Functional Mechanical Design

Cam Mechanisms (1/2)

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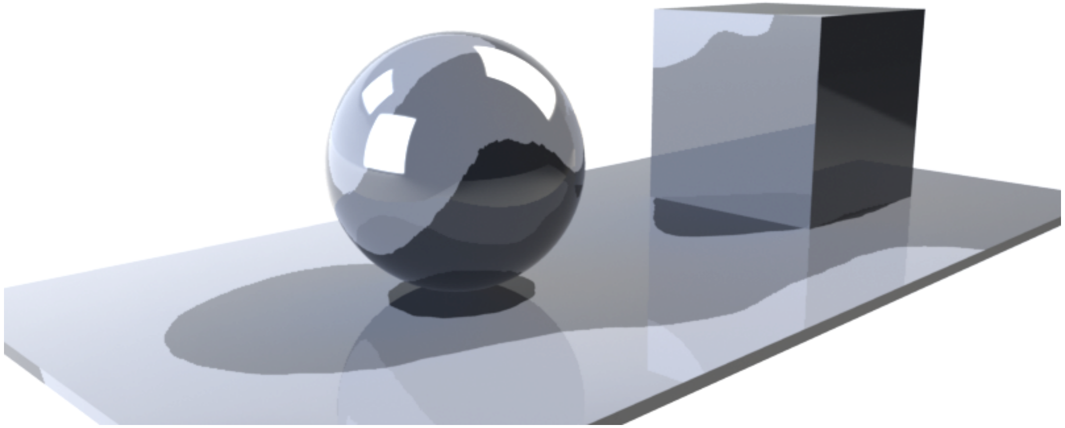
Introduction

A mechanical mechanism is a system made up of rigid bodies (usually called links) connected together by means of kinematic pairs.
There are several way to classify mechanisms, one of these is based on the analysis of the kinematic pair used. We distinguish between two kind of pairs:

- **Higher kinematic pairs**: in this case the contact between the two members is on a point or along a line;
- **Lower kinematic pairs**: the contact occurs on a surface or area of the connected bodies.

The difference of the two pairs concern the relative mobility provided to the connected bodies and the capability of bearing loads.



Note that, when the contact is on a point, the surfaces of the member in contact can have any shape, whereas when the contact is on lines or on surfaces the shape of the body must be appropriate.
In case of lower kinematic pairs the surface of the two body in contact can be only the following:

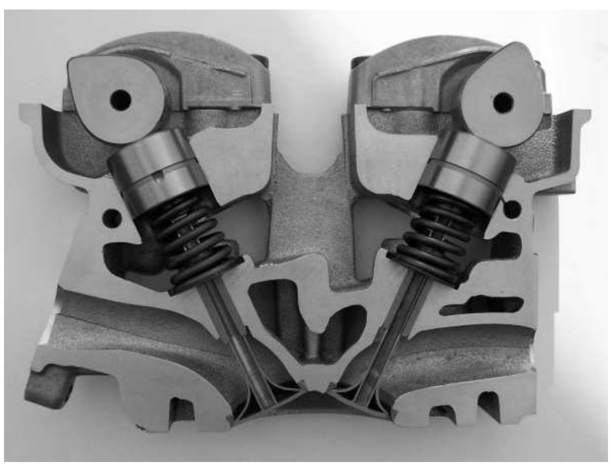
- planar;
- spherical;
- cylindrical;
- helical.

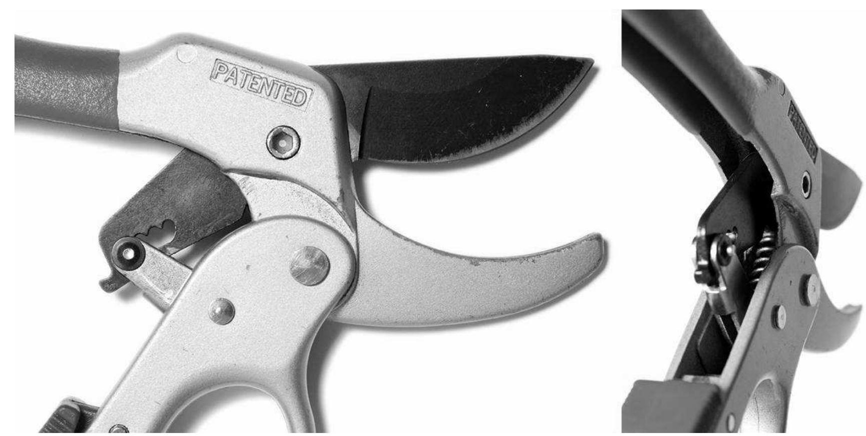
The lower kinematic pairs that allow only one degree of freedom in the relative motion between the two coupled members, are called **elementary pairs** and can be:

- prismatic;
- rotoidal;
- helical;

- Mechanisms with only higher kinematic pairs are called **CAM MECHANISMS**.
- Mechanisms with only lower kinematic pairs are called **LINKAGE MECHANISMS**.

This distinction is only a way to introduce a mechanism classification and it is not so rigid.





Main characteristics of the two mechanisms are:

Cam mechanisms:

- can realize exactly the desired motion profile $y = y(t)$;
- can realize every kind of motion profiles, even impacts;
- are compact;
- have a low load capacity and problems with wear
- generate low displacements;

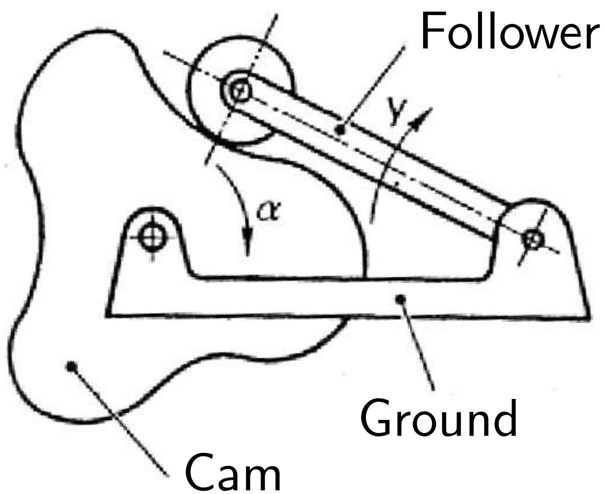
Linkage mechanisms:

- approximate the desired motion profile $y = y(t)$ by means of the elementary movements allowed by kinematic pairs;
- are useful to produce continuous movements;
- are bulky;
- have a high load capacity;
- can generate large displacements;

With a cam mechanisms the motion transmission occurs through the contact between two profiles suitably shaped. The objective is to drive the follower element with the desired motion curve.
The most interesting feature of cam mechanisms is to be able to realize almost any desired motion curve.

The design of these mechanisms is based on three principal steps:

1. Trajectory planning;
2. Computation of the cam profile:
3. Checks on Feasibility and Sizing.



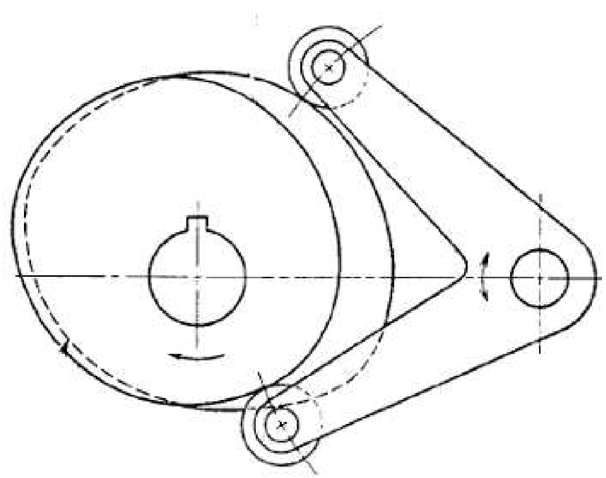
A cam mechanism is made up of:

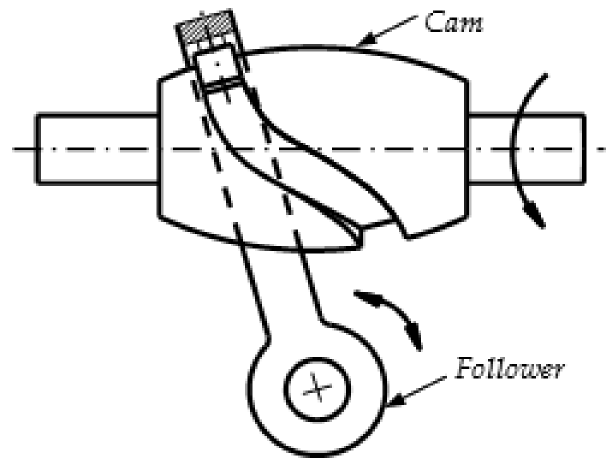
- **Cam**: characterized by a unidirectional rotational motion
- **Follower**: characterized by alternative movement (translation or rotation)
- **Ground or frame**: fixed

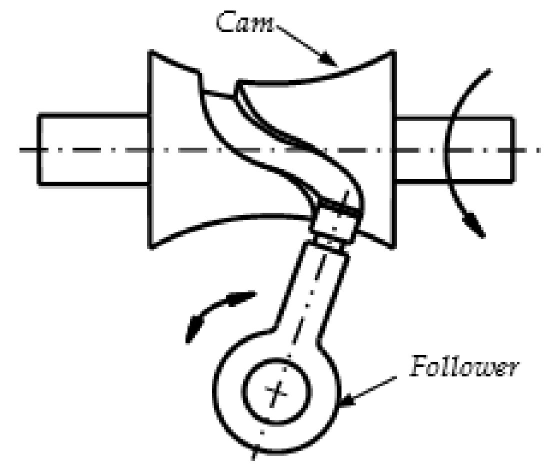
Note: a cam transmits the motion to the follower through contact. The locus of the contact points between the cam and the follower is a result of the designers' studies and normally is a complex profile.

