DIPARTIMENTO DI MECCANICA

POLITECNICO MILANO 1863

MECHANICAL SYSTEM DYNAMICS

Practical application of experimental modal analysis technique

M. Vignati

Target

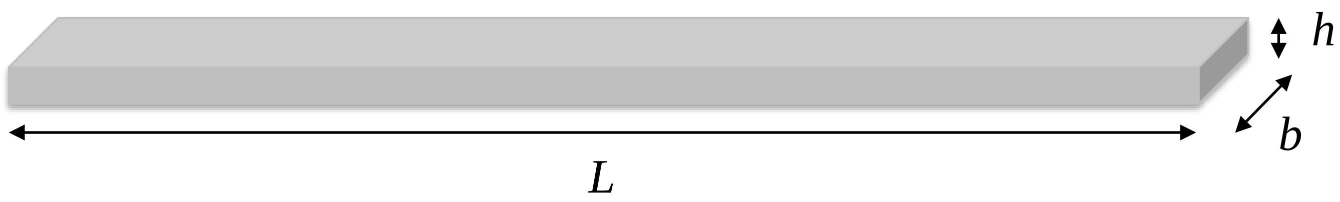
Identify the **natural frequencies** and **mode shapes** of a real system through experimental modal analysis.

Steps:

- System descritption
- Experimental setup design
    - Constraints -> how to fix the system
    - Inputs -> how to excite
    - Outputs -> what to measure
    - Measurement system (analytical tools)
- Data processing
- Comparison with analytical model

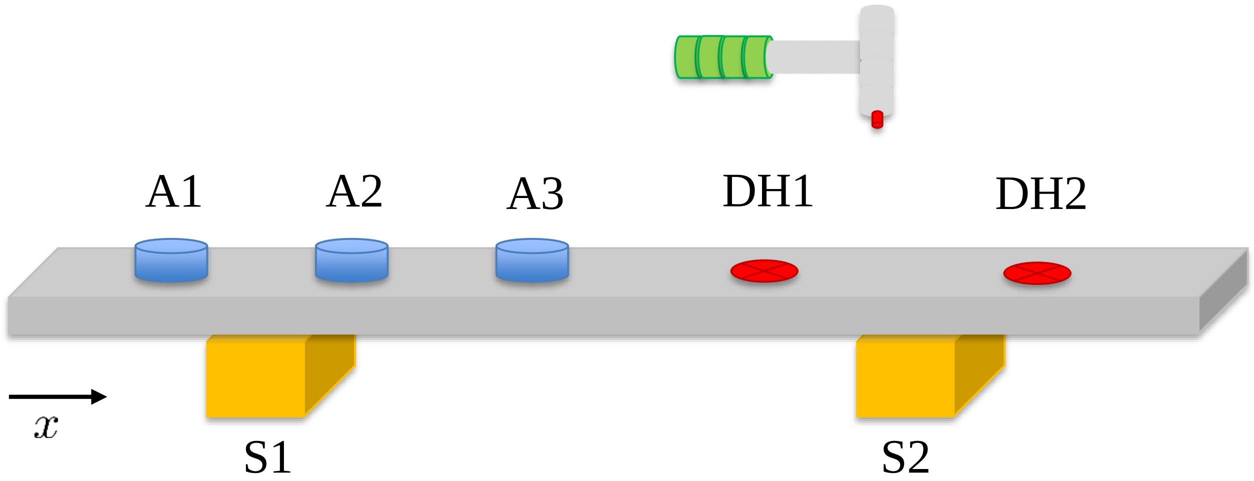
Properties of the system

An aluminum beam with rectangular constant cross-section



Parameter	symbol	unit	value
Lenght	L	mm	1200
Thickness	h	mm	8
Width	b	mm	40
Density	ρ	$\mathrm{kg/m^3}$	2700
Young's Modulus	E	GPa	68

Experimental setup design



Parameter	symbol	x [mm]	Transducer	Sensitivity
Accelerometer	A1	105	Piezo	100 mV/g
Accelerometer	A2	415	Piezo	100 mV/g
Accelerometer	A3	600	Piezo	100 mV/g
Dynamometric Hammer	DH1	815	Piezo	2.17 mV/N
Dynamometric Hammer	DH2	1065	Piezo	2.17 mV/N

