



POLITECNICO
MILANO 1863



Functional Mechanical Design

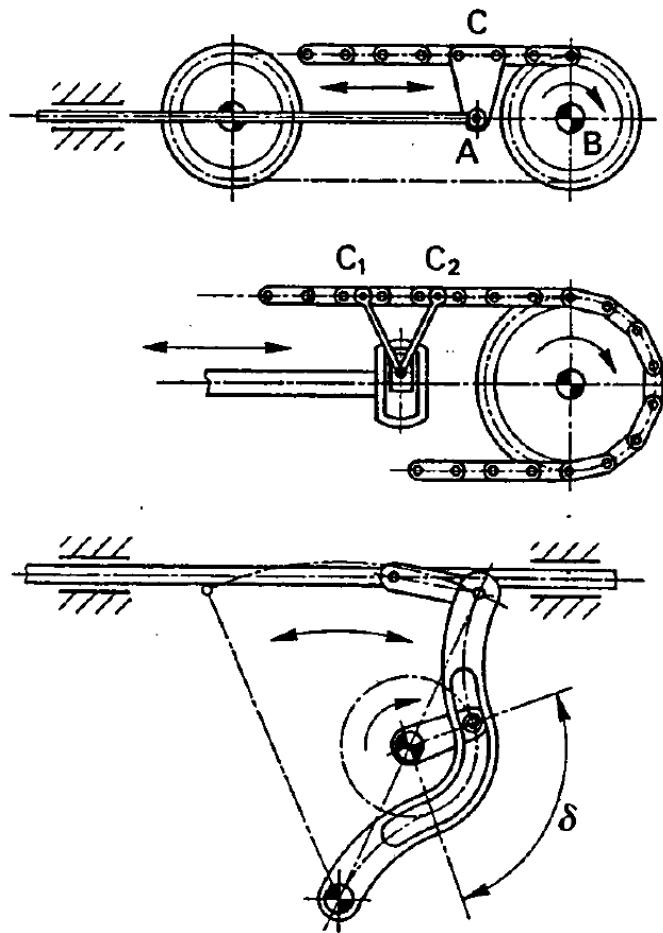
Linkage Mechanisms (3/3)

Simone Cinquemani

Mechanisms with arrest

Mechanisms with arrest

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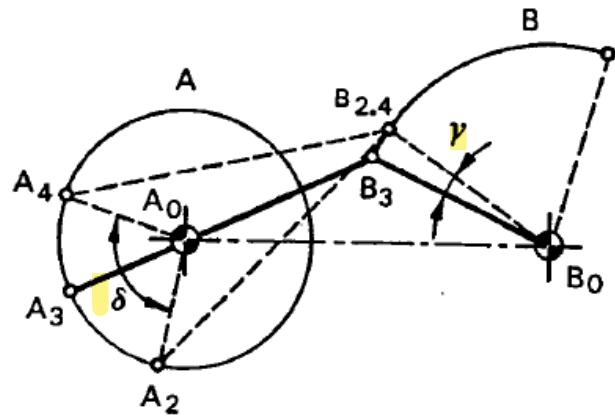


The **motion curves with arrest** are usually realized by means of cam mechanisms: if the stroke is too high, we can use different solutions such as those in the figure. Linkage mechanisms can be used to realize motion curves with arrest: in these applications the arrest is not entirely complete, but rather we are faced with extremely slow movements. Two techniques exist to realize delay mechanisms:

- **Superimposition of dead centres;**
- **Coupler curve methods.**

Dead centre superimposition method

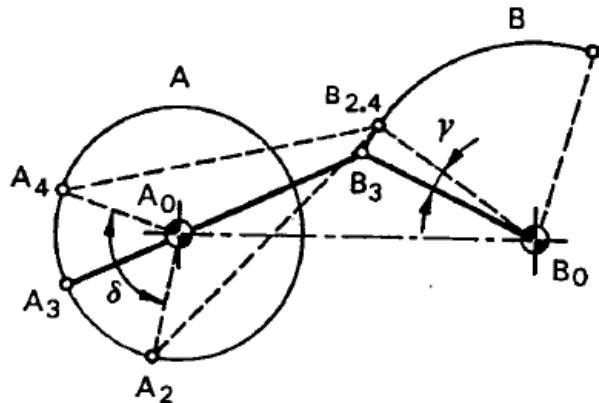
This method is based on connecting a series of elementary mechanisms, such as, for example some four bar linkages. Every time that a four bar linkage reaches a dead centre, all the velocities of all the following four bar linkages are nullified causing the arrest of the follower.



For a four bar linkage while the crank AA_0 traces the rotation $\widehat{A_2A_0A_4}$ the follower BB_0 effects the angle $\gamma = \widehat{B_2B_0B_3}$: a small angle with respect to the complete follower rotation. In the configuration shown in the figure we have $\delta = 95^\circ$ $\gamma = 9^\circ$.

Dead centre superimposition method

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Having indicated with α the crank rotation beginning from the dead centre position and with γ the follower rotation measured starting from the same configuration, we will have:

$$\gamma = \gamma(\alpha) \Rightarrow \text{serie development} \Rightarrow \gamma = \gamma(0) + \gamma'(0)\alpha + \frac{1}{2}\gamma''(0)\alpha^2\dots$$

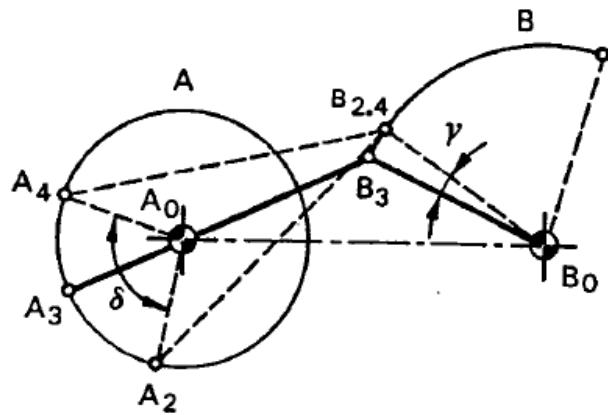
In $\alpha = 0$ we have $\gamma(0) = \gamma'(0) = 0$ and thus, having arrested the series development at the second order, we have:

$$\gamma = \frac{1}{2}\gamma''(0)\alpha^2 = \frac{1}{2}\epsilon\alpha^2$$

where $\epsilon = \gamma''(0)$ is the follower geometrical acceleration in the dead centre.

Dead centre superimposition method

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Whilst the crank goes through the angle δ between $\alpha = 0$ that is the dead centre under analysis, the follower goes through twice the angle γ ; Thus in order to move the follower of an angle γ from the dead centre, the crank must rotate at an angle $\alpha = \delta/2$.

which substituted into the series development provides the following relationship:

$$\gamma = \frac{1}{2}\epsilon \left(\frac{\delta}{2}\right)^2 \quad \Rightarrow \quad \frac{\gamma}{\delta} = \frac{1}{8}\epsilon\delta$$

Dead centre superimposition method

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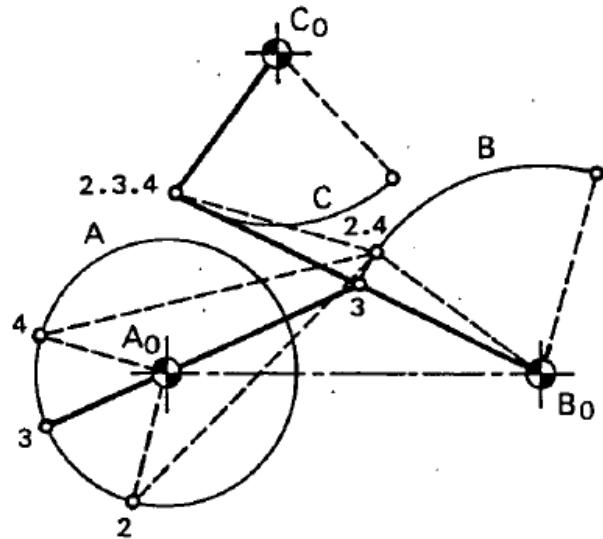
$$\frac{\gamma}{\delta} = \frac{1}{8}\epsilon\delta$$

Note that in order to have γ smaller compared to δ , we need to have ϵ smaller compared to $1/\delta$. It is important to highlight that ϵ cannot be reduced as you want while δ depends on the context, so γ/δ cannot be reduced under a certain value.

We can obtain a greater reduction using several four bar mechanisms in series.

Dead centre superimposition method

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In the figure a four bar mechanism B_0BCC_0 is added in series: CC_0 being the follower of the overall mechanism.

Note that compared to the previous case, for the same rotation δ of the link AA_0 , it is impossible to graphically appreciate the rotation of the link CC_0 . Although the overall rotation of CC_0 is equal to the BB_0 one.

Dead centre superimposition method

Repeating the series development we will have:

$$\left\{ \begin{array}{l} \gamma_1 = \frac{1}{2}\epsilon_1\alpha_1^2 \quad \text{First four bar linkage} \\ \gamma_2 = \frac{1}{2}\epsilon_2\alpha_2^2 \quad \text{Second four bar linkage} \end{array} \right.$$

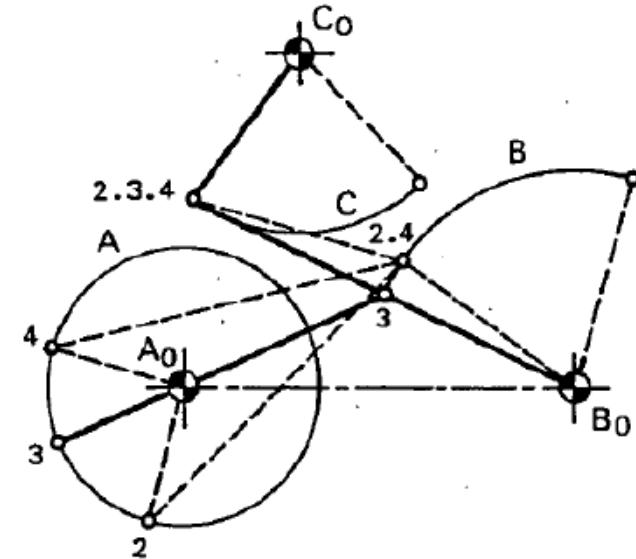
Remembering that $\alpha_2 = \gamma_1$ and that $\alpha_1 = \delta/2$ we obtain:

$$\gamma_2 = \frac{1}{8}\epsilon_1^2\epsilon_2\alpha_1^4 \Rightarrow \frac{\gamma_2}{\delta} = \frac{1}{128}\epsilon_1^2\epsilon_2\delta^3$$

While the ratio between γ_2 and γ_1 is:

$$\frac{\gamma_2}{\gamma_1} = \frac{\frac{1}{8}\epsilon_1^2\epsilon_2\alpha_1^4}{\frac{1}{2}\epsilon_1\alpha_1^2} = \dots = \frac{1}{16}\epsilon_1\epsilon_2\delta^2$$

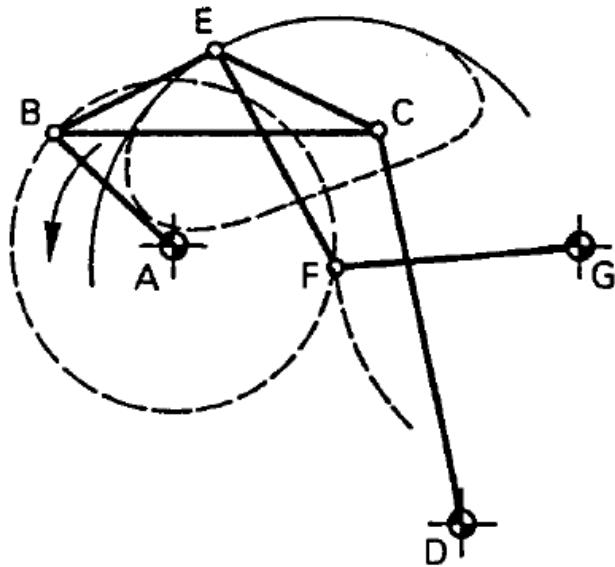
under the hypothesis that ϵ_1 and ϵ_2 are lower compare to $1/\delta$, we obtain $\gamma_2/\gamma_1 \ll 1$. Thus using two four bar mechanisms in series, we can obtain important oscillation reductions of the follower.



Coupler curve method

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It is possible to use circular paths of the coupler curves to obtain delays:



If the connecting rod EF has the same length of the curvature radius of the trajectory of the connecting rod point E (in the circular path), and F is in the curvature centre, the link FG remains fixed while it is traced the circular path.

