ Test image → Subtract mean face → find weight Vector

$$[W_{11}, W_{12}, \dots, W_{1M}]$$

→ Find Euclidian Distance of test image Vector with each of the training image,

↳ select the image with minimum distance.

$$E^2 = \left\| \underset{\substack{\uparrow \\ \text{test}}}{\Omega} - \underset{\substack{\uparrow \\ \text{each training}}}{\Omega_k} \right\|^2$$

weight Vectors



# Fisherface

→ face classification

→ Assume → each face class → enough sample images are available for training.

→ Maximize between class scatters & minimize within class scatters

## Steps

→ Suppose total number of classes are 4 with 2 images for training in each.

$$\begin{bmatrix} 1 \\ a_1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_2 \\ 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 \\ b_1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ b_2 \\ 1 \end{bmatrix}} \underbrace{\begin{bmatrix} 1 \\ c_1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ c_2 \\ 1 \end{bmatrix}} \underbrace{\begin{bmatrix} 1 \\ d_1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ d_2 \\ 1 \end{bmatrix}}$$

→ Find avg. face.

$$\begin{bmatrix} \mu \end{bmatrix} = \frac{1}{N} [a_1 + a_2 + b_1 + \dots + d_2]$$

→ Compute avg. face of each person  
e.g. here for 2 persons per class  
 $\mu_1 = \frac{1}{2} [a_1 + a_2]$  same for  $\mu_2, \mu_3$  &  $\mu_4$ .



→ Subtract avg. face of each class from training faces.

$$a_i' \rightarrow \begin{bmatrix} a_1 - \mu_1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} a_2 - \mu_2 \\ 1 \end{bmatrix} \quad \dots \quad \begin{bmatrix} d_1 - \mu_4 \\ 1 \end{bmatrix} \quad \begin{bmatrix} d_2 - \mu_4 \\ 1 \end{bmatrix}$$

within class scatter matrix

$$S_1 = a_1' a_1'^T + a_2' a_2'^T$$

$$S_4 = d_1' d_1'^T + d_2' d_2'^T$$

$$S_W = S_1 + S_2 + S_3 + S_4.$$

between class scatter matrix

$$S_B = \underset{\substack{\uparrow \\ \text{total} \\ \text{images} \\ \text{per class.}}}{2} (\mu_1 - \mu)(\mu_1 - \mu)^T + \dots + 2(\mu_4 - \mu)(\mu_4 - \mu)^T$$

$$\text{Objective} \rightarrow W_{opt} = \underset{W}{\operatorname{argmax}} \frac{|W^T S_B W|}{|W^T S_W W|}$$

$[W_1, W_2, \dots, W_m]$



If  $S_W$  is non singular

Columns of  $W$  are eigenvectors of  $S_W^{-1}S_B$

If  $S_W$  is singular.

Apply PCA  $\rightarrow$  reduce dimensionality.

$$W_{PCA} = \arg \max |W^T S_T W|$$

$$S_T = \sum_{k=1}^M (X_k - \mu)(X_k - \mu)^T$$

$$W_{f1d} = \arg \max_W \frac{|W^T W_{PCA}^T S_B W_{PCA} W|}{|W^T W_{PCA}^T S_W W_{PCA} W|}$$