S1 and **S2** are flexible supports ≈ free-free beam

Experimental setup design

Accelerometer

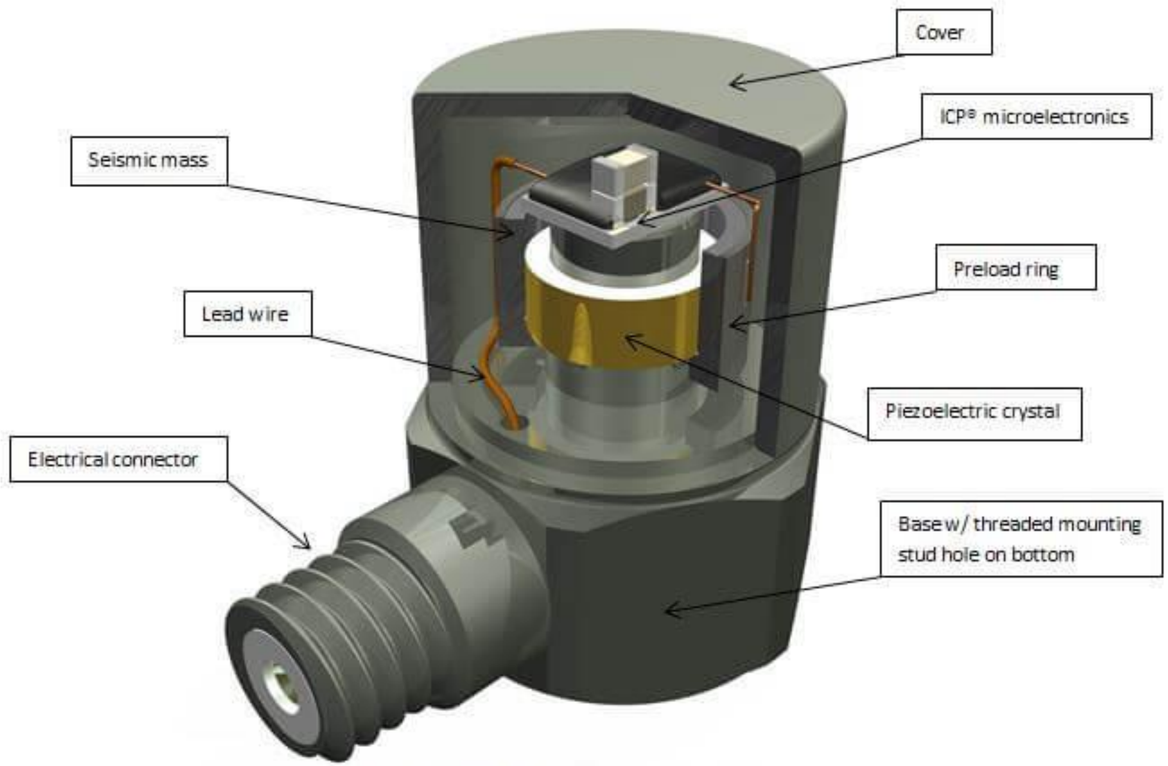
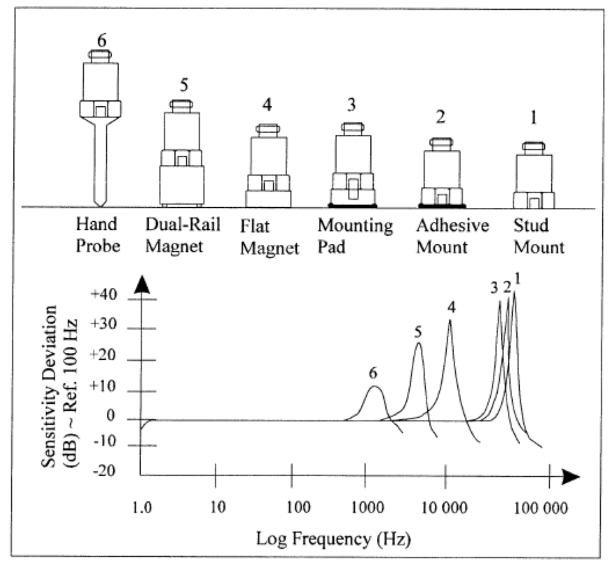


