Lebanese American University - School of Arts & Sciences
CSC447 - Parallel Programming for Multicore and Cluster Systems
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I - Introduction

1. What is a Martix?

Matrices are two-dimensional arrays of numbers, usually organized in *rows* and *columns*.

They are very important and have applications in Computer Graphics, Image Processing, Machine Learning, Physics et cetera.

Example:

$$X = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

2. What is Martix Multiplication?

Multiplication is one of the most important operations to be performed on a matrix.

a. When Can We Multiply Matrices?

Matrix Multipication is only possible when the number of rows r of the first matrix is equal to the number of columns c in the other matrix.

b. Product of Matrix Multiplication

Let A be an $n \times k$ matrix, and B be an $k \times m$ matrix.
The product $A \times B$ is an $n \times m$ matrix, such that:
The $(i, j)^{th}$ entry of $A \times B$ is $[cij]$ where $c_{ij} = (a_{i1} \times b_{1j}) + (a_{i2} \times b_{2j}) + \dots$

3. Why Parallelize Martix Multiplication

We observe that matrix multiplication is often used in computationally intensive tasks, that also often deal with massive data.

Additionally, the best sequential implementation of Matrix Multiplication runs in $O(n^3)$ asymptotic time complexity, which scales horribly with large data.

Thus, we need to take advantage of parallel techniques to reduce the time needed to perform these operations.

II - Parallelization

1. Sequential Implementation

The sequential implementation of matrix multiplication will be used as a benchmark for the evaluation of the parallel implementations discussed later.

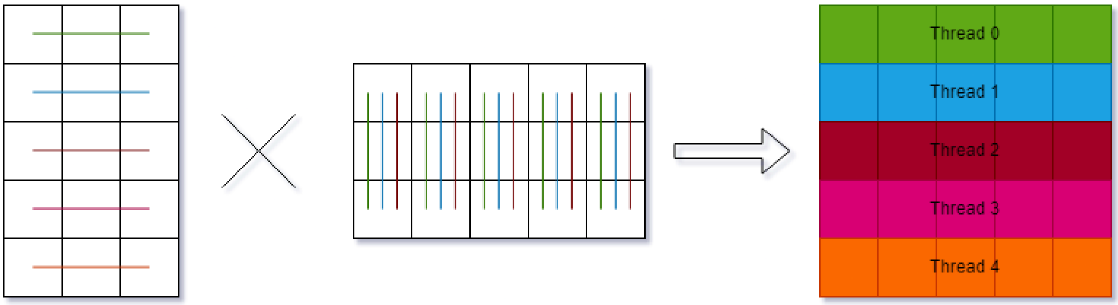
The sequential implementation used here will be the standard implementation in $O(n^3)$ asymptotic time complexity.

Here is the general pseudo-code followed:

```
for every row in C do
    for every column in C do
        for every row in A do
            entry C += entry A * entry B
```

2. General Parallelization Strategy

To parallelize this computation, I plan to distribute the rows among "nodes", whether that be threads accessing shared memory, or threads communicating by passing messages.



3. Parallelization using Multithreading

For multithreading, I will be distrubuting the rows of the result matrix statically and evenly based on the total number of rows and number of available threads.

This will be done by creating the different threads with numbered IDs which I can use later to assign a section of the resulting matrix.

4. Parallelization with OpenMP

The OpenMP implementation here follows the same strategy of distributing the rows among threads though it is executed more loosely.

I used the `#pragma` to parallelize the outermost for loop among some threads that will be decided by the OpenMP runtime library at runtime based on the number of available processors or cores on the system.
The default behavior of the library, as implemented, ses as many threads as there are available processors or cores on the system.

III - Code

The code was written in C++ adhering to C++20 standards.
The standard implementations of the OpenMP library (`omp.h`) and Threads (`std::thread`) were used.
Time measurement was implemented with the `chrono` library for C++ for the highest precision results.

The outputs of the programs are written to textfiles that would be created in the same directory of the source files.

Instructions/Commands for compiling each program is included as a comment in the last 2 lines of every source file.

You can find the code on Github

IV - Performance Evaluation

0. Environment & Testing

a. Environment

All implementations were tested on single machine, with the same performance settings and same power settings to avoid external bias.

Machine Specifications:

Component	Name / Model / Details
Operating System	Windows 11 Pro (22H2)
System Type	x64
Processor	11th Gen Intel Core i5-1135G7 @ 2.40GHz
RAM	8.00GB DDR4 @ 3200MHz
GPU	Intel Iris Xe Graphics
Dedicated GPU	NVIDIA GeForce MX350

b. Test Cases

Both Parallel Implementations ran with the same number of threads.
All implementations were tested using 2 sample matrices of size 1000×1000.
All implementations were tested the same matrices initialized as follows:

```
for (int i = 0; i < a; i++)
{
    for (int j = 0; j < b; j++)
    {
        m1[i][j] = i + j;
    }
}

for (int i = 0; i < b; i++)
{
    for (int j = 0; j < c; j++)
    {
        m2[i][j] = i - j;
    }
}
```

where m_1 is an $a \times b$ matrix, and m_2 is an $b \times c$ matrix.

c. Recording Results

Each implementation was ran 10 times, recording the time elapsed between starting the computation and ending it. Meaning operations like initializing the matrices, including libraries, taking input, printing output et cetera, were excluded from the recorded time.
However, initializing threads is a cost of parallelization, so this operation was included.