Definitions

If the plane, in which the follower is moving, is perpendicular to the rotational axis of the cam, the mechanism is called **planar cam** (or eccentric), otherwise it is called **spatial cam**.

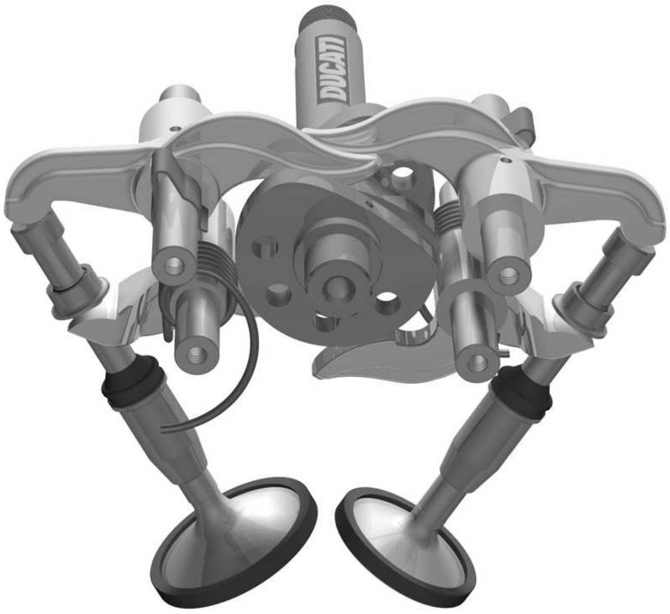






The motion of the follower can be:

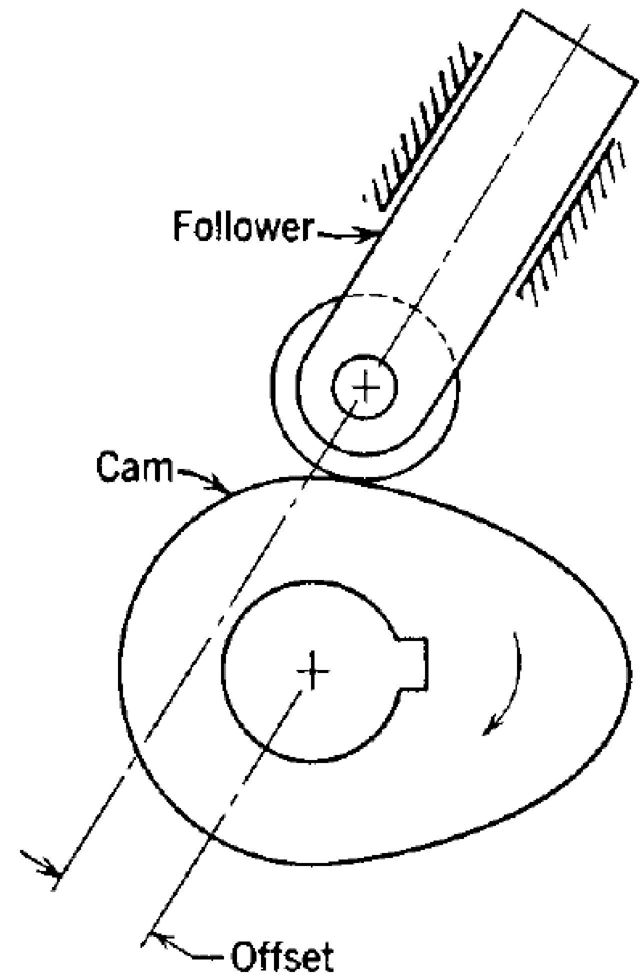
- translational: in this case the mechanism is called *first type mechanism* and the follower is named *tappet*
- oscillatory: it is called *second type mechanism* and the follower is named *oscillating or swinging follower*



In case of translational follower we can distinguish between:

- Radial follower: when the follower translate along an axis passing through the cam center of rotation.
- Offset follower: when the axis of the follower has an offset with respect to the cam center of rotation.

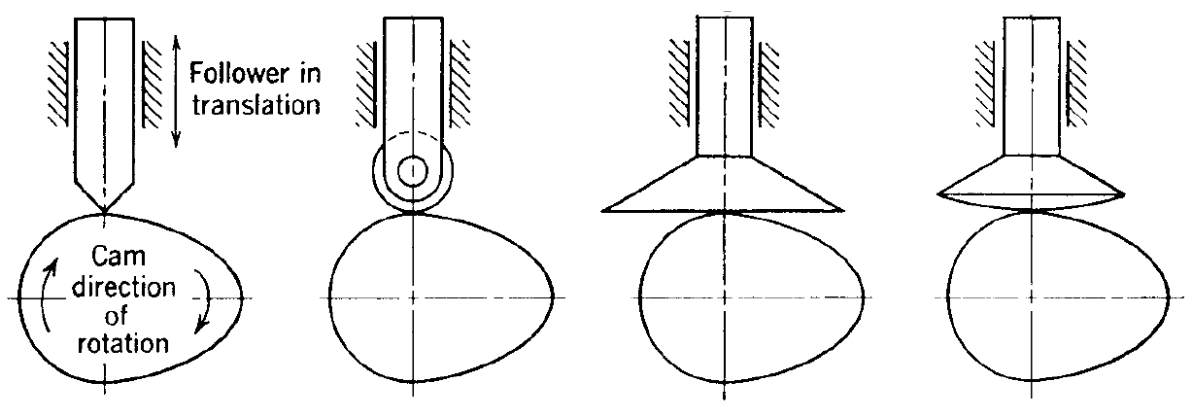
We'll se later that offsetting is often used to improve the cam design in order to reduce force, stress, and also the cam's size.



Definitions

Followers can be grouped by looking at the contact surfaces they have with the cam:

- **Knife-edge** (or point) follower: a sharp edge in contact with the cam (this type of follower is not practical because it results in excessive wear of the contact point)
- **Roller** follower: they are commercially available and use ball or needle bearings supported by a stem. The roller follower has a low coefficient of friction
- **Flat-faced** follower: also called *mushroom* followers in which the contacting surface is flat or spherical.
- **Spherical-faced** follower: *mushroom* followers with spherical contact.