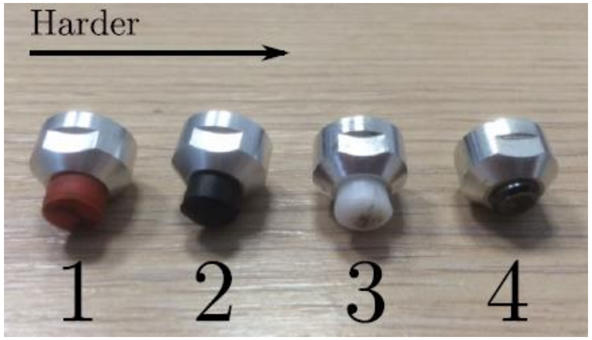
Figure 1: Typical ICP® Accelerometer

Physical	
Sensing Element	Ceramic
Sensing Geometry	Shear
Housing Material	Titanium
Sealing	Welded Hermetic
Size (Hex x Height)	9/32 in x 18.5 mm
Weight	2.0 gm
Electrical Connector	10-32 Coaxial Jack
Electrical Connection Position	Top
Mounting Thread	5-40 Male
Mounting Torque	90 to 135 N-cm

Performance	**ENGLISH**	**SI**
Sensitivity(± 10 %)	100 mV/g	10.2 mV/(m/s²)
Measurement Range	± 50 g pk	± 491 m/s² pk

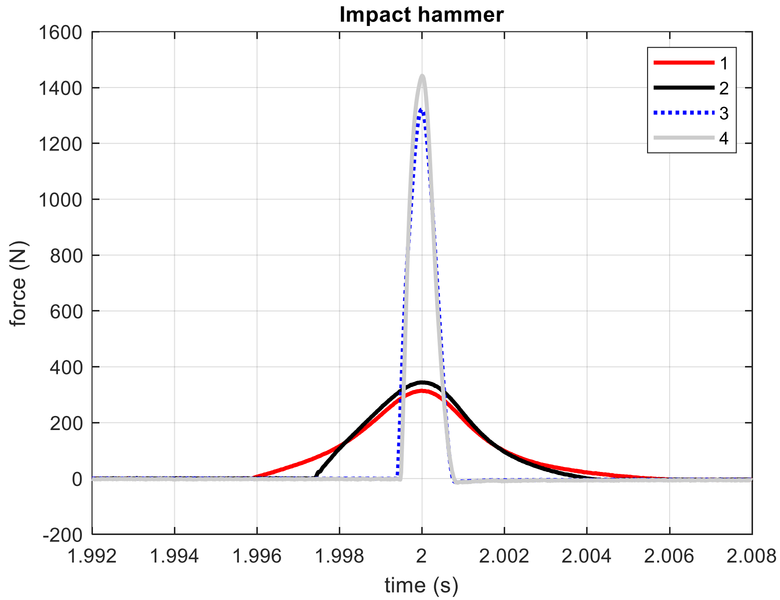


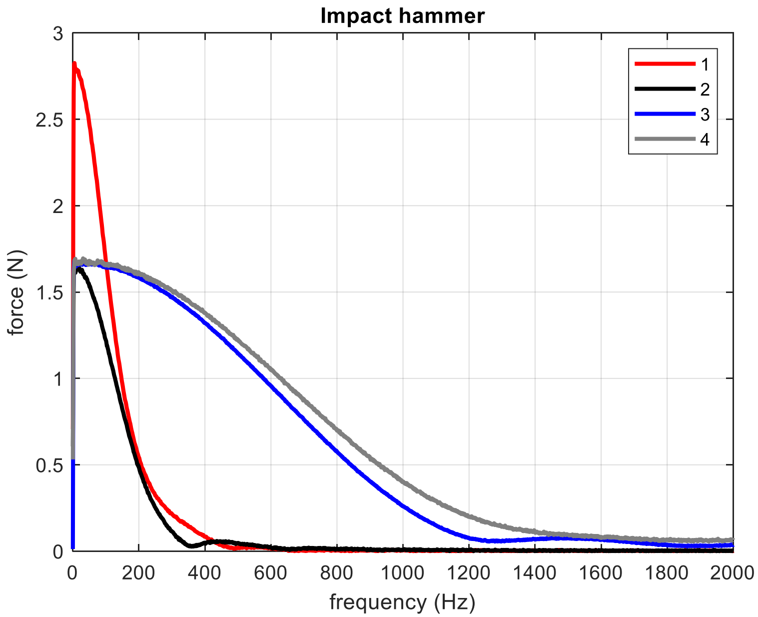
Experimental setup design



Dynamometric Hammer

Why? Impulse input, excites all frequencies (theoretically)



