

Unit-2

Electrical Circuit

Analysis

Prepared by:

Dr.Dharmesh Dabhi

M & V Patel Department of Electrical Engineering
CHARUSAT

dharmeshdabhi.ee@charusat.ac.in

$$I_1=4, I_2=5, I_3=3, I_4=-4, I_5=-2$$

$$I_1+I_2+I_3+I_4+I_5=0$$

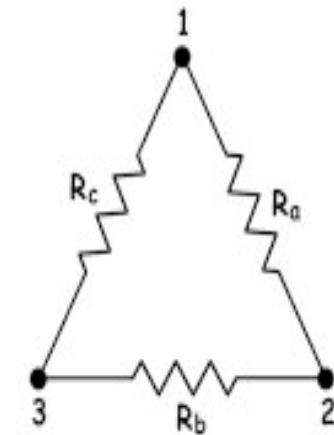
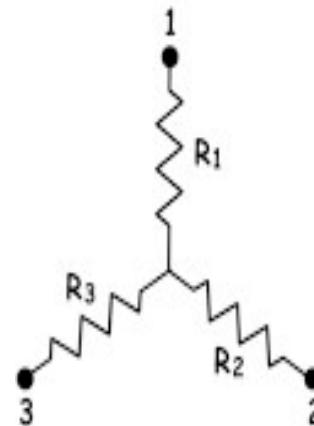
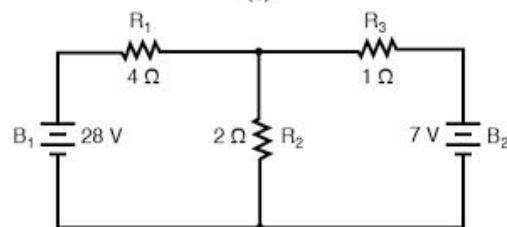
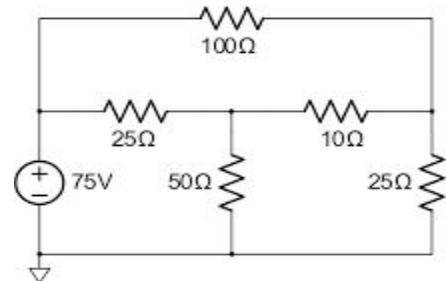
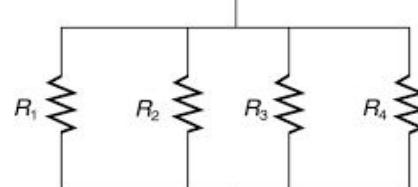
$$V_0=12$$

$$V_1=3$$

$$V_3=5$$

$$V_2=4$$

$$V_0+V_1+V_2+V_3=0$$



Content

Kirchhoff's Current Law (KCL)

Kirchhoff's Voltage Law (KVL)

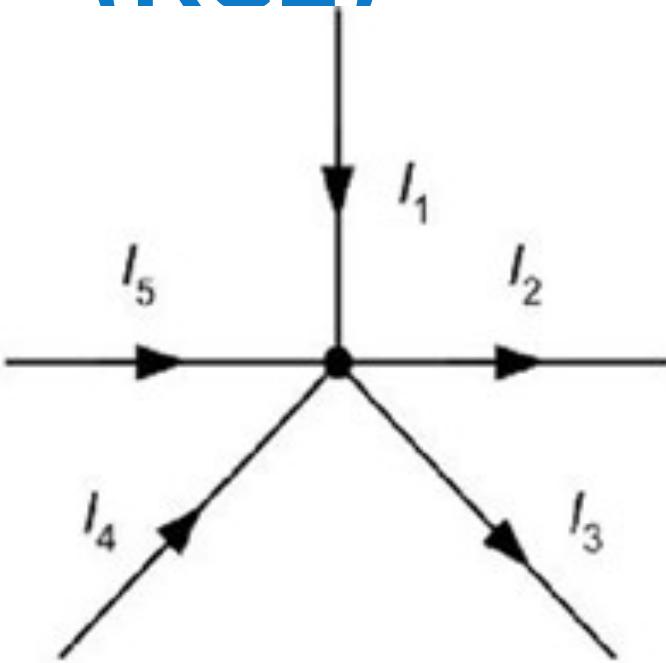
MESH Analysis & its examples

NODAL Analysis & its examples

Series- Parallel Circuit & its examples

Star- Delta Transformation & its examples

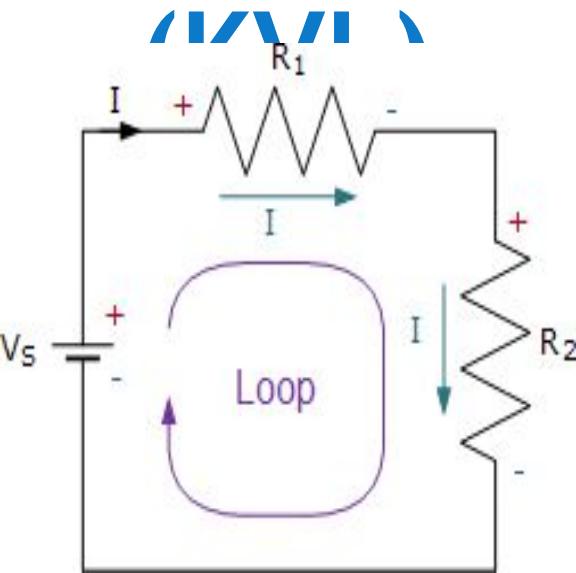
Kirchhoff's Current Law (KCL)



$$I_1 - I_2 - I_3 + I_4 + I_5 = 0$$

- KCL states that “In an electrical circuit, at any point of time, the algebraic sum of the current in all the conductors meeting at any point is zero.”
- In other words, whenever two or more conductors meet at a point then the sum of the current flowing towards the junction point is equal to the sum of current flowing away from it.
- If the current entering into a junction are assigned a positive sign, then the current leaving the junction will be assigned a negative sign.

Kirchhoff's Voltage Law

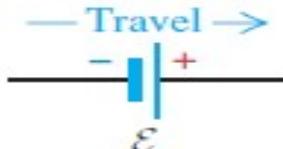


- KVL states that “the algebraic **sum** of the potential differences in any loop must be equal to zero as: $\Sigma V = 0$.”
- Since the two resistors, R_1 and R_2 , are wired together in a series connection, they are both part of the same loop so the same current must flow through each resistor.
- Thus the voltage drop across resistor, $R_1 = I \cdot R_1$ and the voltage drop across resistor, $R_2 = I \cdot R_2$ giving by KVL:

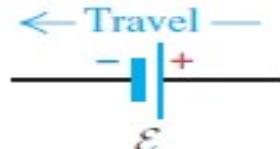
$$V_s + (-IR_1) + (-IR_2) = 0$$

(a) Sign conventions for emfs

$+\mathcal{E}$: Travel direction from $-$ to $+$:

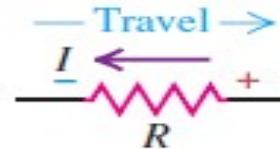


$-\mathcal{E}$: Travel direction from $+$ to $-$:

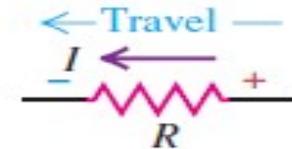


(b) Sign conventions for resistors

$+IR$: Travel *opposite* to current direction:



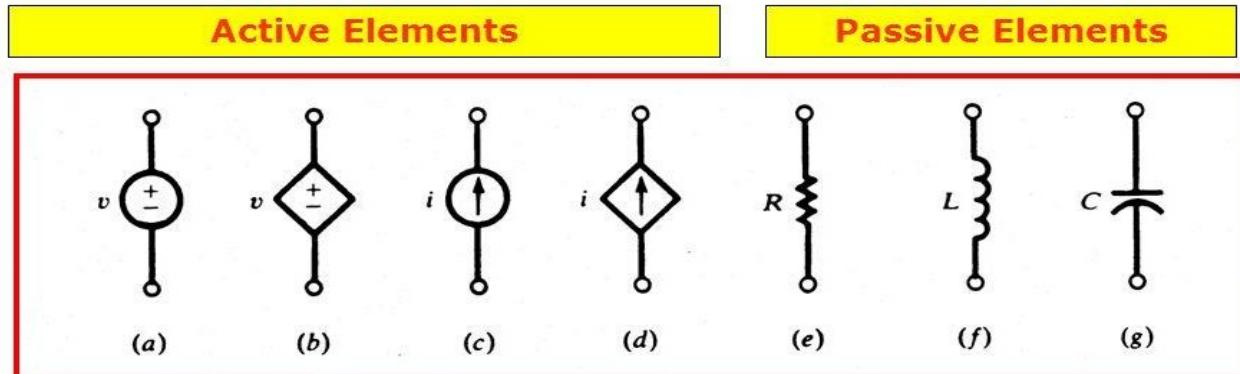
$-IR$: Travel *in* current direction:



Definition

1. Element:- It is a mathematical model of two terminal electrical device which can be completely characterized by its voltage and current.

Circuit Elements



Independent sources

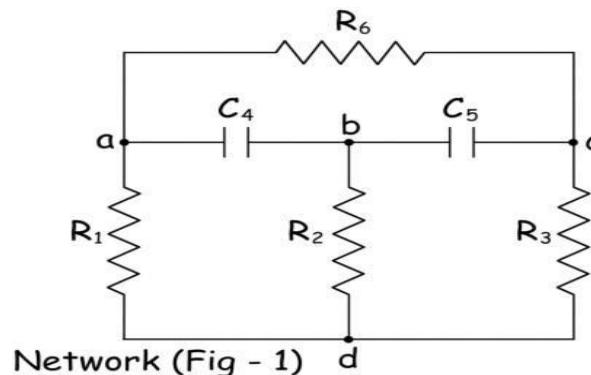
Dependant sources

- A dependent source is an active element in which the source quantity is controlled by another voltage or current.
- They have four different types: VCVS, CCVS, VCCS, CCCS. Keep in minds the signs of dependent sources.

2. Active element:- The element which is capable of delivering an average power greater than zero to some external device over an infinite time interval is called an active element.

3. Passive element:- The element which is capable only of receiving power is called passive element.

4. Network:- The interconnection of two or more elements called an electric network.



5. Circuit:- If a network contains at least one closed path, it is called an electric circuit.

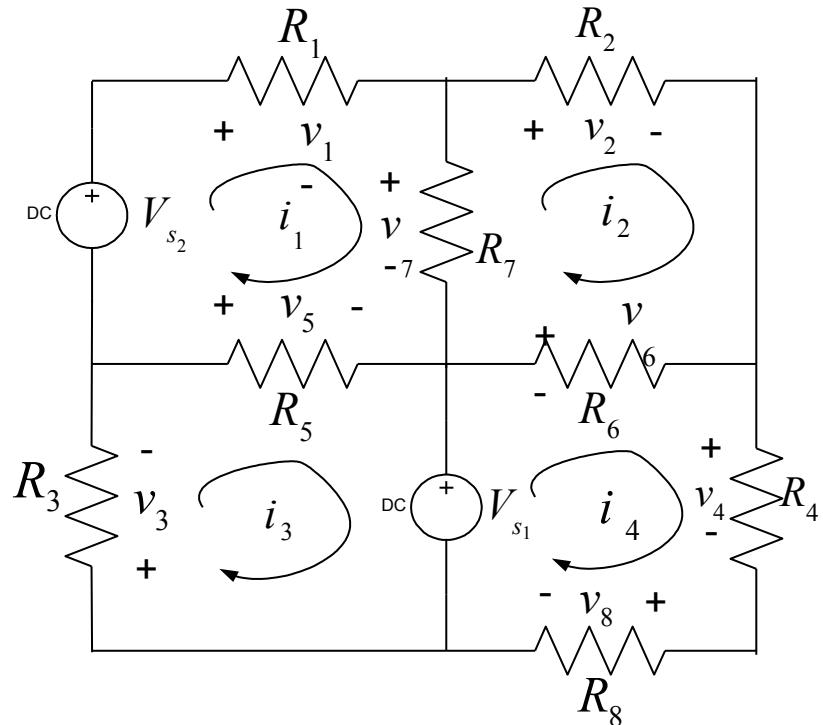
6. **Node:-** The point at which **two or more elements** are connected together is called as node.
7. **Junction:-** It is a point where **three or more elements** are connected together.
8. **Branch:-** A section or portion of a **network or circuit** which lies between two junction points is called as branch.
9. **Loop:-** Any **closed path** in a network is called loop.
10. **Mesh:-** It is the most **elementary form of a loop** and **cannot be further divided into other loops**. A mesh is a loop that has no other circuit paths inside it.

Mesh Analysis

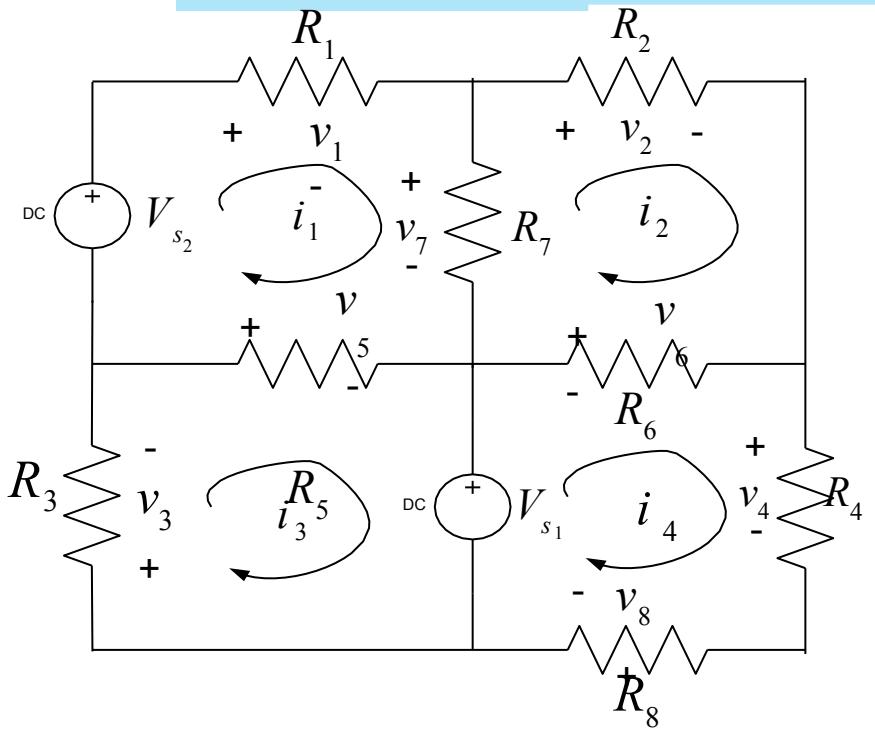
- Mesh analysis applies KVL to find unknown currents.
- It is only applicable to planar circuits (a circuit that can be drawn on a plane with no branches crossing each other).
- A mesh is a loop that does not contain any other loops.
- The current through a mesh is known as the mesh current.
- Assume for simplicity that the circuit contains only voltage sources

Mesh Analysis Steps

1. Assign mesh currents $i_1, i_2, i_3, \dots, i_l$ to the l meshes,
2. Apply KVL to each of the l meshes and use Ohm's law to express the voltages in terms of the mesh currents,
3. Solve the l resulting simultaneous equations to find the mesh currents.



