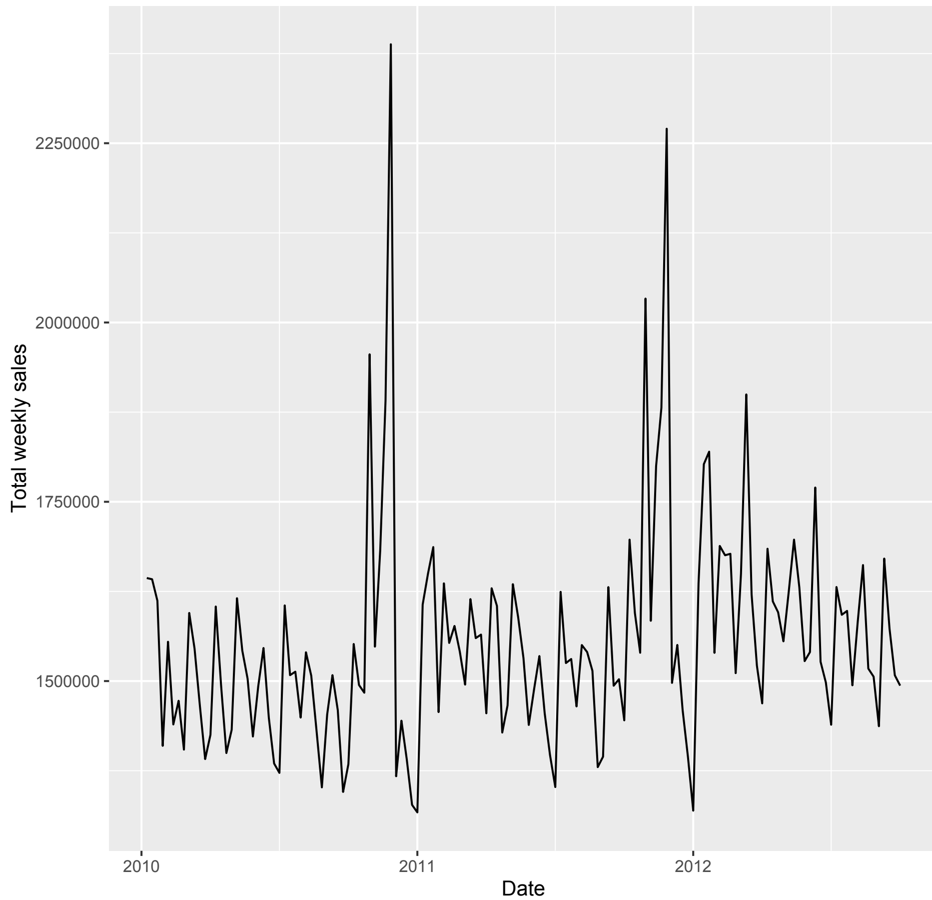
Weekly Sales

Figure 1. Weekly_Sales Time Series from 2010 - 2012




```
        Augmented Dickey-Fuller Test

data:  time_series
Dickey-Fuller = -5.1637, Lag order = 5, p-value =
0.01
alternative hypothesis: stationary
```

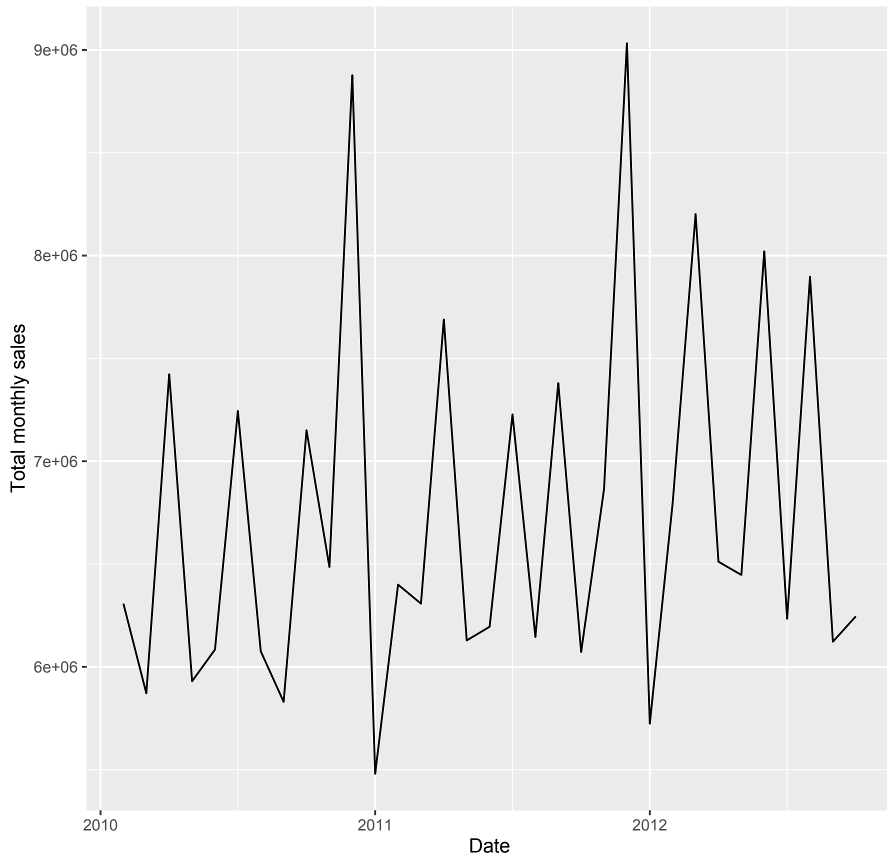
`Week_Sales` is stationary based on the ADF test.

```
        KPSS Test for Level Stationarity

data:  time_series
KPSS Level = 0.47589, Truncation lag parameter = 4, p-value = 0.0471
```

Just further supports that indeed this Time Series is stationary.

Monthly Sales

Figure 2. Monthly_Sales Time Series from 2010 - 2012



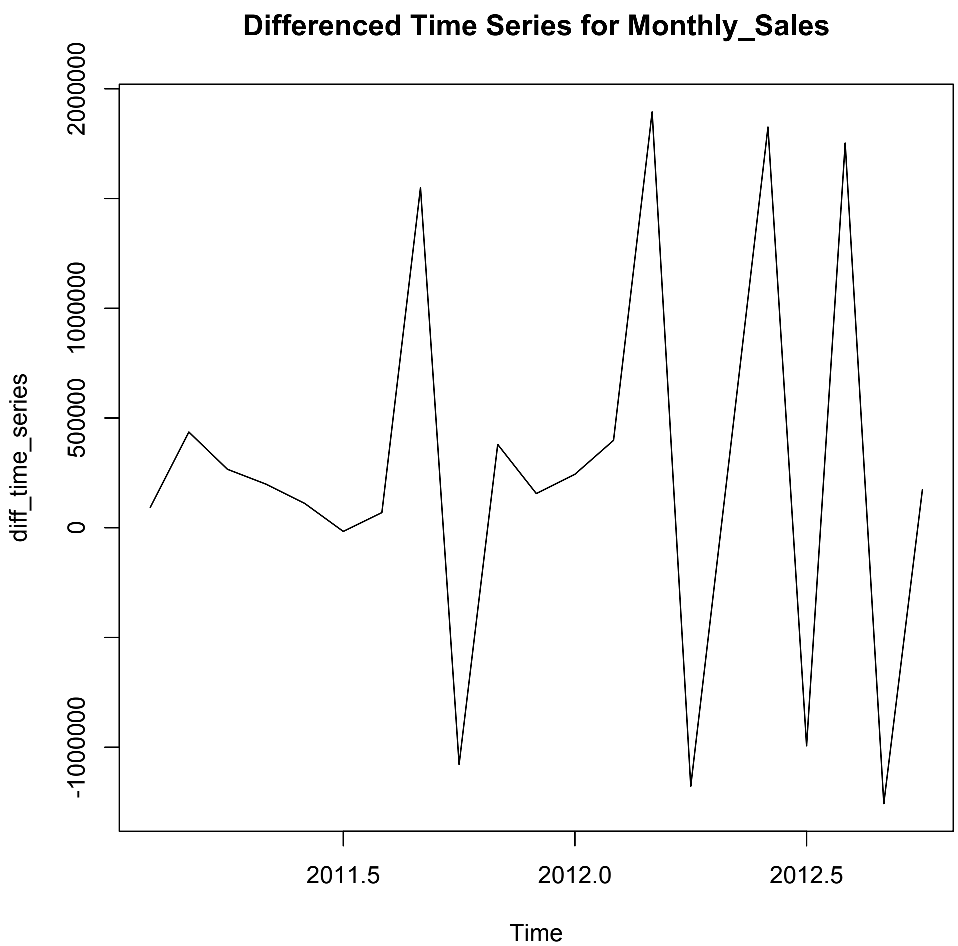

```
        Augmented Dickey-Fuller Test

data:  time_series
Dickey-Fuller = -3.4978, Lag order = 3, p-value = 0.06064
alternative hypothesis: stationary
```

```
        KPSS Test for Level Stationarity

data:  time_series
KPSS Level = 0.32573, Truncation lag parameter = 3, p-value = 0.1
```

`Monthly_Sales` is non-stationary. P-value is > 0.05 so we need to perform differencing to make it stationary.

