Functional Mechanical Design

Linkage Mechanisms (1/3)

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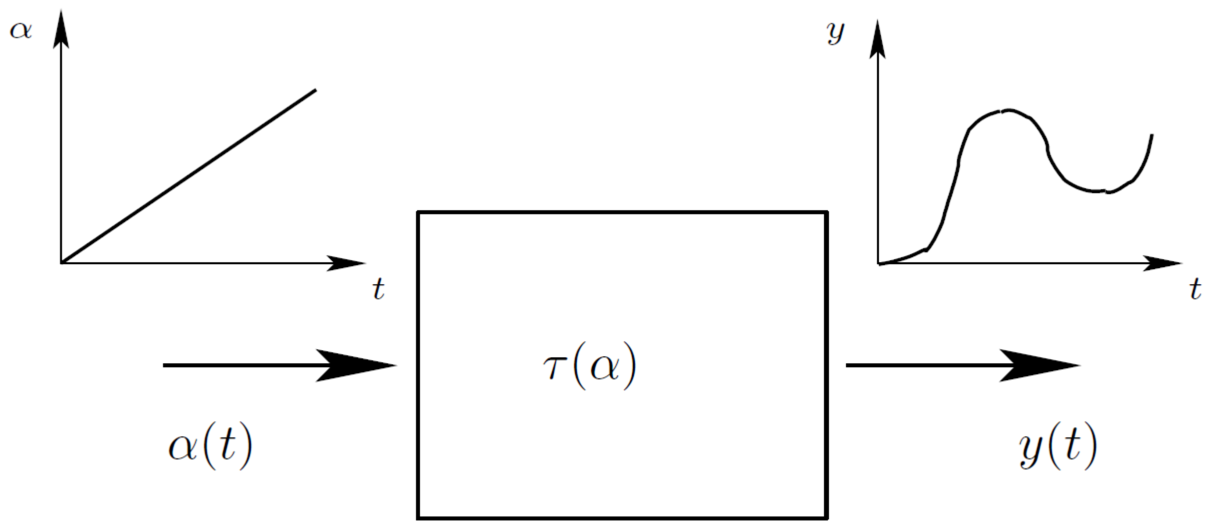
Introduction

There are two approaches to generate motion laws:

1. take movement from a motor with constant angular velocity and transform it by means of **mechanisms** (mechanical transmission with variable transmission ratio $\tau = \tau(\alpha)$). At the output point of such mechanisms (output organ) we have the desired motion. The output organ is usually called "follower".
2. take movement from a motor with variable angular velocity (i.e. controlled position) without a variable transmission ratio between motor and follower. In this case we are talking about **"electronic cams"**.

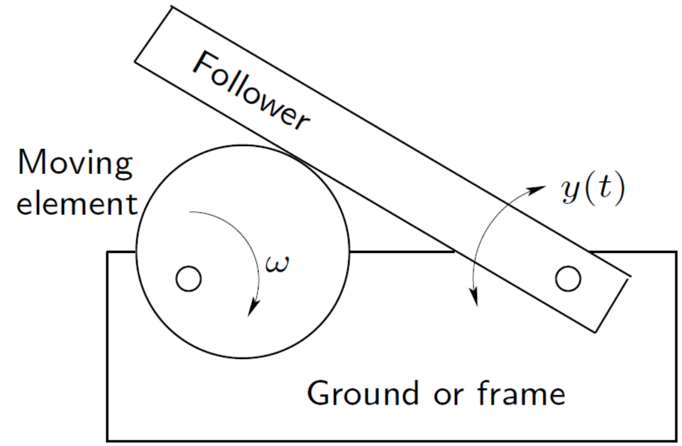


Mechanisms can be considered as machines whose function is to generate in output a particular motion law.

This point of view can be described with the block scheme in the figure. It can also be used to represent a functional group or a single degree of freedom machine.

In general a mechanism for motion generation is made up of at least three fundamental elements:

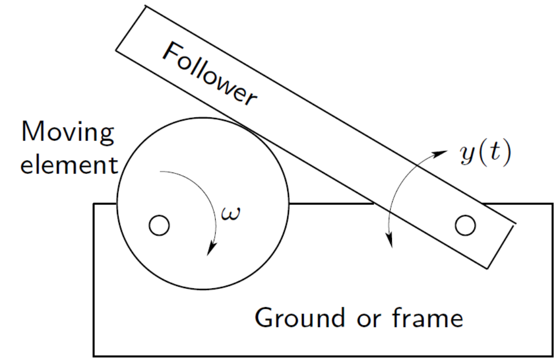
1. Moving element
2. Follower
3. Ground or frame (to which the other two elements are linked)



Note that there are two different problems in the filed of motion law of mechanisms:

1. **Kinematic synthesis**: to invent a mechanism capable of generating a required movement $y(\alpha)$ on its follower supposing all links to be rigid.
2. **Kinematic analysis**: to evaluate the motion law $y(\alpha)$ and its derivative of the follower or of another element of the mechanism.

Remembering the definition of the velocity, we can write:



$$\dot{y} = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}t} = \tau\omega$$

where the term $\tau = \mathrm{d}y/\mathrm{d}\alpha = y'$ is called *geometrical velocity* or *instantaneous transmission ratio*. While for the acceleration we have:

$$\ddot{y} = \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\right) = \frac{\mathrm{d}\tau}{\mathrm{d}t}\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \frac{\mathrm{d}^2\alpha}{\mathrm{d}t^2}\tau = \frac{\mathrm{d}\tau}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}t}\frac{\mathrm{d}\alpha}{\mathrm{d}t} + \tau\dot{\omega} = \gamma\omega^2 + \tau\dot{\omega}$$

where the term γ, called *geometrical acceleration*, is equal to $\gamma = \mathrm{d}^2y/\mathrm{d}\alpha^2 = y''$.

In many automated machines the follower movement is obtained by means of mechanisms that transform the uniform rotational movement ω generated by the moving element.
When $\omega =$ cost there is direct proportionality between the time t and the rotation α of the moving element. In this case it is enough to change t with $\alpha = \omega t$ in order to obtain the motion law in function of α: it is simply an abscissa change.
Being $\mathrm{d}^2\alpha/\mathrm{d}t^2 = \dot{\omega} = 0$

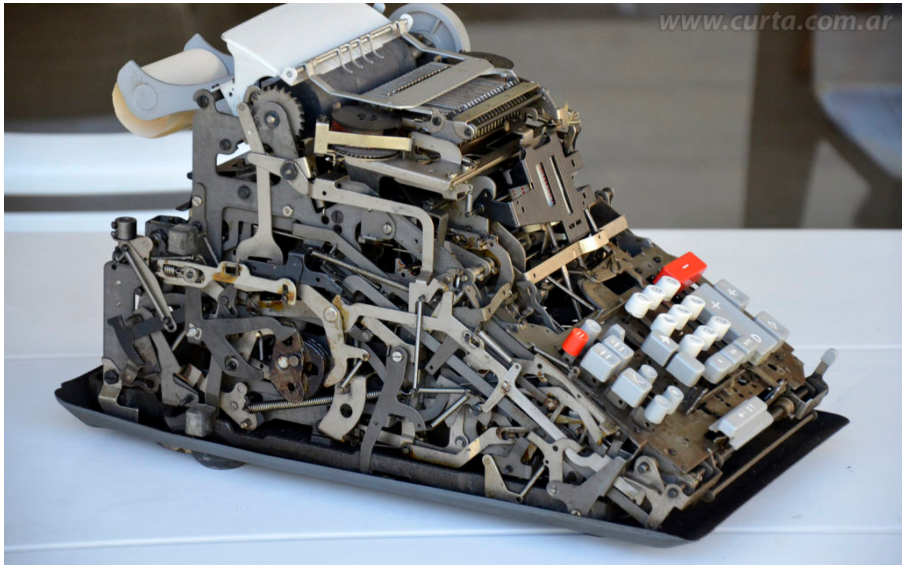
$$\ddot{y} = \gamma\omega^2 = y''\omega^2$$

When the moving element velocity is constant the time is inversely proportional to ω, the velocity proportional to ω and the acceleration proportional to ω^2.