Example



Apply KVL to each mesh

$$\text{Mesh 1: } V_{s_2} - v_1 - v_7 + v_5 = 0$$

$$\text{Mesh 2: } -v_2 + v_6 + v_7 = 0$$

$$\text{Mesh 3: } -v_5 - V_{s_1} - v_3 = 0$$

$$\text{Mesh 4: } -v_4 - v_8 + V_{s_1} - v_6 = 0$$

Mesh 1:

$$V_{s2} - v_1 - v_7 + v_5 = 0$$

Mesh 2:

$$-v_2 + v_6 + v_7 = 0$$

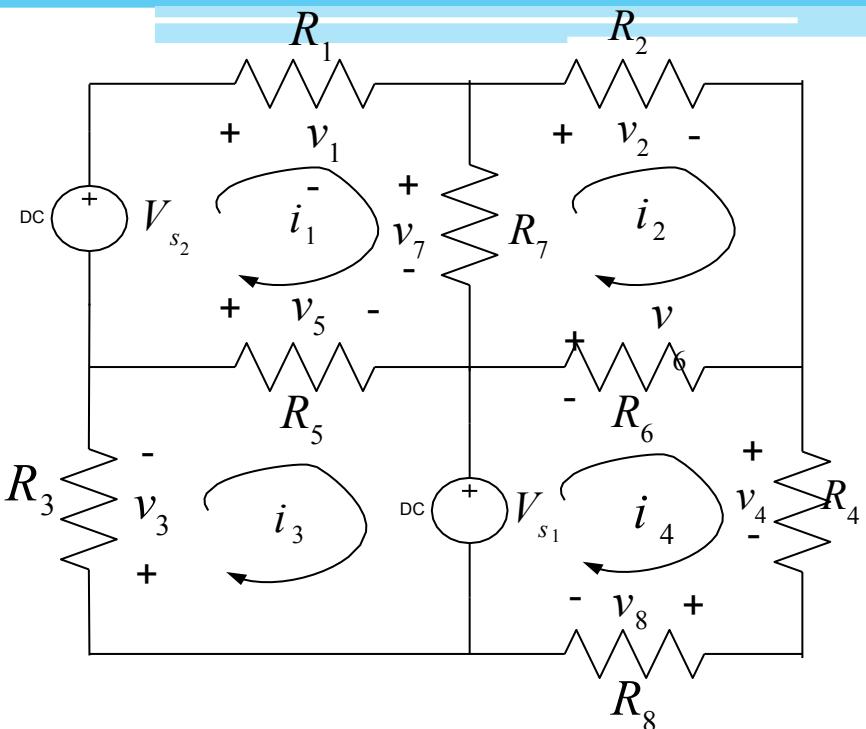
Mesh 3:

$$-v_5 - V_{s1} - v_3 = 0$$

Mesh 4:

$$-v_4 - v_8 + V_{s1} - v_6 = 0$$

Express the voltage in terms of the mesh currents:



Mesh 1: $V_{s2} - i_1 R_1 - (i_1 - i_2) R_7 - (i_1 - i_3) R_5 = 0$

Mesh 2: $-i_2 R_2 - (i_2 - i_4) R_6 - (i_2 - i_1) R_7 = 0$

Mesh 3: $-(i_3 - i_1) R_5 - V_{s1} - i_3 R_3 = 0$

Mesh 4: $-i_4 R_4 - i_4 R_8 + V_{s1} - (i_4 - i_2) R_6 = 0$

Mesh 1: $V_{s_2} - i_1 R_1 - (i_1 - i_2) R_7 - (i_1 - i_3) R_5 = 0$

Mesh 2: $-i_2 R_2 - (i_2 - i_4) R_6 - (i_2 - i_1) R_7 = 0$

Mesh 3: $-(i_3 - i_1) R_5 - V_{s_1} - i_3 R_3 = 0$

Mesh 4: $-i_4 R_4 - i_4 R_8 + V_{sI} - (i_4 - i_2) R_6 = 0$

Mesh 1: $(R_1 + R_5 + R_7) i_1 - R_7 i_2 - R_5 i_3 = V_2$

Mesh 2: $R_7 i_1^s - (R_2 + R_6 + R_7) i_2 + R_6 i_4 = 0$

Mesh 3: $+R_5 i_1 - (R_3 + R_s) i = V_1$

Mesh 4: $-R_6 i_2 - R_4 i_4 + (R_6 + R_4 + R_s) i = V_1$

Mesh 1: $(R_1 + R_5 + R_7)i_1 - R_7i_2 - R_5i_3 = V_2$

Mesh 2: $R_7i_1^s - (R_2 + R_6 + R_7)i_2 + R_6i_4 = 0$

Mesh 3: $+R_5i_3 - (R_3 + R_s)i = V_1$

Mesh 4: $-R_6i_4 + (R_6 + R_4 + R_8)i_1 = V_1$

$$\left\{ \begin{array}{cccc} R_1 + R_5 + R_7 & -R_7 & -R_5 & 0 \\ R_7 & -(R_2 + R_6 + R_7) & 0 & R_6 \\ R_5 & 0 & -(R_3 + R_5) & 0 \\ 0 & -R_6 & 0 & R_4 + R_6 + R_8 \end{array} \right\} \left\{ \begin{array}{c} i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} \right\} = \left\{ \begin{array}{c} V_2 \\ 0 \\ V_1 \\ V_{s_1} \end{array} \right\}$$

$$\left(\begin{array}{cccc}
 R_1 + R_5 + R_7 & -R_7 & -R_5 & 0 \\
 R_7 & -(R_2 + R_6 + R_7) & 0 & R_6 \\
 R_5 & 0 & -(R_3 + R_5) & 0 \\
 0 & -R_6 & 0 & R_4 + R_6 + R_8
 \end{array} \right) \left(\begin{array}{c}
 i_1 \\
 i_2 \\
 i_3 \\
 i_4
 \end{array} \right) = \left(\begin{array}{c}
 V_{s_2} \\
 0 \\
 V_{s_1} \\
 V_{s_1}
 \end{array} \right)$$

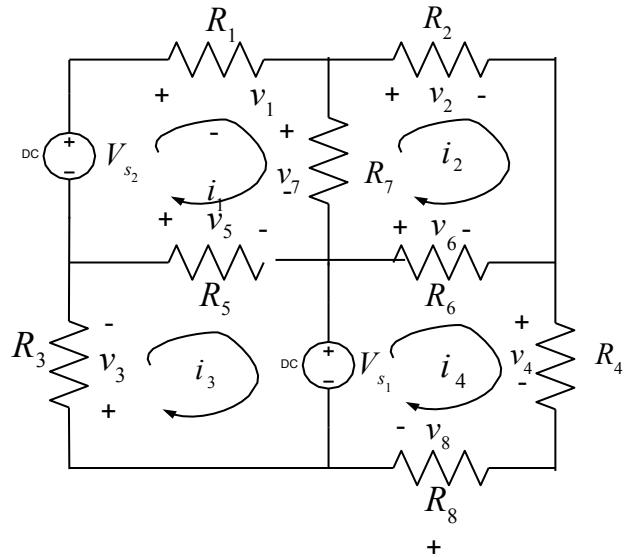
Ri = v

R is an $l \times l$ symmetric resistance matrix

i is a $l \times 1$ vector of mesh currents

V is a vector of voltages representing “known” voltages

Writing the Mesh Equations by Inspection



- The matrix \mathbf{R} is symmetric, $r_{kj} = r_{jk}$ and all of the off-diagonal terms are negative or zero.

The r_{kk} terms are the sum of all resistances in mesh k .

The r_{kj} terms are the negative sum of the resistances common to BOTH mesh k and mesh j .

The v_k (the k^{th} component of the vector \mathbf{v}) = the algebraic sum of the independent voltages in mesh k , with voltage rises taken as positive.

Using Cramer's Rule to Solve a System of Three Equations

Consider the following set of linear equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 =$$

$$b_1 \quad a_{21}x_1 + a_{22}x_2 + a_{23}x_3$$

$$= b_2 \quad a_{31}x_1 + a_{32}x_2 + a_{33}$$

$$x_3 = b_3$$

Using Cramer's Rule to Solve a System of Three Equations

The system of equations above can be written in a matrix form as:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_3 & a_{32} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} b_1 \\ b_2 \\ b_3 \end{vmatrix}$$

Using Cramer's Rule

to Solve a System of Three Equations

Define

$$\begin{aligned} A &= \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \\ x &= \begin{vmatrix} b_1 \\ b_2 \\ b_3 \end{vmatrix} \quad \text{and} \quad B = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} \end{aligned}$$

If $D \neq 0$, then the system has a unique solution as shown below (Cramer's Rule).

$$x^1 = \frac{D_1}{D}, \quad x^2 = \frac{D_2}{D}, \quad x^3 = \frac{D_3}{D},$$

Using Cramer's Rule to Solve a System of Three Equations

where

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{32} & a_{33} \end{vmatrix}$$

$$D_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

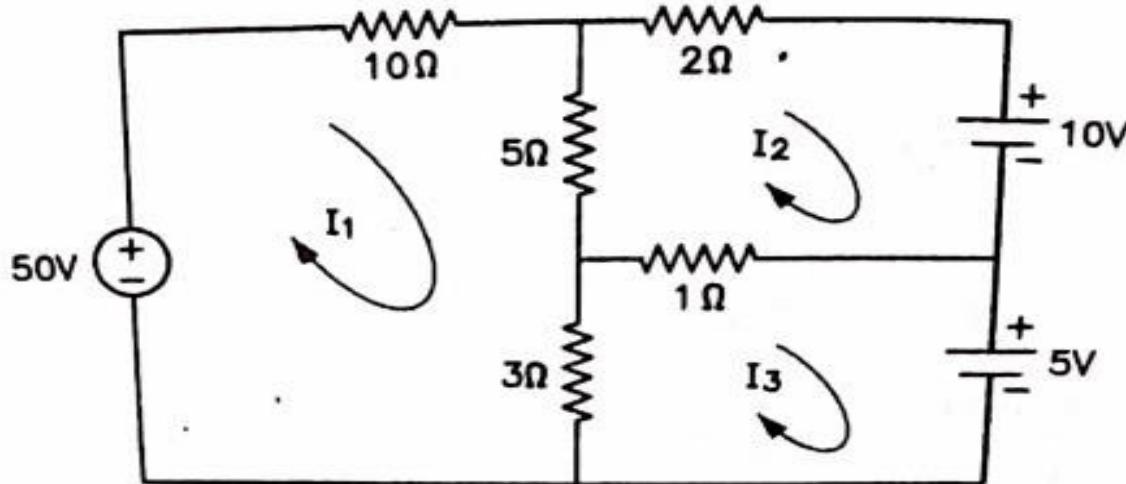
$$\begin{matrix} 2 \\ b \\ 3 \end{matrix}$$

$$D_2 = \begin{vmatrix} a_{11} & b_1 & a_1 \\ a_{12} & b & a_2 \\ a_{13} & b & a_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_{11} & b \\ a_{12} & a_1 \\ a_{13} & b \\ a_2 & b \\ a_3 & b \end{vmatrix}$$

$$\begin{matrix} 2 \\ 3 \end{matrix}$$

1. Determine the mesh currents I_1 , I_2 and I_3 for the network shown



Applying KVL to the above circuit,

The mesh current equations are,

KVL for loop-1,

$$50 - 10 I_1 - 5 (I_1 - I_2) - 3 (I_1 - I_3) = 0$$

$$\therefore 50 = 18 I_1 - 5 I_2 - 3 I_3$$

KVL for loop-2,

$$-2 I_2 - 10 - 1 (I_2 - I_3) - 5 (I_2 - I_1) = 0$$

$$\therefore -10 = -5 I_1 + 8 I_2 - I_3 \quad 2$$

KVL for loop-3,

$$-5 - 3 (I_3 - I_1) - 1 (I_3 - I_2) = 0$$

$$\therefore -5 = -3 I_1 - I_2 + 4 I_3 \quad 3$$

From equation (1), (2) and (3)

$$\begin{bmatrix} 50 \\ -10 \\ -5 \end{bmatrix} = \begin{bmatrix} 18 & -5 & -3 \\ -5 & 8 & -1 \\ -3 & -1 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$\Delta R = \begin{vmatrix} 18 & -5 & -3 \\ -5 & 8 & -1 \\ -3 & -1 & 4 \end{vmatrix} = 356 \Omega$$

$$\cdot \quad \Delta R_1 = \begin{vmatrix} 50 & -5 & -3 \\ -10 & 8 & -1 \\ -5 & -1 & 4 \end{vmatrix} = 1175 \Omega$$

$$\Delta R_2 = \begin{vmatrix} 18 & 50 & -3 \\ -5 & -10 & -1 \\ -3 & -5 & 4 \end{vmatrix} = 355 \Omega$$

$$\Delta R_3 = \begin{vmatrix} 18 & -5 & 50 \\ -5 & 8 & -10 \\ -3 & -1 & -5 \end{vmatrix} = 525 \Omega$$

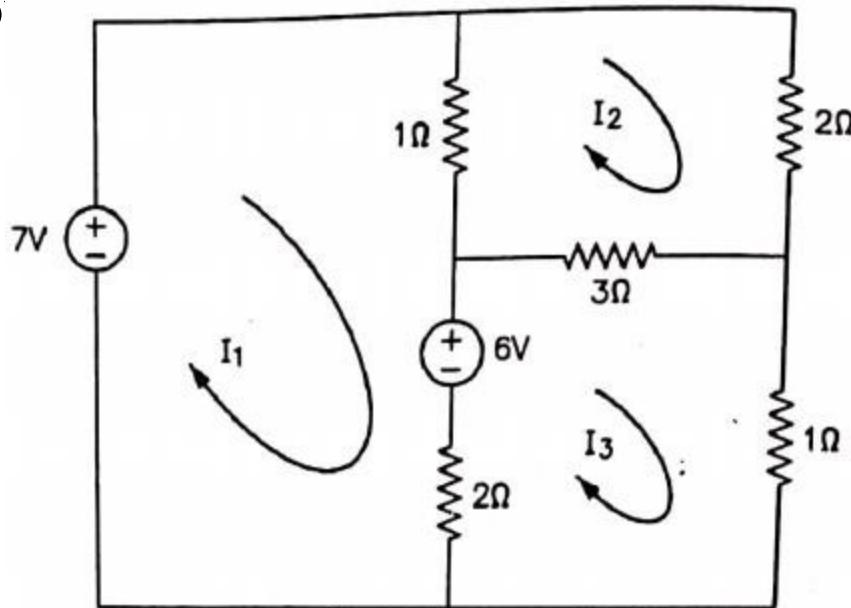
Using cramer's rule,

$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{1175}{356} = \boxed{3.30 \text{ amp.}}$$

$$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{355}{356} = \boxed{0.99 \text{ amp.}}$$

$$I_3 = \frac{\Delta R_3}{\Delta R} = \frac{525}{356} = \boxed{1.47 \text{ amp.}}$$

2. Determine the mesh currents I_1 , I_2 and I_3 for the network shown below



Writing KVL equations for the above circuit, the mesh current equations are.

KVL for loop-1,

$$7 - 1(I_1 - I_2) - 6 - 2(I_1 - I_3) = 0$$

$$\therefore 1 = 3I_1 - I_2 - 2I_3$$

KVL for loop-2,

$$\begin{aligned} -2 I_2 - 3 (I_2 - I_3) - 1 (I_2 - I_1) &= 0 \\ \therefore 0 &= - I_1 + 6 I_2 - 3 I_3 \end{aligned}$$

2

KVL for loop-3,

$$\begin{aligned} - I_3 - 2 (I_3 - I_1) + 6 - 3 (I_3 - I_2) &= 0 \\ \therefore 6 &= - 2 I_1 - 3 I_2 + 6 I_3 \end{aligned}$$

3

From equation (1), (2) and (3),

$$\begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\Delta R = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix} = 39 \Omega$$

$$\Delta R_1 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix} = 117 \Omega$$

$$\Delta R_2 = \begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix} = 78 \Omega$$

$$\Delta R_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix} = 117 \Omega$$

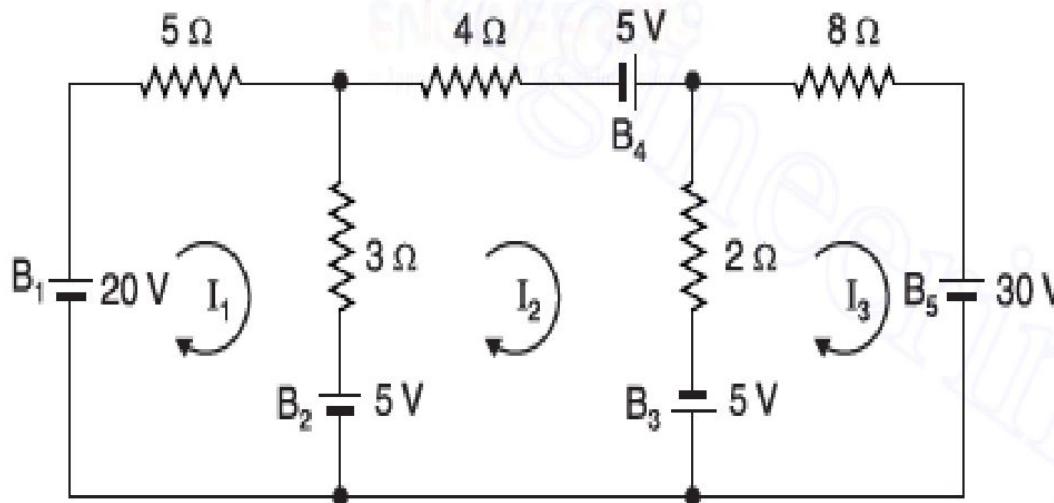
Using cramer's rule,

$$I_1 = \frac{\Delta R_1}{\Delta R} = \frac{117}{39} = \boxed{3 \text{ amp.}}$$

$$I_2 = \frac{\Delta R_2}{\Delta R} = \frac{78}{39} = \boxed{2 \text{ amp.}}$$

$$I_3 = \frac{\Delta R_3}{\Delta R} = \frac{117}{39} = \boxed{3 \text{ amp.}}$$

Example 3.3. By using mesh resistance matrix, determine the current supplied by each battery in the circuit shown in Fig. 3.7.



Solution. Since there are three meshes, let the three mesh currents be I_1 , I_2 and I_3 , all assumed to be flowing in the clockwise direction. The different quantities of the mesh-resistance matrix are :

$$R_{11} = 5 + 3 = 8 \Omega ; \quad R_{22} = 4 + 2 + 3 = 9 \Omega ; \quad R_{33} = 8 + 2 = 10 \Omega$$

$$R_{12} = R_{21} = -3 \Omega ; \quad R_{13} = R_{31} = 0 ; \quad R_{23} = R_{32} = -2 \Omega$$

$$E_1 = 20 - 5 = 15 \text{ V} ; \quad E_2 = 5 + 5 + 5 = 15 \text{ V} ; \quad E_3 = -30 - 5 = -35 \text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \\ 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}}{598} = \frac{1530}{598} = 2.56 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix}}{\text{Denominator}} = \frac{1090}{598} = 1.82 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix}}{\text{Denominator}} = \frac{-1875}{598} = -3.13 \text{ A}$$

The negative sign with I_3 indicates that actual direction of I_3 is opposite to that assumed in Fig. 3.7. Note that batteries B_1 , B_3 , B_4 and B_5 are discharging while battery B_2 is charging.