Force multiplier mechanisms

Force multiplier mechanisms

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Lower kinematic pairs allow to the linkage mechanisms to bear high loads, for this reason they are often used as systems capable of multiplying forces. Using a balancing power principle, we can write:

$$C\omega = Fv \Rightarrow \frac{v}{\omega} = \frac{dy}{d\alpha} = y' \Rightarrow \frac{F}{C} = \frac{1}{y'}$$

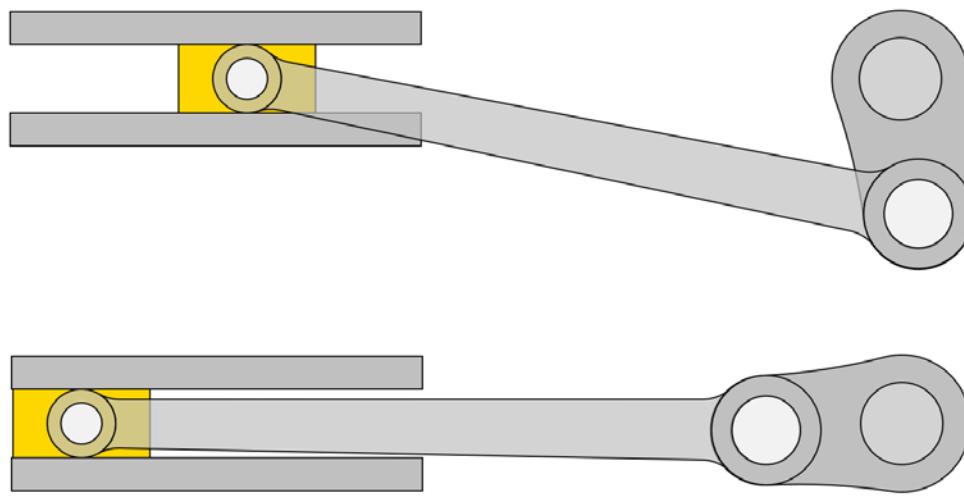
where C represents the motor torque and F the active component of the load force in the v direction.

Note that to obtain high force multiplication it is necessary reduce the velocity or in other terms to work near the dead centre where the geometrical velocity y' is very low.

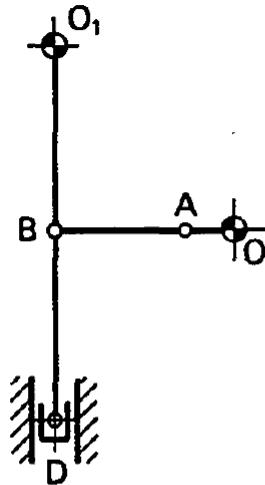
Force multiplier mechanisms

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In particular, when the mechanism reaches the dead centre configuration, we will have that $y' = 0$ and so $F/C \rightarrow \infty$. In other terms with finite value of motor torque, it is possible to bear infinite force on the follower. Actually it is possible to obtain high values of force multiplication but not infinite ones, this is due to the deformation of the bodies that constitute the system and to the friction in the kinematic pairs.



Force multiplier mechanisms

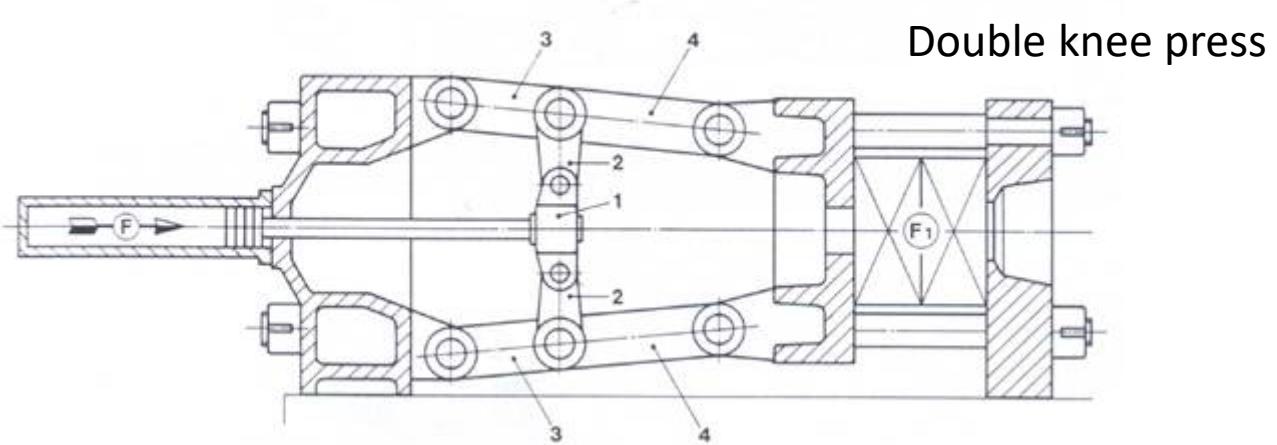
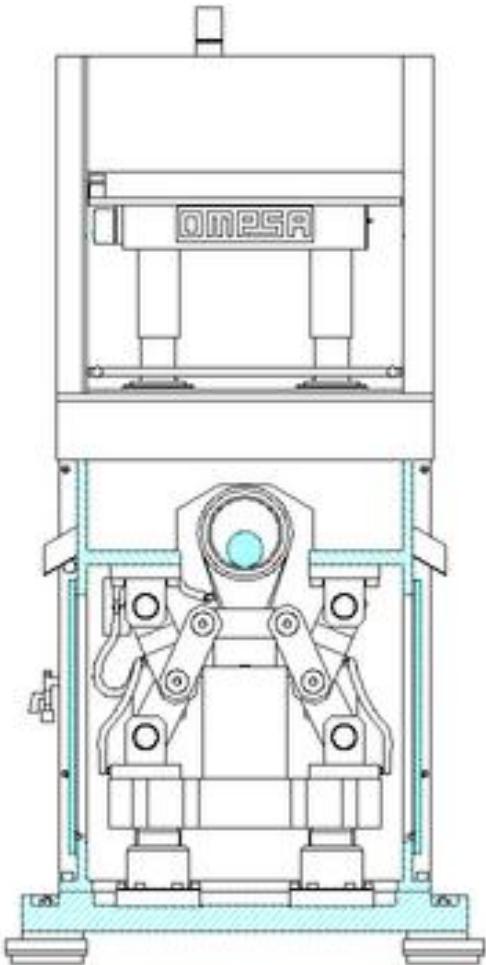


In order to increase the force multiplication effect, a series of mechanisms are used taking advantages from the superimposition of the dead centre configurations. For instance, in the figure is shown a toggle mechanism made up of: a four bar mechanism O_1BAO and a slider crank mechanism O_1BD .

In order to evaluate the increasing of the obtainable multiplication force by means this solution it is worthwhile to take into account the two mechanism O_1BD and OAB separately.

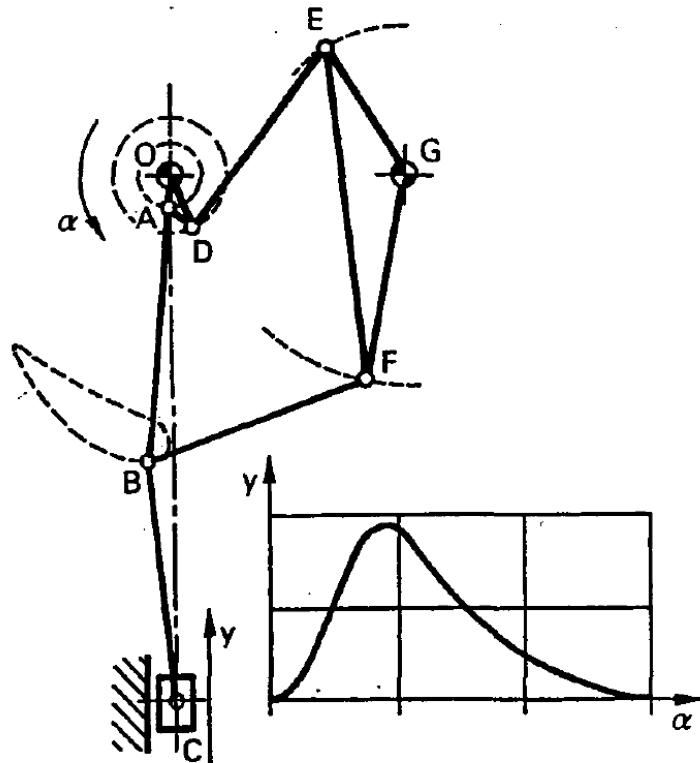
Force multiplier mechanisms

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The concept of using mechanisms in their **dead center configurations** to amplify forces is widely used in industrial machines

Force multiplier mechanisms



In figure is shown a five bar linkage press. The base mechanism is the five bar linkage $OABFG$. The electric motor acts the link OA , while the rod GF is moved by means of the same motor through the four bar mechanism $ODEG$. In this mechanism the ratio between the going time and the return one of the follower is equal to 2 while the time with reduced velocity lasts one third of the overall stroke.

Compliant Mechanisms

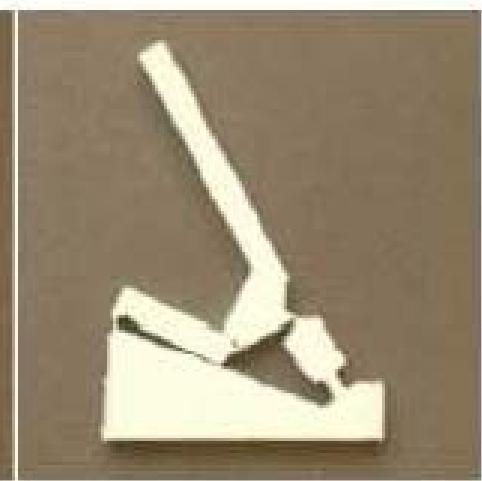
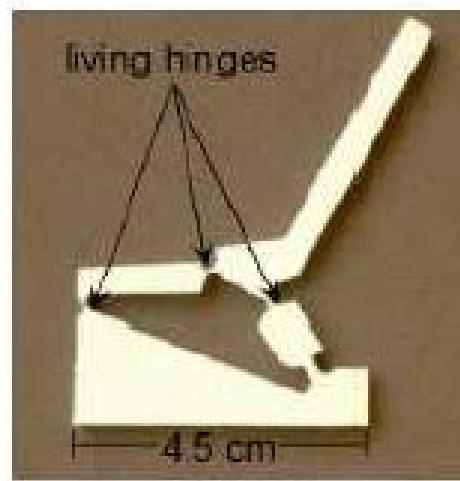
Introduction

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Compliant mechanisms are mechanisms where the mobility is allowed by the deflection of **flexible members** and the deformation is diffused into the mechanism. In **flexure based compliant mechanisms** the flexibility is concentrated in specific areas, named “flexure hinges”, that is the flexible members are very small with respect to the whole mechanism.



A **flexure hinge** is a thin member that provides the relative rotation between two adjacent rigid members through bending.



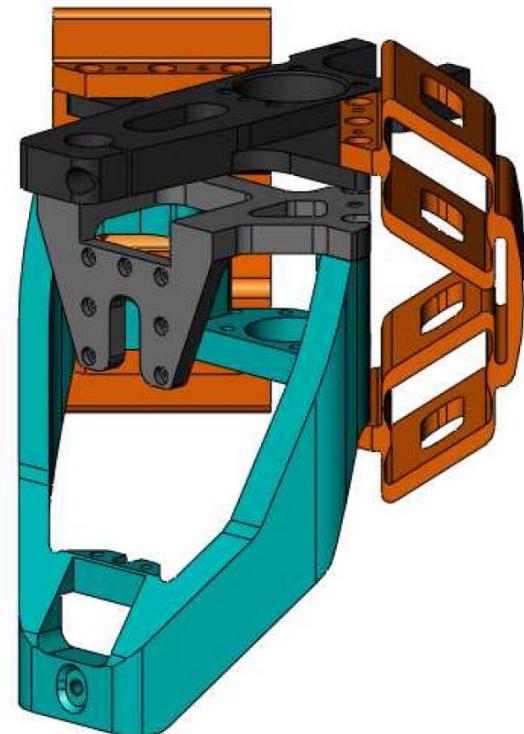
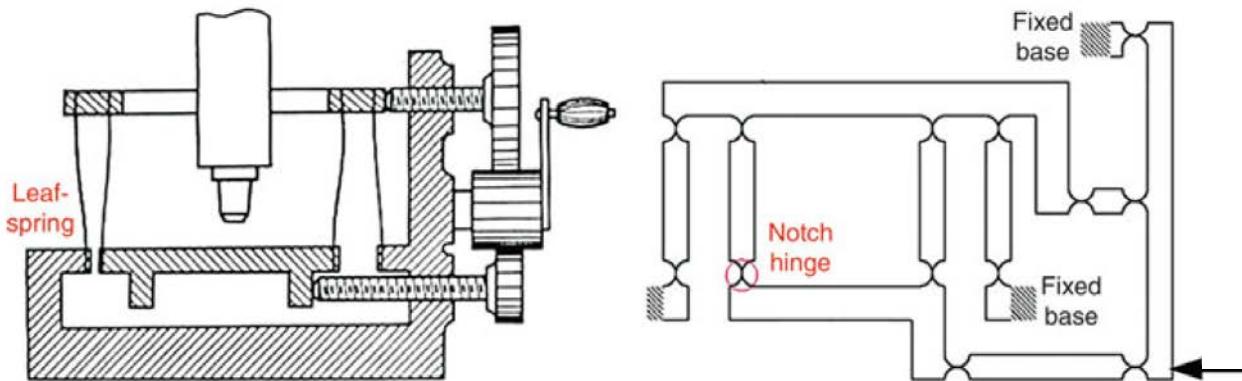
In terms of this rotary function, a flexure hinge can be considered the structural correspondent of a bearing with limited rotation capability.

