POLITECNICO
MILANO 1863

mecc

Functional Mechanical Design
Actuators for automatic machines (1/2)

Simone Cinquemani

Introduction

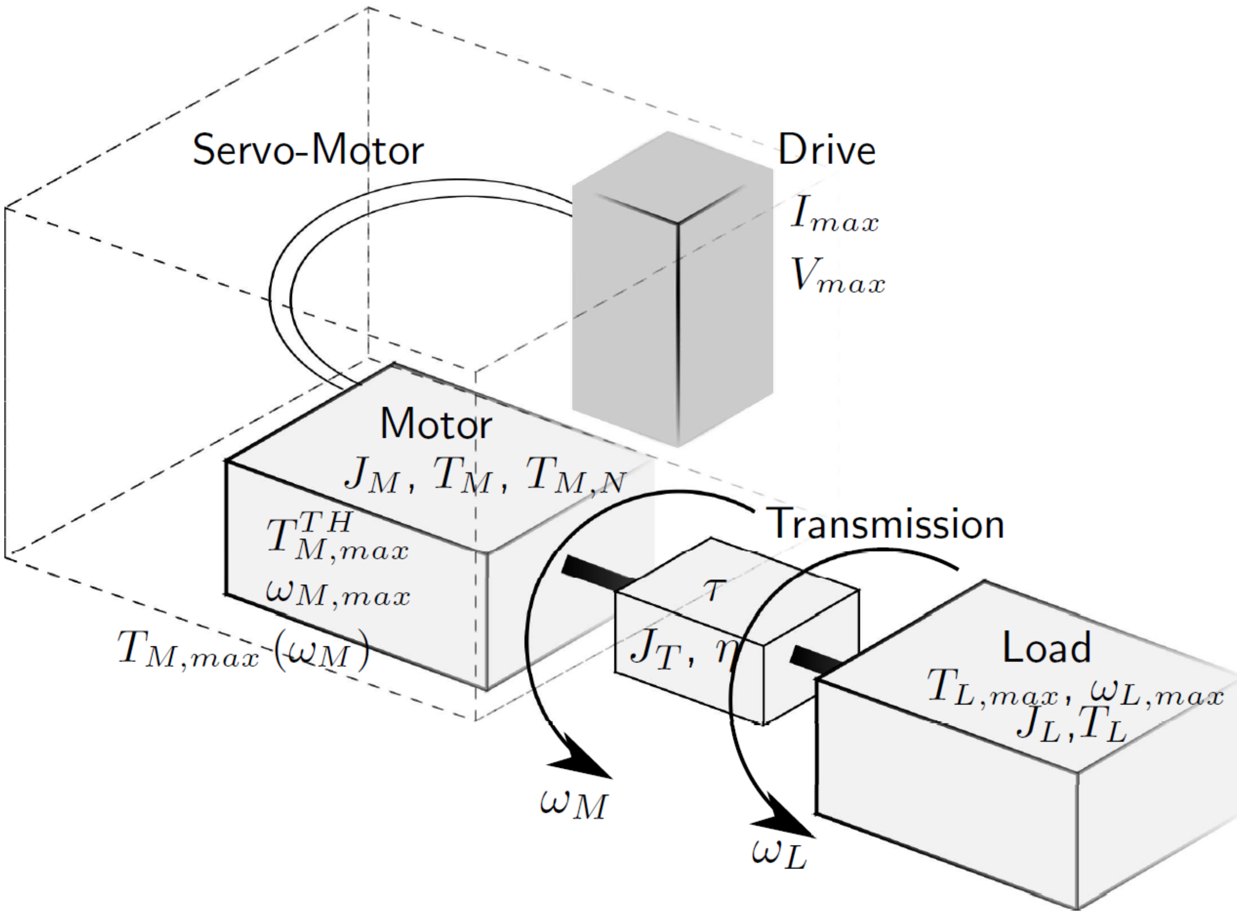
Actuators convert a given kind of energy into mechanical energy. Depending on the physical principle on which the energy conversion is based, different classes of actuators can be identified. The actuator classes which are most relevant to an industrial point of view are:

- electromagnetic actuators
- electrostatic actuators
- hydraulic actuators
- pneumatic actuators
- thermal expansion actuators

There are other classes of emerging actuator technologies like:

- piezoelectric actuators
- magnetostrictive actuators,
- thermally-induced shape memory alloy actuators
- electroactive polymer actuators based on dielectric elastomers

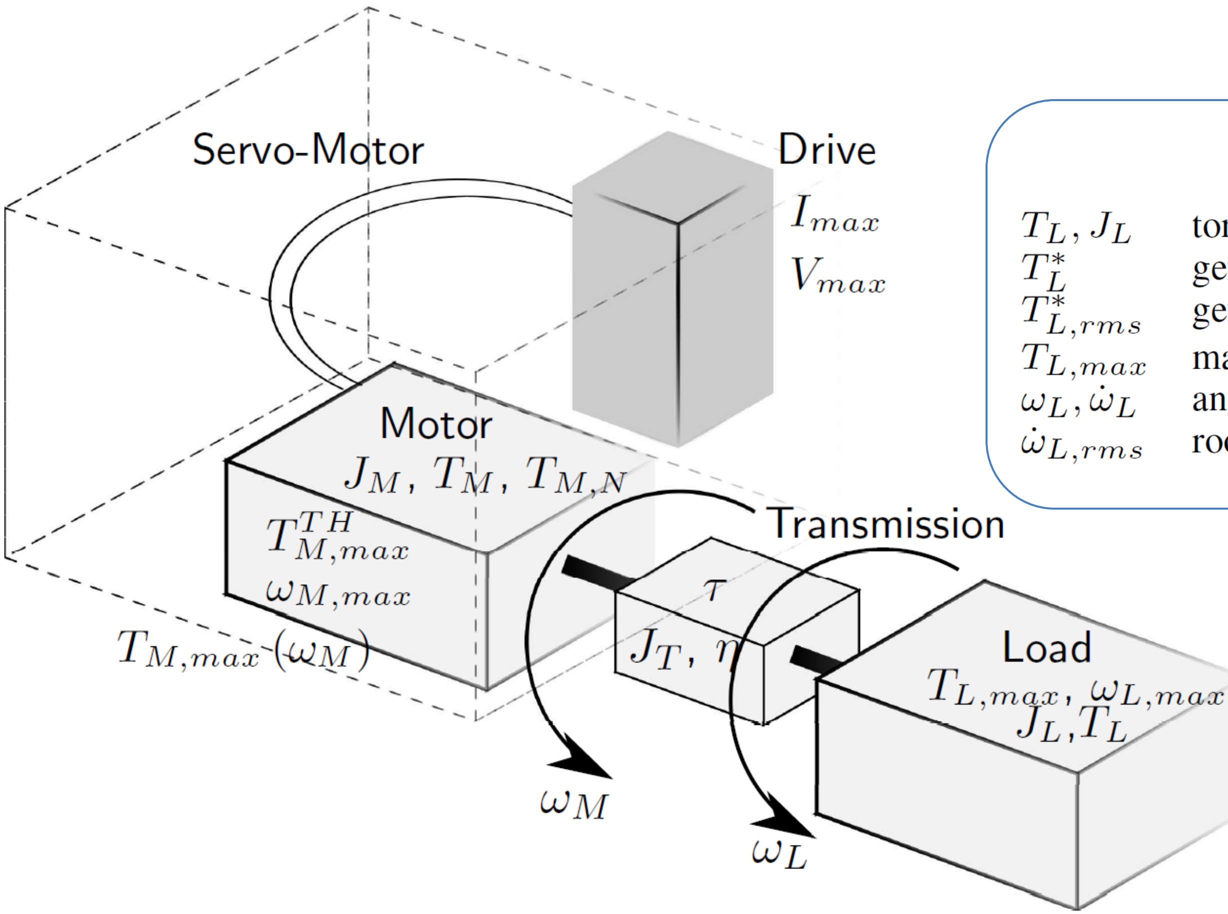
Other, less common, actuator classes are magnetic shape-memory alloys, electrochemical actuators as well as ionic electroactive polymers, or semi-active devices like magnetorheological and electrorheological devices that are not actuators in the above mentioned sense but they are studied for particular industrial applications.



