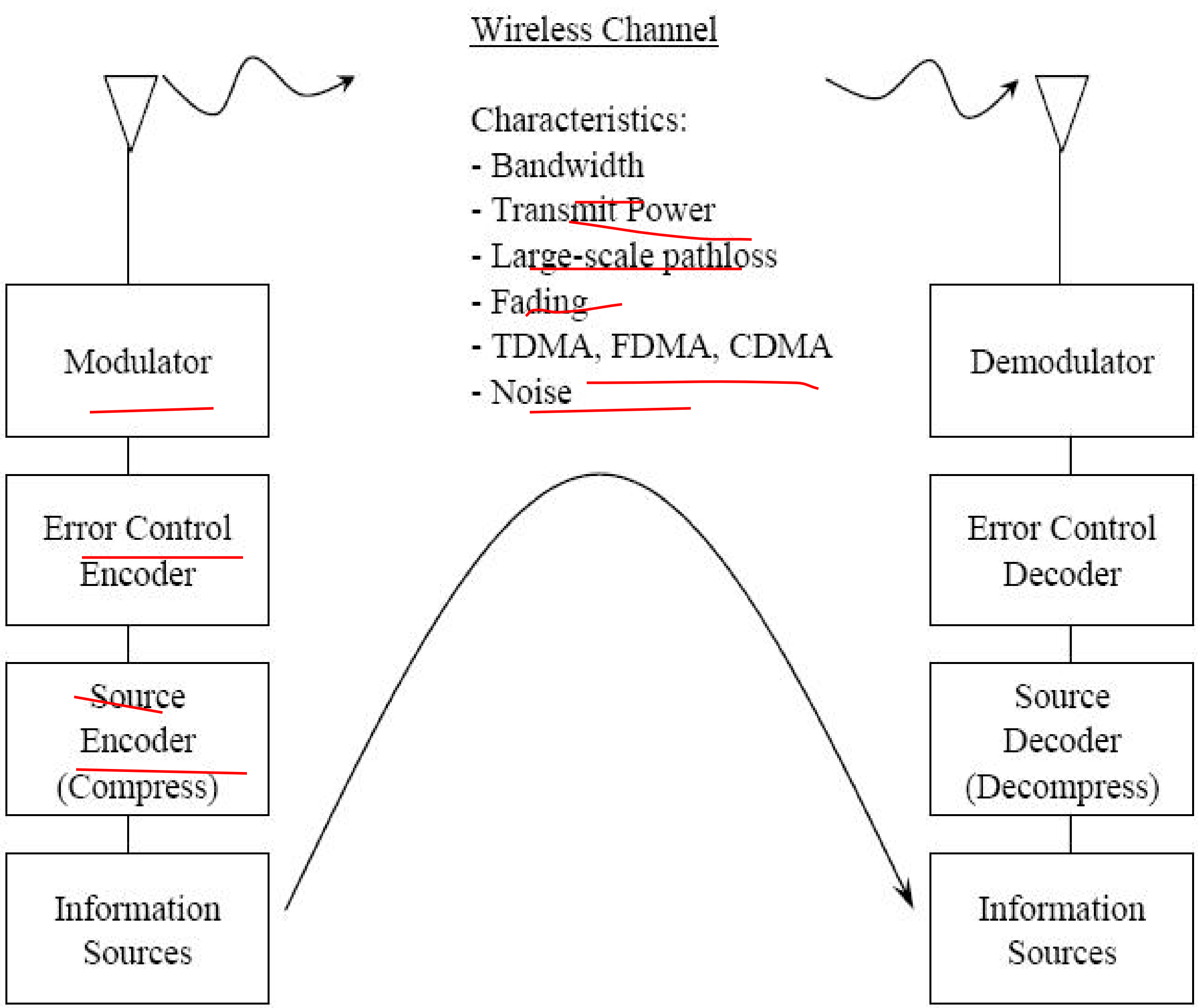
Modulation Techniques for Mobile Radio

Here is a picture of the overall wireless transmission and receiving system:



- Last few weeks:
- Properties of cellular radio systems
  - Reuse by using cells
  - Clustering and system capacity
  - Handoff strategies
  - Co-Channel Interference
  - Adjacent Channel Interference
  - Trunking and grade of service (GOS)
  - Cell splitting
  - Sectoring

- Electromagnetic propagation properties and hindrances
  - Free space path loss
  - Large-scale path loss - Reflections, diffraction, scattering
  - Multipath propagation
  - Doppler shift
  - Flat vs. Frequency selective fading
  - Slow vs. Fast fading

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- Now what are we studying?

  - We are looking at modulation and demodulation.

Introduction

- Modulation: Encoding information in a baseband signal and then translating (shifting) signal to much higher frequency prior to transmission
  - Message signal is detected by observing baseband to the amplitude, frequency, or phase of the signal.
  - Our focus is modulation for mobile radio.
  - The primary goal is to transport information through the MRC with the **best quality** (low BER), **lowest power** & **least amount of frequency spectrum**
    - Must make tradeoffs between these objectives.

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- Must overcome difficult impairments introduced by MRC:
  - Fading/multipath
  - Doppler Spread
  - ACI & CCI
- Challenging problem of ongoing work that will likely be ongoing for a long time.
  - Since every improvement in modulation methods increases the efficiency in the usage of highly scarce spectrum.

I. Analog Amplitude and Frequency Modulation

A. Amplitude Modulation

$$m(t) \quad \rightarrow \quad \text{information signal}$$

$$A_c \cos[2\pi f_c t] \quad \rightarrow \quad \text{carrier}$$

$$f_c \quad \rightarrow \quad \text{carrier frequency}$$

$$S_{AM}(t) = A_c \left[1 + m(t)\right] \cos[2\pi f_c t] \quad \rightarrow \quad \text{transmitted signal}$$

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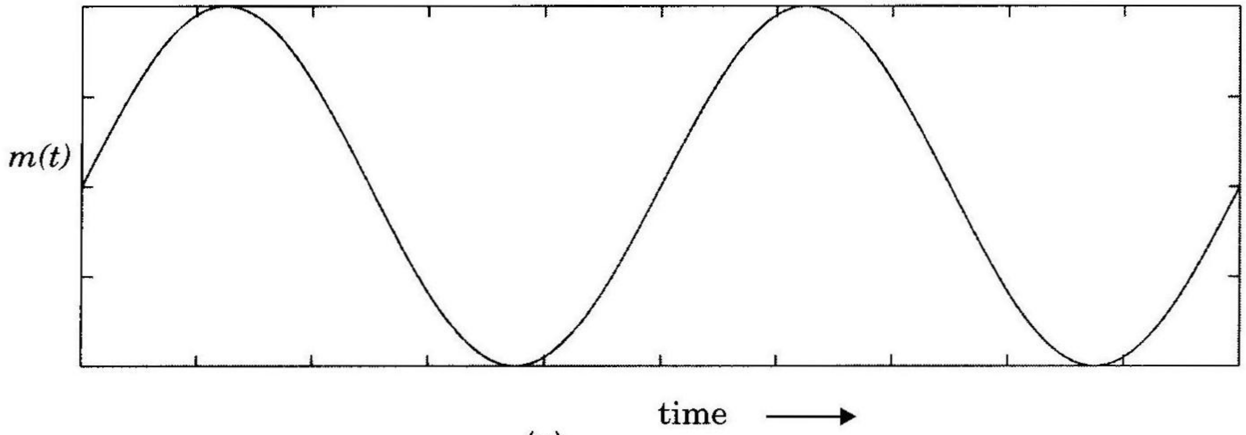


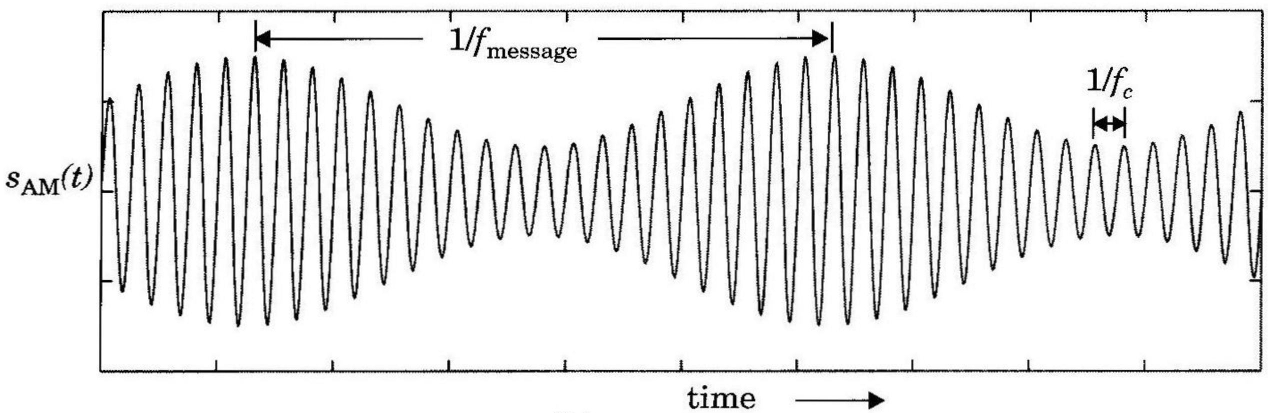
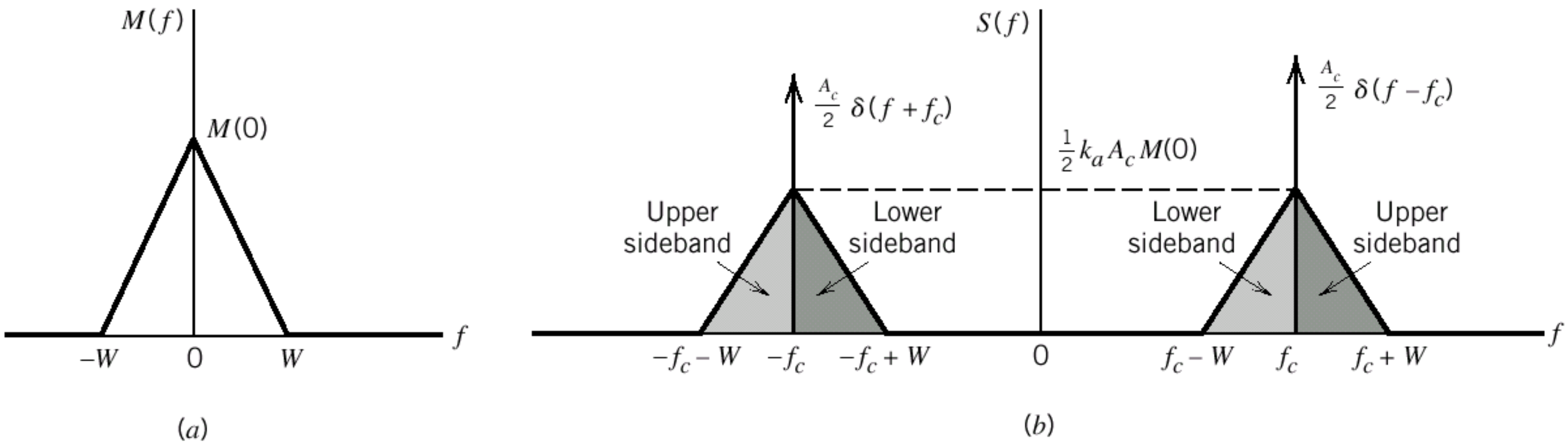


Figure 6.1 (a) A sinusoidal modulating signal and (b) the corresponding AM signal with modulation index 0.5.

Spectrum of AM wave

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{k_a A_c}{2}[M(f - f_c) + M(f + f_c)]$$



Spectrum of baseband signal. Spectrum of AM wave.

B. Frequency Modulation

- Most widely used form of *Angle* modulation for mobile radio applications
  - AMPS
  - Police/Fire/Ambulance Radios
- Generally one form of "angle modulation"
  - Creates changes in the time varying phase (angle) of the signal.
- Many unique characteristics

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- Unlike AM, the amplitude of the FM carrier is kept constant (constant envelope) & the *carrier* frequency is varied proportional to the modulating signal $m(t)$:

$$S_{FM}(t) = A_c \cos[\theta(t)]$$

$$\omega(t) = \text{instatantaneous angular frequency}$$

$$= \frac{d\theta}{dt} = 2\pi f_c + 2\pi k_f \, m(t) \quad \leftarrow \quad \text{desired}$$

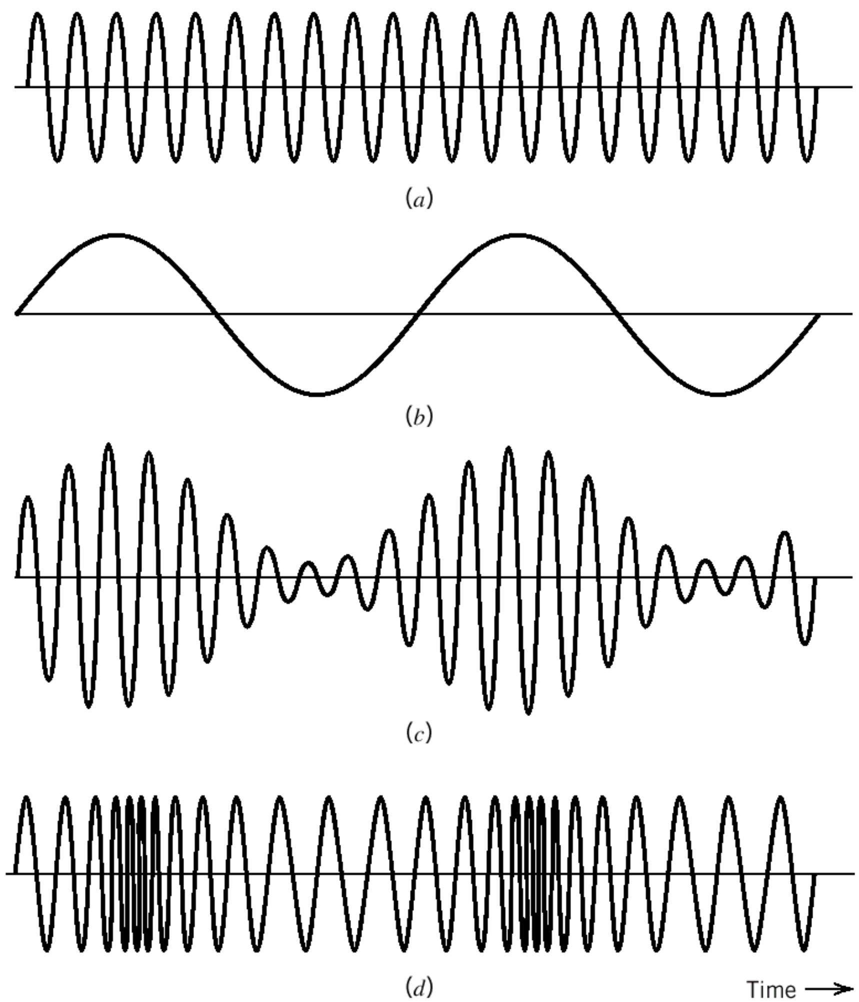
  - f_c plus a deviation of $k_f m(t)$
  - k_f : frequency deviation constant (in Hz/V) - defines amount magnitude of allowable frequency change

(a) Carrier wave.

