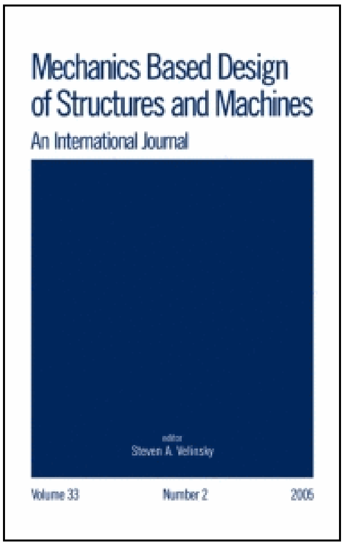

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Hermes Giberti[a]; Simone Cinquemani[a]; Giovanni Legnani[b]
[a] Dipartimento di Meccanica, Politecnico di Milano, Milano, Italy [b] Università degli Studi di Brescia, Brescia, Italy

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A PRACTICAL APPROACH TO THE SELECTION OF THE MOTOR-REDUCER UNIT IN ELECTRIC DRIVE SYSTEMS#

Hermes Giberti[1], Simone Cinquemani[1], and Giovanni Legnani[2]
[1]*Dipartimento di Meccanica, Politecnico di Milano, Milano, Italy*
[2]*Università degli Studi di Brescia, Brescia, Italy*

The selection of a motor-reducer unit in electrical servo-systems has a profound impact on the dynamic performance of a machine. This choice must be made considering all the limits imposed by each component of the system and all the operational constraints. This article proposes a useful and practical methodology for the correct sizing of a motor-reducer unit. The relationships between motor and transmission are investigated by introducing some easily calculated factors useful for comparing all the available motor-reducer couplings and selecting the best solution.

The article suggests an innovative approach for the selection of a motor-reducer unit that involves solving the problem with the use of graphs that would allow showing all the possible alternatives.

Keywords: Electric servo-motor; Motor and reducer pairing; Optimal transmission ratio; Speed reducer.

INTRODUCTION

The evolution of electronics in recent years has led to widespread diffusion of electric drives and their control systems. The ready availability and low cost of electronic devices have allowed rapid diffusion of mechatronic applications, highlighting the need for appropriate methods for selecting a motor-reducer unit. These procedures must be at the same time accurate and easy to use, they should be able to identify the available alternatives, compare them, and help the designer in choosing the most appropriate one for his needs.

The choice of the electric motor required to handle a dynamic load is closely related to the choice of transmission. This operation, in fact, is bound by the limitations imposed by the motor's working range and is subjected to several constraints that depend indirectly on the motor (through its inertia J_M) and on the reducer (through its transmission ratio τ), whose selection is the subject of this article.

A methodology for choosing the gear motor in order to ensure maximum acceleration of the system and reduce execution time for a particular law of motion

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Correspondence: Hermes Giberti, Dipartimento di Meccanica, Politecnico di Milano, 34 via La Masa, Milano, Italy; E-mail: hermes.giberti@polimi.it

is presented in Pasch and Seering (1984). This article introduces the so-called problem of *inertia matching*, showing how best performance can be reached when the inertia of the load, referred to the motor shaft, coincides with the inertia of the motor itself.

In Van De Straete et al. (1998) a procedure for the selection of an AC synchronous motor with permanent magnets and its reducer, for a generic load, is shown. The authors use normalized torques, velocities, and transmission ratios to separate the load from the motor characteristics. By virtue of this normalization, the simulations for one standard motor ($J_M = 1$ [kg m^2]) are applicable to other motors. In Van De Straete et al. (1999a), the same procedure is extended to all types of servo-motors. This methodology produces a chart representing all the usable motors and their corresponding normalized transmission ratios range, but not the available and actually usable commercial transmissions.

In Cusimano (2003) the choice of the motor-reducer unit, required to move a purely inertial load, leads to the identification of the optimal transmission ratio that minimizes the motor root mean square torque. However, depending on the moment of inertia of the chosen motor, the optimal transmission ratio varies, as does the motor root mean square torque. Therefore, the procedure for selecting the motor-reducer unit is iterative and leads to a solution that approximates the desired value of the transmission ratio.

In Cusimano (2005) the choice of the motor-reducer unit is analyzed with regard to the dependence on the law of motion used to operate a generic load, while Van De Straete et al. (1999b) evaluates the gain in motor torque as a consequence of the optimization of trajectories and highlights the effect that a variable transmission ratio has on the performance of the machine.

An in-depth discussion on the problem of motor reducer coupling can be found in Cusimano (2007). Although this work is very accurate, it is extremely hard to use in a real industrial situation. On the other hand, Roos et al. (2006) proposes a simpler approach, that consists in creating a database including the commercial motors and reducers and then trying all possible combinations.

A good trade-off between theory and practice can be found in Legnani et al. (2002), where the choice of motor and reducer is made by comparing two parameters, respectively, related to motor qualities and to load demands.

By developing these concepts, this article identifies a procedure that is rigorous from the theoretical point of view and practical at the same time. Its main advantages are as follows:

1. ease of use, as the steps to follow are defined in a clear and simple way, without requiring extensive mathematical skills or the use of sophisticated tools;
2. development of the problem on the basis of information obtainable from catalogs of motors and transmissions, permitting immediate comparison between all possible pairs.