∴ Current supplied by battery $B_1 = I_1 = 2.56 \text{ A}$

Current supplied to battery $B_2 = I_1 - I_2 = 2.56 - 1.82 = 0.74 \text{ A}$

Current supplied by battery $B_3 = I_2 + I_3 = 1.82 + 3.13 = 4.95 \text{ A}$

Current supplied by battery $B_4 = I_2 = 1.82 \text{ A}$

Current supplied by battery $B_5 = I_3 = 3.13 \text{ A}$

Example 3.4. By using mesh resistance matrix, calculate the current in each branch of the circuit shown in Fig. 3.8.

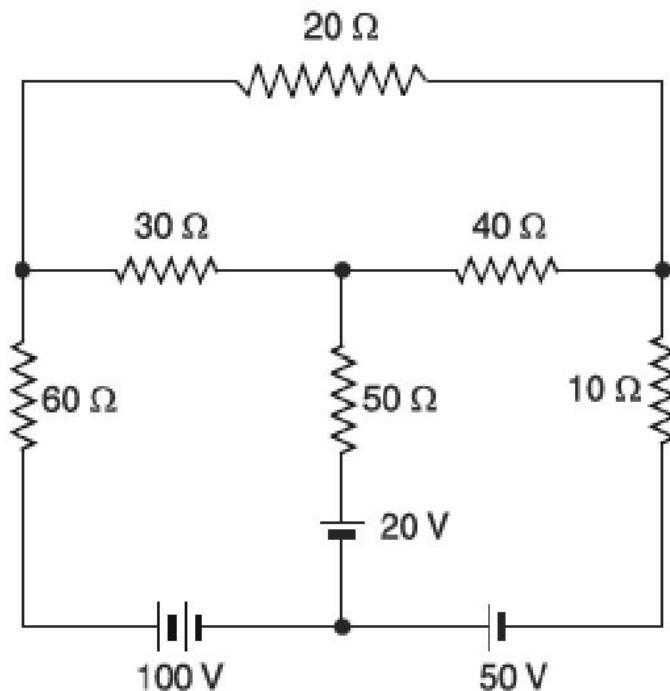


Fig. 3.8

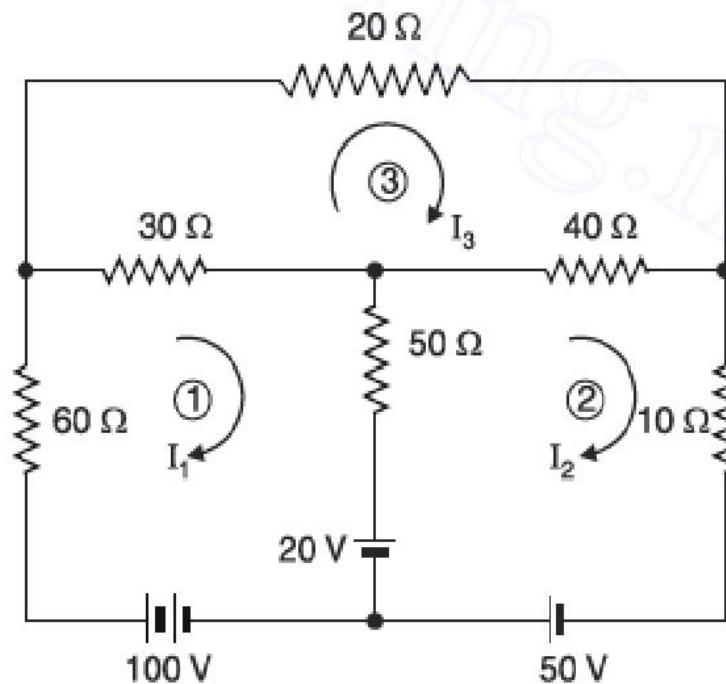


Fig. 3.9

$$R_{11} = 60 + 30 + 50 = 140\ \Omega ; R_{22} = 50 + 40 + 10 = 100\ \Omega ; R_{33} = 30 + 20 + 40 = 90\ \Omega$$

$$R_{12} = R_{21} = -50\ \Omega ; R_{13} = R_{31} = -30\ \Omega ; R_{23} = R_{32} = -40\ \Omega$$

$$E_1 = 100 - 20 = 80\text{ V} ; E_2 = 50 + 20 = 70\text{ V} ; E_3 = 0\text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or $\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 70 \\ 0 \end{bmatrix}$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & -40 & 90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{vmatrix}} = \frac{991000}{601000} = 1.65 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 140 & 80 & -30 \\ -50 & 70 & -40 \\ -30 & 0 & 90 \end{vmatrix}}{\text{Denominator}} = \frac{1275000}{601000} = 2.12 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 140 & -50 & 80 \\ -50 & 100 & 70 \\ -30 & -40 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{897000}{601000} = 1.5 \text{ A}$$

Current in $60\ \Omega$ = $I_1 = 1.65\text{ A}$ in the direction of I_1

Current in $30\ \Omega$ = $I_1 - I_3 = 0.15\text{ A}$ in the direction of I_1

Current in $50\ \Omega$ = $I_2 - I_1 = 0.47\text{ A}$ in the direction of I_2

Current in $40\ \Omega$ = $I_2 - I_3 = 0.62\text{ A}$ in the direction of I_2

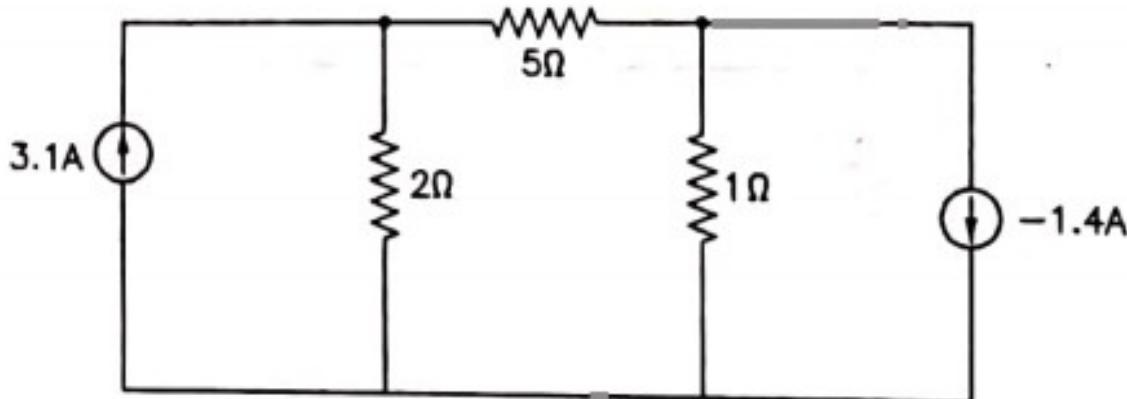
Current in $10\ \Omega$ = $I_2 = 2.12\text{ A}$ in the direction of I_2

Current in $20\ \Omega$ = $I_3 = 1.5\text{ A}$ in the direction of I_3

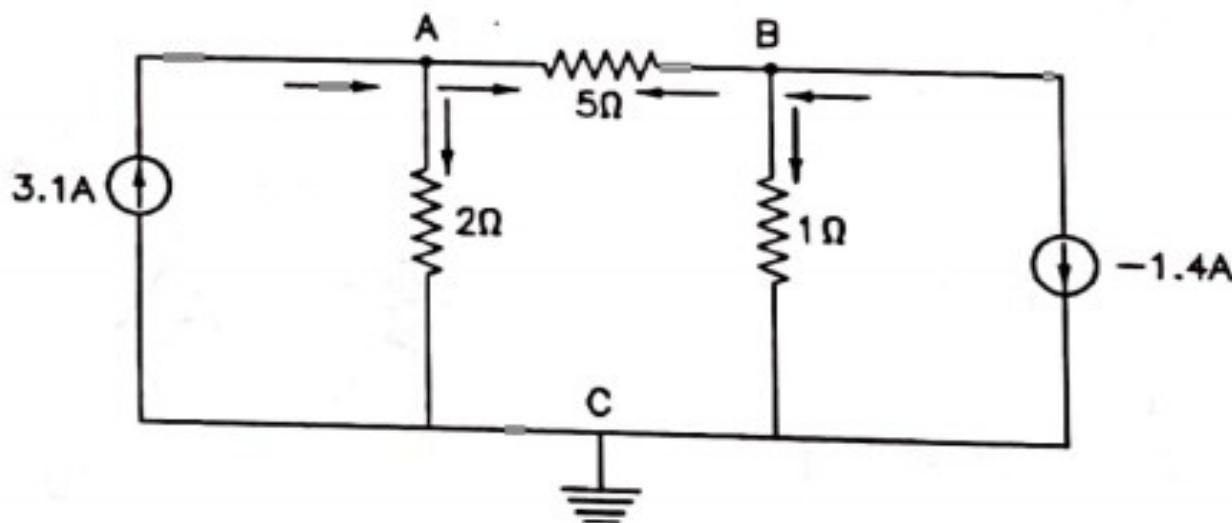
Nodal Analysis

- Nodal analysis is a technique used to analyze circuits with more than one voltage source.
- Based on KCL, it is used to determine the voltages of the nodes in a circuit.
- Once you determine the node voltages, it is possible to find all branch voltages, currents and the power supplied or absorbed by each circuit element.

3. Use nodal analysis to find voltage across 5Ω resistor, for the network shown below.



Step-1 : Identify the no. of nodes in the given circuit.



Step-2 : Apply KCL at each node in the circuit except reference node.

Let us assume potential at node A is V_A and potential at node B is V_B as shown.

Applying KCL at node-A,

$$3.1 = \frac{V_A}{2} + \frac{V_A - V_B}{5}$$

$$\therefore 0.7 V_A - 0.2 V_B = 3.1 \quad \dots\dots (1)$$

Applying KCL at node-B

$$\frac{V_B}{1} + \frac{V_B - V_A}{5} = 1.4$$

$$-0.2 V_A + 1.2 V_B = 1.4 \quad \dots\dots (2)$$

From equations (1) and (2),

$$\begin{bmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3.1 \\ 1.4 \end{bmatrix}$$

$$\Delta G = \begin{vmatrix} 0.7 & -0.2 \\ -0.2 & 1.2 \end{vmatrix} = 0.8 \quad \Delta G_1 = \begin{vmatrix} 3.1 & -0.2 \\ 1.4 & 1.2 \end{vmatrix} = 4 \quad \Delta G_2 = \begin{vmatrix} 0.7 & 3.1 \\ -0.2 & 1.4 \end{vmatrix} = 1.6$$

Using cramer's rule,

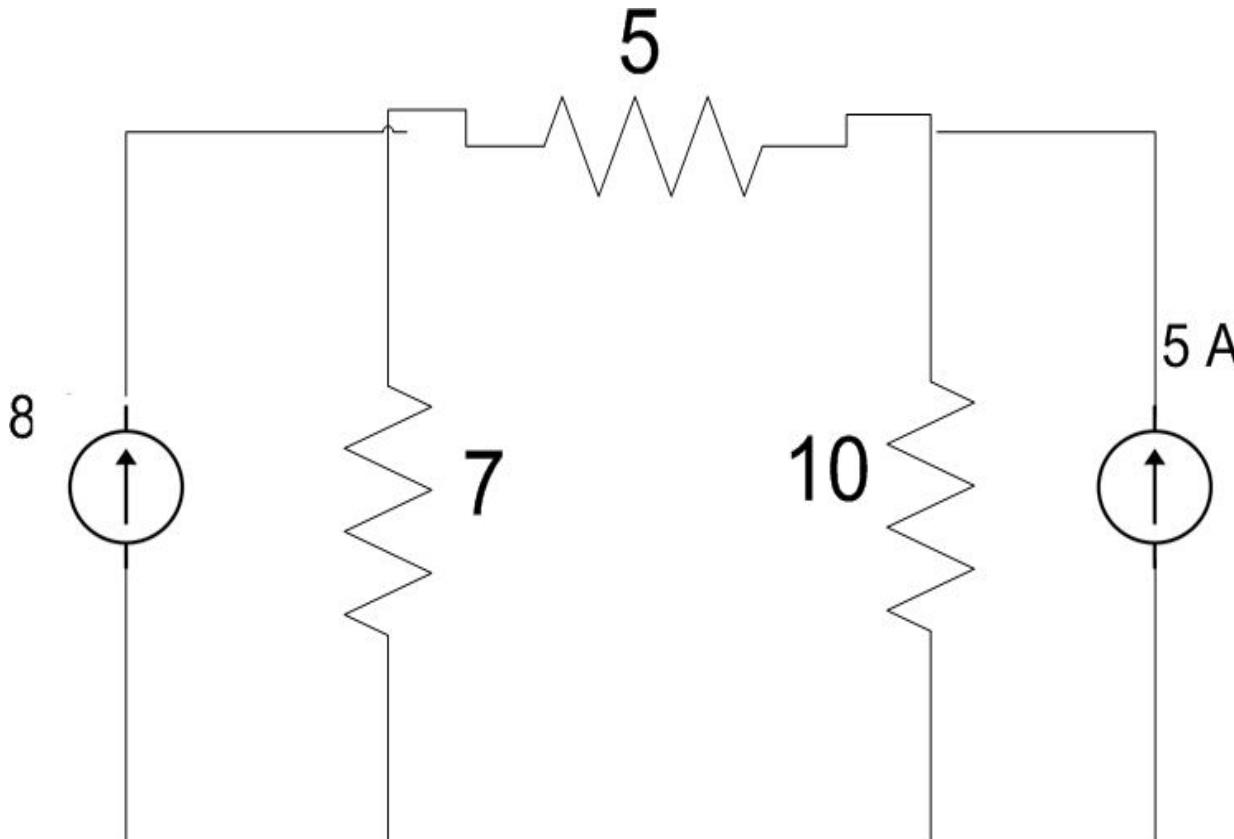
$$\therefore V_A = \frac{\Delta G_1}{\Delta G} = \frac{4}{0.8} = 5 \text{ volts}$$

$$V_B = \frac{\Delta G_2}{\Delta G} = \frac{1.6}{0.8} = 2 \text{ volts}$$

Hence, the voltage across 5Ω resistor is

$$\begin{aligned} V_5 &= V_A - V_B \\ &= \boxed{3 \text{ volt}} \end{aligned}$$

- Use nodal analysis to find voltage across 5Ω resistor, for the network shown below.

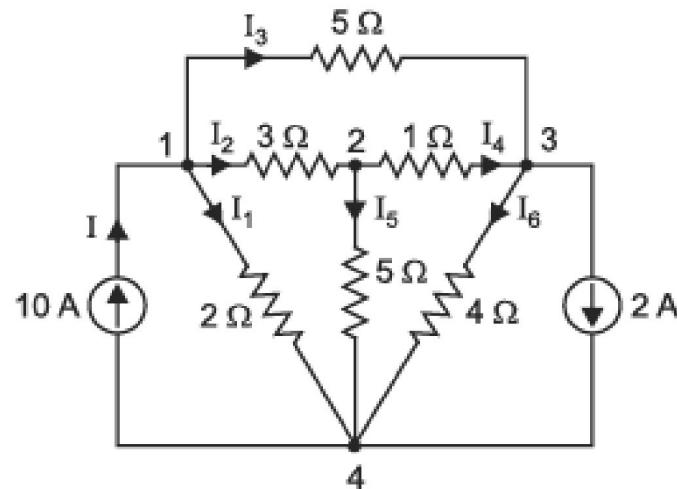
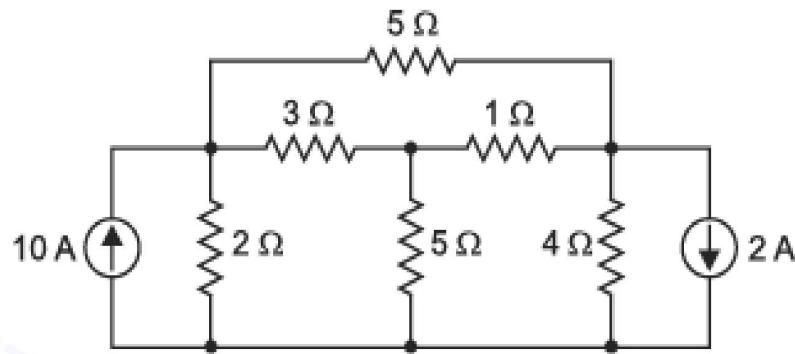


$$\mathbf{VA=54.09 \text{ V}}$$

$$\mathbf{VB=52.27 \text{ V}}$$

$$\mathbf{V5=1.82 \text{ V}}$$

Example 3.10. Use nodal analysis to find the currents in various resistors of the circuit shown in Fig. 3.23 (i).



Solution. The given circuit is redrawn in Fig. 3.23 (ii) with nodes marked 1, 2, 3 and 4. Let us take node 4 as the reference node. We shall apply KCL at nodes 1, 2 and 3 to obtain the solution.

