

ANGLE MODULATION

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OUTLINE

- Introduction
- Angle modulation
- Frequency modulation
- FM waveforms
- FM modulation methods
- FM demodulation methods



INTRODUCTION

- Angle modulation varies a sinusoidal carrier signal in such a way that the **angle of the carrier is varied according to the amplitude** of the modulating baseband signal
- In this method the amplitude of the carrier wave is kept **constant** (*that's why FM is called constant envelope*)



$\theta(t)$ of a signal may be varied in a number of ways according to the baseband signal

Two most important classes of angle modulation are :

- frequency modulation and

- phase modulation



ANGLE MODULATION

Consider a the general carrier

$$v_c(t) = V_c \cos(\omega_c t + \phi_c)$$

$(\omega_c t + \phi_c)$ represents the angle of the carrier

Two ways of varying the angle of the carrier:

- a) By varying the frequency, ω_c – Frequency Modulation.
- b) By varying the phase, ϕ_c – Phase Modulation



FREQUENCY MODULATION

- In FM, the message signal $m(t)$ controls the frequency f_c of the carrier. Consider the carrier

$$v_c(t) = V_c \cos(\omega_c t)$$

then for FM we may write:

FM signal deviation will depend on $m(t)$, where the frequency

$$v_s(t) = V_c \cos(2\pi(f_c + \text{frequency deviation})t)$$

- Given that the carrier frequency will change we may write for an instantaneous carrier signal

- where ϕ_i is the instantaneous angle and f_i is the instantaneous frequency

$$V_c \cos(\omega_i t) = V_c \cos(2\pi f_i t) = V_c \cos(\phi_i)$$

$$\omega_i t = 2\pi f_i t$$



FREQUENCY MODULATION

Since $\phi_i = 2\pi f_i t$ then $\frac{d\phi_i}{dt} = 2\pi f_i$ or $f_i = \frac{1}{2\pi} \frac{d\phi_i}{dt}$

i.e. frequency is proportional to the rate of change of angle

- If f_c is the unmodulated carrier and f_m is the modulating frequency, then we may deduce that

$$f_i = f_c + \Delta f_c \cos(\omega_m t) = \frac{1}{2\pi} \frac{d\phi_i}{dt}$$

- Δf_c is the peak deviation of the carrier

Hence, we have $\frac{1}{2\pi} \frac{d\phi_i}{dt} = f_c + \Delta f_c \cos(\omega_m t)$ i.e.

$$\frac{d\phi_i}{dt} = 2\pi f_c + 2\pi \Delta f_c \cos(\omega_m t)$$



FREQUENCY MODULATION

After integration i.e. $\int (\omega_c + 2\pi\Delta f_c \cos(\omega_m t)) dt$

$$\varphi_i = \omega_c t + \frac{2\pi\Delta f_c \sin(\omega_m t)}{\omega_m}$$

$$\varphi_i = \omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)$$

$$v_s(t) = V_c \cos(\varphi_i)$$

Hence for the FM signal

$$v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)\right)$$



FREQUENCY MODULATION

The ratio $\frac{\Delta f_d}{f_m}$ is called the Modulation Index denoted by β
i.e.

$$\beta = \frac{\text{Peak frequency deviation}}{\text{modulating frequency}}$$

Note – FM, as implicit in the above equation for $v_s(t)$, is a non-linear process – i.e. the principle of superposition does not apply

The FM signal for a message $m(t)$ as a band of signals is very complex

Hence, $m(t)$ is usually considered as a 'single tone modulating signal' of the form