(Factors: frequency of data, seasonality, trends)

Differencing for Monthly_Sales

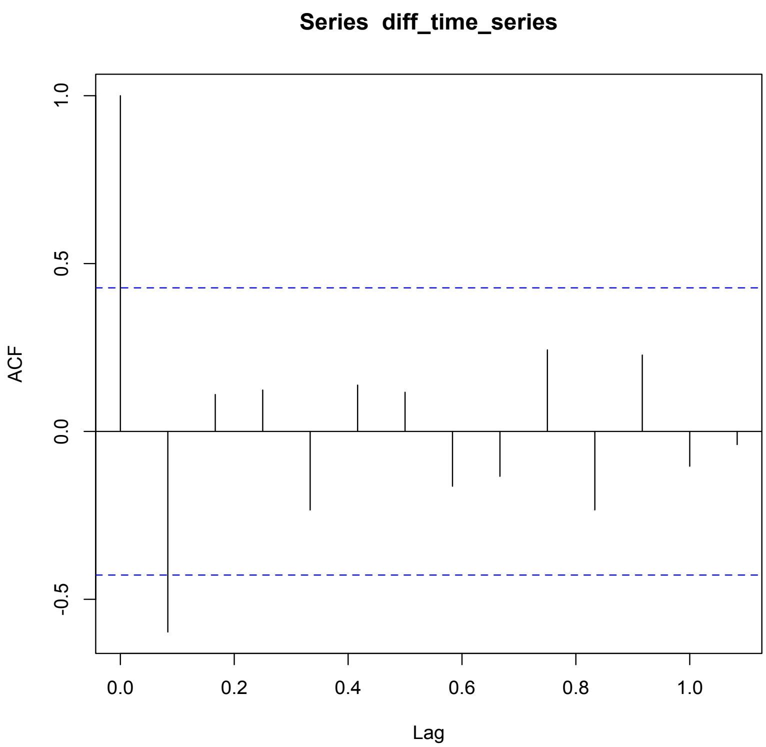



```
        Augmented Dickey-Fuller Test

data:  diff_time_series
Dickey-Fuller = -2.9287, Lag order = 2, p-value = 0.2186
alternative hypothesis: stationary
```

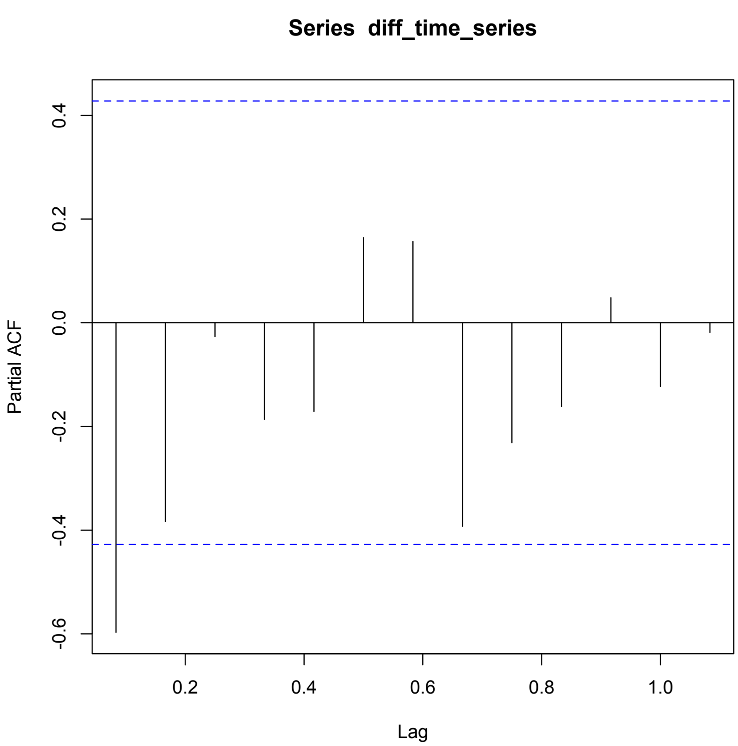
```
        KPSS Test for Level Stationarity

data:  diff_time_series
KPSS Level = 0.087693, Truncation lag parameter = 2, p-value
= 0.1
```

Still showing non-stationary. Meaning we should focus on the Weekly_Sales since it is stationary, and the data exhibits a more consistent and predictable pattern over time regardless of its volatility.

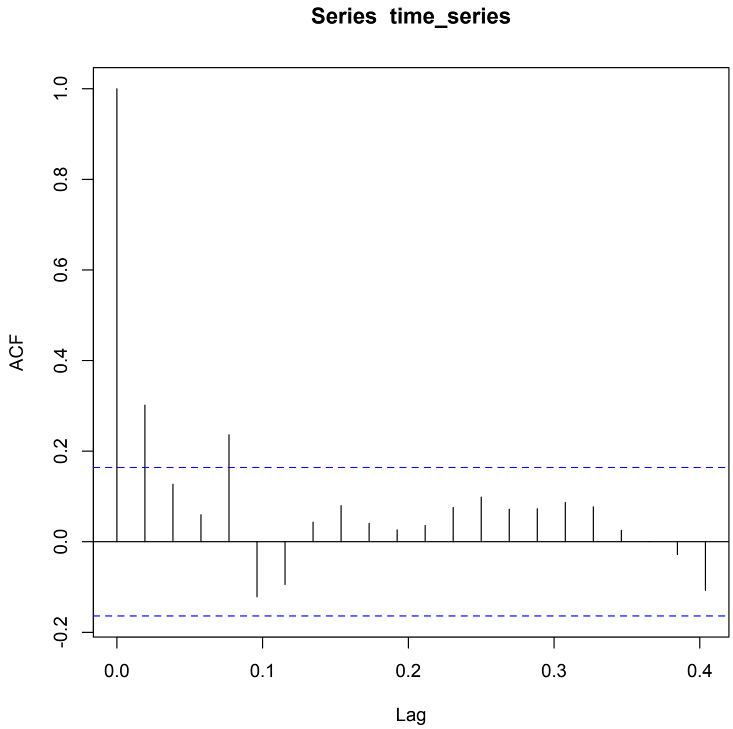
ACF


Series diff_time_series
ACF
1.0
0.5
0.0
-0.5
0.0
0.2
0.4
0.6
0.8
1.0
Lag

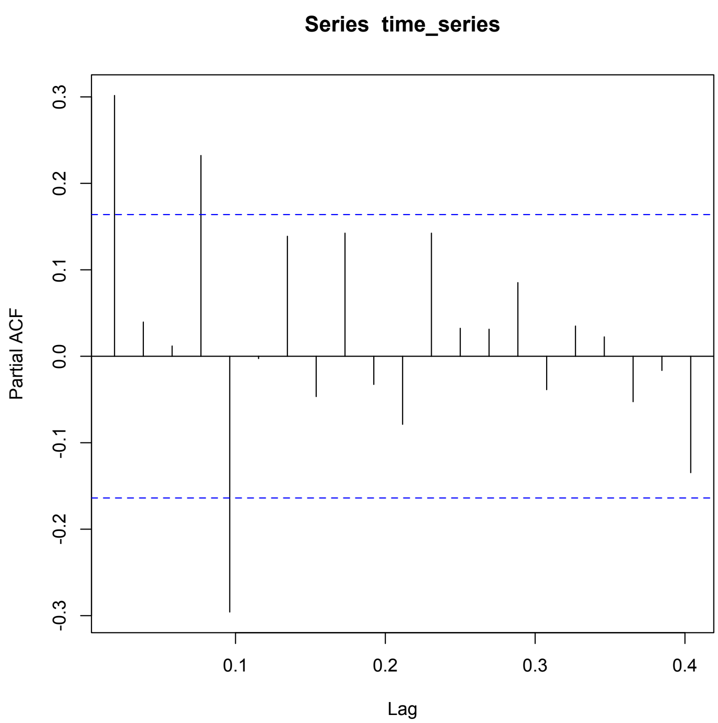
PACF


Series diff_time_series
Partial ACF
0.4
0.2
0.0
-0.2
-0.4
-0.6
0.2
0.4
0.6
0.8
1.0
Lag

Weekly Sales



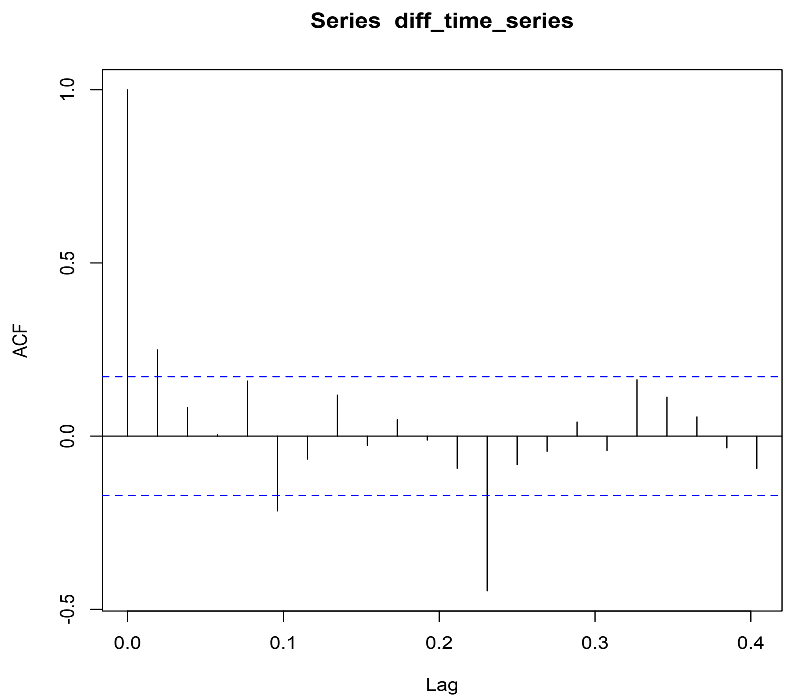
ACF, lag = 4 significance

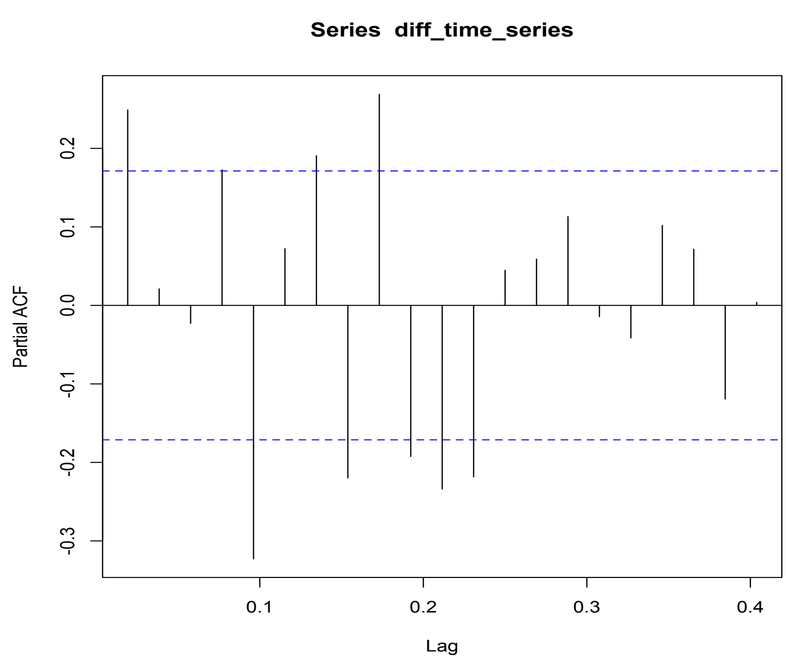


PACF, higher significance at lag = 4

In this case, based on the PACF and ACF we can see a strong correlation at lag 4 suggesting we can use an AR(4) and yule-walker to confirm since it's an autoregressive model.