A scheme of a generic actuated machine. There are three main elements:

- the servo motor
- the transmission
- the load

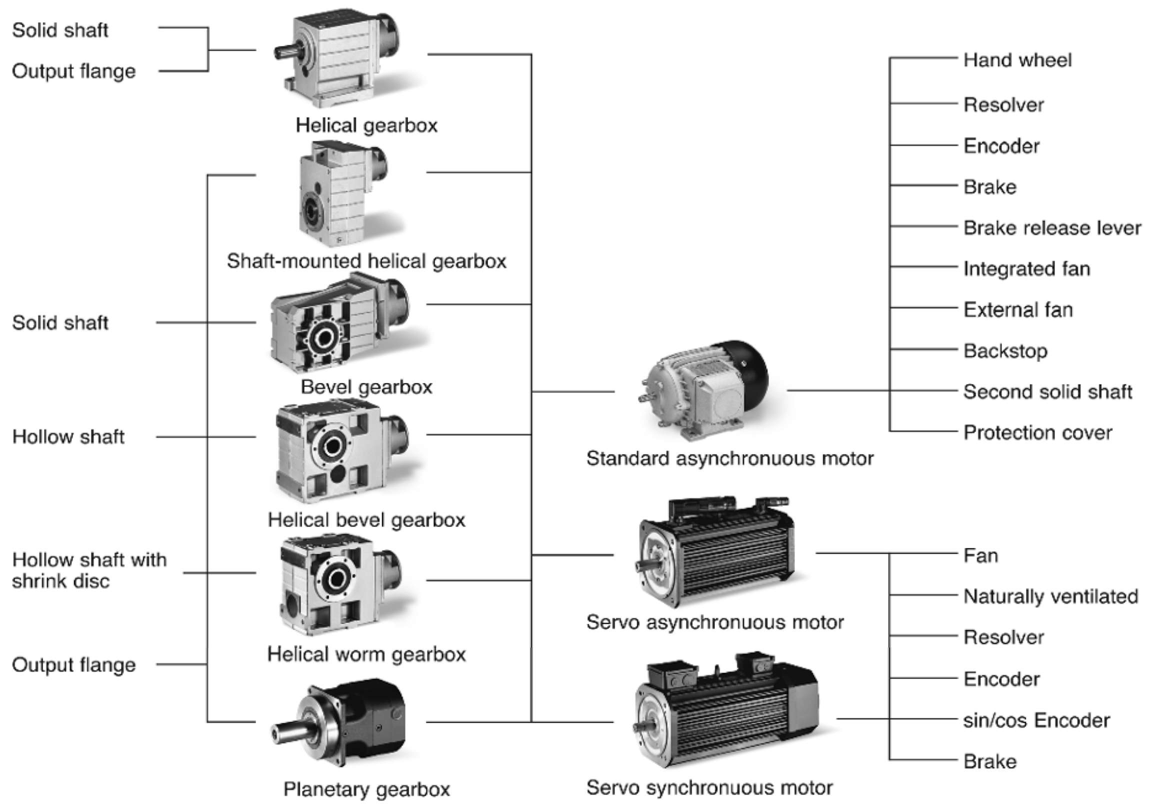
Introduction

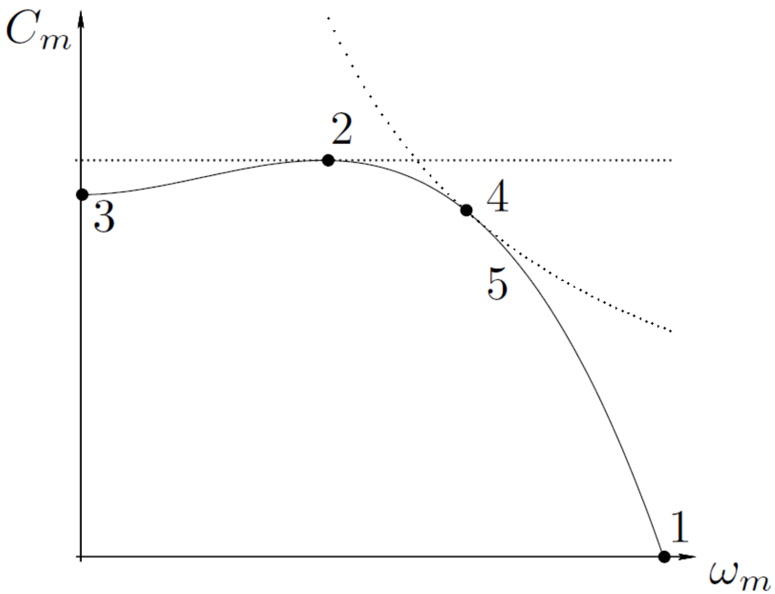


LOAD

T_L, J_L	torque and moment of inertia
T^*_L	generalized torque
$T^*_{L,rms}$	generalized root mean square torque
$T_{L,max}$	maximum torque
ω_L, $\dot{\omega}_L$	angular speed and angular acceleration
$\dot{\omega}_{L,rms}$	root mean square acceleration

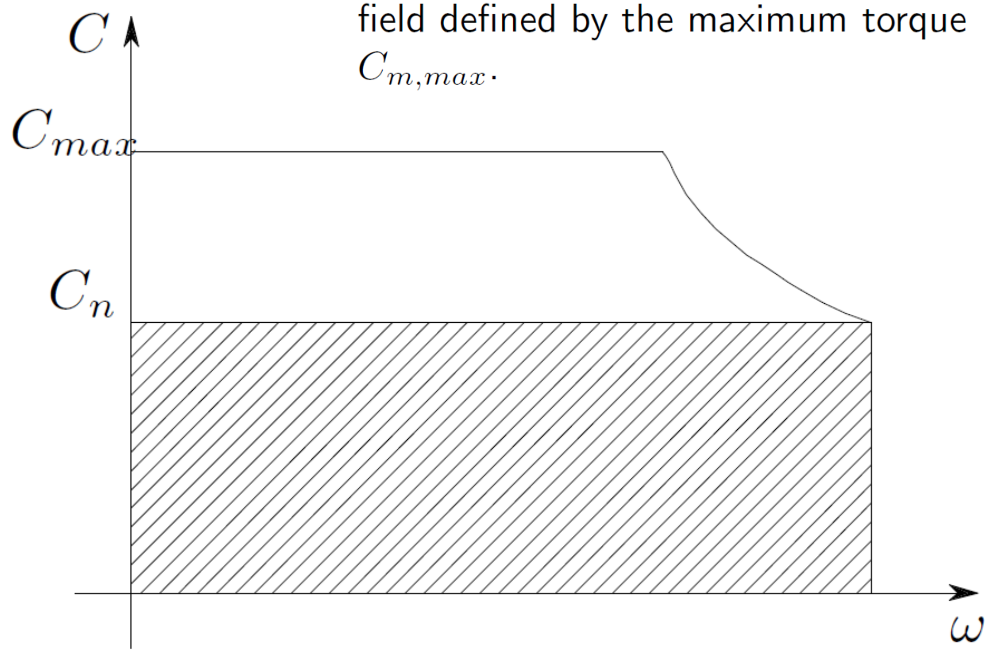
Sizing the gearmotor requires to find a suitable (if we can «the best») **combination of motor and speed reducer** that allows to carry out the desired task.





1: operating point to which the motor tends to bring itself in the absence the motion resistances.
2: maximum torque point
3: stall torque point
4: maximum power point
5: nominal functioning point

The performance curve of a motor can be summarized as shown in the figure: continuous filed defined by the nominal torque C_n and a limit field defined by the maximum torque $C_{m,max}$.



Usually C_n is constant until the maximum available velocity $\omega_{m,max}$, while the torque $C_{m,max}$ decreases starting from a certain value of ω_m.

Motors

The principal information that needs to be looked for in a catalogue is:

- Nominal data: velocity, torque e power.
- rotor inertia and inertia of the auxiliary elements
- no load speed, maximum torque, stall torque, maximum velocity, maximum power.
- Performance curve.
- Insulation class.
- Protection class.

Motors

BLS ~ Technical Specifications

ALL CHARACTERISTICS MEASURED AT 25°C AMBIENT TEMPERATURE	SYMBOLS	UNITS	BLS-40		BLS-55		BLS-71		BLS-72		BLS-73		BLS-74	
			110 VAC	220 VAC	220 VAC	400 VAC	220 VAC	400 VAC	220 VAC	400 VAC	220 VAC	400 VAC	220 VAC	400 VAC
MAX MECHANICAL SPEED	n	rpm	10,000		10,000		11,000		11,000		11,000		11,000	
STALL TORQUE (1) ±10%	M_S	Nm	0.36	0.36	0.7	0.7	0.8	0.8	1.9	1.9	2.7	2.7	3.4	3.4
STALL CURRENT	I_S	A	2.57	1.24	1.4	0.77	2.11	1.13	3.96	2.37	3.91	2.2	4.2	2.25
PEAKTORQUE ±10%	M_J	Nm	1.44	1.44	2.8	2.8	3.2	3.2	7.6	7.6	10.8	10.8	13.6	13.6
TORQUE-WEIGHT RATIO	T_W	Nm/kg	0.6	0.6	0.5	0.5	0.53	0.53	1	1	1.17	1.17	1.21	1.21
EMF CONSTANT ±5%	K_E	Vs/rad	0.08	0.17	0.29	0.53	0.22	0.41	0.28	0.46	0.4	0.71	0.47	0.87
TORQUE CONSTANT ±5%	K_T	Nm/A	0.14	0.29	0.5	0.91	0.38	0.71	0.48	0.8	0.69	1.23	0.81	1.51
RELUCTANCE TORQUE (*)	T_R	Nm	<6%		<4%		<3.5%		<3.5%		<3.5%		<3.5%	
WINDING RESISTANCE ±5%	R	Ω	6	24.4	14.7	47	10.7	33.8	5.3	15.5	6.4	18.9	5.7	18.6
WINDING INDUCTANCE ±5%	L	mH	3.23	12	18.6	61	7.4	24	5.4	13.2	6.4	20	6.7	22
ROTOR INERTIA	J	$kg\ m^2\ 10^{-3}$	0.0024	0.0024	0.017	0.017	0.027	0.027	0.051	0.051	0.074	0.074	0.097	0.097
MECHANICAL TIME CONSTANT	τ_M	ms	1.19	1.19	1.72	1.66	3.46	3.14	2.01	2.15	1.72	1.6	1.45	1.37
ELECTRICAL TIME CONSTANT	τ_E	ms	0.54	0.49	1.27	1.3	0.69	0.71	1.02	0.85	1	1.06	1.18	1.18
THERMAL TIME CONSTANT	τ_{TH}	s	1,190	1,190	1,120	1,120	1,100	1,100	1,280	1,280	1,560	1,560	1,990	1,990
THERMAL RESISTANCE	R_{TH}	°C/W	1.53	1.53	1.99	2.06	1.21	1.34	0.69	0.69	0.59	0.63	0.57	0.61
MASS	M	kg	0.6	0.6	1.4	1.4	1.5	1.5	1.9	1.9	2.3	2.3	2.8	2.8
RADIAL LOAD (at mid-length of shaft)	F_R	N	150		250		216		245		274		314	
AXIAL LOAD	F_A	N	80		100		98		98		98		98	
INSULATION			CLASS-F		CLASS-F		CLASS-F		CLASS-F		CLASS-F		CLASS-F	
PROTECTION			IP-65		IP-65		IP-65		IP-65		IP-65		IP-65	
(1) With an aluminium heat sink plate			300x300x10		300x300x10		300x300x10		300x300x10		300x300x10		300x300x10	