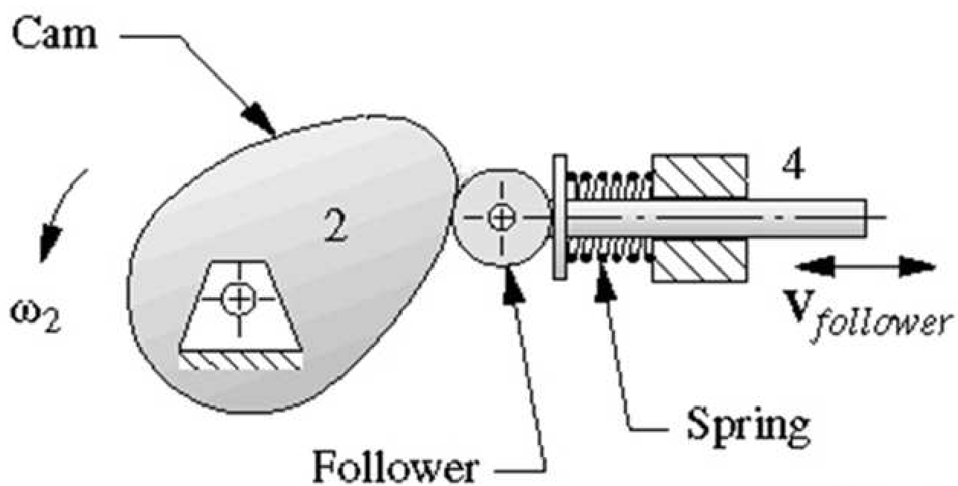


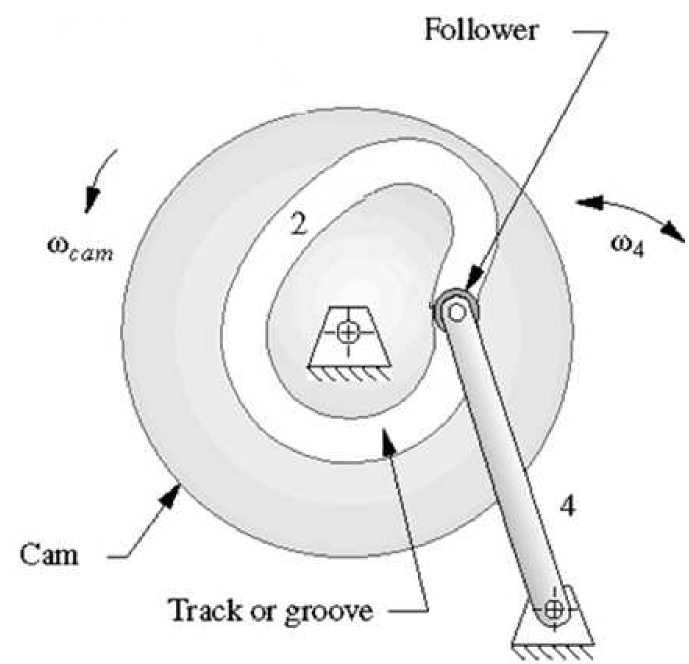
(*a*) Knife-edge follower. (*b*) Roller follower. (*c*) Flat-faced follower. (*d*) Spherical-faced follower.

Definitions

We can mantain the contact between cam and follower in two ways:

- Force closed cam joint: an external force (usually a spring) is used to ensure the contact between follower and cam.
- Form closed cam joint: the follower is trapped into a groove realized on the cam.



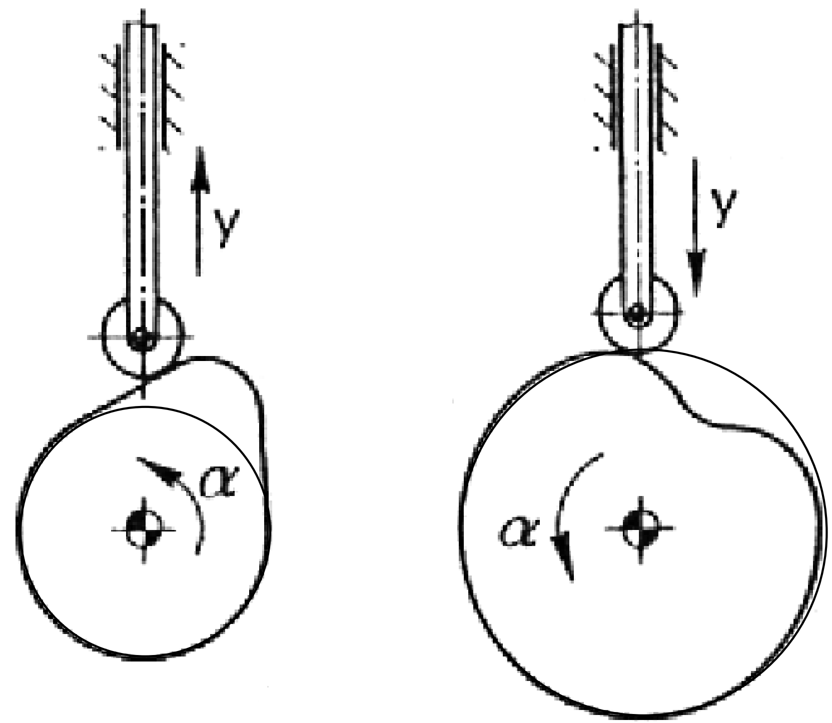


Definitions

Cams can also be divided into:

- **Positive** cam: in this case, the material needed to generate the follower displacement is over the circle centered with the cam center of rotation.
- **Negative** cam: The follower displacement is obtained removing material from the circle centered with the cam center of rotation.

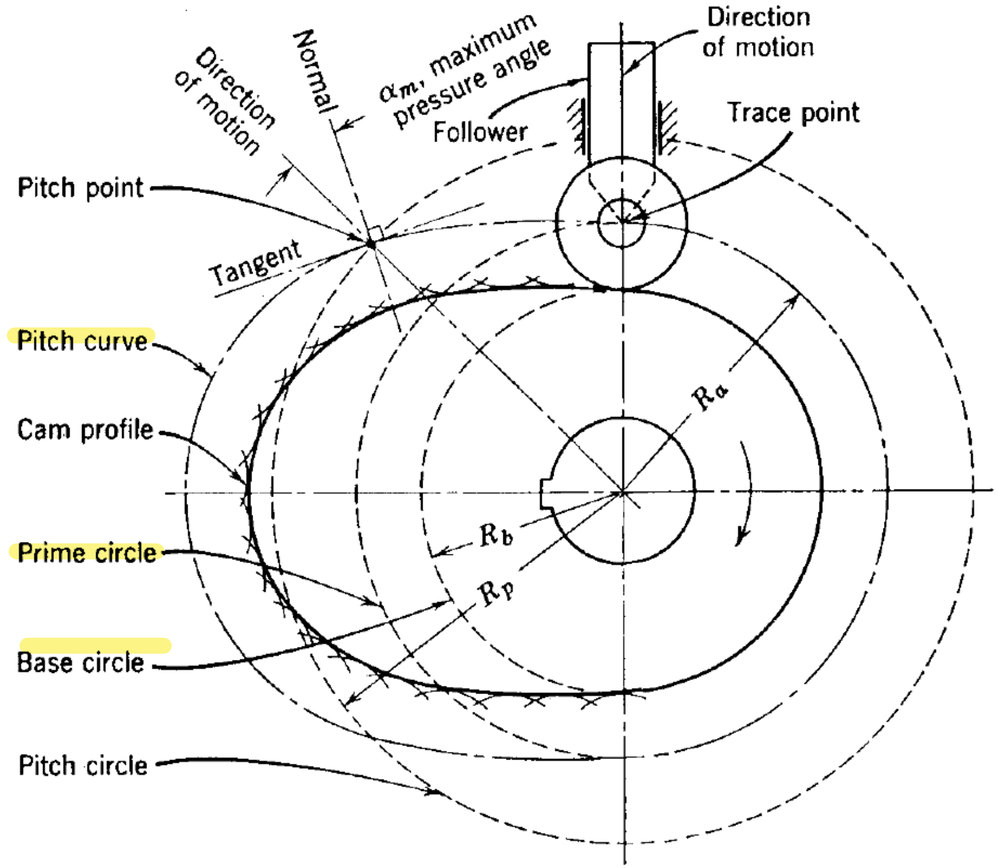
Note: if both positive and negative cams are profiled with the same motion curve, they can be joined together to have a **conjugate** cam (form closed cam).



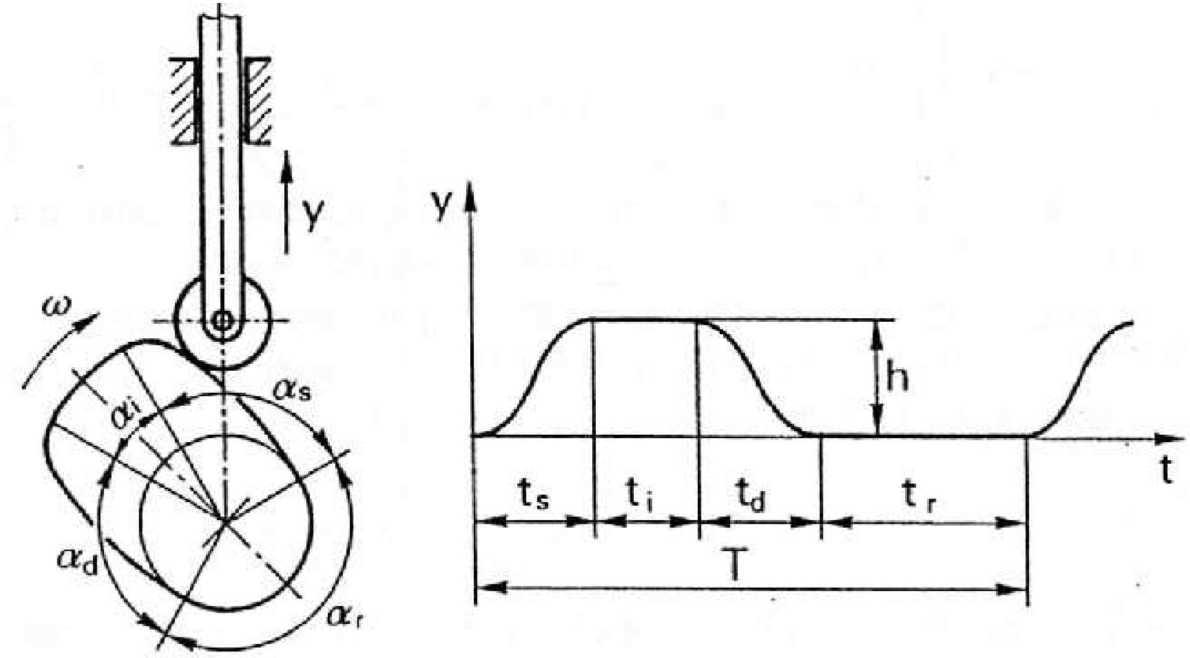
Definitions

Nomenclature:

- **cam profile:** is the surface of the cam (cam profile may be external or internal).
- **base circle**: is the smallest circle tangent to the cam profile from the cam center.
- **pitch profile**: is the path of the trace point (is the point on the follower located at roller center). For knife-edge followers cam profile is equal to pitch profile.
- **prime circle**: is the smallest circle tangent to the pitch curve from the cam center.



We now know that a cyclical motion, performed in a period T, is usually made of 4 main phases: rise (t_s), dwell (t_i), return (t_d) and dwell (t_r), as shown in the figure.



To design a cam means to find the relationship between the motion law and the cam profile

For cam mechanisms it is useful to describe the motion curve of the follower as a function of the cam rotation angle α. As the angular velocity of the cam is constant (ω=const), the relationship between α and t is linear ($\omega t = \alpha$).
The same can be done for velocity ($\dot{y}$) and acceleration ($\ddot{y}$) of the follower that can be expressed as function of α, thus obtaining:

Geometrical velocity $$y' = \frac{\mathrm{d}y}{\mathrm{d}\alpha}$$

Geometrical acceleration $$y'' = \frac{\mathrm{d}^2 y}{\mathrm{d}\alpha^2}$$

These quantities are related with the follower velocity and acceleration by:

$$\dot{y} = \omega y' \qquad \ddot{y} = \omega^2 y''$$

Cam design guidelines

To design a cam mechanism we start from the definition of the follower movement but the choice of the most suitable kind of cam strongly depends on the application and it's based on the **experience** of the designer. In other words, there is not a set of rules to follow to solve the synthesis of a cam mechanism.

As you can imagine, the choice of the kind of cam to use has consequences in terms of **costs** and **performance** of the machine.

For this reason, before describing the steps we have to do to make a cam mechanism from a mathematical point of view, we are going to go through a big picture where all the actors involved with this task are introduced.

Translating or oscillating follower?

First of all, if it is possible, planar radial cams should be preferred rather then spatial ones, because they are less expensive and complicated to manufacture.

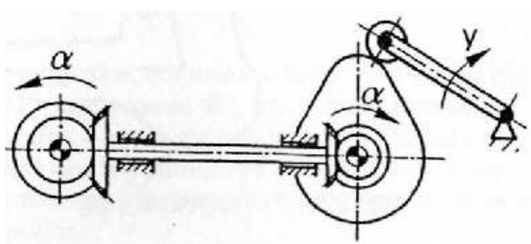
Furthermore the follower movement defines the cam type we have to use: if we need a translational movement the choice will fall on a translating follower otherwise on a swinging one.

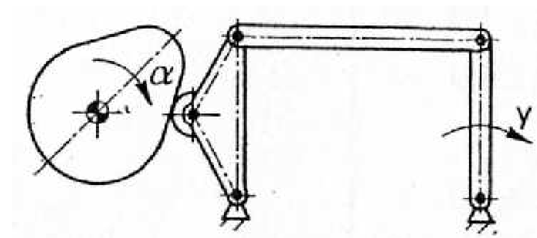
If it is possible to approximate the straight motion of the follower it should be considered the opportunity to use a **oscillating follower** in place of a translating one, because

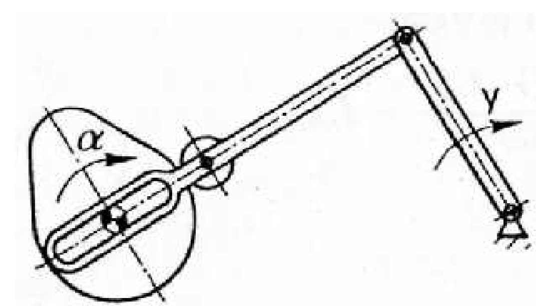
- it is **simpler** realize a pivot rather then a prismatic joint
- the pivot **friction** in an oscillating follower is smaller compared to that in a prismatic joint
- as afterward it will be presented, oscillating follower can work with higher value of **pressure angle**.

Follower train and camshaft design: Sometimes it happens the motor shaft can't be places close to the follower. In this case we can adopt two main solution:

- to use a gear train / shafts to bring the motor movement where it's needed
- to transform the motor movement with a cam and to transmit this movement by means of a mechanism.







The right choice depends on the inertia of the mechanical transmission, its stiffness, clearances and backlashes, etc...

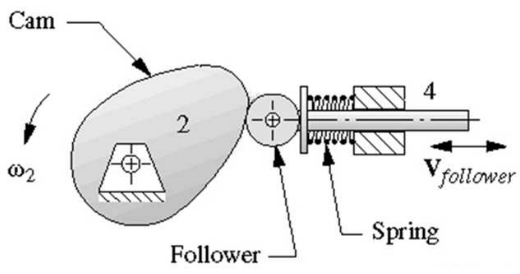
Roller or flat-faced follower?

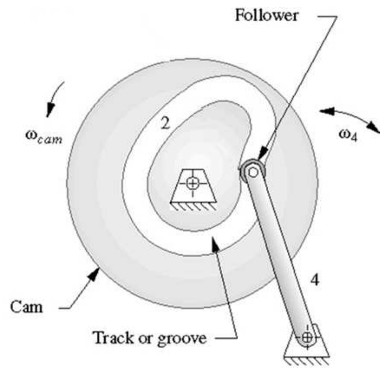
- A flat-faced follower rubs on the surface of the cam so it needs an adequate lubrication to limit wear. These kind of followers are capable of bearing high loads and allowing to save space compare to the roller one.
- A **roller follower is usually a better choice** because it accepts negative radius curvature cams (more variety in terms of motion curves), reduces wear and friction, can work without lubrication (fundamental when oil would cause contamination of products).

The largest users of flat-faced follower are the automotive engine makers: their quantities are high enough to allow custom design and to reduce costs compare to roller ones.
In automatic machinery, roller followers are almost exclusively used, as they are available on the market from several manufacturers and do not require special physical and chemical treatments

Force or form closed?

- Force closed cams are **simpler** to realize and **less expensive** compared to the form closed ones. The precision of the profile is less critical and after a heat treating it can used without grinding. High speed cams are critical as the follower may **detach** because of inertia forces. **Vibration** may occur due to the spring.
- Form closed cams or conjugate cams require two surfaces and this implies **high precision** in manufacturing. This solution does not require return spring thus it can run at **high speed** but it presents the **crossover shock phenomenon** (ie. as the acceleration change sign, the follower could jump from a side of the track to the other thus causing impacts)

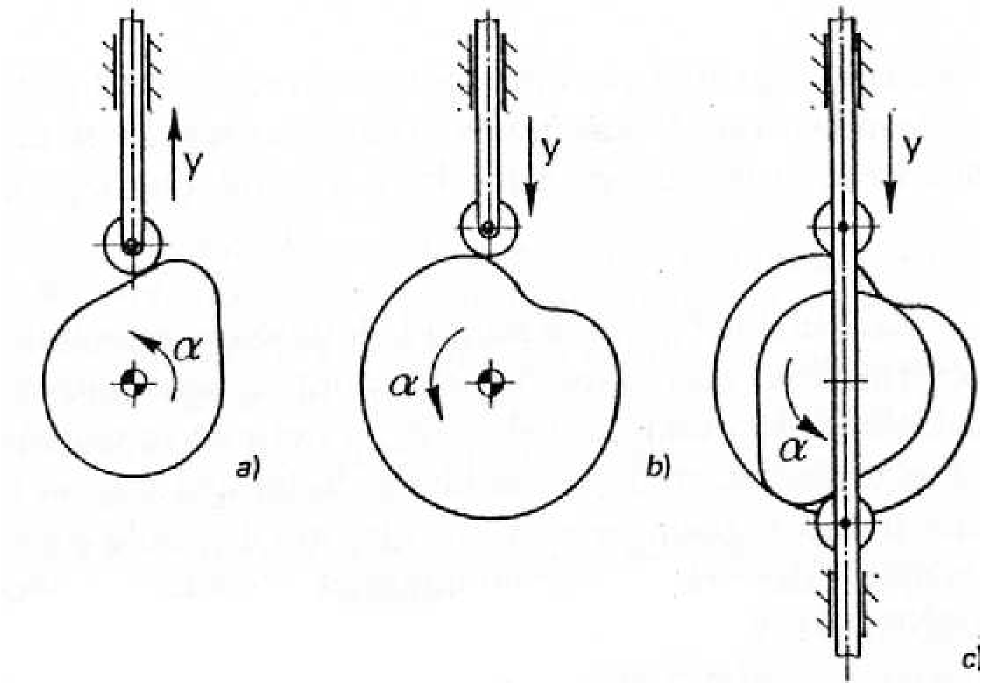




Positive or negative cam?