A mechanism is a mechanical device that transfers motion and/or force from a source to an output.

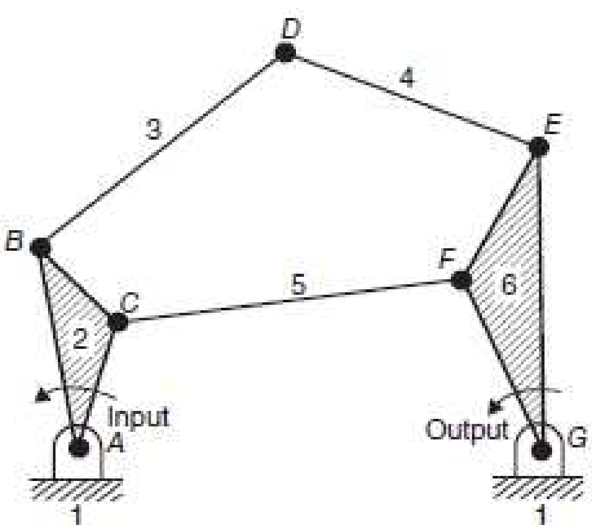
Linkages mechanisms can be designed to perform complex tasks. They consists of links, generally considered as rigid, which are connected by only **lower kinematic pairs**, in particular rotational and prismatic pairs. Usually in automated machines the axes of the rotational pairs are parallel to each other and the directions of the prismatic pairs are orthogonal to these. In this case we are talking about **planar linkage mechanisms**.

Depending on the number of links we may have four bar mechanisms, five bar mechanisms, six bar, etc. Note that among the links we have to consider a fixed link named as “ground” (or frame).



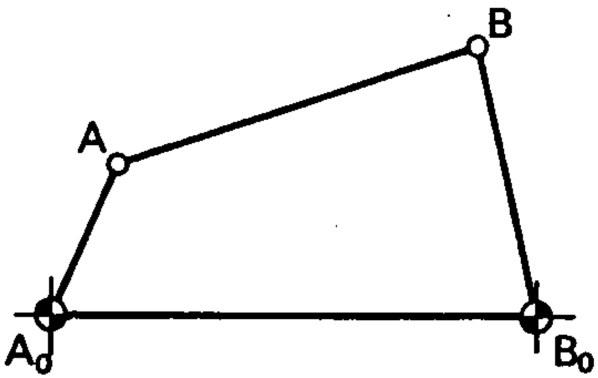
In order to study linkage mechanisms we use **kinematic diagrams**, made up of nodes and straight lines. They serve the same purpose as an electrical circuit scheme used for design and analysis purposes.

They are simplified models of real systems and their main purpose is to replicate the relative motion between links. In these diagrams, links are rigid bodies with at least two nodes. A node is a point on a link that connects it to another link.



Four bar mechanism

The **four bar mechanism** is the simplest linkage mechanism and it is made up of 4 rigid links:



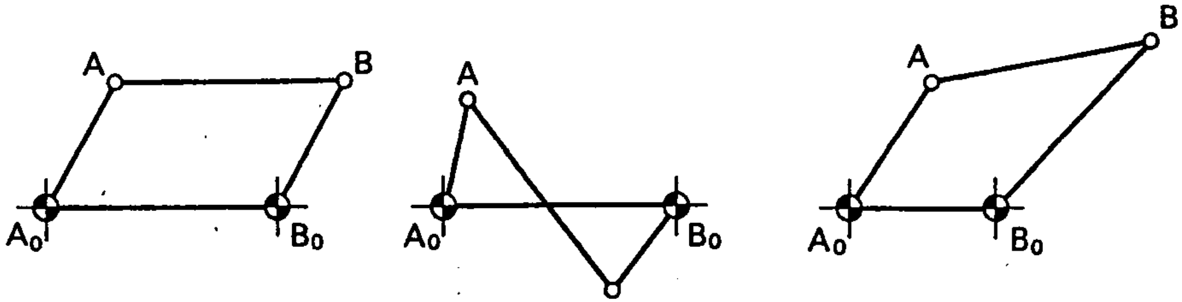
$$\begin{aligned} A_0B_0 &= \text{Frame or ground link} \\ AB &= \text{Connecting rod / Coupler link} \\ AA_0 &= \text{Crank or Rocker} \\ BB_0 &= \text{Crank or Rocker} \end{aligned}$$

While **cranks** have a unidirectional motion, **rockers** alternate the direction of motion moving between two extreme positions called dead centers. This happens when the rocker is aligned with the connecting rod.

Four bar mechanisms are divided into two groups called **Grashof** and **non-Grashof** groups:

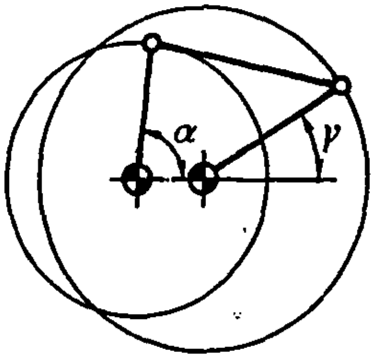
(Longest link) + (Shortest link)	$<$	(sum of the other two links)
	$\Rightarrow$	Grashof
(Longest link) + (Shortest link)	$>$	(sum of the other two links)
	$\Rightarrow$	non-Grashof
(Longest link) + (Shortest link)	$=$	(sum of the other two links)
	$\Rightarrow$	limit situation

The figure shows a number of mechanisms belonging to the last class: parallelogram, antiparallelogram and isosceles (deltoid or kite form) four bar mechanism.

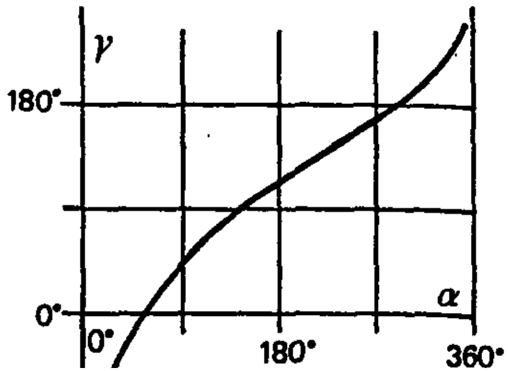


Grashof four bar mechanisms are very interesting because the shorter side of the kinematic chain is a cranck (it carries out complete rotations). We can have three different cases:

- The shorter link constitutes the ground ⇒ double crank four bar mechanism.