(*) Respect to the Stall Torque

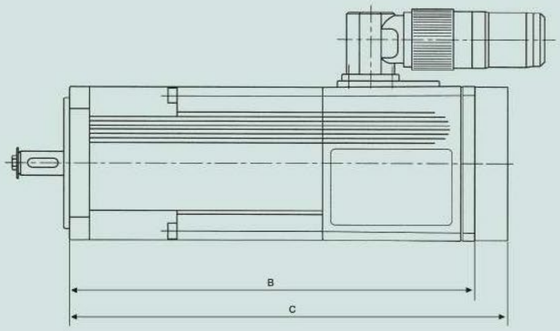
Motors

Resolver Specifications

	UNITS	BL-40	BL-50 BL-70
		2T8 (Transmitter Speed 1)	
Input Voltage/Frecuency	V/kHz	7/10	10/4.5
Primary Element		Rotor	Rotor
Number of Speed		1X	1X
Transformation Ratio		0.5 ± 5%	0.5 ± 5%
Electrical Error	minutes	±10 max	±10 max
Dielectric Strength	VAC/1 minute	500	500
Mass	kg	0.04	0.13
Rotor Moment of Inertia	kg m² 10^{-3}	0.0006	0.0032
Operating Temperature Range	°C	-55 ~ + 155	-55 ~ +155

Brake Specifications

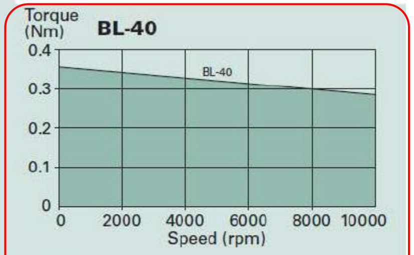
	SIZE	TORQUE Nm	INERTIA kg cm²	MASS kg
BL-40	01	0.4	0.016	0.10
BL-55	02	0.75	0.021	0.15
BL-71 / 72	03	1.5	0.068	0.18
BL-73 / 74	06	3	0.38	0.30

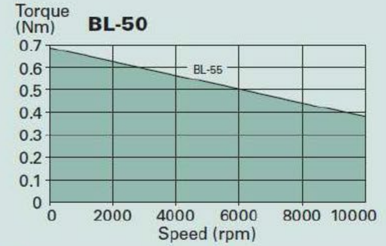


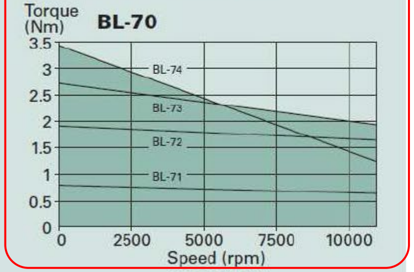
Dimensions

WITHOUT BRAKE		BL-55	BL-71	BL-72	BL-73	BL-74
B		142	128.5	148.5	166.5	184.5
C	Encoder Type "A"	142	147	167	185	203
	Encoder Type "H"	142	128,5	148,5	166.5	184.5
	Encoder Type "K"	—	128,5	148,5	166.5	184.5

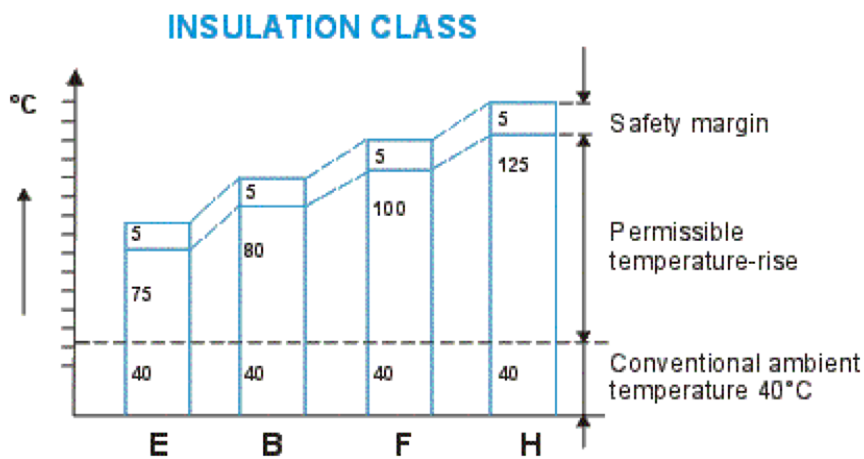
WITH BRAKE		BL-55	BL-71	BL-72	BL-73	BL-74
B		180.5	128.5	148.5	166.5	184.5
C	Encoder Type "A"	180.5	147	167	185	203
	Encoder Type "H"	—	136	156	166.5	184.5
	Encoder Type "K"	—	136	156	166.5	184.5



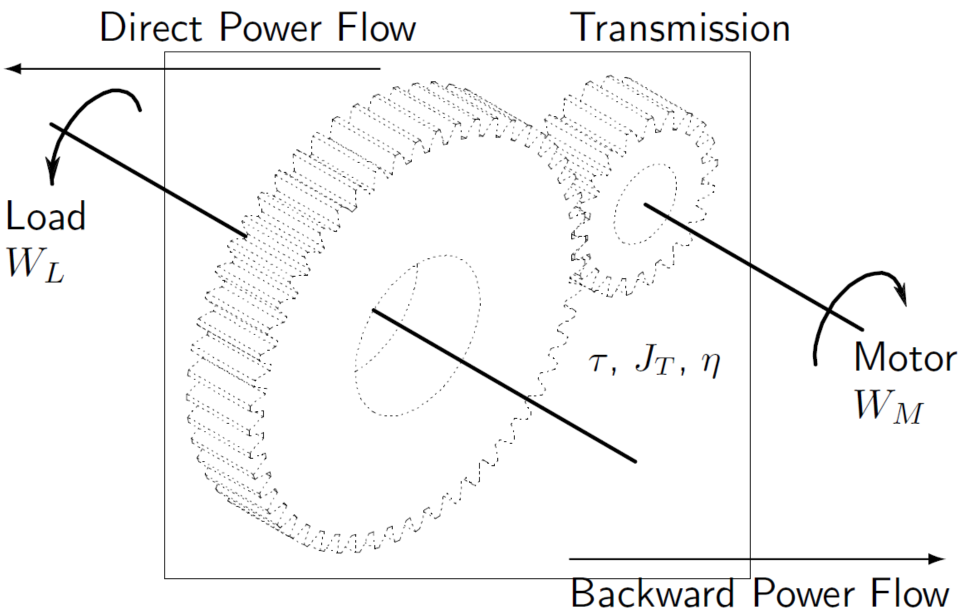


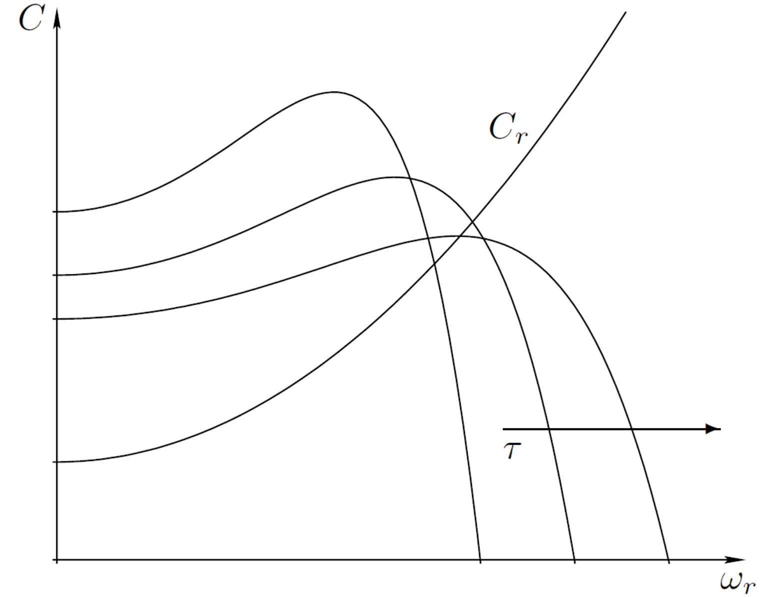


- **Insulation class:** The insulation rating is the maximum allowable winding (hot spot) temperature of a motor operating at an ambient temperature of 40°C.