(b) Sinusoidal modulating signal.

(c) Amplitude-modulated signal.

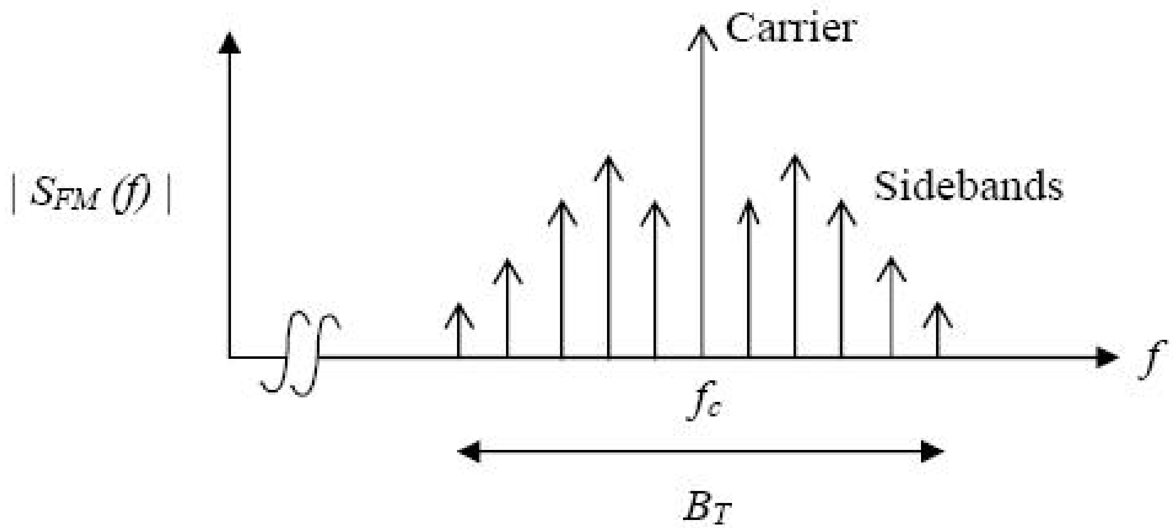
(d) Frequency-modulated signal.



- So

$$s_{FM}(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\eta)\, d\eta\right)$$

- FM signal spectrum → carrier + Message signal frequency # of sidebands



FM Bandwidth and Carson's Rule

- Frequency Deviation: $\Delta f = k_f \max|m(t)|$
  - Maximum deviation of f_i from f_c: $f_i = f_c + k_f m(t)$

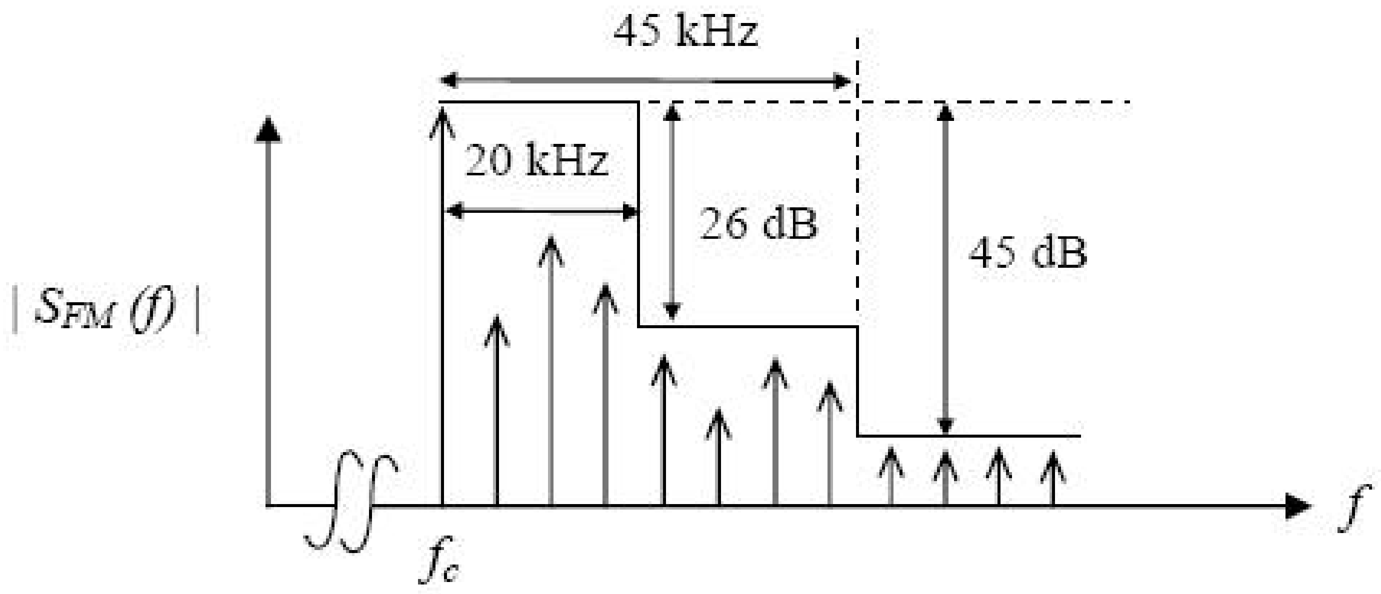
- Carson's Rule:

$$\boxed{B \approx 2\Delta f + 2B_m}$$

  - B depends on maximum deviation from f_c and how fast f_i changes

- Narrowband FM: $\Delta f << B_m \Rightarrow B \approx 2B_m$
- Wideband FM: $\Delta f >> B_m \Rightarrow B \approx 2\Delta f$

- Example: AMPS
  - poor spectral efficiency
  - allocated channel BW = 30 kHz
  - actual standard uses threshold specifications:



- *SNR* vs. BW tradeoff
  - in FM one can increase RF BW to improve *SNR*:

SNR_{out} = SNR after FM detection

$$\approx \Delta f^{3} SNR_{in:\ FM}$$

Δf : peak frequency deviation of Tx the frequency domain

- rapid non-linear, Δf^{3} improvement in output signal quality (SNR_{out}) for increases in Δf
  - "capture effect": FM Rx *rejects* the weaker of the two FM signals (one with smaller SNR_{in}) in the same RF BW → ∴ resistant to CCI
  - Increased Δf requires increasing the bandwidth and spectral occupancy of the signal
  - must exceed the threshold of the FM detector, which means that typically $SNR_{in} \geq 10$ dB (called the capture threshold)

II. Digital Modulation

- Better performance and more cost effective than analog modulation methods (AM, FM, etc.)
- Used in modern cellular systems
- Advancements in VLSI, DSP, etc. have made digital solutions practical and affordable

Performance advantages:

1) Resistant to noise, fading, & interference
2) Can combine multiple information types (voice, data, & video) in a **single** transmission channel
3) Improved security (e.g., encryption) → deters phone cloning + eavesdropping
4) Error coding is used to detect/correct transmission errors
5) Signal conditioning can be used to combat hostile MRC environment
6) Can implement mod/dem functions using DSP software (instead of hardware circuits).

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- Choice of digital modulation scheme
  - Many types of digital modulation methods → subtle differences
  - Performance factors to consider

    1) low Bit Error Rate (BER) at low S/N

    2) resistance to interference (ACI & CCI) & multipath fading

    3) occupying a minimum amount of BW

    4) easy and cheap to implement in mobile unit

    5) efficient use of battery power in mobile unit

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- No existing modulation scheme simultaneously satisfies all of these requirements well.
- Each one is better in some areas with tradeoffs of being worse in others.

- **Power Efficiency** $\rightarrow \eta_p$: ability of a modulation technique to preserve the quality of digital messages at *low power levels* (low *SNR*)
  - Specified as E_b / N_o @ some BER (e.g. 10^{-5}) where E_b : energy/bit and N_o : noise power/bit
  - Tradeoff between *fidelity* and *signal power* $\rightarrow$ BER $\uparrow$ as $E_b / N_o \downarrow$

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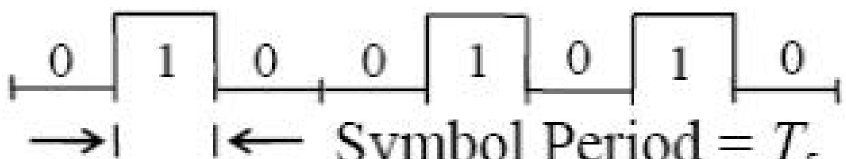
- **Bandwidth Efficiency** $\rightarrow \eta_B$: ability of a modulation technique to accommodate data in a *limited BW*

  - $\eta_B = \dfrac{R}{B}$ bps/Hz $\quad$ R : data rate $\quad$ B: RF BW

  - Tradeoff between data rate and occupied BW
    $\rightarrow$ as $R\uparrow$, then BW $\uparrow$

  - For a digital signal :



$$\text{Symbol Period} = T_s$$

$$\text{Signal BW} = B_s \approx 1/T_s$$

- $R \propto \dfrac{1}{T_s} \propto B \quad \rightarrow \quad$ so as $\quad R\uparrow, T_s\downarrow \quad$ and $\quad B\uparrow$

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- each pulse or "symbol" having *m* finite states represents $n = \log_2 m$ bits/symbol →
  - e.g. m = 0 or 1 (2 states) → 1 bit/symbol (binary)
  - e.g. m = 0, 1, 2, 3, 4, 5, 6, or 7 (8 states) → 3 bits/symbol

- Implementation example: A system is changed from binary to 2-ary.
  - Before: "0" = - 1 Volt, "1" = 1 Volt
  - Now

"0" = - 1 Volt, "1" = - 0.33 volts, "2" = 0.33 Volts, "3" = 1 Volt

  - What would be the new data rate compared to the old data rate if the symbol period where kept constant?

- In general, called M-ary keying

- **Maximum BW efficiency → Shannon's Theorem**
  - Most famous result in communication theory.
  - $$\eta_{B_{\max}} = \frac{C}{B} = \log_2\left(1+\frac{S}{N}\right)$$ where
    - B : RF BW
    - C : channel capacity (bps) of real data (not retransmissions or errors)
    - To produce error-free transmission, some of the bit rate will be taken up using retransmissions or extra bits for error control purposes.
    - As noise power N increases, the bit rate would still be the same, but max $\eta_{B_{\max}}$ decreases.

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- So $$C_{max} = B \log_2\left(1 + \frac{S}{N}\right)$$
- note that $C \propto B$ (expected) but also $C \propto S/N$
  - an increase in signal power translates to an increase in channel capacity
  - lower bit error rates from higher power → more real data
  - large S/N → easier to differentiate between multiple signal states (m) in one symbol ∴ $n \uparrow$
- max $\eta_{B_{max}}$ is fundamental limit that **cannot** be achieved in practice

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- People try to find schemes that correct for errors.
- People are starting to refer to certain types of codes as "capacity approaching codes", since they say they are getting close to obtaining C_{max}.
  - More on this in the chapter on error control.

Example 6.6

If the SNR of a wireless communication link is 20 dB and the RF bandwidth is 30 kHz, determine the maximum theoretical data rate that can be transmitted.

Solution

Given:

$S/N = 20 \text{ dB} = 100$

RF Bandwidth $B = 30000$ Hz

Using Shannon's channel capacity formula (6.37), the maximum possible data rate

$$C = B\log_2\left(1 + \frac{S}{N}\right) = 30000\log_2(1 + 100) = 199.75 \text{ kbps}$$

Example 6.7

What is the theoretical maximum data rate that can be supported in a 200 kHz channel for *SNR* = 10 dB, 30 dB. How does this compare to the GSM standard described in Chapter 1?

Solution

For *SNR* = 10 dB = 10, *B* = 200 kHz.

Using Shannon's channel capacity formula (6.37), the maximum possible data rate

$$C = B\log_2\left(1 + \frac{S}{N}\right) = 200000\log_2(1 + 10) = 691.886 \text{ kbps}$$

The GSM data rate is 270.833 kbps, which is only about 40% of the theoretical limit for 10 dB SNR conditions.

For *SNR* = 30 dB = 1000, *B* = 200 kHz.

The maximum possible data rate

$$C = B\log_2\left(1 + \frac{S}{N}\right) = 200000\log_2(1 + 1000) = 1.99 \text{ Mbps.}$$

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- Fundamental tradeoff between η_B and η_p (in general)
  - If η_B improves then η_p deteriorates (or vice versa)
    - May need to waste more power to get a better data rate.
    - May need to use less power (to save on battery life) at the expense of a lower data rate.
  - η_p vs. η_B is not the only consideration.
    - Use other factors to evaluate → complexity, resistance to MRC impairments, etc.