We might not need to perform Differencing since the data already shows a strong stationary pattern. (I was curious, so I did it anyway as you can see below – very strong correlation at lag 4)





AR(4) Model Process

```
Call:
ar(x = time_series, order.max = 4)

Coefficients:
      1        2        3        4
 0.2865   0.0277  -0.0553   0.2322

Order selected 4  sigma^2 estimated as  2.149e+10
```

AR (4) Model:

$$Y_t = 0.2865y_{t-1} + 0.027y_{t-2} - 0.0553y_{t-3} + 0.2322y_{t-4}$$

AR (1) : 0.2865
AR (2) : 0.0277
AR (3) : -0.0553
AR (4) : 0.2322

Coefficients:

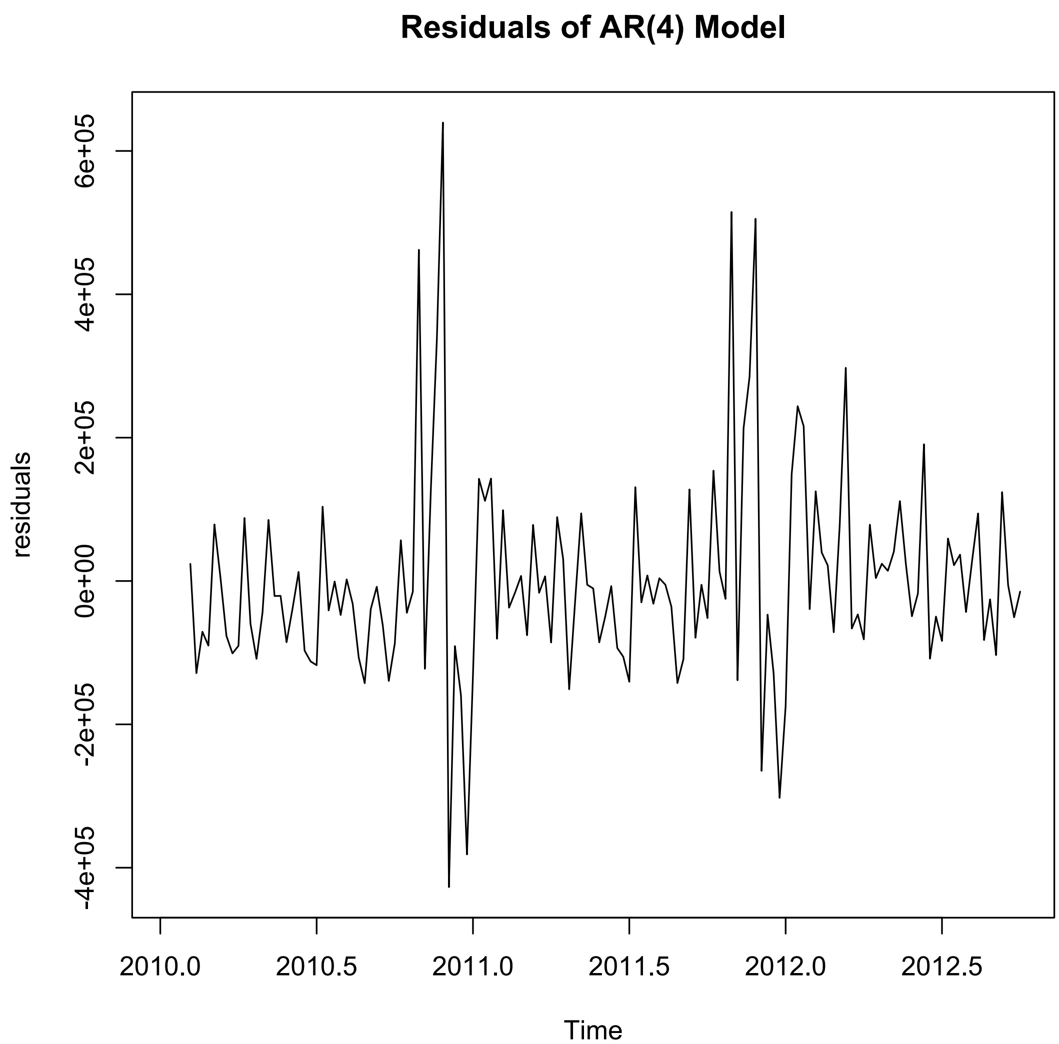
```
> ar_coeffs <- ar_model$ar
> print(ar_coeffs)
[1]  0.28645540  0.02774877 -0.05525683  0.23223569
```

The AR(4) Model process shows there is a strong correlation with the coefficients confirming consistency. This means that the model captures the underlying dynamics of our Weekly_Sales Time Series data. We observed the current values of the Time Series Y_t can be explained by its past values Y_(t-1), Y_(t-2), Y_(t-3), Y_(t-4) with the corresponding coefficients.

	AR(4)
Covariance - Stationary	yes
White noise residuals	yes
Q-tests	0.69493
Residuals (p-value)	0.4045
AIC	3815.099
BIC	3832.876
Coefficients	0.2865, 0.0277, -0.0553, 0.2322
MAPE	6.10 (94% Accuracy)

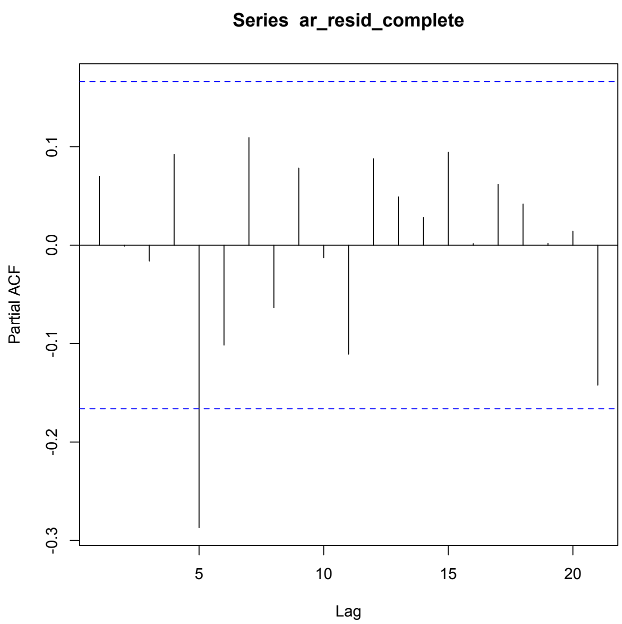
Check Residuals:

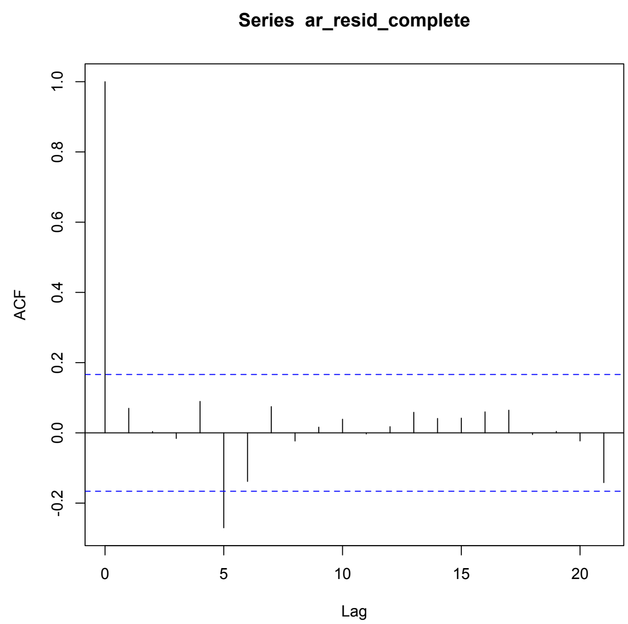
```
	Box-Ljung test

data:  ar_resid
X-squared = 0.69493, df = 1, p-value = 0.4045
```



We have a white noise residual!!! This means that there is no evidence of significant autocorrelation in the residuals of AR(4) model, it does not exhibit any systematic patterns but just randomness.

Residuals: PACF, ACF



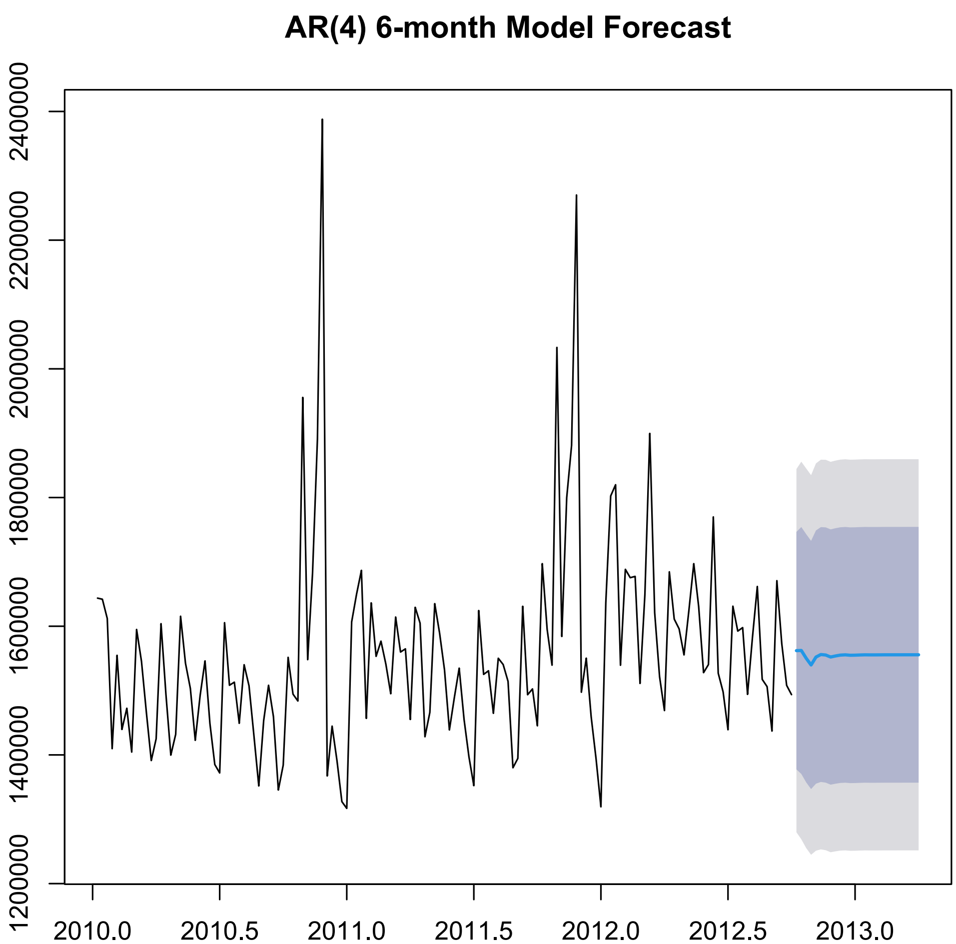


So, it looks like there might be some correlation going on with these residuals as we can see in the PACF spike at lag 4 and ACF spike at lag 5. Our model might not be capturing all of the underlying dynamics of the data which means we need room for improvement. (Possibly consider an AR(5) or AR(6) model.)