- **Protection class:** A two-digit number established by the International Electro Technical Commission, is used to provide an Ingress Protection rating to a piece of electronic equipment or to an enclosure for electronic equipment (First digit: Ingress of solid objects, Second digit: Ingress of liquids).



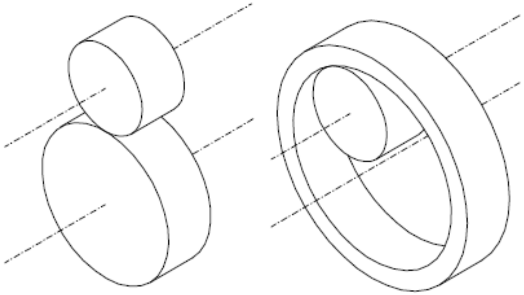


The speed-torque curves can be represented in the motor plane or in the load plane, using the following equation:

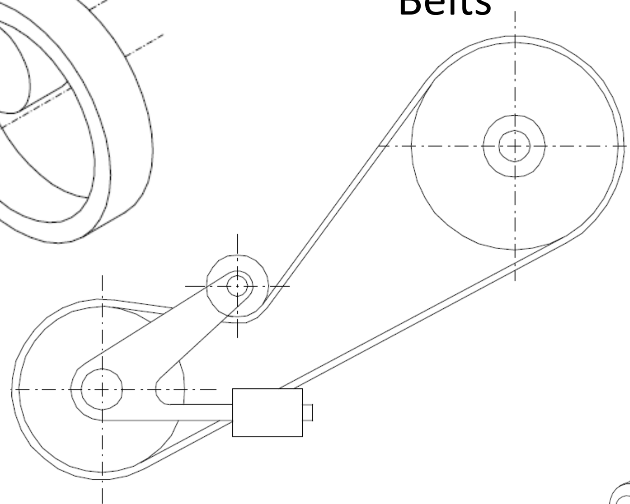
$$\omega_r = \tau\omega_m \qquad C'_m = \frac{C_m}{\tau} \qquad \text{or} \qquad \omega_m = \frac{\omega_r}{\tau} \qquad C'_r = C_r\tau$$

Transmission

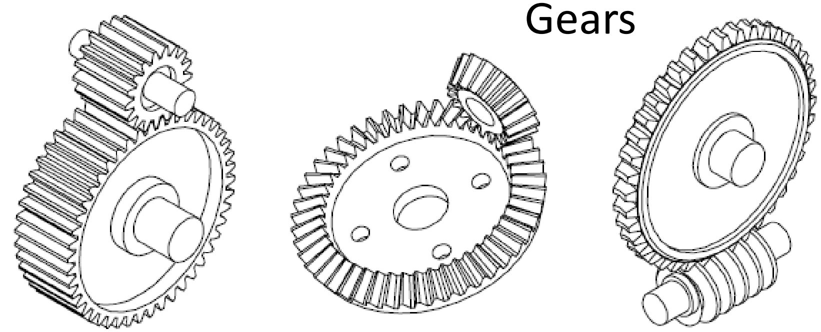
Friction wheels



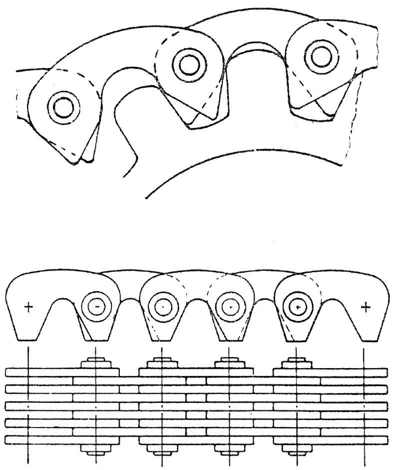
Belts



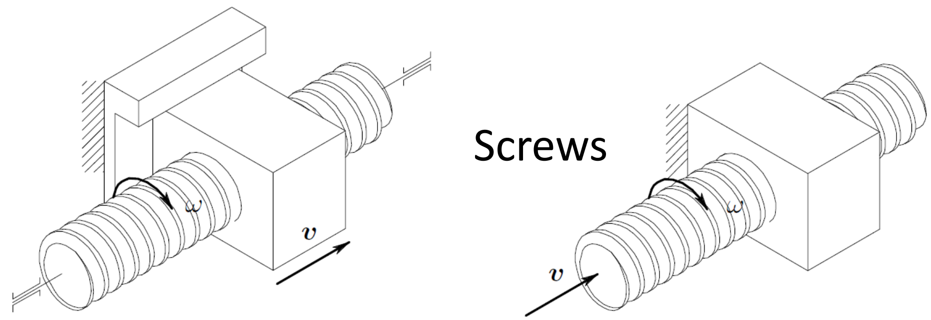
Gears

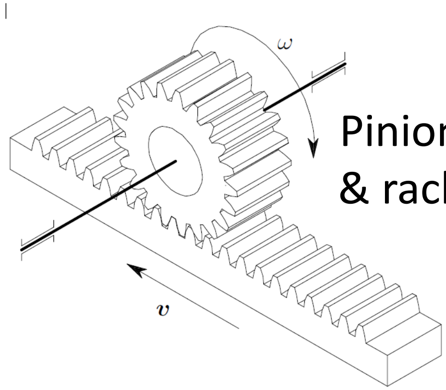


Chains



Screws





Pinions & racks

.... + linkages and cams

Transmission

	Power max (kW)	τ minimum	Dimensions	Cost	Efficiency	Load on bearings
Friction wheels	1/6	20	low	medium	0.90	high
Spur gear	750	1/6	low	high	0.96	low
Helical gears	50000	1/10	low	high	0.98	low
Worm gears	300	1/100	low	high	0.80	medium
Belt	200	1/6	high	low	0.95	high
Trapezoidal belt	350	1/6	medium	low	0.95	high
Toothed belt	100	1/6	medium	low	0.90	low
Linkages	200	1/6	medium	medium	0.90	low

For any application the choice of the suitable mechanical transmission is driven by several factors, for instance, the wheelbase, the bulk, the power, the velocity, the required transmission ratio, the motor or load characteristics, the cost, the maintenance, and the like.

Motor-reducer sizing

The motor reducer choice is a complex task and in general the following conditions must be satisfied:

- Motor checks
  - Instantaneous maximum torque: $C_{max} < C_{m,max}$
  - root mean square torque: $C_{m,q} < C_n$
  - maximum velocity: $\omega_{max} < \omega_{m,max}$
- Gearbox checks
  - maximum input velocity.
  - maximum output torque.

Motor-reducer static sizing

The equation on which the sizing and checking problem are based is the balancing power equation for the all system:

$$\frac{C_m}{\tau} - C_r = \left(\frac{J_m}{\tau^2} + J_r \right) \dot{\omega}_r$$

where τ is the transmission ratio between the actuator and load and this equation is reduced to the load axis by means of the following equations:

$$\omega_r = \tau \omega_m; \qquad \dot{\omega}_r = \tau \dot{\omega}_m$$

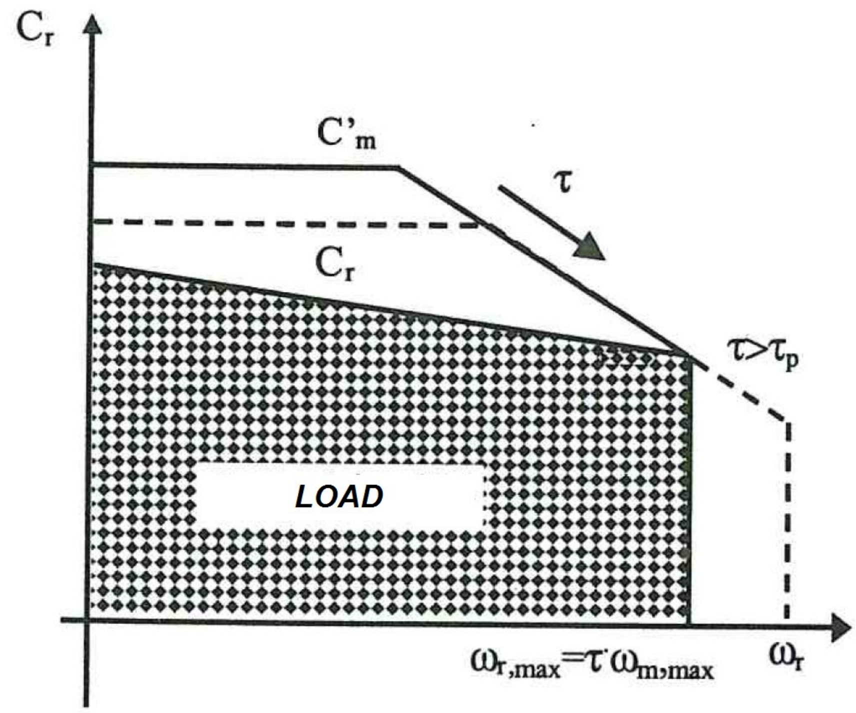
Generally we are faced with two situations:

- $\dot{\omega}_r \simeq 0$ in this case we refer with the definition "static sizing of motor and reducer"
- $\dot{\omega}_r \neq 0$ in this case we refer with the definition "dynamic sizing of motor and reducer"

Motor-reducer static sizing

The mechanical transmission reaches the objective to statically adapt the motor with the load.
This job can be done easily by choosing the right value of τ that allows the motor speed-torque curve to completely cover the load speed-torque curve without exceeding.

