Functional Mechanical Design

Exercise 2

Synthesis of linkage mechanism

In this exercise, the kinematic synthesis of four bar linkage mechanism is required. In particular, the problem of **function generation** is considered: it is required to relate some prescribed rotations of the input link with rotations of output link.
The analysis of a linkage mechanism by an analytical method is carried out using **complex number representation**. For example, a generic link, characterized by length a and oriented with an angle α with respect to the real axis, can represented as:

$$Z = a \cdot e^{i\alpha} \tag{1}$$

If this link pass from the initial position to a generic j-th position, the link generally traslates and rotates: if this rotation is characterized by an angle δ_j, the new position of the link can be represented as:

$$Z_j = a \cdot e^{i(\alpha+\delta_j)} = Z \cdot e^{i\delta_j} \tag{2}$$

Naturally, the rotations of the links of a mechanism can't be arbitrary but must respect the closing vectorial equation of the mechanism. For example, in case of four bar mechanism, this equation for the initial position is:

$$Z_1 + Z_2 - Z_3 - Z_4 = 0 \tag{3}$$

With the purpose of synthetize a mechanism, it is useful to express the difference between the equations for a j-th position and the ones in the initial condition. Here, it is expressed an example for a generic link:

$$Z_j - Z = (e^{i\delta_j} - 1) \cdot Z \tag{4}$$

Now, it is possible to consider the entire mechanism, and the above consideration are applied in figure 1. For the function generation, it is needed to relate the rotations of the input arm (W) with the ones of the output arm (W*): in other words, it is assigned the relationship between the angles ϕ_j and ψ_j.
Notice that the angles ϕ_j, ψ_j and γ_j describes the difference in orientation of the arms between the j-th position and the initial position.
In this case, the closing vectorial equation $B_0B_1A_1A_0A_jB_jB_0$ can be written as follow:

$$\mathbf{W}(e^{i\phi_j} - 1) + \mathbf{AB}(e^{i\gamma_j} - 1) - \mathbf{W^*}(e^{i\psi_j} - 1) = 0 \tag{5}$$

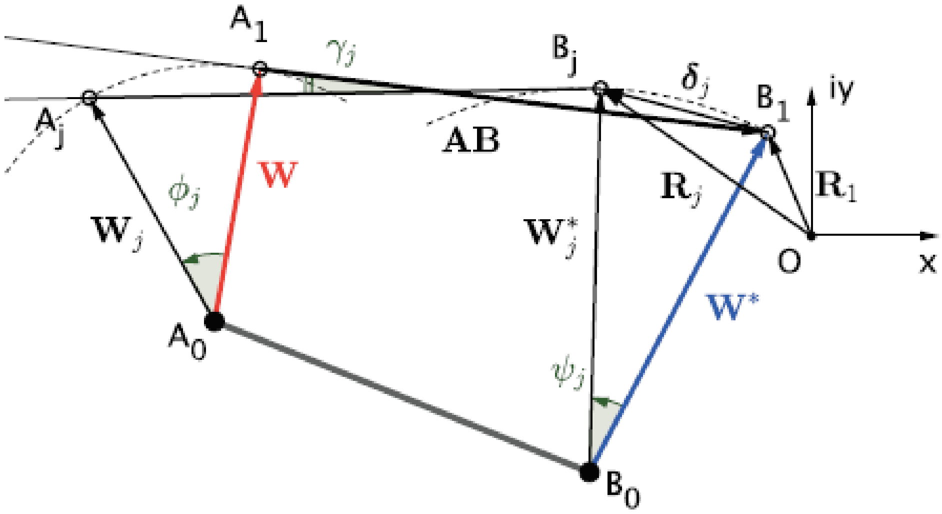

Figure 1: Synthesis of a four bar mechanism for function generation

if n is the number of prescribed position, it is possible to write n-1 vectorial closing equation in the form of equation 5 (one of these position is considered as initial position of the mechanism), which is equivalent to have $2(n-1)$ scalar equations. The number of unknows is 6+n-1 (vectors **W**, **AB**, **W*** and the angles γ_j with $j = 2, 3, ..., n$). Since the number of unknows are higher than the number of equations, it is necessary to make some arbitrary choices, in order to solve the problem. The number of these choices is equal to the difference between the number of unknows and the number of equations, i.e. $6 + n - 1 - 2(n - 1) = 7 - n$. In this way, it is not possible to assign more than seven position.
These concepts are summarized in table 1:

Number of positions $j = 2, 3, ..., n$	Number of scalar equations $e = 2(n-1)$	Number of scalar unknows $i = 6 + n - 1$	Number of solutions (∞^{i-e})
2	2	7 (**W**,**W**,**AB**,γ_2)	∞^5
3	4	8 (previous ones + γ_3)	∞^4
4	6	9 (previous ones + γ_4)	∞^3
5	8	10 (previous ones + γ_5)	∞^2
6	10	11 (previous ones + γ_6)	∞^1
7	12	12 (previous ones + γ_7)	1

Table 1: Number of solutions

Example of a mechanism to move the backrest of a barber chair

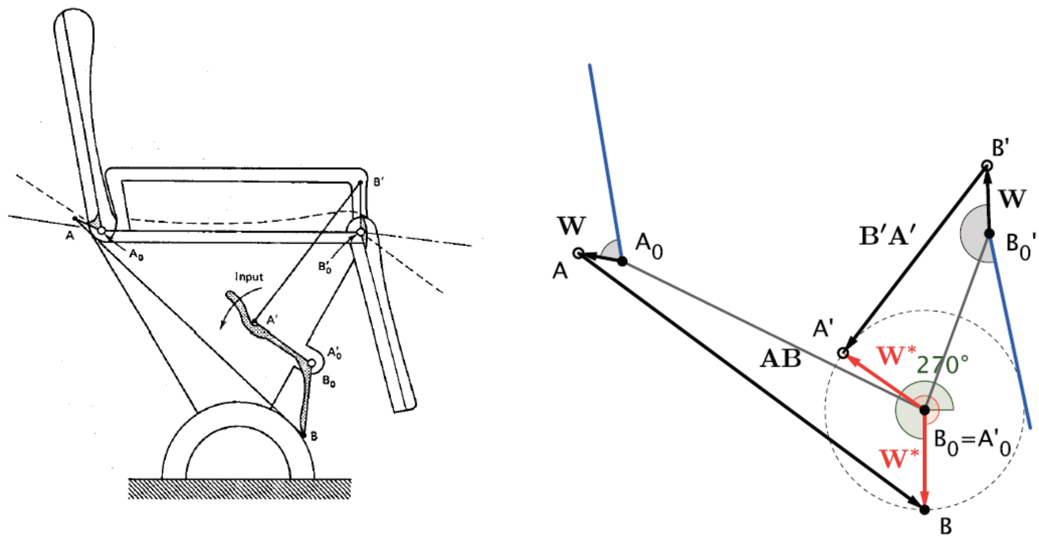

Figure 2: Barber chair and its mechanisms

In figure 2, barber chair and its mechanisms are highlighted. It is characterized by two mechanisms: the first to move the backrest and the other to move the footrest. The two mechanisms are moved by the same input arm. In this example, it is taken into account only the mechanism to move the backrest.
This is a function generation problem with three precision point (n=3).

Data:

$$\phi_2 = 50^\circ \qquad \psi_2 = 22.5^\circ \qquad \phi_3 = 75^\circ \qquad \psi_3 = 45^\circ \tag{6}$$

Suppose that two unknows are choosen in an arbitrary way, in particular **W*** (or in alternative, **W**). The arbitrary choice of **W*** set the scale of the four bar mechanism and its orientation, but this not influence the function that relates the rotations ϕ_j and ψ_j. Once the mechanism is obtained, it can be scaled ed oriented in any way without modifying the input-output relationship. This property is not valid for trajectory generator and motion generator, in which the change in the length of a link modify the generated trajectory of movement.

4 Arbitrary choices:

$$\gamma_2 = 7^\circ \qquad \gamma_3 = 12^\circ \qquad \mathbf{W*} = B_0B = 1\ e^{i270^\circ} \tag{7}$$

In this way, by choosing γ_2 and γ_3 values, the problem becomes linear in the two (four scalar) unknows **W** and **AB**:

$$\begin{cases} \mathbf{W}(e^{i\phi_2} - 1) + \mathbf{AB}(e^{i\gamma_2} - 1) = \mathbf{W*}(e^{i\psi_2} - 1) \\ \mathbf{W}(e^{i\phi_3} - 1) + \mathbf{AB}(e^{i\gamma_3} - 1) = \mathbf{W*}(e^{i\psi_3} - 1) \end{cases} \tag{8}$$

Solution:

$$W = A0A = 0.45e^{i169.47^\circ} \qquad AB = 4.33e^{i323.48^\circ} \tag{9}$$