Negative or positive cam has to be chosen with reference to the material that is supposed to be removed and to the value of the curvature radius of the profile.

Note that the union of a positive and negative cam, realized with the same motion curve, allows us to obtain a conjugate cam solution.



Cam manufacturing

There are several methods to produce a cam. The best way to realize a cam depends on:

- the kind of the profile;
- the precision required;
- the production rate required (occasional,...);
- the volumes and the variety of cams that must be realized;

The principal manufacturing methods to realize cams are:

- manual tracking of the profile;
- elementary machining processes;
- master cam analog duplication method;
- Continuous Numerical Control - CNC - (CNC milling and CNC grinding of cam profiles);
- hot forging, stamping and powder metallurgy;
- electrodischarge machining (EDM);

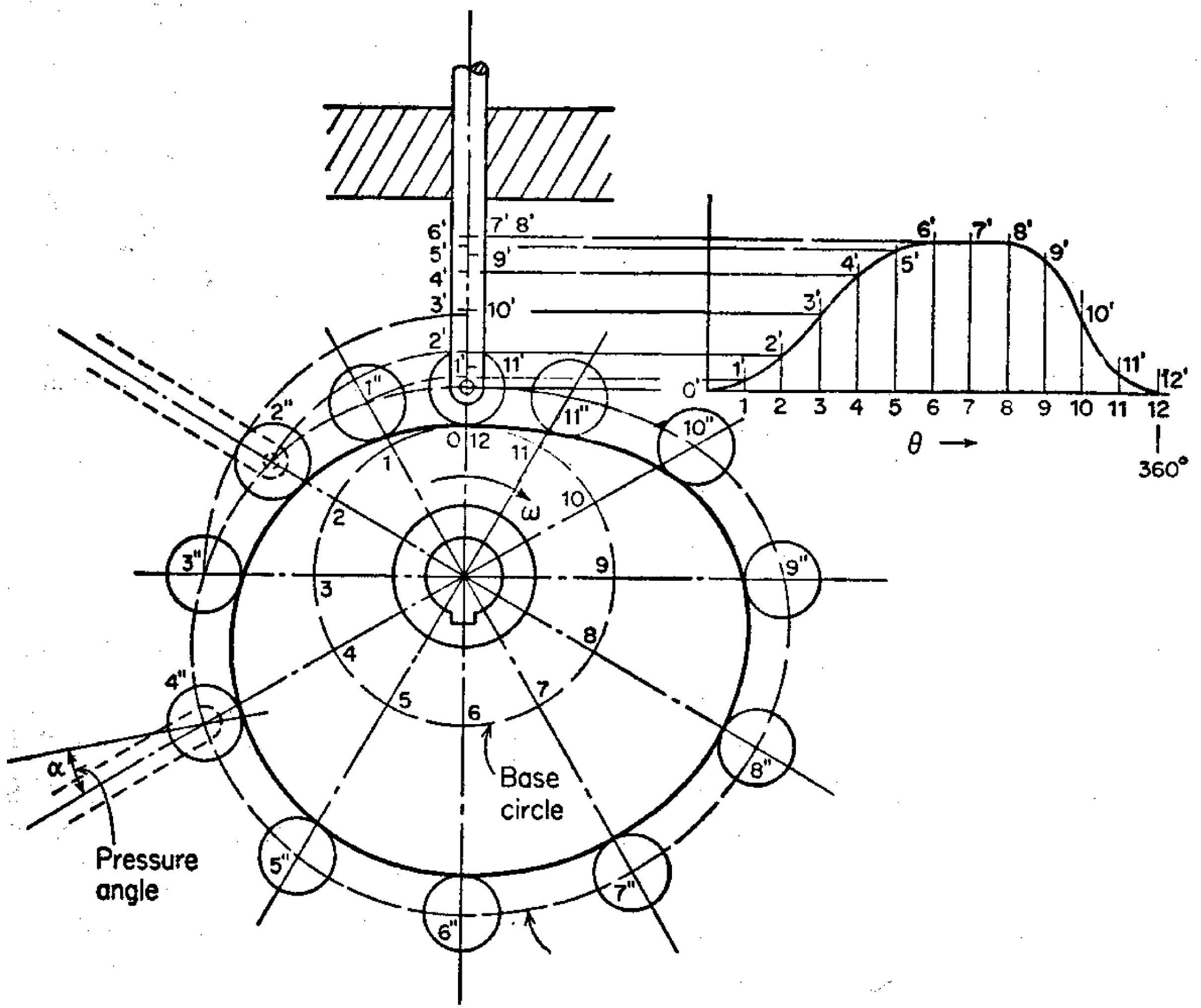
Usually the cam profile needs a grinding machining.

In the design of a cam mechanism, after motion curve profiling we have to "envelop" this curve on the base circle and to be careful with the values of two important parameters: the **curvature radius** and the **pressure angle**. In order to understand the importance of these two parameters, below it will be described the procedure based on a graphical method that was used to draw the cam profile.

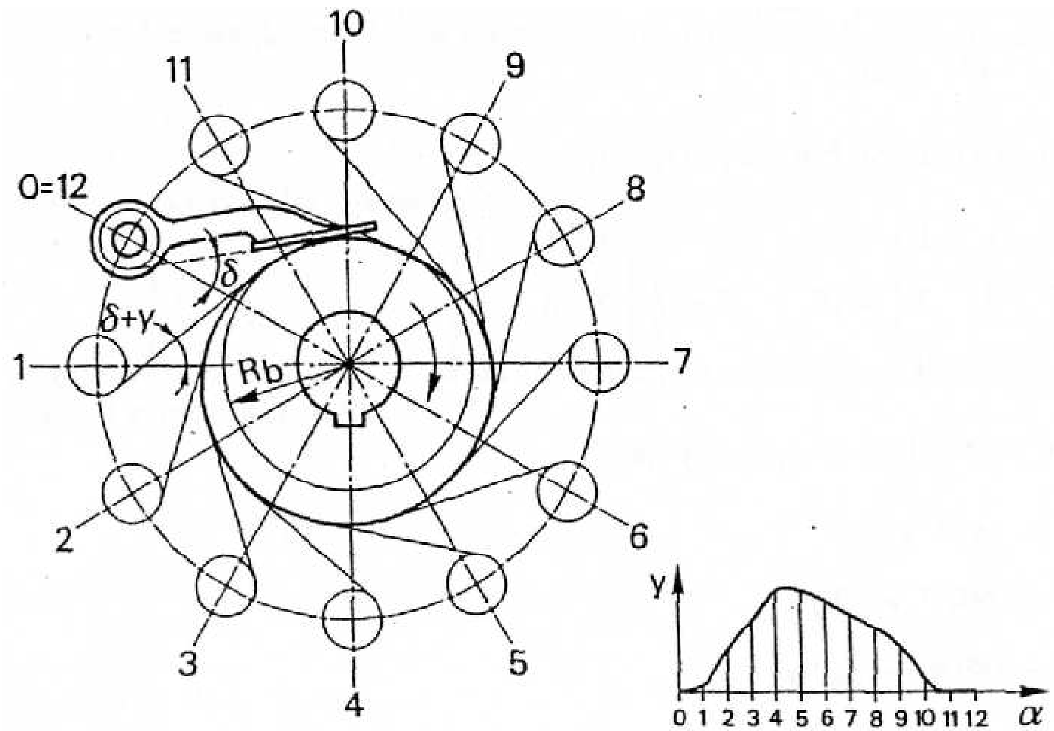
This procedure (graphical layout method) is based on a repeated drawing of the mechanism configuration during its movement. The graphic construction is based on the principle of "**kinematic inversion**" in which the cam is considered the fixed member and the follower is then moved to have the same relative motion. The cam profile is the result of the envelope of the follower profile as the follower is positioned around the cam.