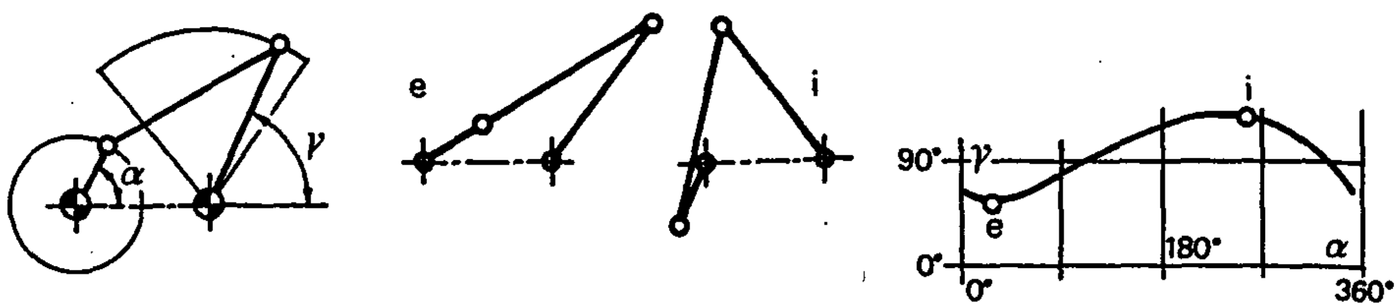
nessun punto morto



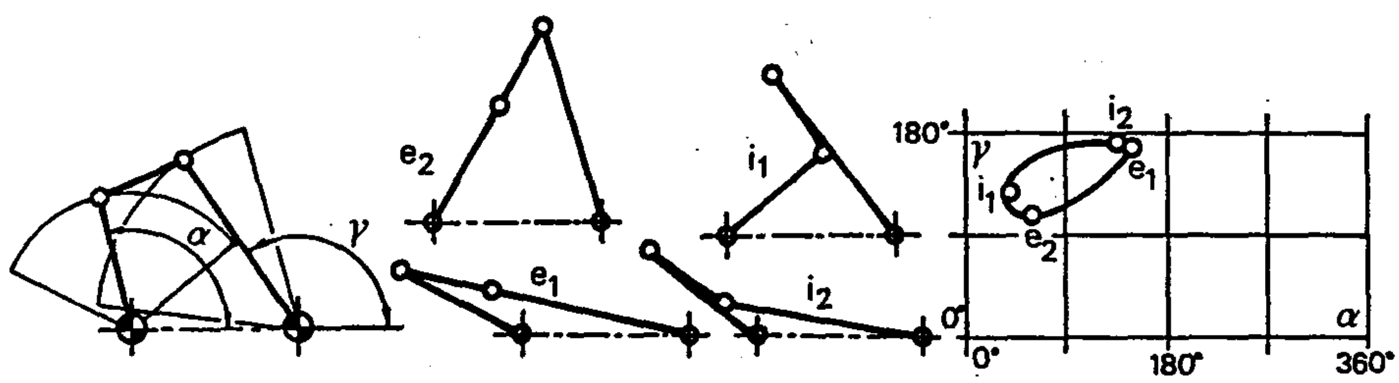
https://www.youtube.com/watch?v=f49-f6p70T0

Four bar mechanism

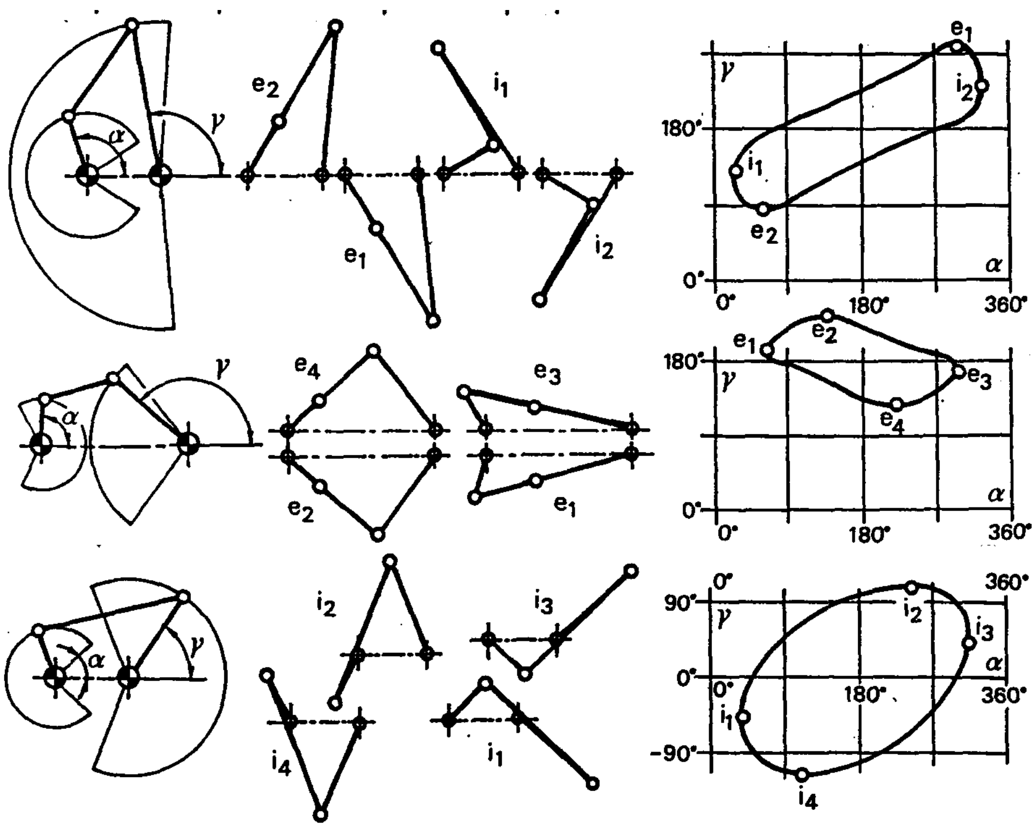
- The shorter link is adjacent to the ground ⇒ four bar mechanism with a crank and a rocker.



- The shorter link is the connection rod ⇒ double rocker four bar mechanism.

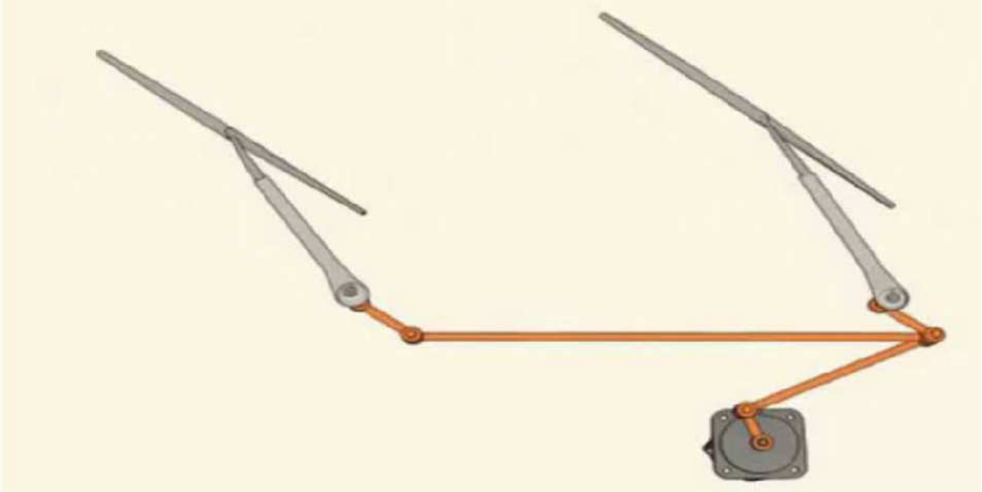


Non Grashof four bar mechanisms have always double-rockers and are also divided into three groups according to the shorter link. The non Grashof four bar mechanisms can attain symmetrical positions with respect to the ground without different mountings

