

# Functional Mechanical Design

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Anno Accademico 2019-2020

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# Chapter 1

## Introduction

The industrial test case is an automatic machine to produce soap. The machine involves 3 subsystems:

- Press
- Upload of raw material
- Turning pad

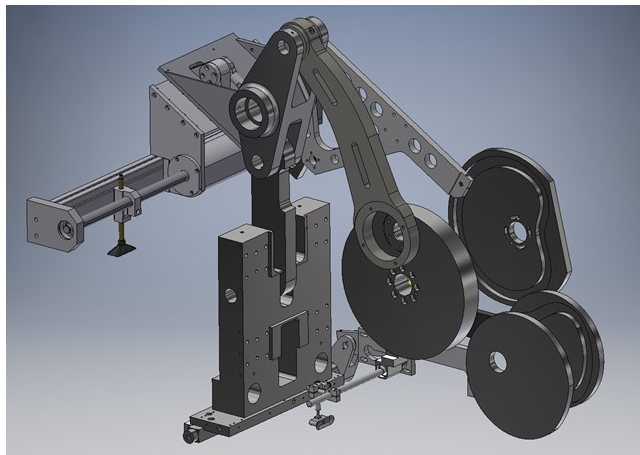


Figure 1.1

## 1.1 Press Subsystem

This subsystem has the purpose of applying a high force on rough pieces of soap in order to deform them and make them assume the desired shape.

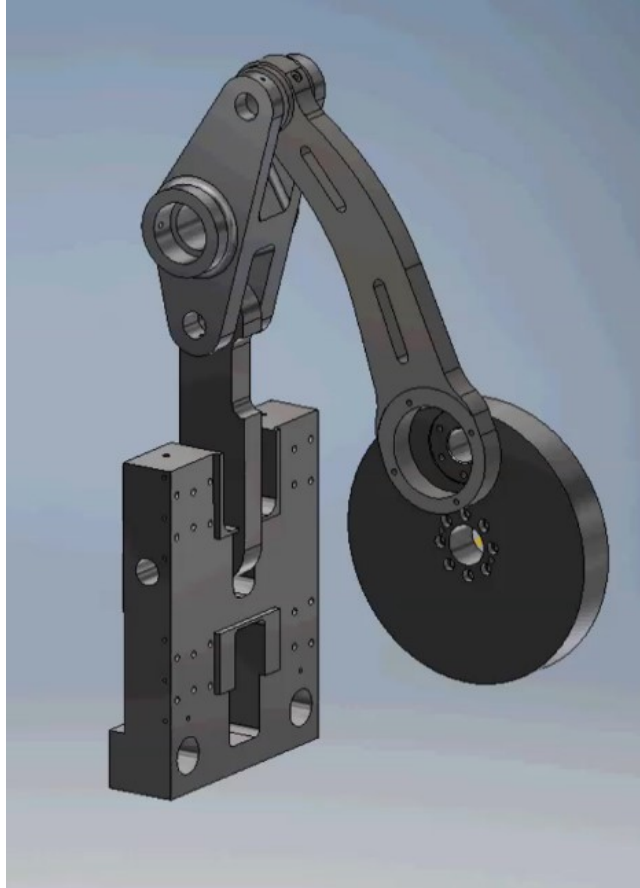


Figure 1.2

## 1.2 Uploading Subsystem

This subsystem is designed to obtain a complex motion in order to charge the raw material onto the tilting pad. This complex motion can be divided into a linear displacement along a horizontal axis ( $x$ ) and a rotation around the orthogonal axis ( $\theta$ ).

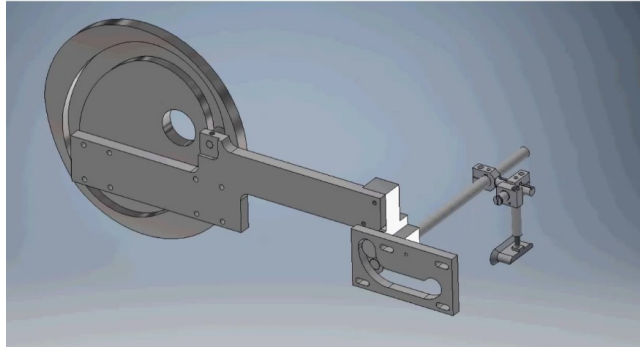


Figure 1.3

## 1.3 Tilting Pad

The porpoise of this subsystem is to support the raw material until is shaped by the press and then rotate to allow the discharge subsystem to pick the final product.

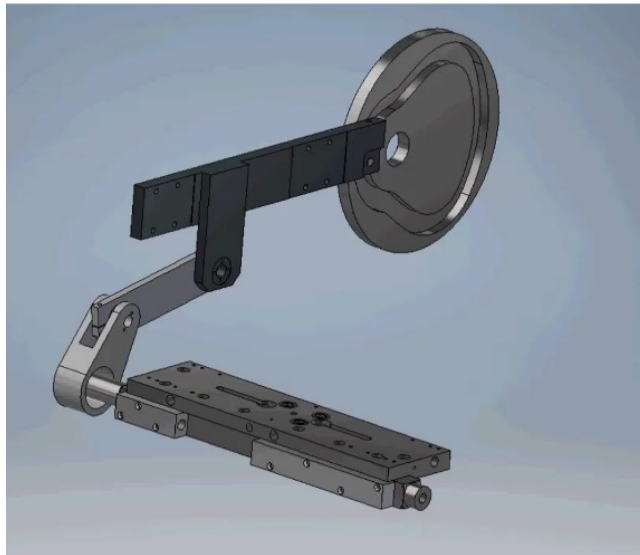


Figure 1.4

The aim of this report is to design one Subsystems at wish and to critically evaluate the feasibility of the solution.

This report is focused on the analysis of the Tilting pad. The tilting pad motion is a pure rotation and takes place around a fixed axis. The movement is realized through a slider crank mechanism which is actuated by a translating cam. Considering that the driving element moves with a constant angular speed, it is necessary to:

1. Design the motion law of the tilting pad
2. Synthesize the slider crank mechanism and the cam mechanisms with a translating follower
3. Analyse the mechanism through kinematic parameters (transmission angle, pressure angle and undercut) and compute the motion law obtained with the designed mechanism
4. Calculate forces transmitted and the motor torque with a multibody model
5. Critically evaluate the feasibility of the system

## Chapter 2

# Motion law of the tilting pad

The motion law of the tilting pad should fulfil some given requirements, for each of these positions of the master angle, the tilting pad must be located at the given angular positions.

Master Angle (deg)	Angular Pos.(deg)
0	45
70	0
200	0
360	45

As it is known, the discharge subsystem needs time to pick the final product, so the tilting pad has to remain still for a while at 45 degrees: another precision point is added.

