# Functional Mechanical Design:

Exercise 3

## Cam Design

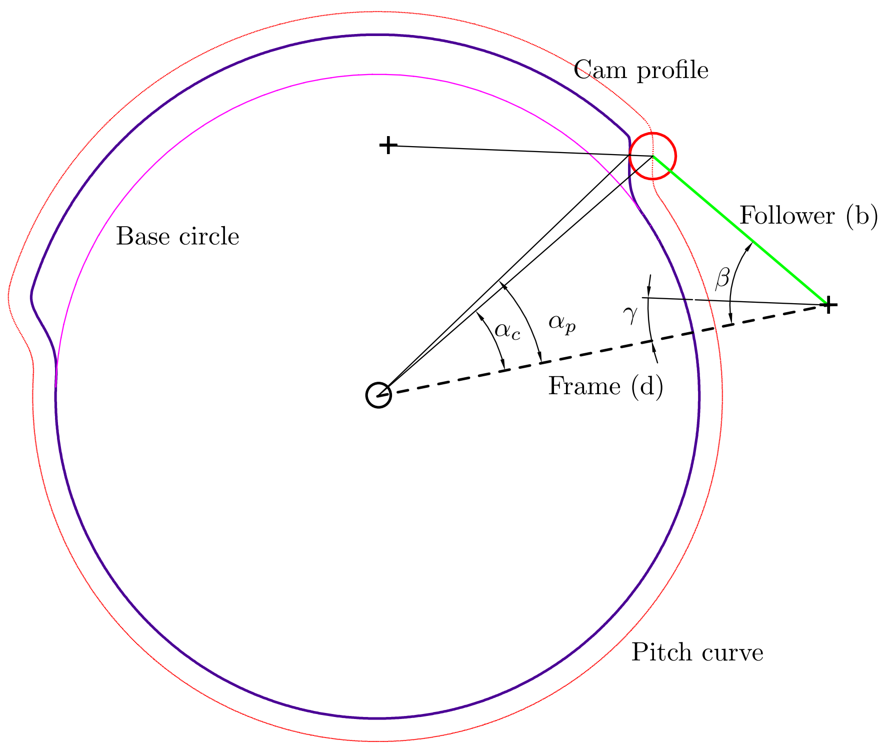



Figura 1: Example of a cam with roller follower

Develop a Matlab function for the design of a cam with a rocker wheel. This function should be able to manage both positive and negative cams with clockwise and counterclockwise directions of the motion. *Input* of the function should be:

- Law of motion of the follower as a function of the cam angular displacement $y(\alpha)$;
- cam base circle;
- roller radius;
- follower length;
- frame length;

The *output* of the function should be:

1. pitch curve of the cam in cartesian coordinates;
2. cam profile in cartesian coordinates;
3. pressure angle as a function of the cam angular displacement;
4. curvature radius as a function of the cam angular displacement.

According to figure 1, we may define:

- $d$ = frame length;
- $b$ = follower length;
- $R_b$= base radius;
- $R_r$ = roller radius;
- $\beta_0$ = angle of the follower, with respect to the horizontal, in the starting condition;
- $\alpha$ = angular displacement of the cam;
- $y(\alpha)$ = displacement of the follower;
- $y'$, $y''$ = geometric speed and accelerations.

The following equations can be used:

Angle between the follower and the frame[1]:

$$\beta = \beta_0 + y(\alpha)$$

Polar coordinates of the pitch curve:

$$\begin{cases} r_c = \sqrt{(b\sin\beta)^2 + (d - b\cos\beta)^2} \\ \varphi_c = \alpha + \alpha_c = \alpha + \arctan\dfrac{b\sin\beta}{d - b\cos\beta} \end{cases}$$

Angle $\gamma$:

$$\tan\gamma = \frac{b\sin\beta(1-y')}{d - b\cos\beta(1-y')}$$

Pressure angle:

$$\theta = \frac{\pi}{2} - \beta - \gamma$$

Polar coordinates of the cam profile:

$$\begin{cases} r = \sqrt{(b\sin\beta - R_r\sin\gamma)^2 + (d - b\cos\beta - R_r\cos\gamma)^2} \\ \varphi = \alpha + \alpha_p = \alpha + \arctan\dfrac{b\sin\beta - R_r\sin\gamma}{d - b\cos\beta - R_r\cos\gamma}; \end{cases}$$

