ADVANCED MATHEMATICS FOR COMPUTER SCIENCE

Final Exam Matrix CS

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CS MATH | CS-AMCS

Matrix Solver Documentation.

-This notebook demonstrates how to solve a system of linear equations using two methods: Matrix Inverse Method and Gaussian Elimination. We'll first generate a random 3x4 matrix, then solve the system both manually and using Python, and finally verify our solutions.

1. Generate Random Matrix

Let's start by importing the necessary libraries and generating our random matrix:

import numpy as np
import matplotlib.pyplot as plt

Set random seed for reproducibility

```
np.random.seed(42)  # for reproducibility
```

Generate a 3x4 random matrix

```
matrix_3x4 = np.random.randint(1, 11, size=(3, 4))  # Range 1-10 for
easier manual calculations
A = matrix_3x4[:, :3]  # Coefficient matrix (first 3 columns)
b = matrix_3x4[:, 3]  # Right-hand side vector (last column)

print("Generated 3x4 Matrix:")
print(matrix_3x4)
print("\nCoefficient Matrix (A):")
print(A)
print("\nRight-hand Side Vector (b):")
print(b)
```

2. Manual Calculations

2.1 Matrix Inverse Method

Manual Calculation

2.1 Matrix Inverse Method

Given:

$$A = \begin{bmatrix} 7 & 8 & 7 \\ 5 & 6 & 3 \\ 9 & 3 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 10 \\ 7 \\ 5 \end{bmatrix}$$

Step 1: Find the determinant of A

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$$\begin{aligned} \det(A) &= (6\cdot 6 - 3\cdot 3) - 8(5\cdot 6 - 3\cdot 9) + 7(5\cdot 3 - 6\cdot 9) \\ &= 7(36-9) - 8(30-27) + 7(15-54) \\ &= 7(27) - 8(3) + 7(-39) \\ &= 189 - 24 - 273 \\ &= -108 \end{aligned}$$

Step 2: Find the adjugate matrix of A

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$$\operatorname{adj}(A) = \begin{bmatrix} (6\cdot 6 - 3\cdot 3) & (3\cdot 7 - 6\cdot 8) & (8\cdot 3 - 7\cdot 6) \\ (3\cdot 9 - 6\cdot 5) & (7\cdot 6 - 9\cdot 3) & (9\cdot 8 - 7\cdot 5) \\ (5\cdot 3 - 6\cdot 9) & (9\cdot 8 - 7\cdot 5) & (7\cdot 6 - 8\cdot 5) \end{bmatrix}$$

$$= \begin{bmatrix} 27 & -27 & -6 \\ 27 & -9 & 37 \\ -39 & 37 & -2 \end{bmatrix}$$

Step 3: Calculate

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$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

$$A^{-1} = \left(-\frac{1}{108}\right) \cdot \begin{bmatrix} 27 & 27 & -6 \\ 27 & -9 & 37 \\ -39 & 37 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -0.25 & 0.25 & 0.0556 \\ -0.25 & 0.0833 & -0.3426 \\ 0.3611 & -0.3426 & 0.0185 \end{bmatrix}$$

Step 4: Calculate x

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$$x = A^{-1} \cdot b$$

where $b = \begin{bmatrix} 10 \\ 7 \\ 5 \end{bmatrix}$

$$x = \begin{bmatrix} -0.25 & 0.25 & 0.0556 \\ -0.25 & 0.0833 & -0.3426 \\ 0.3611 & -0.3426 & 0.0185 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 7 \\ 5 \end{bmatrix}$$

The final A.