Master Angle (deg)	Angular Pos. (deg)
0	45
5	45
70	0
200	0
360	45

The following goals are considered for the realisation of this machine:

- Limit max acceleration
- Limit max velocity
- Limit mechanical power
- Limit vibration

It is considered a symmetric profile to fulfil all this targets, in particular in the table below different motion laws are compared according to the coefficient  $c_v$ ,  $c_a$  and  $c_k$ . A unitary displacement and time is considered.

	$c_v$	$c_a$	$c_k$
Const. symm. Acc.	2	4	8
Cubic	1.5	6	3.46
Cycloidal	2	$2\pi$	8.13
Trap. Vel. ( $\zeta_v \frac{1}{3}$ )	1.5	4.5	6.75

For what concerns the trapezoidal velocity profile, it is chosen  $\zeta_v = \frac{1}{3}$  cause it is the value that minimizes all the coefficient as it is represented in the figure.

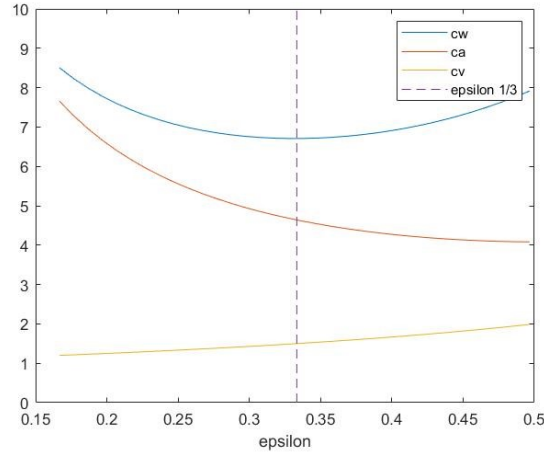


Figure 2.1

Constant symmetric acceleration and the trapezoidal velocity profile seem to be the best choice.

Last point that is important to consider is the discontinuity in the acceleration that can cause vibration. Using the trapezoidal curve method with a ratio  $r = \frac{1}{8}$  the constant symmetric acceleration and the trapezoidal velocity profile are analysed.

First of all it is calculated the modified  $a_{c_{mod}}$  for both the motion laws. Considering that the area below the acceleration plot must remain constant, it is easy to calculate:

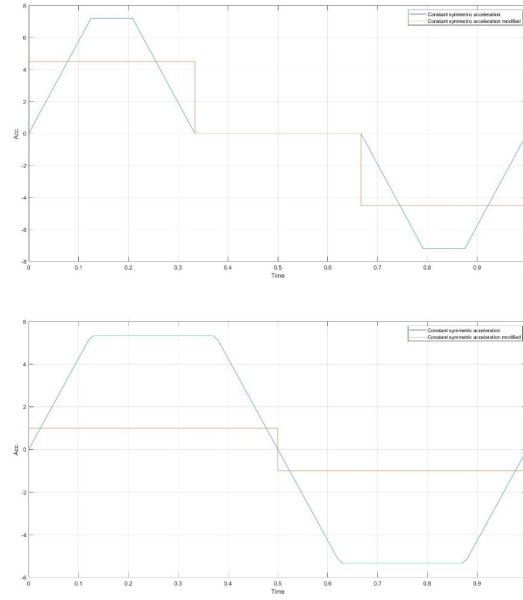
$$c_a * \zeta_v = \frac{1}{2} * (\zeta_v + (\zeta_v - 2r)) * a_{c_{mod}} \quad (2.1)$$

$$a_{c_{mod}} = \frac{c_a * \zeta_v}{\zeta_v - r} \quad (2.2)$$

where  $\zeta_v = 0.5$  for constant Symm. Acc. and  $\zeta_v = 1/3$  for trapezoidal velocity profile.



In the next figures the shapes of the new acceleration are plotted.



(b)

Figure 2.2

The following result are obtained:

	$c_v$	$c_a$	$c_k$
Const. symm. Acc.	2	5.33	8.746
Trap. Vel. ( $\zeta_v \frac{1}{3}$ )	1.5	7.2	7.55

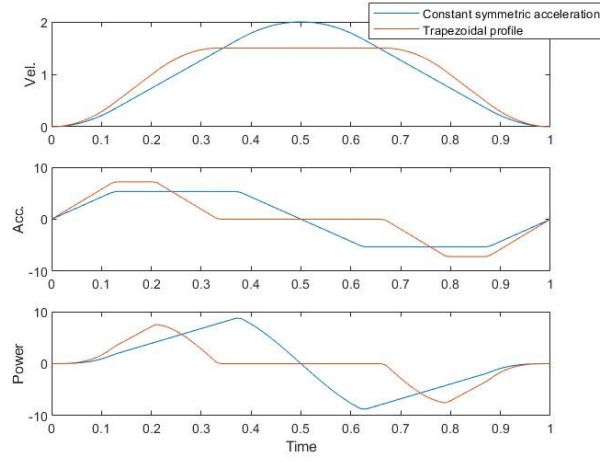


Figure 2.3

Note that  $\zeta_v = 1/3$  is chosen according to the following plot in which it is possible to see that the best performance considering all the coefficients are obtained with this value.

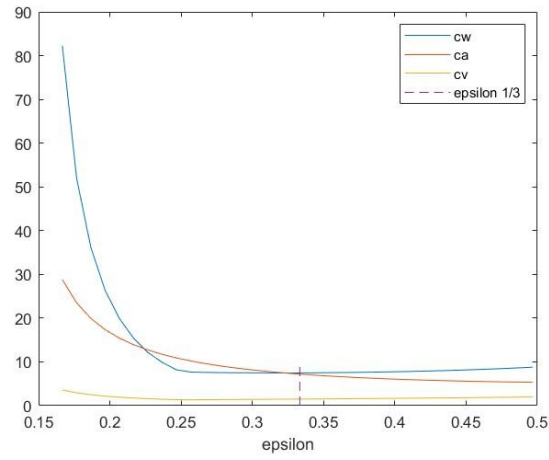


Figure 2.4

For this machine is chosen the modified trapezoidal velocity profile because is the one that minimizes most of the coefficients. In the following figure it is represented the behaviour of the displacement, velocity and acceleration of the crank with respect to the master angle considering a modified trapezoidal velocity profile.