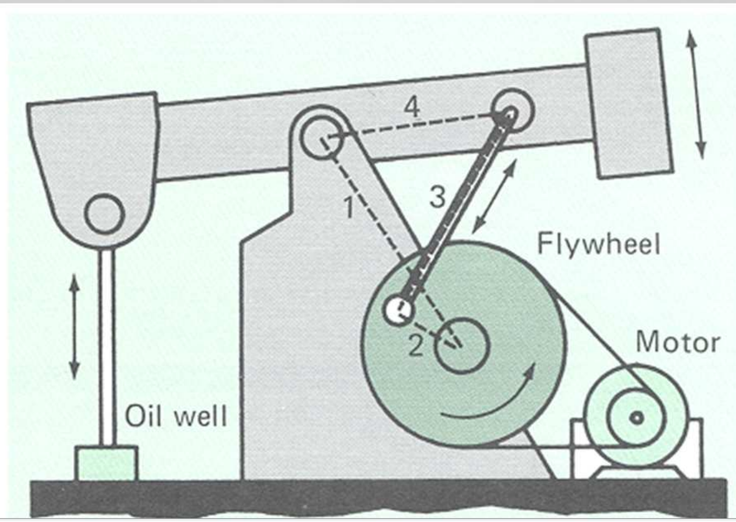


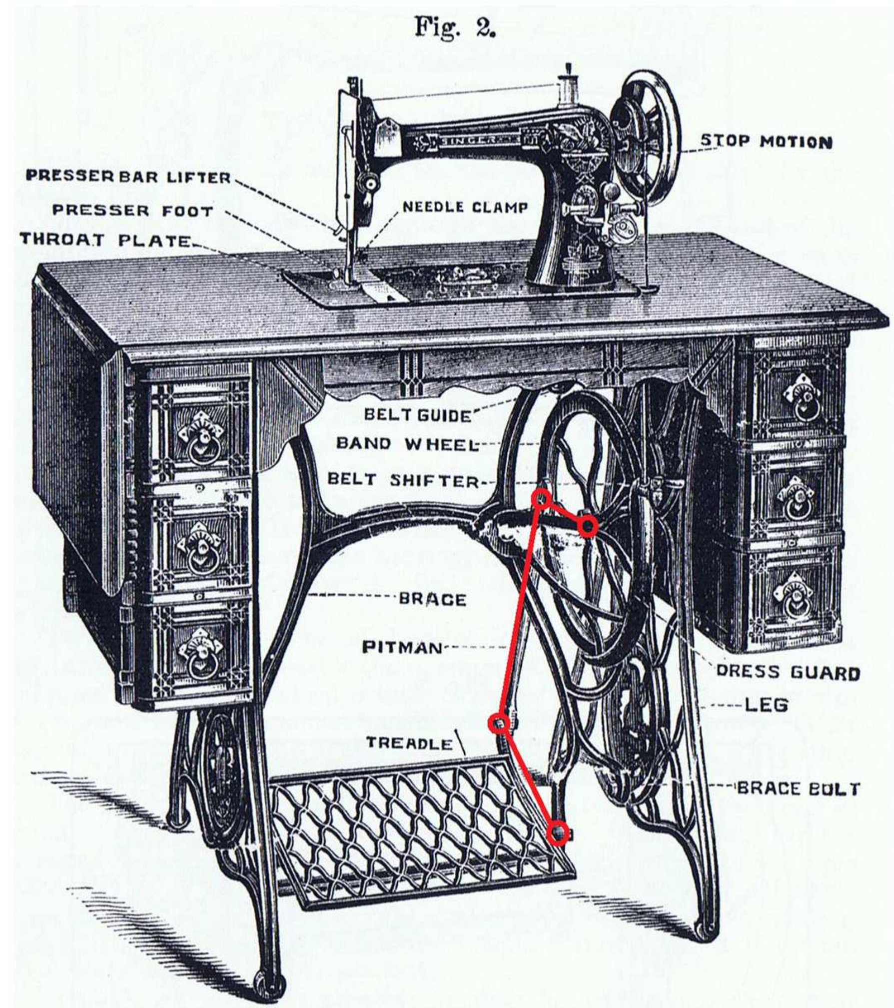
Grashof 4 bar mechanism are interesting because, having at least one crank, this can be directly connected with a motor:

- The double crank mechanisms are useful to transmit unidirectional movements provided with a variable transmission ratio into the period but with average value equal to one.
- 4 bar mechanism with crank and rocker are useful to generate alternative movements.