Note during the dwell phases the cam profile is a circle, and when $y(\alpha) = 0$ usually the follower rests on the base circle. The value of the base circle radius R_b is chosen to define the initial position of the follower and contributes to determinate the final dimension of the cam.

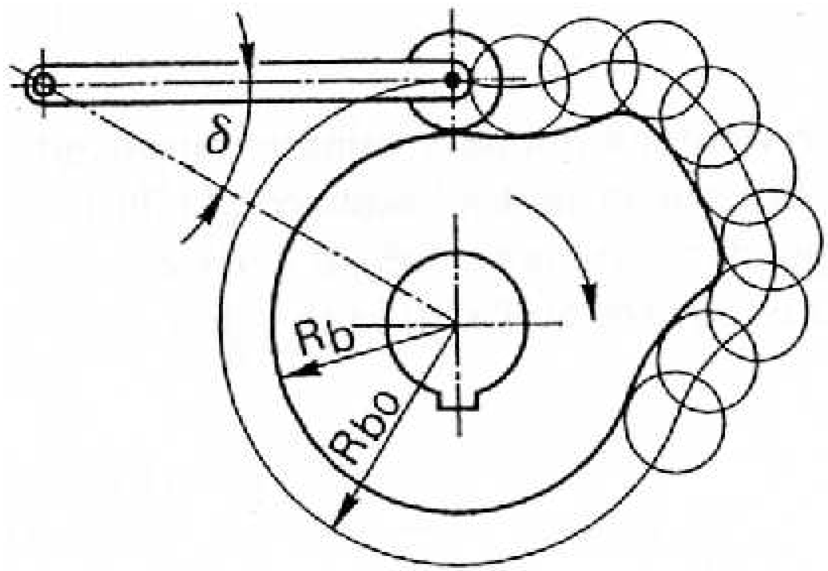
Graphical synthesis



Graphical synthesis



- Sketch the timing chart and divide it into angular sectors.
- draw the cam base circle (smallest circle to cam profile);
- draw the follower at the initial point of the movement (point O);
- define the angle δ_0 (angle between the follower and ground O_1O_2);
- draw the circle with radius O_1O_2;
- divide this circle in the same angular sectors of the timing chart;
- draw the follower in the positions indicated by every sectors of the timing chart $\delta = \delta_0 + y$;
- obtain the cam profile by enveloping the position reached by the follower for every angular sectors.



In case of roller followers, first it is obtained the pitch curve (cam profile if the radius of the roller is nil or knife-edge follower);

Then the cam profile is deduced from the pitch curve by means of the following equation:

$$R_{b0} = R_b + R_r$$

$$R_b = R_{b0} - R_r$$

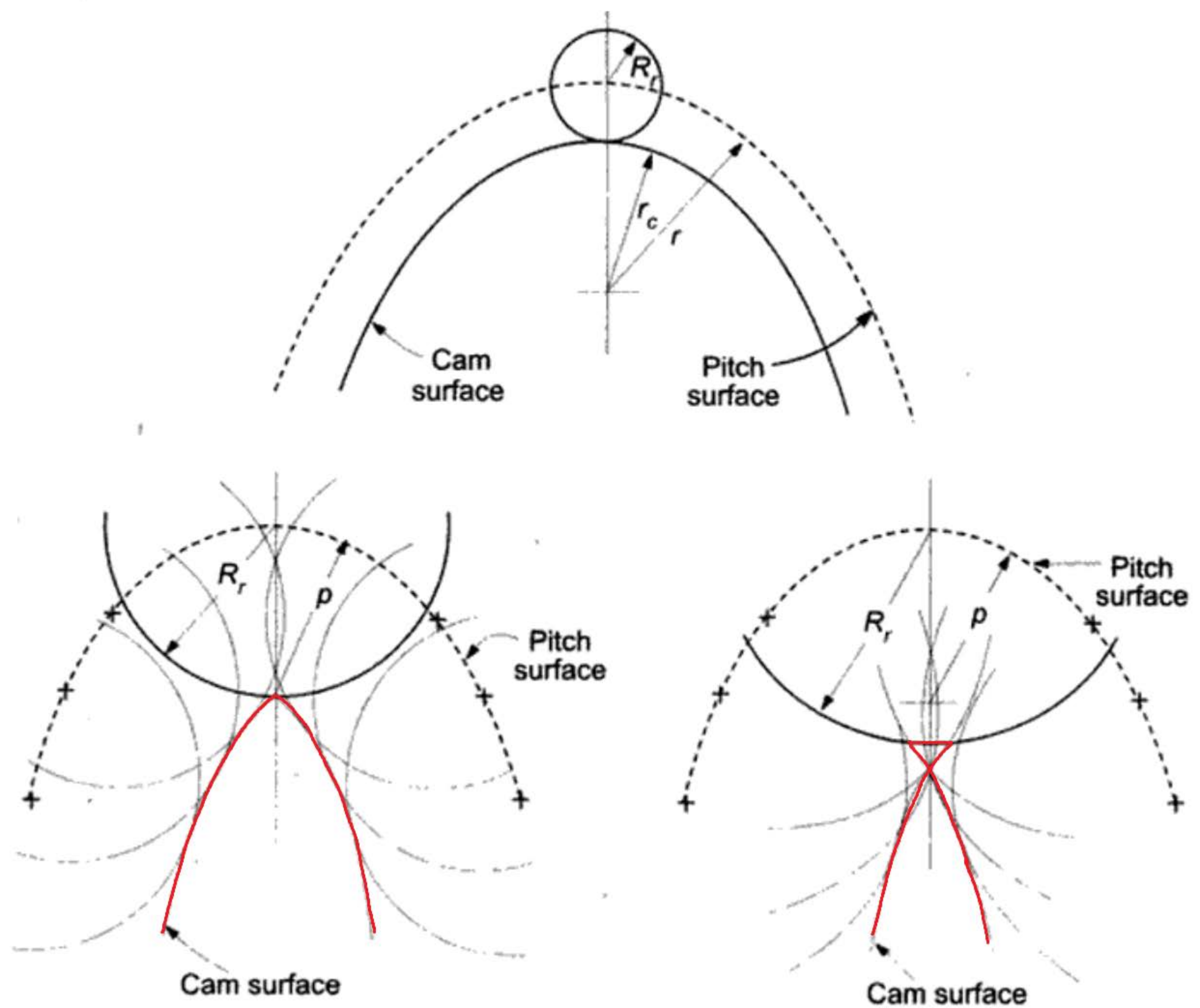
The cam profile is obtained by envelope of all the position of the roller during its movement around the cam.

Undercut and pressure angle

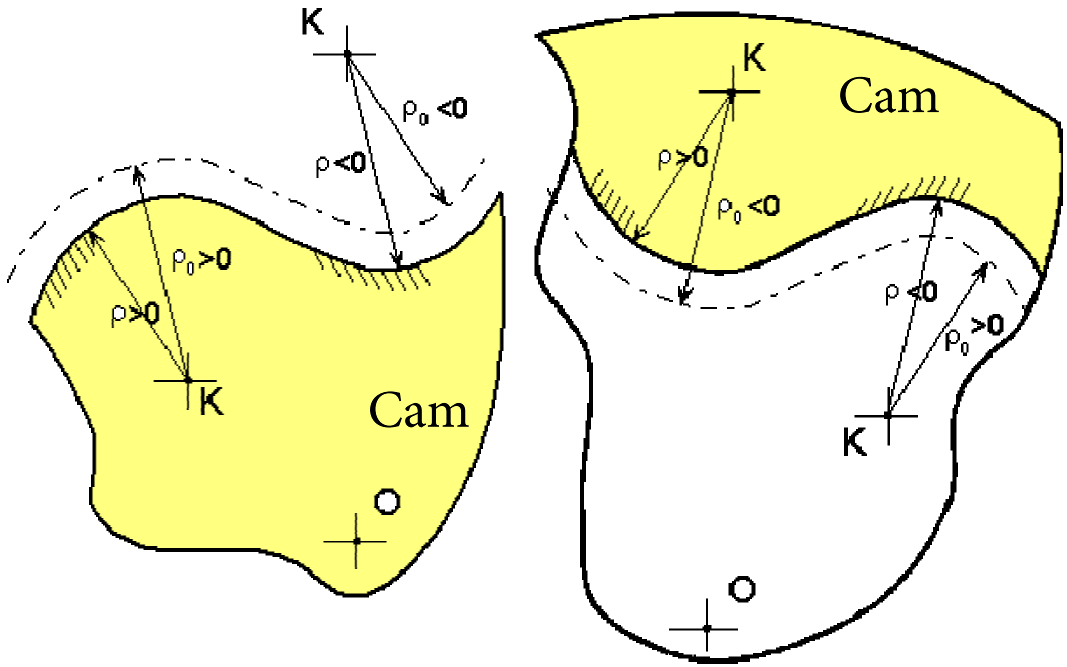
The undercut

There is no restriction on the radius of curvature of the cam profile with a knife-edge follower, but for a roller follower some limitations may arise.