1. Sequential Implementation

Run #	1	2	3	4	5	6	7	8	9	10	Average
Time (microseconds)	8132533	8205830	8094021	8283010	8170052	8361042	8131340	8208902	8127125	8046639	8176049.4

> 8.1760494 seconds on average

2. OpenMP Implementation

Run #	1	2	3	4	5	6	7	8	9	10	Average
Time (microseconds)	997517	1002974	1011741	998237	990064	980729	975253	998024	1029758	1081221	1006551.8

> 1.0065518 second on average

a. Speedup Factor

> Speedup Factor $S(p) = \frac{t_s}{t_p}$
> In this case, $t_s = 8.1760494$, and $t_p = 1.0065518$
> So, $S(p) = 8.122830$

b. Efficiency

> Efficiency $E = \frac{t_s}{t_p \times p} = \frac{S(p)}{p} \times 100$
> In this case, $S(p) = 8.122830$ and $p = 8$
> So $E = 1.015354$

3. Multithreading Implementation

Run #	1	2	3	4	5	6	7	8	9	10	Average
Time (microseconds)	2422249	2425972	2434168	2474027	2493085	2534890	2583405	2571563	2599692	2609645	2514869.6

> 2.5148696 seconds on average

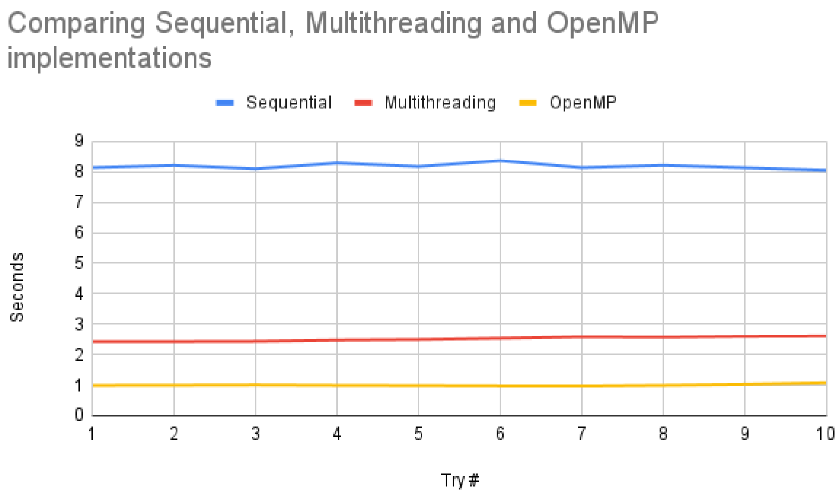
a. Speedup Factor

> Speedup Factor $S(p) = \frac{t_s}{t_p}$
> In this case, $t_s = 8.1760494$, and $t_p = 2.5148696$
> So, $S(p) = 3.251083$

b. Efficiency

> Efficiency $E = \frac{t_s}{t_p \times p} = \frac{S(p)}{p} \times 100$
> In this case, $S(p) = 3.251083$ and $p = 8$
> So $E = 0.406385$

V - Method Comparison



In addition to our calculated metrics, by plotting the elapsed time taken by each implementation to complete multiplying the matrices, we can gain a lot of insight on each algorithm.

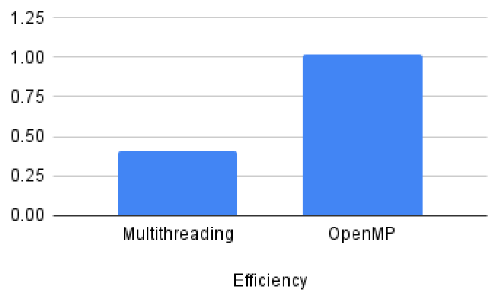
Raw Speed & Speedup Factor

It is clear that the OpenMP is the fastest implementation out of the 3 present.
In fact OpenMP is almost 8 times faster than the sequential algorithm, and 2.5 times faster than manual multithreading!

Efficiency

We have already calculated the efficiency of the OpenMP implementation and the Multithreading implementation, and it is clear that OpenMP is more efficient.

OpenMP has an efficiency of $E = 1.015354$ while Multithreading has an efficiency of $E = 0.406385$



We can see that OpenMP is $2.4985 \approx 2.5$ times more efficient than manual multithreading.

Consistency

We notice that the sequential algorithm has the highest variance in its values, which means it's the least consistent algorithm out of the 3.

On the other hand, OpenMP yeilds the least variance, making it the most consistent implementation.

VI - Discussion & Conclusion

It is clear OpenMP is a better solution to implement a parallelized algorithm for matrix multiplication due to its superior efficiency and speedup.
Moreover, it's simpler to use and is more beginner friendly, which contributes to abstracting away many details that further optimize some algorithms under the hood, to no knowledge of the programmer perhaps!

It is important to note that the reason why OpenMP surpasses multithreading is not explored in this report but could be one of the following reasons:

1. Load balancing: OpenMP can dynamically adjust the workload across threads to balance the processing load, which can lead to better performance. Multithreading typically requires you to manually divide the work among threads, which can be less efficient if the workload is not evenly distributed.

2. Cache coherence: OpenMP can better utilize cache coherence, which can improve performance on multi-core processors. Cache coherence refers to the synchronization of cache data between multiple cores, and OpenMP provides mechanisms to ensure that data is correctly shared between threads, which can reduce cache misses and improve performance.

3. Optimizations: OpenMP compilers can perform more advanced optimizations than multithreading, such as loop unrolling and vectorization. These optimizations can further improve performance and efficiency.