Advantages

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Among the **benefits** provided by flexure hinges mechanisms , the most notable are:

- No backlash
- No friction losses and no need for lubrication
- Compactness
- Usable in small-scale applications
- Ease of fabrication and no need for assembly
- Virtually no maintenance needed



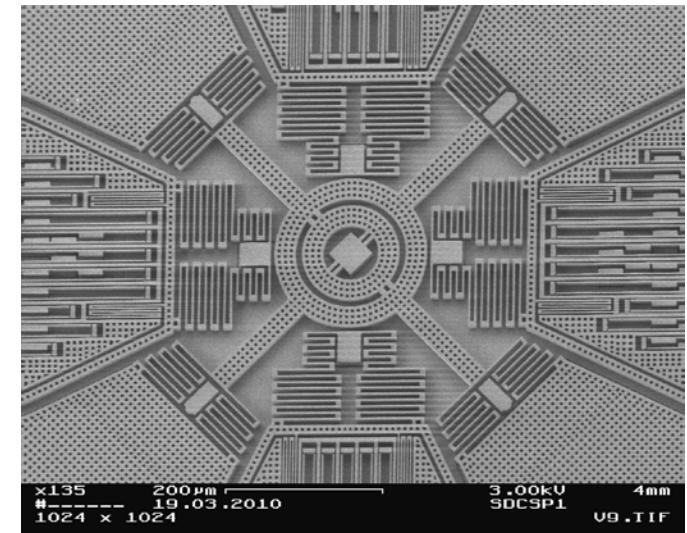
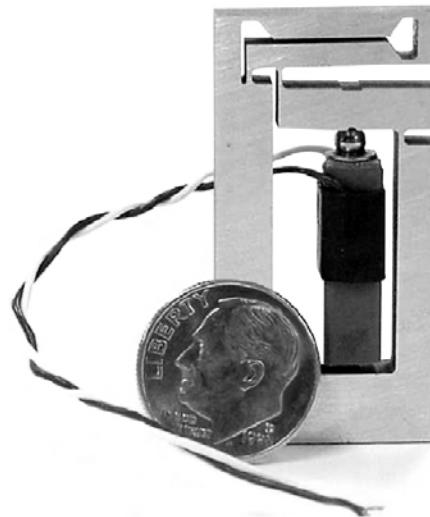
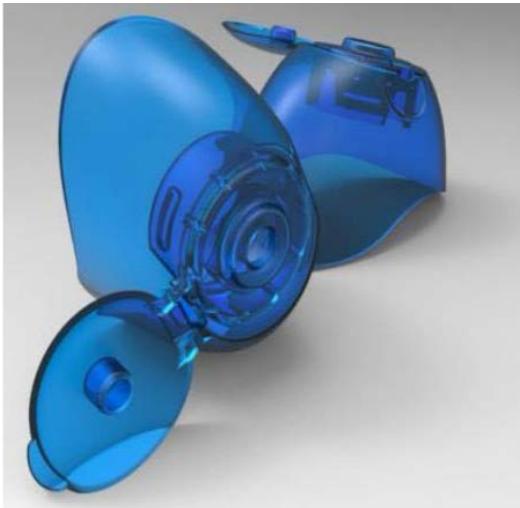
Unfortunately flexure hinges have some drawbacks as:

- The flexure hinges are capable of providing relatively **low levels of rotations**.
- Flexible elements are subjected to **fatigue** limitations

The rotation is not pure because the deformation of a flexure is complex, as it is produced by axial shearing and possibly **torsion loading**, in addition to bending.

- The rotation center (for short flexure hinges, this role is assumed by the symmetry center of the flexure) **is not fixed** during the relative rotation produced by a flexure hinge as it displaces under the action of the combined load.
- The flexure hinge is usually **sensitive to temperature variations**; therefore, its dimensions change as a result of thermal expansion and contraction, which leads to modifications in the original compliance values.

For two-dimensional applications in which the flexure hinge is fabricated by removing material from a blank piece, the manufacturing processes that are being utilized for this purpose include end-milling, electrodischarge machining (EDM), laser cutting, metal stamping, plastic injection or photolithographic.



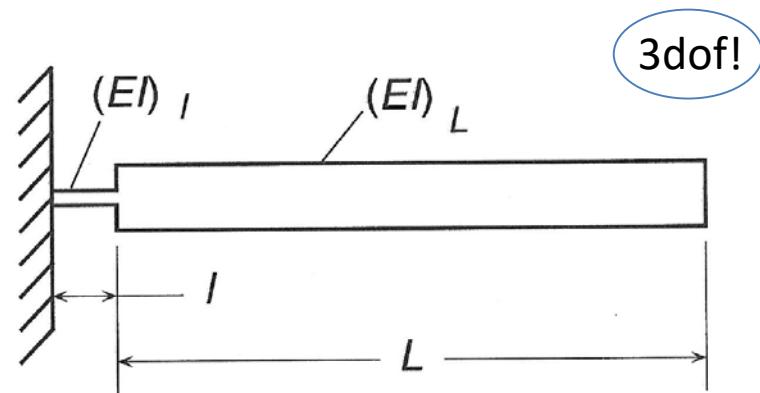
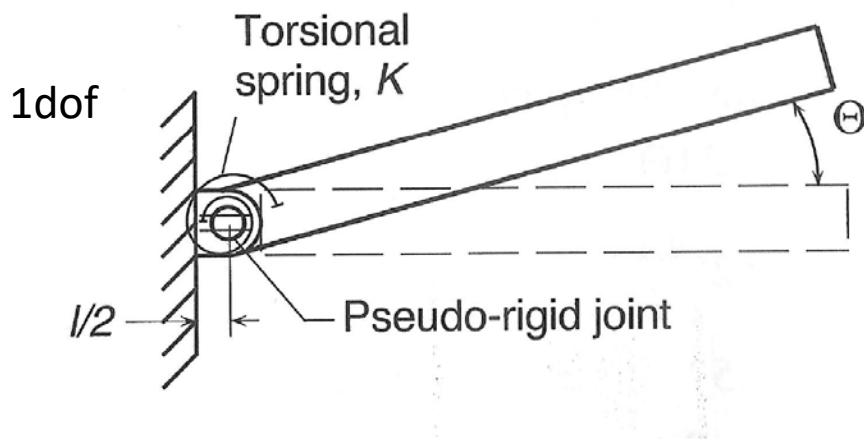
In three-dimensional applications the flexure hinge can be machined by lathe-turning, precision casting or additive manufacturing.

Pseudo-rigid body model

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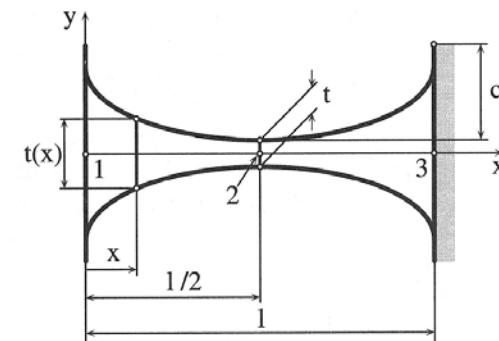
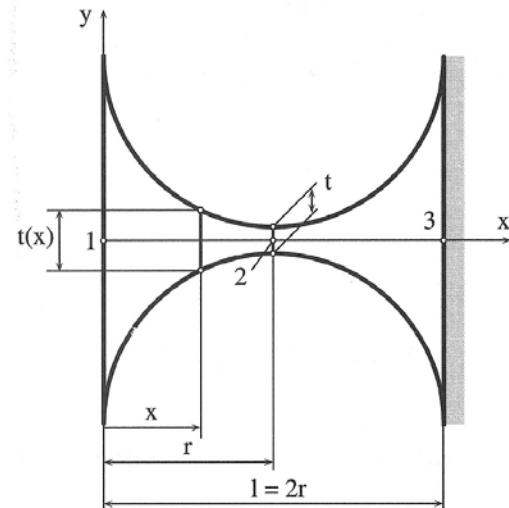
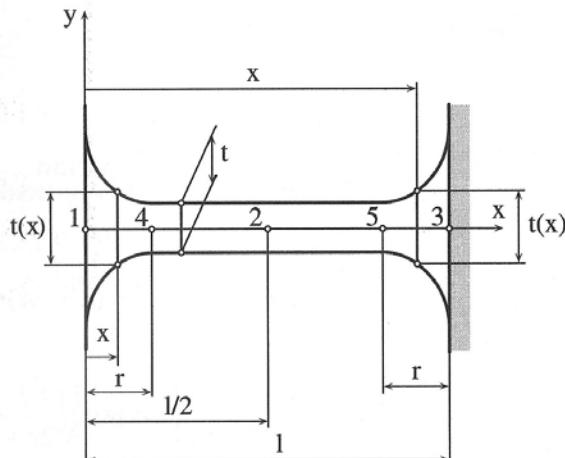
In modeling and analyzing compliant mechanisms, the **pseudo-rigid-body model approach** is the almost-exclusive tool that is currently utilized.

Essentially, the pseudo-rigid-body model treats a **flexible link** (a flexure hinge) as a **torsional spring**, in terms of its compliant behavior. The large deformation assumption is utilized to derive an approximate expression for the flexure's torsional spring rate, and classical methods of rigid-body mechanisms are further used to study the compliant mechanism motion.



Pseudo-rigid body model

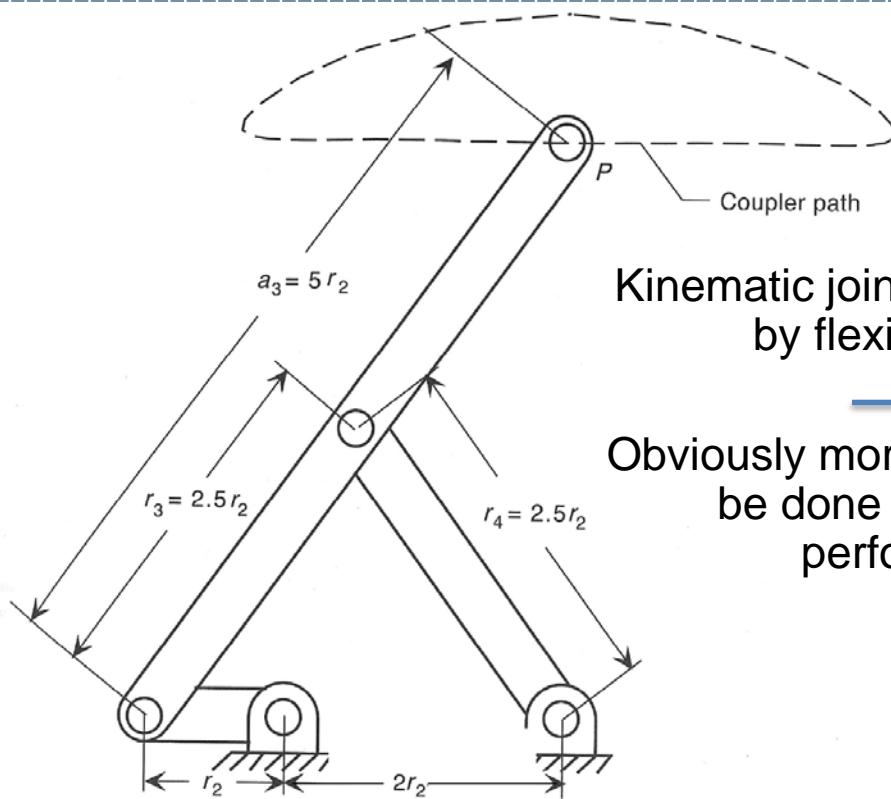
The compliant behavior of flexure hinges and, therefore, the overall response of a flexure-based compliant mechanism largely depend on the specific geometry of the flexure for a given material. Slight alterations or variations in geometry can produce results that are sensible at the response level.



This aspect is particularly important in mechanisms where precision is a key performance parameter or where a finely tuned output is expected in terms of displacement and force.

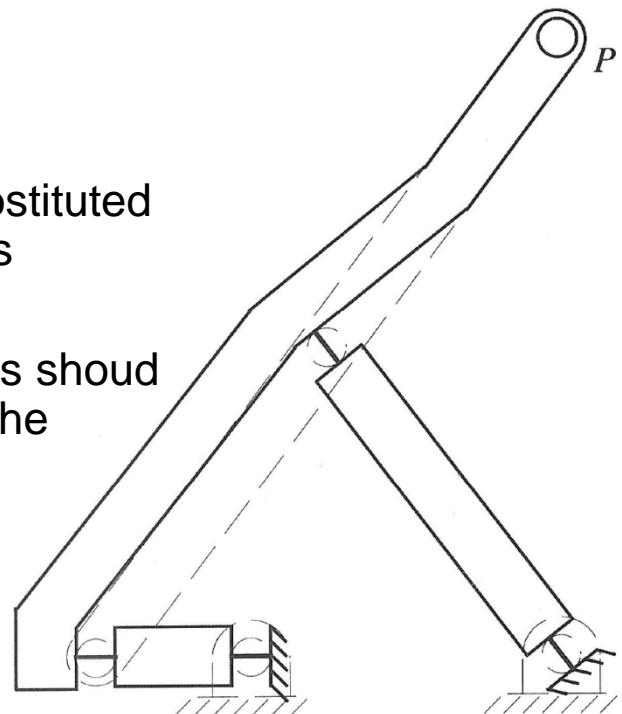
Pseudo-rigid body model

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Kinematic joints are substituted
by flexible beams