6-Month Forecast

Start Date: 2012-10-27
End Date: 2013-04-26

Prediction Intervals: 80% and 95%



AR(4) Model Accuracy = 94% based on MAPE (relative error of 6.103103%)

```
> accuracy(forecast_values)
                  ME     RMSE      MAE        MPE     MAPE    MASE
Training set -352.762 143994.1 97828.72 -0.7679307 6.103103 1.43012
                  ACF1
Training set 0.06956186
```

AR(4) Model Forecasted Summary

```
> summary(forecast_values)

Forecast method: ARIMA(4,0,0) with non-zero mean

Model Information:

Call:
arima(x = time_series, order = c(4, 0, 0), include.mean = TRUE)

Coefficients:
         ar1     ar2      ar3     ar4   intercept
      0.2841  0.0271  -0.0539  0.2279  1555626.77
s.e.  0.0810  0.0839   0.0840  0.0808    23050.99

sigma^2 estimated as 2.073e+10:  log likelihood = -1901.55,  aic = 3815.1

Error measures:
                   ME     RMSE      MAE        MPE     MAPE    MASE
Training set -352.762 143994.1 97828.72 -0.7679307 6.103103 1.43012
                   ACF1
Training set 0.06956186

Forecasts:
         Point Forecast   Lo 80   Hi 80   Lo 95   Hi 95
2012.769        1562040 1377505 1746576 1279817 1844264
2012.788        1562308 1370472 1754144 1268920 1855696
2012.808        1550201 1357336 1743067 1255239 1845163
2012.827        1539797 1346910 1732684 1244802 1834793
2012.846        1552085 1355303 1748866 1251133 1853036
2012.865        1556006 1358014 1753998 1253204 1858809
2012.885        1555256 1356903 1753608 1251901 1858610
2012.904        1552115 1353758 1750471 1248754 1855475
2012.923        1553791 1355258 1752325 1250161 1857422
2012.942        1555117 1356469 1753765 1251311 1858923
2012.962        1555537 1356831 1754243 1251642 1859432
2012.981        1554886 1356175 1753597 1250984 1858788
2013.000        1555023 1356301 1753745 1251104 1858942
2013.019        1555324 1356593 1754055 1251391 1859256
2013.038        1555544 1356806 1754281 1251601 1859486
2013.058        1555459 1356720 1754197 1251514 1859403
2013.077        1555456 1356716 1754195 1251510 1859402
2013.096        1555509 1356769 1754249 1251562 1859456
2013.115        1555579 1356838 1754320 1251631 1859527
2013.135        1555581 1356840 1754322 1251632 1859529
2013.154        1555580 1356838 1754321 1251631 1859528
2013.173        1555588 1356847 1754329 1251639 1859537
2013.192        1555606 1356865 1754347 1251657 1859555
2013.212        1555612 1356870 1754353 1251663 1859561
2013.231        1555613 1356872 1754355 1251665 1859562
2013.250        1555615 1356873 1754356 1251666 1859564
```

Results:

Point forecast ranges from $1,562,040 to $1,555,615.

We see a Lo 80 of $1.3m to a Hi 80 of $1.7m, while on the other hand, we see a low 95 of $1.2 to a Hi 95 of a $1.8m. These values represent the lower and upper bounds of the 80% and 90% prediction interval, respectively.

Overall, the AR(4) model has an accuracy of 94% which suggests that the 6 month forecasted values are, on average within 6.1% of the actual values.

This level of accuracy falls within our predicted intervals of 80-95% which is considered quite good.

ARMA(4,4) Model Process

```
> arma_model <- arima(time_series, order = c(4, 0, 4))
> arma_model

Call:
arima(x = time_series, order = c(4, 0, 4))

Coefficients:
          ar1      ar2      ar3      ar4     ma1     ma2     ma3     ma4   intercept
      -0.2518  -0.5148  -0.2736  -0.0831  0.6421  0.7979  0.4733  0.6377  1555697.04
s.e.   0.1610   0.1985   0.1981   0.1420  0.1339  0.1893  0.2071  0.1443    18359.23

sigma^2 estimated as 1.737e+10:  log likelihood = -1889.59,  aic = 3799.17
```

ARMA (4,4) Model:

$$Y_t = -0.2518y_{t-1} - 0.5148y_{t-2} - 0.2736y_{t-3} - 0.0831y_{t-4}$$
$$+0.6421 \in_{t-1} +0.7979 \in_{t-2} +0.4733 \in_{t-3} +0.6377 \in_{t-4} + \in_t$$

AR (1): -0.2518
AR (2): -0.5148
AR (3): -0.2736
AR (4): -0.0831

MA (1): 0.6421
MA (2): 0.7979
MA (3): 0.4733
MA (4): 0.6377

Coefficients:

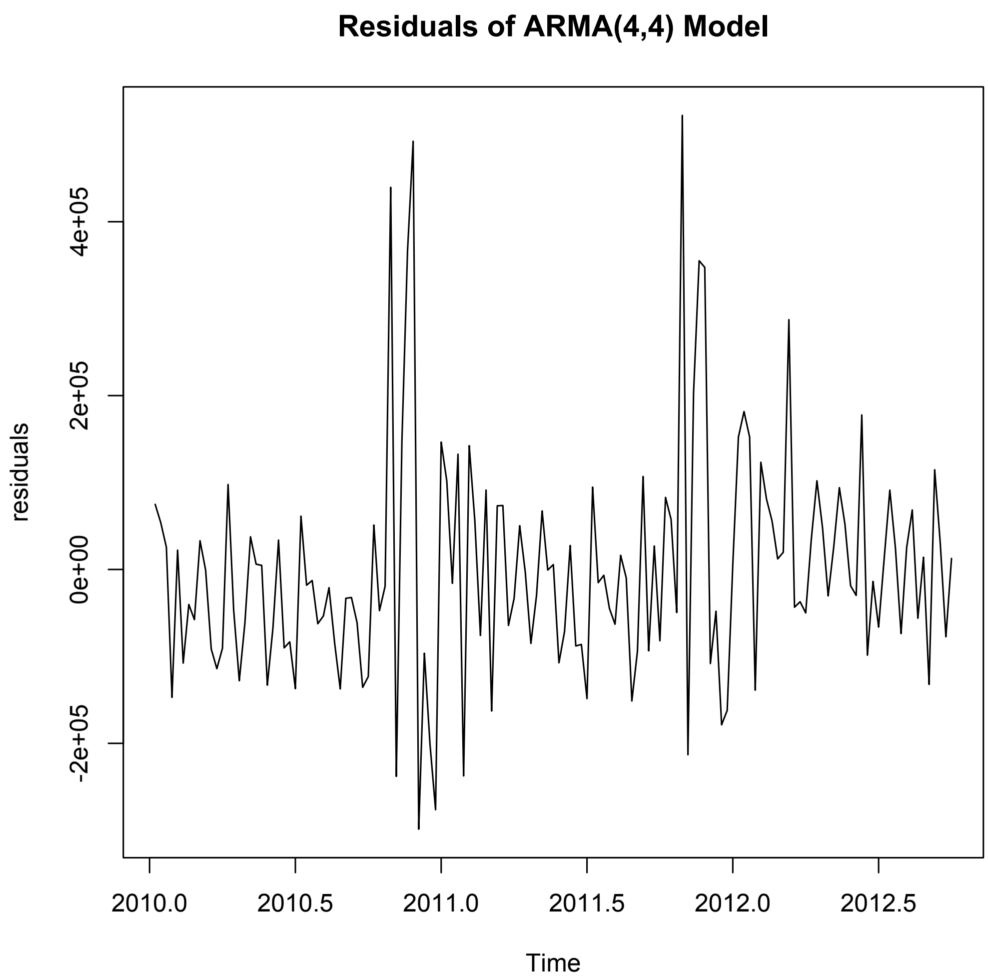
```
> arma_coeffs <- coef(arma_model)
> print(arma_coeffs)
          ar1           ar2           ar3           ar4           ma1           ma2           ma3 
-2.517632e-01 -5.147515e-01 -2.735663e-01 -8.308187e-02  6.421153e-01  7.979333e-01  4.733190e-01 
          ma4     intercept 
 6.377047e-01  1.555697e+06 
```

Based on the ARMA(4,4) coefficients, it is indeed consistent with our model equation for the Time Series meaning that the coefficients in the equation represents the weights of each lagged error term in the model. In other words, it seems like our model is capturing the relationship s present in the data accurately.