$$m(t) = V_m \cos(\omega_m t)$$

FREQUENCY MODULATION

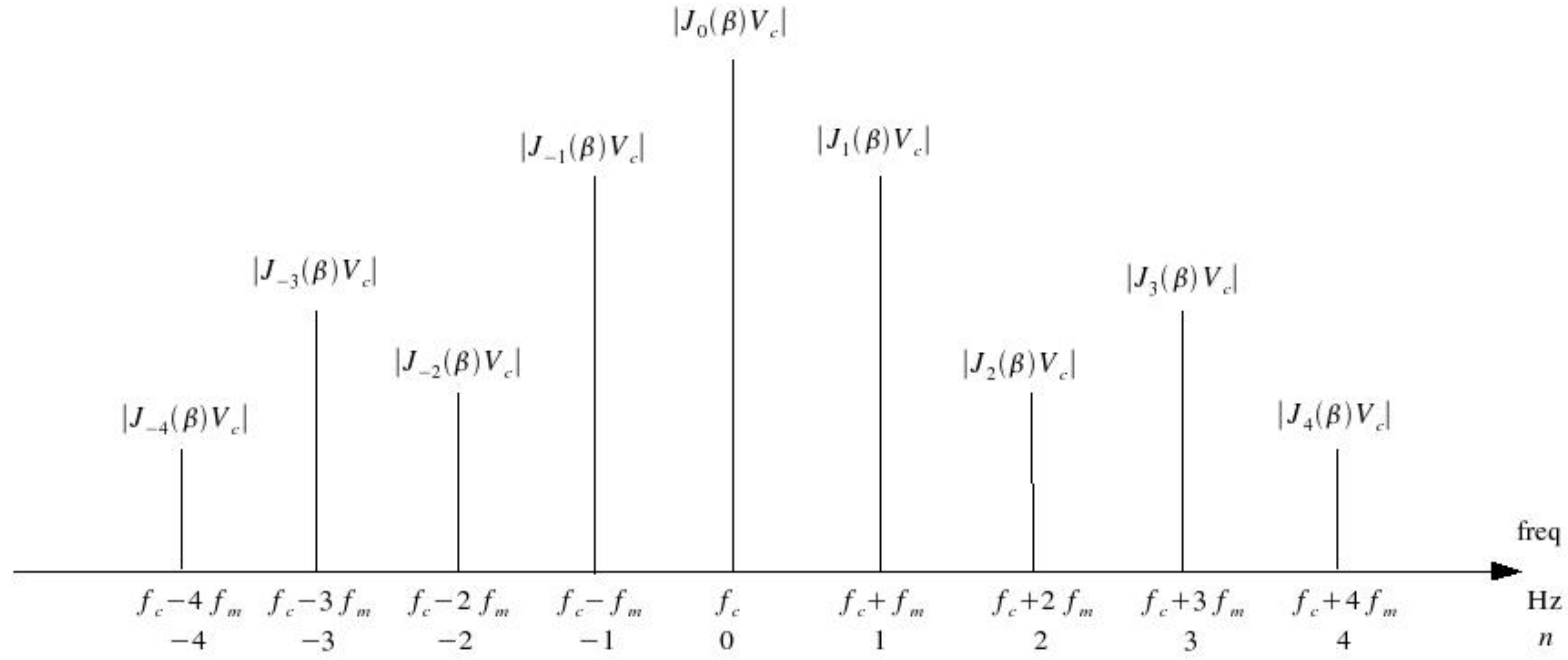
The equation $v_s(t) = V_c \cos\left(\omega_c t + \frac{\Delta f_c}{f_m} \sin(\omega_m t)\right)$ may be expressed as **bessel series (bessel function)**

$$v_s(t) = V_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

□ where $J_n(\beta)$ are **Bessel functions of the first kind**.
Expanding the equation for a few terms we have:

$$\begin{aligned} v_s(t) = & \underbrace{V_c J_0(\beta)}_{\text{Amp}} \underbrace{\cos(\omega_c)}_{f_c} t + \underbrace{V_c J_1(\beta)}_{\text{Amp}} \underbrace{\cos(\omega_c + \omega_m)}_{f_c + f_m} t + \underbrace{V_c J_{-1}(\beta)}_{\text{Amp}} \underbrace{\cos(\omega_c - \omega_m)}_{f_c - f_m} t \\ & + \underbrace{V_c J_2(\beta)}_{\text{Amp}} \underbrace{\cos(\omega_c + 2\omega_m)}_{f_c + 2f_m} t + \underbrace{V_c J_{-2}(\beta)}_{\text{Amp}} \underbrace{\cos(\omega_c - 2\omega_m)}_{f_c - 2f_m} t + \dots \end{aligned}$$