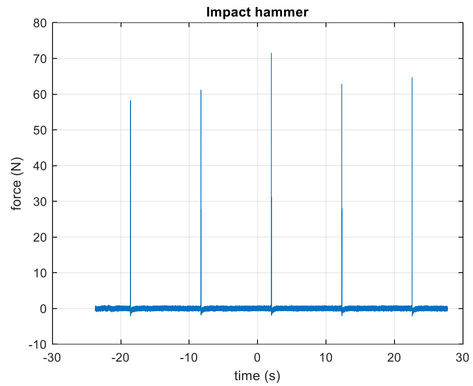


… in these tests we will use hammer tip # 2 (intermediate)

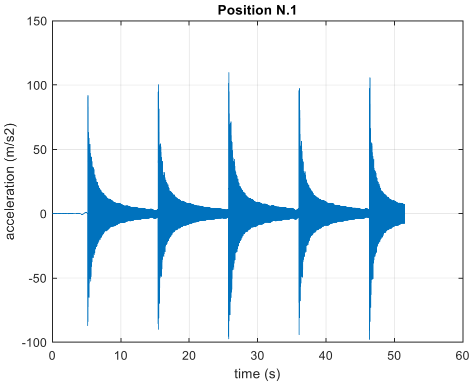
Data processing

Time histories

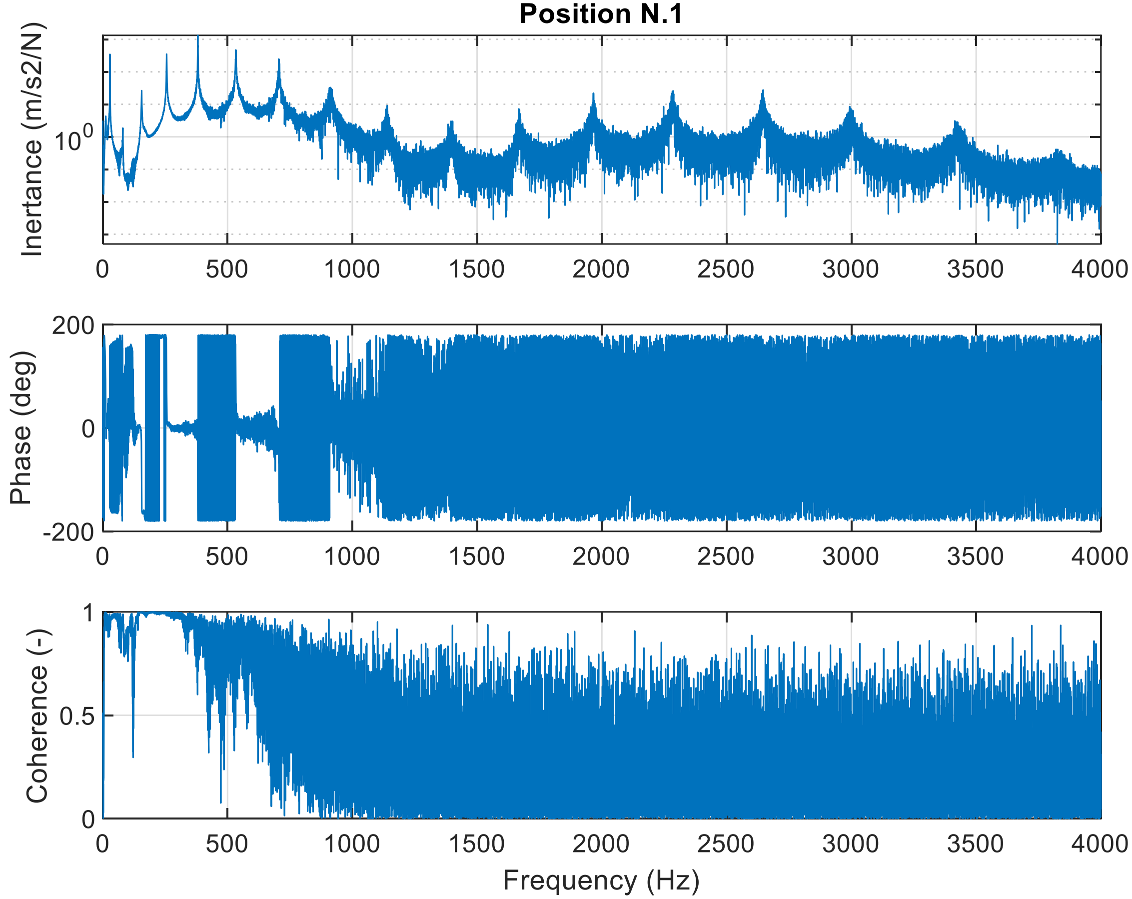
INPUT FORCE



OUTPUT ACCELERATION



Frequency Response Functions



Data processing - procedure

1. Measurements are performed so as to collect a data set of N pairs of sampled time histories for the input force F_k and the output vibration x_j (the length of all the 2N time histories is indicated with T_0)
2. If needed, a Hanning (or other) window, is used to minimize spectral leakage
3. Discrete Fourier Transform is applied to all the signals, thus obtaining 2N discrete spectra F_{k_i} and X_{j_i} with fundamental frequency $\omega_0 = 2\pi/T_0$