$$x = \begin{bmatrix} -0.472 \\ -3.63 \\ 1.3053 \end{bmatrix}$$

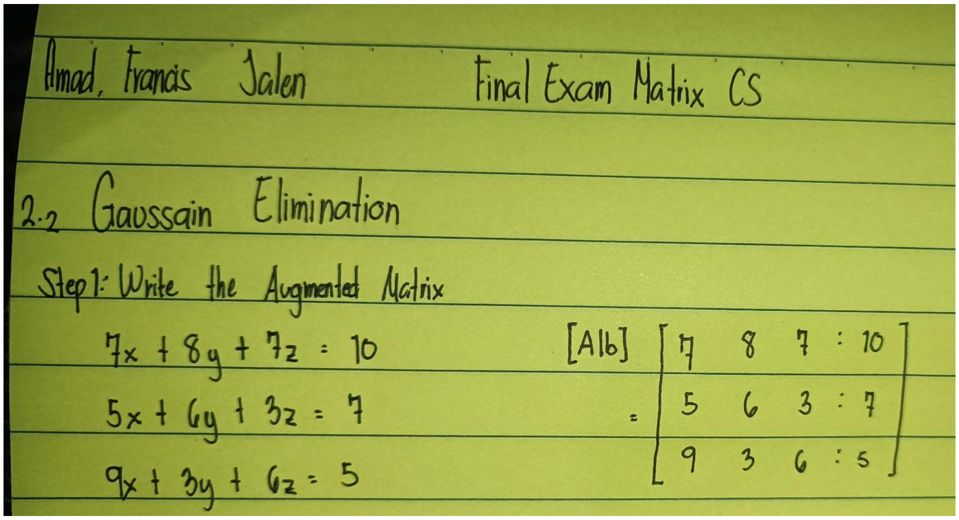
First row: $-0.25 \cdot 10 + 0.25 \cdot 7 + 0.0556 \cdot 5 = -2.5 + 1.75 + 0.278 = -0.472$

Second row: $-0.25 \cdot 10 + 0.0833 \cdot 7 - 0.3426 \cdot 5 = 2.5 + 0.5831 - 1.713 = -3.63$

third row: $0.3611 \cdot 10 + -0.3426 \cdot 7 + 0.0185 \cdot 5 = 3.611 - 2.3982 + 0.0925 = 1.3053$

2.2 Gaussian Elimination

Step 1: Write the augmented matrix



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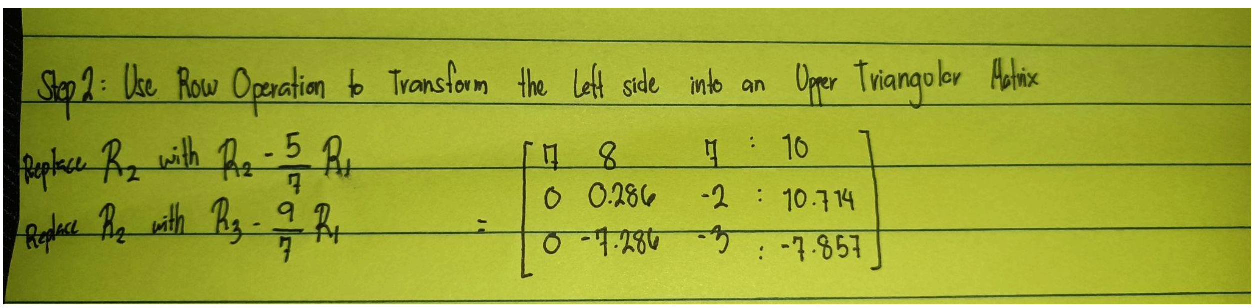
2.2 Gaussain Elimination

Step 1: Write the Augmented Matrix

$$7x + 8y + 7z = 10$$
$$5x + 6y + 3z = 7$$
$$9x + 3y + 6z = 5$$

$$[A|b] = \left[\begin{array}{ccc:c} 7 & 8 & 7 & 10 \\ 5 & 6 & 3 & 7 \\ 9 & 3 & 6 & 5 \end{array}\right]$$

Step 2: Use row operations to transform the left side into an upper triangular matrix



Step 2: Use Row Operation to Transform the Left side into an Upper Triangolar Matrix

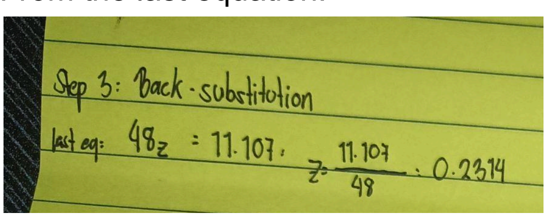
Replace R_2 with $R_2 - \frac{5}{7} R_1$

Replace R_2 with $R_3 - \frac{9}{7} R_1$

$$= \left[\begin{array}{ccc:c} 7 & 8 & 7 & 10 \\ 0 & 0.286 & -2 & 10.714 \\ 0 & -7.286 & -3 & -7.857 \end{array}\right]$$