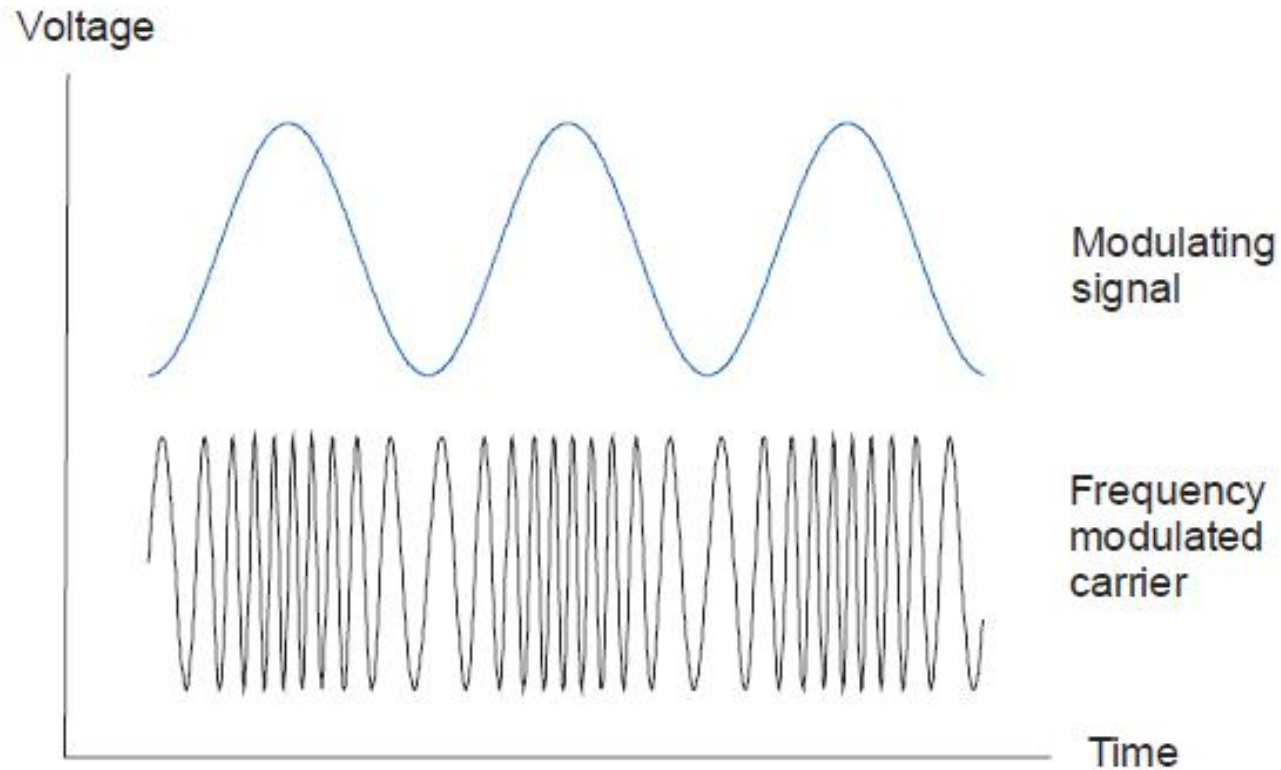
FM SIGNAL SPECTRUM



The amplitudes drawn are completely arbitrary, since we have not found any value for $J_n(\beta)$ – this sketch is only to illustrate the spectrum

FM WAVEFORMS

- Frequency is “wobbled” higher and lower by modulating signal





NUMERICAL



Example

A sinusoidal modulating signal, $m(t) = 4\cos 2\pi 4 \times 10^3 t$, is applied to an FM modulator that has a frequency deviation constant gain of 10 kHz/V. Compute (a) the peak frequency deviation, and (b) the modulation index.

Solution

Given:

Frequency deviation constant $k_f = 10 \text{ kHz/V}$

Modulating frequency, $f_m = 4 \text{ kHz}$

- a) The maximum frequency deviation will occur when the instantaneous value of the input signal is at its maximum. For the given $m(t)$, the maximum value is 4 V, and hence the peak deviation is equal to

$$\Delta f = 4 \text{ V} \times 10 \text{ kHz/V} = 40 \text{ kHz}$$

- b) The modulation index is given by

$$\beta_f = \frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \frac{40}{4} = 10$$



CARSON'S RULE FOR FM BANDWIDTH.

An approximation for the bandwidth of an FM signal is given by $BW = 2(\text{Maximum frequency deviation} + \text{highest modulated frequency})$

Carson's rule

$$\text{Bandwidth} = 2(\Delta f_c + f_m)$$



FM MODULATION METHODS

Direct method :

The carrier frequency is directly varied in accordance with the input modulating signal

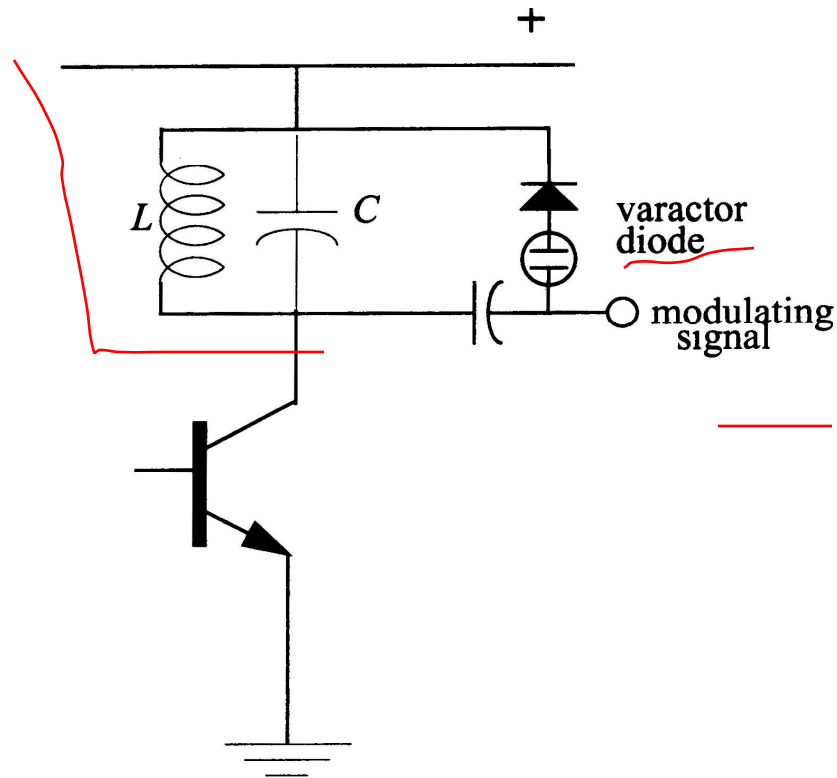
Indirect method:

- A narrowband FM signal is generated using a balanced modulator
- frequency multiplication is used to increase both the frequency deviation and the carrier frequency to the required level,

can be expressed as

$$S_{FM}(t) \approx A_c \cos 2\pi f_c t - A_c \theta(t) \sin 2\pi f_c t$$

FM MODULATION METHOD- DIRECT METHOD



Figure

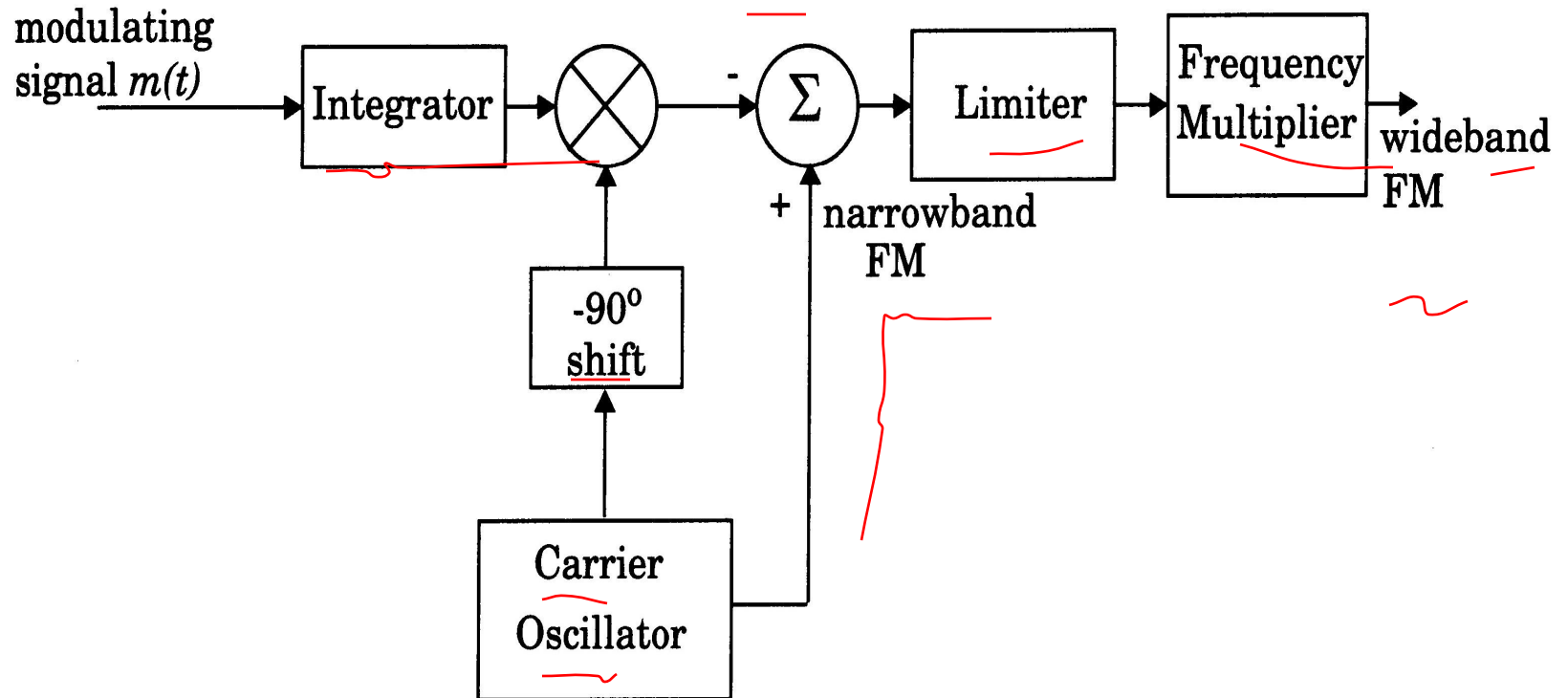
A simple reactance modulator in which the capacitance of a varactor diode is changed to vary the frequency of a simple oscillator. This circuit serves as a VCO.



DIRECT METHOD

- In this method **voltage controlled oscillators (VCOs)** are used to vary frequency of carrier signal in accordance with the baseband signal amplitude variations
- Oscillators use devices whose reactance may be varied by the application of a voltage
- Most commonly used variable reactance device is **the voltage-variable capacitor called a varactor**
- voltage variable capacitor may be obtained by **using reverse biased p-n junction diode**
- The larger the reverse voltage applied to such **a diode the smaller the transition capacitance will be of the diode**
- FM signal may be obtained by incorporating such a device into a **Hartley or Colpitts oscillator**

FM MODULATION METHODS- INDIRECT METHOD



Figure

Indirect method for generating a wideband FM signal. A narrowband FM signal is generated using a balanced modulator and then frequency multiplied to generate a wideband FM signal.



INDIRECT METHOD...

- A simple block diagram of indirect method FM transmitter can be seen on previous slide
- It's a direct implementation of the equation

$$S_{FM}(t) \approx A_c \cos 2\pi f_c t - A_c \theta(t) \sin 2\pi f_c t$$

Where the first term represents the carrier and the second term represents the sideband

- A narrowband FM signal is generated using a balanced modulator which modulates a crystal controlled oscillator
- The maximum frequency deviation is kept constant and small hence the output is a narrowband FM signal
- The wideband FM signal is then produced by multiplying in frequency the narrowband FM signal using frequency multipliers



FM DETECTION TECHNIQUE



- Slope Detector
- Zero-crossing Detector
- PLL for FM Detection
- Quadrature Detection