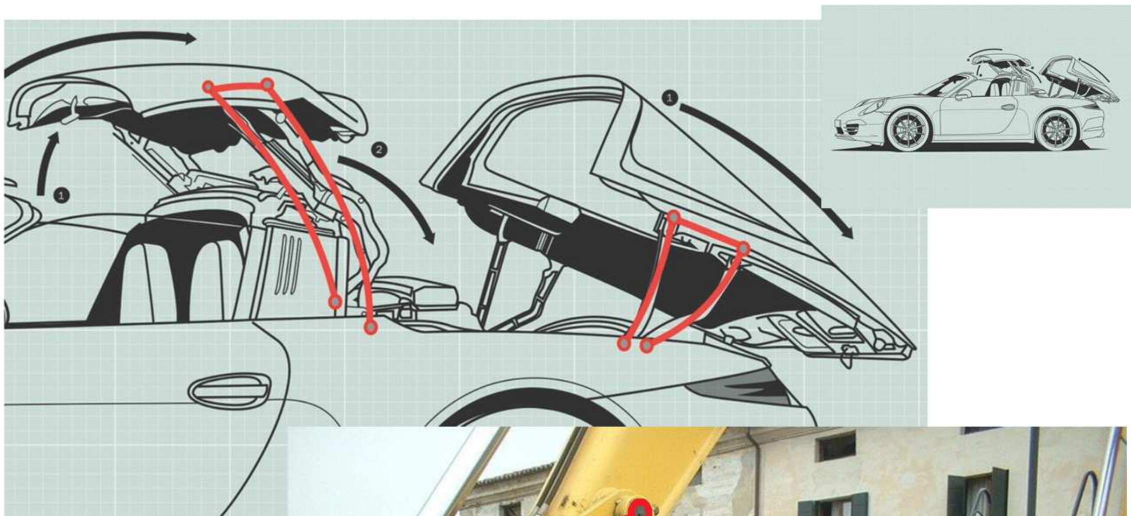


Four bar mechanism

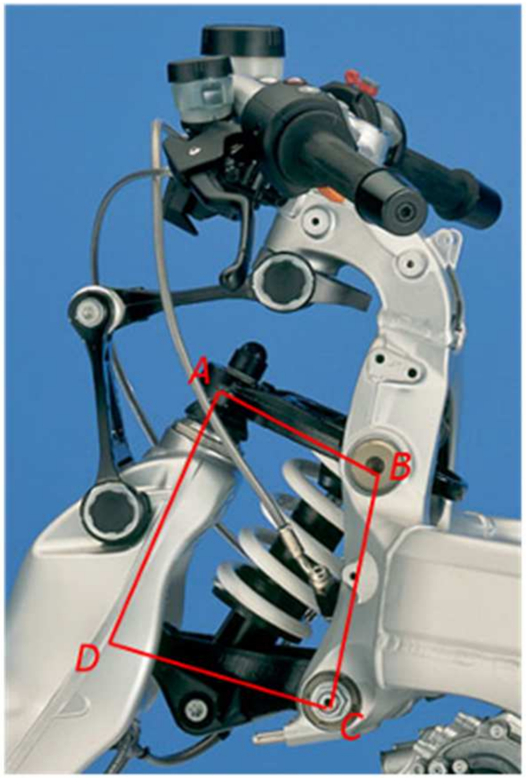


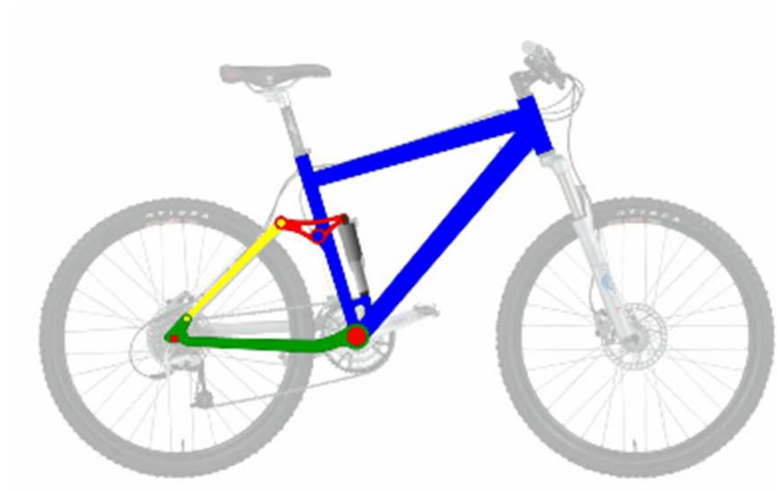


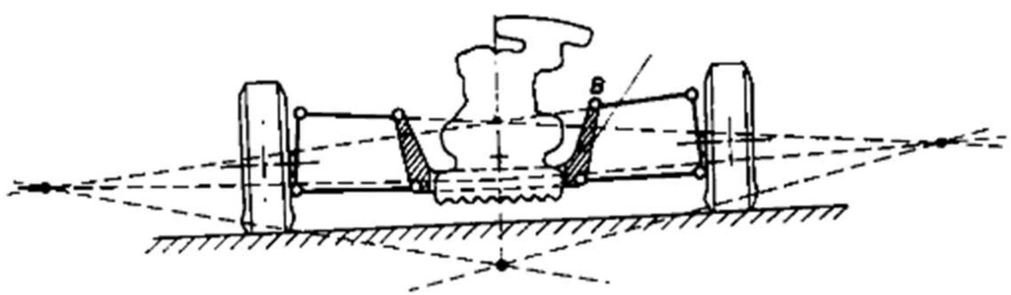
Four bar mechanism



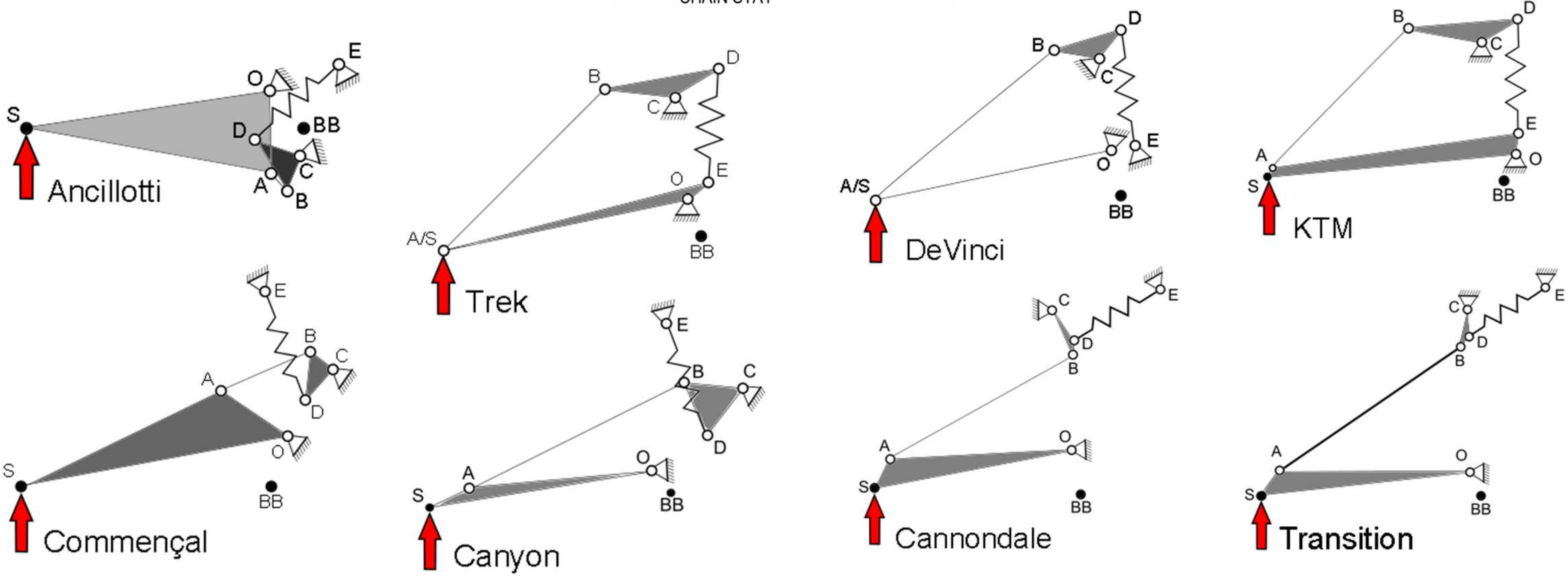
Four bar mechanism



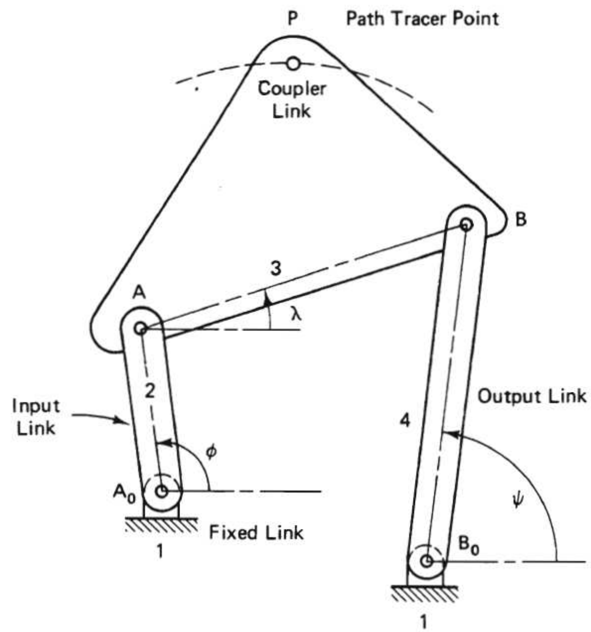