SERVO-SYSTEM MODEL

A simple but general model of a servo system can be characterized by three key elements: servo-motor, transmission, and load (Fig. 1). The load characteristics are

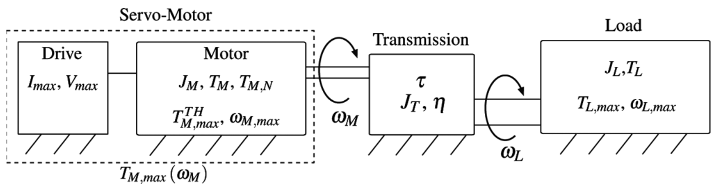

Figure 1 Model of a generic machine.

completely known as they depend on the task, while the motor and the transmission are unknown until they are selected. Symbols used in the article are described in Table 1.

The Load

The power supplied by the motor depends on the external load applied (T_L) and on the inertia acting on the system ($J_L \dot{\omega}_L$). Since different patterns of speed (ω_L)

Table 1 Nomenclature

Symbol	Description
T_M	Motor torque
J_M	motor moment of inertia
$T_{M,rms}$	Motor root mean square torque
$T_{M,N}$	Motor nominal torque
$T^{TH}_{M,max}$	Motor theoretical maximum torque
$T_{M,max}$	Servo-motor maximum torque
ω_M	Motor angular speed
$\dot{\omega}_M$	Motor angular acceleration
T_L	Load torque
J_L	Load moment of inertia
T^*_L	Generalized load torque
$T^*_{L,rms}$	Generalized load root mean square torque
$T_{L,max}$	Load maximum torque
ω_L	Load angular speed
$\dot{\omega}_L$	Load angular acceleration
$\dot{\omega}_{L,rms}$	Load root mean square acceleration
$\tau = \omega_L/\omega_M$	Transmission ratio
τ_{opt}	Optimal transmission ratio
η	Transmission mechanical efficiency
α	Accelerating factor
β	Load factor
τ_{min}	Minimum acceptable transmission ratio
τ_{max}	Maximum acceptable transmission ratio
$\tau_{M,lim}$	Minimum kinematic transmission ratio (defined for each motor)
$\omega_{M,max}$	Maximum speed achievable by the motor
$\omega_{L,max}$	Maximum speed achieved by the load
J_T	Transmission inertia
t_a	Cycle time

and acceleration ($\dot{\omega}_L$) generate different loads, the choice of a proper law of motion is the first project parameter that should be taken into account when sizing the motor-reducer unit. For this purpose, specific texts (Magnani and Ruggieri, 1986; Melchiorri, 2000) are recommended.

Otherwise, it may be that the law of motion has been already defined and therefore represents a problem datum, and not a project variable. Once the law of motion is defined, all the characteristics of the load are known.

The Servo-Motor

Brushless motors are the most common electrical actuators in automation field, and their working range (Fig. 2) could be approximately subdivided into a *continuous* working zone (delimited by motor rated torque) and a *dynamic* zone (delimited by the maximum motor torque $T_{M,max}$). Usually the motor rated torque decreases slowly with motor speed ω_M from T_{M,N_c} to $T_{M,N}$. To simplify the rated torque trend and to take a cautious approach, it is usually considered constant and equal to $T_{M,N}$ up to maximum allowed motor speed $\omega_{M,max}$ (Li and Lipo, 1995). Motor torque $T_{M,N}$ is obtainable from catalogs distributed by motor manufacturers and is defined as the torque that can be supplied by the motor for an infinite time, without overheating.

The maximum torque achieved by the servo-motor $T_{M,max}$ closely depends on the drive associated with it and it is generally different from the theoretical maximum torque $T^{TH}_{M,max}$ of the motor. At low speeds, the constraint introduced by the drive system is related to the maximum current supplied to the motor. Since torque depends on the current, this limit translates into a horizontal line on the motor working field corresponding to a maximum torque which would be different from the theoretical one. At higher speed, this constraint is overcome by the condition on the maximum supportable voltage, which causes a reduction of the motor maximum torque with its speed.

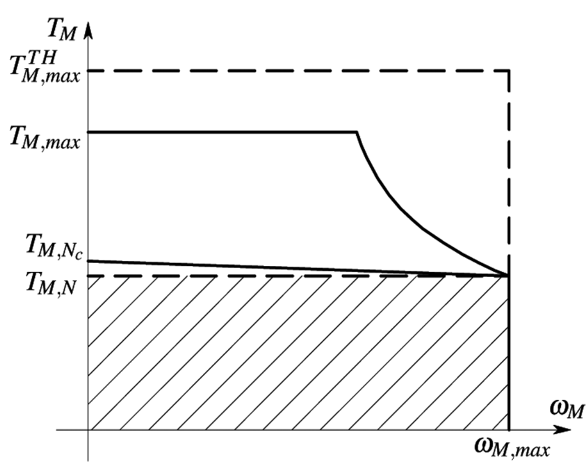

Figure 2 A simplified speed/torque curve of a common brushless motor.