By increasing R_r the cam may become **undercut** and the motion of the follower will not be as prescribed.



The undercut



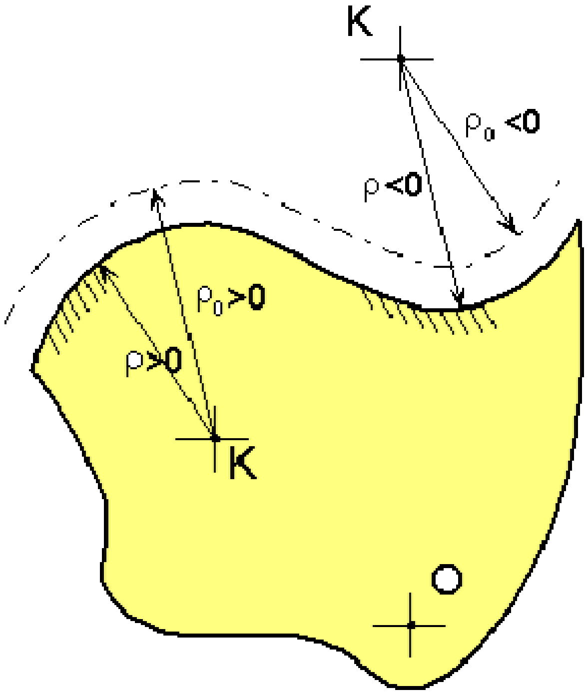
The curvature radius ρ_0 of the pitch profile is positive if the cam center O is into the same part of the curvature center K.

The curvature radius ρ of the cam profile is positive if the cam material is into the same part of the curvature center K. (ie. the cam is convex).

The undercut

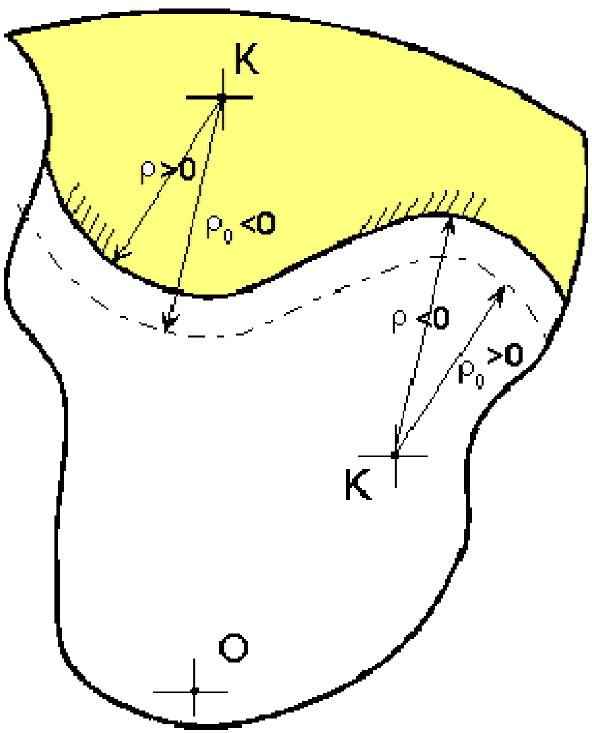
Observing that the curvature radius of the cam profile is equal to: $\rho = \rho_0 - R_r$ where ρ_0 is the curvature radius of the pitch profile. For internal profile we have:

- $\rho_0 > 0$
  1. $R_r < \rho_0$ no undercut ($\rho > 0$, it has the same sign of ρ_0)
  2. $R_r = \rho_0$ limit condition
  3. $R_r > \rho_0$ we have undercut ($\rho < 0$, then it has the opposite sign compare to ρ_0)
- $\rho_0 < 0$
  1. $R_r > \rho_0$ no undercut ($\rho < 0$, it has the same sign of ρ_0)
  2. $R_r = \rho_0$ limit condition
  3. $R_r < \rho_0$ no undercut ($\rho < 0$, it has the same sign of ρ_0)



If the profile is external ($\rho = -(\rho_0 + R_r)$):

- $\rho_0 < 0$
  1. $R_r > \rho_0$ we have undercut ($\rho > 0$ so discorde compare to ρ_0)
  2. $R_r = \rho_0$ limit condition
  3. $R_r < \rho_0$ no undercut ($\rho < 0$ it has the same sign of ρ_0)
- $\rho_0 > 0$
  1. $R_r > \rho_0$ no undercut ($\rho > 0$ it has the same sign of ρ_0)
  2. $R_r = \rho_0$ limit condition
  3. $R_r < \rho_0$ no undercut ($\rho > 0$ it has the same sign of ρ_0)



Summarizing:

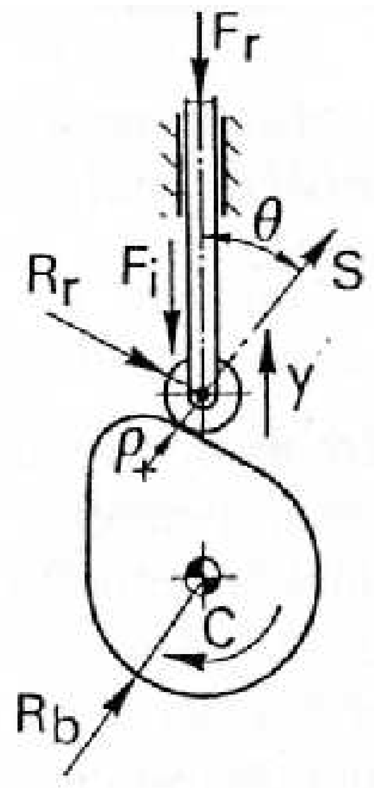
Undercut can be easily observed by considering the product $\rho \cdot \rho_0$. If

- $\rho \cdot \rho_0 > 0$ we don't have undercut
- $\rho \cdot \rho_0 = 0$ there is a limit situation
- $\rho \cdot \rho_0 < 0$ we have undercut

The simplest way to avoid undercut is to decrease the radius of the roller, or to increase the base radius.

Design parameters:

Considering a particular kind of cam, translating roller follower, we describe the principal design parameters involved into the project:



With reference to the figure:

- F_r = load;
- F_i = inertial force;
- R_r = radius of the roller;
- C = motor torque;
- ρ = curvature radius of the cam profile;
- R_b = radius of the base circle;
- S = force between cam and follower;
- V = velocity of the follower;
- θ = pressure angle;

With reference to the quantities described in the previous slide it is easy to write the equilibrium equation of the forces acting on the follower along the moving direction. The result of this equation is:

$$S = \frac{F_r + F_i}{\cos\theta}$$

Knowing the maximum **admissible load** for the roller S_{amm}, it will be:

$$S < S_{amm}$$

and using the Herz relationship, it is possible to evaluate the **contact pressure** between the cam and the roller:

$$p = \sqrt{0.175\frac{SE}{bR_r}\left(1 + \frac{R_r}{\rho}\right)} \quad \Rightarrow \quad p < p_{amm}$$

Cam design guidelines

It is obvious that in cam design it is important to keep the values of p and S limited. Lets consider that:

$$S = f\left(F_i, \theta, \dots\right) \qquad p = p\left(R_r, \rho, b, \dots\right)$$

To limit S we can:

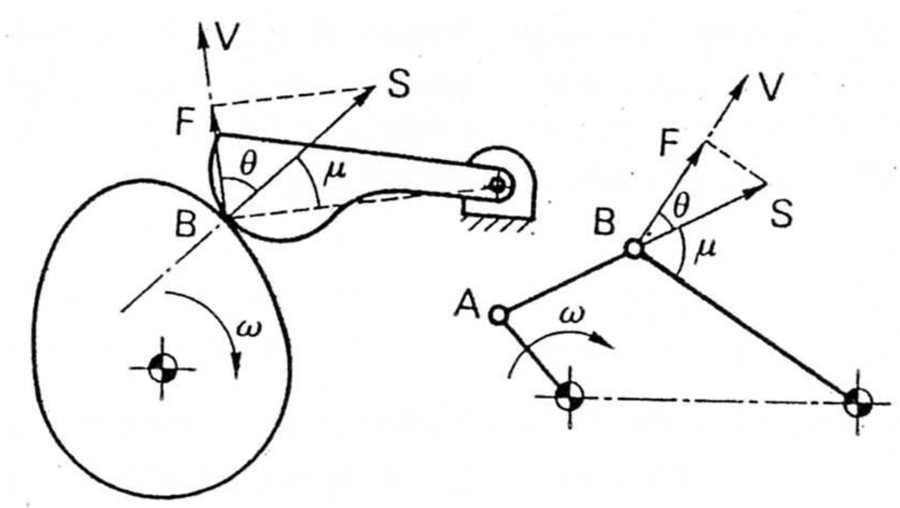
- reduce F_i acting on the motion curve design;
- limit θ by modifying the cam geometry (for example increasing R_b);
- limit the spring force;

while to limit p we can:

- change R_r (roller dimension) and ρ that is linked with the motion curve design and the cam geometry;
- increase the value of b reduces the contact pressure (but amplifies misalignment problems);

The movement transmission

Pressure angle and Transmission angle are two parameters describing the ability of a mechanism to transmit the movement



Pressure angle

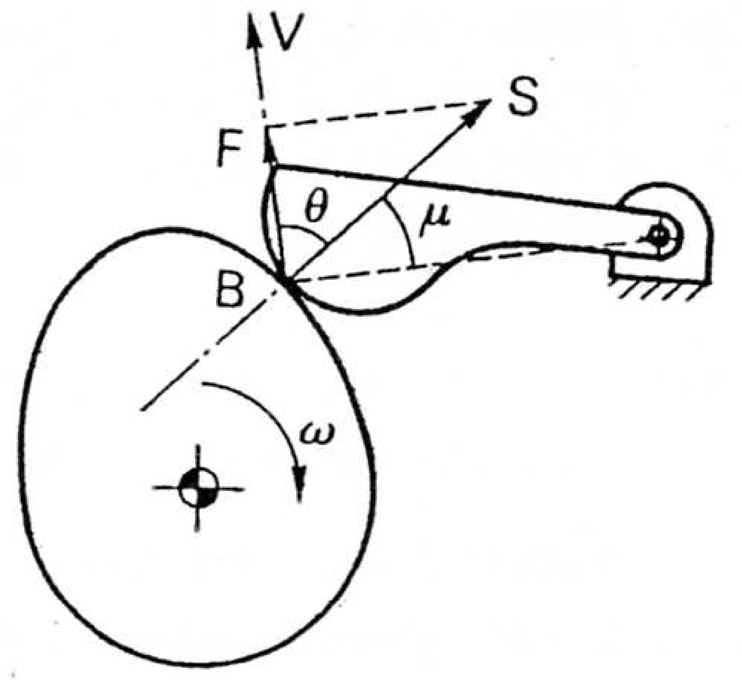
Indicated with the letter θ, it is mainly used for cam mechanisms. It is the lower angle between the direction of the force S and the velocity V of the contact point of the follower.

Transmission angle

Indicated with the letter μ, it is used for linkage mechanism.

Best performance are obtained for small values of θ, while there is no movement when $\theta = \frac{\pi}{2}$ (or $\mu = 0$).

Transmission of motion



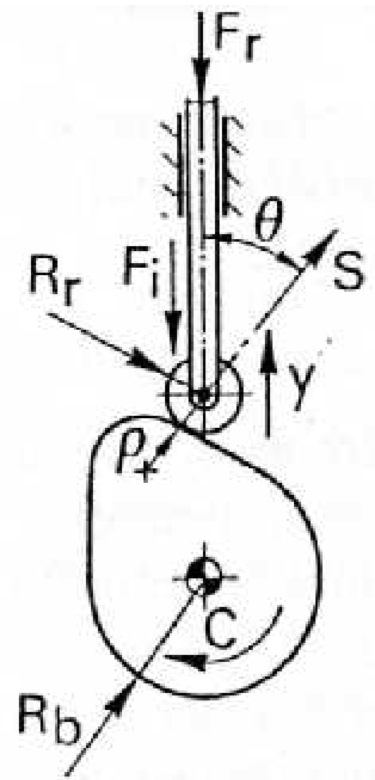
Note that if the pressure angle θ is small, it means the system is able to transmit most of the power from the cam to the follower and the motion is obtained without useless overstress.
The component of the force useful to transmit motion is:

$$S = \frac{F}{\cos\theta}$$

Pressure angle

As already observed the pressure angle is used to evaluate the quality of the motion transmission

Neglecting friction we have:

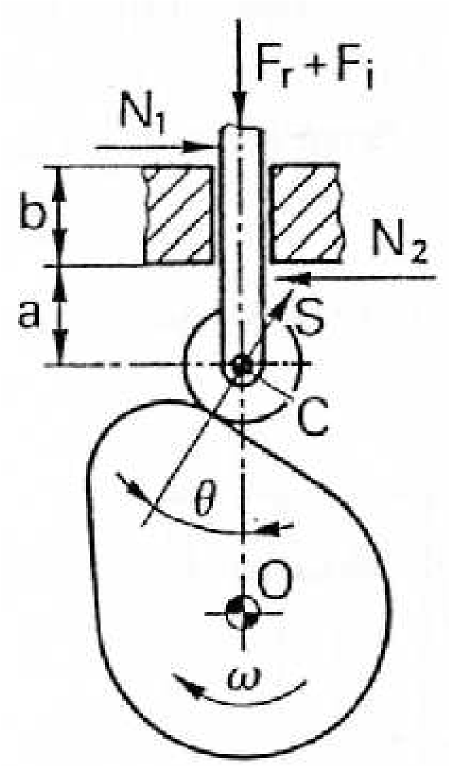


$$S = \frac{F_r + F_i}{\cos\theta} = \frac{F}{\cos\theta}$$

If $\theta = \frac{\pi}{2}$ the force is $S = \infty$. To limit the value of S it is necessary have $\theta < \frac{\pi}{2}$. In particular having $\theta = 45°$ the value of the force S is about 40% more than the value of F.

Pressure angle

With friction the situation changes for the translating follower, in particular:



$$(M_1 = 0) \quad N_2 b = S(a+b)\sin\theta$$
$$N_2 = S\frac{a+b}{b}\sin\theta$$

$$(M_2 = 0) \quad N_1 b = Sa\sin\theta$$
$$N_1 = S\frac{a}{b}\sin\theta$$

$$(R_y = 0) \quad T_1 + T_2 + F = S\cos\theta$$
$$f(N_1 + N_2) + F = S\cos\theta$$
$$S = \frac{F}{\cos\theta - f(\frac{2a+b}{b})\sin\theta}$$

With friction the value of the pressure angle (θ_0) that makes the force S infinite is:

$$\cos\theta - f\left(\frac{2a+b}{b}\right)\sin\theta = 0$$

$$\Downarrow$$

$$\left[f\left(\frac{2a+b}{b}\right)\right]^{-1} = \frac{\sin\theta_0}{\cos\theta_0} = \tan\theta_0$$

Then with translating follower and friction in order not to have infinite value of S it is necessary $\theta < \theta_0$. For example if $f = 0.15$ and $b = 2a$ we will have $\theta_0 = 73°$.

Pressure angle

Substituting the value of the pressure angle's limit θ_0 into the expression of the force S, we obtain:

$$S = \frac{F_r + F_i}{\cos\theta - \frac{\cos\theta_0}{\sin\theta_0}\sin\theta} = \frac{(F_r + F_i)\sin\theta_0}{\cos\theta\sin\theta_0 - \cos\theta_0\sin\theta} = \frac{(F_r + F_i)\sin\theta_0}{\sin(\theta_0 - \theta)}$$

It is possible to calculate the value of θ with which the force S doesn't exceed the 140% of the value of F. We will have:

$$\theta_0 - \theta > 45° \quad \Rightarrow \quad \theta < (\theta_0 - 45°)$$

where θ_0 depends on f, a, b.

Summarizing we have obtained some formulas by means of which we can choose the pressure angle:

1. For swinging cams generally the friction effect is negligible: $\theta_{max} = 45^\circ$
2. For translating cams:

$$\begin{cases} \theta_{max} = 30^\circ & \text{for rise phase} \\ \theta_{max} = 50^\circ & \text{for return phase} \end{cases}$$

   The difference is due to the fact that the friction forces change direction and the block conditions are difference in the rise phase compared to the return one.
3. For flat faced follower if there isn't friction between cam and flat faced follower, θ is always constant and nil.