At node 1. Applying KCL, we have,

$$I_1 + I_2 + I_3 = I$$

$$\text{or } \frac{V_1}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{5} = 10$$

$$\text{or } 31V_1 - 10V_2 - 6V_3 = 300 \quad \dots(i)$$

At node 2. Applying *KCL*, we have,

$$I_2 = I_4 + I_5$$

or $\frac{V_1 - V_2}{3} = \frac{V_2 - V_3}{1} + \frac{V_2}{5}$

or $5V_1 - 23V_2 + 15V_3 = 0$

...(ii)

At node 3. Applying *KCL*, we have,

$$I_3 + I_4 = I_6 + 2$$

or $\frac{V_1 - V_3}{5} + \frac{V_2 - V_3}{1} = \frac{V_3}{4} + 2$

or $4V_1 + 20V_2 - 29V_3 = 40$

...(iii)

From eqs. (i), (ii) and (iii), $V_1 = \frac{6572}{545} \text{ V}$; $V_2 = \frac{556}{109} \text{ V}$; $V_3 = \frac{2072}{545} \text{ V}$

∴ Current $I_1 = \frac{V_1}{2} = \frac{6572}{545} \times \frac{1}{2} = 6.03 \text{ A}$

Current $I_2 = \frac{V_1 - V_2}{3} = \frac{1}{3} \left[\frac{6572}{545} - \frac{556}{109} \right] = 2.32 \text{ A}$

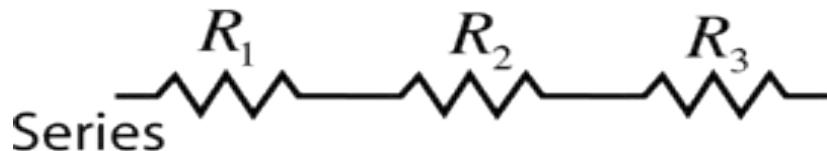
Current $I_3 = \frac{V_1 - V_3}{5} = \frac{1}{5} \left[\frac{6572}{545} - \frac{2072}{545} \right] = 1.65 \text{ A}$

$$\text{Current } I_4 = \frac{V_2 - V_3}{1} = \frac{556}{109} - \frac{2072}{545} = 1.3\text{A}$$

$$\text{Current } I_5 = \frac{V_2}{5} = \frac{556}{109} \times \frac{1}{5} = 1.02\text{A}$$

$$\text{Current } I_6 = \frac{V_3}{4} = \frac{2072}{545} \times \frac{1}{4} = 0.95\text{A}$$

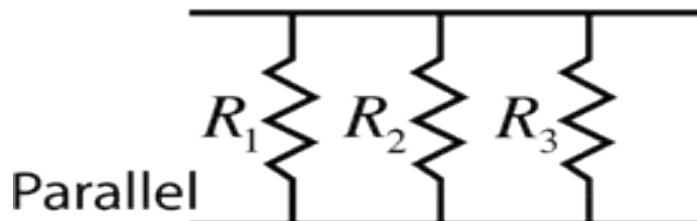
Series- Parallel Circuits



$$R_{equivalent} = R_1 + R_2 + R_3 + \dots$$

$$R_{equivalent} = \frac{V}{I} = \frac{V_1 + V_2 + V_3 + \dots}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \dots = R_1 + R_2 + R_3 + \dots$$

Series key idea: The current is the same in each resistor by the current law.



$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

Parallel:

$$\frac{V}{R_{equivalent}} = I = I_1 + I_2 + I_3 + \dots = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \dots$$

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

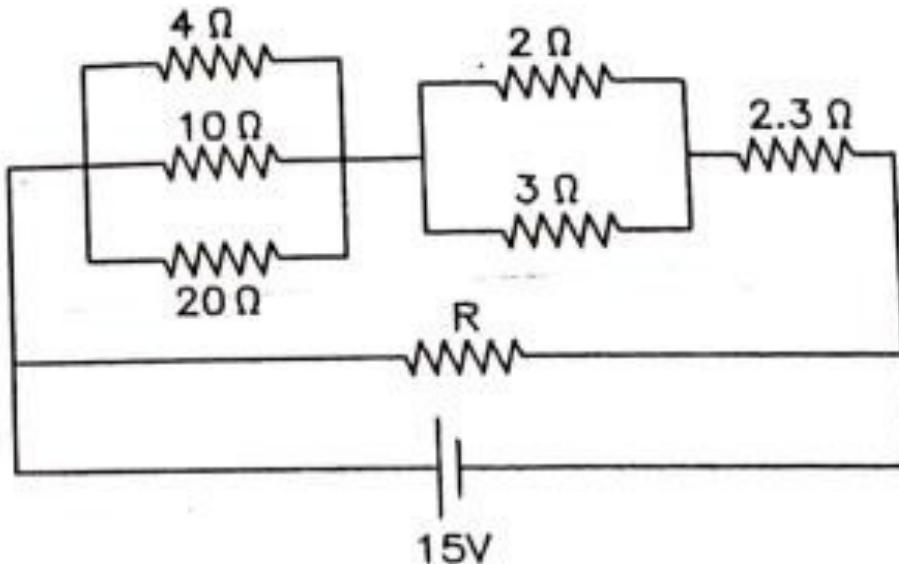
Parallel key idea: The voltage is the same across each resistor by the voltage law.

Parallel and Series - Formulas

	Capacitor 	Resistor 	Inductor*
Series	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$	$R = R_1 + R_2$	$L = L_1 + L_2$
Parallel	$C = C_1 + C_2$	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$
Fundamental Formula	$\Delta V = \frac{Q}{C}$	$\Delta V = IR$	$E_L = -L \frac{dI}{dt}$

* To be defined in a later chapter

4. Determine the value of R so that the current supplied by the battery is :



The equivalent of this combination is given by

$$\frac{1}{R_1} = \frac{1}{4} + \frac{1}{10} + \frac{1}{20} = \frac{8}{20}$$

$$\therefore R_1 = \frac{20}{8} = 2.5 \Omega$$

Similarly Resistances 2Ω and 3Ω are in parallel. Hence their equivalent resistance is given by

$$R_2 = \frac{2 \times 3}{2 + 3} = 1.2 \Omega$$

series combination = $2.5 + 1.2 + 2.3 = 6 \Omega$.

This resistance of 6Ω is in parallel with unknown resistance R . So total resistance of

$$\text{the circuit} = 6 \parallel R = \frac{6R}{R+6} \Omega \quad \dots \text{(i)}$$

Equating equations (i) and (ii)

Again total current = 5 A

$$\frac{6R}{R+6} = 3$$

and battery voltage = 15 V

$$R = 6 \Omega$$

,

$$\therefore \text{Total resistance} = \frac{15}{5} = 3 \Omega \quad \dots \text{(ii)}$$

Example 2.28. Find R_{AB} in the circuit shown in Fig. 2.34.

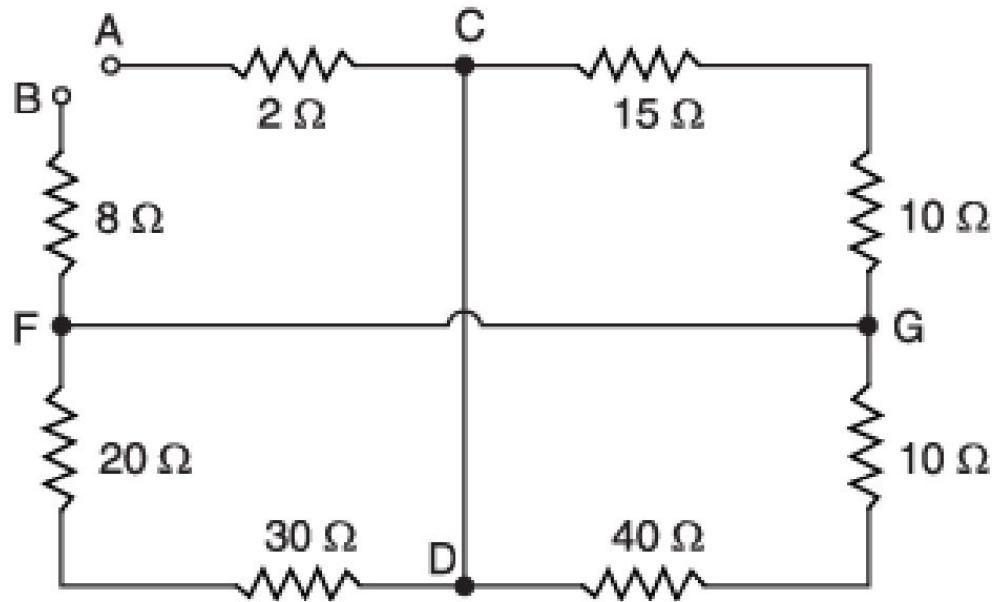
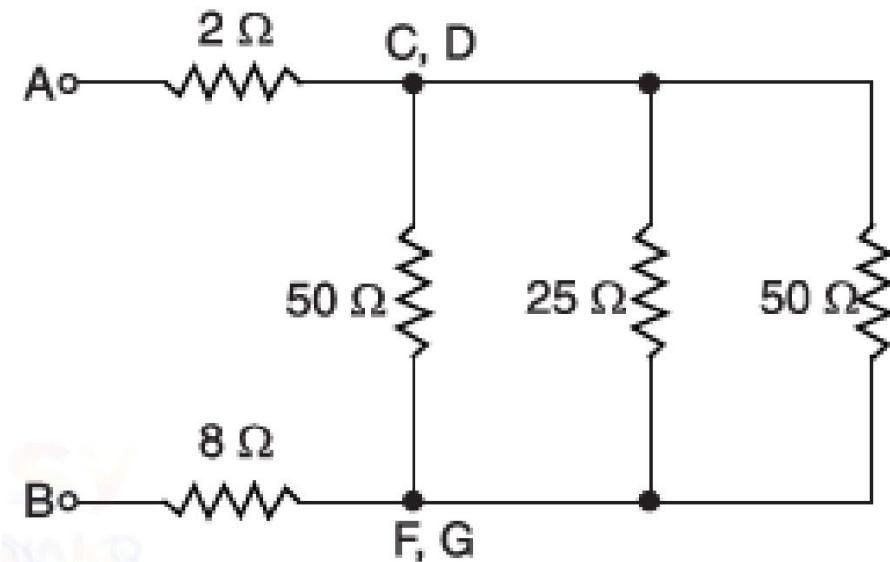
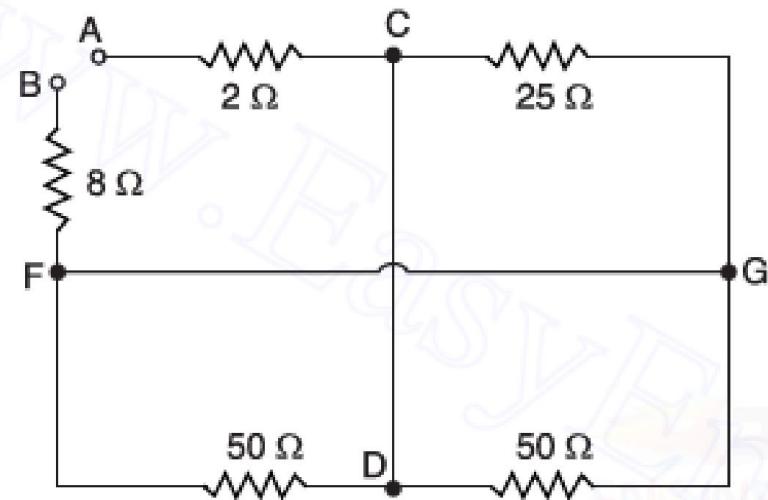


Fig. 2.34



$$\begin{aligned}R_{AB} &= 2 + (50 \parallel 25 \parallel 50) + 8 \\&= 2 + (25 \parallel 25) + 8 \\&= 2 + 12.5 + 8 = 22.5 \Omega\end{aligned}$$

Example 2.34. What is the equivalent resistance of the ladder network shown in Fig. 2.41?

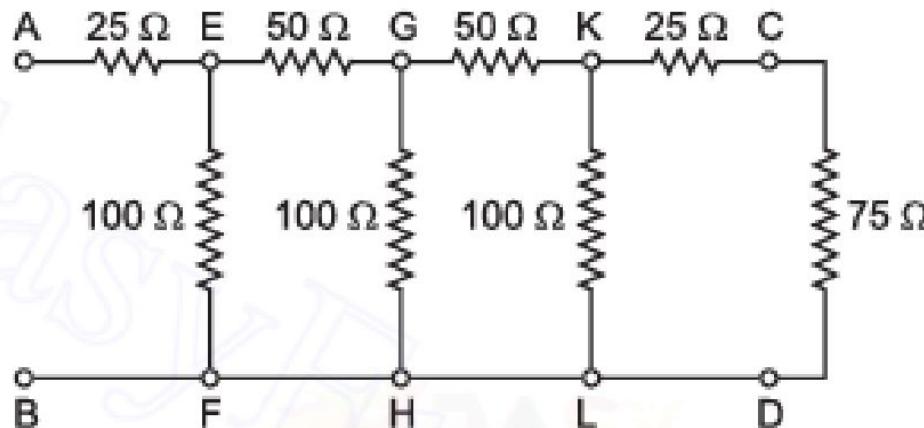
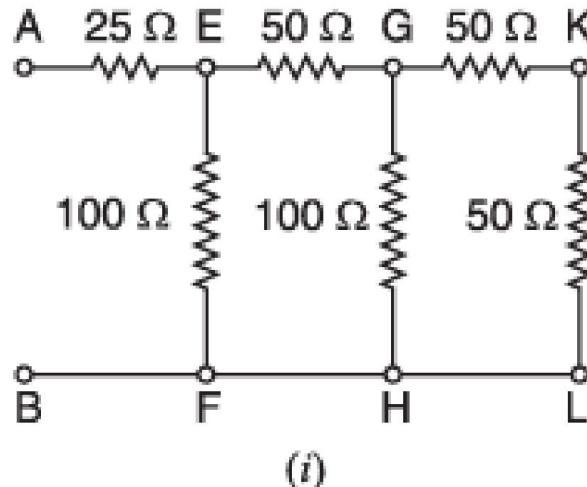


Fig. 2.41

Solution. Referring to Fig. 2.41, the resistance between points K and L is

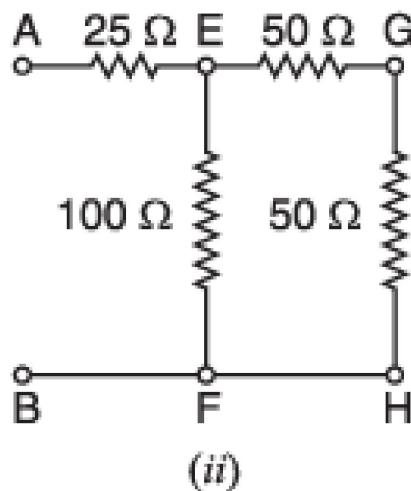
$$R_{KL} = (25 + 75) \Omega \parallel 100 \Omega = \frac{100 \times 100}{100 + 100} = 50 \Omega$$

The circuit of Fig. 2.41 then reduces to the one shown in Fig. 2.42 (i). Referring to Fig. 2.42 (i),

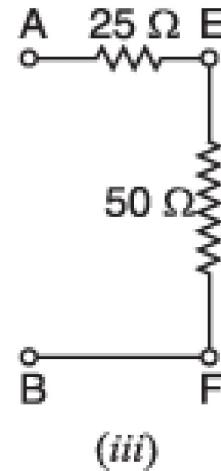


$$R_{GH} = (50 + 50) \Omega \parallel 100 \Omega = \frac{100 \times 100}{100 + 100} = 50 \Omega$$

The circuit of Fig. 2.42 (i) then reduces to the one shown in Fig. 2.42 (ii). Referring to Fig. 2.42 (ii),



(ii)



(iii)

$$R_{EF} = (50 + 50) \Omega \parallel 100 \Omega = \frac{100 \times 100}{100 + 100} = 50 \Omega$$

The circuit of Fig. 2.42 (ii) then reduces to the one shown in Fig. 2.42 (iii). Referring to Fig. 2.42 (iii),

Equivalent resistance of the ladder network
= $25 + 50 = 75 \Omega$

Example 2.33. All the resistances in Fig. 2.40 are in ohms. Find the effective resistance between the points A and B.