Frequently, in industrial applications, the machine task is cyclical with a period t_a which is normally much smaller than the motor thermal time constant: in this case the motor, because of its capacity and thermal resistance, is unable to follow the fast heat oscillations of the power dissipated, which are then filtered: the temperature of the motor evolves as if it were subject to constant dissipated power, equal to the mean power dissipated in the cycle. Assuming that the heat dissipation is due to the Joule effect and the torque is proportional to the current, the motor behavior can be analyzed through the root mean square (rms) value of T_M defined as

$$T_{M,rms} = \sqrt{\frac{1}{t_a}\int_0^{t_a} T_M^2 \,\mathrm{d}t}, \tag{1}$$

namely the torque, acting steadily over the cycle, which is attributable to the total energy dissipation that actually occurred in the cycle.

As is well known (Van De Straete et al., 1998), the selection of the actuator means checking the following conditions:

- rated motor torque:

$$T_{M,rms} \le T_{M,N}; \tag{2}$$

- maximum motor speed:

$$\omega_M \le \omega_{M,max}; \tag{3}$$

- maximum servo-motor torque:

$$T_M(\omega_M) \le T_{M,max}(\omega_M)\,. \tag{4}$$

The Transmission

The terms on the right side of inequalities (2), (3), (4) are characteristic of each motor. On the other hand, the quantities $T_{M,rms}$, ω_M, and T_M depend on the load and, therefore, on the reducer transmission ratio τ.

In fact, the gear ratio adapts the torque and speed values required by the load to that available from the motor. Because of the mutual dependence between the motor and the transmission the selection of the two components should be performed in parallel.

Moreover, the choice of transmission also depends on other factors, such as the torque applied on the transmission shaft, the maximum achievable speed, potential clearances and the system's mechanical efficiency η (Roos et al., 2006). In this article the transmission is approximated to a system with no moment of inertia and no loss of power ($\eta = 1$). This assumption is frequently considered acceptable (Cusimano, 2005, 2007; Pasch and Seering, 1984; Roos et al., 2006; Van De Straete et al., 1998, 1999a) so long as the efficiency and inertia of the reducer are considered in the final stage of the selection procedure.

Note that in mechatronic applications, where there is a need to ensure high dynamics, the designer's preference is for transmissions with high efficiency and a low moment of inertia, thus approximating the system with an ideal transmission.

THE EFFECTS OF THE TRANSMISSION RATIO ON THE TORQUE AND SPEED REQUIRED OF THE MOTOR

Conditions expressed by inequalities (2), (3), and (4) are well known in the literature and represent the starting point of all the procedures for motor and reducer selection.

In this article these conditions will be rewritten by introducing certain parameters related to motor and load features. It is important to highlight that all the parameters used have a physical meaning and are easily obtained. In this way the designer will have a clear idea of the needs and of the steps to follow.

Once these parameters are calculated, conditions (2), (3), can be easily expressed as functions of τ, thus obtaining a range of acceptable transmission ratios for each acceptable motor. However, since it is difficult to express the constraint imposed on the servo-motor maximum torque (4) as a function of τ, this condition will be checked after the motor and its transmission have been chosen.

The Maximum Speed

Since each motor has a maximum achievable speed ($\omega_{M,max}$), we have

$$\omega_{L,max} \leq \tau\omega_{M,max}, \tag{5}$$

where $\omega_{L,max}$ is the maximum speed achieved by the load. Considering a specific motor, the condition on the maximum achievable speed (3) can be written in terms of τ

$$\tau \geq \tau_{M,lim} = \frac{\omega_{L,max}}{\omega_{M,max}}, \tag{6}$$

where $\tau_{M,lim}$ is defined, for each motor, as the ratio between the maximum speed achieved by the load and the one achievable by the motor. For a specific motor, $\tau_{M,lim}$ is the minimum transmission ratio value that can be employed to drive the given load.

The Motor Torque and Its Root Mean Square Value

The motor torque T_M can be written as

$$T_M = \tau T_L^* + J_M\dot{\omega}_M = \tau T_L^* + J_M\frac{\dot{\omega}_L}{\tau}, \tag{7}$$

where

$$T_L^* = T_L + J_L\dot{\omega}_L \tag{8}$$

is the generalized resistant torque at the load shaft. Equation (7) highlights the dependence of the applied torque on the gear ratio and on the inertia of the motor, while from (8) we can see that all the terms related to the load are known.

The root mean square torque is obtained from (1):

$$T^2_{M,rms} = \int_0^{t_a} \frac{T^2_M}{t_a} \mathrm{d}t = \int_0^{t_a} \frac{1}{t_a} \left(\tau T^*_L + J_M \frac{\dot{\omega}_L}{\tau} \right)^2 \mathrm{d}t. \tag{9}$$

and then

$$T^2_{M,rms} = \tau^2 {T^*}^2_{L,rms} + J^2_M \frac{\dot{\omega}^2_{L,rms}}{\tau^2} + 2J_M (T^*_L \dot{\omega}_L)_{mean}. \tag{10}$$

Inequality (2) can be written as

$$\frac{T^2_{M,N}}{J_M} \geq \tau^2 \frac{{T^*}^2_{L,rms}}{J_M} + J_M \frac{\dot{\omega}^2_{L,rms}}{\tau^2} + 2(T^*_L \dot{\omega}_L)_{mean}. \tag{11}$$

Let's introduce two parameters, the *accelerating factor* of the motor:

$$\alpha = \frac{T^2_{M,N}}{J_M}, \tag{12}$$

which describes the performances of each motor, and the *load factor*:

$$\beta = 2 \left[\dot{\omega}_{L,rms} T^*_{L,rms} + (\dot{\omega}_L T^*_L)_{mean} \right], \tag{13}$$

that defines the performance required by the task. The unit of measurement of both factors is W/s. The coefficient α is exclusively defined by parameters related to the motor and therefore it does not depend on the machine's task: it can be easily calculated for each motor using the information provided in the manufacturer catalogs. It could in fact be quoted in them, thus providing a classification of commercial motors on the basis of this standard. This coefficient can be traced back to quantities used in Van De Straete et al. (1998), Legnani et al. (2002), and Cusimano (2007). Otherwise, the coefficient β depends only on the working conditions (applied load and law of motion) and is a measure that defines the power rate required by the system.