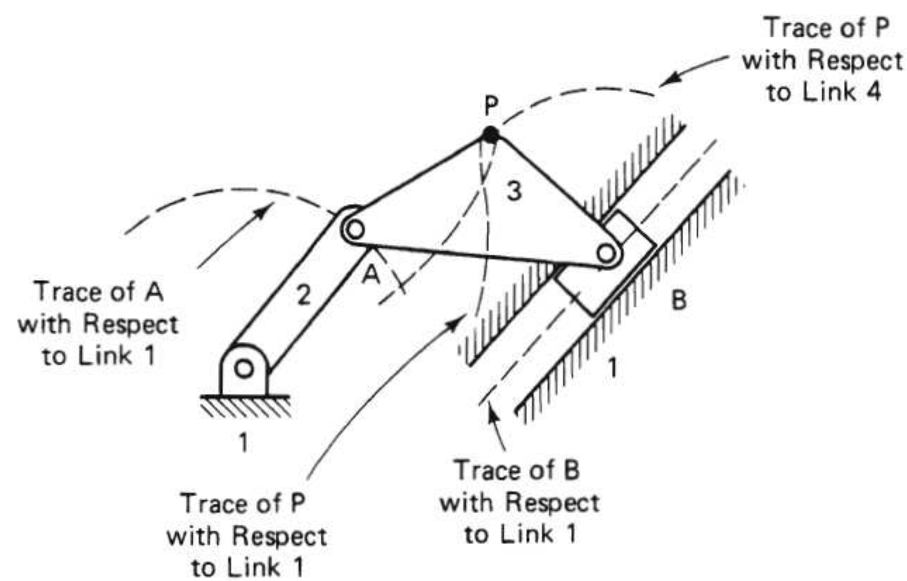


Four bar mechanism



Each point of the coupler link traces a different path with respect to the ground.

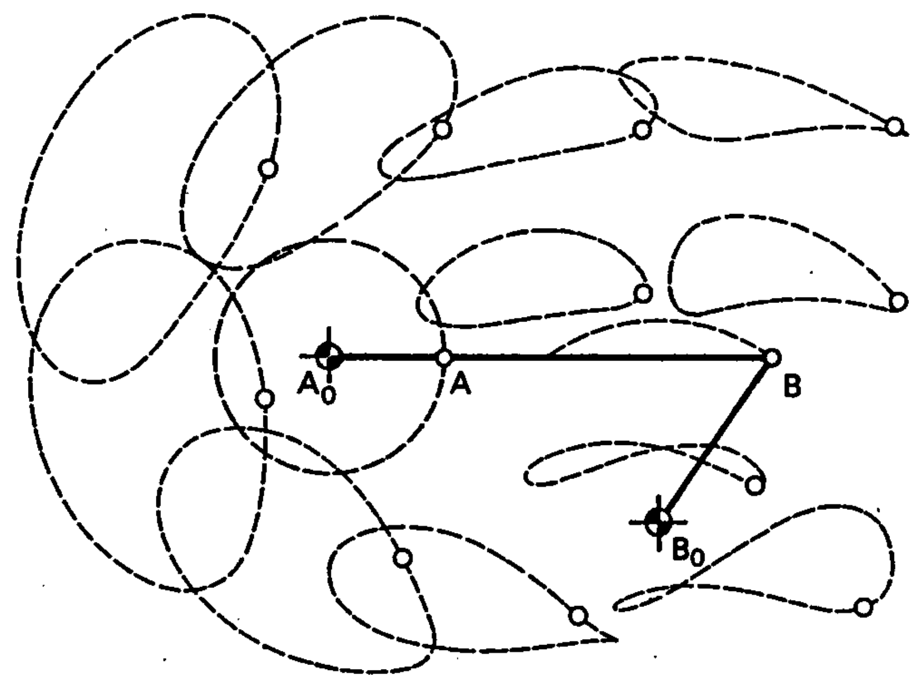




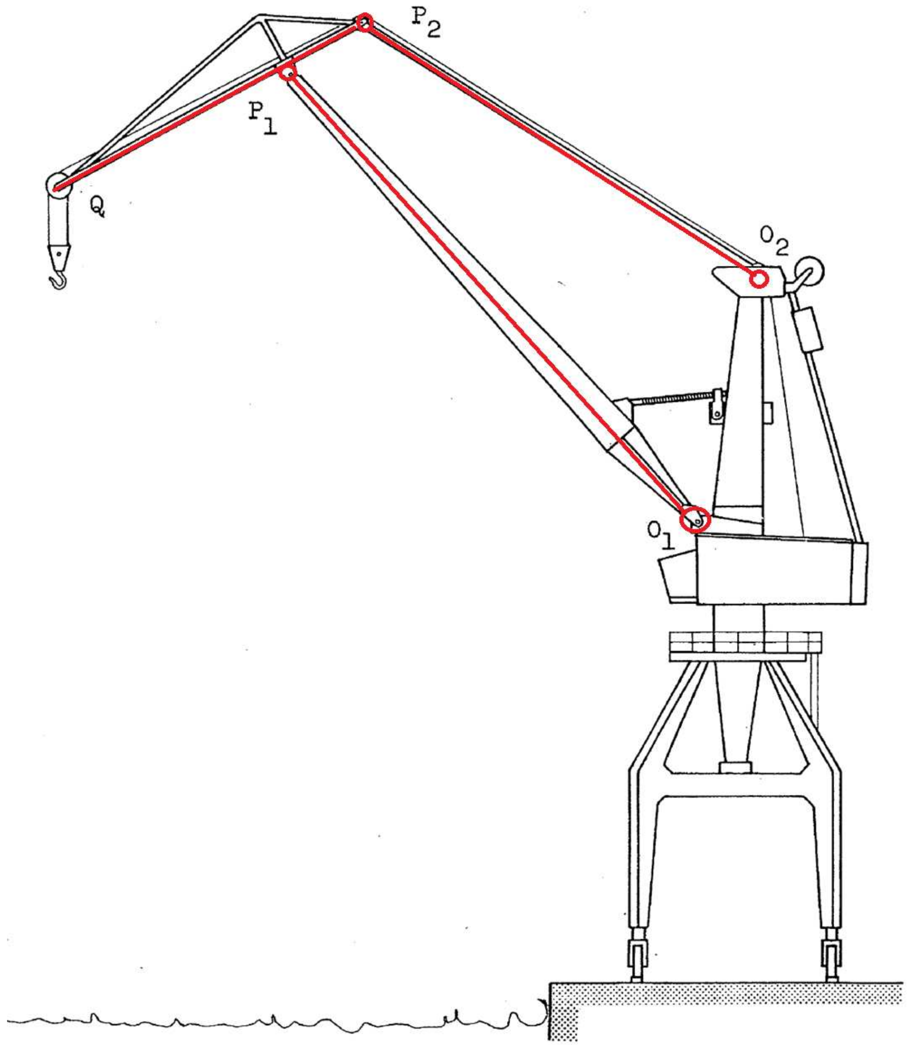
The trajectories of the point belonging of a connection rod in a four bar mechanism are called **coupler curves**.