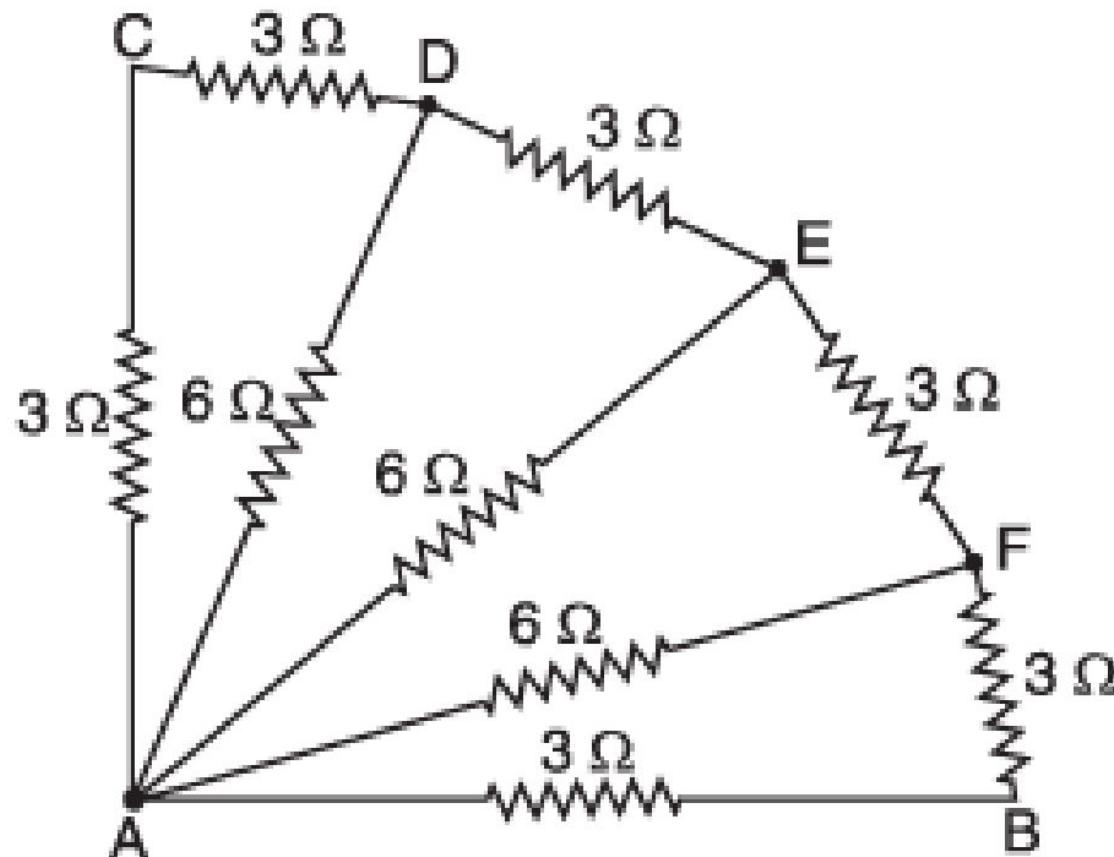


Fig. 2.40

Solution. Resistance between points A and D is

$$R_{AD} = (3 + 3) \Omega \parallel 6 \Omega = \frac{6 \times 6}{6 + 6} = 3 \Omega$$

$$R_{AE} = (R_{AD} + 3) \Omega \parallel 6 \Omega = \frac{6 \times 6}{6 + 6} = 3 \Omega$$

$$R_{AF} = (R_{AE} + 3) \Omega \parallel 6 \Omega = \frac{6 \times 6}{6 + 6} = 3 \Omega$$

\therefore Resistance between points A and B is

$$R_{AB} = (R_{AF} + 3) \Omega \parallel 3 \Omega$$

$$= \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2 \Omega$$

Example 2.30. An electrical network is arranged as shown in Fig. 2.38. Find the value of current in the branch AF.

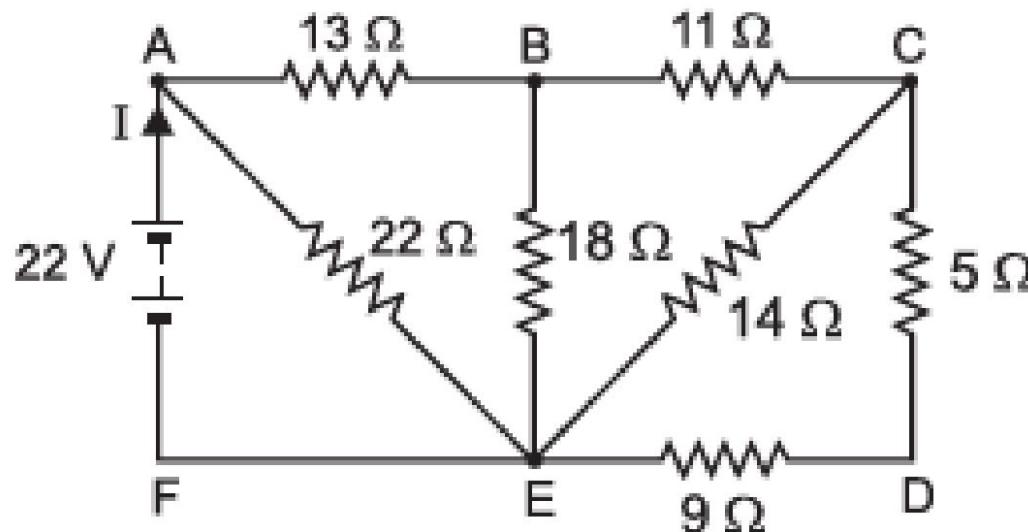


Fig. 2.38

Solution. Resistance between E and C,

$$R_{EC} = \frac{(5+9) \times 14}{(5+9)+14} = 7 \Omega$$

Resistance between B and E,

$$R_{BE} = \frac{(11+7) \times 18}{(11+7)+18} = 9 \Omega$$

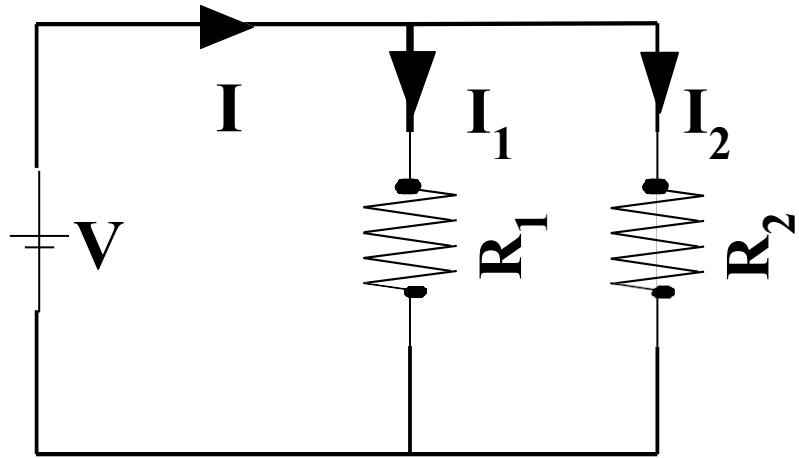
Resistance between A and E,

$$R_{AE} = \frac{(13+9) \times 22}{(13+9)+22} = 11 \Omega$$

i.e., Total circuit resistance, $R_T = 11 \Omega$

∴ Current in branch AF, $I = V/R_T = 22/11 = 2 \text{ A}$

Current Division in Parallel Circuits

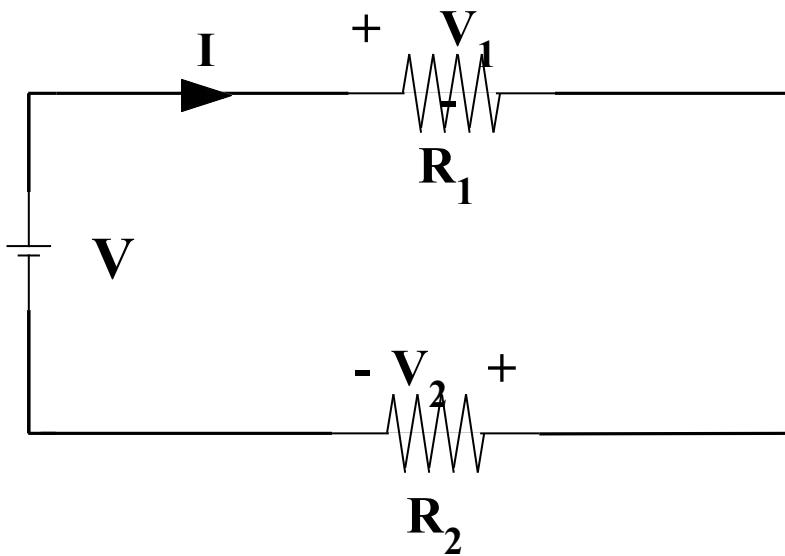


$$I = I_1 + I_2$$

$$I_1 = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \frac{R_1}{R_1 + R_2}$$

Voltage Division in Series Circuits



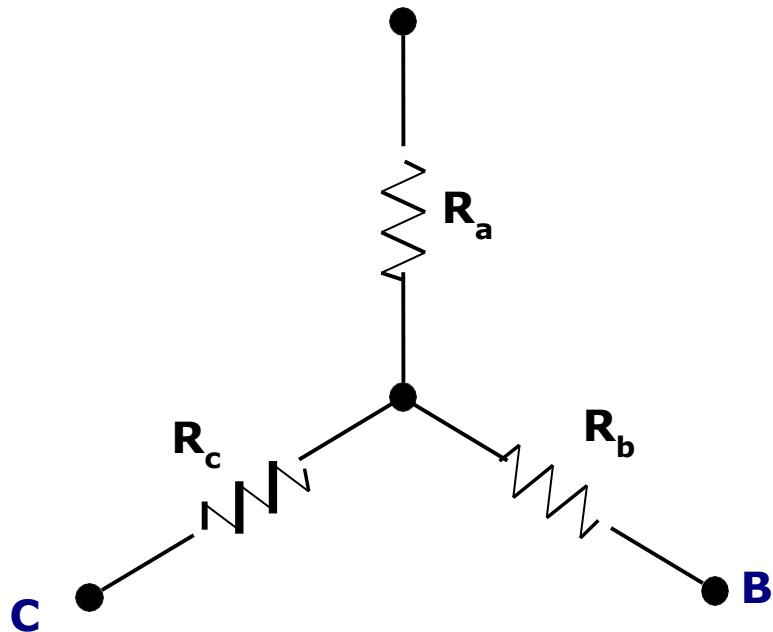
$$V = V_1 + V_2$$

$$V_1 = V \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \frac{R_2}{R_1 + R_2}$$

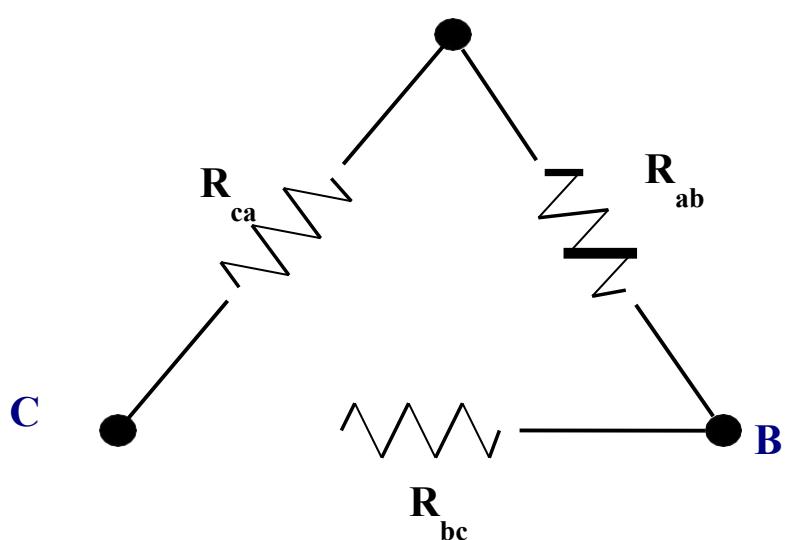
Star- Delta Transformation

A



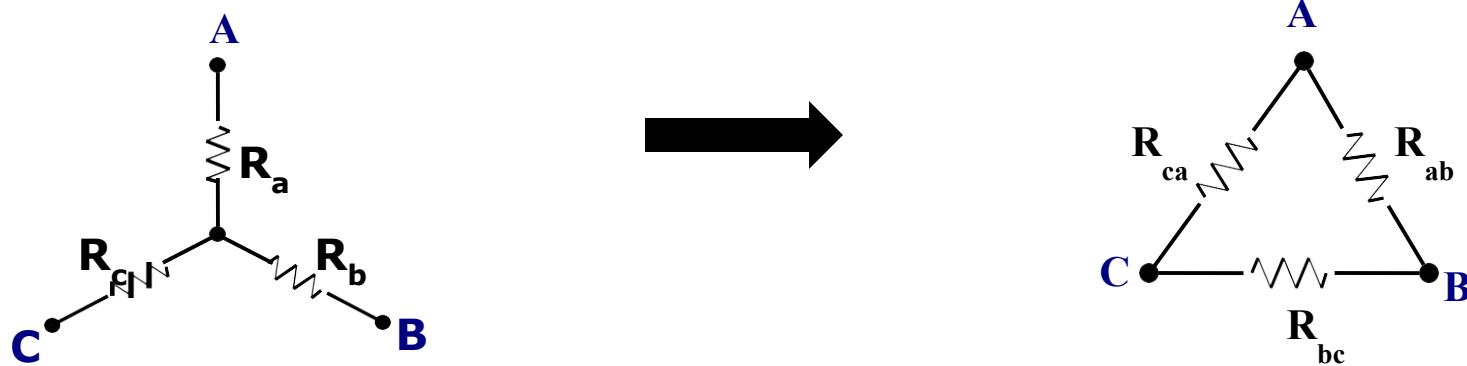
Star Connection

A



Delta Connection

Star to Delta Transformation:

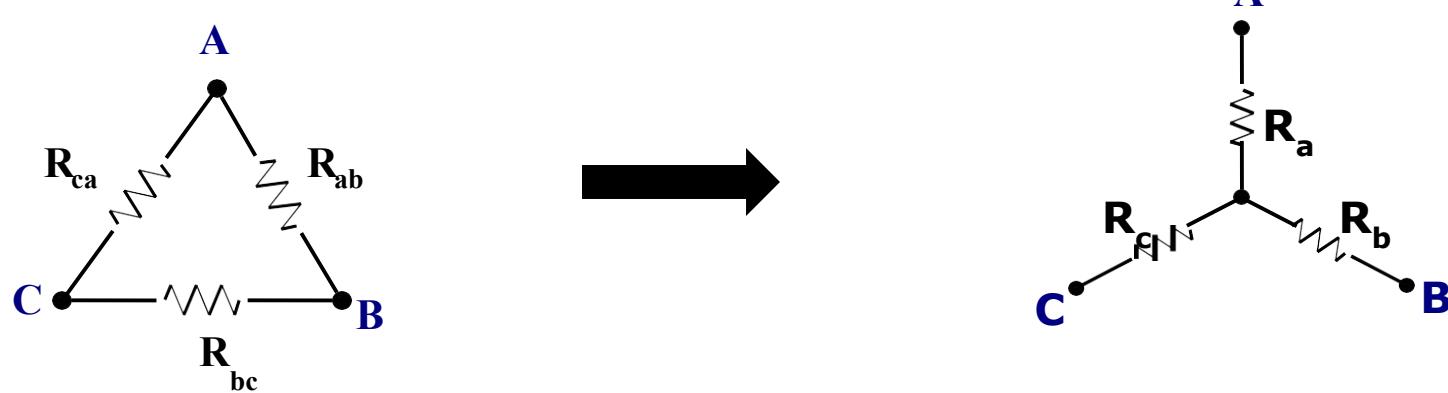


$$R_{ab} = R_a + R_b + \frac{R_a R_b}{R_c}$$

$$R_{bc} = R_b + R_c + \frac{R_b R_c}{R_a}$$

$$R_{ca} = R_c + R_a + \frac{R_c R_a}{R_b}$$

Delta to Star Transformation:

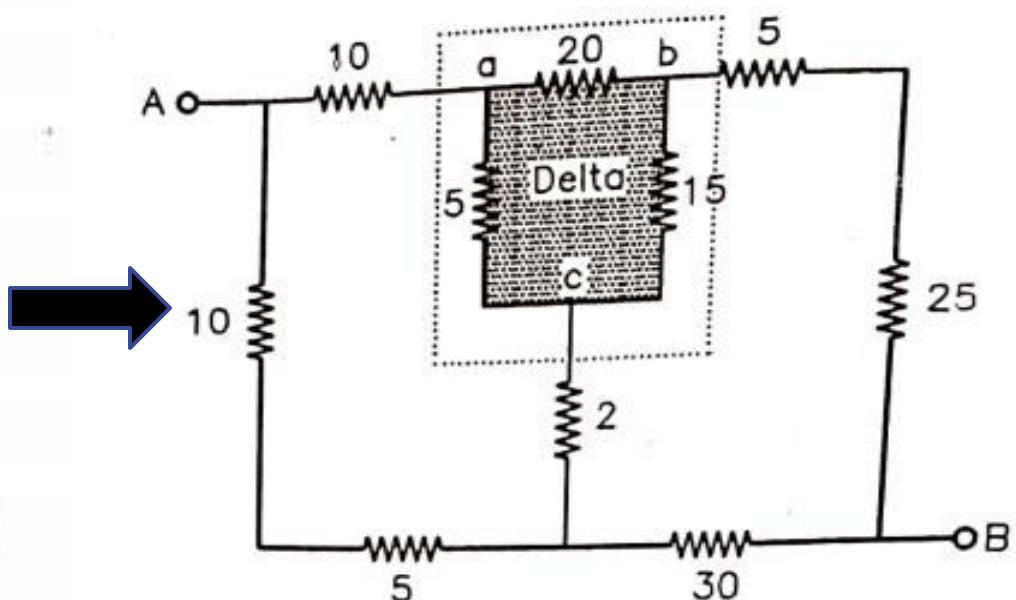
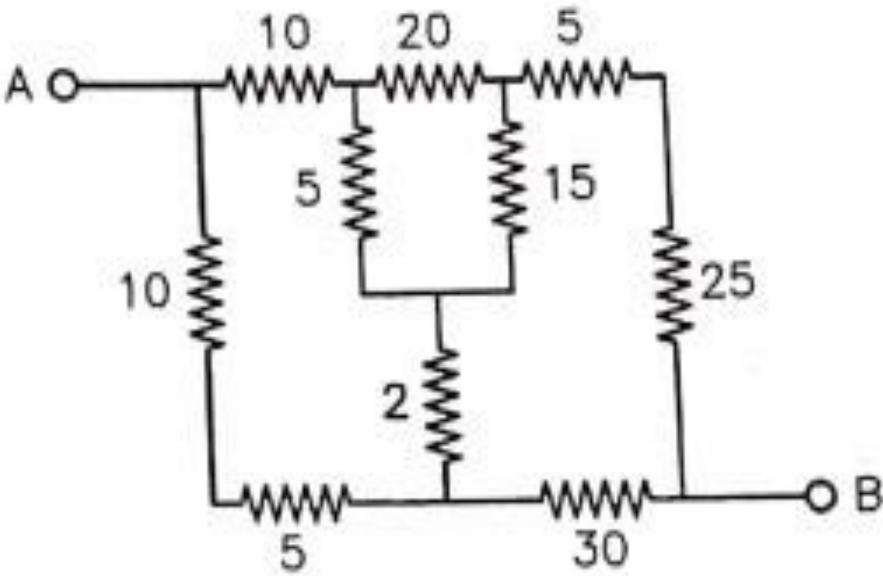