Using α and β, Eq. (11) becomes

$$\alpha \geq \beta + \left[T^*_{L,rms} \left(\frac{\tau}{\sqrt{J_M}} \right) - \dot{\omega}_{L,rms} \left(\frac{\sqrt{J_M}}{\tau} \right) \right]^2. \tag{14}$$

Since the term in brackets is always positive, or null, the load factor β represents the minimum value of the right-hand side of Eq. (14). This means that the motor accelerating factor α must be sufficiently greater than the load factor β, for the inequality (11) to be verified.

The preliminary choice of motor is made by comparing only the values α and β; these values are easily calculated if we know the mechanical properties of the motor and the load features. A motor must be rejected if $\alpha < \beta$, while if $\alpha \geq \beta$ the motor can have enough rated torque if τ is chosen properly (see Section 4).

Figure 3 shows the accelerating factors α_i of commercial motors available for the industrial case analyzed in Section 7 and the load factor β. Motors M1, M2,

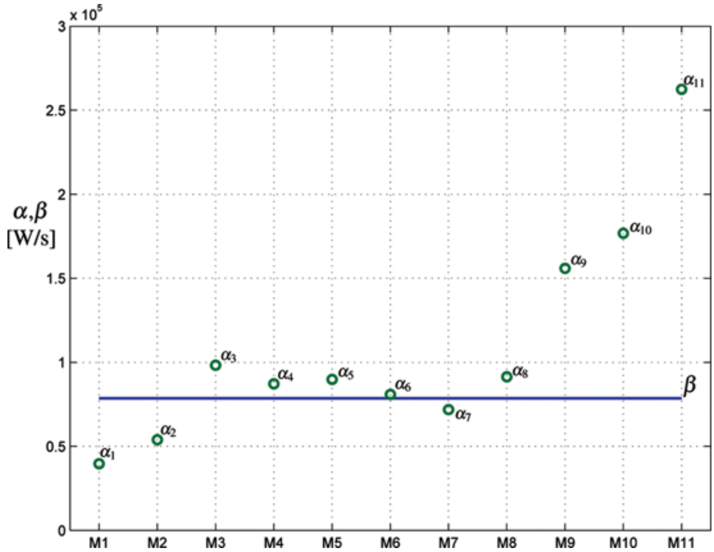

Figure 3 Comparison between the accelerating factors (α_i) and the load factor β of the industrial case discussed in Section 7. Motors M1, M2, and M7 do not pass this test and are immediately rejected.

and M7 have to be discarded. Note that the condition $\alpha_i \geq \beta$ is necessary but not sufficient to satisfy the constraint on the root mean torque, since it assumes that the term in brackets in Eq. (14) is null.

TRANSMISSION SELECTION

As described by Eq. (14), the suitability of a motor depends on the speed ratio τ. Once we have identified some potentially usable motors in the previous step, the gear ratio value has to be chosen carefully, in order to satisfy inequalities (6) and (11).

The Optimal Transmission Ratio, τ_{opt}

As mentioned in (Section 3.2), the right-hand side of Eq. (14) has a minimum if a suitable transmission is chosen, whose gear ratio τ erases the terms in brackets. In this case

$$\tau = \tau_{opt} = \sqrt{\frac{J_M \dot{\omega}_{L,rms}}{T^*_{L,rms}}}. \tag{15}$$

The value τ_{opt} is called *optimal transmission ratio* and, in the case of purely inertial load ($T_L = 0$), the gear ratio $\tau'_{opt} = \sqrt{\frac{J_M}{J_L}}$ introduced in Pasch and Seering (1984) allows maximum system acceleration. Moreover, for this value of τ, $T_{M,rms}$ is also at a minimum (see the Appendix). Under these conditions ($\tau = \tau'_{opt}$) the motor's moment of inertia and the load's moment of inertia, both related to the same transmission shaft, assume the same value. This means that the kinetic energy of the motor and that of the load are equal. In other words, the power supplied by the motor to accelerate the system is equally distributed on the two transmission shafts. In this condition the inertia of the load reflected to the motor shaft equals the motor inertia; it is said that "motor and load are balanced."