4. PSD (Power Spectral Density - real) $G_{XX}(n\omega_0)$ and $G_{FF}(n\omega_0)$, as well as CSD (Cross-Spectral Density - complex) functions $G_{XF}(n\omega_0)$ are computed.
5. Finally the experimental FRF G_{jk}^{EXP} and the coherence function γ_{jk}^2 are estimated.

The experimental FRF is computed according to H1 estimator

$$G_{jk}^{EXP} = \frac{X_j}{F_k} = \frac{G_{XF}}{G_{FF}}$$

Discrete Cross-spectral-density

$$G_{YZ} = \frac{1}{M} \sum_{m=1}^{M} \frac{Y(m\omega_0) Z^*(m\omega_0)}{2\omega_0}$$

Coherence function

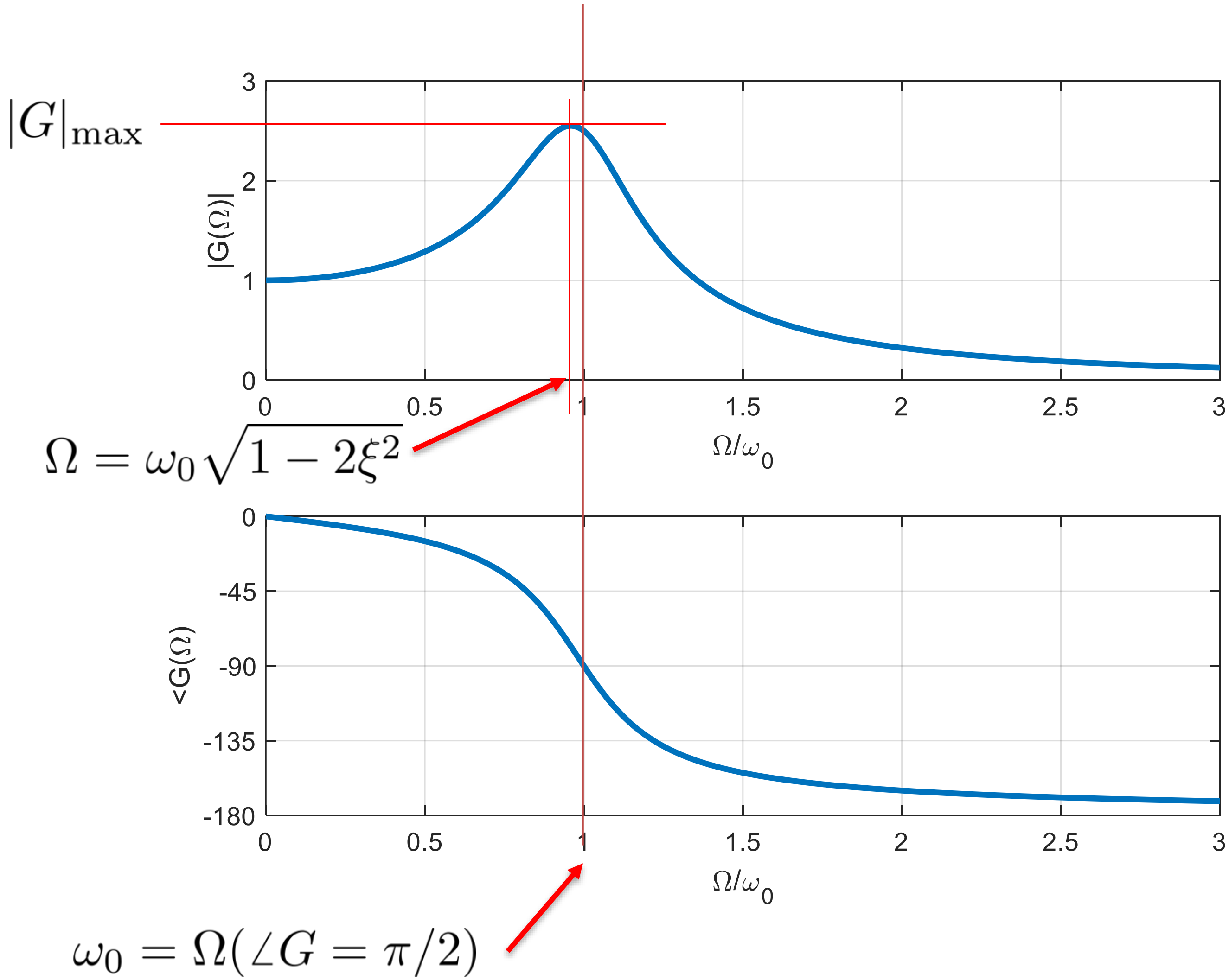
$$\gamma_{jk}^2 = \frac{|G_{XF}|^2}{G_{XX} G_{FF}}$$