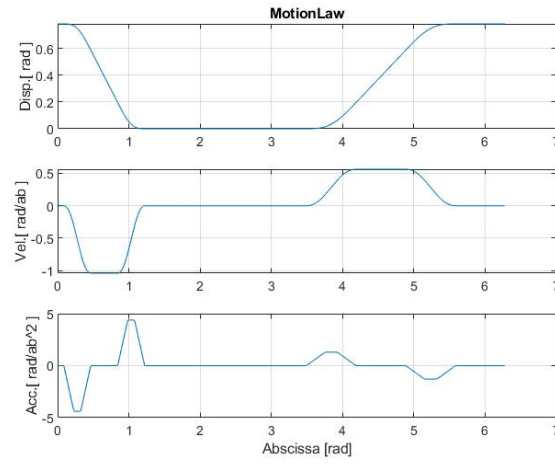


Figure 2.5

## Chapter 3

# Kinematic Analysis

The porpoise of the subsystems is focused on the rotary motion of the tilting pad in which project constraints are given in order to satisfy the synchronism within the whole machine. Table 2 resume those points that can be also represented in a discrete master angle dependant plot as follow:

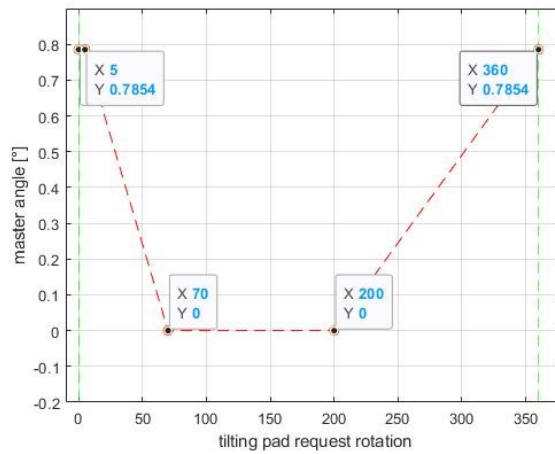


Figure 3.1

It is clear that the cyclical motion of the tilting pad is bounded between a maximum span of  $45deg$  (0.7854 radians) thus the most suitable way to obtain such a motion is a slider crank mechanism in which the rotation of the pad is driven by the crank.

Such a mechanism is composed by a hinged rotating link (the crank) and a slider link connected with the first one. For our porpoise, the driver function is done by the slider and the follower is the crank.

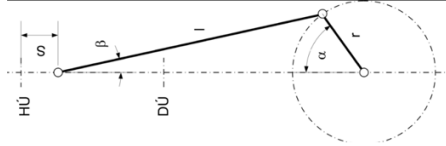


Figure 3.2

The slider crank needs to be drawn so that it complies with the precision points. In order to do that, the closure equations of the mechanism are carried out:

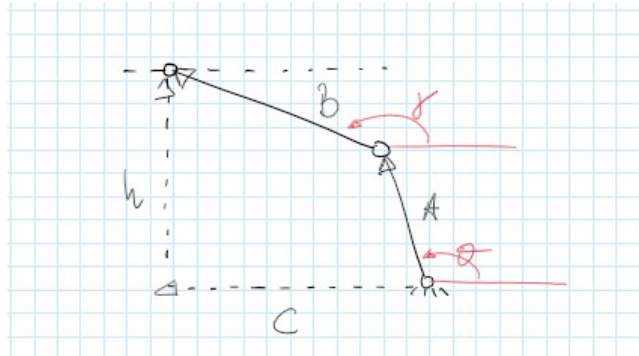


Figure 3.3

$$\begin{cases} A + B &= C + h \\ h &= 1 \end{cases}$$

Where A, B and C are the vectorial representations of the link in the complex plane and h is a fixed length that is the distance between the sliding axis of B and the ground revolute joint of A.

One can notice that the total variable are 6 (the length of each link, four, and the three angles of A, B), that should be reduced to 5 if it is considered h equal to 1 or better, if it is normalized with respect to h. Than since there is not the absolute angle for any configuration, but only the difference between two extreme configurations, one equation may be added considering the angles of the second one as relative with respect to the first one so that one can impose this relative span of  $45deg$  and get at the end 6 unknowns in four trigonometric equations. To solve them at least 2 other variables must be arbitrary chosen. Just as an example, the system below is solved by imposing gamma in the two configurations:

$$\begin{bmatrix} C\theta_1 & C\gamma_1 & 1 & 0 \\ S\theta_1 & S\gamma_1 & 0 & 0 \\ C\theta_2 & C\gamma_2 & 0 & 1 \\ S\theta_2 & S\gamma_2 & 0 & 0 \end{bmatrix} \begin{pmatrix} A \\ B \\ C_1 \\ C_2 \end{pmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Figure 3.4

Where  $\theta_2 - \theta_1 = 45deg$  and  $\gamma_{1,2}$  are chosen freely.

### 3.1 Effect of $\gamma$ on transmission angles

The transmission angle is related to the active component of the force transmitted by the slider to the crank so that at a value of  $90deg$  corresponds a complete transmission of the force.

The same concept could be watched from the opposite: since the task of the subsystem does not involve considerable external forces, the only significant ones are friction, that has to be considered somehow, and inertia, so that a "good" transmission angle means that the force required to drive all the forces from the tilting pad side is reduced to the minimum (geometrically speaking).

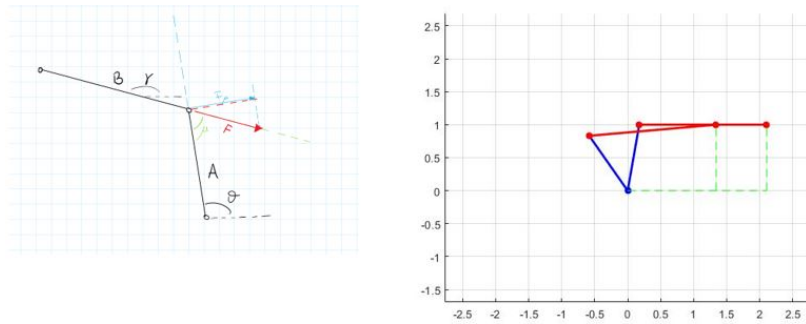


Figure 3.5

Values of gamma near to  $90deg$  shifted with respect to theta are suitable

for our porpoise. The table below shows our project choices in the two extreme configurations.

Config.	$\gamma(deg)$	$\theta(deg)$	A	B	C
1	180	100	1.0154	1.9300	2.1063
2	175	55	1.0154	1.9300	1.3402

Notice that the values of the length are all normalized with respect to  $h$ .