Range of Allowed Transmission Ratios

For the reasons illustrated in (3.2), after classifying the motors on the basis of their accelerating factor, it will be better to select those motors whose coefficient α is suitably higher than the load factor β. In this case, for each motor, there is a range of gear ratios satisfying the constraint on motor rated torque. This range can be calculated by solving the biquadratic inequality (14).

$$\left(\frac{{T^*_{L,rms}}^2}{J_M}\right)\tau^4 + (\beta - \alpha - 2T^*_{L,rms}\dot{\omega}_{L,rms})\tau^2 + J_M\dot{\omega}^2_{L,rms} \leq 0, \tag{16}$$

whose solution is

$$\tau_{min} \leq \tau \leq \tau_{max}, \tag{17}$$

with

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T^*_{L,rms}}\left[\sqrt{\alpha - \beta + 4\dot{\omega}_{L,rms}T^*_{L,rms}} \pm \sqrt{\alpha - \beta}\right]. \tag{18}$$

It is evident that an acceptable range of transmission ratios exists only for $\alpha > \beta$. For such ratios the condition expressed in (2) is satisfied.

The range width $\Delta\tau = \tau_{max} - \tau_{min}$ is a function of the difference between the two factors α and β.

$$\Delta\tau = \frac{\sqrt{J_M}}{T^*_{L,rms}}\sqrt{\alpha - \beta}. \tag{19}$$

If $\alpha = \beta$, we can see from Eqs. (14) and (19) that the only gear ratio allowed is τ_{opt}. However, this combination, even if acceptable from a theoretical point of view, should be considered inadequate in practice because of unavoidable approximations and uncertainties.

Moreover, remembering Eq. (6), the constraint imposed on the maximum speed required by the load $\omega_{L,max}$ (Section 3.1) could reduce this range. It becomes

$$\tau_{max} \geq \tau \geq \max\left(\tau_{min}; \tau_{M,lim}\right). \tag{20}$$

Note how the inequality in Eq. (20) could have no solution.

The Appendix shows some frequent cases, representing certain operating conditions of the system, in which the problem is simplified.

SELECTION THROUGH GRAPHICAL REPRESENTATION

To make the selection procedure easy to use, a suitable graphical representation can be employed. The main steps are as follows:

Step 1. Creation of a database containing all the commercially available motors and reducers useful for the application. For each motor the accelerating factor (α_i) must be calculated. Once the database has been completed it can be re-used and updated each time a new motor-reducer unit selection is needed.

Step 2. Calculation of the load factor β, on the basis of the features of the load (T_L^*).

Step 3. Preliminary choice of useful motors: all the available motors can be shown on the graph in Fig. 3 which displays their acceleration factors, while the horizontal line represents the load factor β. The graph is related to the industrial example discussed in Section 7. All the motors for which $\alpha < \beta$ can be immediately rejected, because they cannot supply sufficient torque, while the others are admitted to the next selection phase.

Step 4. Identification of the ranges of useful transmission ratios for each motor preliminarily selected in step 3. For these motors a new graph is produced (Fig. 4) displaying, for each of them, the value of the transmission ratios τ_{max}, τ_{min}, τ_{opt}, and $\tau_{M,lim}$. The graph in Fig. 4 is generally drawn using a logarithmic scale for the y-axis, so τ_{opt} is always the midpoint of the adoptable transmission ratios range. In fact

$$\tau_{opt}^2 = \tau_{min}\tau_{max} \Leftrightarrow \log\tau_{opt} = \frac{\log\tau_{min} + \log\tau_{max}}{2}.$$

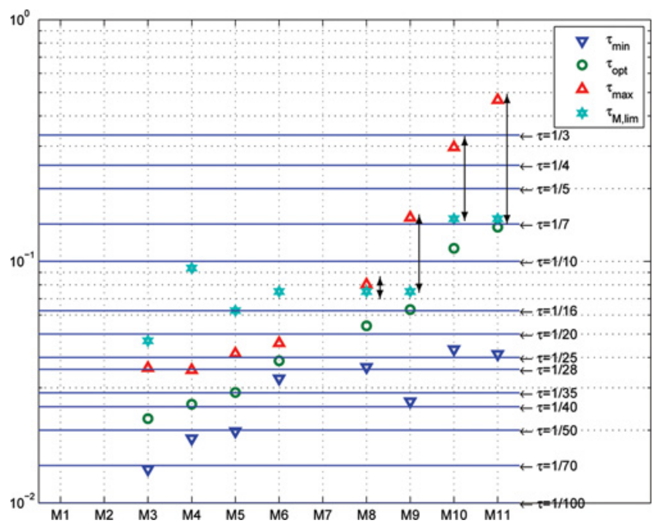

Figure 4 Overview of available motor-reducer couplings for the industrial example discussed in Section 7.

Table 2 Combination of suitable motors and speed reducers for the industrial example discussed in Section 7

Motor	Speed reducer
M9	$\tau = 1/10$, $\tau = 1/7$
M10	$\tau = 1/5$, $\tau = 1/4$
M11	$\tau = 1/5$, $\tau = 1/4$, $\tau = 1/3$

From Eqs. (6) and (17), a motor is acceptable if there is at least a transmission ratio τ for which Eq. (20) is verified. These motors are highlighted by a vertical line on the graph.

In the example in Fig. 4, only these motors pass the test: M8, M9, M10, and M11.

Step 5. Identification of useful commercial speed reducers: the speed reducers available are represented by horizontal lines. If one of them intersects the vertical line of a motor, this indicates that the motor can supply the required torque if that specific speed reducer is selected. Table 2 sums up the acceptable combination of motors and speed reducers for the case shown in Fig. 4. These motors and reducers are admitted to the final selection phase.