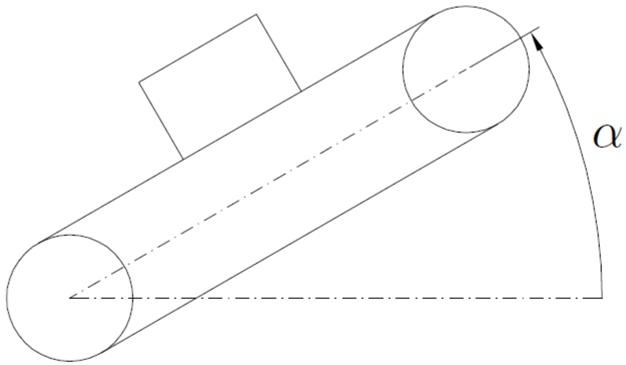
To obtain this result we have to plot the two curves on the same plane and we adapt one over the other multiplying and dividing by τ.



If $\omega_{m,max}$ is the maximum motor velocity and $\omega_{r,max}$ the maximum load velocity, the transmission ratio must be at least:

$$\tau_p = \frac{\omega_{r,max}}{\omega_{m,max}}$$

The most used motor in this case is the asynchronous.



In this example we have to sizing the actuation system of a conveyor belt shown in figure. For the problem solution we know: the maximum pay load P, the pulley diameter D and its inertia moment J, the overall equivalent friction coefficient f_{eq} and the range of the operating velocity V_{min}-V_{max}.

The first step of the solution is evaluate the load field. In order to do that it is necessary to calculate the overall resistance force:

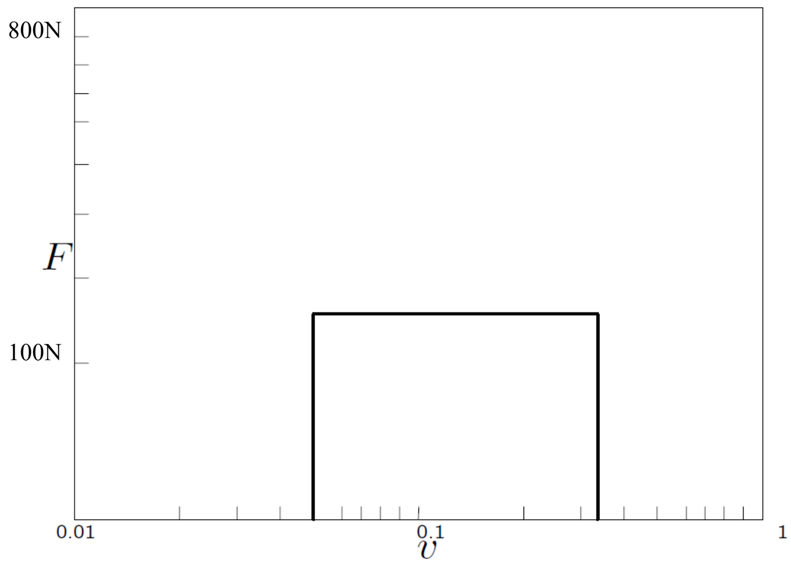
$$F_r = P\sin(\alpha) + P f_{\text{eq}}\cos(\alpha)$$

and the maximum power required by the load:

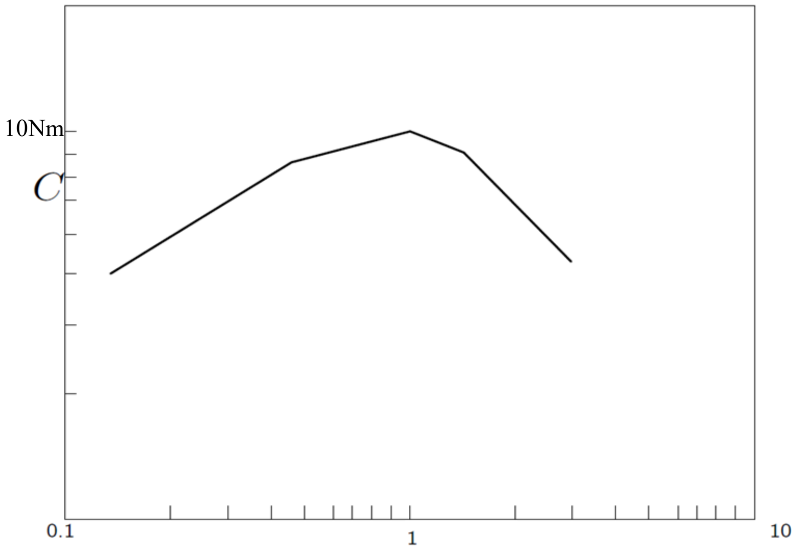
$$W_{\text{max}} = F_r V_{\text{max}}$$

Motor-reducer static sizing

Load field

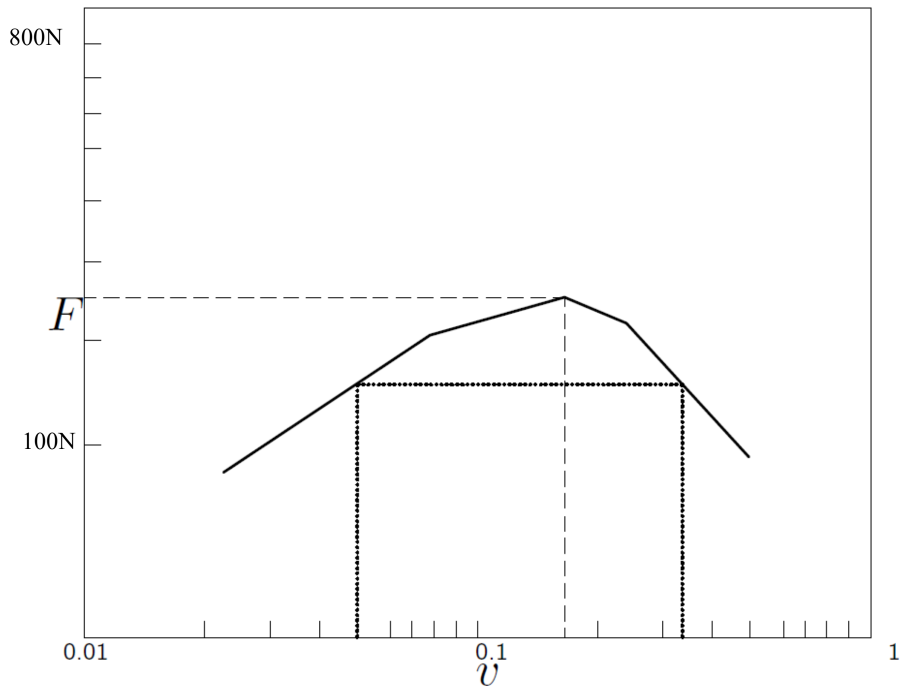


Motor field



The load field is constant for the overall range of the velocity variation.

Motor-reducer static sizing



Transmission ratio can be easily obtained by overlapping the working fields of the motor and the load.