	ARMA(4,4)
Covariance - Stationary	yes
White noise residuals	yes
X-squared	0.0026134
Residuals (p-value)	0.9592
AIC	3799.174
BIC	3828.802
Coefficients	-0.2518, -0.5148, -0.2736, -0.0831 0.6421, 0.7979, 0.4733, 0.6377
MAPE	5.8994 (95% accuracy)

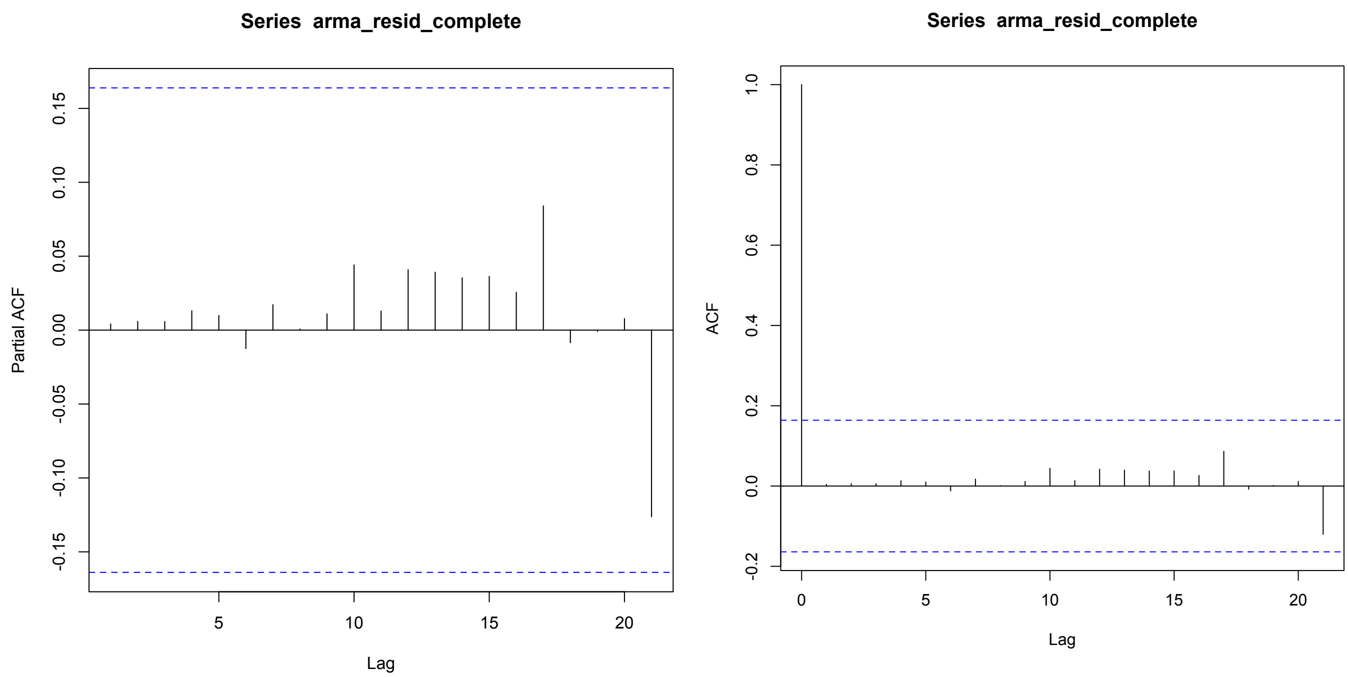
Check Residuals:

```
	Box-Ljung test

data:  arma_resid
X-squared = 0.0026134, df = 1, p-value = 0.9592
```



ARMA(4,4) model have white noise residuals, which is what we want to see. As we can observe the p-value here is 0.95 meaning that there is no evidence of autocorrelation in the model. (We can reject the null) Having white noise is good for our model because it adequately captures the underlying patterns and randomness in the data as you can see the similarities in our model equation and our coefficients. Overall, I think we have a very good model here.

Residuals: PACF, ACF

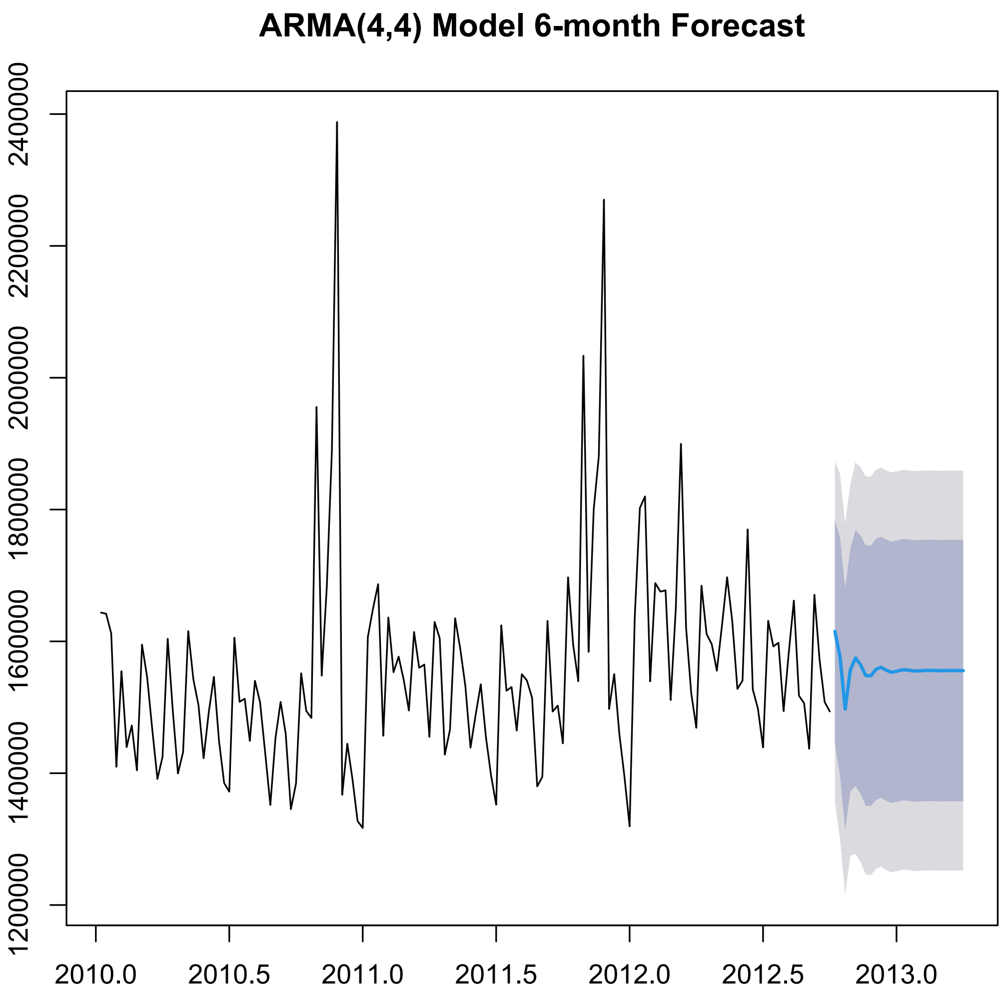


We can conclude here that the residuals of the ARMA(4,4) model do not show any autocorrelation, indicating that again our model has adequately captured the underlying dynamics of the data. The residuals that you see here are all within the significance level. Overall, we have a very good data so far.

6-Month Forecast

Start Date: 2012-10-27
End Date: 2013-04-26

Prediction Intervals: 80% and 95%



ARMA(4,4) Model Accuracy = 95% based on MAPE (relative error of 5.8994%)

```
> accuracy(forecast_values)
                   ME     RMSE     MAE        MPE   MAPE     MASE        ACF1
Training set -116.8959 131804.8 94220.6 -0.6337962 5.8994 1.377374 0.004230539
```

ARMA(4,4) Model Forecasted Summary

```
> summary(forecast_values)

Forecast method: ARIMA(4,0,4) with non-zero mean

Model Information:

Call:
arima(x = time_series, order = c(4, 0, 4), include.mean = TRUE)

Coefficients:
          ar1      ar2      ar3      ar4     ma1     ma2     ma3     ma4   intercept
      -0.2518  -0.5148  -0.2736  -0.0831  0.6421  0.7979  0.4733  0.6377  1555697.04
s.e.   0.1610   0.1985   0.1981   0.1420  0.1339  0.1893  0.2071  0.1443    18359.23

sigma^2 estimated as 1.737e+10:  log likelihood = -1889.59,  aic = 3799.17

Error measures:
                    ME     RMSE     MAE        MPE   MAPE     MASE        ACF1
Training set -116.8959 131804.8 94220.6 -0.6337962 5.8994 1.377374 0.004230539

Forecasts:
         Point Forecast   Lo 80   Hi 80   Lo 95   Hi 95
2012.769        1615036 1446122 1783951 1356704 1873369
2012.788        1576642 1395315 1757970 1299325 1853959
2012.808        1497465 1313467 1681463 1216065 1778866
2012.827        1556605 1372430 1740779 1274934 1838276
2012.846        1574783 1380581 1768986 1277776 1871791
2012.865        1564615 1368760 1760469 1265082 1864148
2012.885        1548217 1350683 1745751 1246115 1850319
2012.904        1547693 1350131 1745256 1245547 1849839
2012.923        1557537 1359464 1755610 1254611 1860464
2012.942        1560659 1362549 1758770 1257676 1863643
2012.962        1556312 1358075 1754548 1253135 1859488
2012.981        1553150 1354865 1751434 1249900 1856399
2013.000        1554512 1356212 1752812 1251238 1857785
2013.019        1556726 1358400 1755053 1253412 1860040
2013.038        1556694 1358367 1755020 1253380 1860008
2013.058        1555452 1357116 1753788 1252124 1858780
2013.077        1555062 1356726 1753399 1251733 1858392
2013.096        1555625 1357286 1753963 1252292 1858957
2013.115        1556026 1357687 1754365 1252693 1859360
2013.135        1555845 1357506 1754185 1252512 1859179
2013.154        1555563 1357223 1753903 1252228 1858897
2013.173        1555570 1357231 1753910 1252236 1858905
2013.192        1555730 1357390 1754070 1252395 1859065
2013.212        1555778 1357438 1754118 1252443 1859113
2013.231        1555705 1357365 1754045 1252371 1859040
2013.250        1555655 1357315 1753995 1252320 1858989
```

Results:

Point forecast ranges from $1,615,036 to $1,555,655.

We see a Lo 80 of $1.4m to a Hi 80 of $1.7m, while on the other hand, we see a low 95 of $1.3 to a Hi 95 of a $1.8m. These values represent the lower and upper bounds of the 80% and 90% prediction interval, respectively.

Overall, the ARMA(4,4) model has an accuracy of 95% which suggests that the 6 month forecasted values are, on average within 5.8% of the actual values.

This level of accuracy falls within our predicted intervals of 80-95% which is considered very good.