Step 6. Optimization of the selected alternatives: the selection can be completed using such criteria as economy, overall dimensions, space availability, or any other criteria considered important depending on the specific needs.

Step 7. Checks (see Section 6).

CHECKS

On the basis of the content of Sections (3.2) and (4.2), a list of motors with the corresponding speed reducers that satisfy the constraints described in Eqs. (2) and (3) can be compiled. These candidates must now be considered in a final check. For each combination of motor and reducer unit, their moment of inertia J_M and J_T and the transmission mechanical efficiency η are fully known.

Let's define W_M and W_L, respectively, as the power upstream and downstream of the transmission. When the power flows from the motor to the load it is said that the machine works with *direct power flow*, otherwise, the process is described as *reverse*. Transmission power losses are described by two different mechanical efficiency values $\eta_d \leq 1$ and $\eta_r \leq 1$, where

$$\eta(t) = \begin{cases} \eta_d & \text{if } W_r > 0 \text{ (direct power flow functioning)} \\ 1/\eta_r & \text{if } W_r < 0 \text{ (reverse power flow functioning).} \end{cases} \tag{21}$$

Equation (7) can be improved as

$$T_M = (J_M + J_T)\,\frac{\dot{\omega}_L}{\tau} + \frac{\tau T_L^*}{\eta}. \tag{22}$$

Now it's possible to check:

- The maximum torque supplied by the servo-motor for each angular velocity achieved:

$$T_{M,max}(\omega_M) \geq \max\left|\frac{\tau T_L}{\eta} + \left(\frac{J_M + J_T}{\tau} + \frac{J_L \tau}{\eta}\right)\dot{\omega}_L\right| \quad \forall \omega. \tag{23}$$

This test is easily performed by superimposing the motor torque $T_M(\omega_M)$ on the motor torque/speed curve (cf. Fig. 6).

- The effect of the transmission's mechanical efficiency (η) and its moment of inertia (J_T) on the root mean square torque:

$$T_{M,N}^2 \geq T_{M,rms}^2 = \int_0^{t_a} \frac{T_M^2}{t_a}\mathrm{d}t = \int_0^{t_a} \frac{1}{t_a}\left((J_M + J_T)\frac{\dot{\omega}_L}{\tau} + \frac{\tau T_L^*}{\eta}\right)^2 \mathrm{d}t. \tag{24}$$

- The resistance of the transmission as supplied by the manufacturer.

Note that the effect of nonidealness of the transmission may already be introduced in Eq. (7). However, this would make it impossible to define β as dependent only on the load. This would rob the procedure of the immediacy desired, with only marginal improvement in the results obtained.

AN INDUSTRIAL EXAMPLE OF MOTOR REDUCER SELECTION

Figure 5 shows a computer numerical control (CNC) wire bending machine. The system automatically performs the task of bending in the plane, or 3D, a wire (or tape) giving it the desired geometry. The machine's operation is simple: semifinished material is stored in a coil and is gradually unrolled by the unwinding unit. The straightened wire is guided along a conduit to the machine's bending unit, which consists of a rotating arm on which one or more bending heads are mounted.

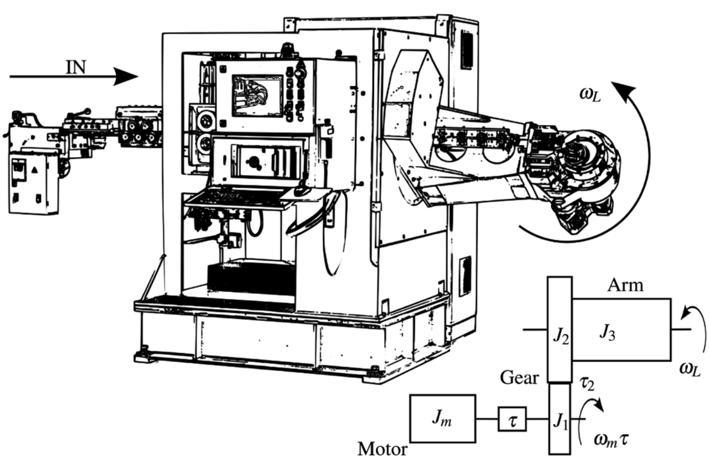

Figure 5 CNC wire bending machine.

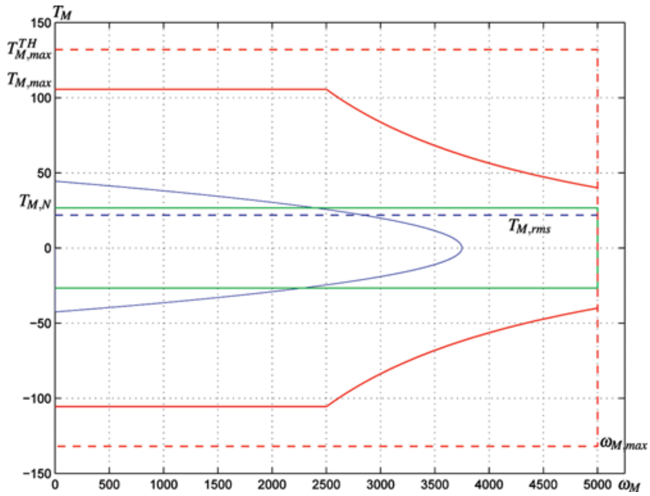

Figure 6 Checkouts on maximum and nominal torque.