Motor-reducer dynamic sizing

The motor reducer dynamic sizing procedure is different from the previous one because we have to take into account the effect of the **motor inertia** (that is unknown till the sizing is carried out). In this case we have:

$$C_m = \tau C_r^* + J_m \frac{\dot{\omega}_r}{\tau}$$

while the root mean square torque is:

$$C_{m,q}^2 = \int \frac{{C_m}^2 dt}{t_a} = \int \left(\tau C_r^* + J_m \frac{\dot{\omega}_r}{\tau}\right)^2 \frac{dt}{t_a} =$$

$$= \int \left(\tau^2 {C_r^*}^2 + J_m^2 \frac{\dot{\omega}_r^2}{\tau^2} + 2 J_m C_r^* \dot{\omega}_r\right) \frac{dt}{t_a}$$

from which we can obtain:

$$C_{m,q}^2 = \tau^2 {C^*}^2_{r,q} + J_m^2 \frac{\dot{\omega}_{r,q}^2}{\tau^2} + 2 J_m (\dot{\omega}_r C_r^*)_{\text{average}}$$

Motor-reducer dynamic sizing

In order to pass the thermal check, the actuator must have a nominal torque C_n greater than:

$$C_n{}^2 \geq C^2_{m,q} = \tau^2 C^{*2}{}_{r,q} + J^2_m \frac{\dot{\omega}^2_{r,q}}{\tau^2} + 2J_m(\dot{\omega}_r C^*_r)\ \text{medio}$$

or in other terms:

$$\frac{C^2_n}{J_m} \geq \tau^2 \frac{C^*_{r,q}{}^2}{J_m} + J_m \frac{\dot{\omega}^2_{r,q}}{\tau^2} + 2(\dot{\omega}_r C^*_r)_{\text{medio}} =$$

$$\tau^2 \frac{C^*_{r,q}{}^2}{J_m} + J_m \frac{\dot{\omega}^2_{r,q}}{\tau^2} + 2(\dot{\omega}_r C^*_r)_{\text{medio}} - 2C^*_{r,q}\dot{\omega}_{r,q} + 2C^*_{r,q}\dot{\omega}_{r,q} =$$

$$\left(\frac{\tau}{\sqrt{J_m}} C^*_{r,q} - \frac{\sqrt{J_m}}{\tau}\dot{\omega}_{r,q}\right)^2 + 2(\dot{\omega}_r C^*_r)_{\text{medio}} + 2C^*_{r,q}\dot{\omega}_{r,q}$$

Motor-reducer dynamic sizing

$$\frac{C_n^2}{J_m} \geq \left(\frac{\tau}{\sqrt{J_m}} C^*_{r,q} - \frac{\sqrt{J_m}}{\tau} \dot{\omega}_{r,q} \right)^2 + 2 \left[(\dot{\omega}_r C^*_r)_{\text{medio}} + C^*_{r,q} \dot{\omega}_{r,q} \right]$$

We can now introduce two new quantities, the accelerating factor:

$$\alpha = \frac{C_n^2}{J_m}$$

that is capable of characterizing the actuator performance, and the load factor:

$$\beta = 2 \left[\dot{\omega}_{r,q} C^*_{r,q} + (\dot{\omega}_r C^*_r)_{\text{medio}} \right]$$

that is capable of characterizing the load requirement. Both of them are expressed in W/s

Motor-reducer dynamic sizing

Introducing α and β, into the previous equation, we obtain:

$$\alpha > \beta + \left[C^*_{r,q} \left(\frac{\tau}{\sqrt{J_m}} \right) - \dot{\omega}_{r,q} \left(\frac{\sqrt{J_m}}{\tau} \right) \right]^2$$

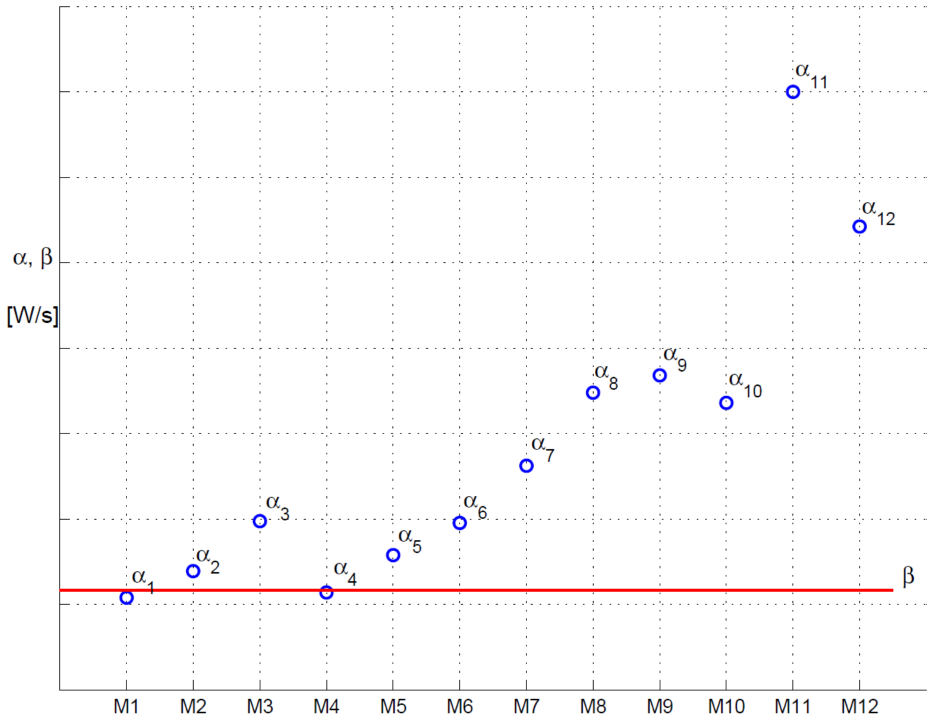
The τ value that minimises the accelerating factor α makes it equal to the load factor β is the one that nullifies the terms into the brackets, and it is:

$$\tau^2_{opt} = J_m \frac{\dot{\omega}_{r,q}}{C^*_{r,q}}$$

it is called **optimum transmission ratio**.

Motor-reducer dynamic sizing

Having classified the motors with respect to their accelerating factor, we have to chose the one with α greater than the load factor β. Note that β depends on only the load parameters usually well known.



$$\alpha > \beta + \left[C^*_{r,q} \left(\frac{\tau}{\sqrt{J_m}} \right) - \dot{\omega}_{r,q} \left(\frac{\sqrt{J_m}}{\tau} \right) \right]^2$$

Motor-reducer dynamic sizing

- If $\alpha = \beta$ the only transmission ration available is τ_{opt}.
- If $\alpha > \beta$, exists a range of available τ values between a minimum τ_{min} and a maximum τ_{max}.

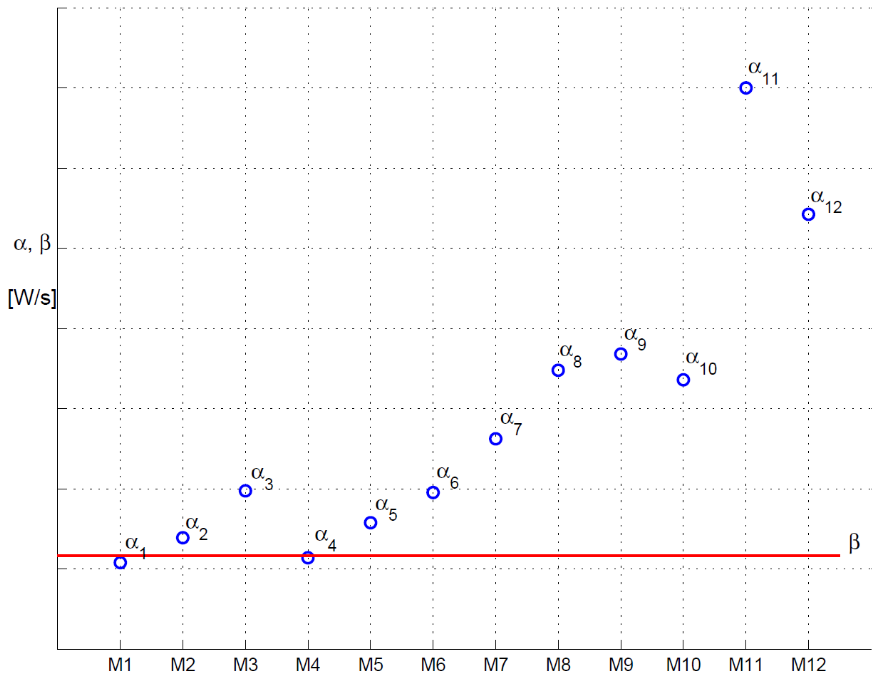
These values are obtained solving the previous disequation in function of τ:

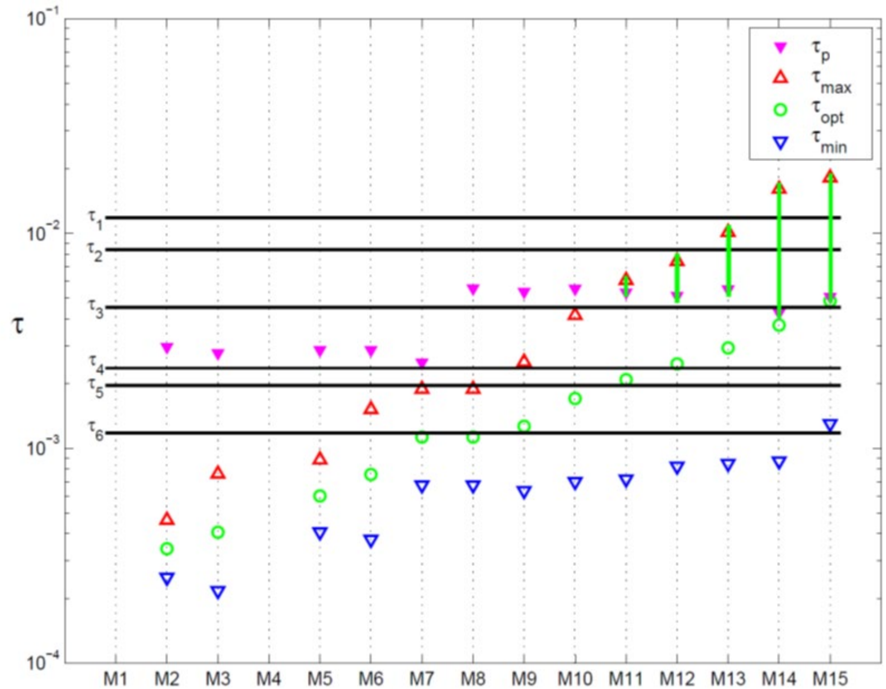
$$\tau_{min}, \tau_{max} = \sqrt{J_m} \frac{\sqrt{\alpha - \beta + 4\dot{\omega}_{r,q} C^*_{r,q}} \pm \sqrt{\alpha - \beta}}{2C^*_{r,q}}$$

Due to a kinematic constrain there is a limit transmission ratio under which the motor is not capable of reaching the required velocity to the load:
$\tau > \tau_p = \omega_{r,max}/\omega_{m,max}$.