Curvature radius $\rho$ of the cam profile:

$$\begin{cases} \gamma' = \dfrac{b(1-y')y'\cos(\beta+\gamma) - by''\sin(\beta+\gamma)}{d\cos\gamma - b(1-y')\cos(\beta+\gamma)} \\ \rho_0 = \dfrac{-b\cos(\beta_0+y)(1-y')+d}{(1+\gamma')\cos\gamma} \\ \rho = \rho_0 - R_r \end{cases}$$

Finally note that, if one has a negative cam, quantities relate do the follower displacement, speed and acceleration should be considered as negative ($y = -y$, $y' = -y'$ e $y'' = -y''$). When the direction of motion is counterclockwise ($\alpha = -\alpha$), the geometric speed should be considered with an opposite sign.

[1]Note once $R_b$ is defined, $\beta_0$ can be obtained and viceversa. Usually $R_b$ is used as a design parameter.

# Case study

Let's consider a system described by these main parameters:

| Nome | Simbolo | Valore [mm] |
|---|---|---|
| Frame length | d | 30 |
| Follower length | b | 30 |
| Base radius | Rb | 20 |
| Roller radius | Rr | 5 |

According to figure 1, the initial position of the follower is: $\beta_0 = 0.8596$ [rad].
The cam profile should be designed to have the follower law of motion depicted in figure 2 whose main information are collected in table:

| Master angle | Name | Motion law |
|---|---|---|
| $0 \Rightarrow 60$ | Dwell | - |
| $60 \Rightarrow 105$ | Rise | Cycloidal |
| $105 \Rightarrow 180$ | Dwell | - |
| $180 \Rightarrow 270$ | Return | Cycloidal |
| $270 \Rightarrow 360$ | Dwell | - |

