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CSCI 264-01
Homework 6

HW6 Problem 4 Writeup

1. **Reduce problem *P* to problem *Q_1* :**
   a. Given G, s, and t from problem P, we can simply run the longest path algorithm on the same input by letting $G_1 = G$, $s_1 = s$, and $t_1 = t$. If the result l is equal to one less than the number of vertices, then G has an s-t Hamiltonian path.

   b. *Explanation*: Problem Q_1 specifies that the longest path can go through each vertex at most one time. This means that a path with length l must pass through $l + 1$ unique vertices. Thus, if there exists a path from s to t with a length one less than the number of vertices, then that path must pass through every vertex exactly once.

2. **Reduce problem *P* to problem *Q_2* :**
   a. Given G, s, and t from problem P, we can create the input for input for problem Q_2 by creating two new vertices, which will become s_2 and t_2. The new vertex s_2 is connected only to s by a single edge with a very large positive weight. Similarly, t_2 is connected only to t by a single edge with a very large positive weight. G_2 is constructed identically to G, but each edge is assigned a weight of -1. With this input, if the result of the shortest path algorithm l is equal to the number of vertices minus three, then G has an s-t Hamiltonian path.

   b. *Explanation*: By creating new vertices with only one edge connecting them to the rest of the graph, the algorithm is forced to take the edges with large positive values. Seeking a path with a low cost, the algorithm will attempt to reduce this large positive weight as much as it can. If every edge in G_2 has a value of -1, the shortest path will be the one that traverses as many of these negative edges as possible. Since each the path can go through every vertex at most once, a path with length l will pass through $l + 1$ unique vertices. Thus, a path that contains $n – 3$ edges will pass through $n – 2$ vertices. Accounting for the two extra vertices added to the original input, such a path would be an s-t Hamiltonian path, proving that one does exist on G.