Motor-reducer dynamic sizing

- If $\tau_p > \tau_{max}$ we have to chose a motor with a greater α
- If $\tau_{min} < \tau_p < \tau_{max}$ we have to chose a reducer with transmission ratio between $\tau_p < \tau < \tau_{max}$;
- Only if $\tau_p < \tau_{min}$ the τ chosing can be done in the all range $\tau_{min} < \tau < \tau_{max}$.





Motor-reducer dynamic sizing

Now it's possible to check:

- The maximum torque supplied by the servo-motor for each angular velocity achieved:

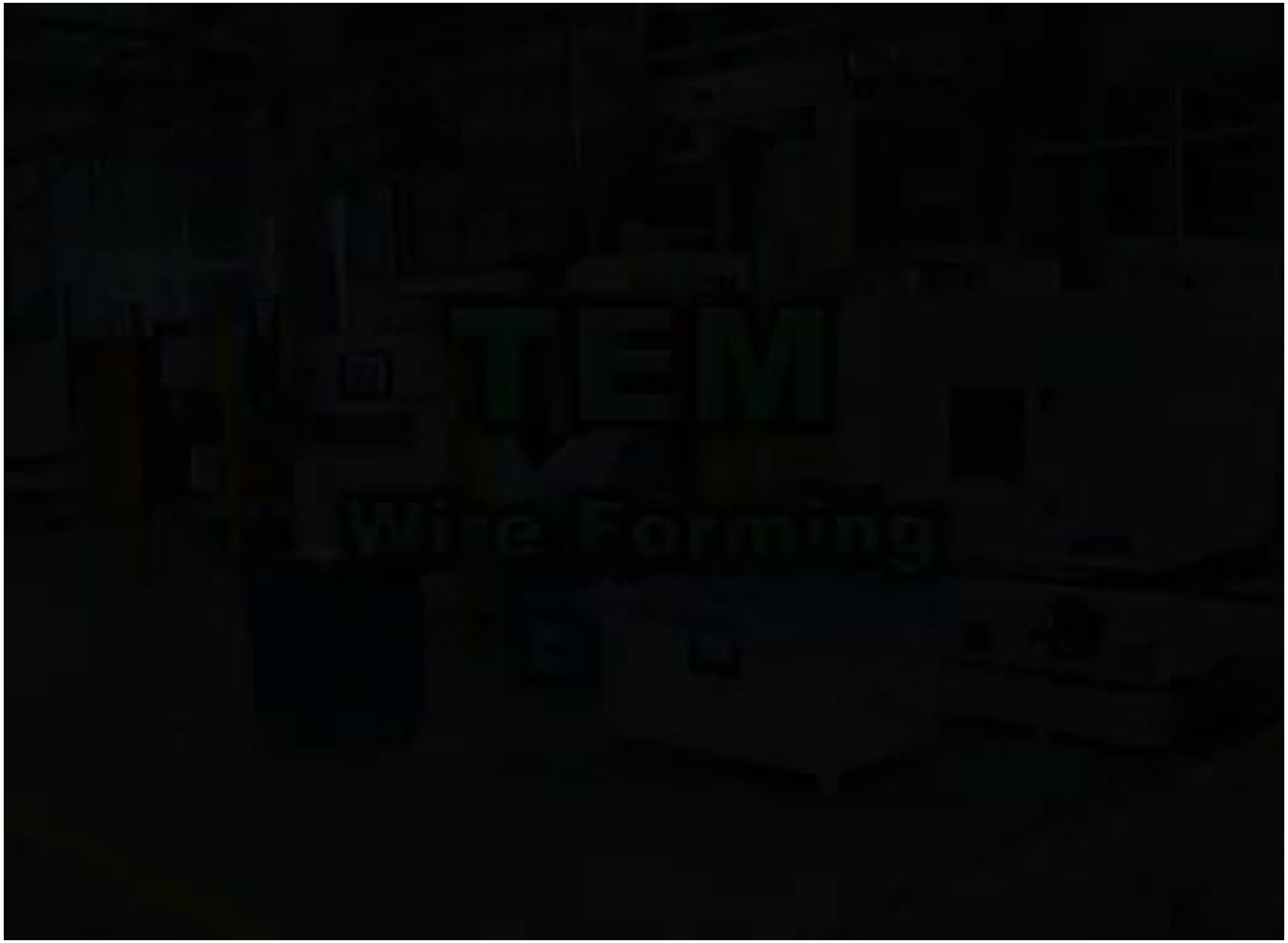
$$T_{M,max}(\omega_M) \geq \max \left| \frac{\tau T_L}{\eta} + \left(\frac{J_M + J_T}{\tau} + \frac{J_L \tau}{\eta} \right) \dot{\omega}_L \right| \quad \forall \omega.$$

  This test is easily performed by superimposing the motor torque $T_M(\omega_M)$ on the motor torque/speed curve (cf. Fig. 6).
- The effect of the transmission's mechanical efficiency (η) and its moment of inertia (J_T) on the root mean square torque:

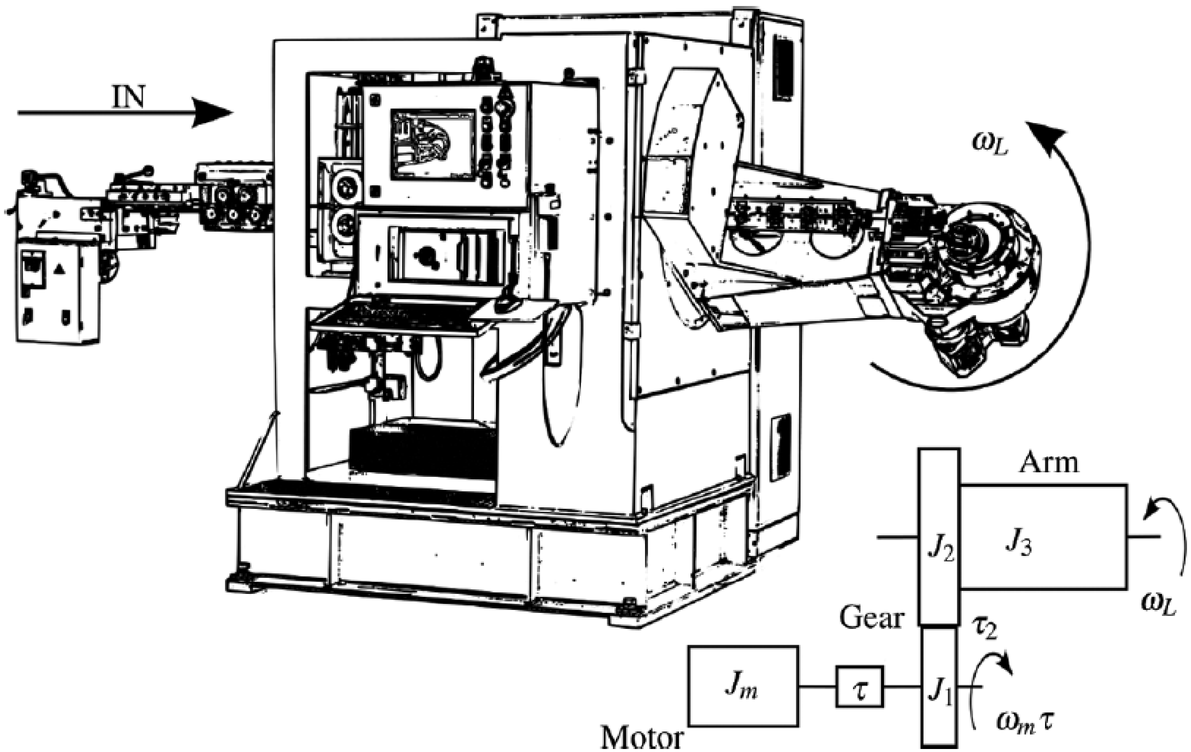
$$T_{M,N}^2 \geq T_{M,rms}^2 = \int_0^{t_a} \frac{T_M^2}{t_a} \mathrm{d}t = \int_0^{t_a} \frac{1}{t_a} \left((J_M + J_T) \frac{\dot{\omega}_L}{\tau} + \frac{\tau T_L^*}{\eta} \right)^2 \mathrm{d}t.$$

- The resistance of the transmission as supplied by the manufacturer.