Obviously more analyses should
be done to check the
performance



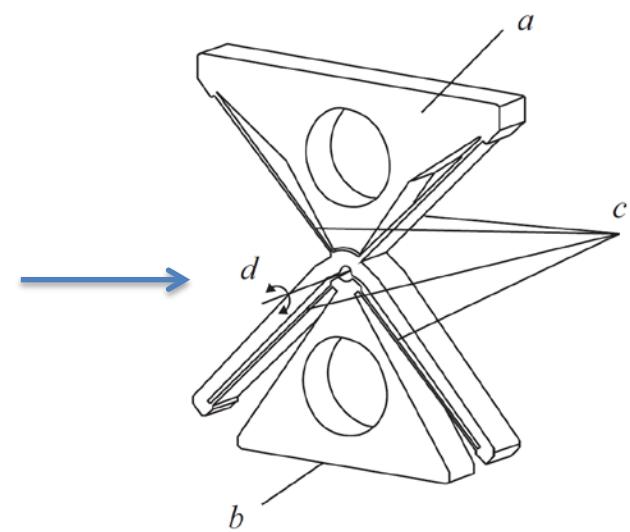
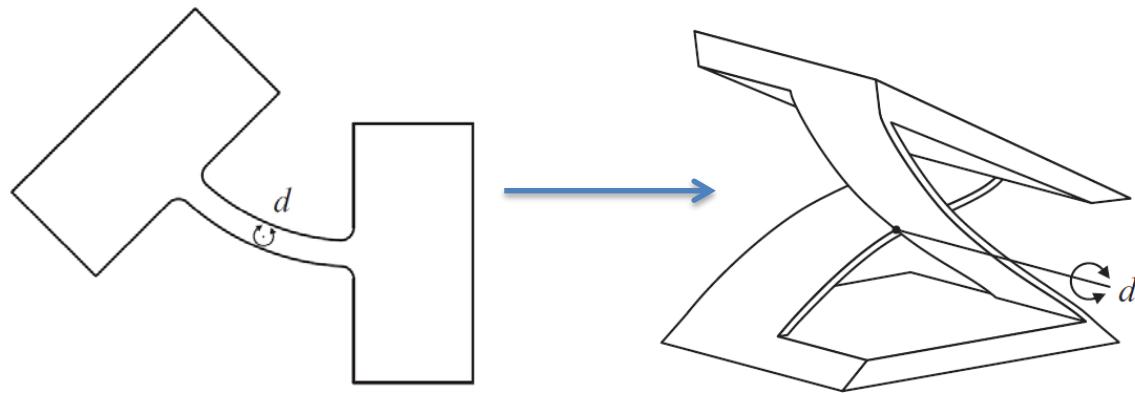
Checks should be related to: output displacement/force (mechanical advantage), stiffness, energy consumption, precision of output motion, fatigue.

Pseudo-rigid body model

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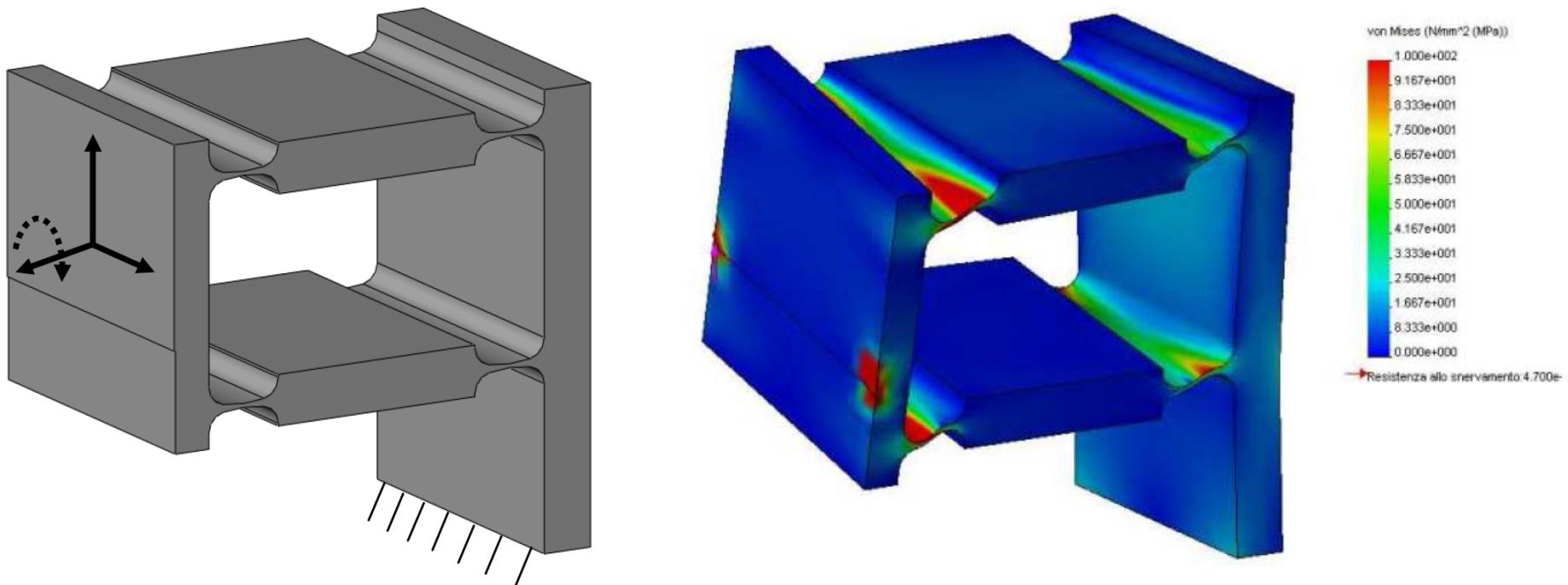
If used for **small displacements** and rotations, compliant mechanisms can profitably substitute traditional linkages. Their kinematic synthesis is usually carried out on pseudo rigid body model, therefore it's easy, at least at the very beginning, to move from kinematic mechanisms to compliant ones.

To guarantee high precision or better fatigue life, some improvements can be done on simple flexure hinges mechanisms, but this developments require numerical analysis (eg. F.E.A.)



Pseudo-rigid body model

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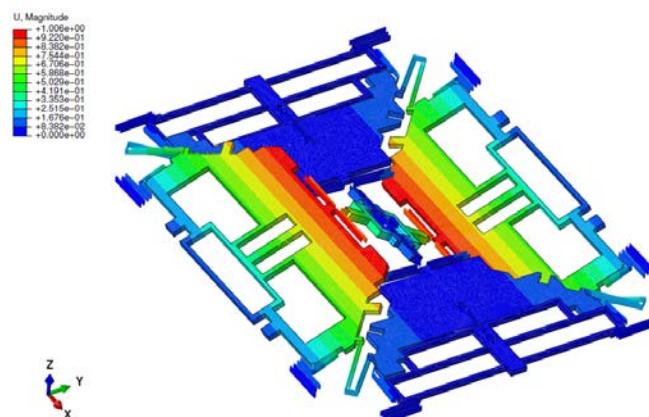
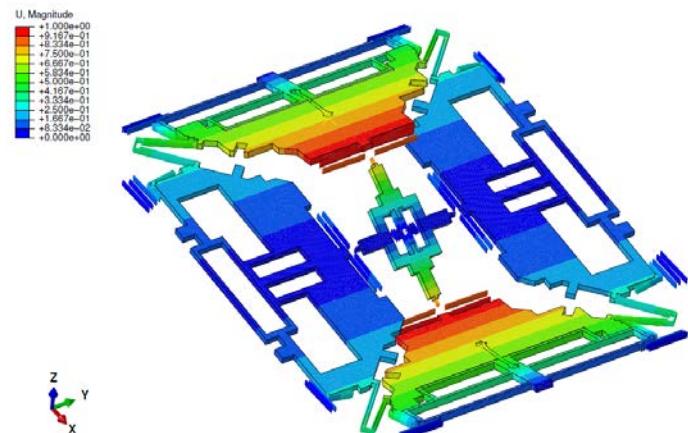
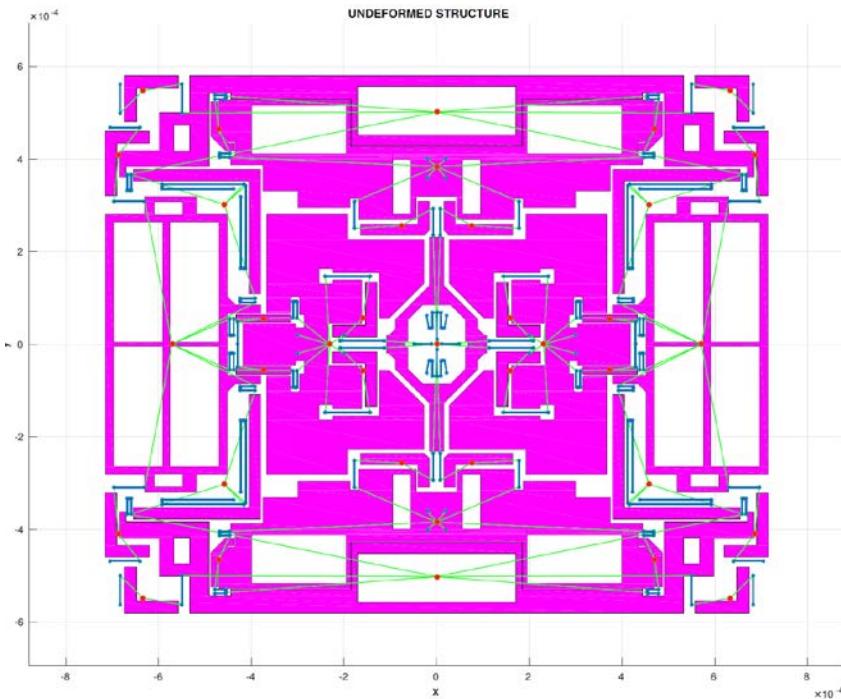


Precision of motion and fatigue strength can be evaluated with a finite element approach

Applications

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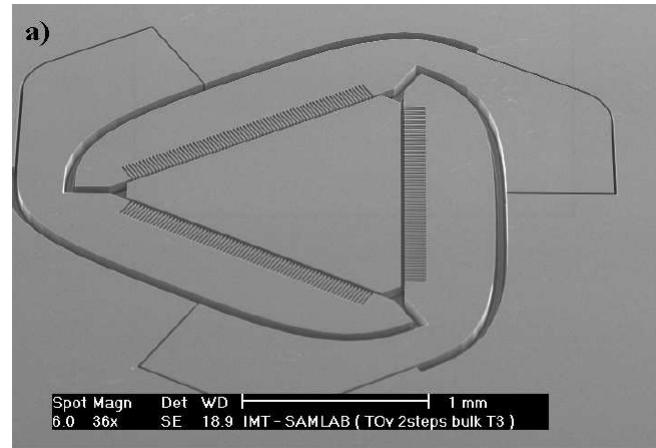
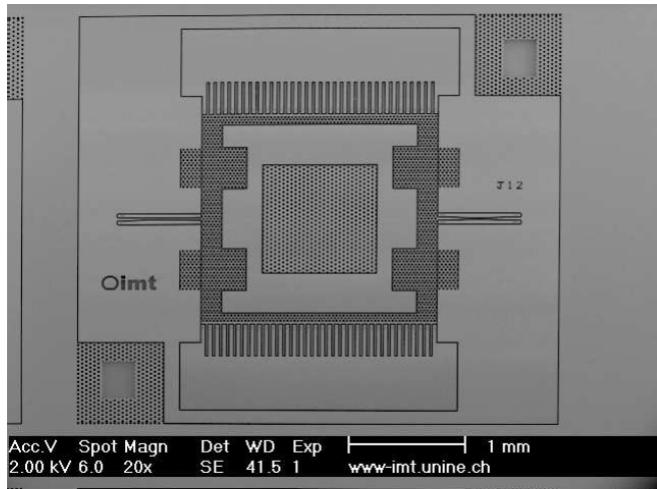
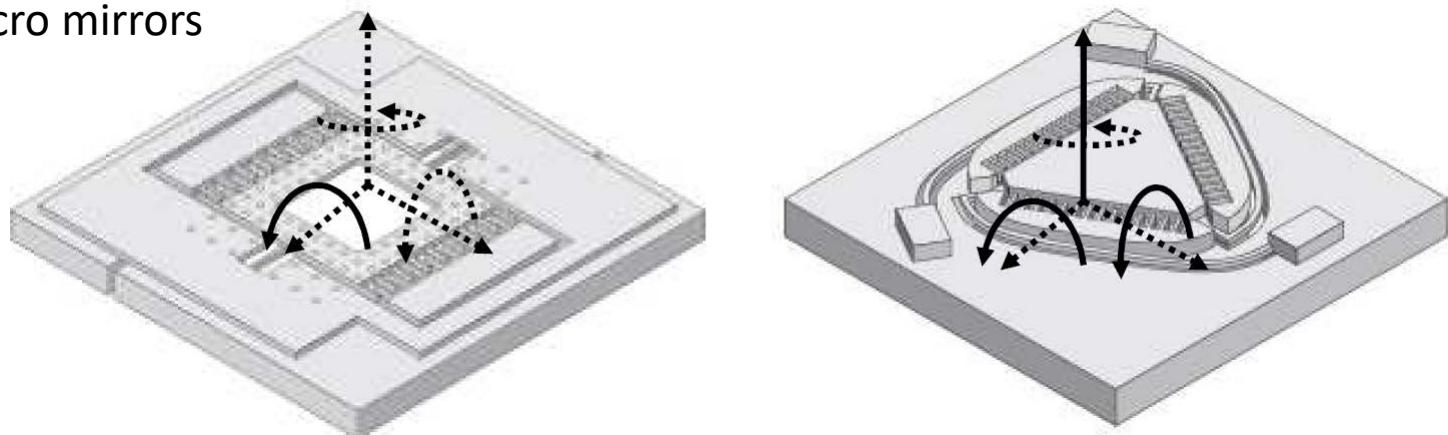
Micro ElectroMechanical devices (**MEMS**) are all based on compliant mechanisms to realize flexible structural elements and allow movements of inertial masses