In modal approach the FRF can be written as

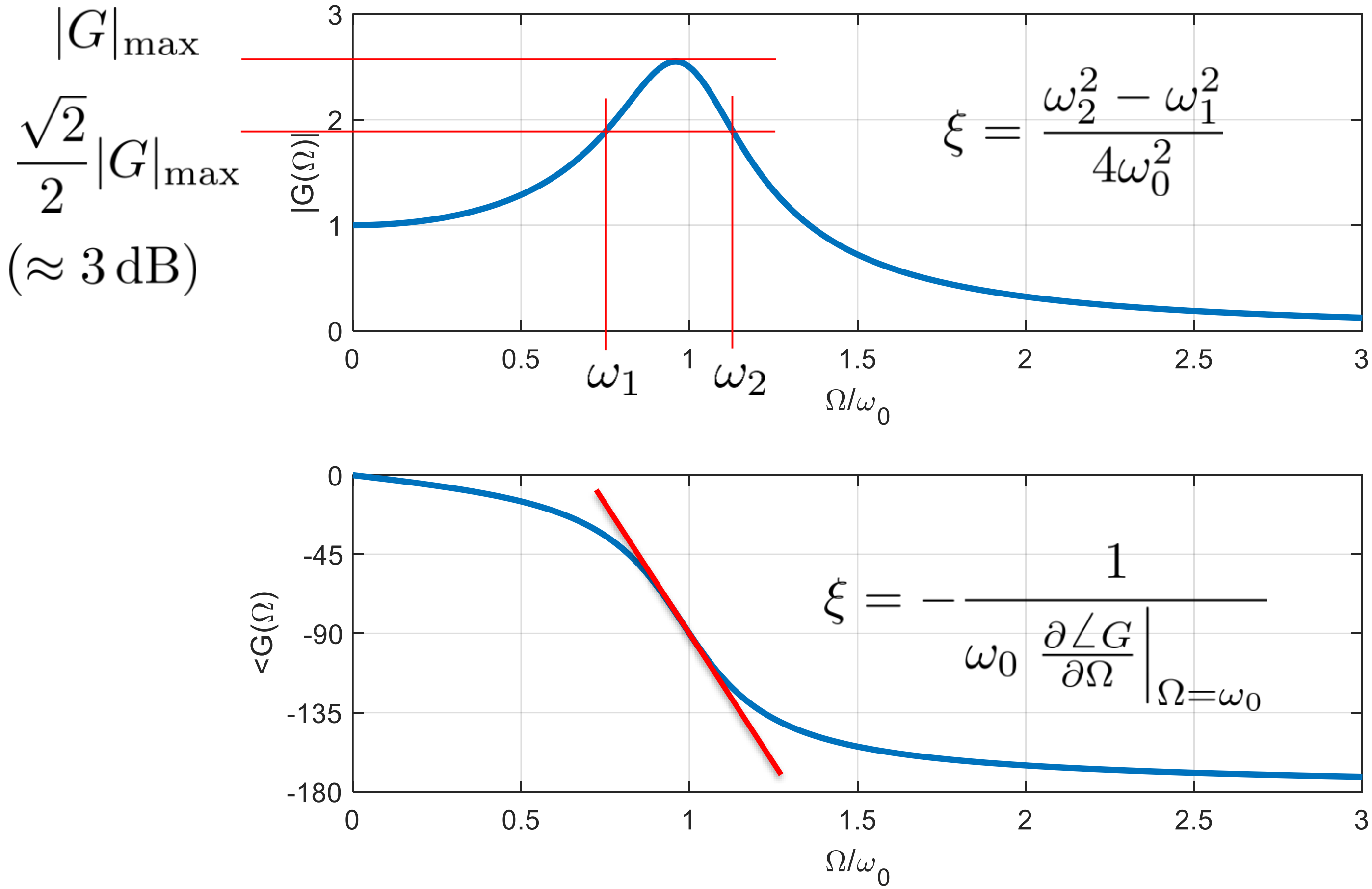
$$G_{jk} = \frac{X_{j0}}{F_k} = \sum_{i=1}^{N} \frac{X_j^{(i)} X_k^{(i)} / m_i}{-\Omega^2 + j2\xi_i \omega_i \Omega + \omega_i^2}$$