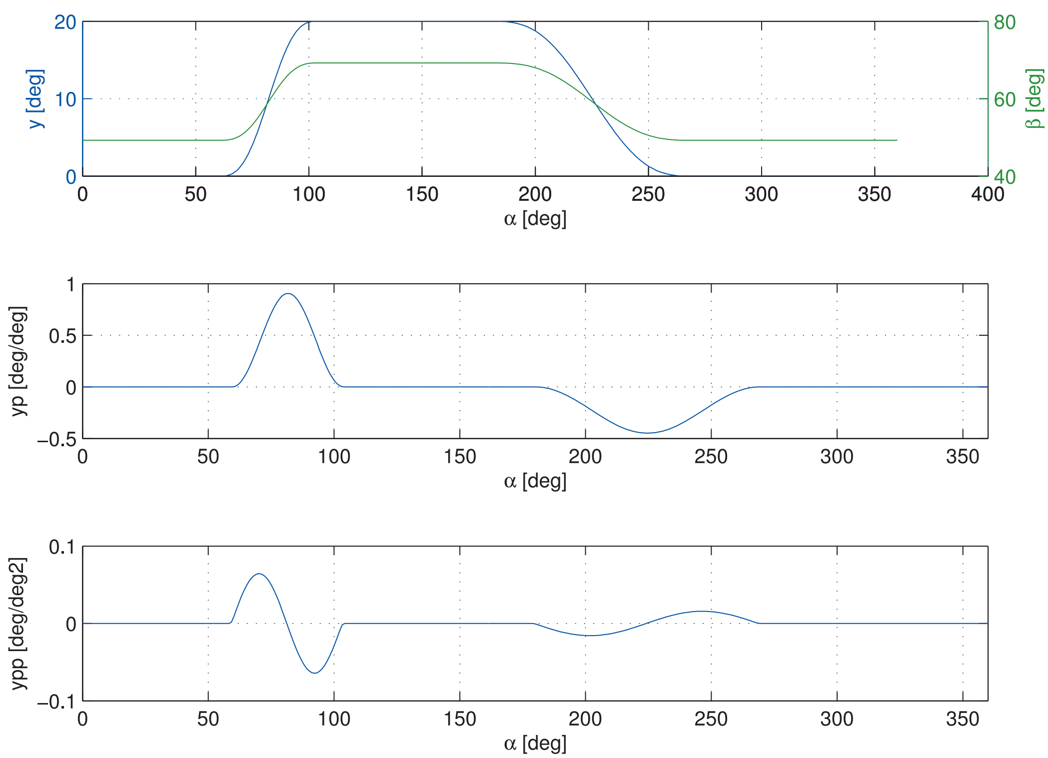



Figura 2: Follower law of motion

The law of motion shown in figure 2 is dimensional and the stroke of the follower is $h = 20$ [deg]. Pressure angle and cam curvature radius are depicted in figures 3, 4 respectively.
Figure 5 shows the base curve, the cam profile and the pitch curve.
According to the functions reported in figures 3, 4, some consideration about the designed cam can be drawn:

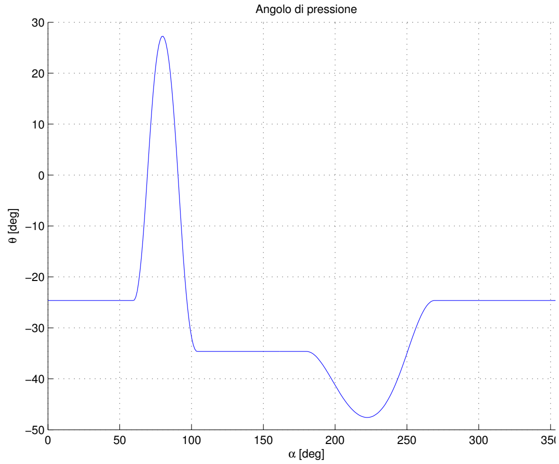



Figura 3: Pressure angle.

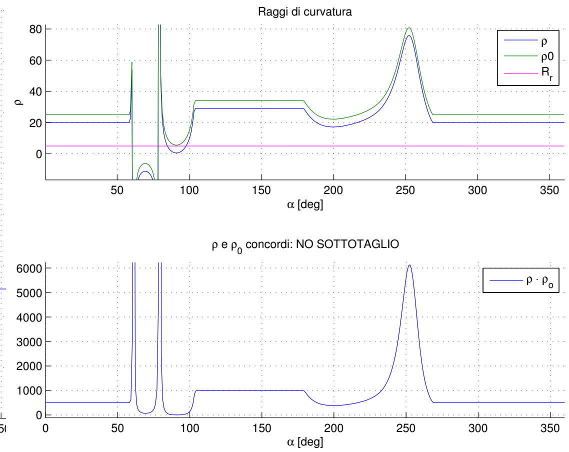



Figura 4: Curvature radius.

- the cam has a high pressure angle in the return phase;
- undercut condition in the rising phase may be problematic

The cam can be optimized to avoid undercut condition and reducing the pressure angle.

By increasing the roller radius, and keeping the base radius as constant) the initial cposition of the follower is changing, thus changhing the pressur angle too. This modification amplifies the risk of undercut as demonstrated in figures 6(b) and 6(a) (optained for a roller radius $R_r = 10$ mm).

Figures 7(a) and 7(b) depict the corresponding cam profile where undercut condition can be easily noticed.