ARMA(3,4) Model Process

```
> arma_model2 <- arima(time_series, order = c(3, 0, 4))
> arma_model2

Call:
arima(x = time_series, order = c(3, 0, 4))

Coefficients:
          ar1      ar2      ar3     ma1     ma2     ma3     ma4   intercept
      -0.2562  -0.4572  -0.2095  0.6527  0.7589  0.4094  0.5652  1555721.38
s.e.   0.1638   0.1645   0.1631  0.1401  0.1752  0.1861  0.1035    19336.95

sigma^2 estimated as 1.742e+10:  log likelihood = -1889.75,  aic = 3797.5
```

ARMA (3,4) Model:

$$Y_t = -0.2562y_{t-1} - 0.4572y_{t-2} - 0.2095y_{t-3}$$
$$+0.6527 \in_{t-1} + 0.7589 \in_{t-2} + 0.4094 \in_{t-3} + 0.5652 \in_{t-4} + \in_t$$

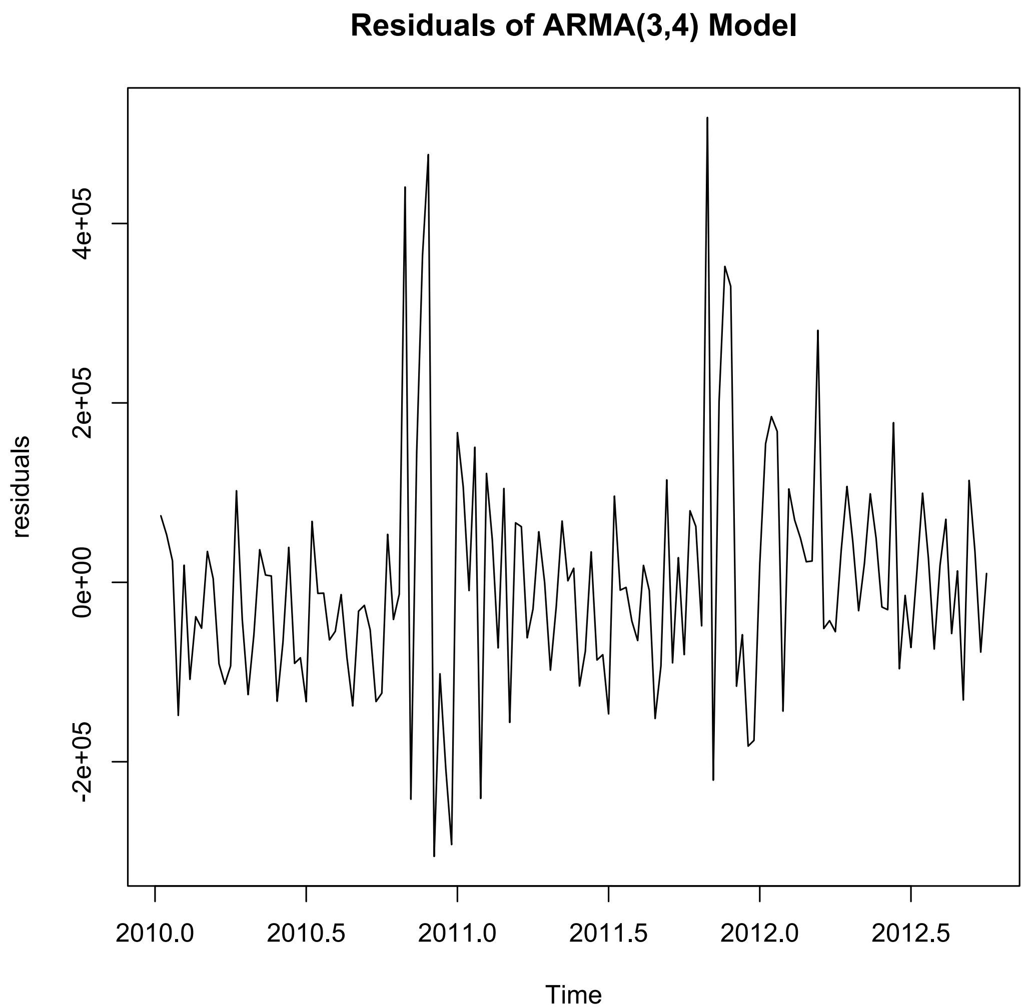
AR (1) :	-0.2562
AR (2) :	-0.4572
AR (3) :	-0.2095
MA (1) :	0.6527
MA (2) :	0.7589
MA (3) :	0.4094
MA (4) :	0.5652

Coefficients:

	ARMA(4,4)
Covariance - Stationary	yes
White noise residuals	yes
X-squared	0.0011205
Residuals (p-value)	0.9733
AIC	3797.502
BIC	3824.168
Coefficients	-0.2562, -0.4572, -0.2095, 0.6527, 0.7589, 0.4094, 0.5652
MAPE	5.941535 (95% accuracy)

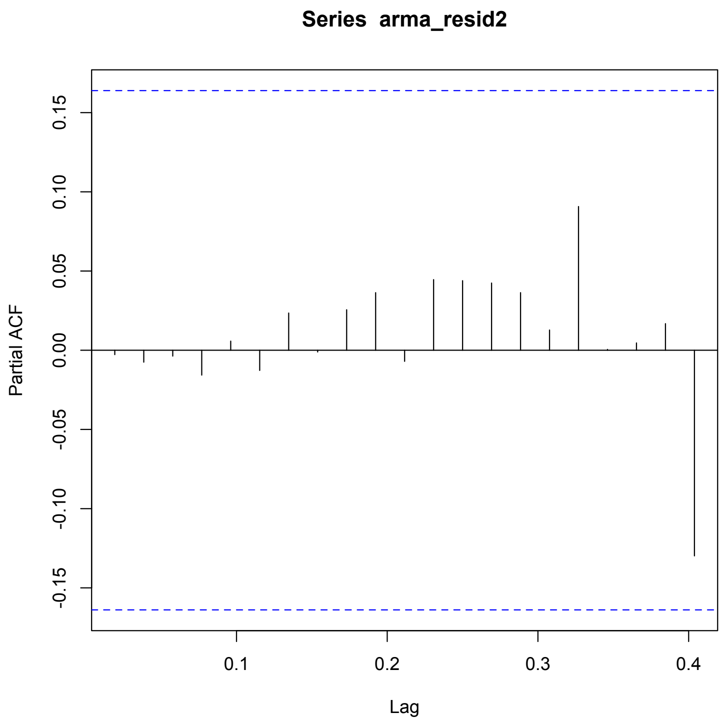
Check Residuals:

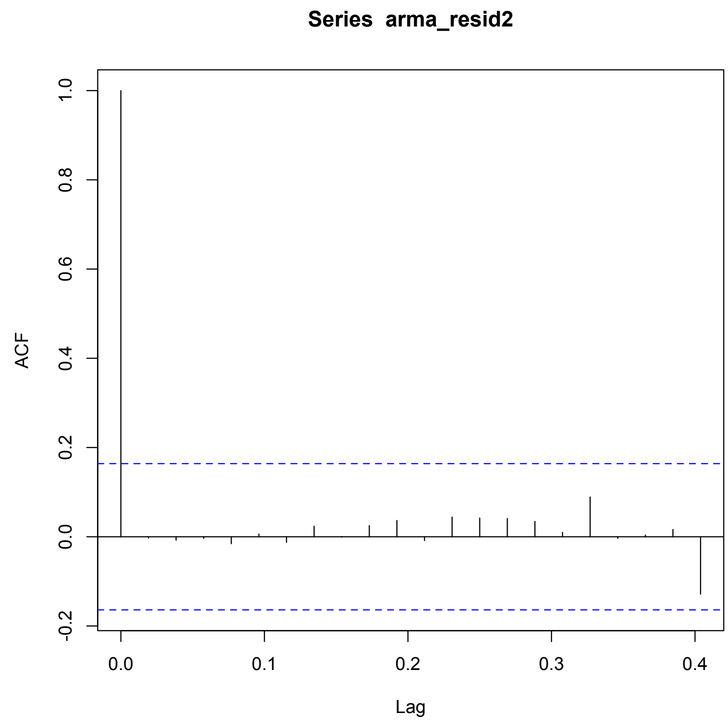
```
        Box-Ljung test

data:  arma_resid
X-squared = 0.0011205, df = 1, p-value = 0.9733
```



We have white noise residuals which is what we want!!!

Residuals: PACF, ACF



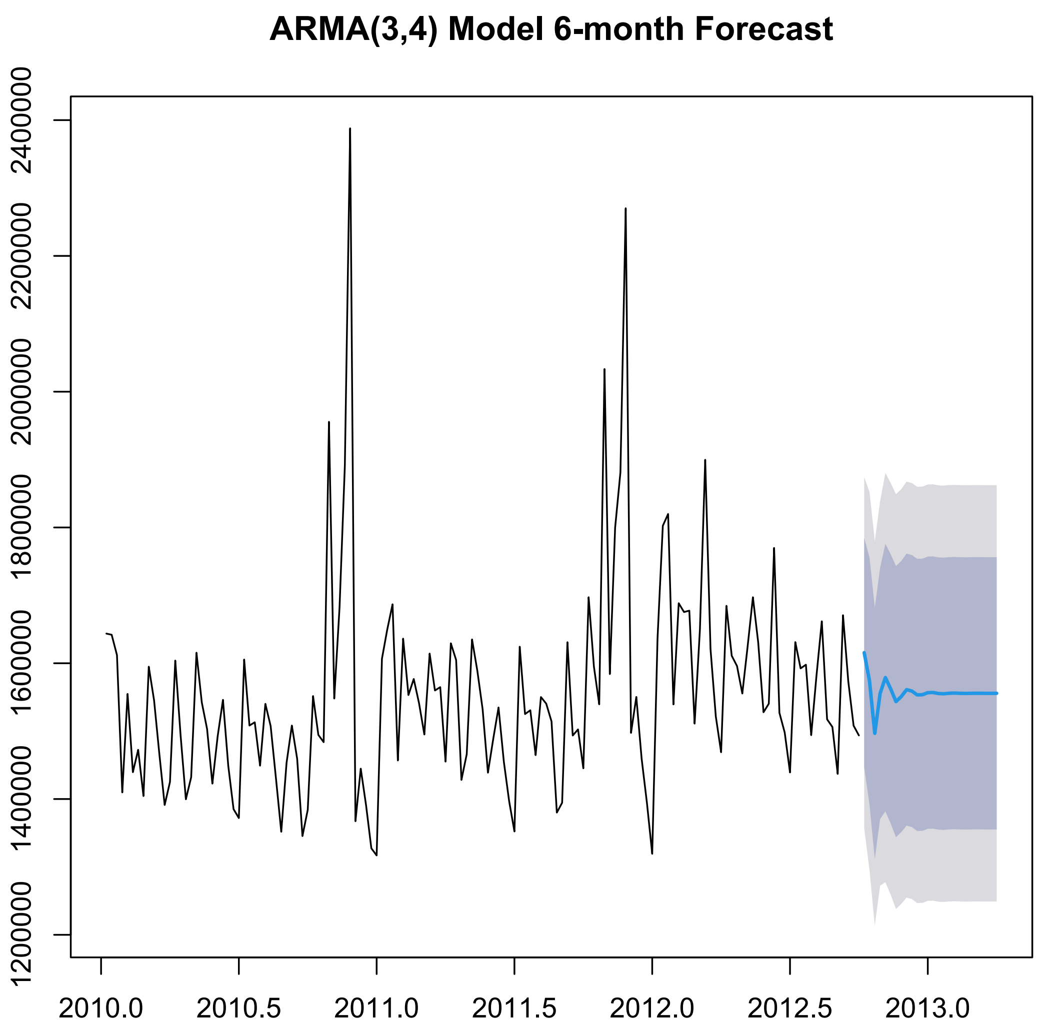


We can see that the residuals are within the significance level here – which is what we want to see!!!