$W_{opt} = W_{f1d} W_{PCA}$

$\rightarrow$  classify new face by nearest neighbor

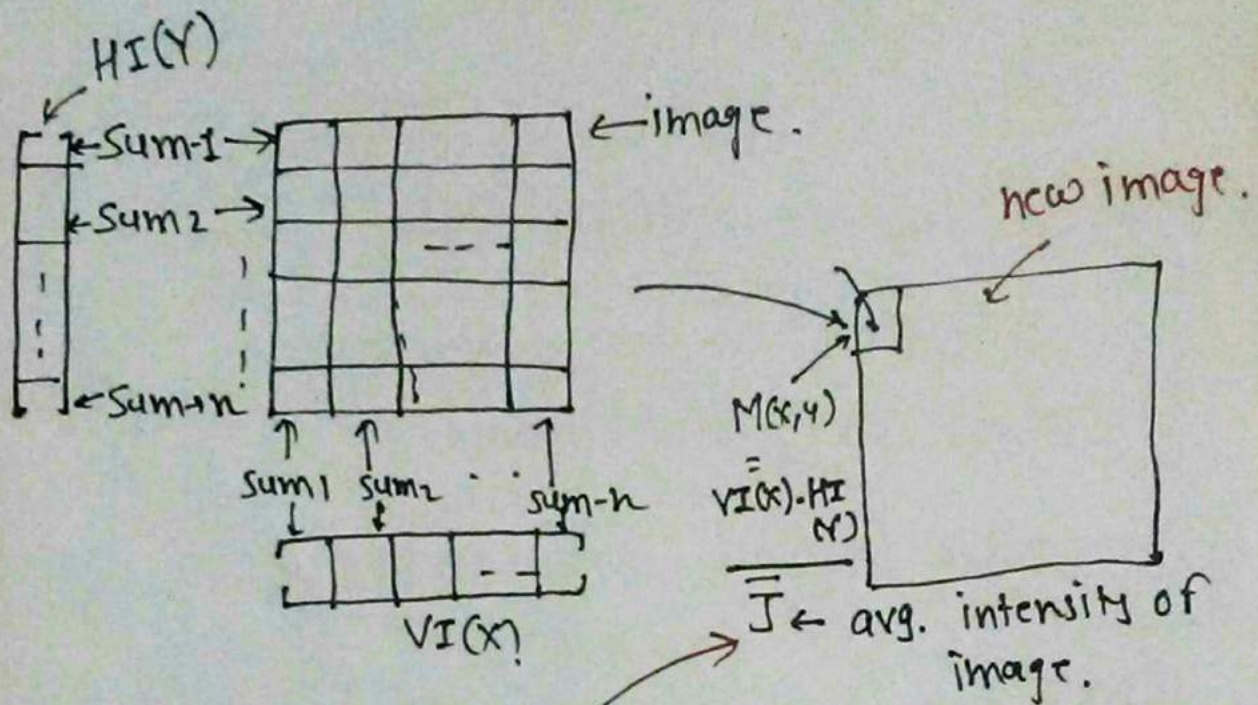
Disadvantage :  $\rightarrow$  Expensive calculation time  
 $\rightarrow$  Large storage.

$\rightarrow$  fail when each person's class contain one training face

$\rightarrow$  It perform best in variation in lighting & facial expression.



$$\frac{(PC)^2 A}{=}$$



Pre Processing

→ unimportant features → faded out  
and important features become more  
Silent after Pre-Processing.

→ Results → improved when tested  
for one sample per person.



## 2DPCA

### Idea

2D matrix  $\rightarrow$  not 1D vector.

$\rightarrow$  No need to transform  
Image Matrix to Vector.

Suppose each image size  
 $m \times n$

& total images are  $M$

then Covariance Matrix

$$G_t = \frac{1}{M} \sum_{j=1}^M (A_j - \bar{A})^T (A_j - \bar{A})$$

$\uparrow$   $n \times n$  matrix       $\uparrow$  training images       $\nwarrow$  mean face.

$$J(X) \rightarrow X^T G_t X$$

$\uparrow$  generalized total scatter criterion       $\uparrow$  eigen vectors of size  $n \times 1$        $\rightarrow$  Maximize criteria (Optimal Projection Axis)  $X_{opt}$

eigen vectors of  $G_t$  corresponding to largest eg. value.



## Advantages

→ Size of image covariance matrix → much smaller.

↳ easy to evaluate

→ Less time to determine corresponding eigenvectors.

## Disadvantages

Storage → PCA →  $w_1, w_2 \dots w_n$

2DPCA →

Principle Component Vectors.