Applications

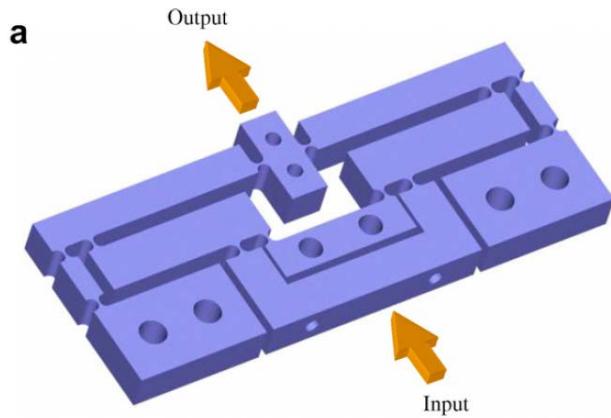
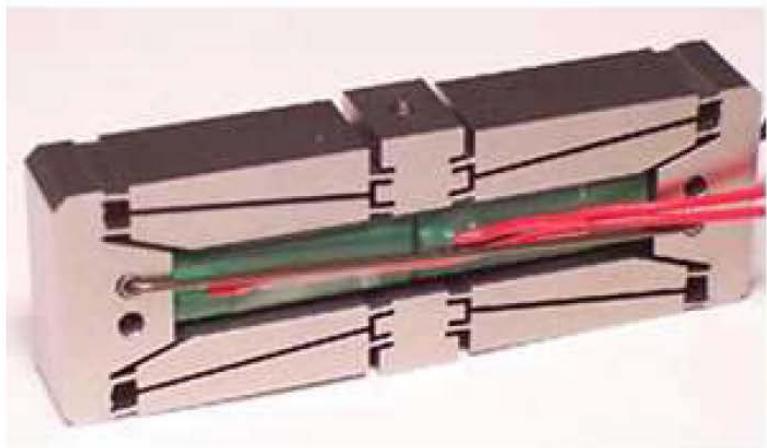
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MEMS Micro mirrors



Applications

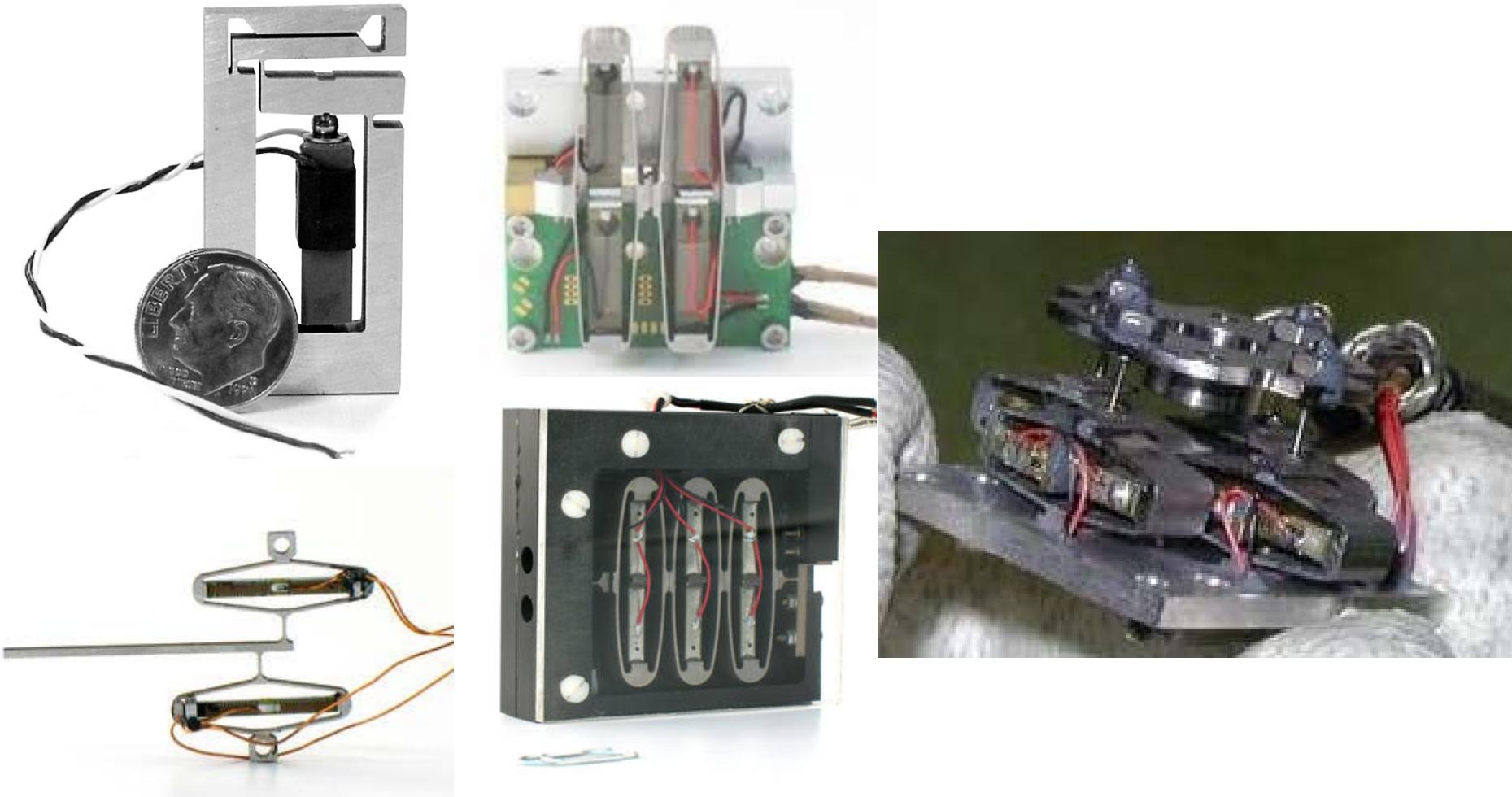
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Compliant mechanisms are used to amplify the displacement of **solid state actuators** whose stroke is limited

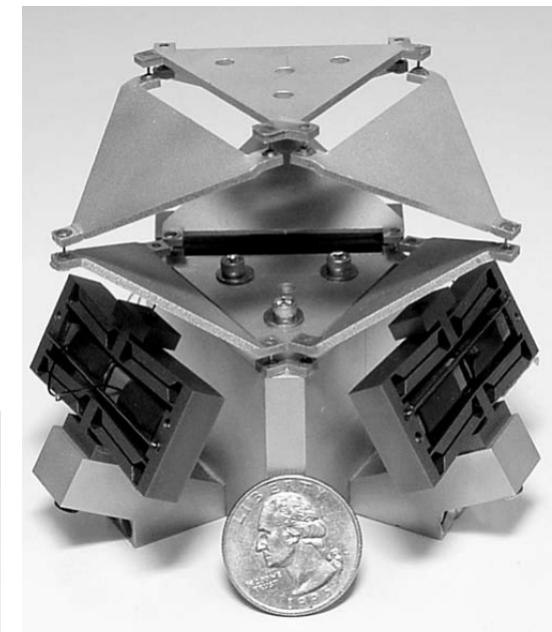
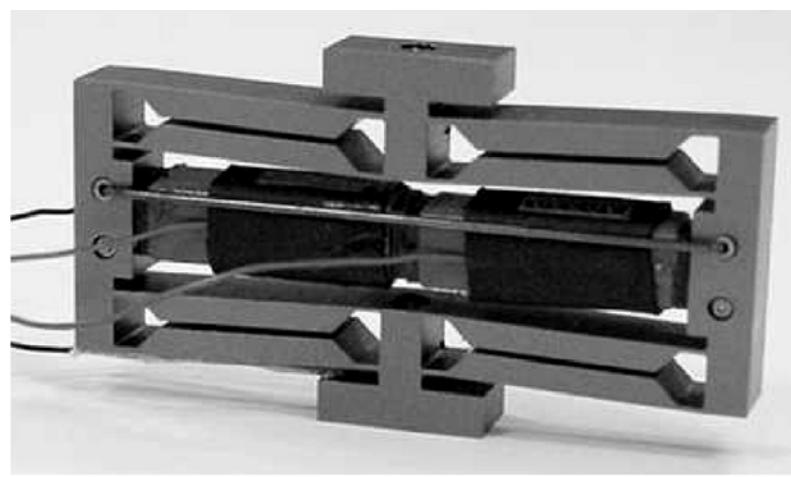
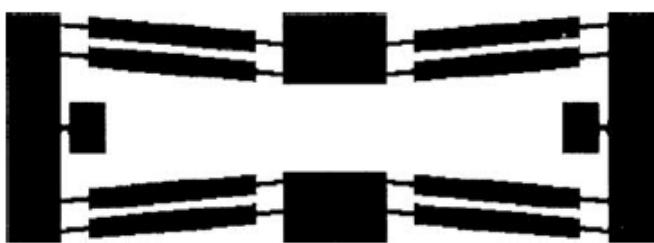
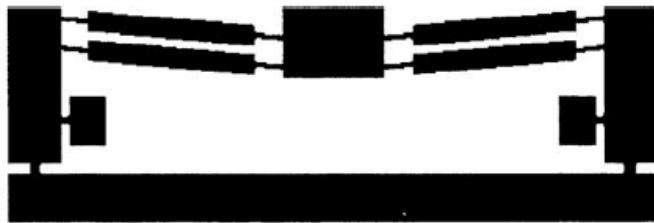
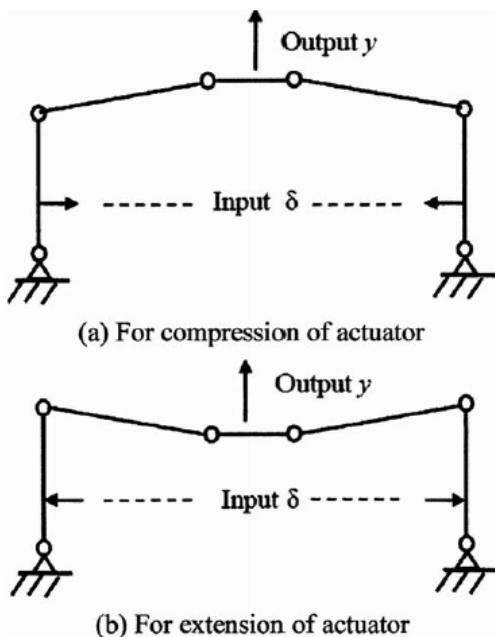
Applications

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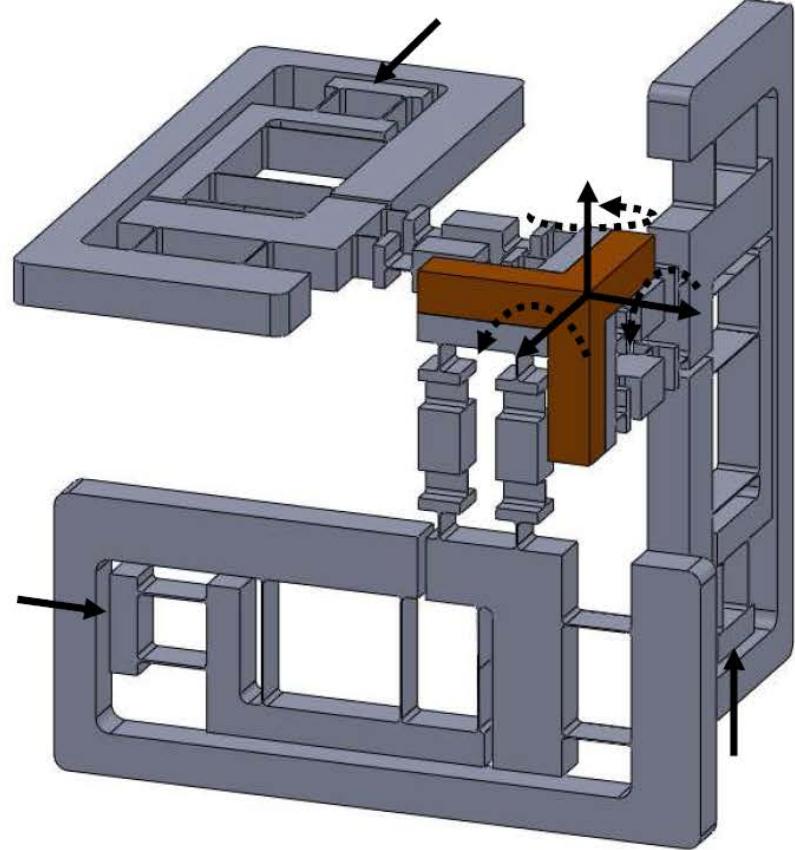
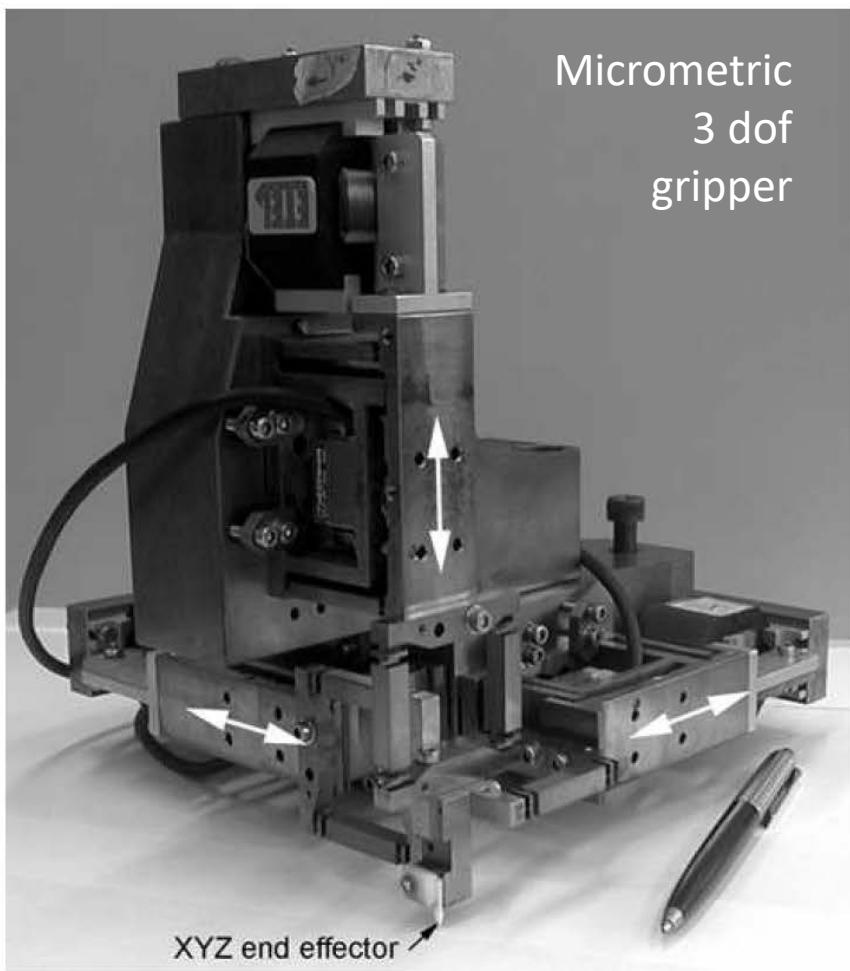
Applications