6-month Forecast

Start Date: 2012-10-27
End Date: 2013-04-26

Prediction Intervals: 80% and 95%



ARMA(3,4) Model Accuracy = 95% based on MAPE (relative error of 5.941535%)

```
> accuracy(forecast_values)
                    ME     RMSE      MAE        MPE     MAPE    MASE         ACF1
Training set -149.0374 131970.9 94707.41 -0.6291275 5.941535 1.38449 -0.002770136
```

ARMA(3,4) Model Forecasted Summary

```
> summary(forecast_values)

Forecast method: ARIMA(3,0,4) with non-zero mean

Model Information:

Call:
arima(x = time_series, order = c(3, 0, 4), include.mean = TRUE)

Coefficients:
          ar1      ar2      ar3     ma1     ma2     ma3     ma4   intercept
      -0.2562  -0.4572  -0.2095  0.6527  0.7589  0.4094  0.5652  1555721.38
s.e.   0.1638   0.1645   0.1631  0.1401  0.1752  0.1861  0.1035    19336.95

sigma^2 estimated as 1.742e+10:  log likelihood = -1889.75,  aic = 3797.5

Error measures:
                    ME     RMSE      MAE        MPE     MAPE    MASE          ACF1
Training set -149.0374 131970.9 94707.41 -0.6291275 5.941535 1.38449 -0.002770136

Forecasts:
         Point Forecast   Lo 80   Hi 80   Lo 95   Hi 95
2012.769        1615582 1446455 1784710 1356924 1874241
2012.788        1573978 1392042 1755914 1295731 1852225
2012.808        1496751 1311691 1681811 1213727 1779775
2012.827        1555519 1370377 1740661 1272368 1838669
2012.846        1578907 1381853 1775961 1277539 1880275
2012.865        1562228 1363965 1760490 1259011 1865444
2012.885        1543497 1343777 1743218 1238051 1848943
2012.904        1551022 1351291 1750753 1245560 1856484
2012.923        1561151 1360852 1761450 1254821 1867481
2012.942        1559040 1358741 1759339 1252709 1865370
2012.962        1553374 1352939 1753808 1246835 1859912
2012.981        1553668 1353229 1754108 1247123 1860214
2013.000        1556626 1356152 1757099 1250028 1863223
2013.019        1556920 1356443 1757397 1250317 1863523
2013.038        1555431 1354947 1755915 1248817 1862045
2013.058        1555058 1354572 1755545 1248441 1861676
2013.077        1555773 1355285 1756261 1249153 1862392
2013.096        1556072 1355584 1756561 1249451 1862693
2013.115        1555747 1355258 1756236 1249126 1862368
2013.135        1555544 1355055 1756033 1248922 1862165
2013.154        1555682 1355193 1756171 1249060 1862303
2013.173        1555807 1355318 1756297 1249186 1862429
2013.192        1555755 1355265 1756244 1249133 1862377
2013.212        1555682 1355193 1756171 1249060 1862304
2013.231        1555698 1355209 1756188 1249076 1862320
2013.250        1555738 1355249 1756228 1249117 1862360
```

Results:

Point forecast ranges from $1,615,582 to $1,555,738.

We see a Lo 80 of $1.4m to a Hi 80 of $1.7m, while on the other hand, we see a low 95 of $1.3 to a Hi 95 of a $1.8m. These values represent the lower and upper bounds of the 80% and 90% prediction interval, respectively.

Overall, the ARMA(3,4) model has an accuracy of 95% which suggests that the 6 month forecasted values are, on average within 5.9% of the actual values.

This level of accuracy falls within our predicted intervals of 80-95% which is considered very good.

Re-evaluation of AR(4):

Because of the pattern of autocorrelation from the residuals in AR(4), I've decided to investigate further beyond lag = 4, which in this case I'm looking at both AR(5) and AR6).

(I didn't include the plots for AR(6) but its commented in the code since I Just wanted to take a look at the residuals plot for both ACF and PACF whether they fall under the significance level, but AR(6) don't so I'm only including results for AR(5) this time.)

Results:

The residuals for AR(6) ACF falls within the significance but PACF has a spike in lag 6 so this may not be a good model compared to AR(5). Considering their 6-month forecast accuracy they are both at 94% accuracy with roughly 6% relative error. Therefore, we will go with **AR(5)** instead considering it has better metrics results and it's residuals falls within the significance level for both PACF and ACF.

AR(5) model process will be demonstrated below.

AR(5) Model Process

```
> ar_model <- ar(time_series, order = 5)
> ar_model

Call:
ar(x = time_series, order.max = 5)

Coefficients:
      1        2        3        4        5
 0.3552   0.0114  -0.0470   0.3170  -0.2958

Order selected 5  sigma^2 estimated as  1.976e+10
```

AR (5) Model :

$$Y_t = 0.3552y_{t-1} + 0.0114y_{t-2} - 0.0470_{y+-3} + 0.3170y_{t-4}$$
$$-0.2958y_{t-5} + \in_t$$

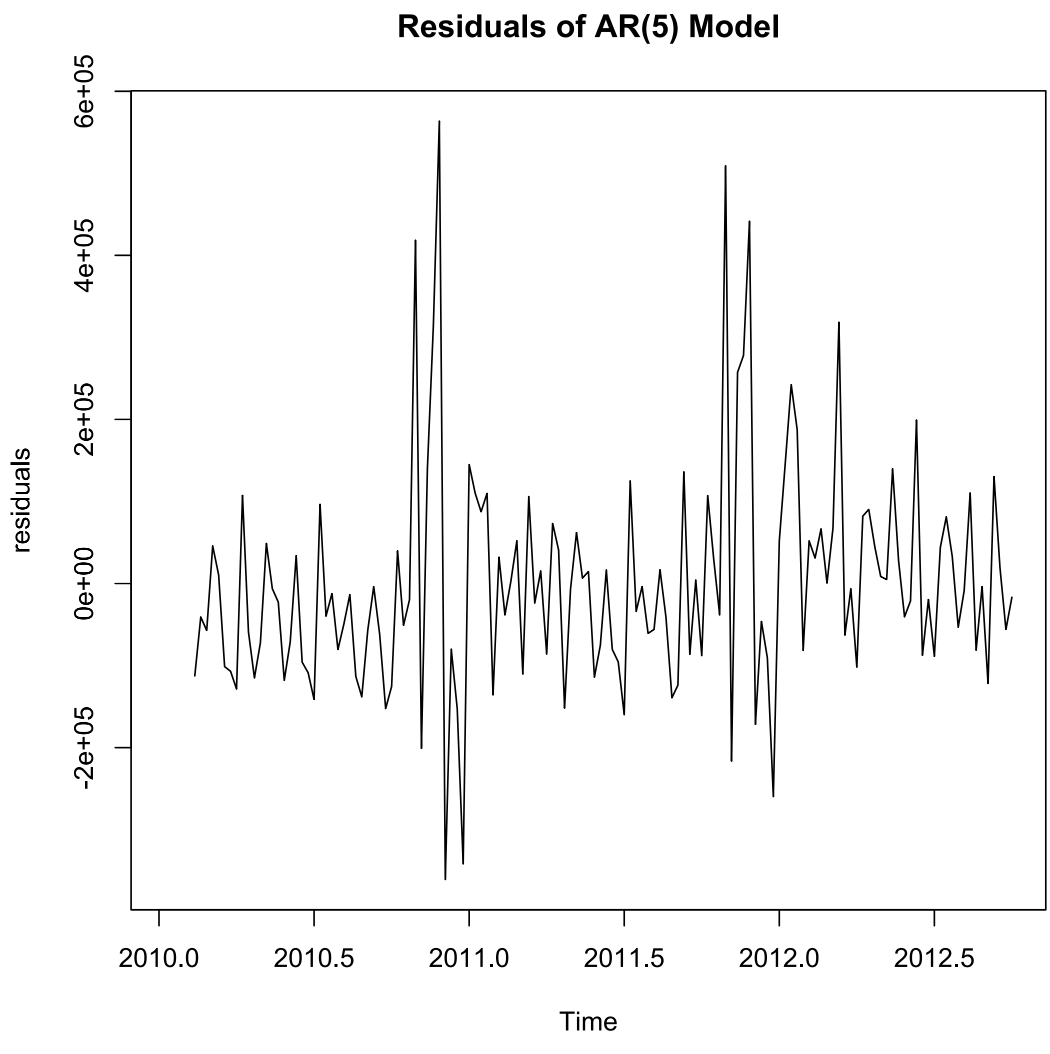
AR(1) : 0.3552
AR(2) : 0.0114
AR(3) : -0.0470
AR(4) : 0.3170
AS(5) : -0.2958

Coefficients:

	AR(5)
Covariance - Stationary	yes
White noise residuals	yes
X-squared	0.00018199
Residuals (p-value)	0.9892
AIC	3804.093
BIC	3824.833
Coefficients	0.3552, 0.0114, -0.0470, 0.3170, -0.2958
MAPE	6.095725 (94% accuracy)