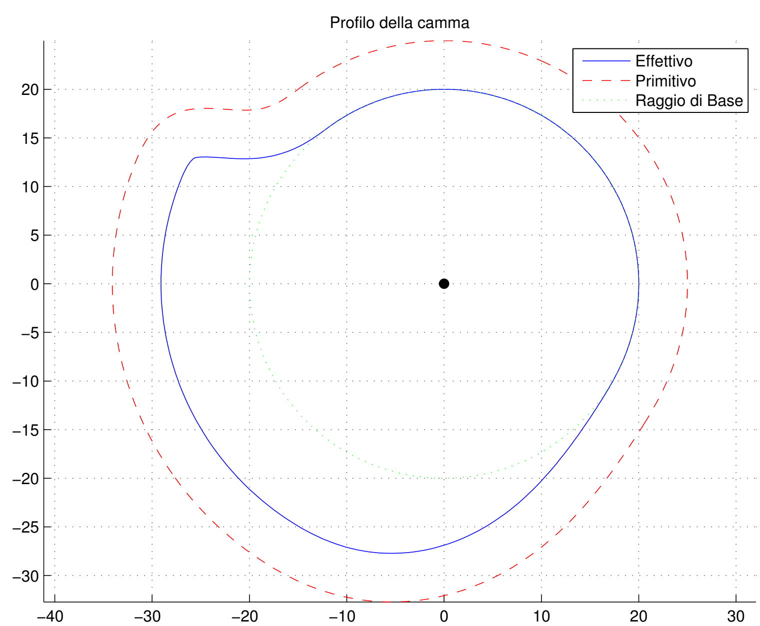
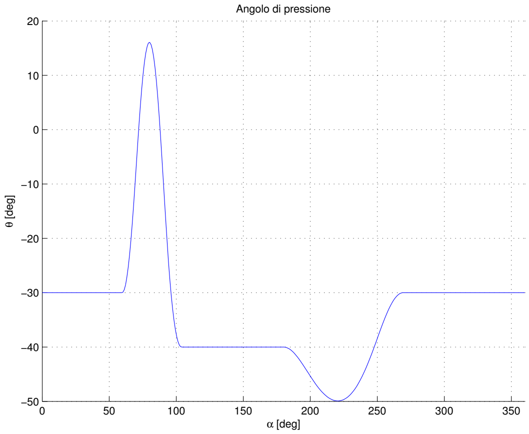
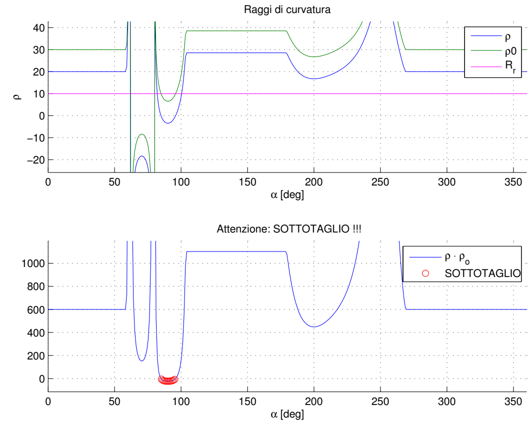



Figura 5: Cam profile.

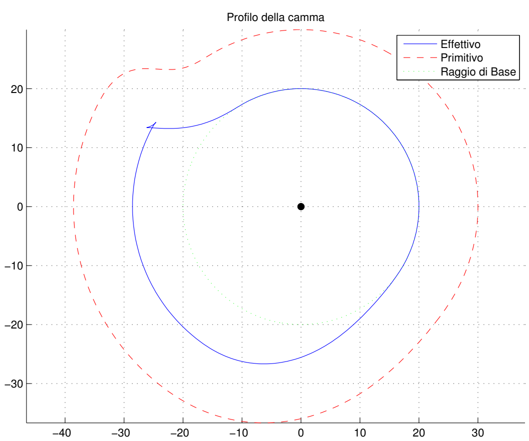



(a)

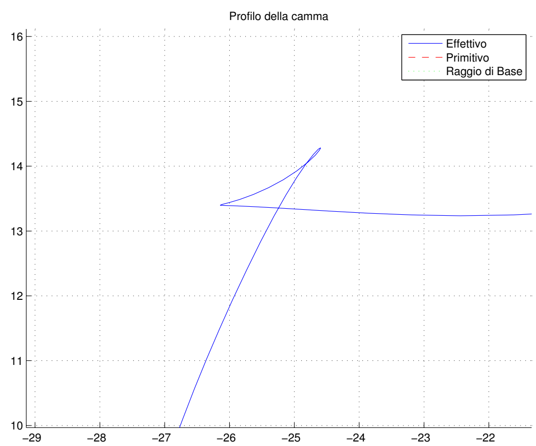



(b)

Figura 6: Pressure angle and curvature radius for $R_r = 10$ mm.