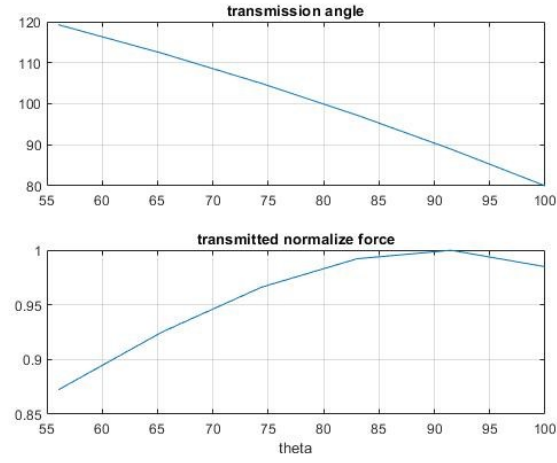


Figure 3.6: Value of the transmission angle with respect to theta and percentage of the force that is transmitted.

## Chapter 4

# Cam design

### 4.1 Introduction

A member that is in contact with the cam profile and transmits power and achieves a predetermined motion, generally reciprocating linear motion or swing, is called a follower. The basic feature of the cam mechanism in the application is that the follower can obtain a more complex motion. Since the motion law of the follower depends on the cam profile curve, it is only necessary to design the contour curve of the cam according to the motion law of the follower. Cam mechanisms are widely used in a variety of automatic machinery, instruments, and steering controls. The reason why the cam mechanism is so widely used is that the cam mechanism can realize various complicated motion requirements, and the structure is simple and compact.

### 4.2 Main Features

#### 4.2.1 Cams Working Principle

Cams are mechanisms for propelling the reciprocating movement or swing by the rotary motion or the reciprocating motion of the cam. The cam has a curved profile or groove, and has a disc cam, a cylindrical cam, a moving cam, etc., wherein the groove curve of the cylindrical cam is a space curve and thus belongs to a space cam. The follower is in point contact or line contact with the cam, and has a roller follower, a flat bottom follower, and a tip follower. The tip follower can keep in contact with any complex cam profile for any movement. However, the tip is easy to wear and is suitable for low-speed mechanisms with low transmission force. To keep the follower in contact with the cam always, a spring or gravity can be applied.

A cam having a groove allows the follower to transmit a determined motion as one of the operative cams. In general, the cam is active, but there are also driven or fixed cams. Most cams are single-degree-of-freedom, but there are also double-degree-of-freedom cone cams. The cam mechanism is compact and ideal for applications requiring intermittent movement of the follower. Compared with hydraulic and pneumatic similar mechanisms, it is reliable in motion and





follower with respect to the cam. This position corresponds to the configuration in which the follower begins to rise on the cam. Knowing the follower motion curve,  $y(\alpha)$ , we can add the roller radius  $R_{b0}$ , so to obtain the pitch profile:

$$r_0(\alpha) = R_{b0} + y(\alpha) \quad (4.1)$$

Considering a cam mechanism, with a follower having a translating movement (which is the configuration of the studied project), we can define the pressure angle as following, considering C as the center of the cam and K as the center of the curvature:

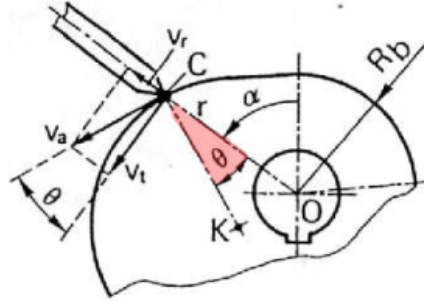


Figure 4.2

The pressure angle is:

$$\theta = K\hat{C}O \quad (4.2)$$

To calculate  $\theta$  we rely on the relative kinematic approach. the motion of the contact point C, is described by two components:

$$\nu_r = \dot{y} = \frac{\partial y}{\partial \alpha} \frac{\partial \alpha}{\partial t} = y' \omega \quad (4.3)$$

$$\nu_r = \omega \overline{OC} = \omega(R_{b0} + y) \quad (4.4)$$

So, the absolute velocity is given by:

$$\nu_a = \sqrt{\nu_r^2 + \nu_t^2} = \omega \sqrt{y'^2 + (R_{b0} + y)^2} \quad (4.5)$$

Since the absolute velocity is tangent to the pitch profile, and so, normal to the CK which is the radius of curvature, the pressure angle is given by the angle between the absolute velocity and the drag velocity:

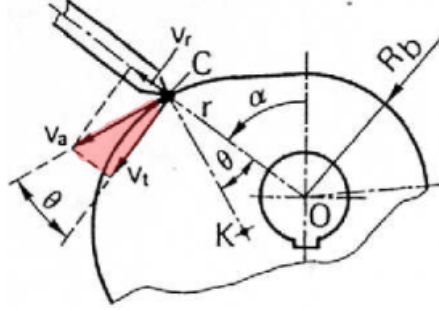


Figure 4.3

$$\tan\theta = \frac{\nu_r}{\nu_t} = \frac{y'}{R_{b0} + y} \quad (4.6)$$

For evaluating the radius of curvature, we use the Coriolis theorem. obtaining the acceleration in point C, we get:

$$\bar{a} = \bar{a}_r + \bar{a}_{tr} + \bar{a}_c \quad (4.7)$$

The projection of  $\bar{a}$  on the direction of the radius of curvature will give of us a new drag acceleration, considering the radius of curvature, as the radius of rotation:

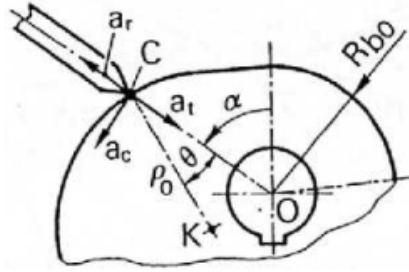


Figure 4.4

So,

$$a_N = \frac{(a_{tr} - a_r)\nu_t + a_c\nu_r}{\nu_a} = \frac{\nu_a^2}{\rho_0} \quad (4.8)$$

$$\rho_0 = \frac{\nu_a^3}{a_t\nu_t - a_r\nu_t + a_c\nu_r} \quad (4.9)$$

substituting the radial, drag and absolute velocity, we will obtain:

$$\rho_0 = \frac{(y'^2 + (R_{b0} + y))^3}{(R_{b0} + y)y'' + 2y'^2} \quad (4.10)$$

Note that,  $\rho_0$  can be negative and in that case the cam profile is concave. The cam profile radius of curvature, now, can easily be obtained by subtracting the



In the end the diagrams are plotted for a positive and negative cam:

1. Cam and pitch Profile

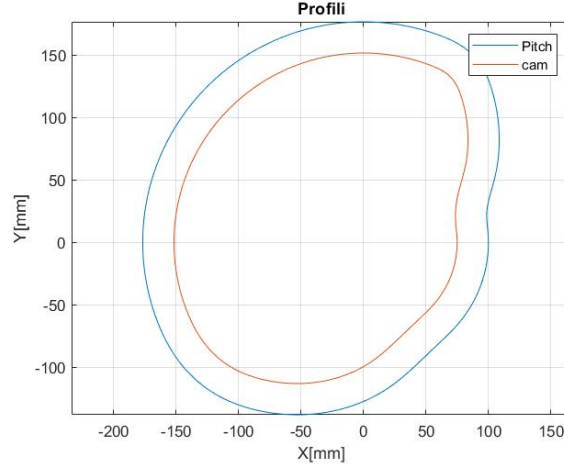


Figure 4.6: Plot of positive cam and pitch profile

2. Pressure angle  $\theta$  with respect to the master angle

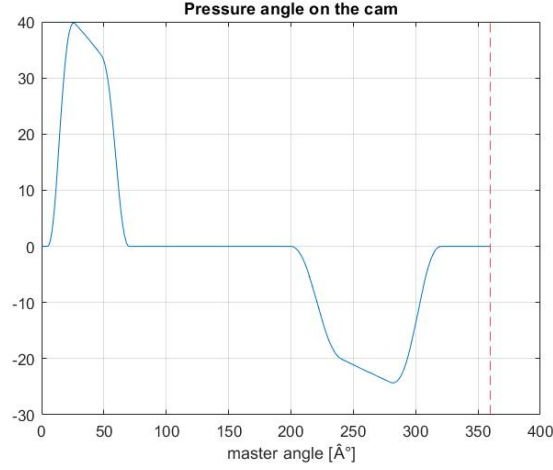


Figure 4.7

Here we have the plot of the pressure angle for the cam mechanism. As it is shown here, the pressure angle is less than  $45deg$  for the positive cam, which demonstrates that a reasonable result in terms of the pressure angle regarding our system is obtained. The study of the effect of the friction on the pressure angle and in general on the system is available in the study of the feasibility of the system.

3. The product of  $\rho$  and  $\rho_0$  for positive and negative cam

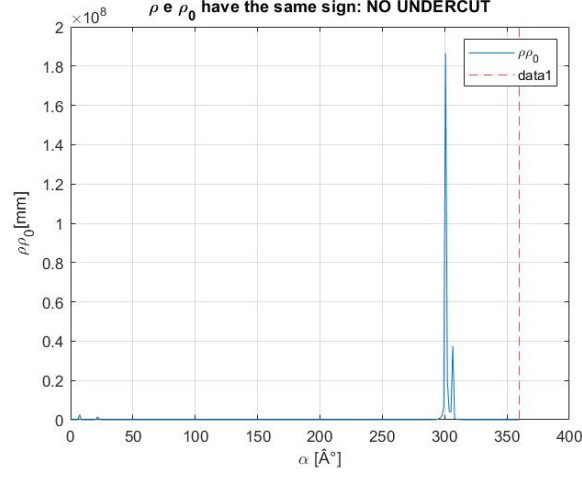


Figure 4.8

Since the product of the product of  $\rho$  and  $\rho_0$  is positive in all values of master angle, we do not have any undercut.

4. Changes of radius OC with respect to the master angle

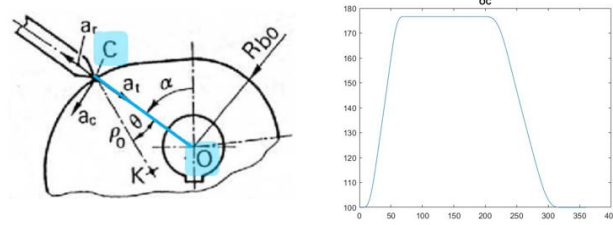


Figure 4.9: Plot of the radius  $r$  with respect to the master angle

In the above plot, the minimum on the  $y$  axis is equal to the base radius which is equal to 100.

## Chapter 5

# Calculate forces transmitted and the motor torque with a multibody model

It is designed our mechanism using Adams View.

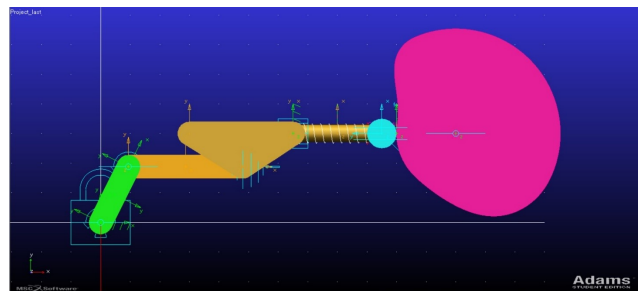


Figure 5.1

As shown in the figure above, the mechanism is made up of:

- two links
- a mass
- a cam
- a follower
- a roller

The green link is the crank whereas the yellow one is the coupler link. The lengths of the links are taken from the Matlab code and the material selected is steel. The mass is located in correspondence of the origin of the crank and it accounts for the inertia of the tilting pad. The plate is used to connect the

coupler and the follower, and from a computational point of view is part of the follower.

The cam was designed using the Cam Machinery tool of Adams.

The first step is creating the follower motion, therefore it is imported the data of the translation of the follower from Matlab. Knowing the displacement, Adams calculates acceleration and jerk.

The second step is creating the cam profile. We selected a disk shape, a minimum radius of 100 mm and a circular end radius of the follower of 25mm. It is decided to neglect the spring of the follower because we want a kinematic simulation.

At last, the cam is created.

The data imported from Matlab is as shown in the following table:

Crank length	105 mm
Coupler length	193 mm
Crank orientation	80 deg
Coupler orientation	0 deg

The joints between the different parts of the mechanism are:

JOINT	BODIES	DOC
Revolute	Crank/Graund	5
Fixed	Mass/Crank	6
Hooke	Crank/Coupler	4
Sherical	Coupler/Follower	3
Cylindrical	Follower/Ground	4
Revolute	Cam/Ground	5
Revolute	Roller/Follower	5
Point to curve	Cam/Follower	2

With these constraints we have a system with 2 degrees of freedom: the rotation of the roller and the rotation of the cam. Therefore, it is imposed two motions: a null rotation to the roller and a displacement of  $180deg * time$  to the cam.

The pressure of the press subsystem on the tilting pad is neglected, as well as friction.

In this way, it is possible to run a kinematic simulation of the model because the Groubler count shows 0 degrees of freedom. Here it is reported some interesting results.