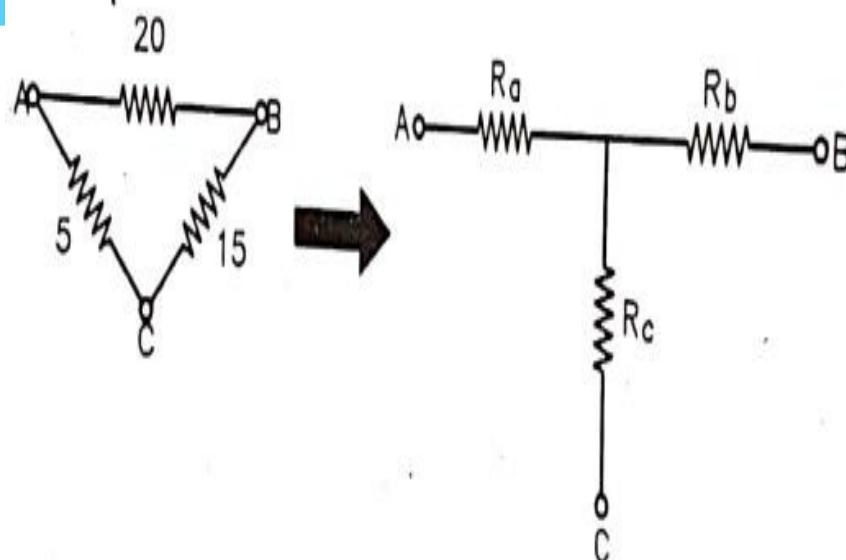
$$R_a = \frac{R_{ab} R_{ca}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_b = \frac{R_{bc} R_{ab}}{R_{ab} + R_{bc} + R_{ca}}$$

$$R_c = \frac{R_{ca} R_{bc}}{R_{ab} + R_{bc} + R_{ca}}$$

1. Using star- delta transformation , determine the resistance between the terminal A & B in the given circuit.

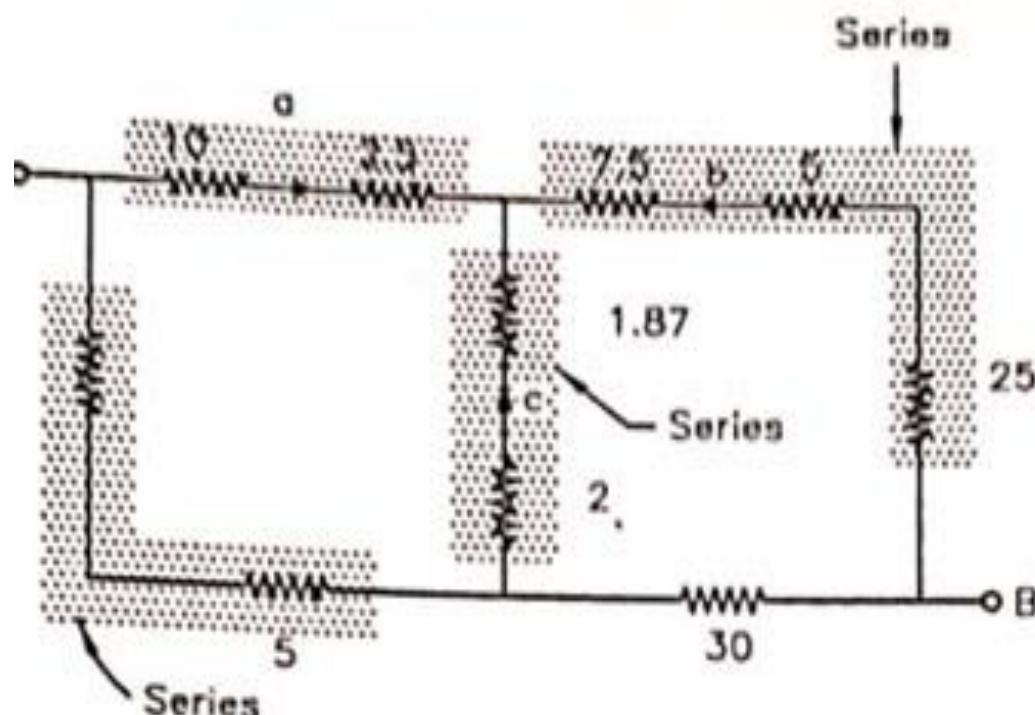
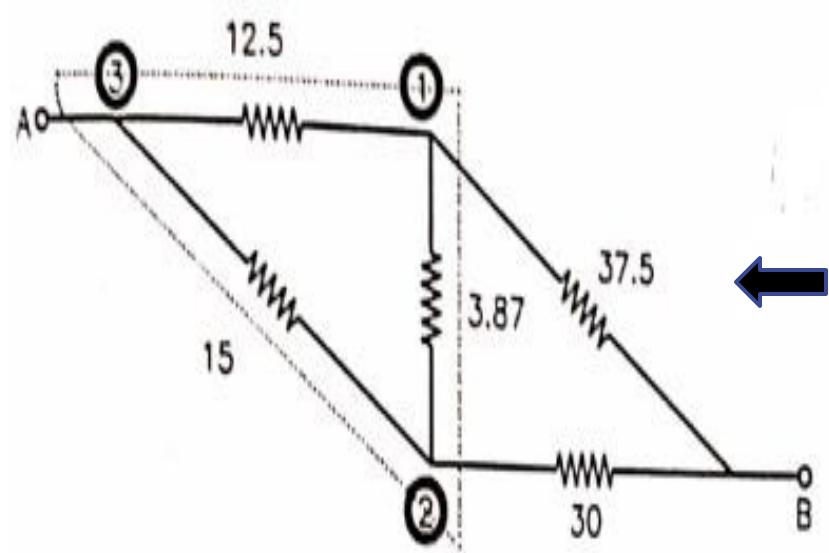




$$R_a = \frac{R_{ab} \times R_{ac}}{R_{ab} + R_{bc} + R_{ac}} = \frac{20 \times 5}{20 + 15 + 5} = \frac{100}{40} = 2.5 \Omega$$

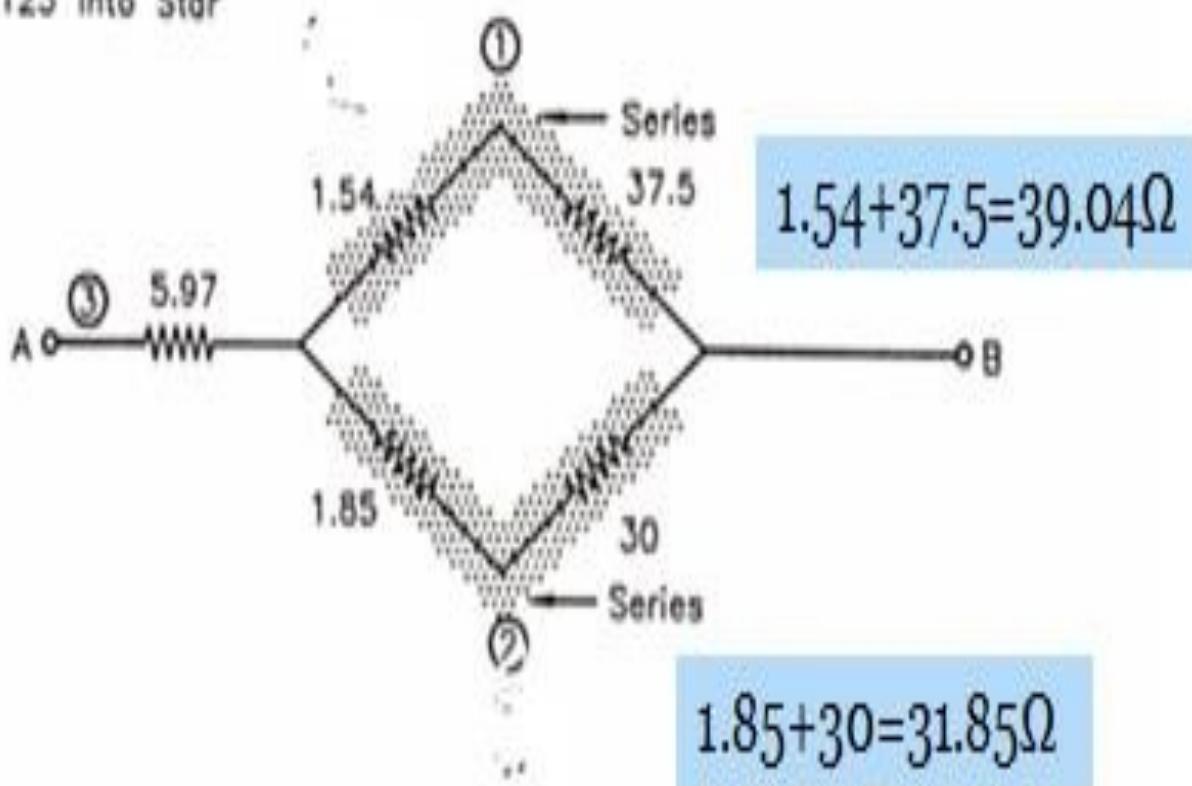
$$R_b = \frac{R_{ab} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}} = \frac{20 \times 15}{20 + 15 + 5} = \frac{300}{40} = 7.5 \Omega$$

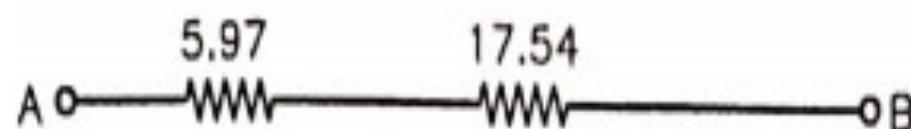
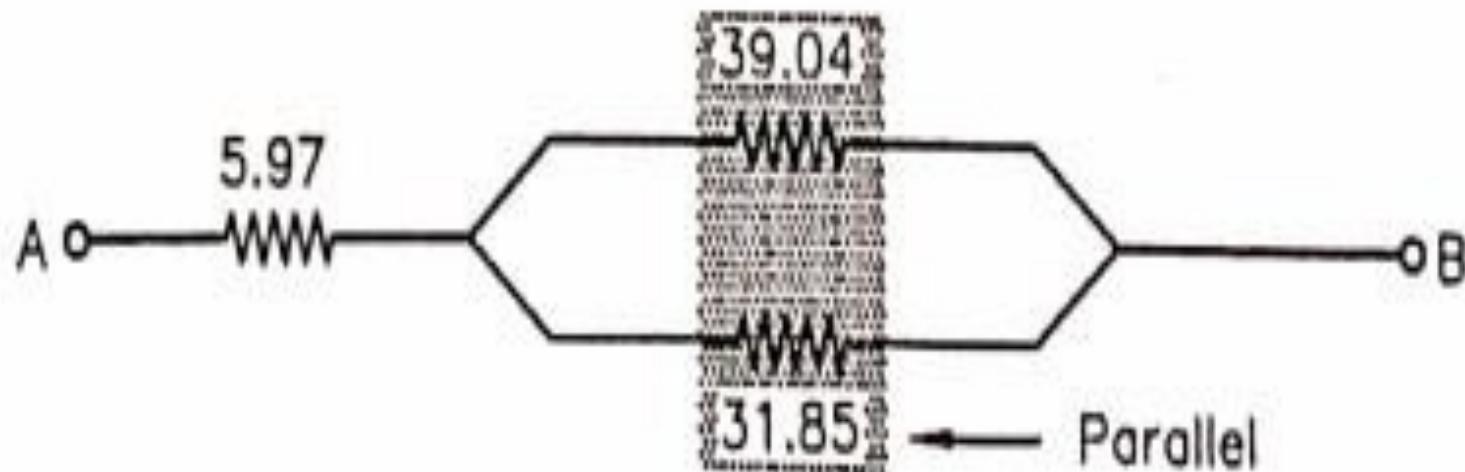
$$R_c = \frac{R_{ab} \times R_{bc}}{R_{ab} + R_{bc} + R_{ac}} = \frac{5 \times 15}{20 + 15 + 5} = \frac{75}{40} = 1.87 \Omega$$



Convert Δ 123 into Star

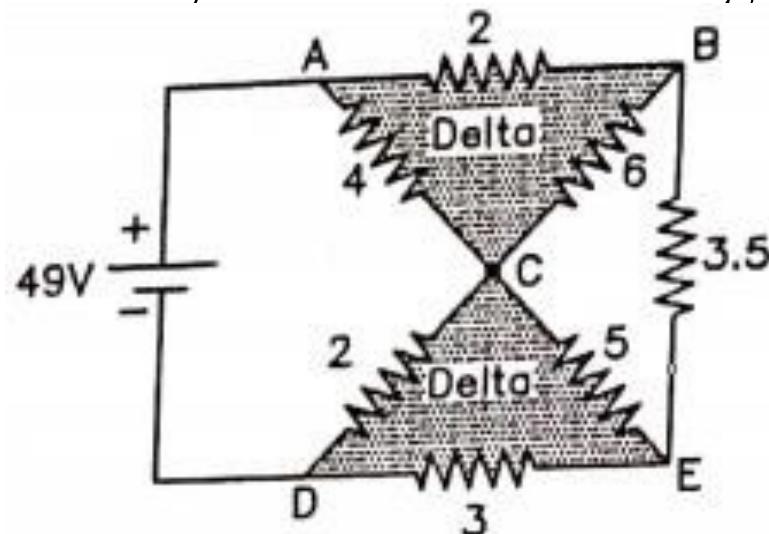
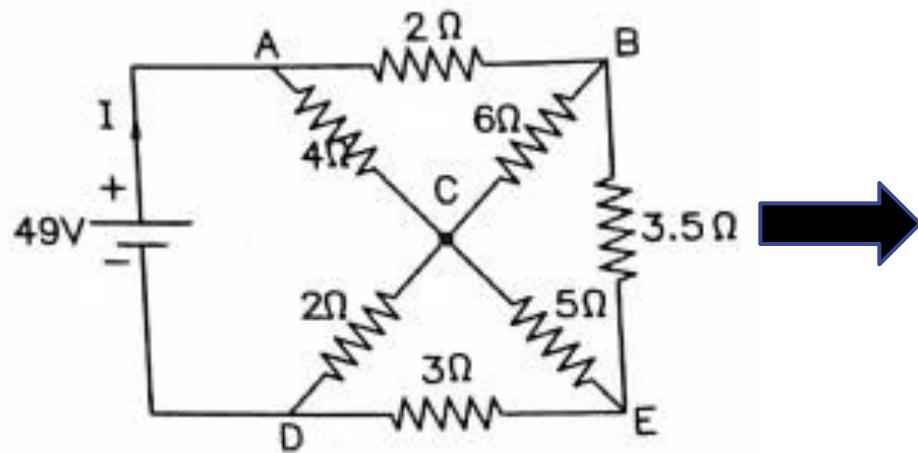
Convert Δ 123 into Star :





$$R_{AB} = 5.97 + 17.54 = \boxed{23.51 \Omega}$$

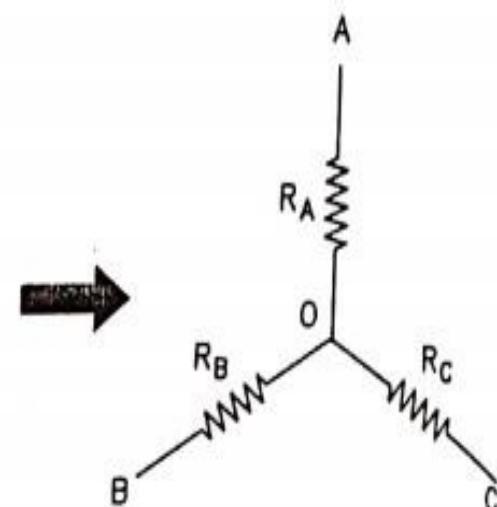
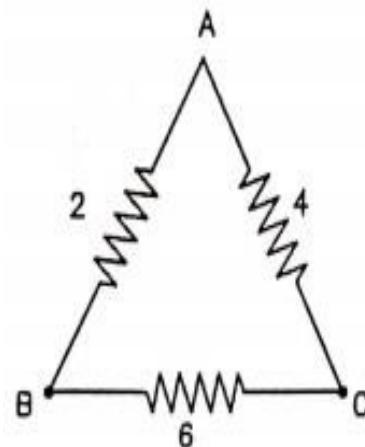
2. Calculate the current supplied by the battery in the network of fig shown below.

