Billiards

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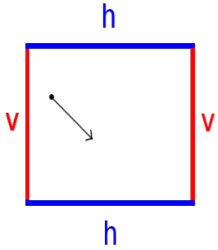
Introduction

- Billiard ball bouncing in a square
- Assume no gravity or friction

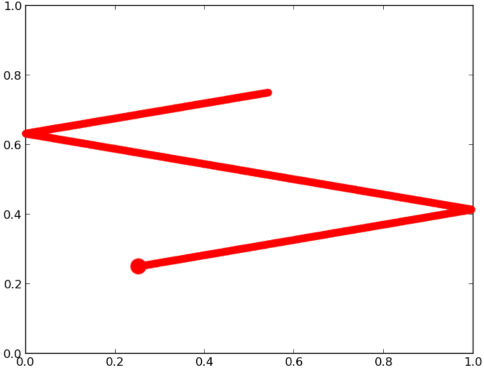
Basic Notation

Definition

A table $T \subset \mathbb{R}^2$ is the unit square. Vertical sides are labelled with a v. Horizontal sides are labelled with an h.

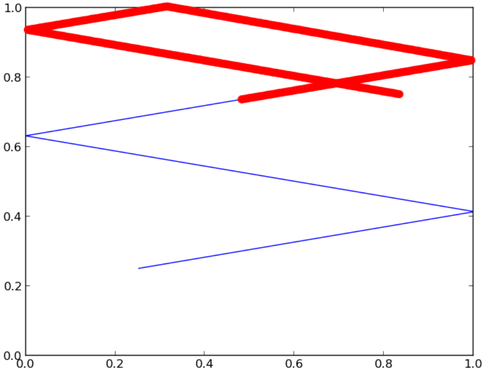


Example



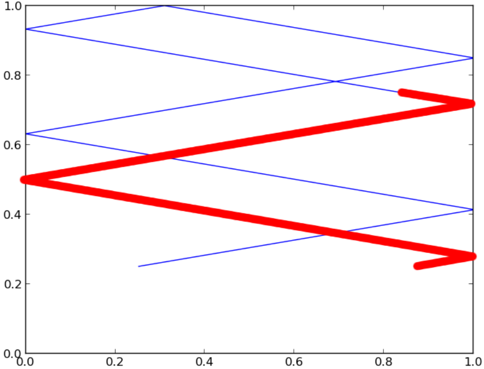
$$vv \tag{1}$$

Example



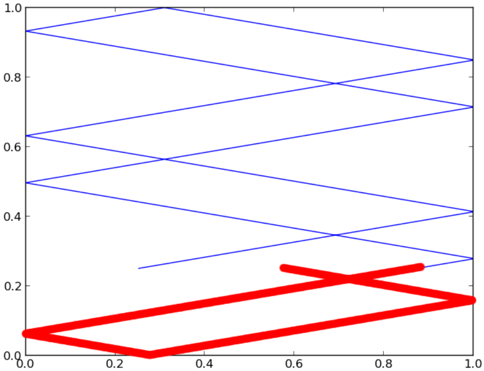
$$vv\,{\color{red}vhv} \tag{2}$$

Example



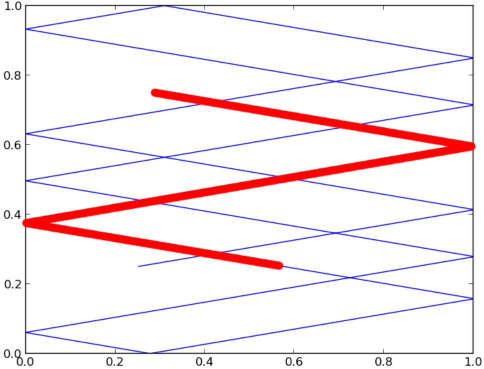
$$vvvhv\textcolor{red}{vvv} \tag{3}$$

Example



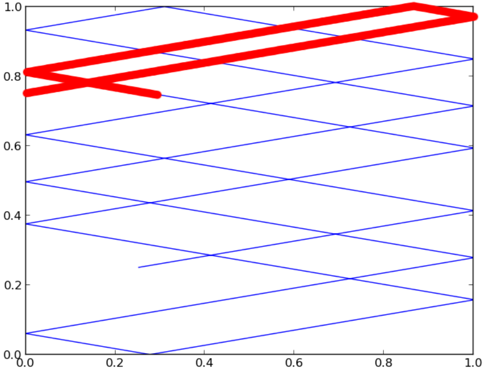
$$vvvhvvvvvhv \tag{4}$$

Example



$$vvvhvvvvvhv\textcolor{red}{vv} \tag{5}$$

Example



$$vvvhvvvvvhvvv\,vhv \tag{6}$$

Resulting Sequence

$$vvvhvvvvvhvvvvhv \tag{7}$$

Presentation Outline

Problem Statement

Problem: Given a sequence of v and h collisions, determine if it is a valid collision sequence.

Basic Notation

Definition

v collision: when the ball collides with a *v* side

Definition

h collision: when the ball collides with an *h* side

Definition

Collision sequence (α): a sequence of *v* and *h* collisions which starts and ends with an *h* collision.

Representing Collision Sequences

Indexing Definitions

$I(A, b)_k$: index of k$^{\text{TH}}$ b in A

β_i: # of v's between i$^{\text{TH}}$ and (i+1)$^{\text{TH}}$ collisions ($[I(\alpha, h)]_{i+1} - [I(\alpha, h)]_i$)

Theorem

$$\beta_i \geq 1 \quad i \in$$

Extensions to Cubes

- Assign x, y, and z as the opposite pairs of faces of a cube.
- Characterize sequences of x, y, and z collisions.

Intuition

- Examine collisions in xy, yz, and xz planes.
- Movement in each plane is independent.
- Combine xy, yz, and xz collision sequences to get final sequence.

Example

Example

xy sequence: $xxyyxx$ yz sequence: $zzyzyz$ xz sequence: $xxzzzx$