The plots of displacement, velocity and acceleration of the crank are here shown:

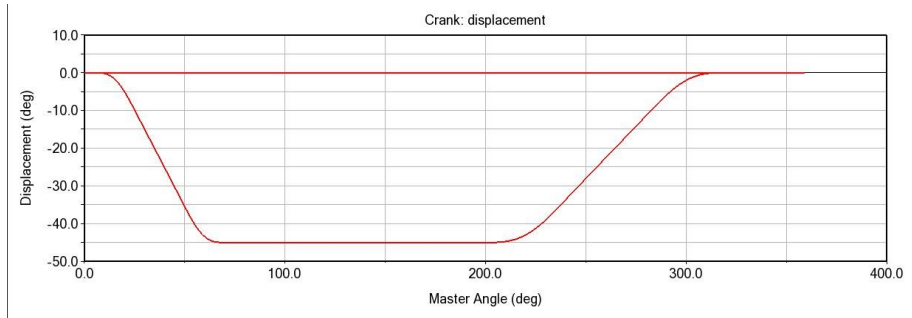


Figure 5.2

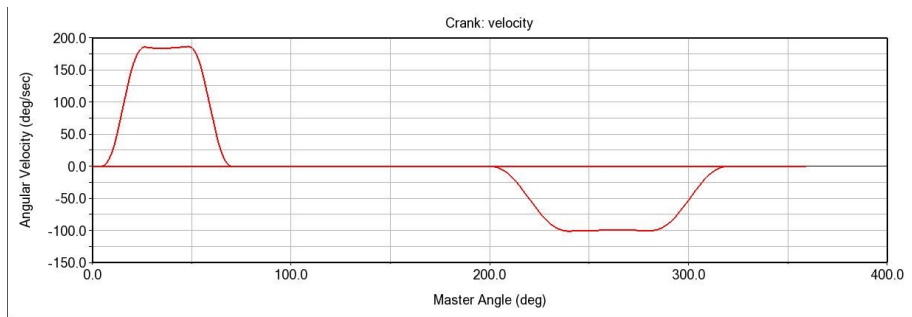


Figure 5.3

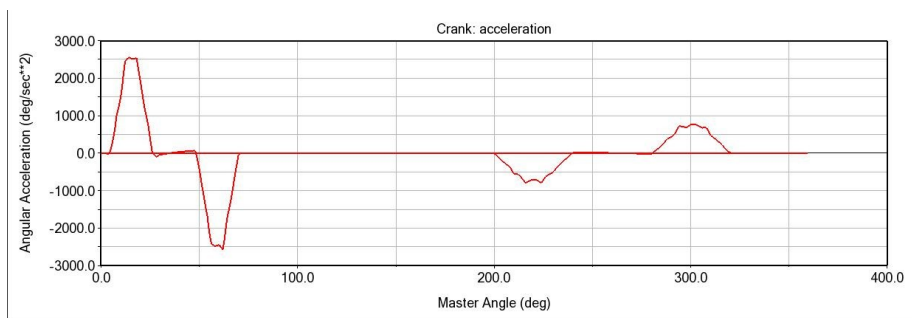


Figure 5.4

The torque transmitted to the tilting pad:

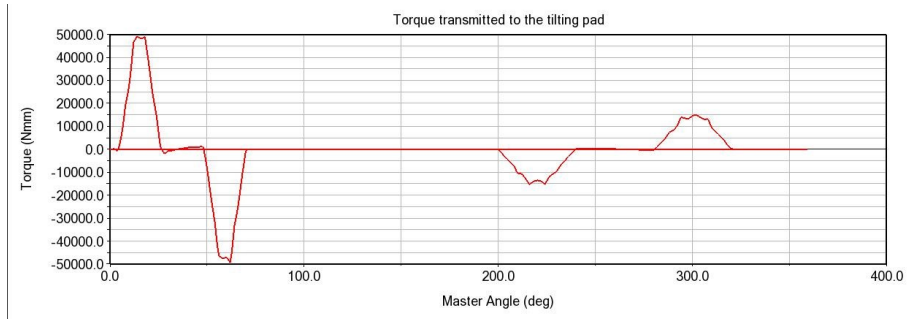


Figure 5.5

Regarding the cam, it is possible to find the torque applied by the cam, that can be used to size the motor needed to move our mechanism:

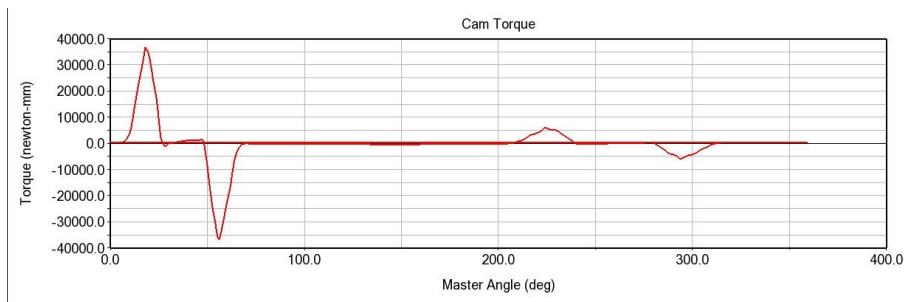


Figure 5.6

The Pressure Angle of the cam:



Figure 5.7

As for the forces transmitted, the cam transmits to the follower these forces along x and y axes:

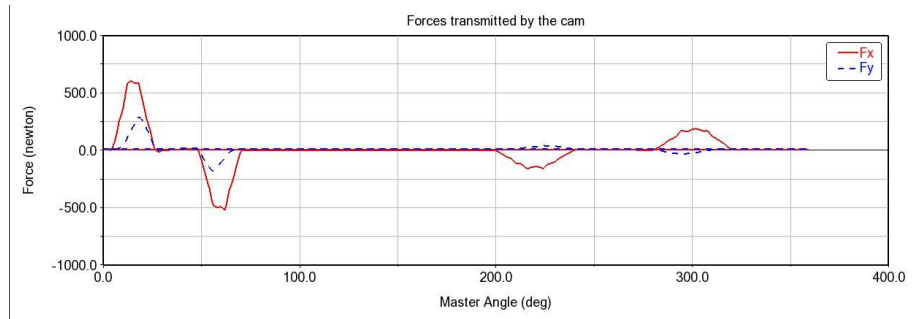


Figure 5.8

The S force resulting in the plane is:

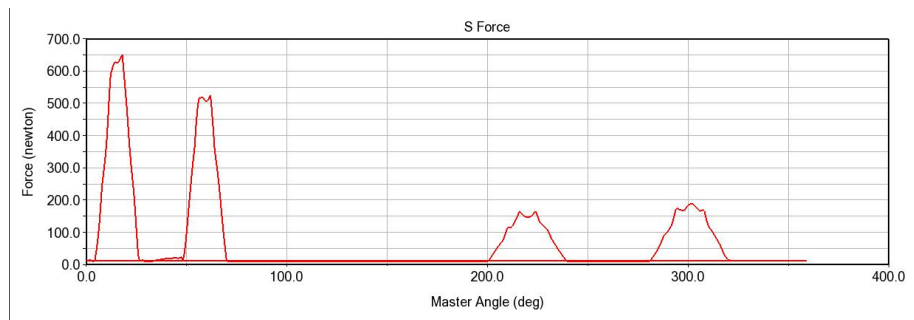


Figure 5.9

The Force along the z axes should be null. However, it can be noticed from the following plot that this is not true.