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Applications

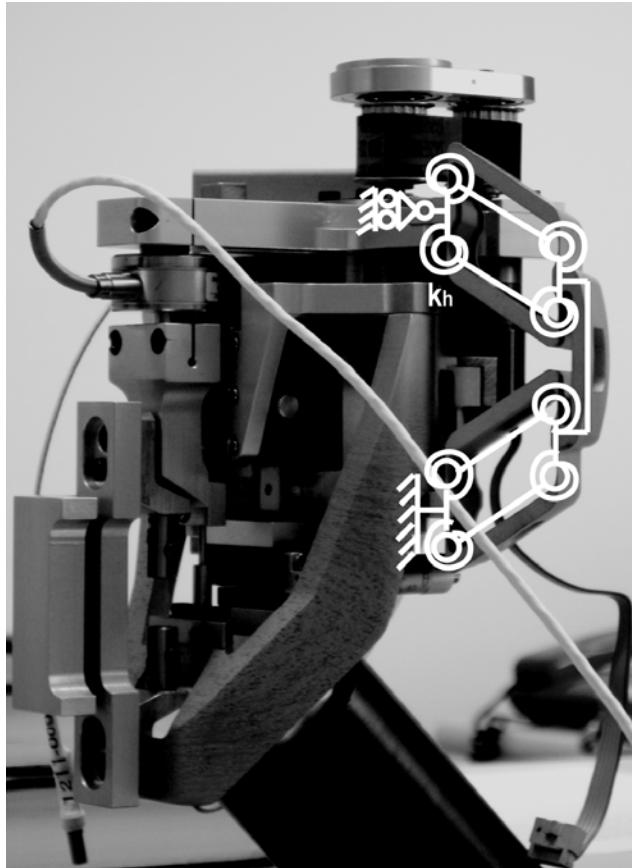
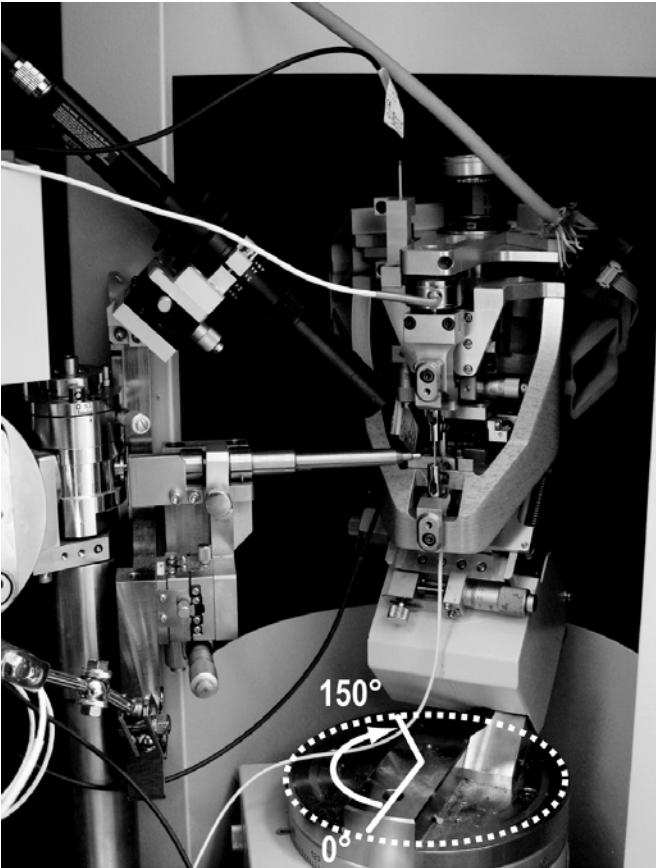
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Compliant mechanisms allows very precise small motion as they don't have backlashes.

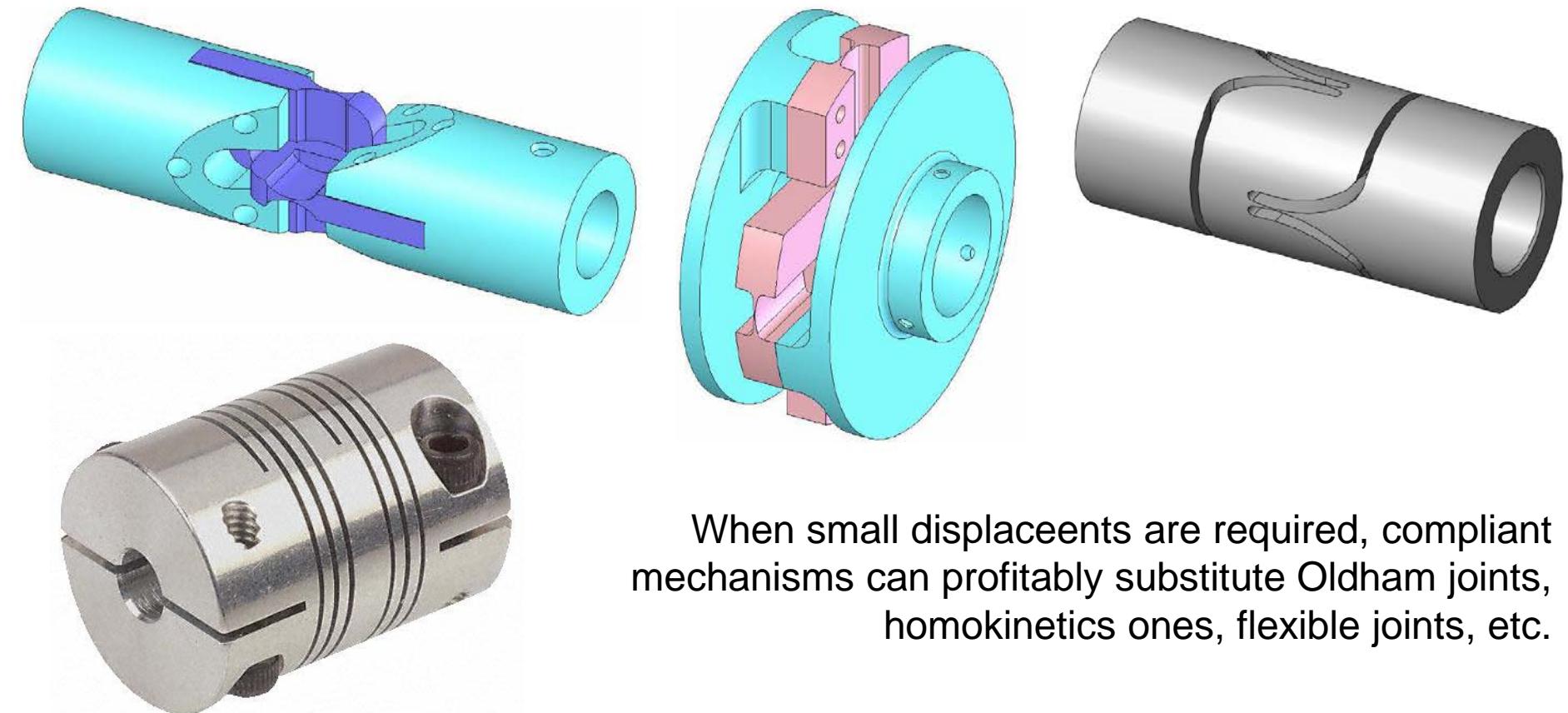
Applications

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Applications

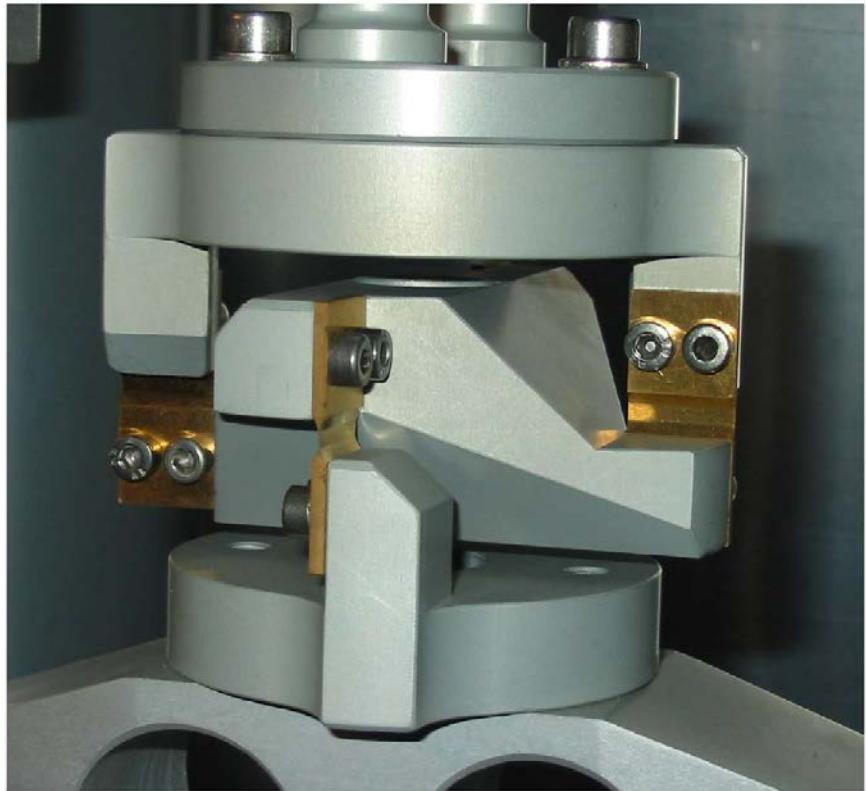
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When small displacements are required, compliant mechanisms can profitably substitute Oldham joints, homokinetics ones, flexible joints, etc.