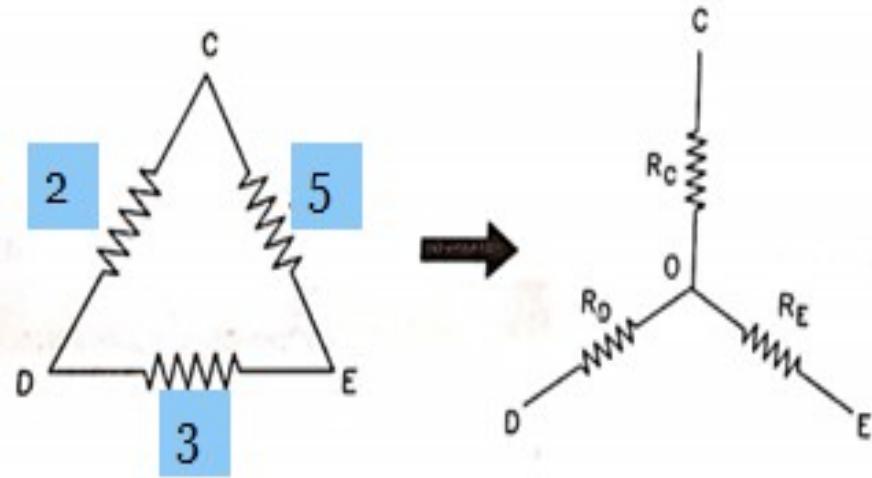


$$R_A = \frac{2 \times 4}{2+4+6} = \frac{2}{3} \Omega$$

$$R_B = \frac{2 \times 6}{2+4+6} = 1 \Omega$$

$$R_C = \frac{6 \times 4}{6+4+2} = 2 \Omega$$

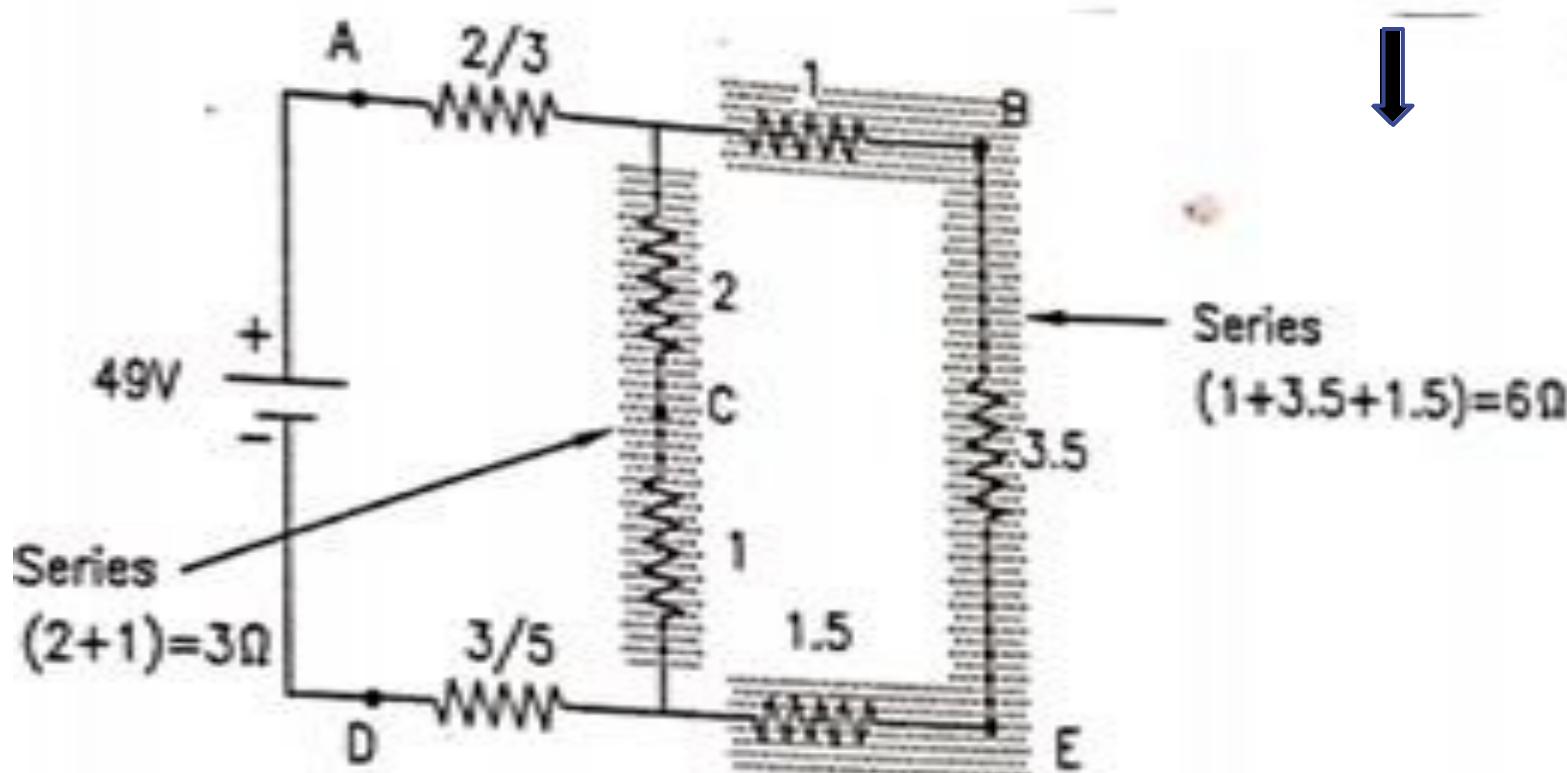


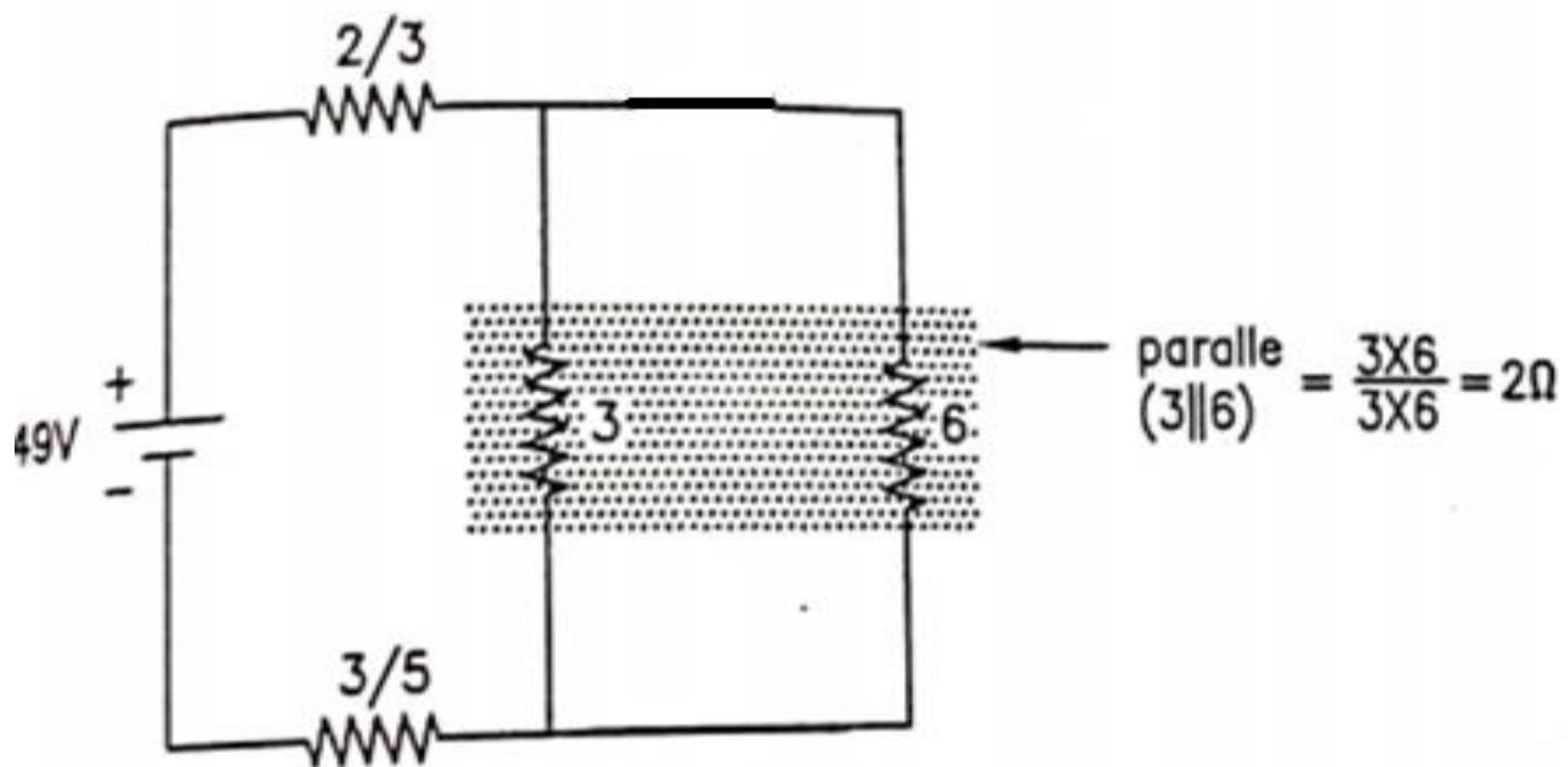


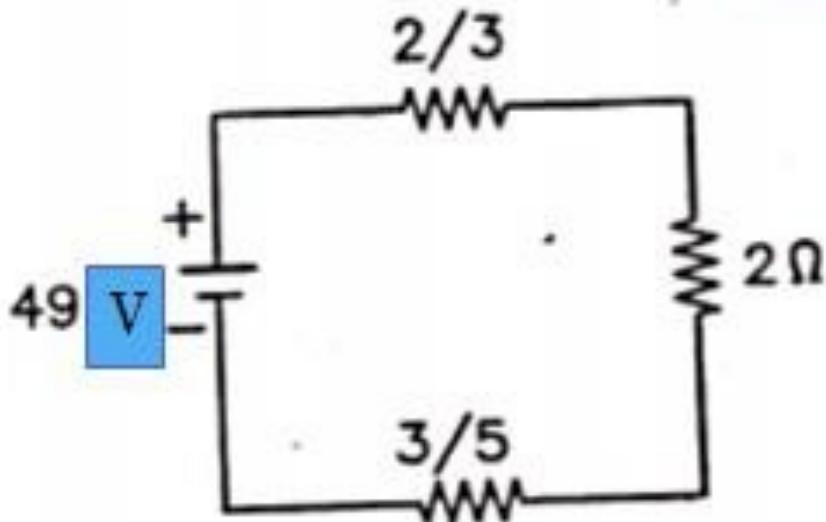
$$R_C = \frac{2 \times 5}{2 + 5 + 3} = 1\Omega$$

$$R_D = \frac{2 \times 3}{2 + 5 + 3} = \frac{3}{5}\Omega$$

$$R_E = \frac{3 \times 5}{2 + 5 + 3} = 1.5\Omega$$





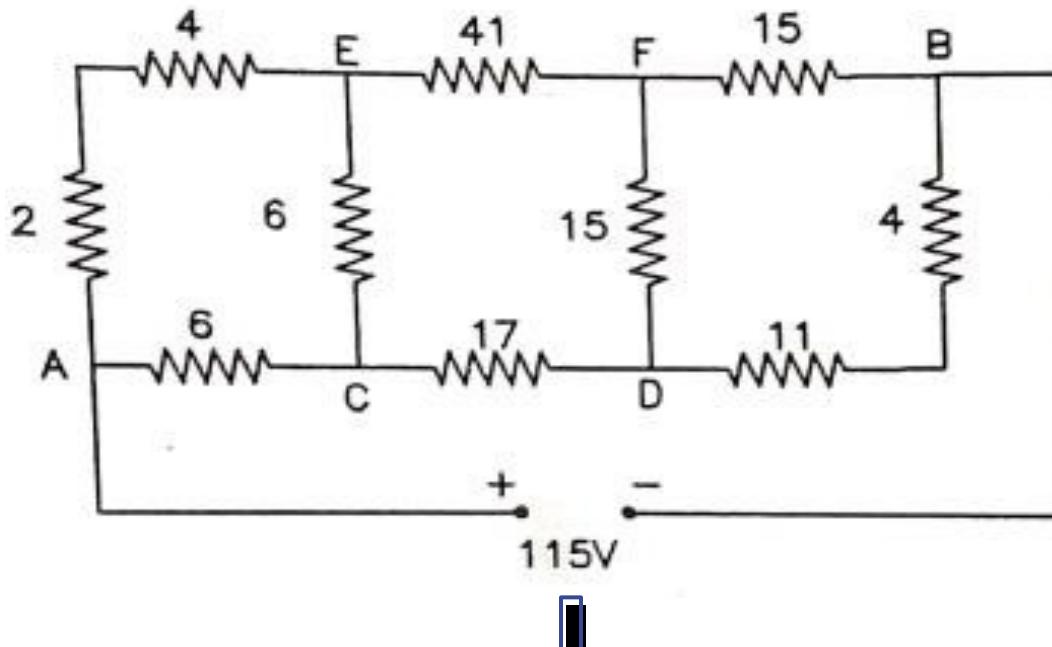


Now resistances $\frac{2}{3}\Omega$, 2Ω and $\frac{3}{5}$ are in series, so total resistance of the circuit is given by,

$$R_{eq} = \frac{2}{3} + 2 + \frac{3}{5} = \frac{49}{15} \Omega$$

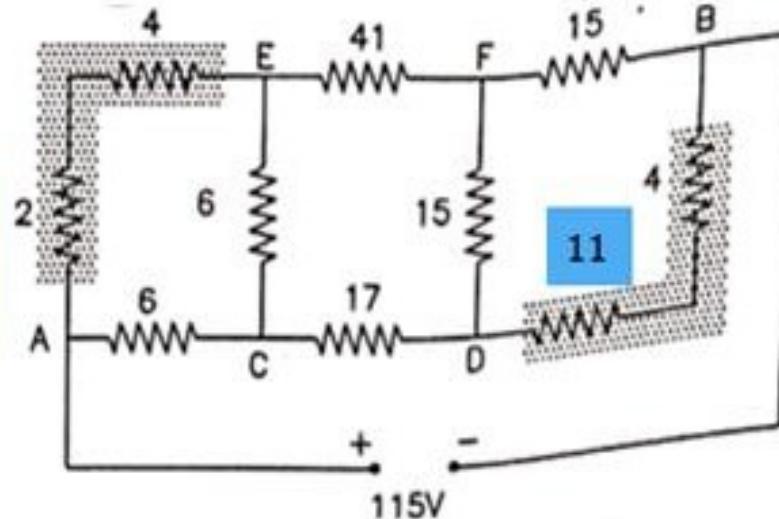
\therefore Current supplied by the battery $= \frac{49}{49/15} = \boxed{15 A}$

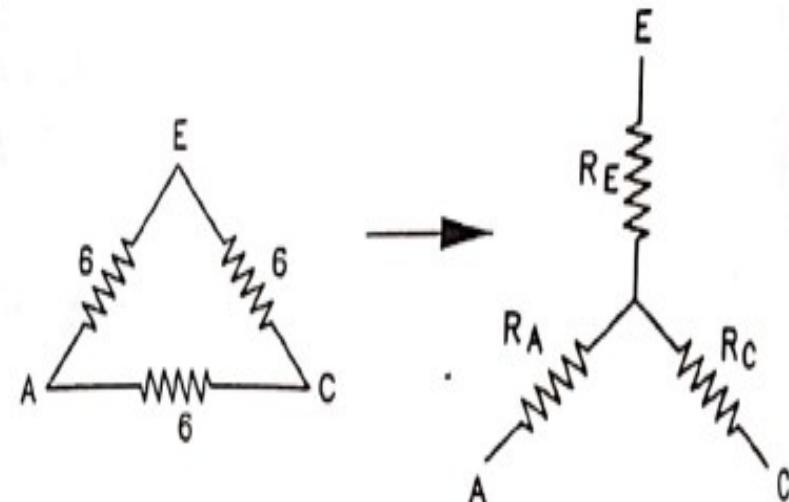
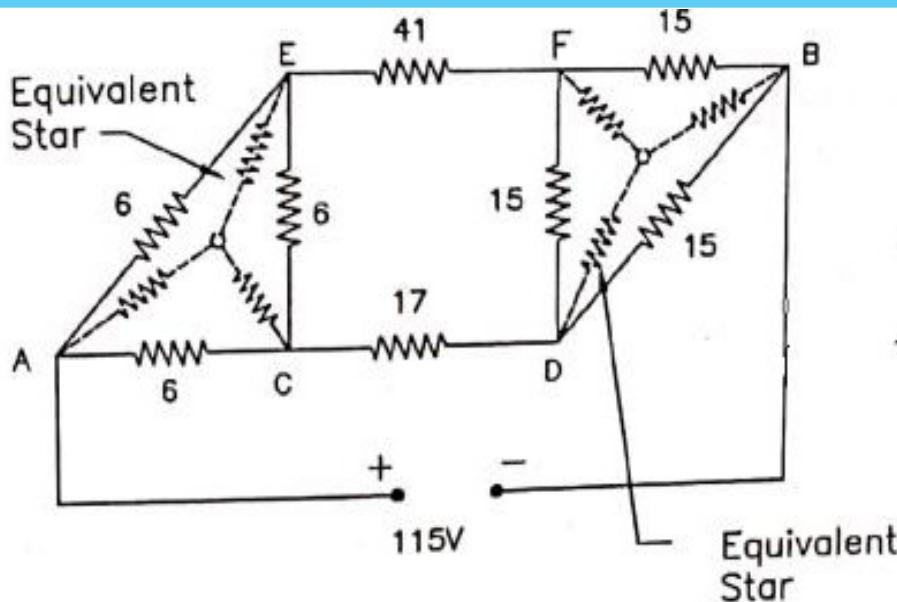
5. Determine the current in the $17\ \Omega$ resistor in the network shown below.