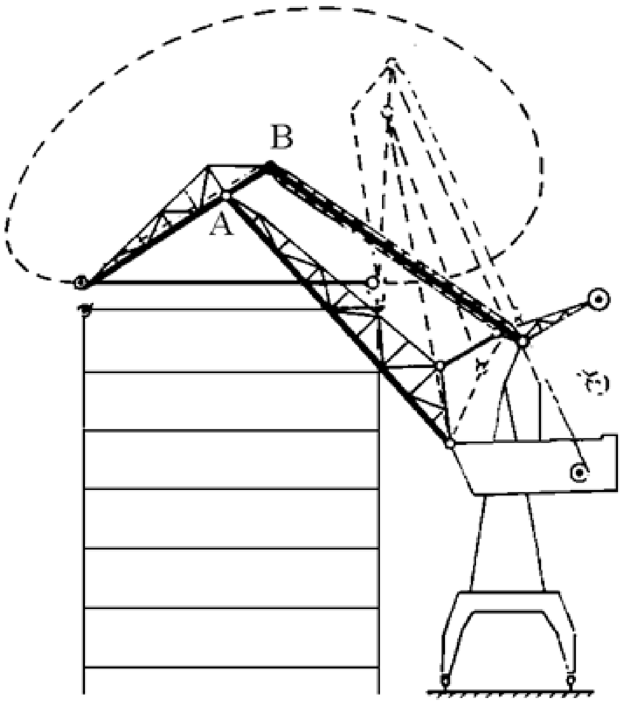
Four bar coupler curves

Coupler curves were collected in atlases that are a useful starting point for design and analysis. Linkages are defined by their link ratios based on a unit length crank. The trajectories of a connection rod point is represented by means of a dashed line. Each dash represents five degrees of crank rotation. So for an assumed constant crank velocity, the dash spacing is proportional to path velocity. The changes in velocity and the quick-return nature of the coupler path motion can be clearly seen from the dash spacing.



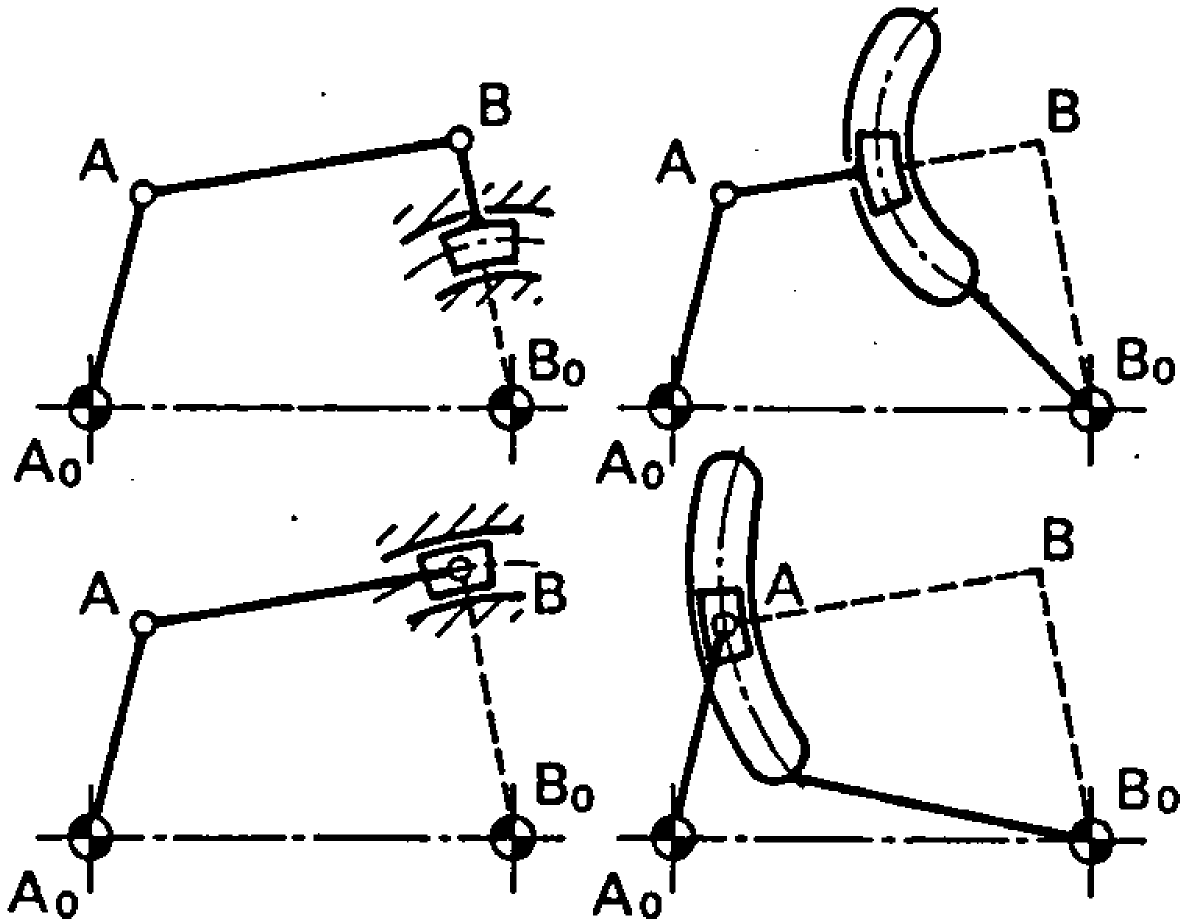
Four bar mechanism



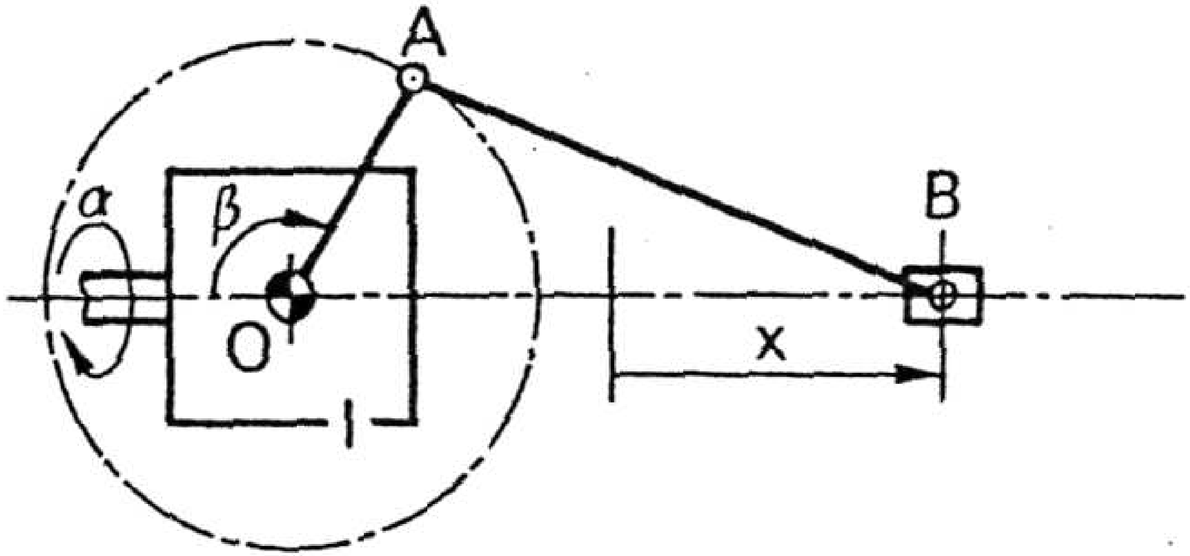


Particular configurations

From a kinematic point of view, four bar linkages can be obtained with different layouts.