Applications

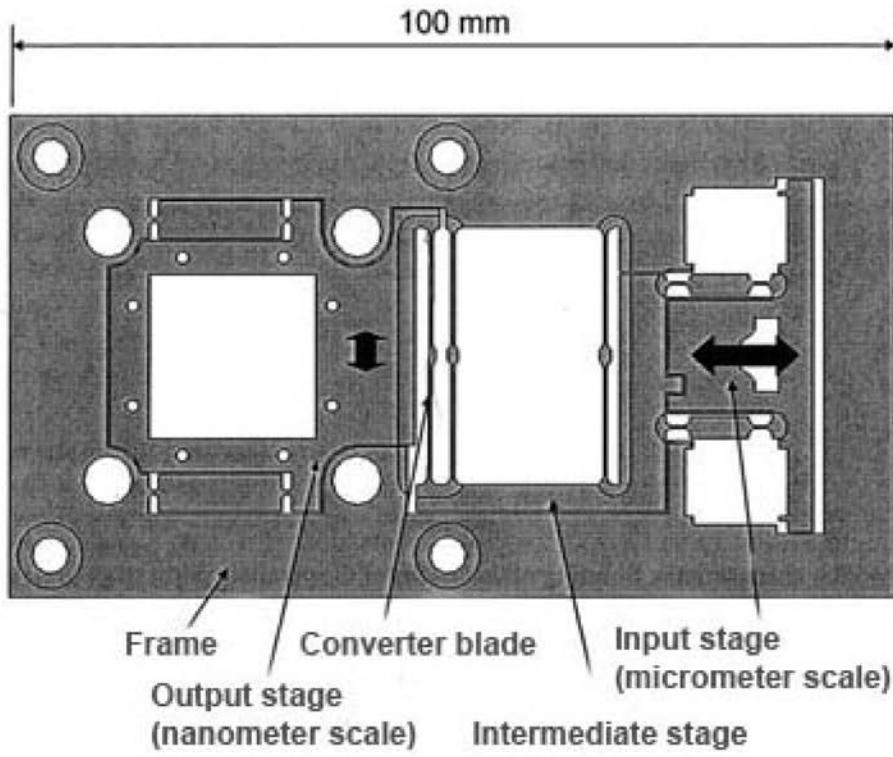
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Applications

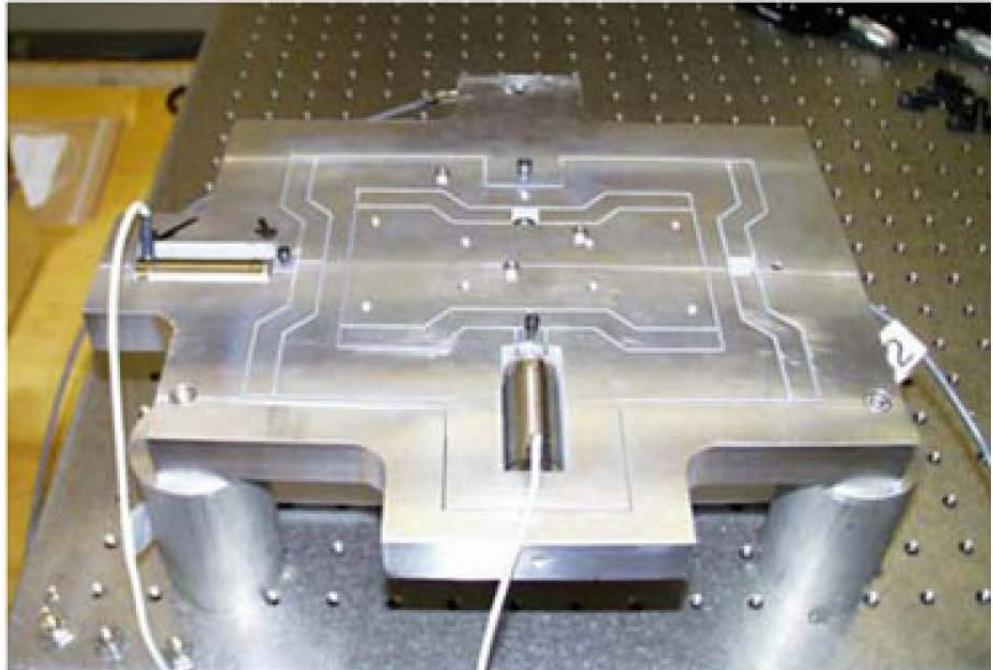
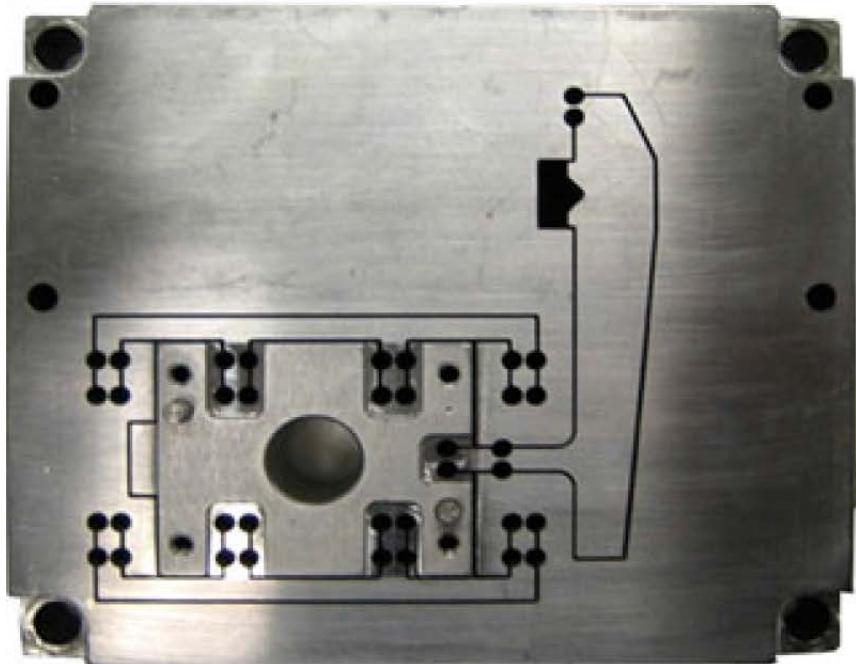
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The nanoconverter is a mechanism with a transmission ratio up to 1/1000 in a very small volume. In this way it is possible to use commercial actuator with micrometric accuracy to have a nanometric accuracy.



Applications

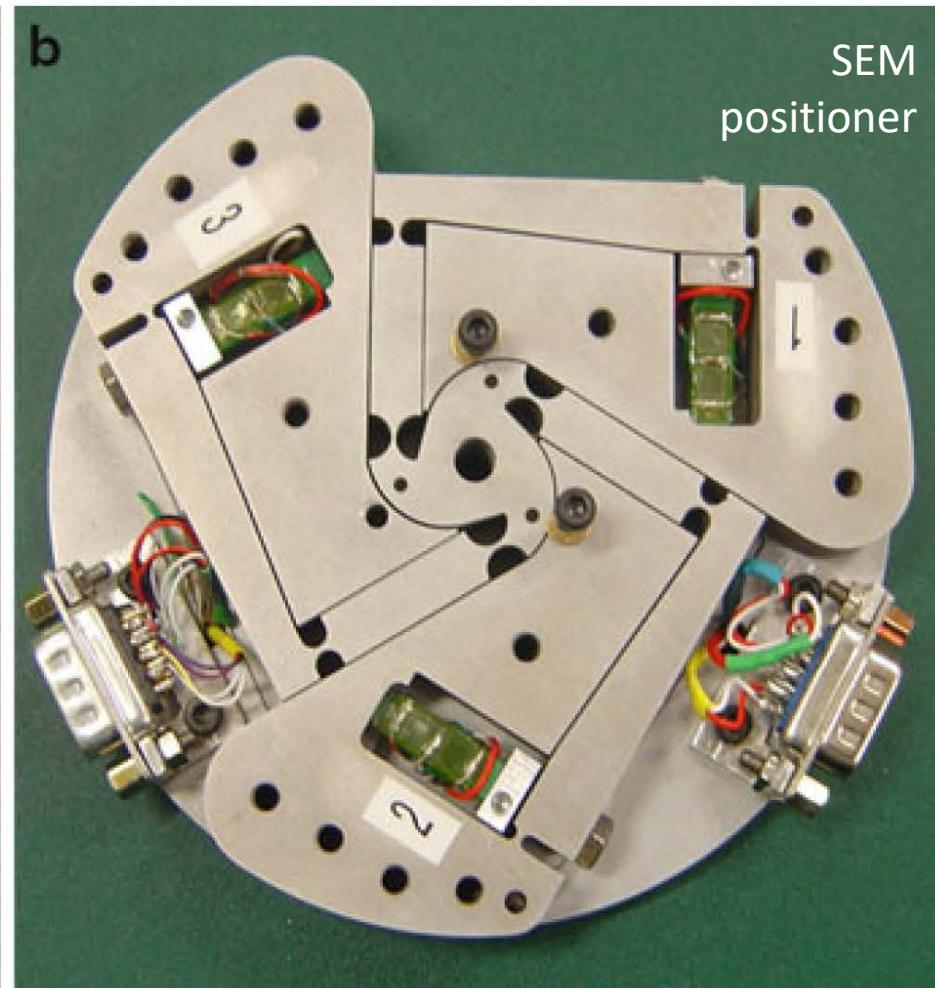
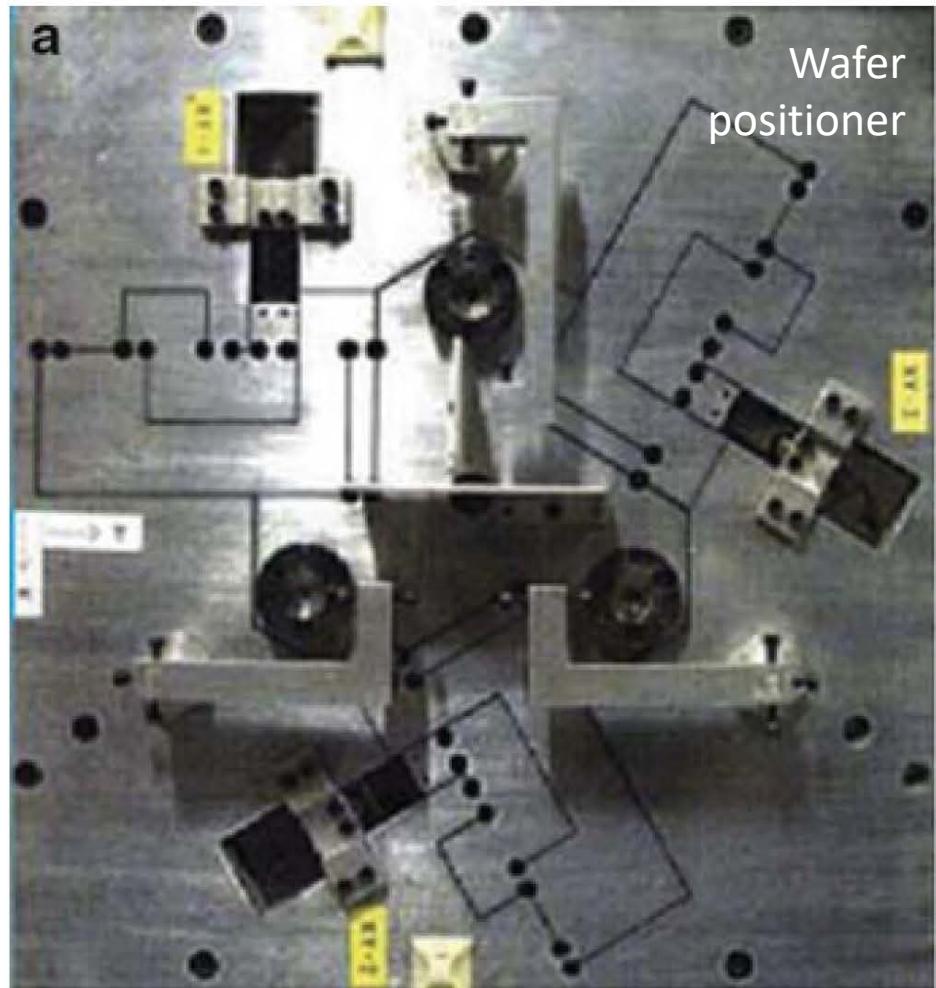
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Monolithic compliant mechanism are used for nano displacements. They are claimed for high stiffness, accuracy and precision.

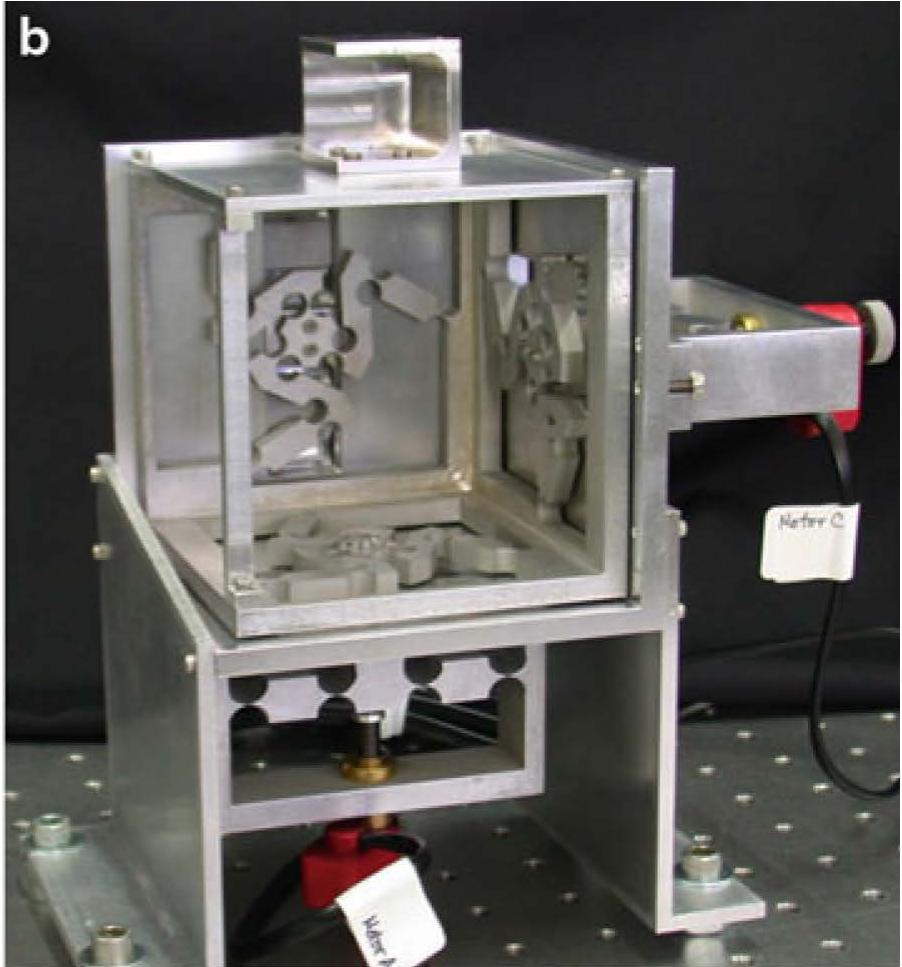
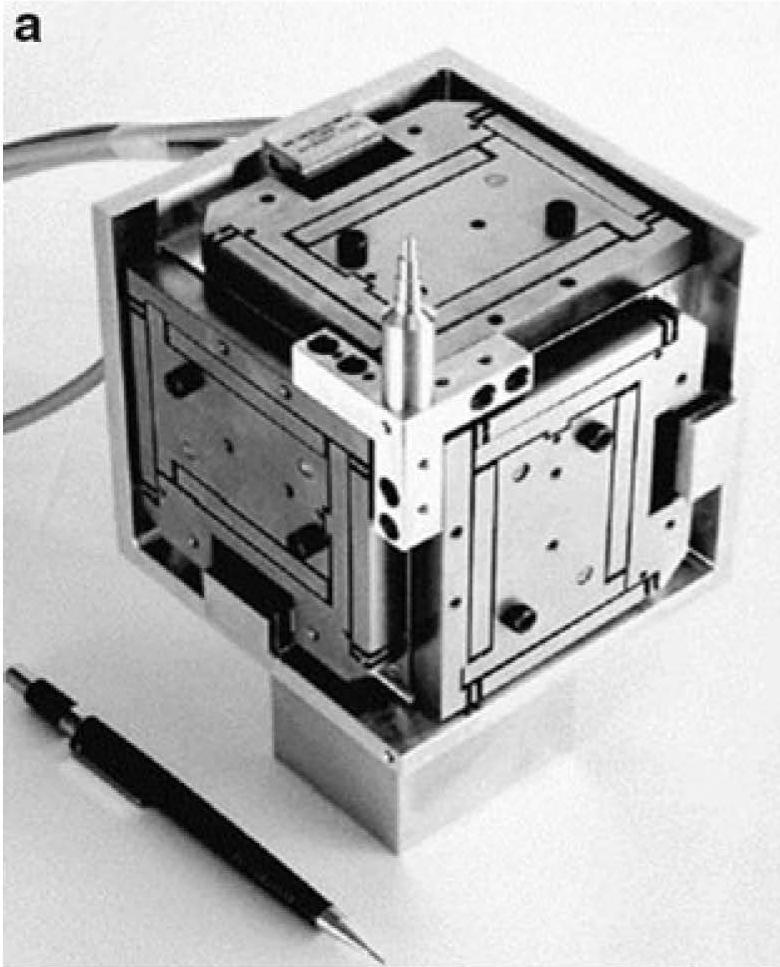
Applications