Bandwidth Specifications

- **Many** definitions depending on application → all use **Power Spectral Density (PSD)** of modulated bandpass signal

$$S_W(f) = \lim_{T\to\infty}\left(\frac{\overline{|W_T(f)|^2}}{T}\right)$$

- Many signals (like square pulses) have some power at all frequencies.

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Why Carrier?

- Effective radiation of EM waves requires antenna dimensions comparable with the wavelength:
  - Antenna for 3 kHz would be ~100 km long
  - Antenna for 3 GHz carrier is 10 cm long
- Sharing the access to the telecommunication channel resources

Modulation Process

$$f = f\left(a_1, a_2, a_3, \ldots a_n, t\right) \text{ (= carrier)}$$

$$a_1, a_2, a_3, \ldots a_n \text{ (= modulation parameters)}$$

$$t \text{ (= time)}$$

- Modulation implies varying one or more characteristics (modulation parameters a_1, a_2, … a_n) of a carrier *f* in accordance with the information-bearing (modulating) baseband signal.
- Sinusoidal waves, pulse train, square wave, etc. can be used as carriers

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Continuous Carrier

Carrier: $A\sin[\omega t+\varphi]$

- A = const
- ω = const
- φ = const

- Amplitude modulation (AM)
  - $A = A(t)$ – carries information
  - ω = const
  - φ = const

- Frequency modulation (FM)
  - A = const
  - $\omega = \omega(t)$ – carries information
  - φ = const

- Phase modulation (PM)
  - A = const
  - ω = const
  - $\varphi = \varphi(t)$ – carries information

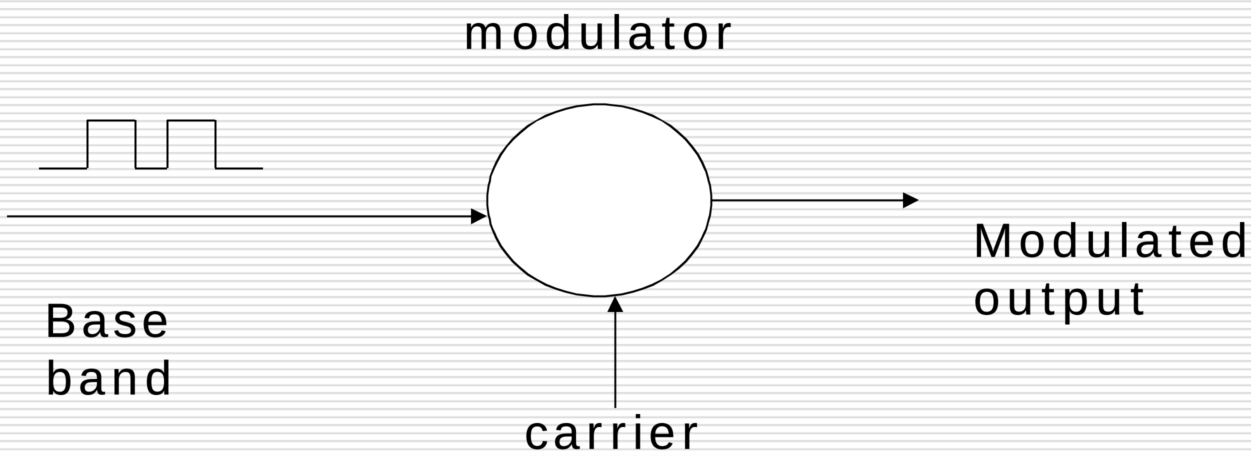
Basic Concepts

- The basic form of 3 different digital modulation methods used for transmitting digital signals methods are:
  - Amplitude Shift Keying
  - Frequency Shift Keying
  - Phase Shift Keying

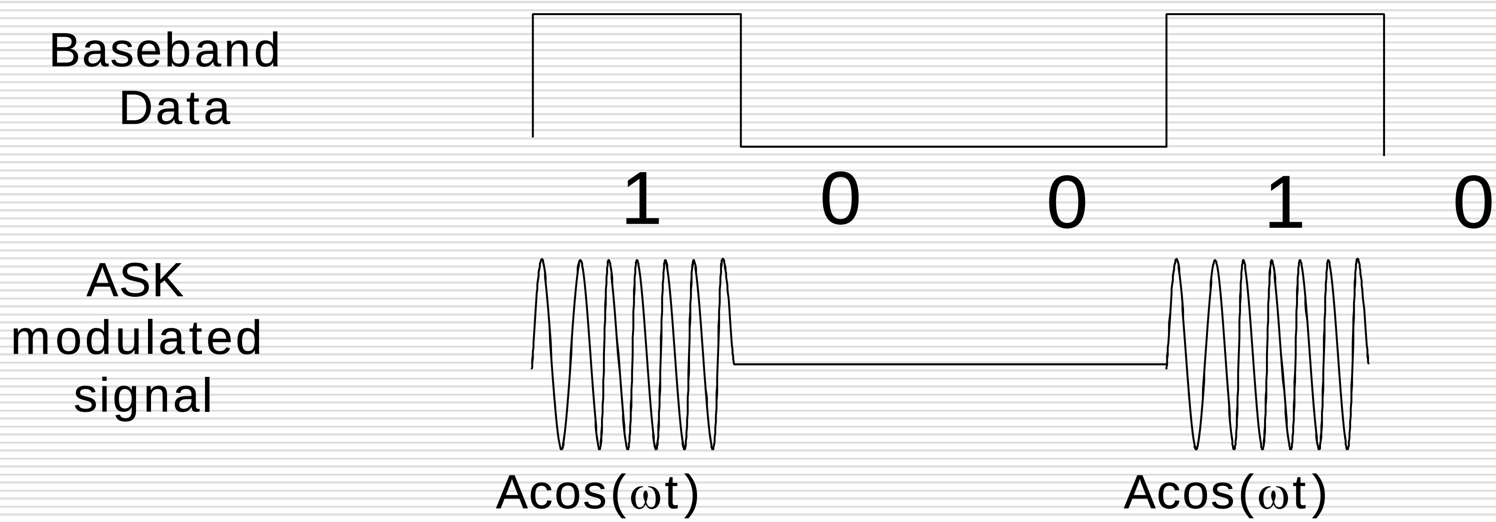
Digital Modulation methods

Amplitude Shift Keying (ASK)

- The amplitude of the carrier is varied in accordance with the amplitude of the modulating signal

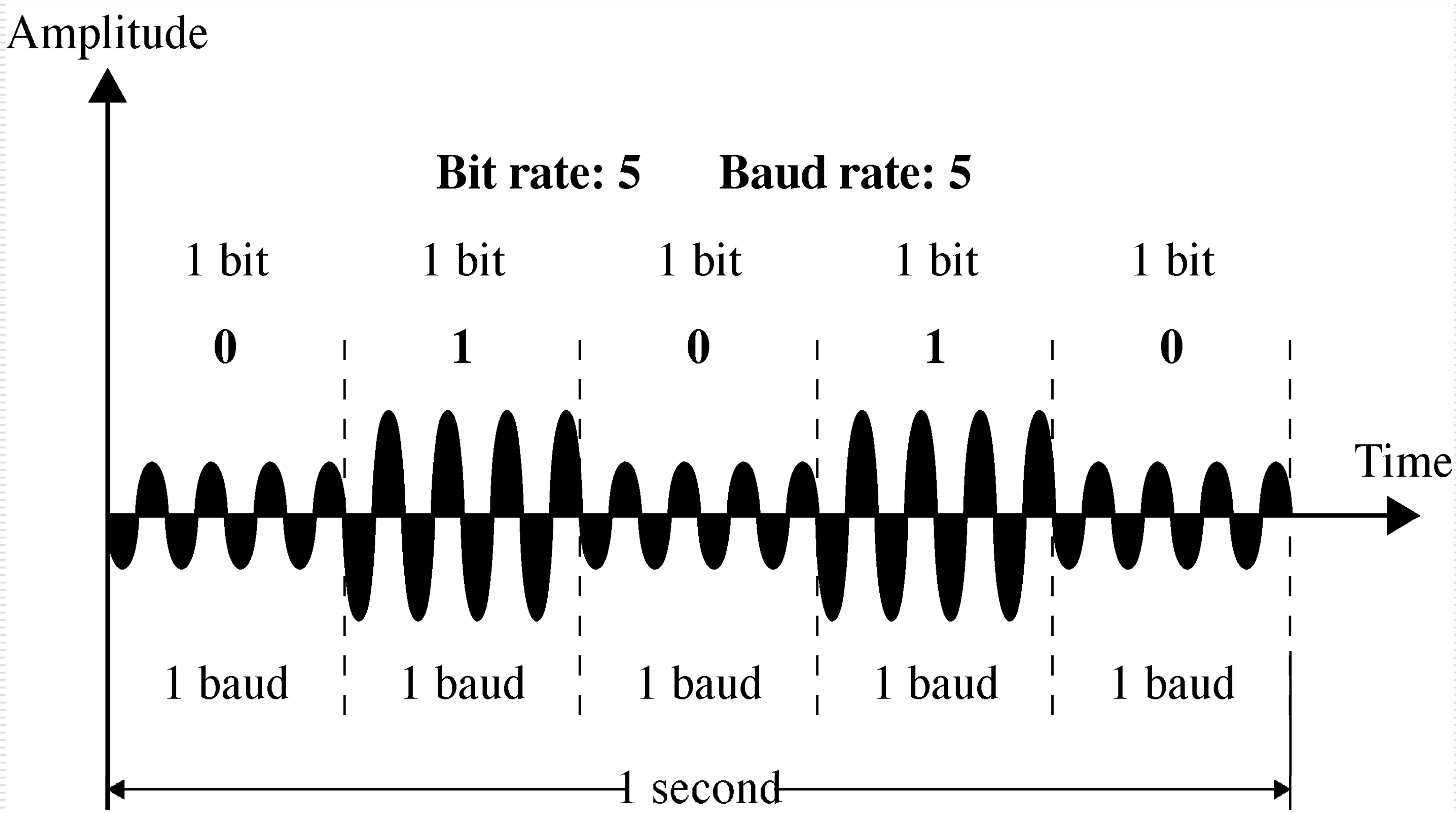


Amplitude Shift Keying (ASK)



- Pulse shaping can be employed to remove spectral spreading
- ASK demonstrates poor performance, as it is heavily affected by noise, fading, and interference

ASK

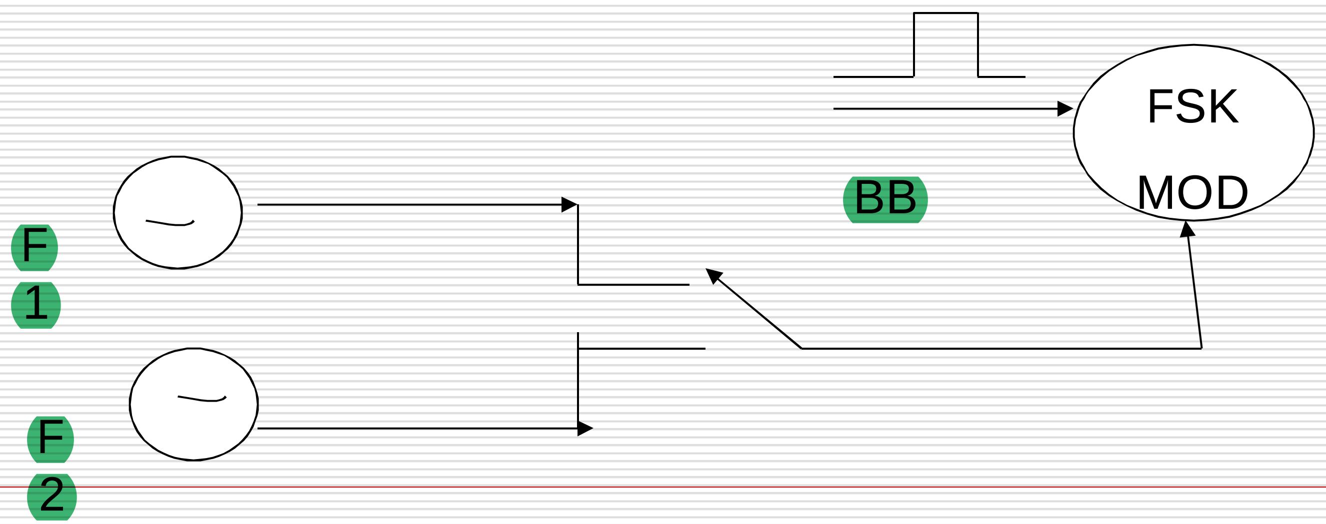


Amplitude
Bit rate: 5
Baud rate: 5
1 bit
1 bit
1 bit
1 bit
1 bit
0
1
0
1
0
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

Digital Modulation methods

Frequency Shift Keying

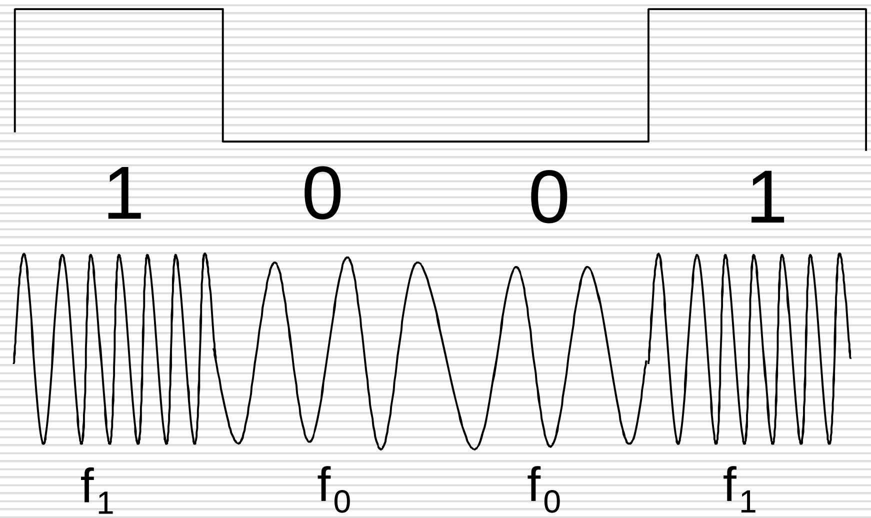
- Frequency of the carrier is varied in accordance with the amplitude of the modulating signal and the carrier amplitude remains constant.