Resistor in series
($2+4=6\Omega$)

Resistor in series
($11+4=15\Omega$)

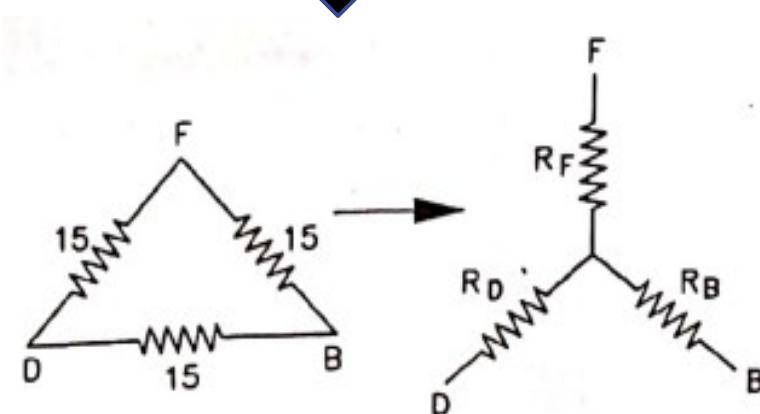


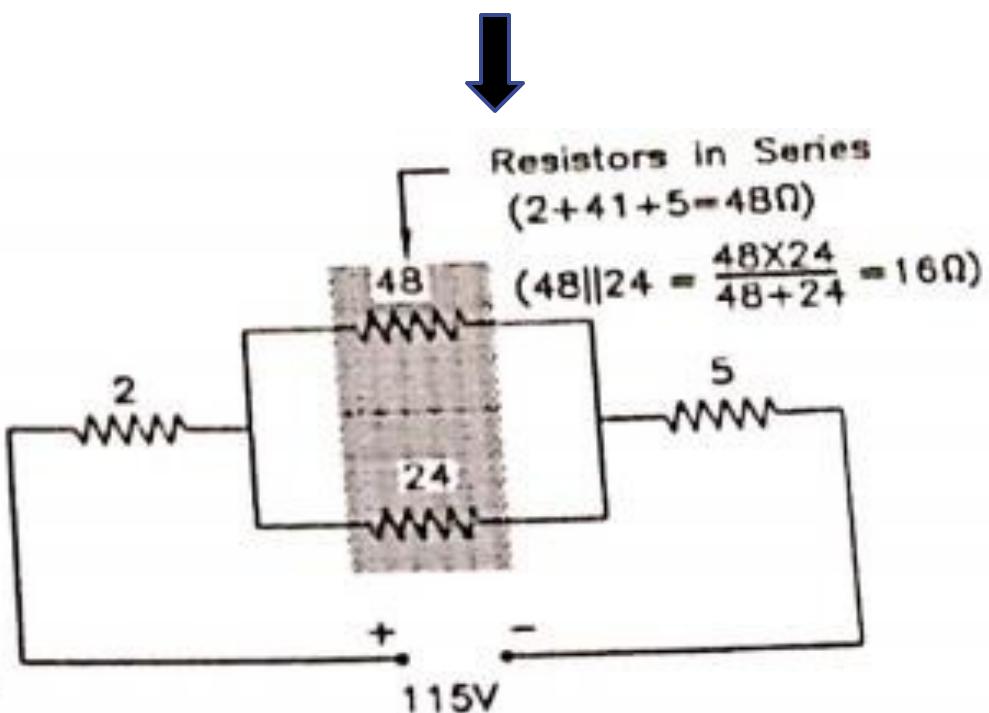
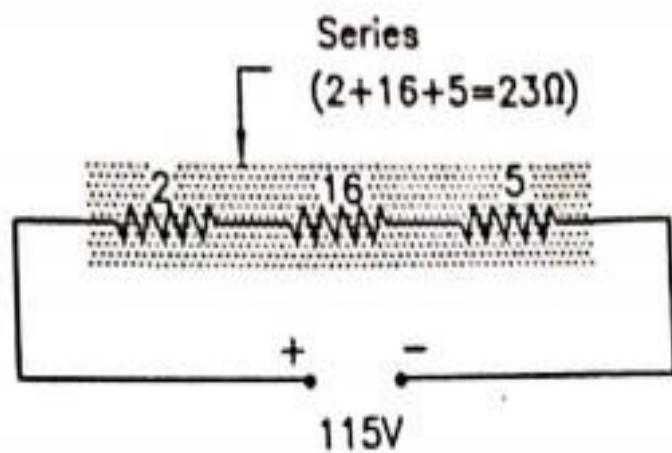
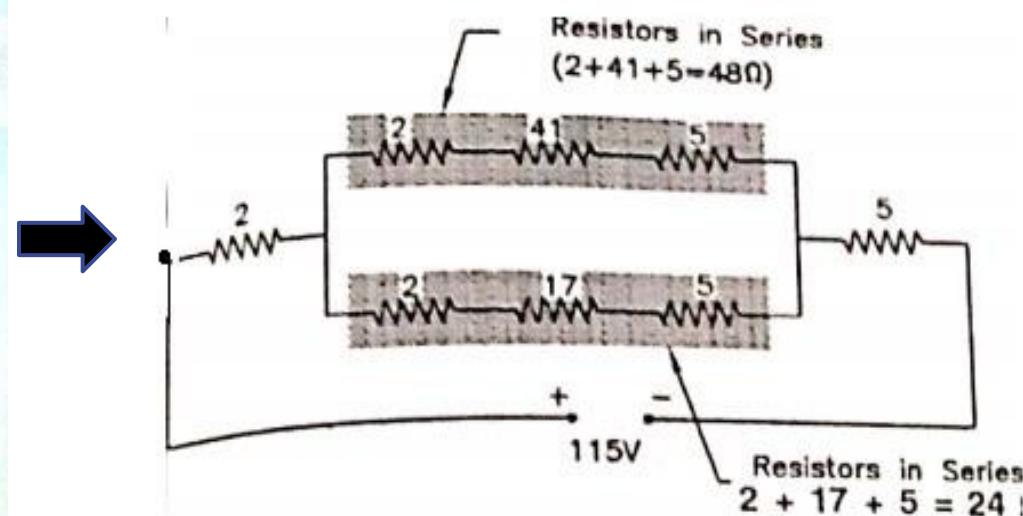
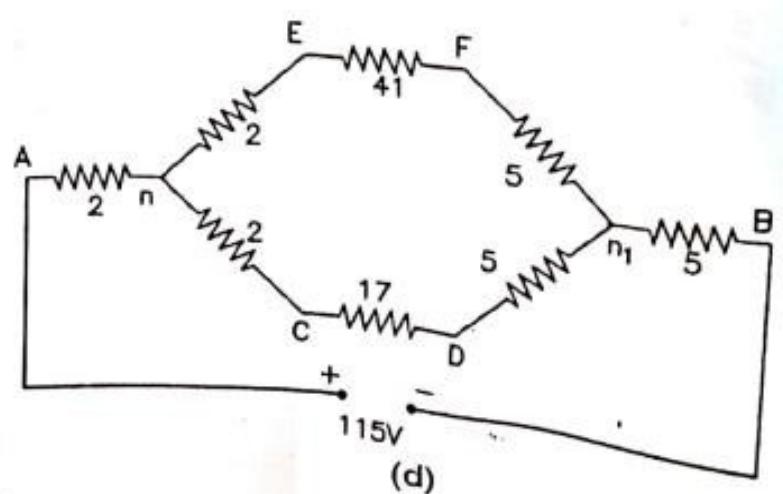


Applying Δ/Y transformation,

$$R_E = R_A = R_C = \frac{6 \times 6}{6 + 6 + 6} = 2 \Omega$$

$$R_D = R_F = R_B = \frac{15 \times 15}{15 + 15 + 15} = 5 \Omega$$





$$\text{The current taken by the circuit} = \frac{115}{23} = 5 \text{ A}$$

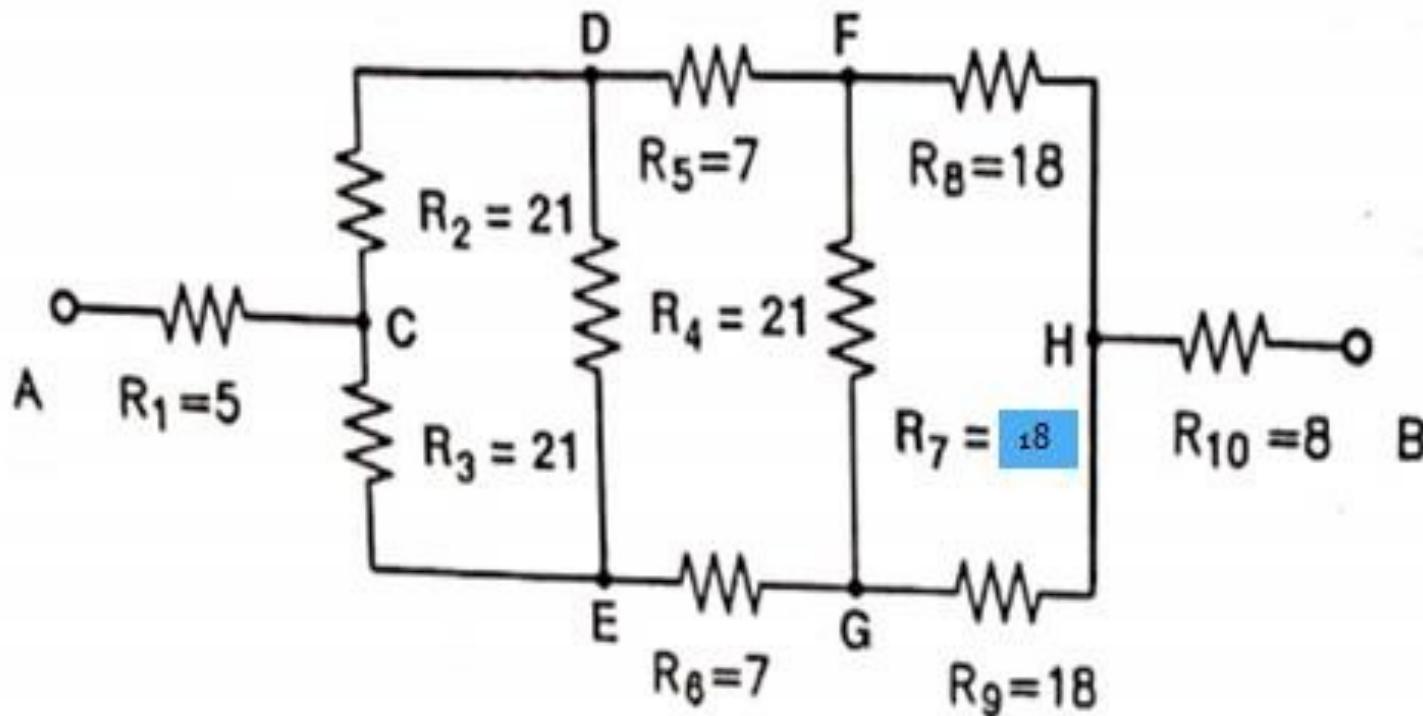
The current through $17\ \Omega$ resistor is the same as the current in $24\ \Omega$ resistor

The circuit current divides in $24\ \Omega$ and $48\ \Omega$. So current in $17\ \Omega$ resistor is given by,

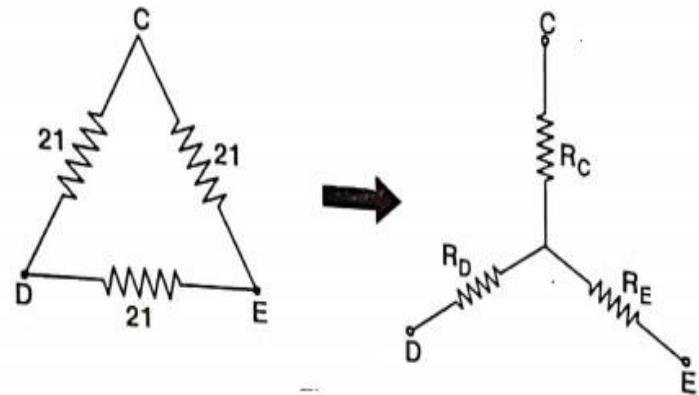
$$I = 5 \times \frac{48}{48 + 24}$$

$$I = 5 \times \frac{48}{48 + 24} = \frac{10}{3} \text{ A}$$

6. Find the resistance between terminals A and B as shown in fig below.

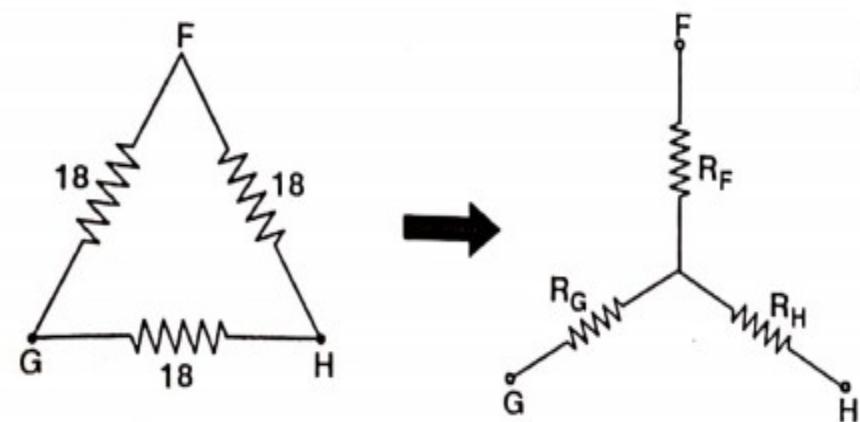


The two delta network (21, 21, 21 and 18, 18, 18) can be converted into star Δ-Y conversion (21,21,21)



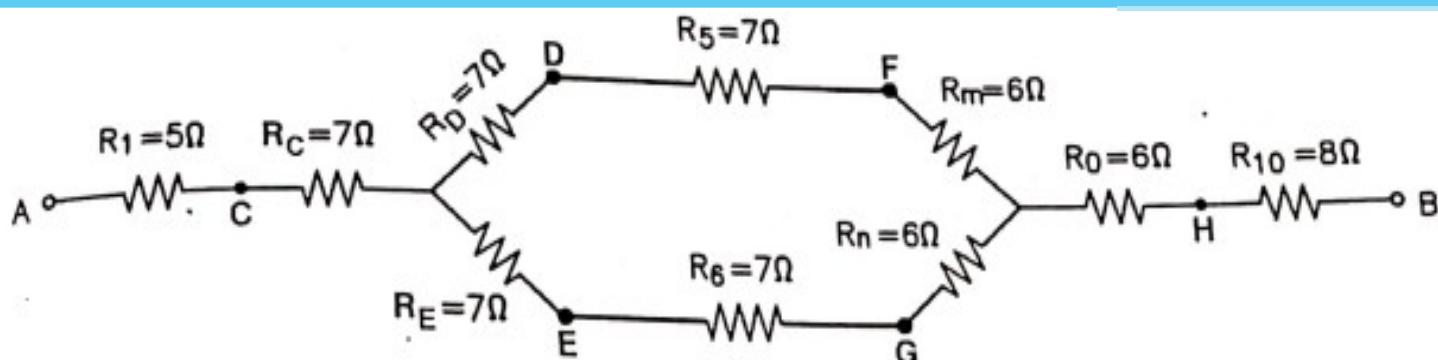
$$R_C = \frac{R_2 \times R_3}{R_2 + R_3 + R_4} = \frac{21 \times 21}{21 + 21 + 21} = 7 \Omega$$

Similarly R_D & R_E can be found which are also $R_D = 7 \Omega$ & $R_E = 7 \Omega$

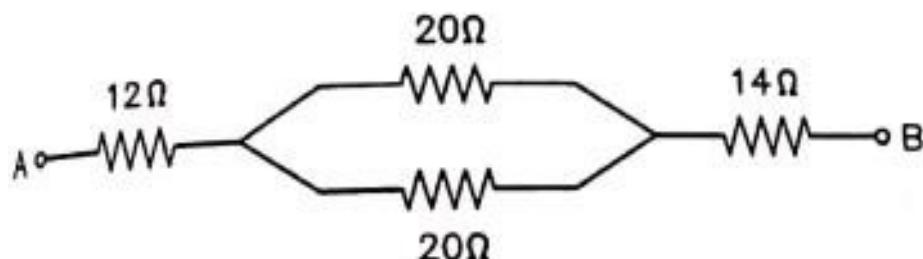


$$R_F = \frac{R_7 \times R_8}{R_7 + R_8 + R_9} = \frac{18 \times 18}{18 + 18 + 18} = 6 \Omega$$

Similarly R_G & R_H can be found which are also $R_F = 7 \Omega$ and $R_H = 7 \Omega$, So the circuit becomes like.



So the circuit between A & B Becomes



So the resistance between A & B is 36Ω

$$R_{AB} = 36\Omega$$