Each head is positioned in space by a rotation of the arm around the axis along which the wire is guided in order to shape it in all directions.

The production capacity of the machine is related to the functionality of the heads, while bending productivity depends strongly on arm speed, which allows the heads to reach the position required for bending. The design of the system actuating the rotating arm (selection of motor and speed reducer) is therefore one of the keys to obtaining high performance.

Consider now only the bending unit: the motor, with its moment of inertia J_M, is connected through a planetary reducer with transmission ratio τ to a pair of gear wheels that transmits the rotation to the arm.

The pair of gear wheels has a ratio $\tau_2 = 1/5$ which is dictated by the overall dimension of the gearbox and cannot be modified. Putting $J_1 = 0.0076\,\text{kg}\,\text{m}^2$, $J_2 = 1.9700\,\text{kg}\,\text{m}^2$, and $J_3 = 26.5\,\text{kg}\,\text{m}^2$, respectively, as the moment of inertia referred to the axes of rotation of the two wheels and of the arm, the comprehensive moment of inertia referred to the output shaft of the planetary gear is

$$J_L = J_1 + (J_2 + J_3)\tau_2^2 = 1.1464 \text{ kg m}^2.$$

Since the load is purely inertial (in the selecting phase frictions are not considered), the load factor can easily be calculated using Eq. (A2):

$$\beta = 4J_L\dot{\omega}_{L,rms}^2, \tag{25}$$

where $\dot{\omega}_{L,rms}$ is a function of the law of motion used. The choice of the law of motion depends on the kind of operation requested and, in the most extreme case, it consists of a rotation of $h = 180°$ in $t_a = 0.6\,[\text{s}]$. After this, a stop of $t_s = 0.2\,[\text{s}]$ before the next rotation is normally scheduled.

The value of $\dot{\omega}_{L,rms}$ can be expressed through the mean square acceleration coefficient ($c_{a,rms}$) using the equation:

$$\dot{\omega}_{L,rms} = c_{a,rms}\frac{h}{t_a^2}\frac{1}{\tau_2}\sqrt{\frac{t_a}{t_a + t_s}}.$$

As is known (Van De Straete et al., 1999b), the minimum mean square acceleration law of motion is the cubic equation whose coefficient is $c_{rms} = 2\sqrt{3}$. Moreover, this law of motion has the advantage of higher accelerations, and therefore high inertial torques, corresponding to low velocities. Substituting numerical values in Eq. (25) we get: $\beta = 7.8573 \cdot 10^4\,\mathrm{W/s}$. Considering the same law of motion, maximum acceleration and maximum speed can easily be obtained by

$$\dot{\omega}_{L,max} = c_a\frac{h}{t_a^2}\frac{1}{\tau_2} \simeq 261.8\ [\mathrm{rad/s^2}]; \quad \omega_{L,max} = c_v\frac{h}{t_a}\frac{1}{\tau_2} \simeq 39.3\ [\mathrm{rad/s}], \tag{26}$$

where $c_a = 6$ and $c_v = 1.5$.

If we know the load factor β and have selected the motors and transmissions available from catalogs, the graph shown in Fig. 4 can be plotted. Minimum and maximum transmission ratios are obtained using the simplified expression for the purely inertial load case, as shown in Eq. (A3):

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T^*_{L,rms}}\left[\sqrt{\alpha} \pm \sqrt{\alpha - \beta}\right],$$

while the optimum and the minimum kinematic transmission ratios, respectively, are calculated using Eqs. (15) and (6).

Available motors for this application are synchronous sinusoidal brushless motors (produced by "Mavilor," *http://www.mavilor.es/*). Manufacturer's catalogs give information on motor starting torque and maximum torque, while the nominal torque can be obtained from the working zone shown. The motor moment of inertia is increased to consider the resolver. The motors considered have no brake. Commercial transmissions considered for the selection are planetary reducers (produced by "Wittenstein," *http://www.wittenstein.it/*).

The graph in Fig. 4 shows all the available pairings between the motors and transmissions considered. Three of the 11 motors (M1, M2, and M7) are immediately discarded, since their accelerating factors α are too small compared with the load factor β. Motors M3, M4, M5, and M6 are eliminated because their maximum speed is too low. Suitable motors are M8, M9, M10, and M11. The selection can be completed evaluating the corresponding available commercial speed reducers whose ratio is within the acceptable range. Motor M8 is discarded since no transmission can be coupled to it. Suitable pairings are shown in Table 2.

The final selection can be performed using the criterion of cost: the cheapest solution is motor M9 and a reducer with a transmission ratio $\tau = 1/10$. The main features of the selected motor (model Mavilor BLS 144) and transmission (model Alpha SP+140) are shown in Table 3.