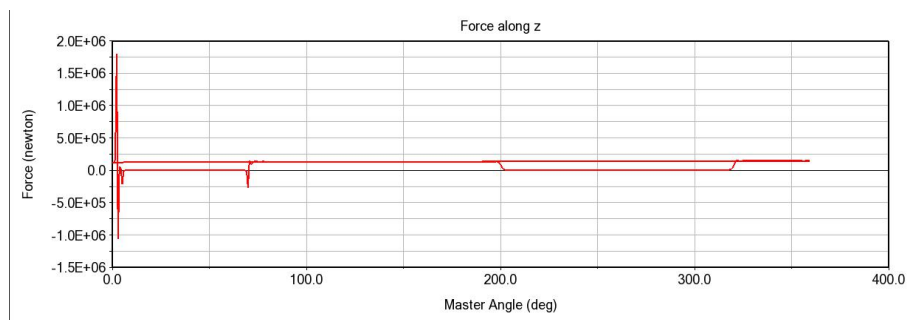


Figure 5.10

This is related to the fact that the center of mass of the follower has a non null z component. Since this force does not affect the kinematic and dynamic of the plane it is possible to neglect this problem.

Comparing the ratio between these forces to the tangent of the pressure angle of the cam, almost same results are obtained. So this condition is verified. It is now taken into account how the forces are transmitted, and in the end what percentage of the force transmitted by the cam is perceived by the crank. Let's begin with the forces transmitted by the follower to the coupler link:

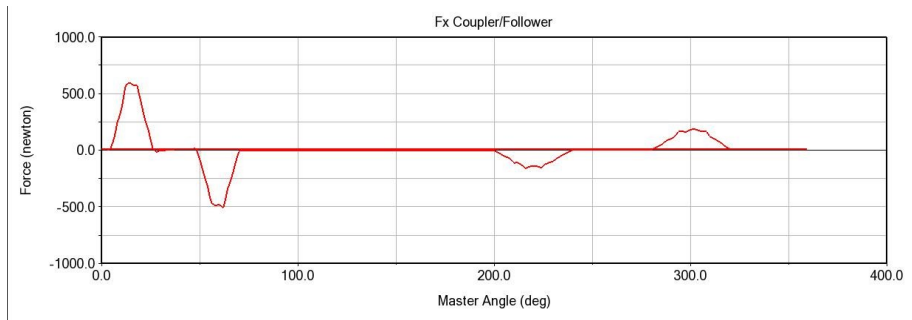


Figure 5.11

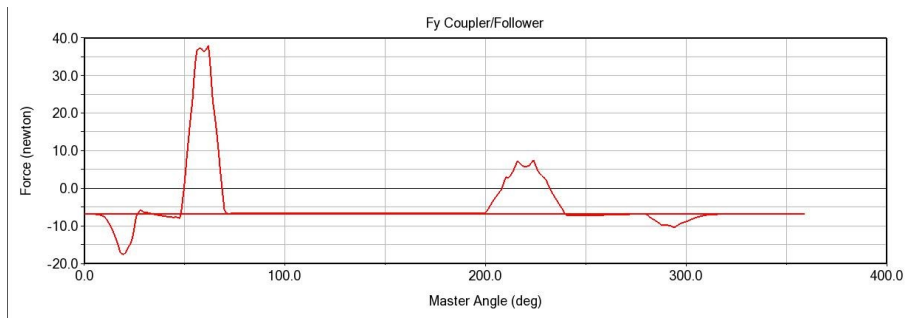


Figure 5.12

Finally, the force that arrives at the crank:

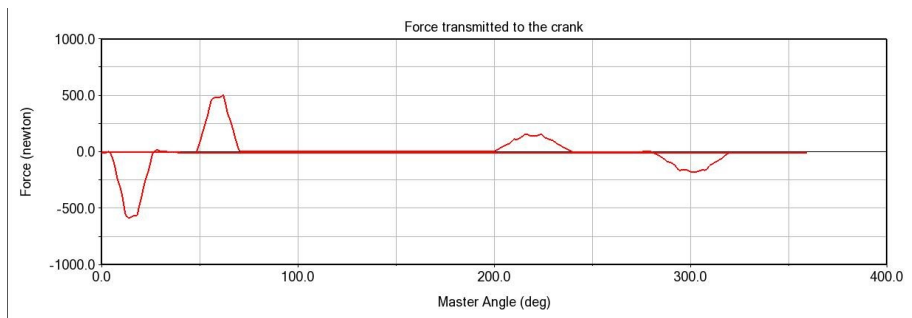


Figure 5.13

The maximum value of the force that arrives to the crank is 503N, whereas the initial maximum value of the force transmitted by the cam is 667, 5N, calculated

as the square root of the two components  $F_x$  and  $F_y$  represented in the plot shown before.

## Chapter 6

# Feasibility of the system

Now it is possible to discuss the feasibility of our system.

### 6.1 Condition

The main condition that now are considered considering the cam-follower system are two.

#### 6.1.1 S condition

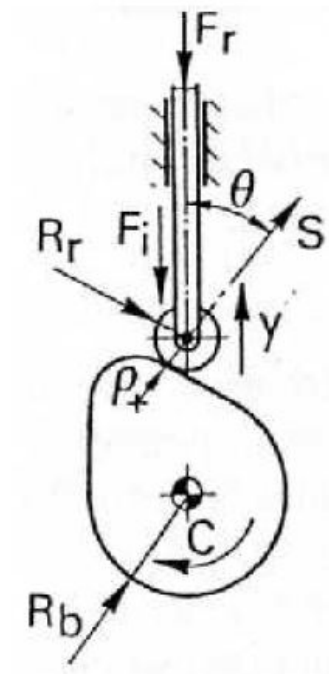


Figure 6.1

First of all:

$$S = \frac{F}{\cos(\theta)} < S_{amm} \quad (6.1)$$

Where  $F$  is the force of inertia of the follower,  $S$  is the force transmitted by the cam and  $\theta$  is the pressure angle. It is considered null the load force of the follower. From the first equation it is possible to notice that to avoid  $S = \infty$ ,

$$\theta < \frac{\pi}{2} = \theta_0 \quad (6.2)$$

However it is a good practice to limit the value of the pressure angle below  $45deg$  in order to have

$$S < 1.4F \quad (6.3)$$

The presence of a friction force between slider and ground introduces a further limitation on the pressure angle. The equation for the equilibrium force and