SLOPE DETECTOR

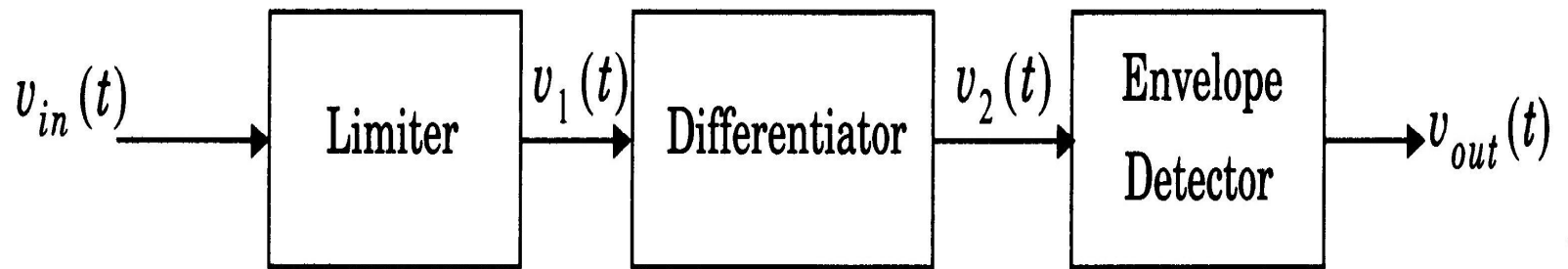


Figure
Block diagram of a slope detector type FM demodulator.

SLOPE DETECTOR

- FM signal is first passed through an amplitude limiter to remove any amplitude perturbations due to fading
- After limiter we get a constant envelope signal and it may be represented by

$$v_1(t) = V_1 \cos[2\pi f_c t + \theta(t)] = V_1 \cos\left[2\pi f_c t + 2\pi k_f \int m(\eta) d\eta\right]$$

- This signal is then passed through a slope detector which is nothing but a differentiator, its output may be represented as:

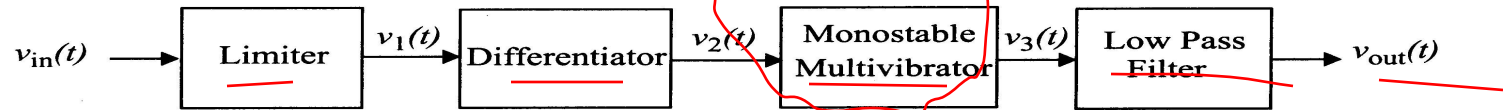
$$v_2(t) = -V_1 \left[2\pi f_c t + \frac{d\theta}{dt} \right] \sin(2\pi f_c t + \theta(t))$$

- Then its fed to envelope detector and finally the output we get is of the form:

$$v_{out}(t) = V_1 \left[2\pi f_c + \frac{d}{dt} \theta(t) \right] = V_1 2\pi f_c + V_1 2\pi k_f m(t)$$

- This equation shows that the output of the envelope detector contains a dc term proportional to the carrier frequency and a time-varying term proportional to the original message signal $m(t)$