Frequency Shift Keying (FSK)

Baseband Data

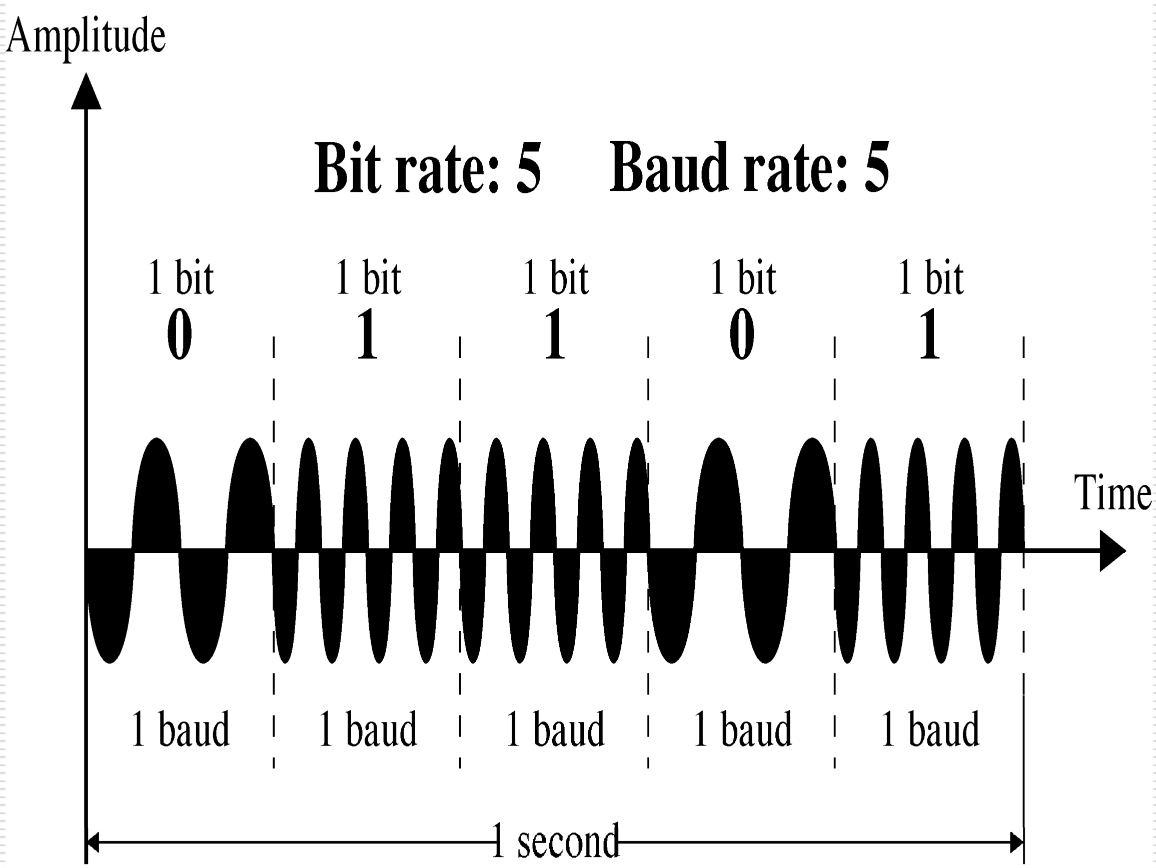
BFSK modulated signal



where $f_0 = A\cos(\omega_c - \Delta\omega)t$ and $f_1 = A\cos(\omega_c + \Delta\omega)t$

- Example: The ITU-T V.21 modem standard uses FSK
- FSK can be expanded to a M-ary scheme, employing multiple frequencies as different states

FSK



Amplitude
Bit rate: 5
Baud rate: 5
1 bit
0
1 bit
1
1 bit
1
1 bit
0
1 bit
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

Digital Modulation methods

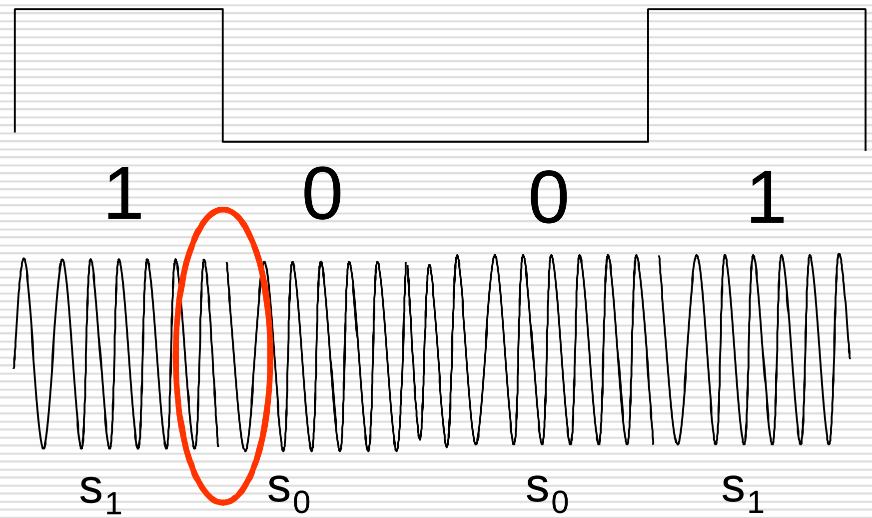
PHASE SHIFT KEYING

- The phase of the carrier is varied in accordance with the information.
- PSK is divided into two level and multilevel systems (M-ary schemes).

Phase Shift Keying (PSK)

Baseband Data

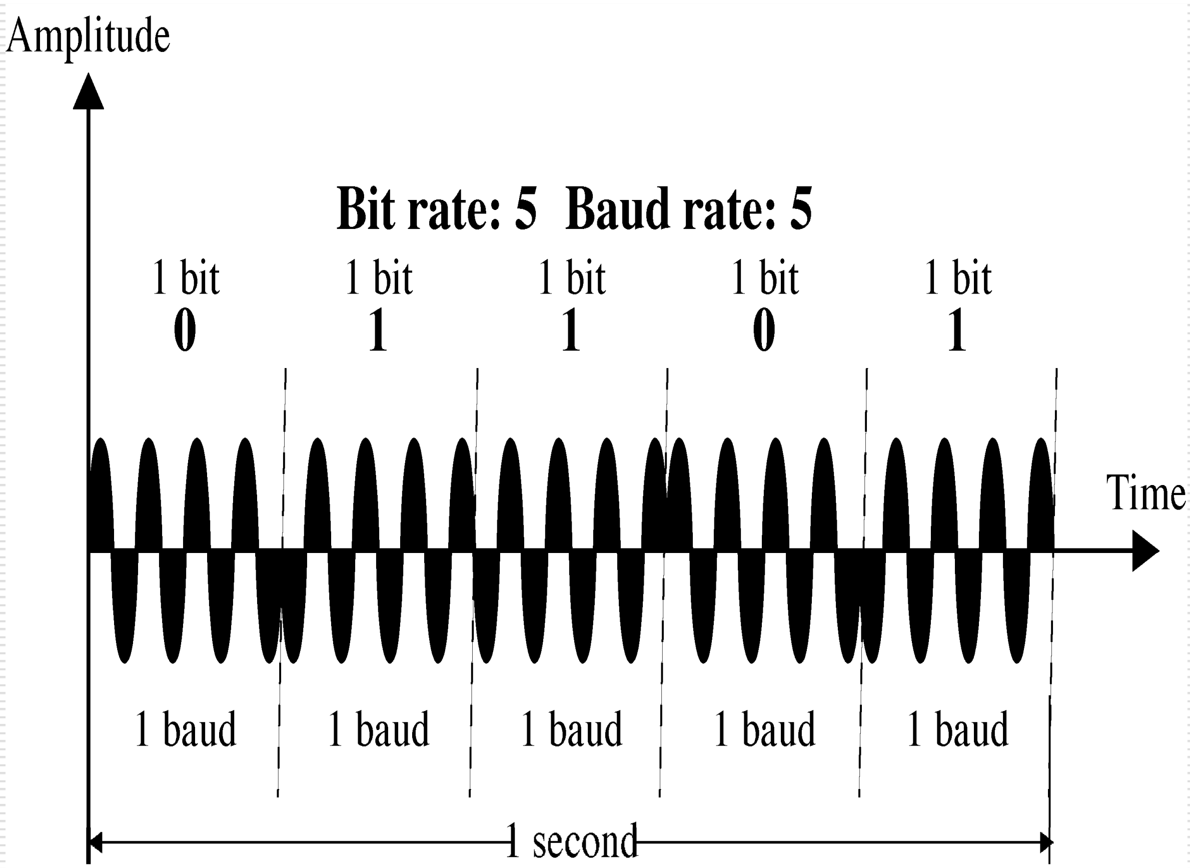
BPSK modulated signal



where $s_0 = -A\cos(\omega_c t)$ and $s_1 = A\cos(\omega_c t)$

- Major drawback – rapid amplitude change between symbols due to phase discontinuity, which requires infinite bandwidth. Binary Phase Shift Keying (BPSK) demonstrates better performance than ASK and BFSK
- BPSK can be expanded to a M-ary scheme, employing multiple phases and amplitudes as different states

PSK


Amplitude
Bit rate: 5 Baud rate: 5
1 bit
0
1 bit
1
1 bit
1
1 bit
0
1 bit
1
Time
1 baud
1 baud
1 baud
1 baud
1 baud
1 second

Differential Modulation

- In the transmitter, each symbol is modulated relative to the previous symbol and modulating signal, for instance in BPSK
  0 = no change, 1 = $+180^0$
- In the receiver, the current symbol is demodulated using the previous symbol as a reference.
- The previous symbol serves as an estimate of the channel. A no-change condition causes the modulated signal to remain at the same 0 or 1 state of the previous symbol.

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DPSK

- Differential modulation is theoretically 3dB poorer than coherent. This is because the differential system has 2 sources of error: a corrupted symbol, and a corrupted reference (the previous symbol)
- DPSK = Differential phase-shift keying:
  - In the transmitter, each symbol is modulated relative to
    - (a) the phase of the immediately preceding signal element and
    - (b) the data being transmitted.

PSK Generalities--Symbols Bits & Bauds

Bit – Refers to the unit of information.

Bit rate is the frequency of a system bit stream.

Symbol – Refers to the unit of transmission energy.

--Representation of bits that the medium transmits to convey information.

-- A symbol can contain one or more bits

Bits are transmitted in the form of Symbols.

$$\textit{Symbol rate} = \frac{\textit{bit rate}}{\textit{the number of bits transmitted with each symbol}}$$

Baud – Rate of change of symbols is known as Baud Rate.

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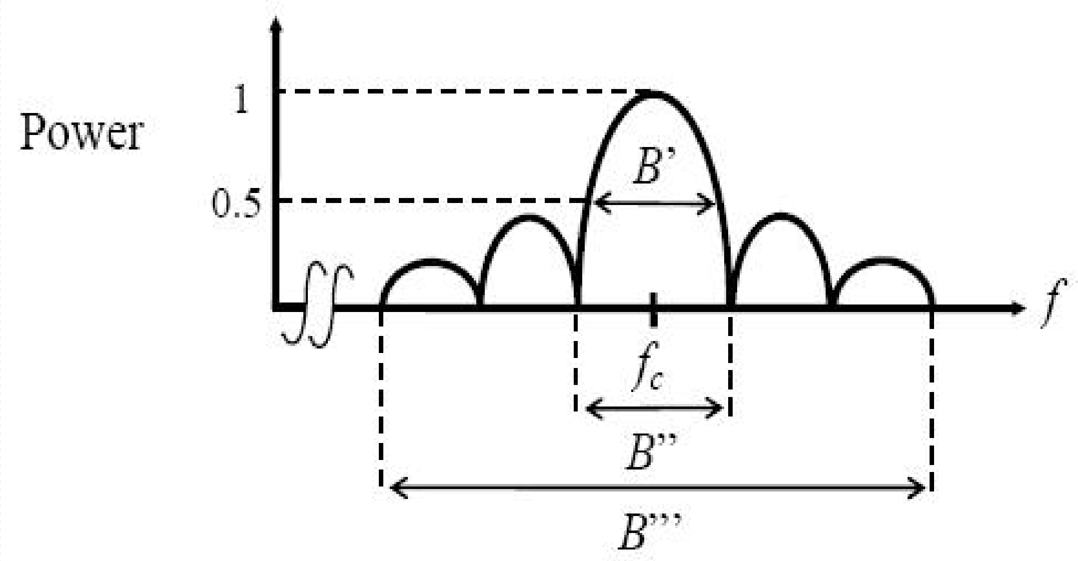

- *B'* : half-power (-3 dB) BW
- *B"* : null-to-null BW
- *B'"* : absolute BW
  → range where PSD > 0

- FCC definition of occupied BW → BW contains 99% of signal power

III. Geometric Representation of Modulation Signal

- Geometric Representation of Modulation Signals - Constellation Diagrams
  - Graphical representation of complex (A & θ) digital modulation types
    - Provide insight into modulation performance
  - Modulation set, S, with M possible signals

$$S = \{ S_1(t),\ S_2(t),\ \cdots\ S_M(t) \}$$

    - Binary modulation → $M = 2$ → each signal = 1 bit of information
    - M-ary modulation → $M > 2$ → each signal > 1 bit of information