↓  
How to reduce the Dimension → not clear.



Noise model

Real world

↳ Certificate photographs  
→ Corrupted by Various  
Scratches, blur or Dis-  
coloration.

Paper

Authentication System to  
handle the problem with one  
training image per person.

Idea

Synthesize multiple new face  
images which imitate the corrupt<sup>ed</sup>  
images for recognition.



Imitation  $\rightarrow$  Noise model

```
graph TD
    A[Imitation  $\rightarrow$  Noise model] --> B[Contrast]
    A --> C[Brightness]
    A --> D[Gaussian Blur.]
```

$\rightarrow$  By changing the values of noise parameters  $\rightarrow$  several corrupted images corresponding to one sample are imitated.

error rate  $\rightarrow$  in 137 scanned Id-cards  
 $\hookrightarrow$  1.32%.

$\downarrow$   
indicates  
method can significantly improve  
the similarity between corrupted images  
& training images.



For Better recognition ?

↳ enlarge the size.  
of training set.

Synthesizing Virtual Samples

New representations  
for single training  
Samples.

New Visual  
Samples → not  
existed in Database.

↳ ROCA → Representational  
Oriented Component analysis.

↳ One sample per person

Preprocessing ∴ apply linear & non-linear  
filters to → 150 representations

↳ OCA classifier is built  
on each representation.

→ All OCA classifiers are combined  
to give weighted linear sum for final Dec.



## Image Perturbation

↓  
Construct new Representation

→ For given image  $X$  →  
error range is set for  
Vertical & Horizontal localization  
Error.

A new sample is generated  
by changing Vertical & or Horizontal  
coordinate value.

## SVD Perturbation

face recognition → Single  
Image Per Person.

$I \rightarrow$  Input image  $\rightarrow N_1 \times N_2$

$P = U \Sigma^N V^T \rightarrow N_1 \times N_2 \rightarrow$  Diagonal Matrix  
↳  $N_1 \times N_1 \rightarrow$  orthogonal matrix  
↳  $N_2 \times N_2 \rightarrow$  orthogonal matrix



→ Combine  $I$  linearly with  $P$  to generate new image with following equation.

$$J = \frac{I + \alpha P}{1 + \alpha} \rightarrow 0 < \alpha < 1$$

→ Change the values of  $\alpha$  to generate additional images.

$J$  works better by minor changes in expression, illumination & occlusions.

### Drawback

→ Generated Virtual images may be highly correlated.

→ New sample should not be considered as independent training images.



Generate      Novel Views  
=                      = goal  
Virtual      samples → Should  
=                      =

↳ occupy different locations  
in the face space & represent  
specific variations of face images

Solution

① apply geometrical transformation  
→ rotation  
→ scale etc.

② Synthesizing the face image  
under different pose &  
different illumination condition

Technique

Parallel Deformation

→ Generate novel view of single  
face image under different poses.



Need to learn transformation

$\Delta X \rightarrow$  Difference between original face  $X$  & its reference face

$\Delta X_i \rightarrow$  Difference between images of other prototypical face to the same reference face

Assume.  $\rightarrow$  linear class Assumption

$$\Delta X = \sum_{i=1}^q \lambda_i \Delta X_i$$

Obtain  $\lambda_i$  by minimizing.

$$\left\| \Delta X - \sum_{i=1}^q \lambda_i \Delta X_i \right\|$$

$\rightarrow$  One can use it to generate novel images of each single face.

Disadvantages

$\rightarrow$  Difficult to generate for diff. pose.

$\rightarrow$  Novel image need  $\rightarrow$  Correspondance between  $\delta$  real & reference image



## Disadvantages of Holistic methods

- One single feature vector to represent each face image.
- Sensitive to large appearance change due to
  - expression
  - illumination
  - Pose & partial occlusion

## Solution

↳ Local features

- Original face → represented by set of low dimensional feature vectors.
- Problem of Dimensionality can be removed.
- Recognize face Based on its parts
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