(a)



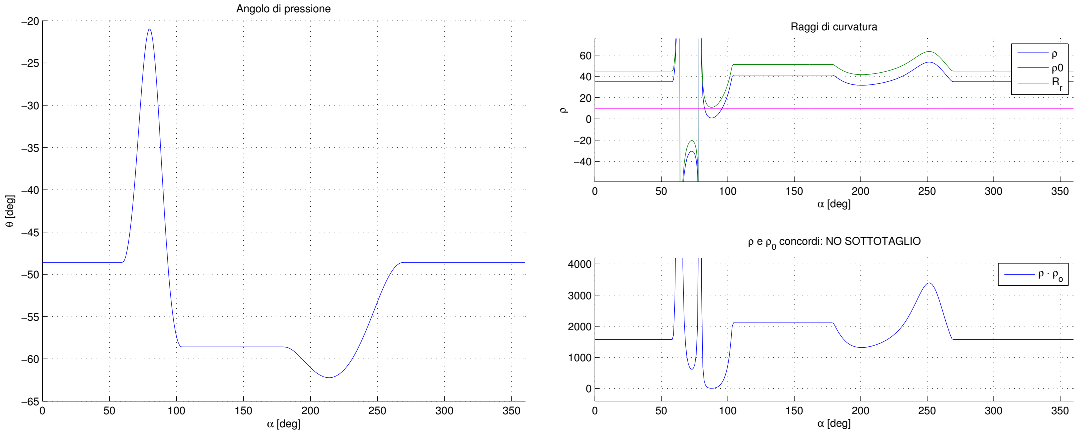

(b)

Figura 7: Undercut condition.

To reduce the pressure angle, keeping the rolled radius as $R_r = 10$, the base radius may be increased from 20 [mm] to 35 [mm]. This modification has effects on both the rpessure angle and the curvature radius.

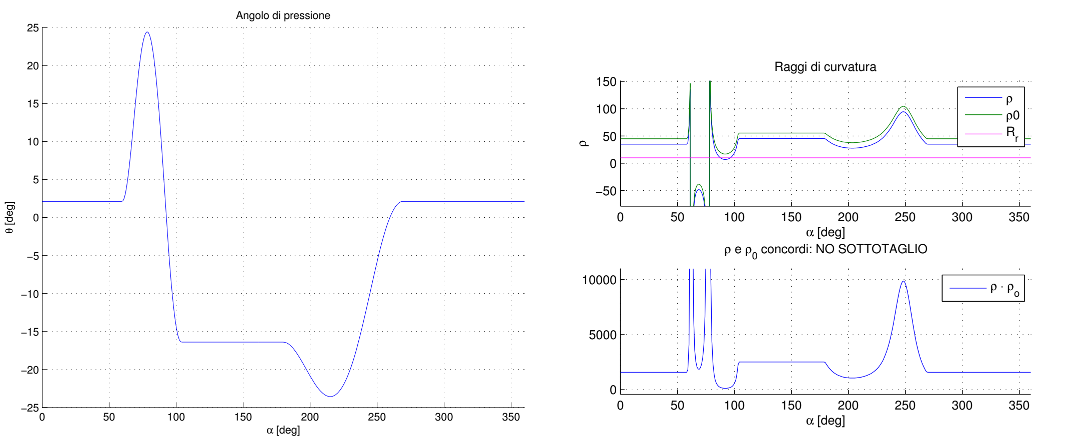
As shown in figure 8(a), increasing the base radius the pressure angle increases but its changes are now reduced. By changing the position of the follower (thus changing $\beta_0$), eg. by increasing the frame length, the mean value of the pressure angle can be minimized, thus reducing its maximum value. For example, frame length can be set as 55 mm. Figures 9(a) and 9(b) show the effect.

Finally it's interesting to evaluate the effects of different laws of motion of the follower (expecially related to the maximum speed) on the cam main properties. For example, by changing the cycloidal motion law with a constant symmetric one (1/3, 1/3, 1/3) as depicted in figure 10, we may have a better pressure angle (Fig. 12(a)) and a different trend of the curvature radius (Fig. 12(b)).
The cam profile obtained with the new law of motion is depicted in figure 11. While the new profile is very close to the previous one, we obtained better results in terms of pressure angle (-13%).




(a) (b)

Figura 8: Effects due to the base radius modification.




(a) (b)

Figura 9: Effects due to changes in the frame length.

