Billiards

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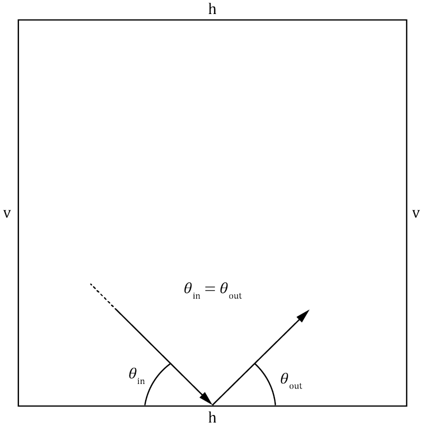
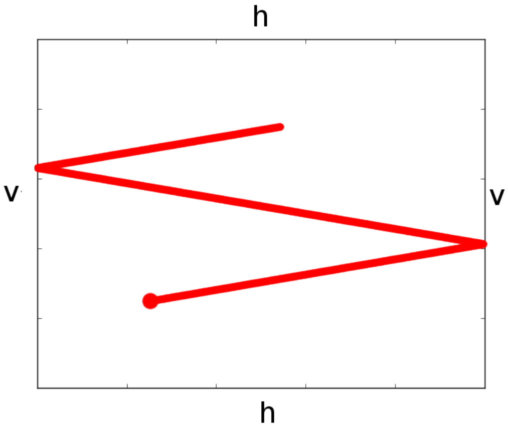
Introduction

Frictionless, massless, point-sized billiard ball bouncing in a square.