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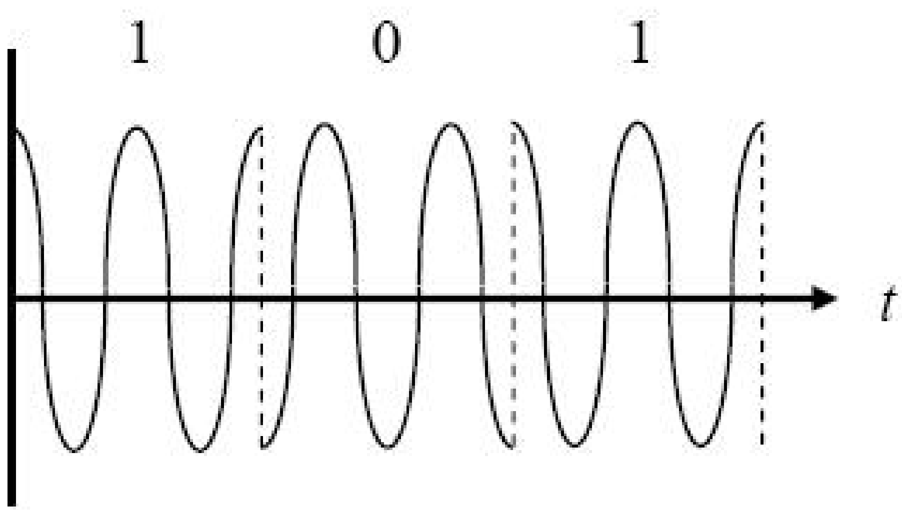
Example: Binary Phase Shift Keying (BPSK)

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

$$S_2(t) = -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$$

for $0 \leq t \leq T_b$

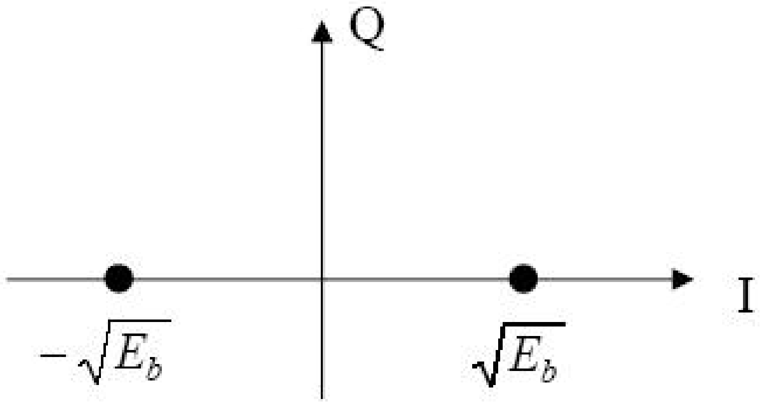


- Phase change between bits → Phase shifts of 180° for each bit.
- Note that this can also be viewed as AM with +/- amplitude changes
- Let $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \rightarrow$ basis signal

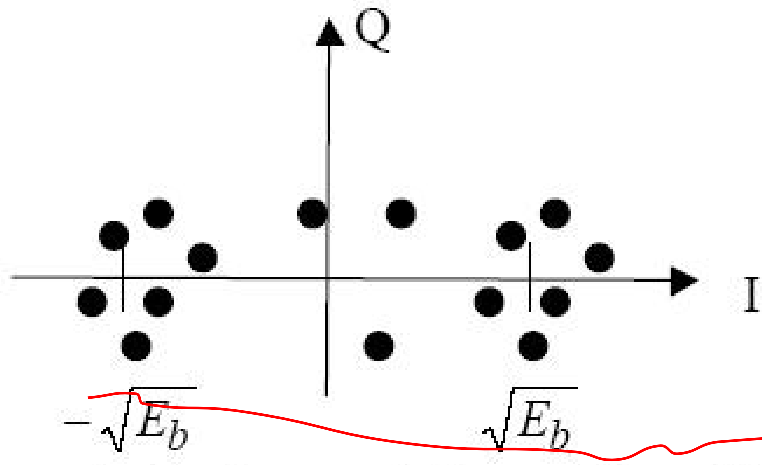
$$S_{BPSK} = \left\{\sqrt{E_b}\,\phi_1(t), -\sqrt{E_b}\,\phi_1(t)\right\}$$

- Dimension of the vector space is the # of basis signals required to represent *S*.

- Plot amplitude & phase of S in vector space :



two states with $A = \sqrt{E_b}$ that are 180° out of phase



Actual received values as influenced by additive noise

Constellation diagram properties :

1) Distance between signals is related to differences in modulation waveforms

- Large distance → "sparse" → easy to discriminate → good BER @ low $SNR\ (E_b / N_o)$
- From above, noise of $-2\sqrt{E_b}$ added to $\sqrt{E_b}$ would make the received signal look like $s_2(t)$ → error.
- From $\sqrt{E_b}$, noise of $> -\sqrt{E_b}$ would make the result closer to $-\sqrt{E_b}$ and would make the decoder choose $s_2(t)$ → error.

∴ Above example is **Power Efficient** (related to density with respect to # states/N)

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2) Occupied BW ↓ as # signal states ↑

- If we can represent more bits per symbol, then we need less BW for a given data rate.
- Small separation → "dense" → more signal states/symbol → more information/Hz !!

∴ **Bandwidth Efficient**

IV. Linear Modulation Methods

- In linear modulation techniques, the amplitude of the transmitted signal varies linearly with the modulating digital signal.
- Performance is evaluated with respect to E_b / N_o

$$E_b = \text{signal energy per bit}$$

$$= \int_{\substack{\text{over one}\\ \text{bit time}}} power\ dt = \int_0^{T_b} s^2(t)\, dt$$

where $s(t)$ is the transmitted signal (assuming voltage across a 1Ω resistor)

If $s(t) = A_c$ for $0 \le t \le T_b$, then $E_b = A_c{}^2 T_b$.

If $s(t) = A_c \cos(2\pi \frac{1}{T_b} t)$ for $0 \le t \le T_b$, then $E_b = \frac{1}{2} A_c{}^2 T_b$ so $A_c = \sqrt{\frac{2E_b}{T_b}}$.

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BPSK

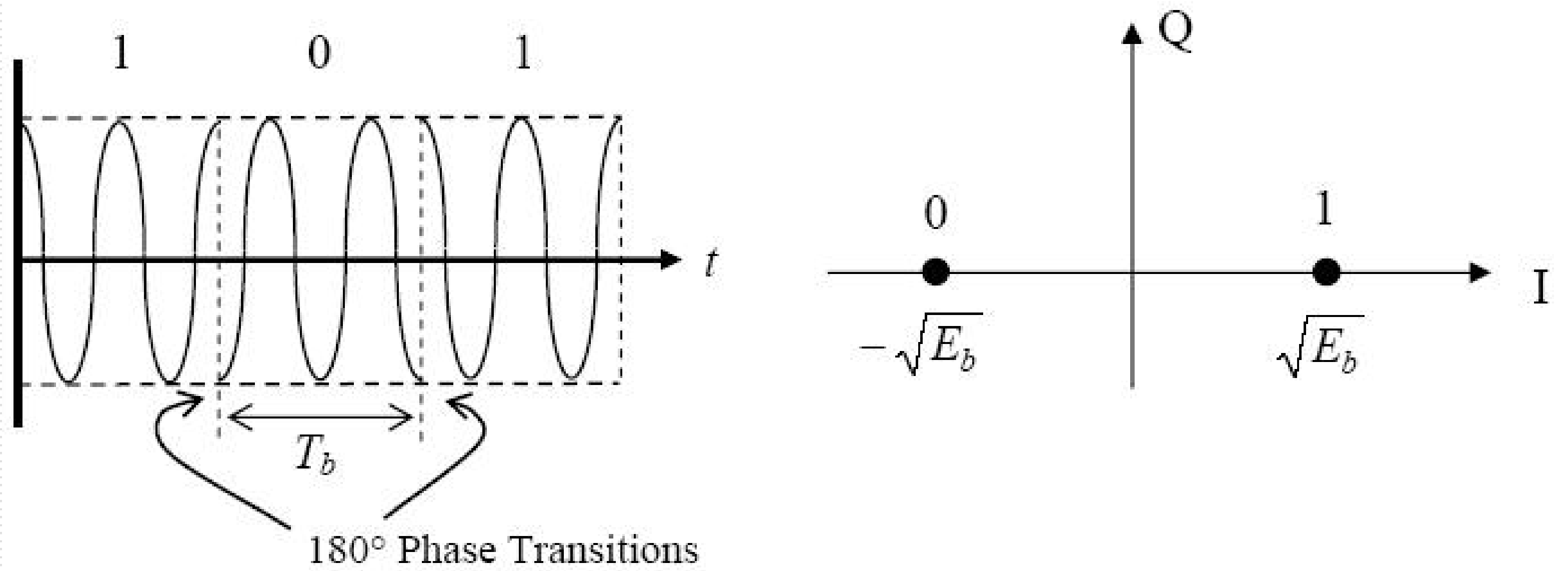
- BPSK → Binary Phase Shift Keying

$$S_{BPSK} = \pm A_c \cos(2\pi f_c t + \theta_0) = \pm\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_0)$$

$$or \quad = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \theta_0 + i\pi),\ i = \{0,1\}$$

$$S_{BPSK} = \pm\sqrt{E_b}\,\phi_1(t) \quad \text{where } \phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t + \theta_0)$$

"+" = "1" and "–" = "0" $\rightarrow \phi_1(t) =$ basis function



- Phase transitions force carrier **amplitude** to change from "+" to "–".
  - Amplitude varies in time

BPSK RF signal BW

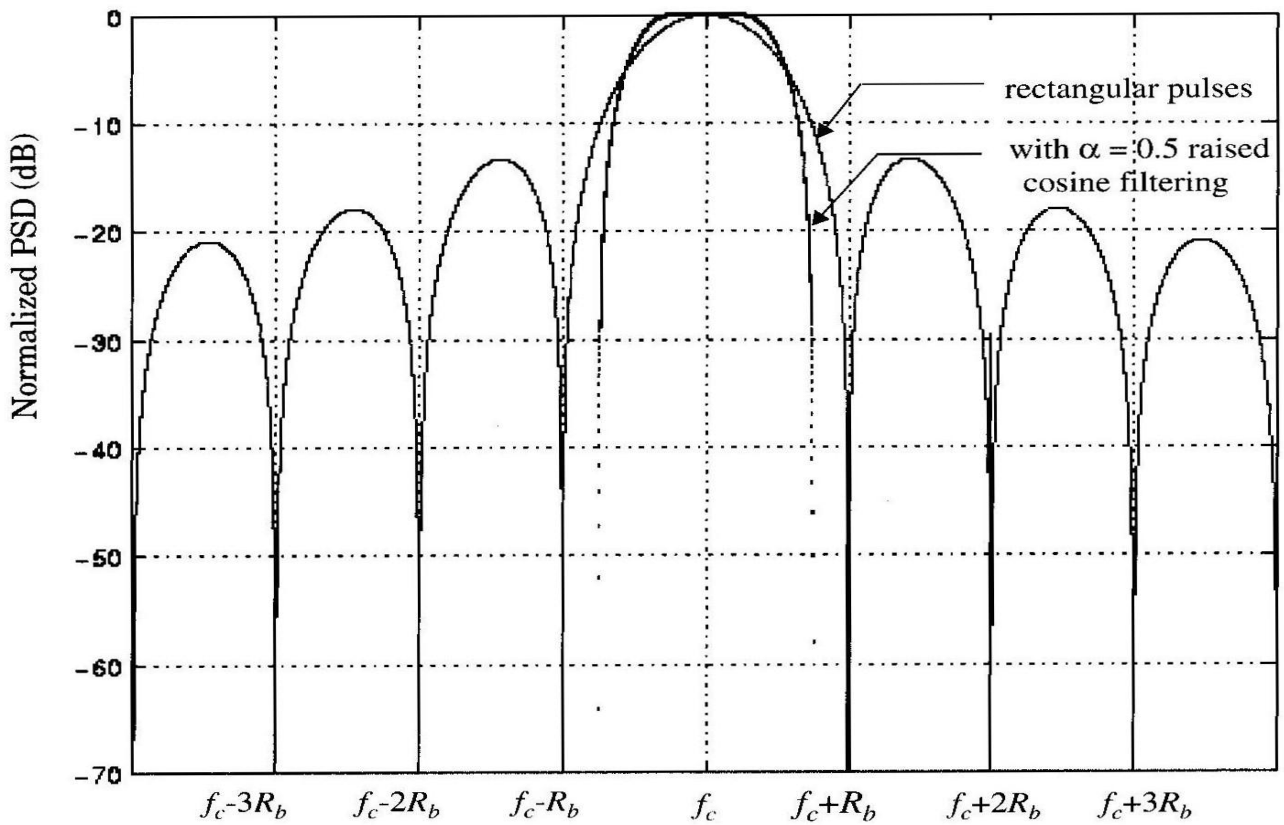


Figure 6.22 Power spectral density (PSD) of a BPSK signal.

- Null-to-null RF BW = 2 R_b = 2 / T_b
- 90% BW = 1.6 R_b for rectangular pulses

- Probability of Bit Error is proportional to the distance between the closest points in the constellation.
  - A simple upper bound can be found using the assumption that noise is additive, white, and Gaussian.

$$\mathrm{Prob}\{\text{bit error}\} \le Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

  - d is distance between nearest constellation points.

- *Q(x)* is the Q-function, the area under a normalized Gaussian function (also called a Normal curve or a bell curve)

$$Q(z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

- Appendix F, Fig. F.1
- Fig. F.2, plot of Q-function
- Tabulated values in Table F.1.

