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CSCI 264
Homework 2

Problem 4

1) $T(n) = 3T(n/9) + \sqrt{n}$

$a = 3,\ b = 9,\ f(n) = \sqrt{n}$

$\log 3 / \log 9 = 1.585 / 3.17 = 1/2$

$n^{1/2} = \Theta(n^{\log 3/\log 9}) \rightarrow$ Case 2 applies

$T(n) = \Theta(n^{\log 3/\log 9} \log n) = \Theta(n^{1/2} \log n) = \boxed{\Theta(\sqrt{n} \log n)}$

2) $T(n) = 4T(n/2) + n^3$

$a = 4,\ b = 2,\ f(n) = n^3$

$\log 4 / \log 2 = 2/1 = 2$

$n^3 = \Omega(n^{2+\epsilon}) \rightarrow$ Case 3 could apply

$4f(\frac{n}{2}) \overset{?}{\leq} c\,f(n) \rightarrow 4(\frac{n}{2})^3 \overset{?}{\leq} cn^3 \rightarrow \frac{n^3}{2} \overset{?}{\leq} cn^3 \rightarrow$ holds for $c = \frac{1}{2}$

Case 3 applies.

$\boxed{T(n) = \Theta(n^3)}$

3) $T(n) = 5T(n/4) + n \log n$

$a = 5,\ b = 4,\ f(n) = n \log n$

$\log 5 / \log 4 = 2.322 / 2 = 1.161$

$n \log n = \Omega(n^{1.161+\epsilon}) \rightarrow$ case 3 could apply

$5f(\frac{n}{4}) \overset{?}{\leq} c\,f(n) \rightarrow 5(\frac{n}{4}\log(\frac{n}{4})) \overset{?}{\leq} cn\log n \rightarrow \frac{5}{4}n(\log n - \log 4) \overset{?}{\leq} cn\log n$

$\frac{5}{4}n(\log n - 2) \overset{?}{\leq} cn\log n \rightarrow \frac{5}{4}n\log n - \frac{5}{2}n \overset{?}{\leq} cn\log n \rightarrow$ holds for $c = \frac{5}{4}$

Case 3 applies.

$\boxed{T(n) = \Theta(n \log n)}$