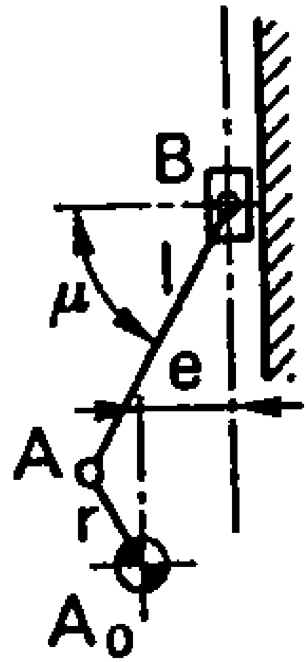
Slider crank mechanism



The **slider-crank** mechanism is a four bar chain with a slider replacing an infinitely long rocker. Slider-crank mechanisms are used to transform rotational motion into alternative linear one or vice-versa. If the slider is connected with the ground the slider-crank mechanism is called **ordinary**.

=> Example

Calling e the distance between A_0 and the axis of the pin B, if $l > r + e$ the crank r can realize a complete revolution. The quantity e is called eccentricity.

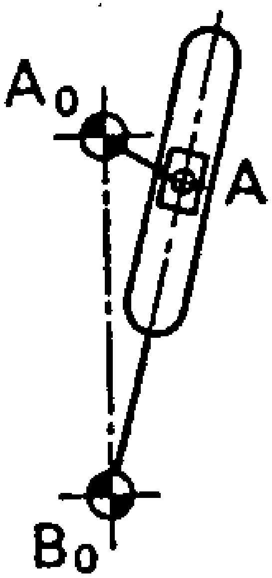


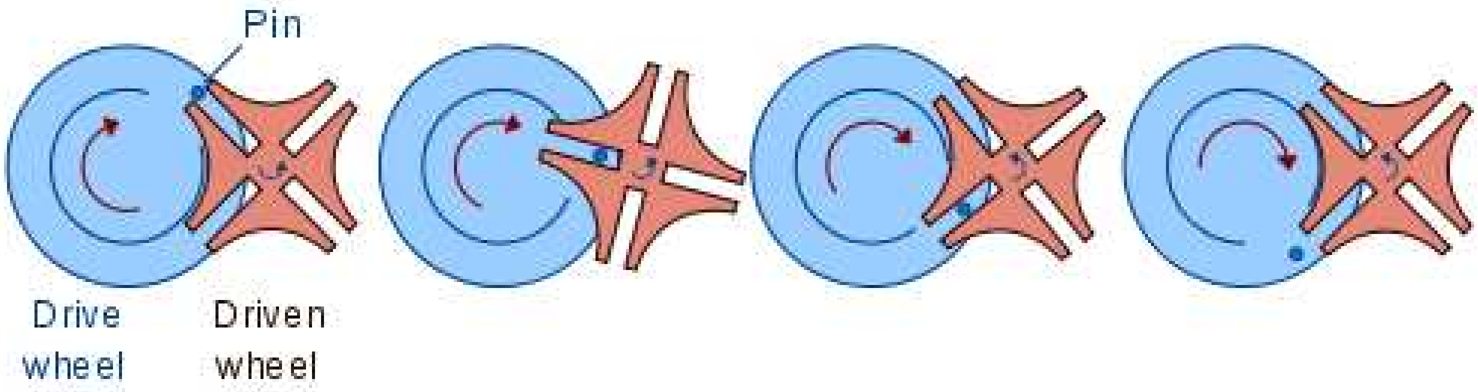
- If $e = 0$ Slider-crank ordinary and **centered**;
- If $e \neq 0$ Slider-crank ordinary and **deviated**.

Slider crank mechanism

Slider-crank is **not ordinary** when the slider is connected to a moving link. Figure shows the slider-crank with yoke also named "maltese cross".

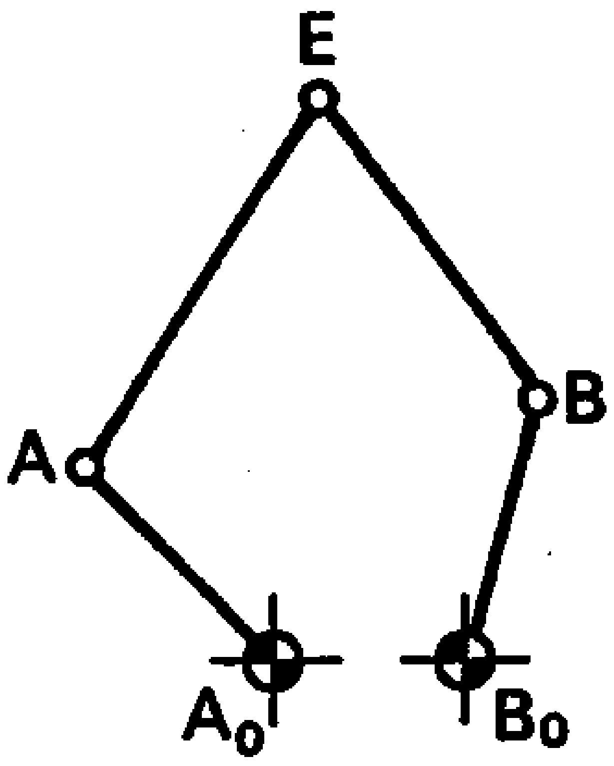
This mechanism is very used to realize intermittors

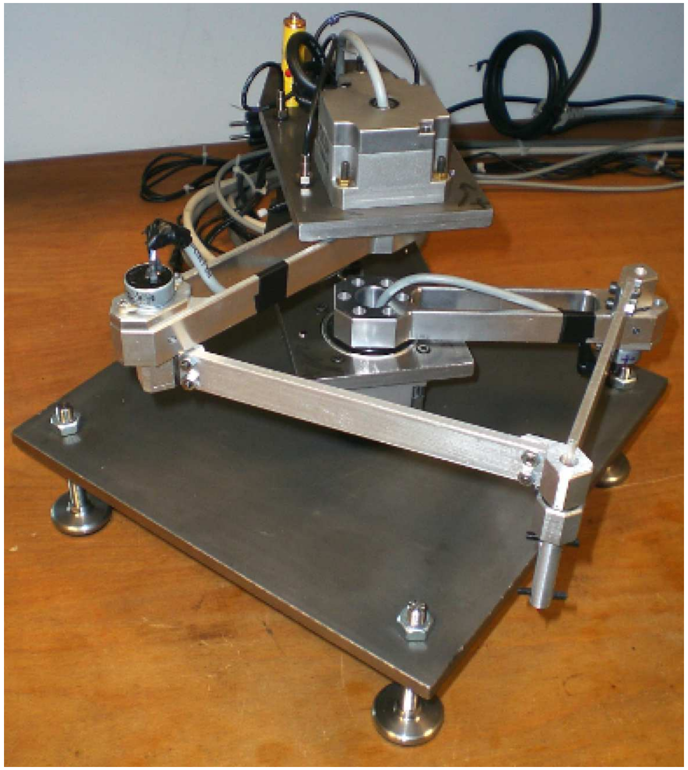




Five bar mechanism

To generate any trajectories in a plane (2 *d.o.f.*), five bar mechanisms can be used. By moving the links A_0A and B_0B, the point E translates in the plane into a specific working area.





Five bar mechanism