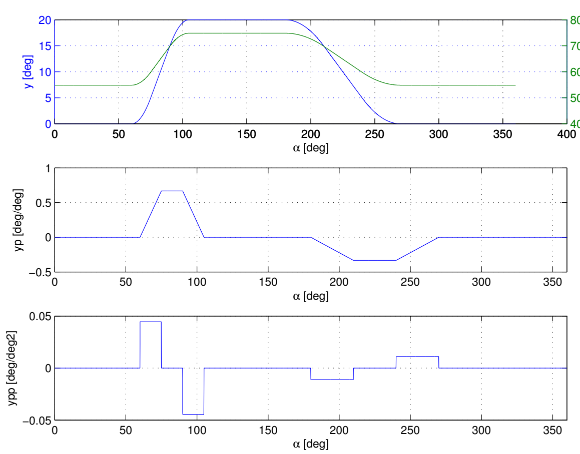



Figura 10: Motion law 1/3-1/3-1/3.

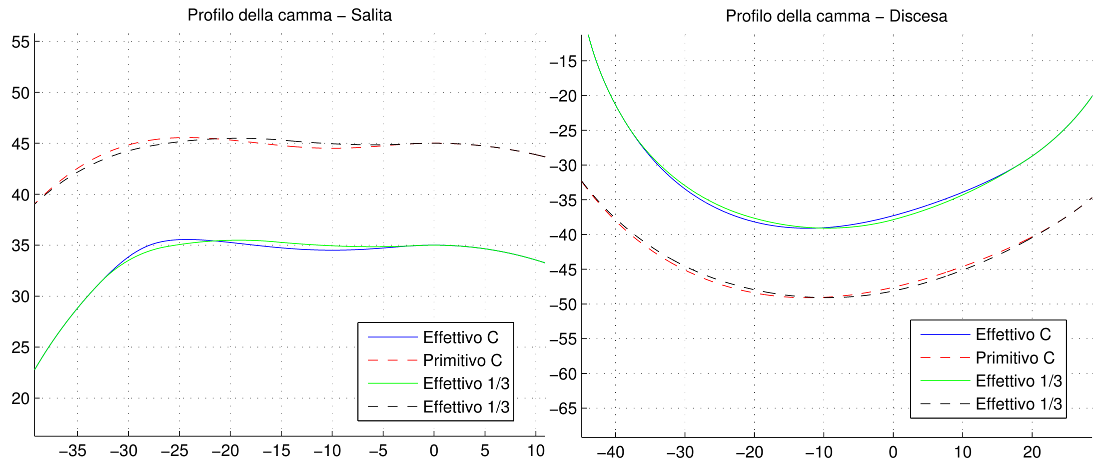
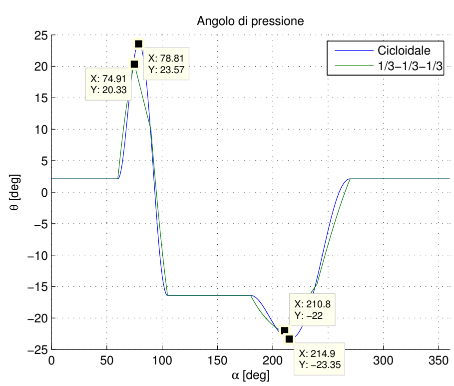
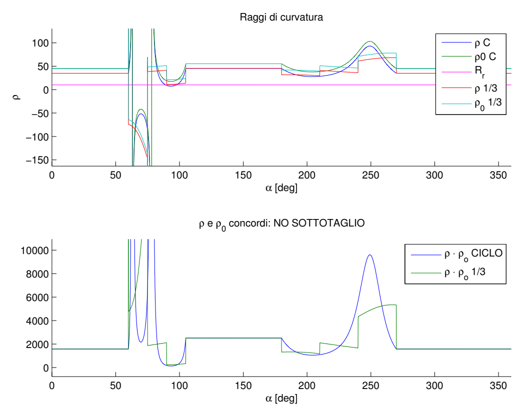



Figura 11: Comparison between cam profiles obtained with different laws of motion




(a) Comparison between pressure angles obtained with different laws of motion.




(b) Comparison between curvature radii obtained with different laws of motion.

Figura 12: Pressure angles and curvature radii obtained with differen laws of motion