

Running with the specified configuration, the worst-case brute-force algorithm performs as expected, displaying $O(n^2)$ behavior and consistently taking longer than the other two algorithms. However, in general the basic $O(n(\log n)^2)$ algorithm method performs faster than the optimal $O(n \log n)$ algorithm, and unexpected result. The cause of this inconsistency is likely that the basic algorithm only sorts points by y-coordinate when they are present within the vertical strip around the division point between recursion sets. Due to the wide x-distribution of the values, very few points will fall within this strip, meaning that the number of points that must be y-sorted during this step is much lower than the worst case “n.” On the other hand, the optimal algorithm goes to the trouble of sorting *all* of the points by y-coordinate, something that the basic algorithm does not have to do. If this explanation is true, it demonstrates the practical difference between worst-case complexity and average complexity because, while the worst-case complexity of the basic algorithm is greater than that of the optimal algorithm, its average complexity is likely better.

The experimental data is presented below along with a graph of run time against log (base 10) of the number of points.

n		Brute	Basic	Optimal
10	1	0.003	0.004	0.003
100	2	0.004	0.004	0.002
1000	3	0.04	0.016	0.015
10000	4	1.956	0.078	0.134
100000	5	191.865	0.754	0.937
1E+06	6		8.5	9.451
1E+07	7		92.888	104.293