ZERO CROSSING DETECTOR



Zero-crossing detector

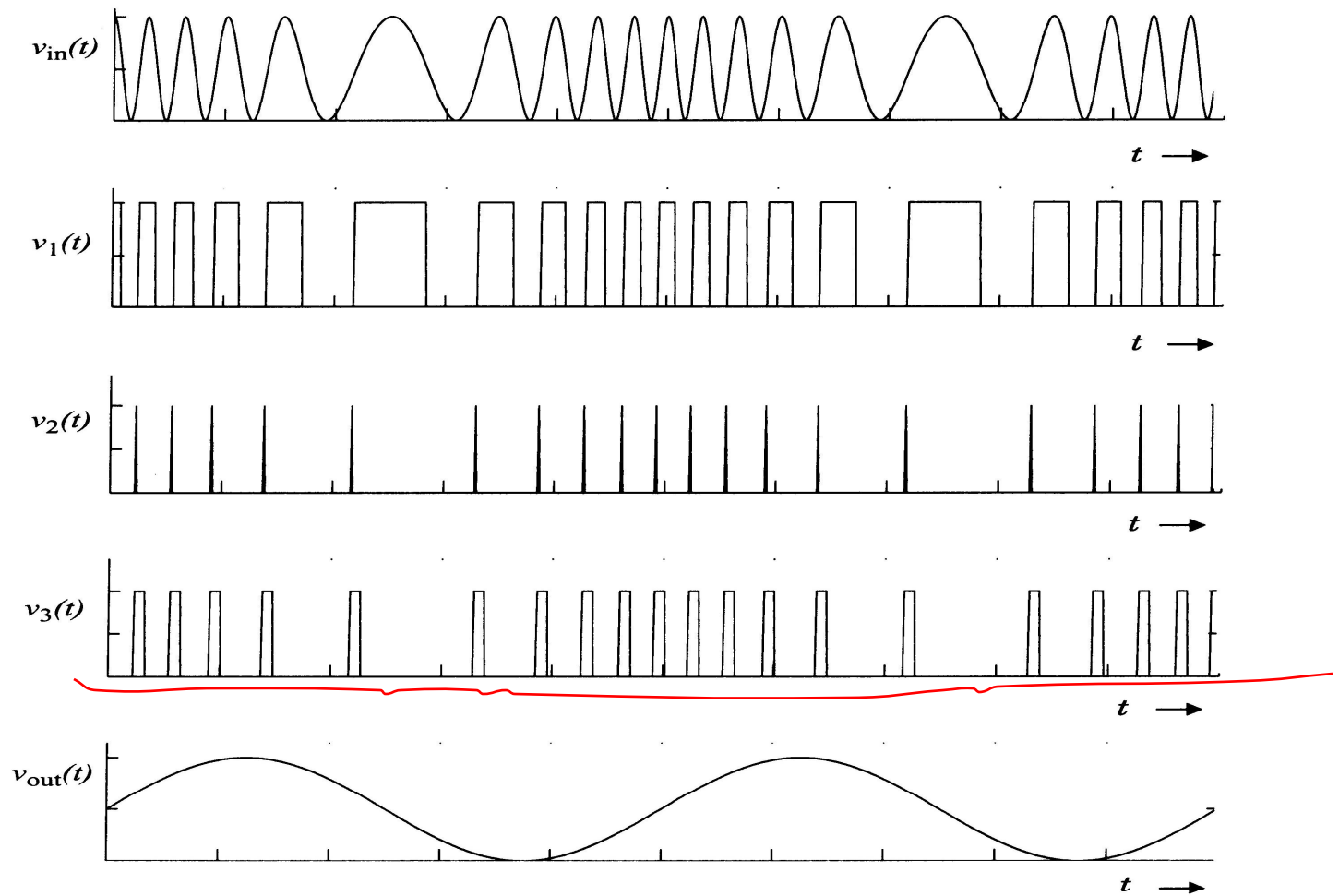


Figure
Block diagram of a zero-crossing detector and associated waveforms



ZERO CROSSING DETECTOR

- When linearity is required e.g. in data communication a zero crossing detector is used to perform frequency to amplitude conversion by directly counting the number of zero crossing in the input FM signal
- The reason behind using it is to generate a pulse train with an average value that is proportional to the frequency of input signal
- A block diagram can be seen on the previous slide
- Input FM signal is passed through a limiter circuit it converts the input signal to a frequency modulated pulse train
- Pulse train $v_1(t)$ is then passed through a differentiator whose output is used to trigger a monostable multivibrator (also called "one shot")
- Output of one shot consists of train of pulses with average duration proportional to the desired signal
- A low pass filter output is the desired signal