j = A1, A2, A3
k = DH1, DH2

Modal identification – natural frequency



Modal identification - damping



Half-power points

Phase tangent

Modal identification – mode shape

$$G_{jk} = \frac{X_{j0}}{F_k} = \sum_{i=1}^{N} \frac{X_j^{(i)} X_k^{(i)} / m_i}{-\Omega^2 + j2\xi_i\omega_i\Omega + \omega_i^2}$$

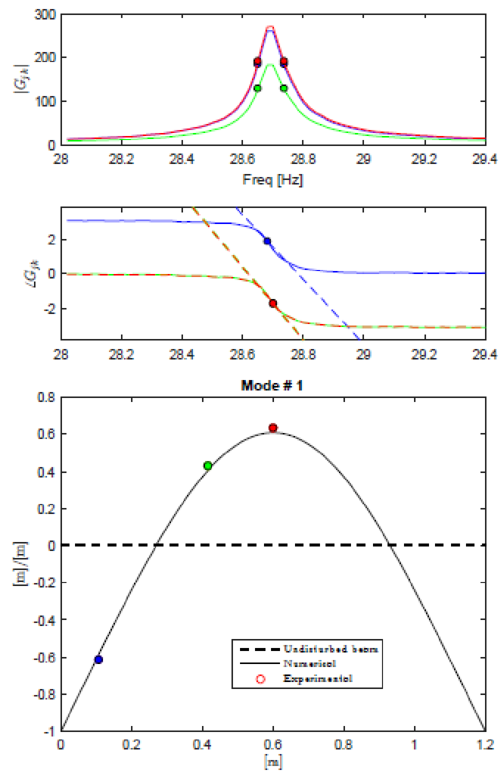
j = A1, A2, A3
k = DH1, DH2

Mode shapes are given by:

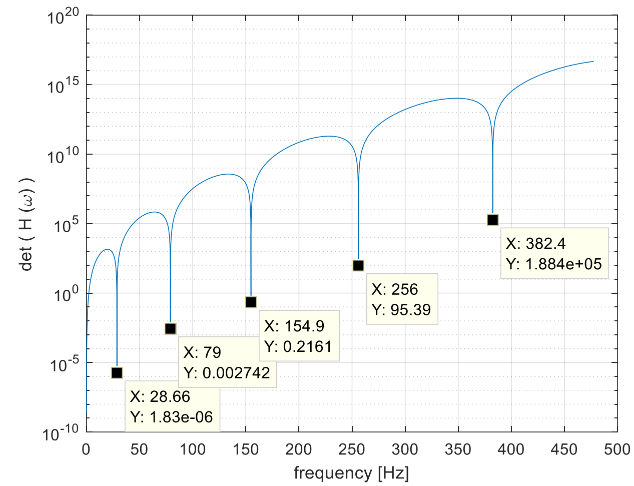
- Relative amplitudes
- Phase

In resonance

$$\boxed{G_{jk}(\omega_i) \approx \left(-j\frac{X_k^{(i)}}{c_i\omega_i}\right) X_j^{(i)}}$$



