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CSCI 264-01
Homework 3

HW3 Problem 1 Writeup

a) Verbal Description
       The location with the minimum possible distance to each police car can be
found with the median value of the x coordinates and the median value of the y
coordinates. This algorithm implements randomized select to search for those median
values, then calculates the minimum possible distance sum by adding the
displacement of each police car's x and y coordinates from those of the target
location.

b) Pseudocode
       Let X be the array of x coordinates
       Let Y be the array of y coordinates
       Find the median values of X and Y using the randomized Select algorithm
       Let minSum = 0
       For every (X[i], Y[i]) pair in X and Y:
              minSum += abs(X[i] - median value of X)
              minSum += abs(Y[i] - median value of Y)
       Return minSum

c) Proof of Correctness
       Using the median value as the target location will always produce the
smallest distance sum because it guarantees that there will be an equal number of
cars in either direction, because it appropriately handles the ratio of cars on
either side of the median value by ensuring the shop is placed closer to the side
with more values (if the coordinates are mostly low numbers with a few high
numbers, the median value will place the shop closer to the lower values), and
because it will reduce at least one distance to zero, as it will always pick a spot
on which a car already exists.
       Since this problem takes place on a grid system and diagonal movement is not
possible, x and y distances can be calculated separately and then added together to
get the total distance: on a grid, there is always a minimum x displacement and y
displacement to get from one point to another, which remains the same no matter
what path is taken to get there. Thus, once the median values are found, the
algorithm can calculate the minimum distance sum on the x-axis and the minimum
distance sum on the y-axis and sum them together to get the total and always be
correct.

d) Running Time Estimate
       O(n)

e) Running Time Estimate Reasoning
       This algorithm uses the randomized Select algorithm to find the median, which
we can assume has an expected running time of O(n). Once the data has been loaded
(which in itself is an O(n) operation), the medians are selected, and the ideal
locations are found, finding the minimum distance requires iterating through each
(x,y) pair once more. Thus, the final estimated running time is O(n) + O(n) + O(n)
= O(n).