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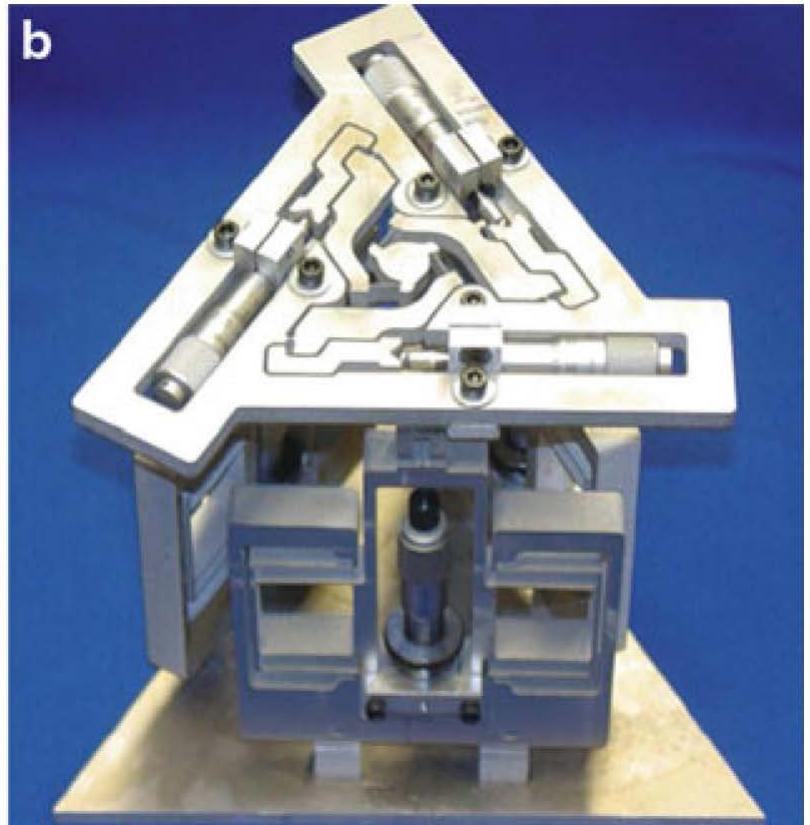
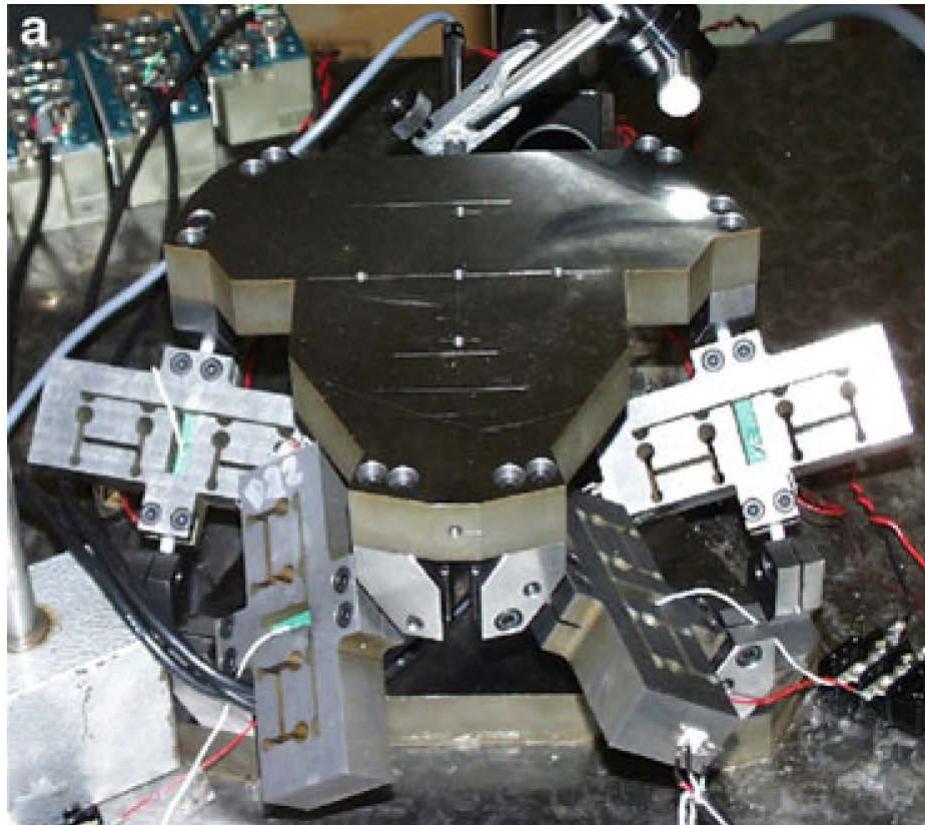
Applications

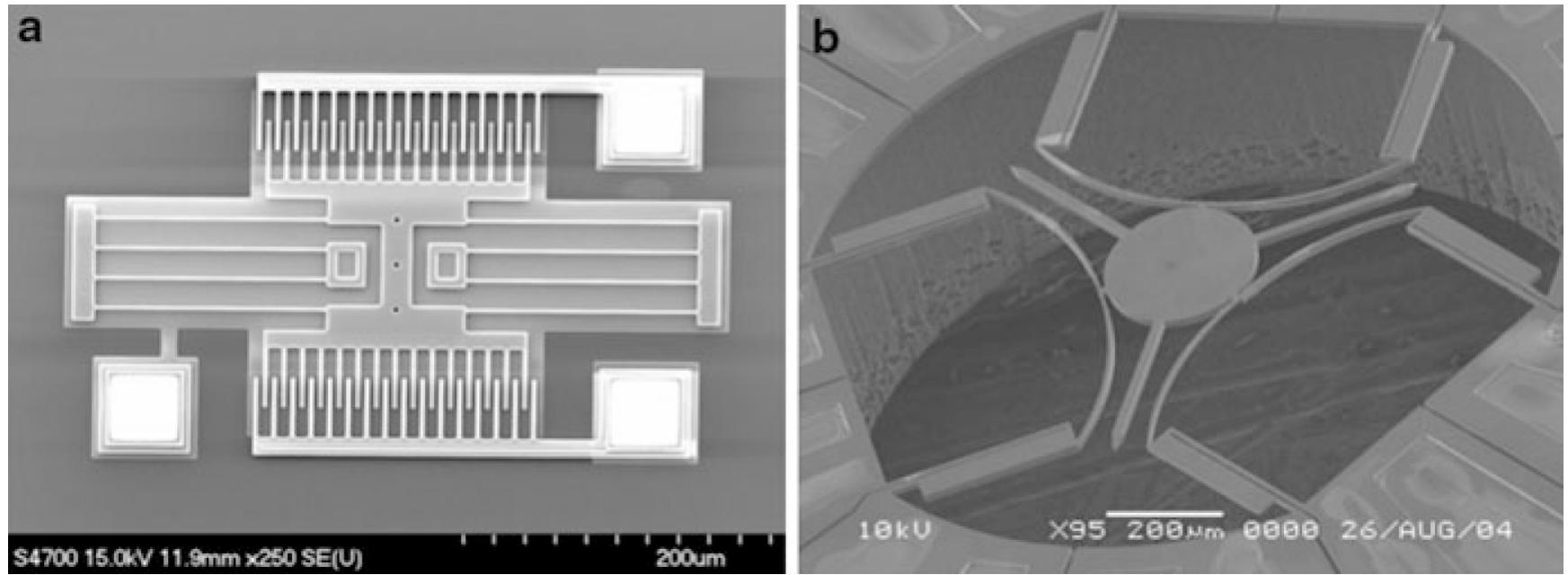
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Applications

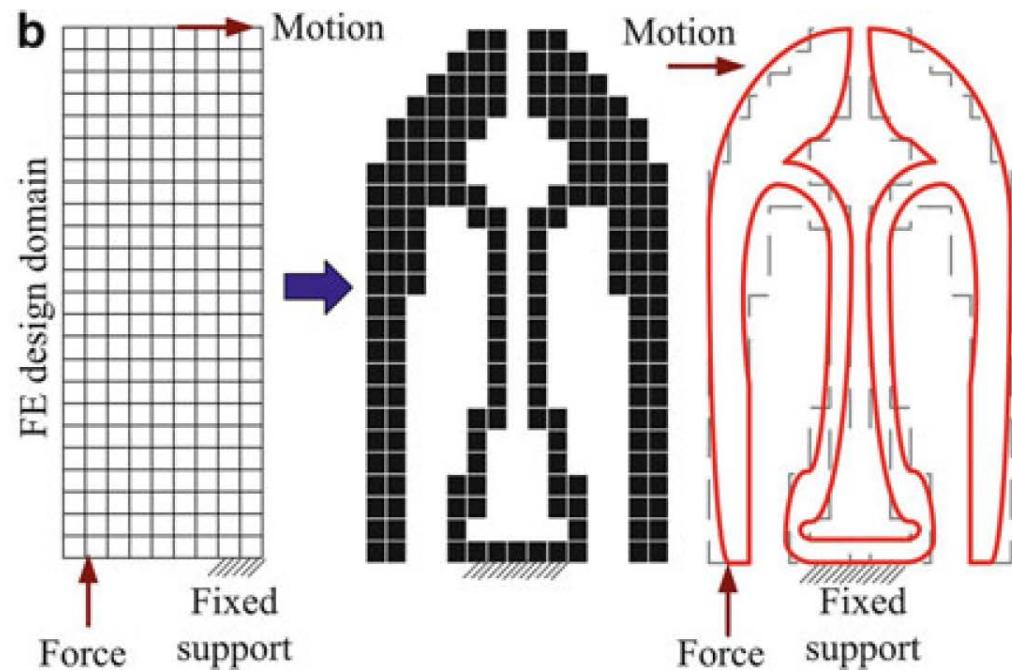
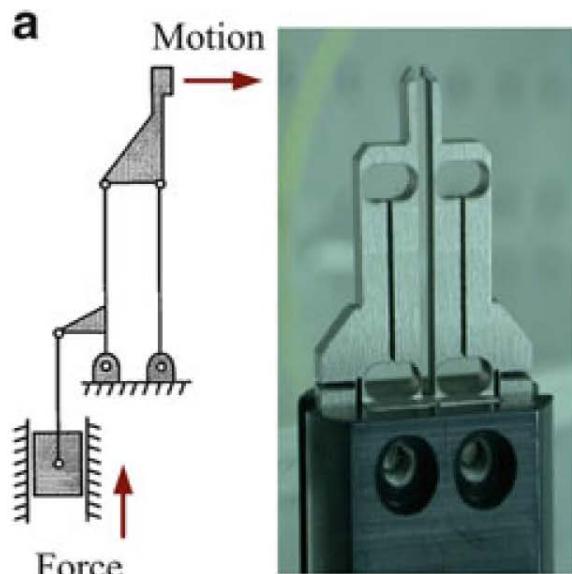
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Applications

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Micro gripper and mechanisms to amplify the displacement are commonly developed as monolithic compliant devices.