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CSCI 264-01

Homework 6

HW6 Problem 4 Writeup

1. **Reduce problem *P* to problem *Q1* :**
   1. Given G, s, and t from problem P, we can simply run the longest path algorithm on the same input by letting G1 = G, s1 = s, and t1 = t. If the result *l* is equal to one less than the number of vertices, then G has an s-t Hamiltonian path.
   2. *Explanation*: Problem Q1 specifies that the longest path can go through each vertex at most one time. This means that a path with length *l* must pass through *l* + 1 unique vertices. Thus, if there exists a path from *s* to *t* with a length one less than the number of vertices, then that path must pass through every vertex exactly once.
2. **Reduce problem *P* to problem *Q2*:**
   1. Given G, s, and t from problem P, we can create the input for input for problem Q2 by creating two new vertices, which will become s2 and t2. The new vertex s2 is connected only to s by a single edge with a very large positive weight. Similarly, t2 is connected only to t by a single edge with a very large positive weight. G2 is constructed identically to G, but each edge is assigned a weight of -1. With this input, if the result of the shortest path algorithm *l* is equal to the number of vertices minus three, then G has an s-t Hamiltonian path.
   2. *Explanation*: By creating new vertices with only one edge connecting them to the rest of the graph, the algorithm is forced to take the edges with large positive values. Seeking a path with a low cost, the algorithm will attempt to reduce this large positive weight as much as it can. If every edge in G2 has a value of -1, the shortest path will be the one that traverses as many of these negative edges as possible. Since each the path can go through every vertex at most once, a path with length *l* will pass through *l* + 1 unique vertices. Thus, a path that contains *n* – 3 edges will pass through *n* – 2 vertices. Accounting for the two extra vertices added to the original input, such a path would be an s-t Hamiltonian path, proving that one does exist on G.