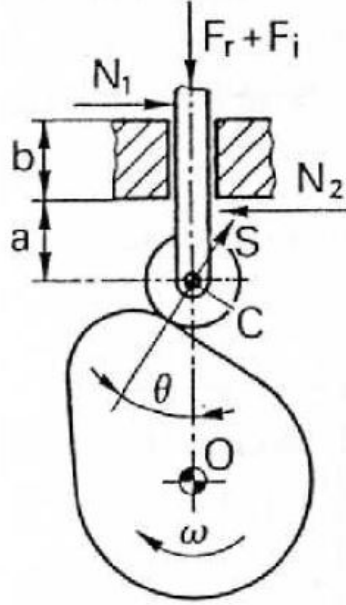


Figure 6.2

momentum are written in that way it is possible to find:

$$S = \frac{F}{\cos(\theta) - f\left(\frac{2a+b}{b}\right)\sin(\theta)} \quad (6.4)$$

Where  $f = 0.15$  is the coefficient of friction and  $b = 2a$ . Now it is possible to calculate the  $\theta_0$  that nullify the denominator:

$$\theta_0 = \left(f \frac{2a+b}{b}\right)^{-1} = 73deg \quad (6.5)$$

Substituting the value of  $\theta_0$  into the expression of  $S$ . After some simplification the limit value of  $\theta$  that satisfy this condition  $S = \frac{F}{\cos(\theta)} < S_{amm}$  is:

$$\theta < \theta_0 - 45deg \quad (6.6)$$

Due to the changing of the friction force and different block condition in the rise compared to the return of the cam:

$$\theta_{max} = 30deg \text{ RISE}$$

$$\theta_{max} = 50deg \text{ RETURN}$$

Comparing the value of  $\theta$  presented before, it is observed that  $\theta_{rise}$  is bigger than the limit value. To avoid this problem it is possible to increase the value of  $R_{b0}$ .

### 6.1.2 p condition

The pressure generated between the cam and the follower considering the Herz equation must be

$$p = \sqrt{0.175 * \frac{SE}{bR_r} (1 + \frac{R_r}{\rho})} < p_{amm} \quad (6.7)$$

where  $S = 667,5N$  (it is considered the maximum value of  $S$ ),  $R_r = 25mm$  is the roller radius,  $E = 2,07 * 10^5 N/mm^2$  elastic module steel,  $\rho = 73mm$  curvature radius of the cam and  $b = 50mm$  thickness of the cam. From the equation, it is possible to calculate the pressure of contact between the cam and the follower. In our case is equal to  $161.148MPa < 250MPa$  (yielding stress of the steel).

## 6.2 Motor consideration

The choice of the motor is carried out by evaluating the torque history in the centre of cam rotation.

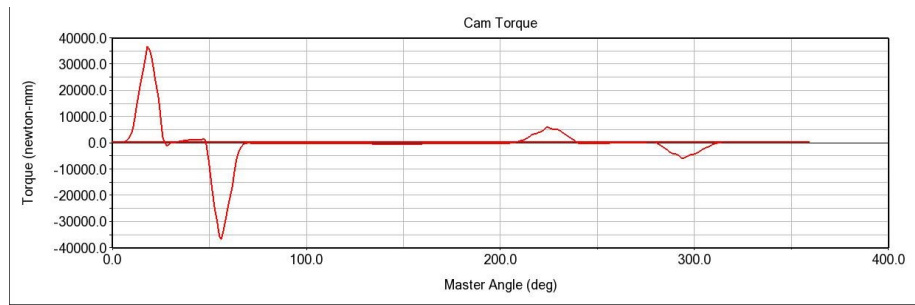


Figure 6.3

Since the torque is not constant along the angular position of the cam, the minimum requirement for motor dimensioning should be:

$$T_{motor_N} > RMS_{torque} \quad (6.8)$$

$$T_{motor_{max}} > max(torque_{load}) \quad (6.9)$$



The values are:

- $torque_{max} = 36.682Nm$
- $RMS = 7.2Nm$

The rotation speed is  $30rpm$  so a gearbox (or a generic reduction) must be adopted, the value of  $\tau$  is performed after the selection of the motor specific so for example by selecting the motor BLS-40 we have a stall torque of  $0.36Nm$  and a max torque of  $1.44Nm$ . A value of reduction 1:28 is sufficient to maintain the maximum value of the torque under the peak of the motor and the RMS under the nominal torque of the motor. Instead if we want to over dimension the motor, a value of reduction 1:105 always maintains the nominal torque of the motor above the maximum load torque. It is possible to check also that  $\omega_{motor}$  multiplied by the two different  $\tau$  is lower than the max mechanical speed of the motor.

BLS ~ Technical Specifications			
ALL CHARACTERISTICS MEASURED AT 25°C AMBIENT TEMPERATURE		SYMBOLS	UNITS
			BLS-40
			110 VAC
			220 VAC
			10,000
MAX MECHANICAL SPEED	n	rpm	
STALL TORQUE (1) ±10%	$M_s$	Nm	0.36
STALL CURRENT	$I_s$	A	2.57
PEAK TORQUE ±10%	$M_p$	Nm	1.44
TORQUE-WEIGHT RATIO	$T_w$	Nm/kg	0.6
EMF CONSTANT ±5%	$K_e$	Vs/rad	0.08
TORQUE CONSTANT ±5%	$K_t$	NmA	0.14
RELUCTANCE TORQUE (2)	$T_R$	Nm	<5%
WINDING RESISTANCE ±5%	R	$\Omega$	6
WINDING INDUCTANCE ±5%	L	mH	3.23
ROTOR INERTIA	J	$kg \cdot m^2 \cdot 10^{-3}$	0.0024
MECHANICAL TIME CONSTANT	$T_M$	ms	1.19
ELECTRICAL TIME CONSTANT	$T_E$	ms	0.54
THERMAL TIME CONSTANT	$T_{TH}$	s	1,190
THERMAL RESISTANCE	$R_{TH}$	°C/W	1.53
MASS	M	kg	0.6
RADIAL LOAD (at mid-length of shaft)	$F_R$	N	150
AXIAL LOAD	$F_A$	N	80
INSULATION			CLASS-F
PROTECTION			IP-65
(1) With an aluminium heat sink plate			300x300x10
(2) Respect to the Stall Torque			

Figure 6.4



Figure 6.5