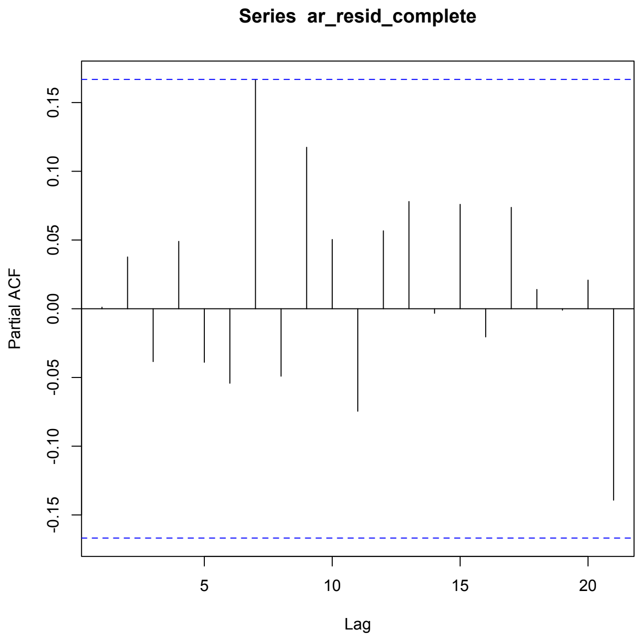
Check Residuals:

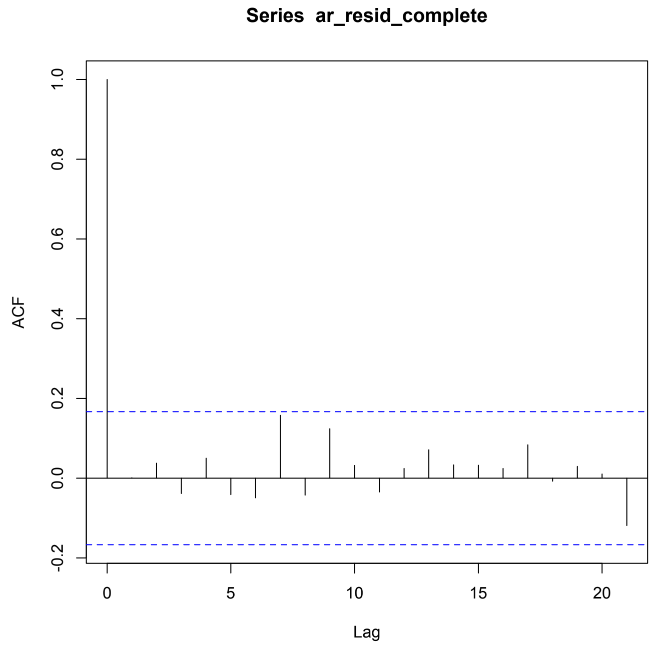
```
        Box-Ljung test

data:  ar_resid
X-squared = 0.00018199, df = 1, p-value = 0.9892
```



We have white noise residuals which is what we want to see again so this is good!

Residuals: PACF, ACF



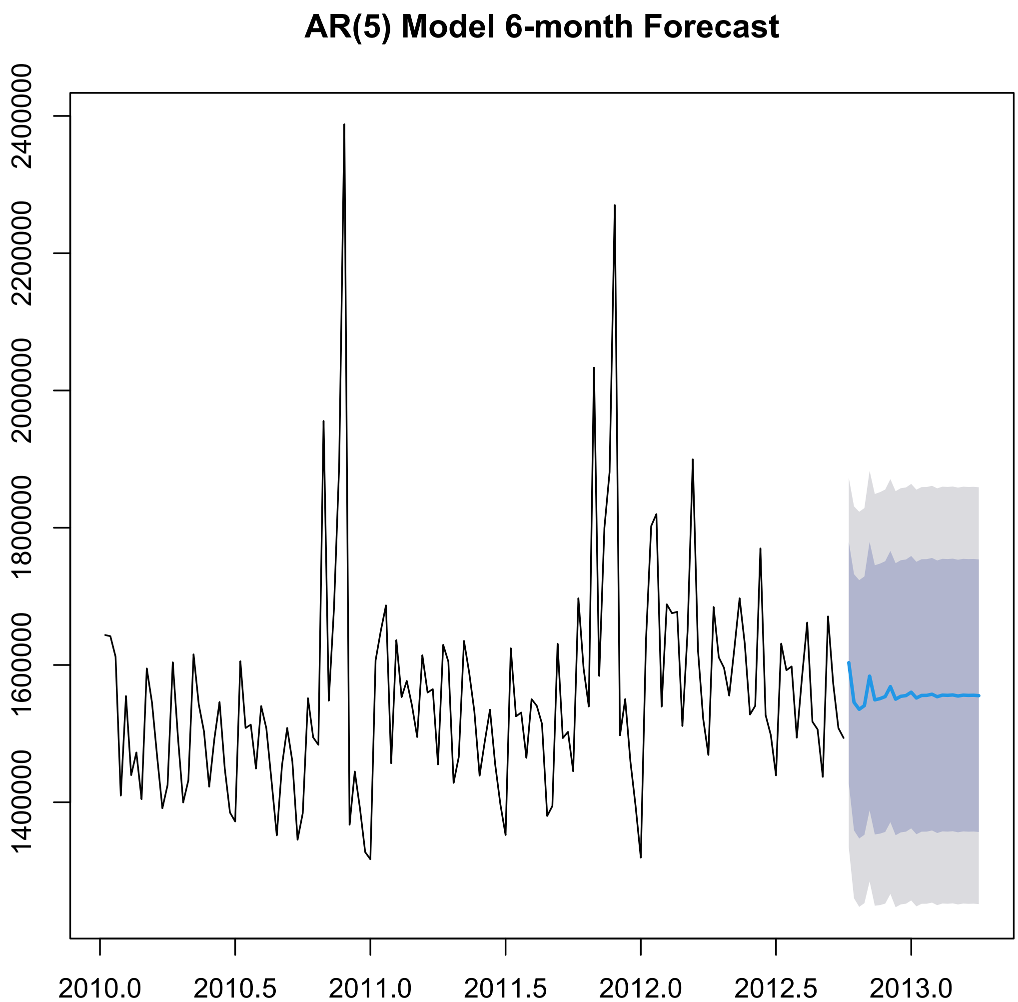


Based on the results, we can conclude that this model AR(5) is a lot better than the AR(4) model that we started with since all of the lags fall within the significance level for both PACF and ACF.

6-month Forecast

Start Date: 2012-10-27
End Date: 2013-04-26

Prediction Intervals: 80% and 95%



ARMA(3,4) Model Accuracy = 94% based on MAPE (relative error of 6.095725%)

```
> accuracy(forecast_values)
                   ME     RMSE      MAE      MPE     MAPE     MASE       ACF1
Training set -207.908 137373.3 97350.98 -0.68917 6.095725 1.423136 0.00191953
```

AR(5) Model Forecasted Summary

```
> summary(forecast_values)

Forecast method: ARIMA(5,0,0) with non-zero mean

Model Information:

Call:
arima(x = time_series, order = c(5, 0, 0), include.mean = TRUE)

Coefficients:
         ar1     ar2      ar3     ar4      ar5  intercept
      0.3527  0.0114  -0.0447  0.3138  -0.2932  1555707.7
s.e.  0.0794  0.0799   0.0799  0.0803   0.0792    17402.6

sigma^2 estimated as 1.887e+10:  log likelihood = -1895.05,  aic = 3804.09

Error measures:
                   ME     RMSE      MAE      MPE     MAPE     MASE       ACF1
Training set -207.908 137373.3 97350.98 -0.68917 6.095725 1.423136 0.00191953

Forecasts:
         Point Forecast   Lo 80   Hi 80   Lo 95   Hi 95
2012.769        1603409 1427358 1779460 1334162 1872656
2012.788        1545660 1358979 1732342 1260156 1831165
2012.808        1535438 1347231 1723644 1247600 1823275
2012.827        1540808 1352597 1729019 1252965 1828652
2012.846        1583834 1388250 1779419 1284714 1882955
2012.865        1549224 1353109 1745340 1249291 1849157
2012.885        1550992 1354285 1747699 1250154 1851830
2012.904        1553981 1356738 1751224 1252324 1855639
2012.923        1568531 1370943 1766118 1266346 1870715
2012.942        1550140 1351945 1748334 1247027 1853252
2012.962        1554388 1356124 1752652 1251170 1857606
2012.981        1555447 1357166 1753728 1252202 1858692
2013.000        1560380 1361952 1758808 1256911 1863849
2013.019        1551904 1353403 1750405 1248323 1855485
2013.038        1555650 1357145 1754154 1252063 1859236
2013.058        1555740 1357234 1754246 1252152 1859329
2013.077        1557431 1358891 1755971 1253791 1861072
2013.096        1553755 1355197 1752313 1250087 1857423
2013.115        1556134 1357574 1754695 1252462 1859806
2013.135        1555786 1357225 1754347 1252114 1859458
2013.154        1556359 1357795 1754922 1252682 1860036
2013.173        1554801 1356231 1753371 1251115 1858487
2013.192        1556098 1357528 1754669 1252411 1859786
2013.212        1555706 1357135 1754276 1252018 1859393
2013.231        1555933 1357362 1754504 1252245 1859621
2013.250        1555294 1356722 1753867 1251604 1858984
```

Results:

Point forecast ranges from $1,603,409 to $1,555,294.

We see a Lo 80 of $1.4m to a Hi 80 of $1.7m, while on the other hand, we see a low 95 of $1.3 to a Hi 95 of a $1.8m. These values represent the lower and upper bounds of the 80% and 90% prediction interval, respectively.

Overall, the ARMA(3,4) model has an accuracy of 94% which suggests that the 6 month forecasted values are, on average within 6.09% of the actual values.

This level of accuracy falls within our predicted intervals of 80-95% which is considered very good.

Final Thoughts:

For our models, we choose AR(5), ARMA(3,4) and ARMA(4,4) based on their metrics results.

	AR(5)	**ARMA(3,4)**	**ARMA(4,4)**
Covariance - Stationary	yes	yes	yes
White noise residuals	yes	yes	yes
X-squared	0.00018199	0.0011205	0.0026134
Residuals (p-value)	0.9892	0.9733	0.9592
AIC	3804.093	3797.502	3799.174
BIC	3824.833	3824.168	3828.802
Coefficients	0.3552, 0.0114, -0.0470, 0.3170, -0.2958	-0.2562, -0.4572, -0.2095, 0.6527, 0.7589, 0.4094, 0.5652	-0.2518, -0.5148, -0.2736, -0.0831 0.6421, 0.7979, 0.4733, 0.6377
MAPE	6.095725 (94% accuracy)	5.941535 (95% accuracy)	5.8994 (95% accuracy)