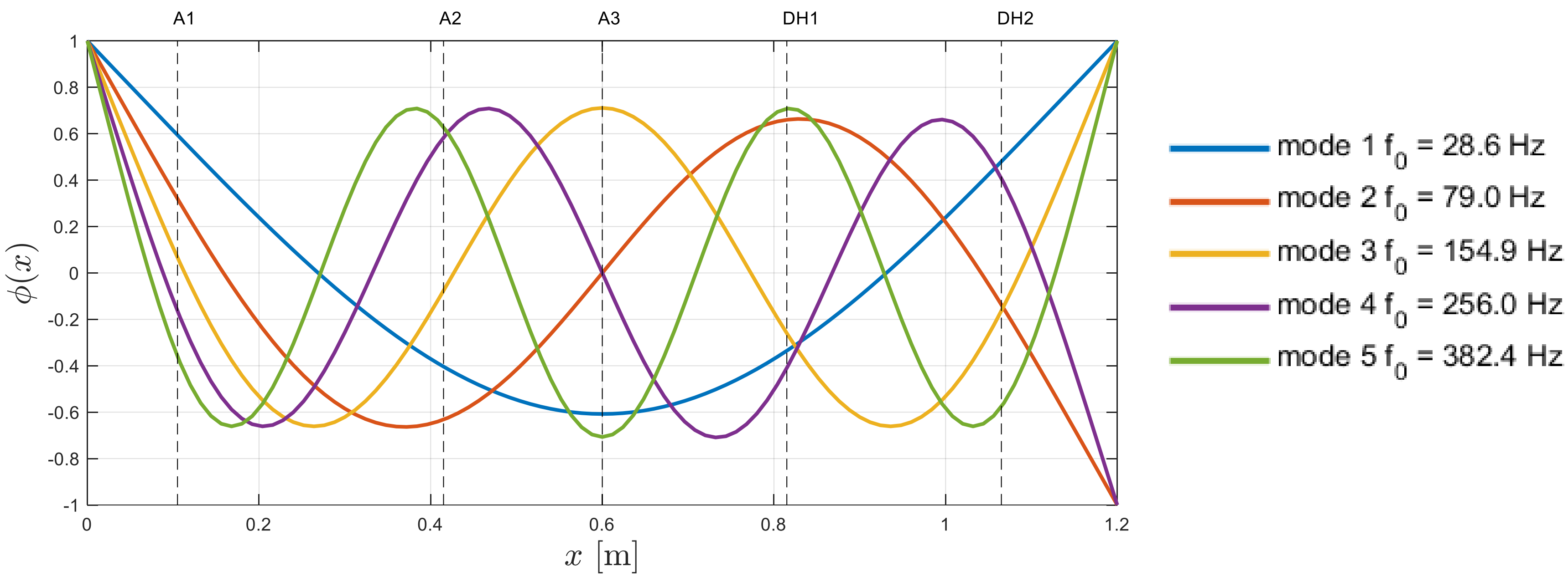
Analytical solution for free-free beam



$$w(x,t) = (A\sin\gamma x + B\cos\gamma x + C\sinh\gamma x + D\cosh\gamma x)\cos(\omega t + \varphi)$$

Boundary conditions

$$M_z(0,t) = 0 \qquad -B + D = 0$$
$$Q(0,t) = 0 \qquad -A + C = 0$$
$$M_z(L,t) = 0 \qquad -A\sin\gamma L - B\cos\gamma L + C\sinh\gamma L + d\cosh\gamma L = 0$$
$$Q(L,t) = 0 \qquad -A\cos\gamma L + B\sin\gamma L + C\cosh\gamma L + d\sinh\gamma L = 0$$



Available Experimental Data

- `DH1.mat` hammer in DH1 position
- `DH2.mat` hammer in DH2 position
- `RDH1.mat` Acc1 and DH1 position interchanged (**reciprocity** against DH1.mat)

Each *.mat file cointains:

- **freq** frequency vector (resolution 0.02 Hz)
- **frf** frequency responce functions (complex), collected by columns (A1, A2, A3)
- **cohe** coherence function, collected by columns (A1, A2, A3)

Single mode identification (up to 5-th mode)

1. Identification of the natural **frequencies**
2. Identification of the damping ratio by the **“half-power points”** method
3. Identification of the damping ratio by the **“slope of the phase diagram”** method
4. Comparison Analytical Vs Experimental **mode shapes**

For the oral examination...

...short report, for each mode, the identification results (items 1 to 4), for at least one test configuration among DH1, DH2 and RDH1. Collect the results in table form (for each sensor, items 1 to 3) and plot a diagram for the comparison (item 4)