PLL FOR FM DETECTION

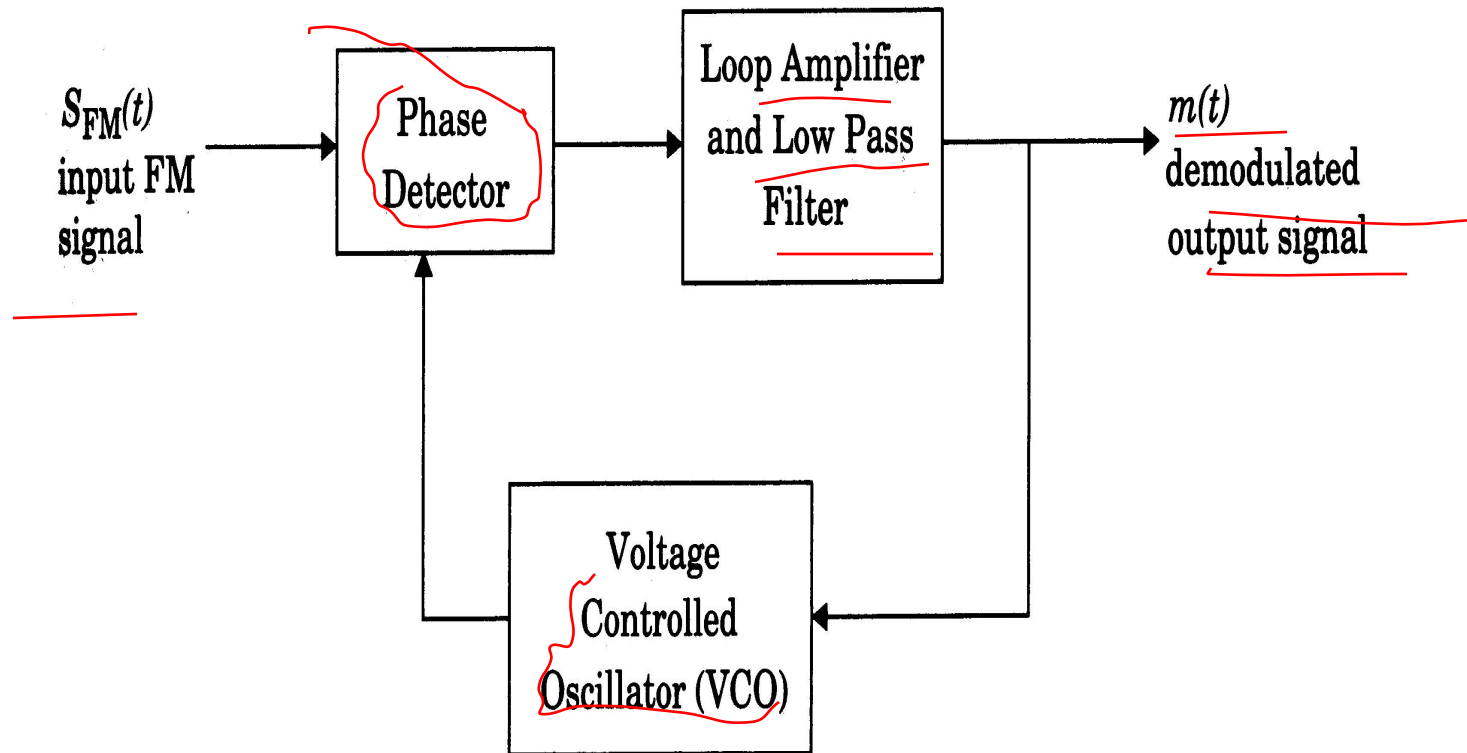


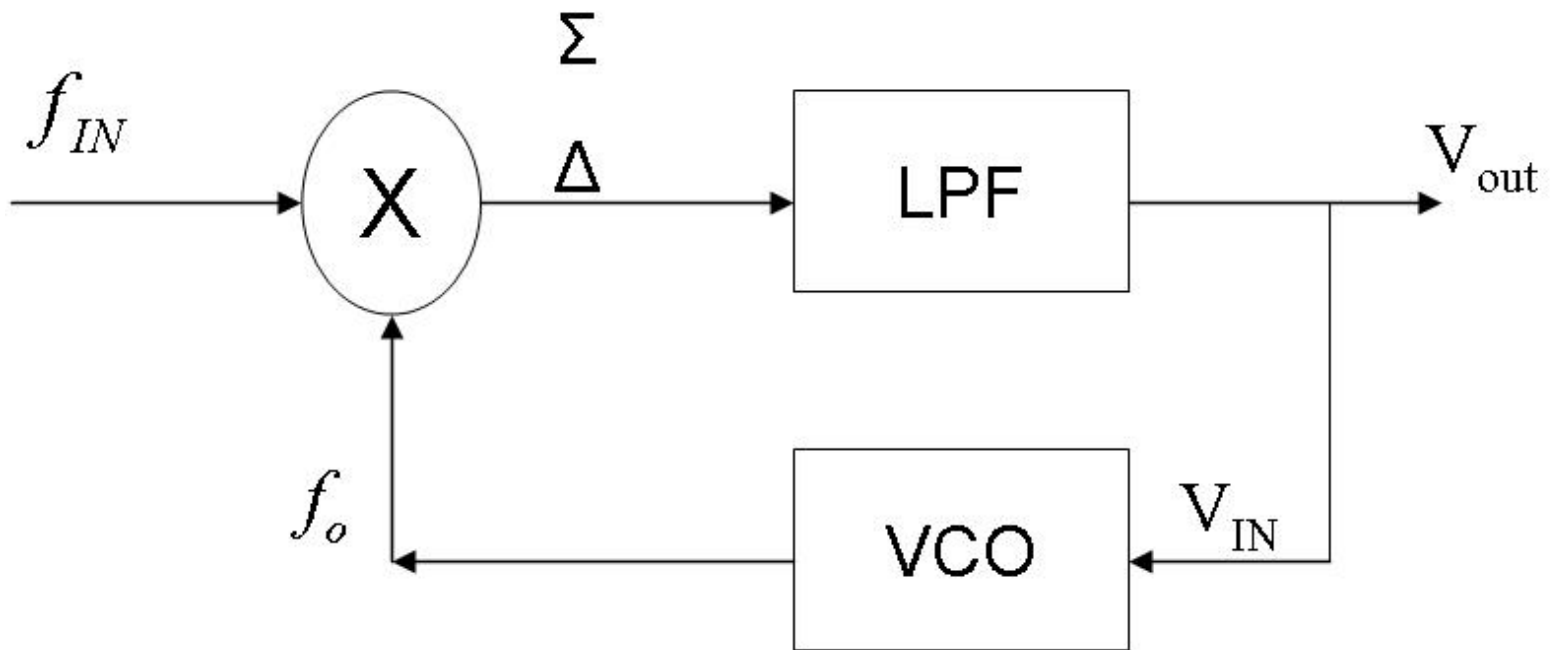
Figure
Block diagram of a PLL used as a frequency demodulator.



PLL FOR FM DETECTION

- Phase locked loop is yet another popular technique for the FM detection
- It's a closed loop control system which can track the variations in the received signal phase and frequency
- As can be seen on previous slide it consists of a VCO in the feedback loop with an output frequency which is varied in accordance with the demodulated output voltage level
- The output of the VCO is compared with the input signal using a phase comparator which produces an output voltage proportional to the phase difference
- The feed back loop functions in a manner that facilitates locking of the VCO frequency to the input frequency
- Once the VCO frequency is locked to the input frequency the VCO continues to track the variations in the input frequency
- Once this tracking is achieved the control voltage to the VCO is simply the demodulated FM signal