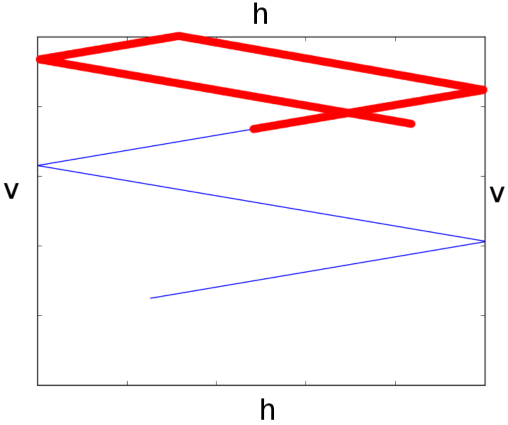
Example

vv



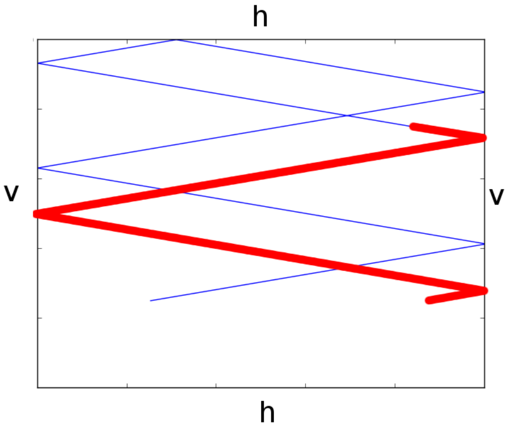
Example

$vv\,vhv$



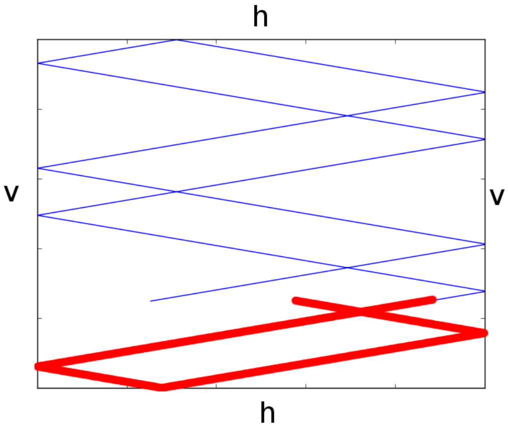
Example

vvvhvvvv



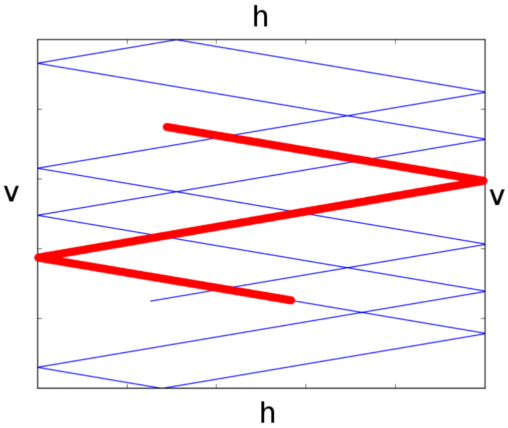
Example

$vvvhvvvv\,vhv$



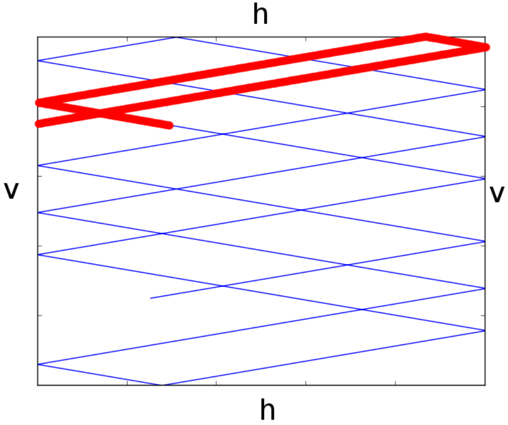
Example

vvvhvvvvvhvvv



Example

vvvhvvvvvhvvv vhv



Resulting Sequence

$vvvhvvvvvhvvvvhv$

Presentation Outline

Problem Statement

Problem: Given a sequence of v and h collisions, determine if it is a valid collision sequence.

Basic Notation

Definition

v collision: when the ball collides with a *v* side

Definition

h collision: when the ball collides with an *h* side

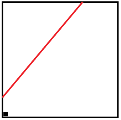
Definition

Collision sequence (α): a sequence of *v* and *h* collisions which starts and ends with an *h* collision.

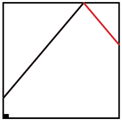
Tiling Representation

- Tile the table in the plane for a more powerful representation of the problem
- Tiling will reflect the table about each side
- After tiling, we only need to deal with straight line trajectories in a tiled plane

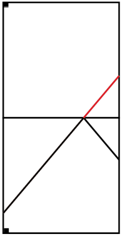
Tiling Tables



Tiling Tables



Tiling Tables



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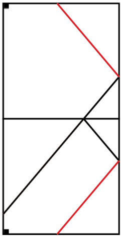

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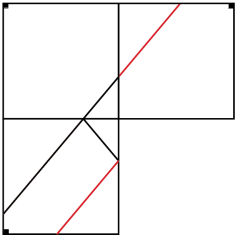

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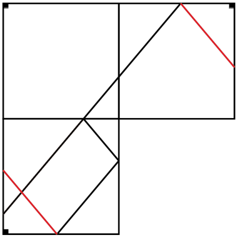
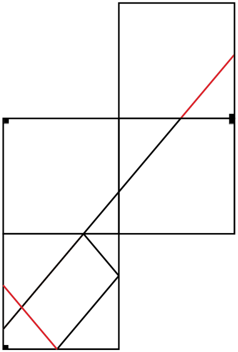


Tiling Tables



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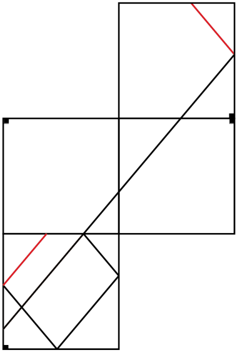

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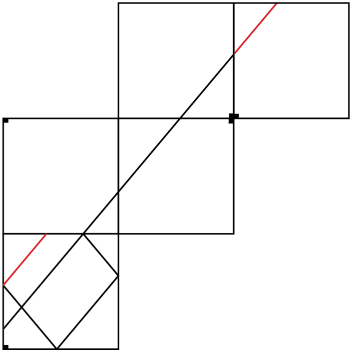

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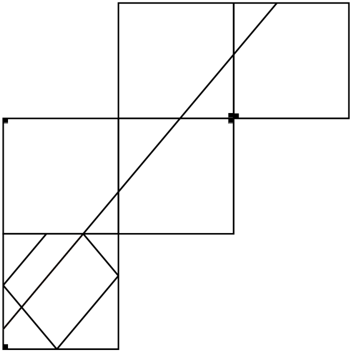

Indexing Definitions

$I(A, b)_k$: index of k^{TH} b in A

β_i: # of v's between i^{TH} and $(\mathrm{i}+1)^{\mathrm{TH}}$ collisions $([I(\alpha, h)]_{i+1} - [I(\alpha, h)]_i)$

Theorem

$$\beta_i \geq 1 \quad i \in \{0, \ldots, length(I(\alpha, h)) - 2\}$$

Extensions to Cubes

- Assign x, y, and z as the opposite pairs of faces of a cube.
- Characterize sequences of x, y, and z collisions.

Intuition

- Examine collisions in xy, yz, and xz planes.
- Movement in each plane is independent.
- Combine xy, yz, and xz collision sequences to get final sequence.

Example

Example

xy sequence: $xxyyxx$

yz sequence: $zzyzyz$

xz sequence: $xxzzzx$