- Here

$$d = 2\sqrt{E_b} \quad \text{so} \quad \text{Prob}\{\text{bit error}\} \le Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

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- Demodulation in Rx
  - Requires reference of Tx signal in order to properly determine phase
    - carrier must be transmitted along with signal
  - Called Synchronous or "**Coherent**" detection
    - complex & costly Rx circuitry
    - good BER performance for low *SNR* → **power efficient**

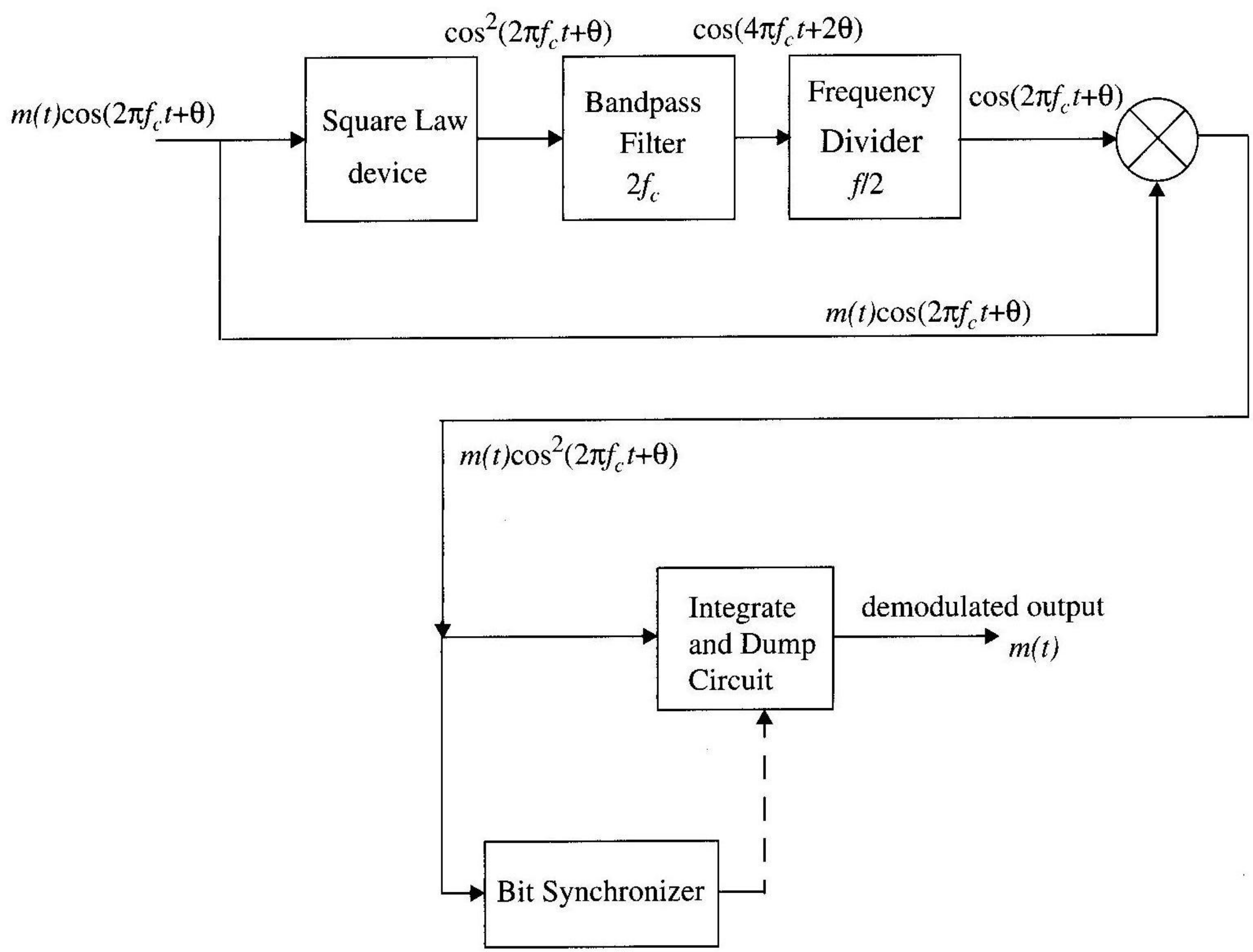


Figure 6.23 BPSK receiver with carrier recovery circuits.

DPSK

- DPSK → Differential Phase Shift Keying
  - Non-coherent Rx can be used
    - easy & cheap to build
    - no need for coherent reference signal from Tx
  - Bit information determined by **transition** between two phase states
    - incoming bit = 1 → signal phase stays the same as previous bit
    - incoming bit = 0 → phase switches state

- If $\{m_k\}$ is the message, the output $\{d_k\}$ is as shown below.

Table 6.1 Illustration of the Differential Encoding Process

$\{m_k\}$		1	0	0	1	0	1	1	0
$\{d_{k-1}\}$		1	1	0	1	1	0	0	0
$\{d_k\}$	1	1	0	1	1	0	0	0	1

- can also be described in modulo-2 arithmetic $d_k = \overline{m_k \oplus d_{k-1}}$

- Same BW properties as BPSK, uses same amount of spectrum
- Non-coherent detection → all that is needed is to compare phases between successive bits, not in reference to a Tx phase.
- power efficiency is 3 dB **worse** than coherent BPSK (higher power in E_b / N_o is required for the same BER)

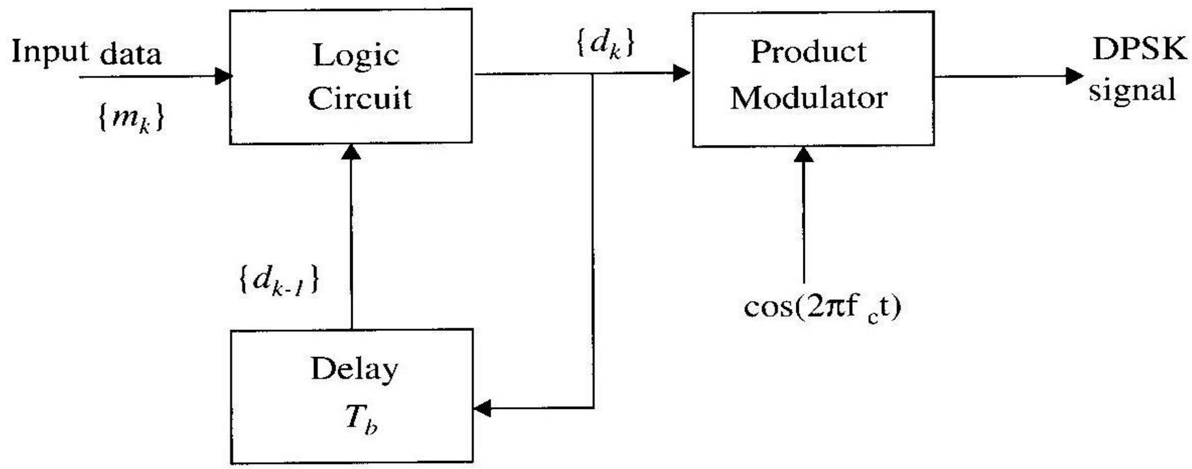


Figure 6.24 Block diagram of a DPSK transmitter.

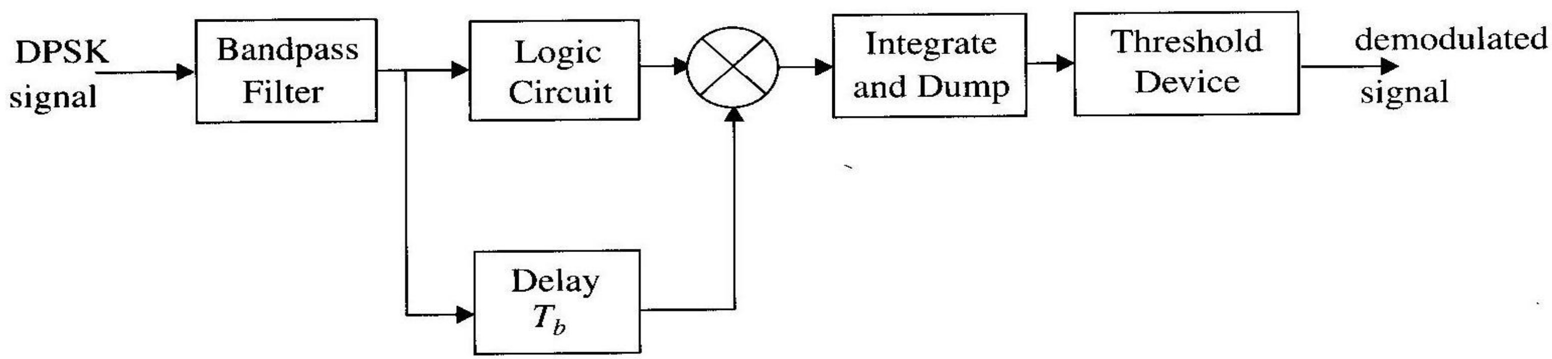


Figure 6.25 Block diagram of DPSK receiver.

QPSK

- QPSK → Quadrature Phase Shift Keying
  - **Four** different phase states in **one** symbol period
  - **Two** bits of information in each symbol

Phase:	0	$\pi/2$	π	$3\pi/2$	→ possible phase values
Symbol:	00	01	11	10	

- Note that we choose binary representations so an error between two adjacent points in the constellation only results in a single bit error
  - For example, decoding a phase to be π instead of $\pi/2$ will result in a "11" when it should have been "01", only one bit in error.

$$S_{QPSK} = \sqrt{\frac{2E_S}{T_s}} \cos\left(2\pi f_c t + (i-1)\frac{\pi}{2}\right) \quad \text{for } i = 1,2,3,4$$

- **Constant** amplitude with four different phases
- remembering the trig. identity

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$S_{QPSK} = \left\{ \sqrt{\frac{2E_s}{T_S}} \cos(2\pi f_c t) \cos[(i-1)\frac{\pi}{2}] - \sqrt{\frac{2E_s}{T_S}} \sin(2\pi f_c t) \sin[(i-1)\frac{\pi}{2}] \right\}$$

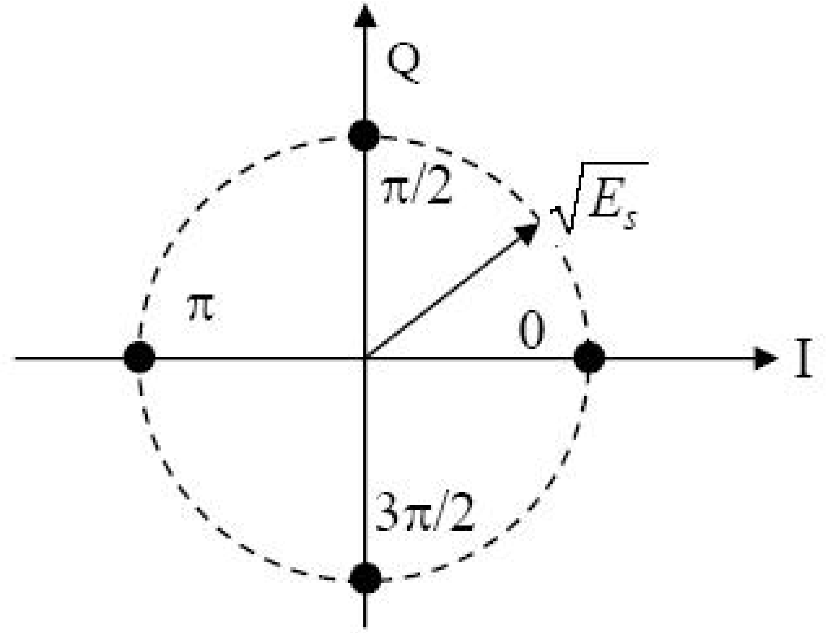
Then we can make the following definition:

$$S_{QPSK} = \left\{ \sqrt{E_s} \cos[(i-1)\frac{\pi}{2}]\phi_I(t) - \sqrt{E_s} \sin[(i-1)]\frac{\pi}{2}\phi_Q(t) \right\} \text{ for } i = 1, 2, 3, 4$$

where $\phi_I(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t)$ and $\phi_Q(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t)$

E_s = signal energy per symbol

$$= \int_{\substack{\text{over one}\\ \text{symbol time}}} power\ dt = \int_0^{T_s} s^2(t)\, dt$$



- Now we have two basis functions
- $E_s = 2\,E_b$ since 2 bits are transmitted per symbol
- I = in-phase component from $s_I(t)$.
- Q = quadrature component that is $s_Q(t)$.