PHASE LOCKED LOOP



PHASE LOCKED LOOP

- The input f_{IN} is applied to the multiplier and multiplied with the VCO frequency output f_O , to produce $\Sigma = (f_{IN} + f_O)$ and $\Delta = (f_{IN} - f_O)$.
- The low pass filter passes only $(f_{IN} - f_O)$ to give V_{OUT} which is proportional to $(f_{IN} - f_O)$.
- If $f_{IN} \approx f_O$ but not equal, $V_{OUT} = V_{IN} \propto f_{IN} - f_O$ is a low frequency (beat frequency) signal to the VCO.
- This signal, V_{IN} , causes the VCO output frequency f_O to vary and move towards f_{IN} .
- When $f_{IN} = f_O$, $V_{IN} (f_{IN} - f_O)$ is approximately constant (DC) and f_O is held constant, i.e locked to f_{IN} .
- As f_{IN} changes, due to deviation in FM, f_O tracks or follows f_{IN} . $V_{OUT} = V_{IN}$ changes to drive f_O to track f_{IN} .
- V_{OUT} is therefore proportional to the deviation and contains the message signal $m(t)$.

QUADRATURE DETECTION

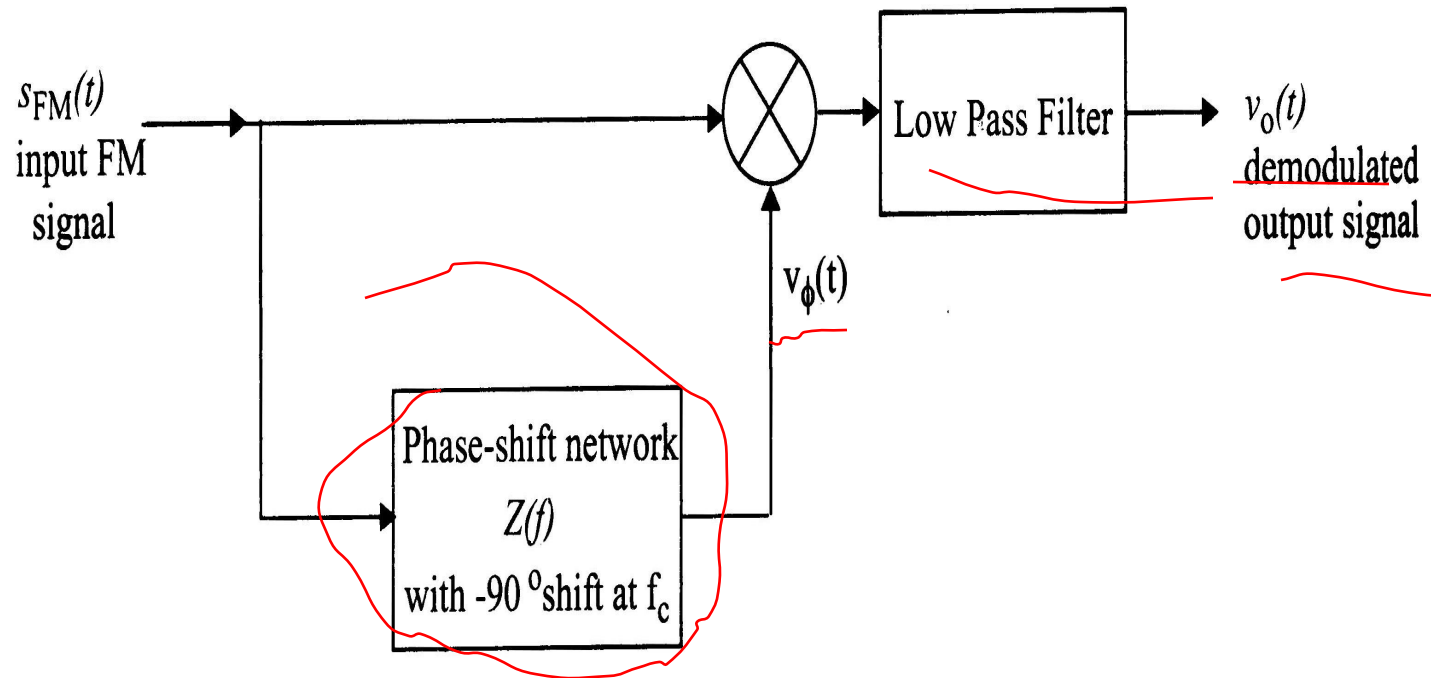


Figure
Block diagram of a quadrature detector.



QUADRATURE DETECTION

- The detector consists of a network which shifts the phase of the incoming FM signal by an amount proportional to its instantaneous frequency and uses a product detector (phase detector) to detect the phase difference between the original FM signal and the signal at the output of the phase-shift network
- As the phase shift introduced by the phase-shift network is proportional to the instantaneous frequency of the FM signal, the output voltage of the phase detector will also be proportional to the instantaneous frequency of the input FM signal
- In this manner frequency to amplitude conversion is achieved and the FM signal is demodulated



FREQUENCY MODULATION VS. AMPLITUDE MODULATION

FM signals have all their information in phase or frequency of the carrier

AM signals have all their information in the amplitude of the carrier

Frequency modulation has better noise immunity when compared to amplitude modulation

AM signals are able to occupy less bandwidth as compared to FM signals



REFERENCES

Wireless communications: by Theodore S. Rappaport
Internet