Step 3: Back-substitution

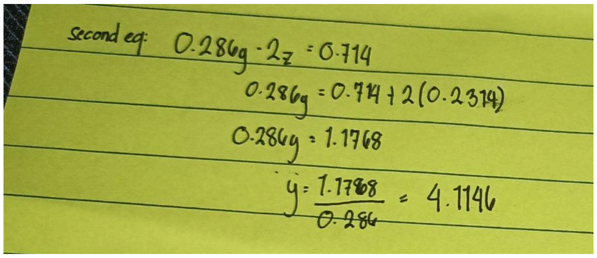
From the last equation:



Step 3: Back-substitution

last eq: $48z = 11.107$, $z = \frac{11.107}{48} = 0.2314$

From the second equation:



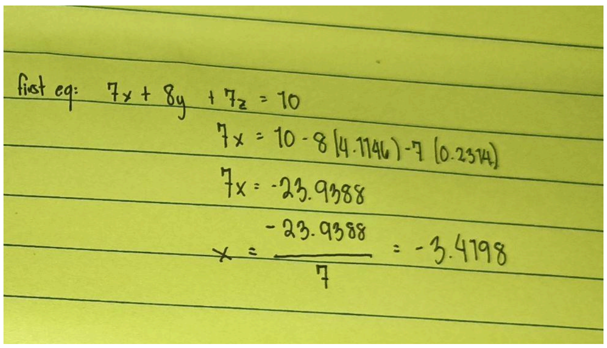
Second eq: $0.286y - 2z = 0.714$

$0.286y = 0.714 + 2(0.2314)$

$0.286y = 1.1768$

$y = \frac{1.1768}{0.286} = 4.1146$

From the first equation:



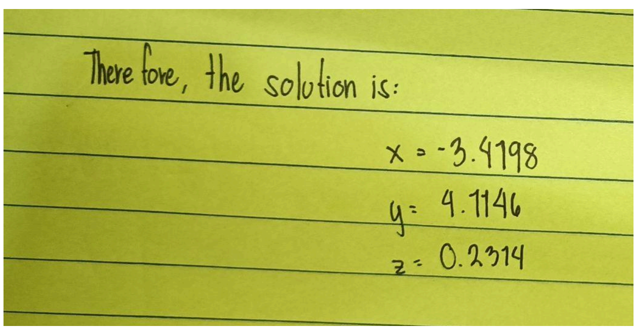
first eq: $7x + 8y + 7z = 10$

$7x = 10 - 8(4.1146) - 7(0.2314)$

$7x = -23.9388$

$x = \frac{-23.9388}{7} = -3.4198$

Therefore, the solution is:



Therefore, the solution is:

$x = -3.4198$

$y = 4.1146$

$z = 0.2314$

3. Python Solution

Now let's solve the system using Python and compare with our manual calculations:

Solve using Matrix Inverse Method

```
try:
    A_inv = np.linalg.inv(A)   # Compute A^-1
    x_inverse = np.dot(A_inv, b)   # x = A^-1 * b
    print("\nSolution using Matrix Inverse Method:")
    print(x_inverse)
except np.linalg.LinAlgError:
    print("\nMatrix A is singular and cannot be inverted.")
```

Solve using Gaussian Elimination

```
x_gaussian = np.linalg.solve(A, b)  # Direct Gaussian elimination
print("\nSolution using Gaussian Elimination (NumPy):")
print(x_gaussian)
```

4. Verification

Let's verify that our solution satisfies the equation Ax = b:

Verify the solution

```
b_check = np.dot(A, x_gaussian)  # Recompute b using A and the computed x
print("\nVerification (Ax = b):")
print(b_check)
print("\nOriginal b:")
print(b)
print("\nAre the results equal?", np.allclose(b_check, b))
```

5. Visualization

Finally, let's visualize our results:

Visualize the results

```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 5))

ax1.imshow(A, cmap='viridis')
ax1.set_title('Coefficient Matrix A')
for i in range(3):
    for j in range(3):
        ax1.text(j, i, A[i, j], ha='center', va='center', color='w')

ax2.bar(range(3), x_gaussian)
ax2.set_title('Solution Vector x')
ax2.set_xlabel('Variable')
ax2.set_ylabel('Value')
for i, v in enumerate(x_gaussian):
    ax2.text(i, v, f'{v:.2f}', ha='center', va='bottom')
```

```
plt.tight_layout()
plt.show()
```

Github link:
https://github.com/jalen8/Final-Exam-Matrix-CS-AMAD.git