Figure 6 shows the required motor torque as a function of speed during the working cycle. It has been calculated considering the inertia of both the motor

Table 3 Main features of selected motor and transmission

Motor M9	
Moment of inertia	$J_M = 0.0046$ [kg m^2]
Nominal torque	$T_{M,N} = 26.7$ [Nm]
Maximum Torque	$T^{TH}_{M,max} = 132$ [Nm]
Maximum achievable speed	$\omega_{M,max} = 5000$ [rpm]
Speed reducer $\tau = 1/10$	
Moment of inertia	$J_T = 5.8 \cdot 10^{-4}$ [kg m^2]
Nominal torque	$T_{T,N} = 220$ [Nm]
Maximum Torque	$T_{T,max} = 480$ [Nm]
Maximum endurable speed	$\omega_{T,max} = 4000$ [rpm]
Nominal speed	$\omega_{T,N} = 2600$ [rpm]
Mechanical efficiency	$\eta = 0.97$

and the gearbox and the mechanical efficiency of the transmission as in Eq. (22). Since the mechanical efficiency of the speed reducer in reverse power flow mode is not available, it is assumed to be equal to that in direct power flow. To verify the condition on the maximum torque reported in Eq. (23), the curve has to be contained within the dynamic working field. Note how the maximum torque achieved by the motor is limited by the drive associated with it. We can see from Fig. 6 that the maximum torque condition is verified.

The motor root mean square torque can be updated considering the inertia of the transmission and its mechanical efficiency. The new value is shown on the graph and is lower than the nominal motor torque in Eq. (24).

Finally, checks should be carried out on the reducers following the manufacturer's guidelines. In this case they mainly consist of verifying that both the maximum and the nominal torque applied to the transmission incoming shaft are lower than the corresponding limits shown in the catalog ($T_{T,max}$, $T_{T,N}$).

$$T_{max} \simeq 300 \text{ [Nm]} < T_{T,max} = 480 \text{ [Nm]},$$
$$T_n \simeq 150 \text{ [Nm]} < T_{T,N} = 220 \text{ [Nm]}.$$

In addition, the maximum and the mean angular speed of the incoming shaft have to be lower than the corresponding limits on velocity ($\omega_{T,max}$, $\omega_{T,N}$).

$$n_{max,rid} \simeq 3750 \text{ [rpm]} < \omega_{T,max} = 4000 \text{ [rpm]},$$
$$n_{mean,rid} \simeq 1873 \text{ [rpm]} < \omega_{T,N} = 2600 \text{ [rpm]}.$$

The selected motor-transmission pairing satisfies all the checks and provides margins for both the engine ($\approx$20% on the nominal torque) and the reducer, allowing the resistant load not taken into consideration earlier to be overcome.

CONCLUSIONS

The article presents a practical procedure for the selection of the most suitable motor- and speed-reducer pairing to drive a machine. The methodology is based on information achievable on manufacturers' catalogs and it is carried out through a graphical representation of the characteristics of the machine and of available motors and speed reducers. The designer has then a useful procedure to compare all the feasible solutions and to choose the best one. Theoretical aspects are applied to a case study, showing the easiness of use of the methodology and its practical approach.

APPENDIX

In this section some frequent cases, representing specific operating conditions of the system in which the problem is simplified, are reported. For these cases, the simplified formula of the load factor β and the range of available transmission ratios are shown.

Negligible Motor Moment of Inertia ($J_M \ll J_L$)

Where, regardless of the gear ratio, the motor moment of inertia is negligible compared to the applied load and its actions of inertia, on the slow shaft of the transmission the applied torque can be approximated to $T_M^* = T_M/\tau$.

It must match the torque required by the load:

$$T_L^* = T_L + J_L \dot{\omega}_L. \tag{A1}$$

To derive maximum advantage from the reducer, the transmission ratio τ should dovetail with the minimum allowed value $\tau_{M,lim}$. In this case, if $J_M/\tau_{M,lim}^2 \ll J_L$, the assumption is acceptable. Once we have defined the root mean square torque $T_{L,rms}^* = \sqrt{\int T_L^{*2}\, \mathrm{d}t/t_a}$, the motor will be chosen so that

$$T_{M,N} > \tau_{lim} T_{L,rms}^*.$$

Purely Inertial Load ($T_L = 0$)

If the load is purely inertial ($T_L = 0$), Eq. (A1) becomes:

$$T_{L,rms}^* = J_L \dot{\omega}_{L,rms}$$

then

$$\tau_{opt} = \sqrt{\frac{J_m}{J_r}}$$

whereas the load factor is

$$\beta = 4 J_L \dot{\omega}_{L,rms}^2. \tag{A2}$$

For each motor, the values of τ_{min} and τ_{max} can be obtained by

$$\tau_{min}, \tau_{max} = \frac{\sqrt{J_M}}{2T_{L,rms}^*}\left[\sqrt{\alpha} \pm \sqrt{\alpha - \beta}\right]. \tag{A3}$$

Constant Load ($T_L = cost$)

In the frequent case in which T_L is constant during the task, defining

$$\gamma = \sqrt{1 + \left(\frac{T_L}{J_L \dot{\omega}_{L,rms}}\right)^2},$$

it is easy to obtain

$$\begin{aligned} T^*_{L,rmd} &= J_L \dot{\omega}_{L,rms} \gamma, \quad (\dot{\omega}_L T^*_L)_{mean} = J_L \dot{\omega}^2_{L,rms}, \\ \tau_{opt} &= \sqrt{\frac{1}{\gamma}\left(\frac{J_M}{J_L}\eta\right)}, \quad \beta = 2 J_L \omega^2_{L,rms}(1+\gamma). \end{aligned} \tag{A3}$$

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