QPSK RF Signal BW

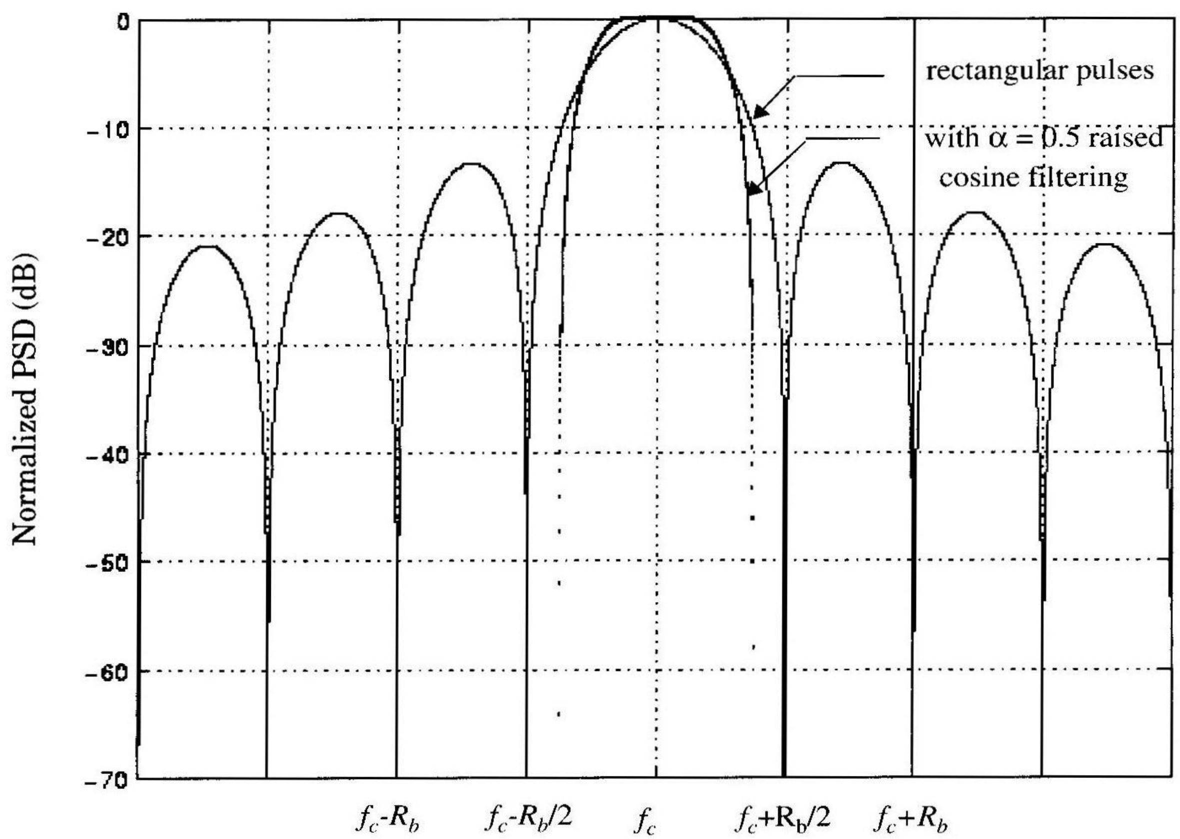


Figure 6.27 Power spectral density of a QPSK signal.

- null-to-null RF BW = R_b = $2R_S$ (2 bits / one symbol time) = 2 / T_s
- double the BW efficiency of BPSK → or **twice** the data rate in same signal BW

- BER is once again related to the distance between constellation points.

$$\text{Prob}\{\text{bit error}\} \le Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

- d is distance between nearest constellation points.

- Here $d = \sqrt{2E_s}$ so $\text{Prob}\{\text{bit error}\} \le Q\left(\sqrt{\frac{E_s}{N_0}}\right)$

  But $E_s = 2\,E_b$ so $\text{Prob}\{\text{bit error}\} \le Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

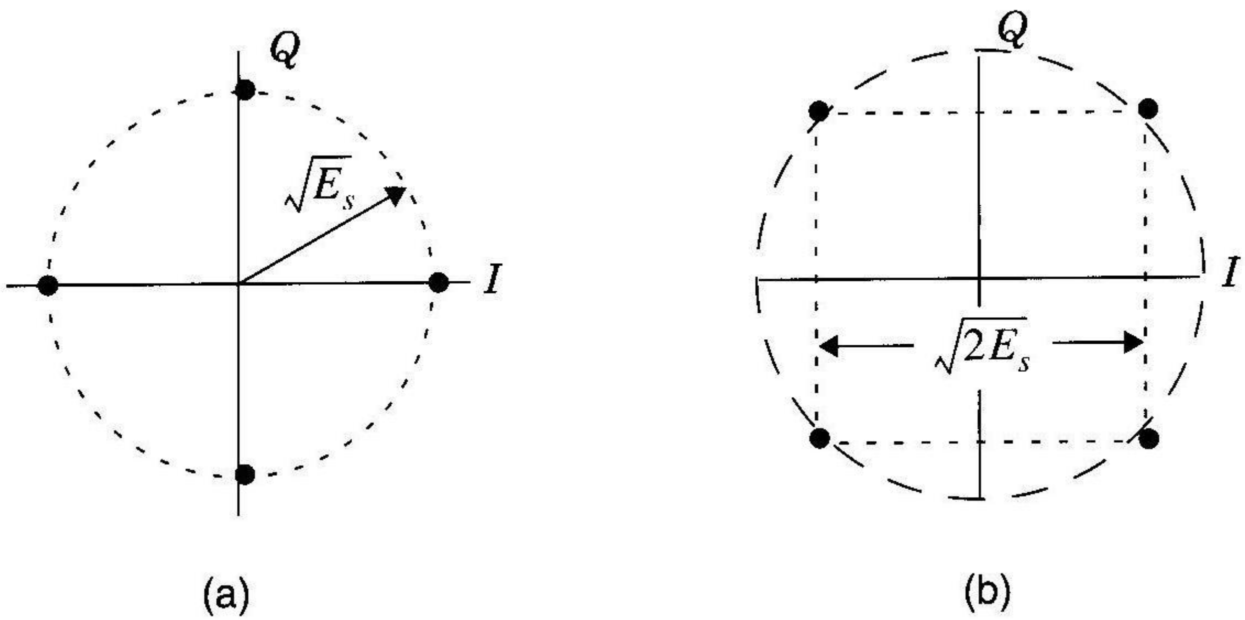


Figure 6.26 (a) QPSK constellation where the carrier phases are 0, π/2, π, 3π/2; (b) QPSK constellation where the carrier phases are π/4, 3π/4, 5π/4, 7π/4.

- How does BER performance compare to BPSK?
  - Why? same # of states per number of basis functions for both BPSK **and** QPSK (2 states per one function or 4 states per 2 functions)
  - **same** power efficiency

    (same BER at specified E_b / N_o)
  - **twice** the bandwidth efficiency

    (sending 2 bits instead of 1)

QPSK Transmission and Detection Techniques

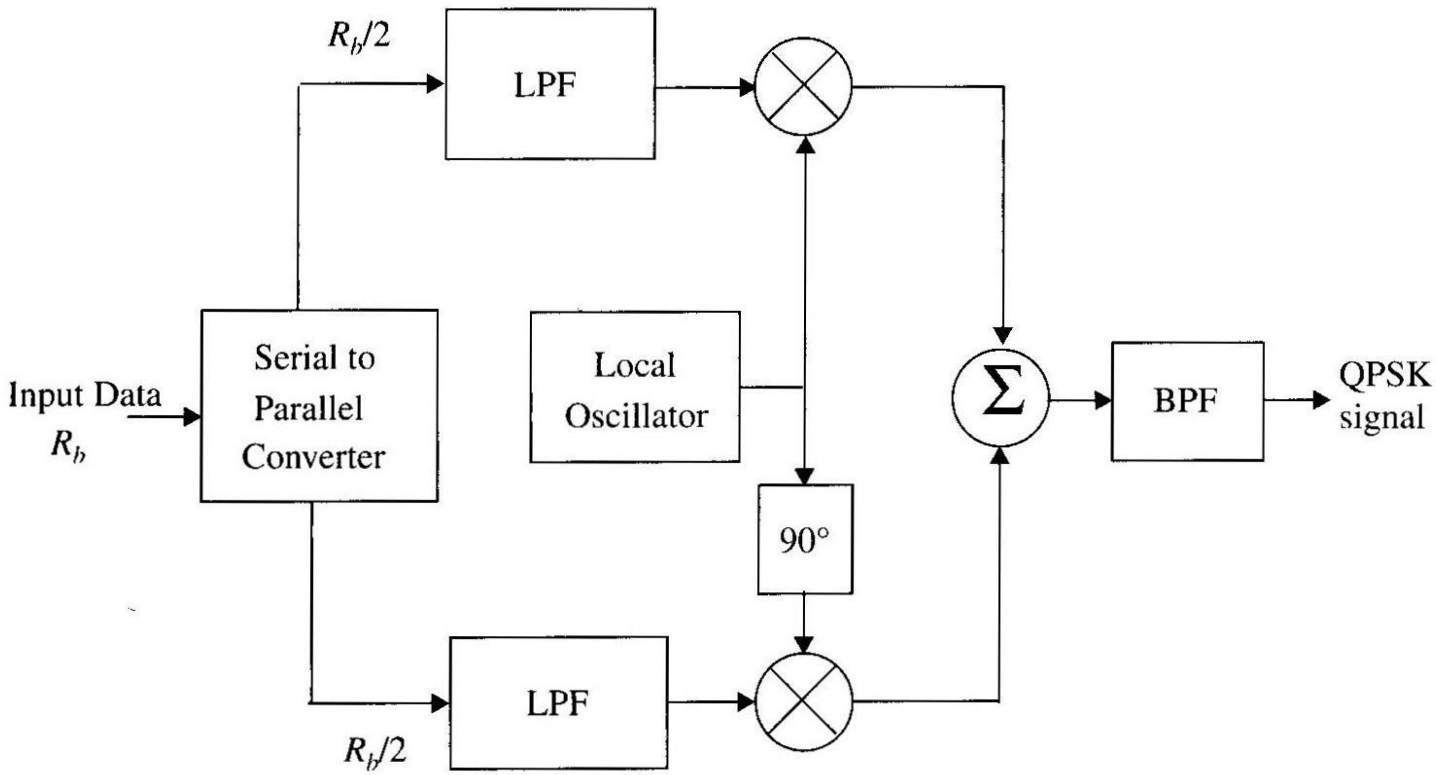


Figure 6.28 Block diagram of a QPSK transmitter.

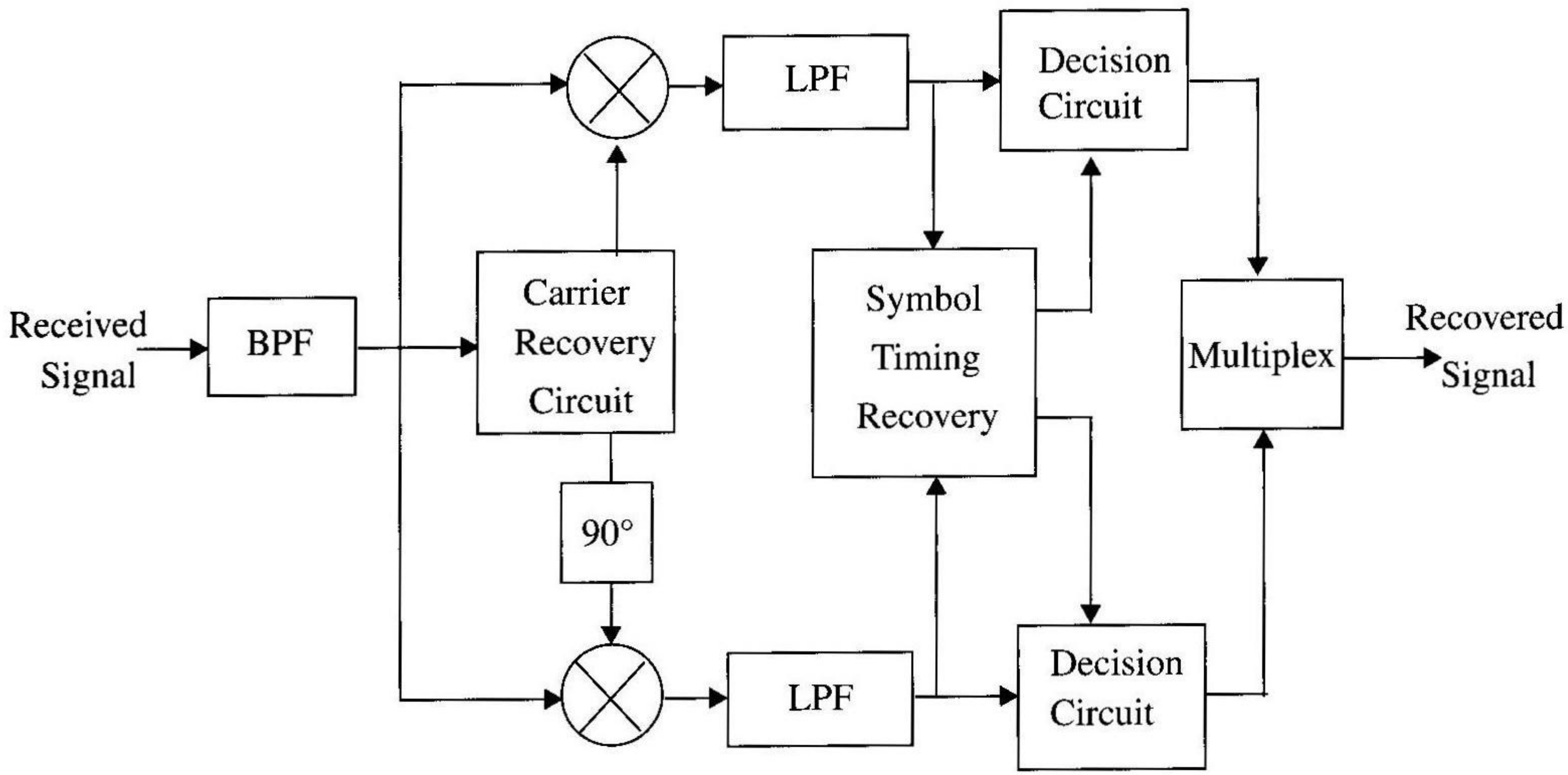
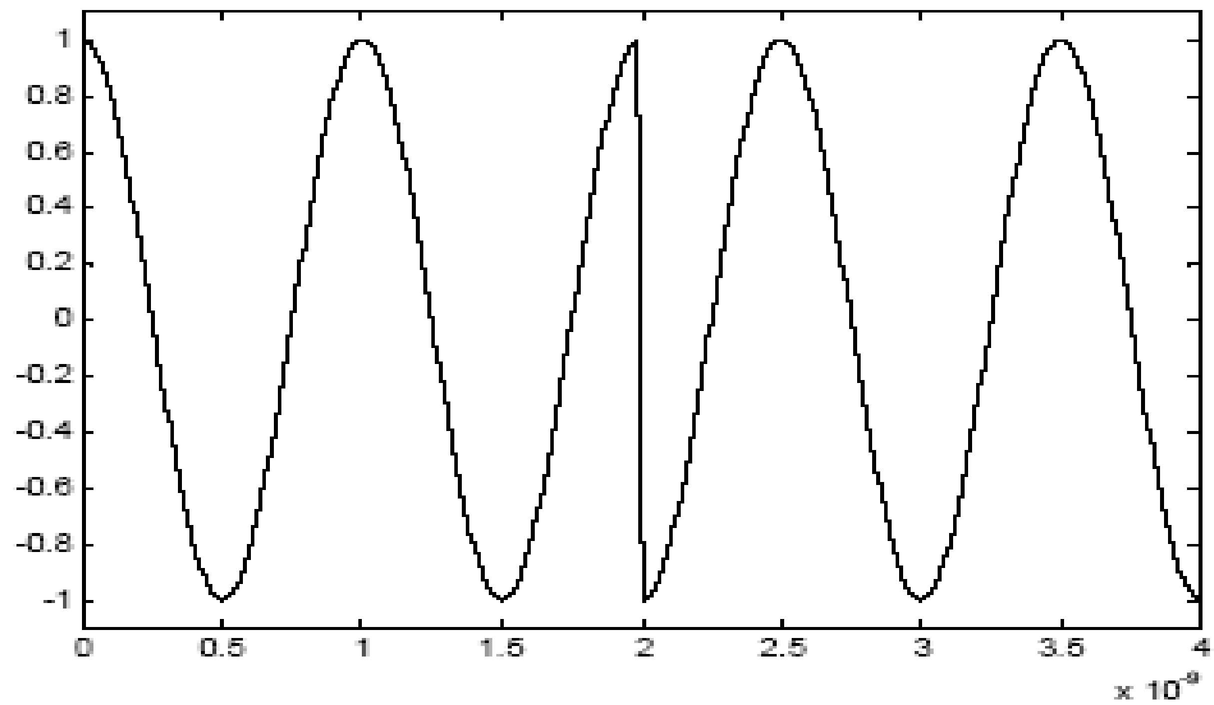