Example

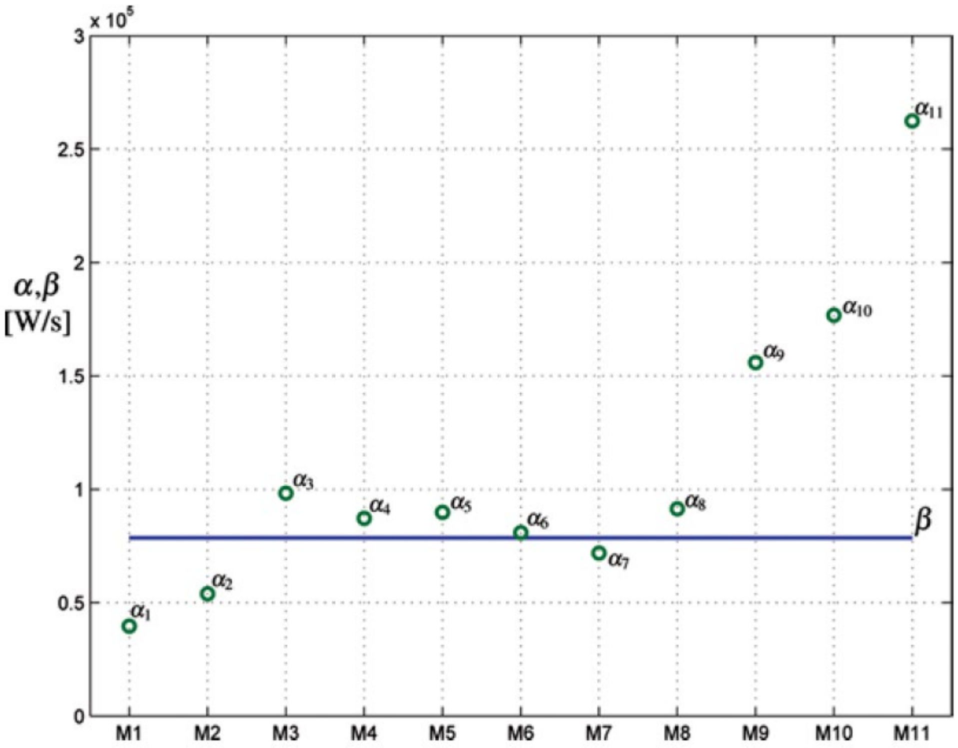


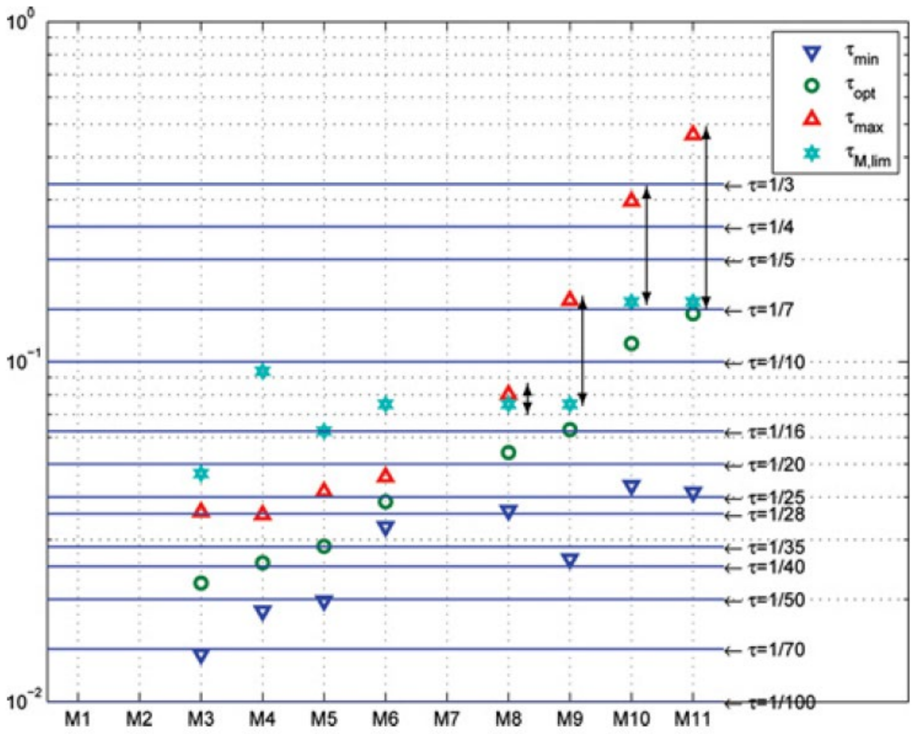
Example



IN
ω_L
Arm
J_2
J_3
ω_L
Gear
τ_2
J_m
τ
J_1
$\omega_m \tau$
Motor

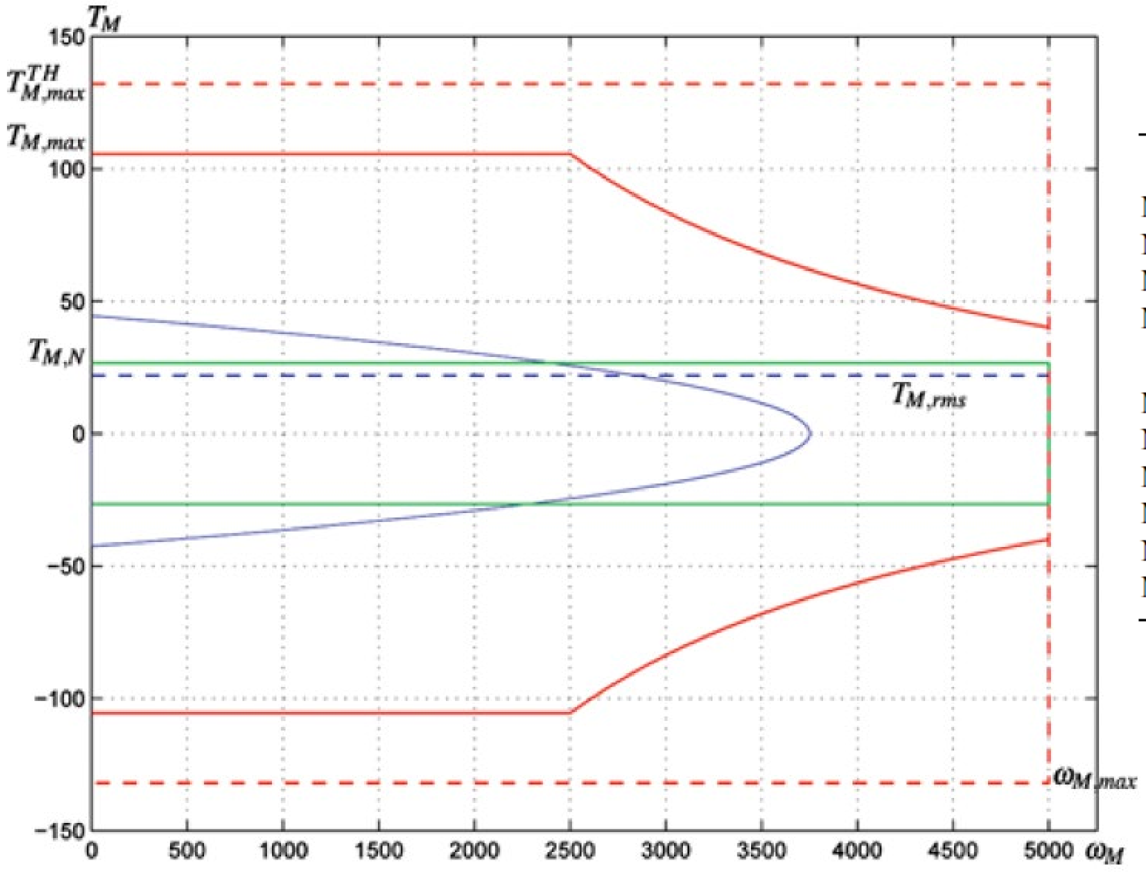
Example





Motor	Speed reducer
M9	$\tau = 1/10,\ \tau = 1/7$
M10	$\tau = 1/5,\ \tau = 1/4$
M11	$\tau = 1/5,\ \tau = 1/4,\ \tau = 1/3$

Example



Motor M9	
Moment of inertia	$J_M = 0.0046$ [kg m^2]
Nominal torque	$T_{M,N} = 26.7$ [Nm]
Maximum Torque	$T_{M,max}^{TH} = 132$ [Nm]
Maximum achievable speed	$\omega_{M,max} = 5000$ [rpm]
Speed reducer $\tau = 1/10$	
Moment of inertia	$J_T = 5.8 \cdot 10^{-4}$ [kg m^2]
Nominal torque	$T_{T,N} = 220$ [Nm]
Maximum Torque	$T_{T,max} = 480$ [Nm]
Maximum endurable speed	$\omega_{T,max} = 4000$ [rpm]
Nominal speed	$\omega_{T,N} = 2600$ [rpm]
Mechanical efficiency	$\eta = 0.97$