Exercises with REDUCE
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1 Installing Software

Firstly, you need to install the Computer Algebra software called Reduce. This can be found at `https://reduce-algebra.sourceforge.io/`. There is also a Reduce manual [HS18] here, which I will refer to. It is over 1000 pages, so I *don't* suggest printing it, but you may wish to download it.

Windows This will give you a command called `reduce` that you can run: see Figure 2. You probably want the version called "CSL Reduce" (built on Codemist Standard Lisp).

Macintosh The way we have found to work is to use the PSL version — the file `redpsl`.

Linux You get a `reduce` command.

Once started, you get a screen (where I typed in `(x+1)^2` and hit 'Enter') like Figure 1.

Figure 1: Reduce startup

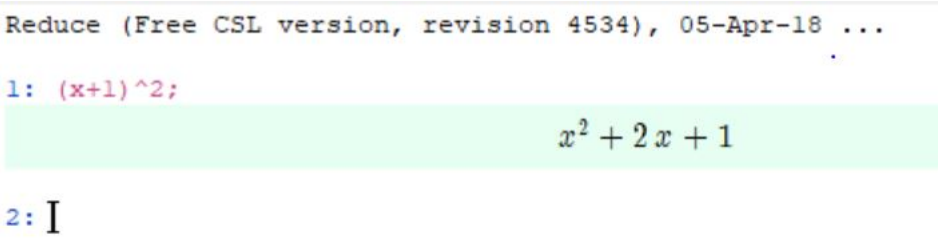
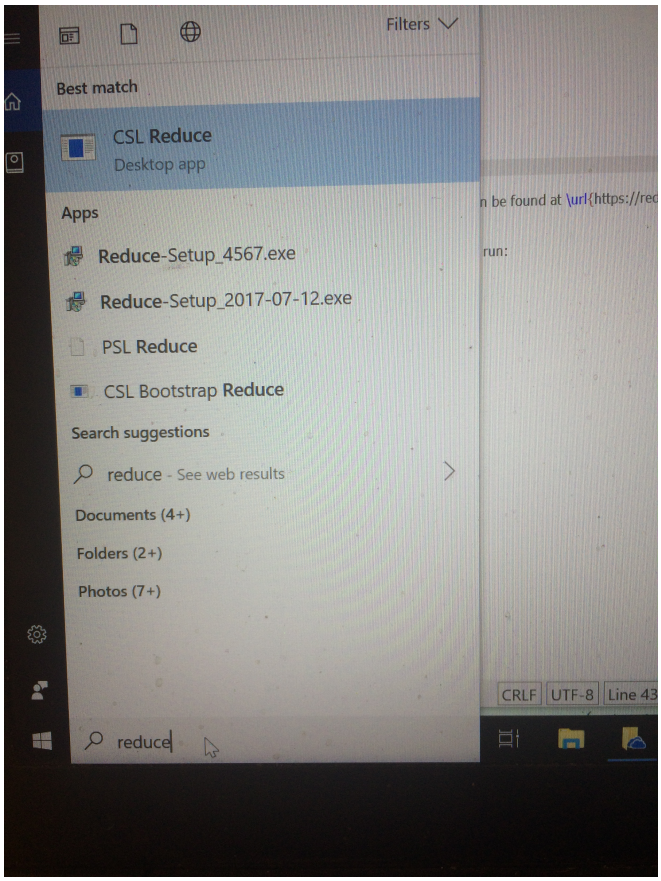


Figure 2: Choosing Reduce on Windows



2 Easy Exercises

Note that, while Reduce tries to give you two-dimensional "mathematics-looking" output, the input is strictly one-dimensional keyboard.

Table 1: One-dimensional input

Mathematics	You type	English
if $x = 7$	`if x=7`	
let $x = 7$	`x:=7`	
$x + y$, $x - y$	`x+y`, `x-y`	
xy	`x*y`	
$\frac{x}{y}$	`x/y`	
x^2	`x^2`	
$\sqrt{x}$	`sqrt(x)`	SQuare RooT
$\log x$	`log(x)`	
$\frac{\mathrm{d}f}{\mathrm{d}x}$	`df(f,x)`	DiFferentiate
$\int f \mathrm{d}x$	`int(f,x)`	INTegral

1. Repeat Figure 1
2. What happens if you replace `+1` by other numbers?
3. Type `a*b-ab`;

Reduce treats "juxtaposition" of letters (placing two together) as making a multiletter variable. If you look carefully at the screen, you can see that the spacing and the exact shapes[1] of the letters are different, at least on my version. The same is true if you use digits *after* the first letter[2].

4. Try `(x-a)^2;`.
5. What happens if you replace `a` by `X`?

Reduce doesn't care about upper/lower case: `x` and `X` are the same. **This is unusual: most other systems make the difference.**

6. What happens if you replace `a` by `I`?

Reduce treats `I` (or `i`) as $\sqrt{-1}$. **Other systems have different conventions, but this is often a trap**.

7. Try replacing `a` by `GOLDEN_RATIO`.

The full set of special names is at [HS18, pp. 36–37]. Note in particular that, for technical reasons, `t` is dangerous.

[1] The technical term is "font".

[2] The full rules are at [HS18, p. 35]. Note that going outside "letters and numbers" can lead to very confusing results: try `x1!+1-x1!+2`.

8. Try `z=x+y;` then ask for the value of z, ie. type `z;`.


Reduce treats `=` as a "test of equality". To get assignment we need `:=` instead of `=`.

9. Reduce knows about some functions: try `log(x)`.

10. Try expressions like `log(x^4)` or `log(x*y)` You might think that Reduce doesn't know very much about log. Surely we all know

$$\log(x^4) = 4\log(x). \tag{1}$$

11. Try `z:=log(x^4)-4*log(x)` (when you'll see Reduce doesn't simplify this), then `sub(x=i,z)`.


Reduce (or more accurately its authors) know more about log than you might think, and in particular that (1), or almost all such beliefs about log, are not true for complex numbers (or indeed negative real ones[3]). Knowing what to do *algorithmically* in this area is still an ongoing research question. See [BBDP04, for example].

References

[BBDP04] J.C. Beaumont, R.J. Bradford, J.H. Davenport, and N. Phisanbut. A Poly-Algorithmic Approach to Simplifying Elementary Functions. In J. Gutierrez, editor, *Proceedings ISSAC 2004*, pages 27–34, 2004.

[HS18] A.C. Hearn and R. Schöpf. REDUCE User's Manual (Free Version; June 8, 2018). `http://reduce-algebra.sourceforge.net/`, 2018.

[3] So declaring `z` to be real [HS18, p. 145] doesn't help.