Figure 6.29 Block diagram of a QPSK receiver.

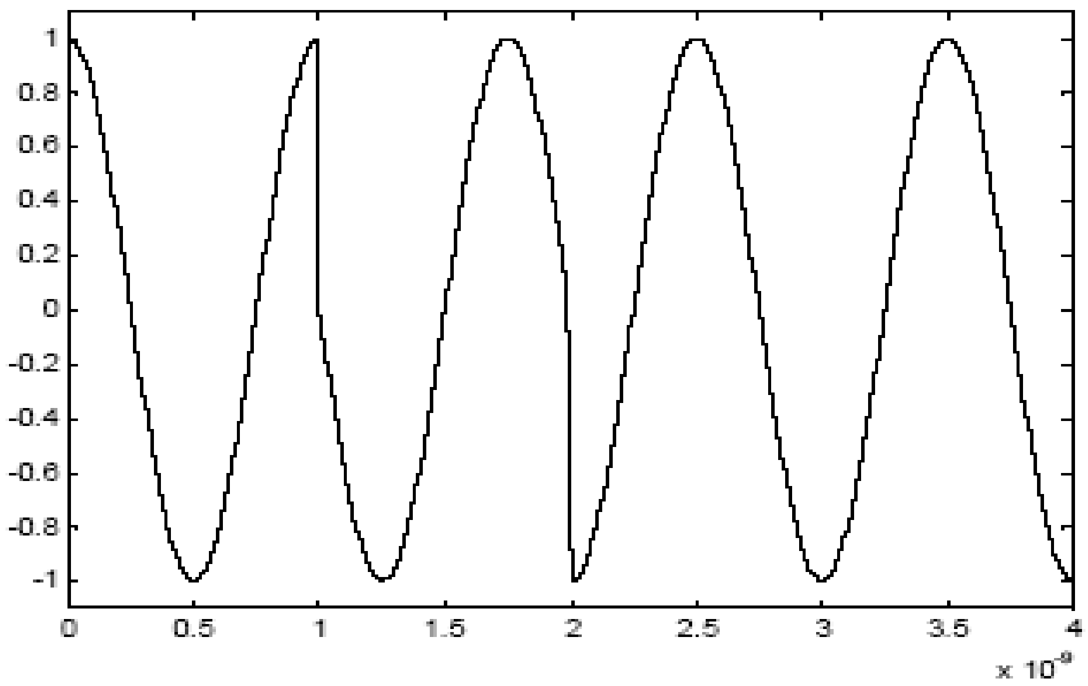
OQPSK

- Offset QPSK
  - The occasional phase shift of π radians can cause the signal envelope to pass through zero for just in instant.
  - Any kind of hard limiting or nonlinear amplification of the zero-crossings brings back the filtered sidelobes
    - since the fidelity of the signal at small voltage levels is lost in transmission.
  - OQPSK ensures there are fewer baseband signal transitions applied to the RF amplifier,
    - helps eliminate spectrum regrowth after amplification.



- Example above: First symbol (00) at 0º, and the next symbol (11) is at 180º. Notice the signal going through zero at 2 microseconds.
  - This causes problems.

- Using an offset approach: First symbol (00) at 0º, then an intermediate symbol at (10) at 90º, then the next full symbol (11) at 180º.
  - The intermediate symbol is used halfway through the symbol period.
  - It corresponds to allowing the first bit of the symbol to change halfway through the symbol period.
  - The figure below does have phase changes more often, but no extra transitions through zero.
  - IS-95 uses OQPSK, so it is one of the major modulation schemes used.


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
x 10^-9

- In QPSK signaling, the bit transitions of the even and odd bit streams occur at the same time instants.
- but in OQPSK signaling, the even and odd bit Streams, $m_I(t)$ and $m_Q(t)$, are offset in their relative alignment by one bit period (half-symbol period)

- the maximum phase shift of the transmitted signal at any given time is limited to ± 90º

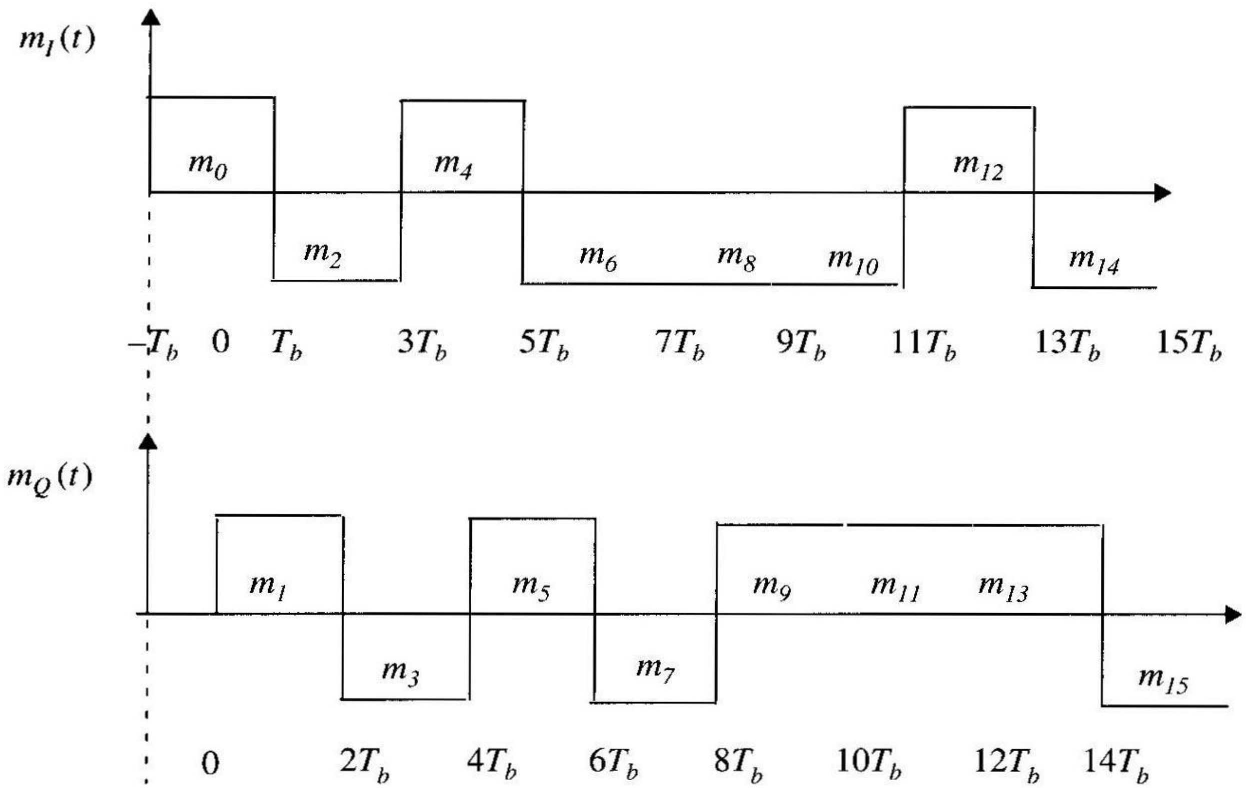


Figure 6.30 The time offset waveforms that are applied to the in-phase and quadrature arms of an OQPSK modulator. Notice that a half-symbol offset is used.

- The spectrum of an OQPSK signal is identical to that of a QPSK signal, hence both signals occupy the same bandwidth

π/4 QPSK

- π/4 QPSK
  - The π/4 shifted QPSK modulation is a quadrature phase shift keying technique
    - offers a compromise between OQPSK and QPSK in terms of the allowed maximum phase transitions.
  - It may be demodulated in a coherent or noncoherent fashion.
    - greatly simplifies receiver design.
  - In π/4 QPSK, the maximum phase change is limited to $\pm 135^{o}$
  - in the presence of multipath spread and fading, π/4 QPSK performs better than OQPSK

TABLE 6.2 *Correspondence between input dibit and phase change for π/4-shifted DQPSK*

Gray-Encoded Input Dibit	*Phase Change, Δθ (radians)*
00	$\pi/4$
01	$3\pi/4$
11	$-3\pi/4$
10	$-\pi/4$

Table 6.3 $\pi/4$-shifted DQPSK results for Example 6.2

Step k	Phase θ_{k-1} (radians)	Input Dibit	Phase Change $\Delta\theta_k$ (radians)	Transmitted Phase θ_k (radians)
1	$\pi/4$	00	$\pi/4$	$\pi/2$
2	$\pi/2$	10	$-\pi/4$	$\pi/4$
3	$\pi/4$	10	$-\pi/4$	0
4	0	01	$3\pi/4$	$3\pi/4$

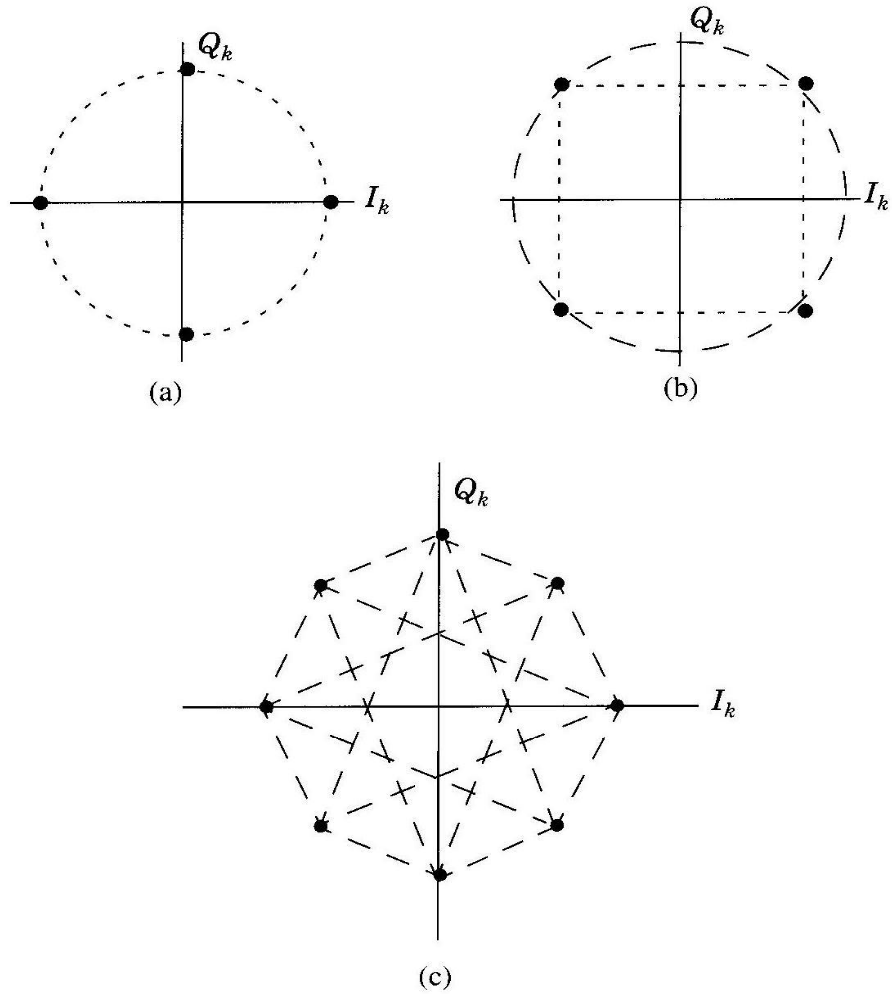


Figure 6.31 Constellation diagram of a $\pi/4$ QPSK signal: (a) possible states for θ_k when $\theta_{k-1} = n\pi/4$; (b) possible states when $\theta_{k-1} = n\pi/2$; (c) all possible states.