Billiards

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November 22nd, 2013

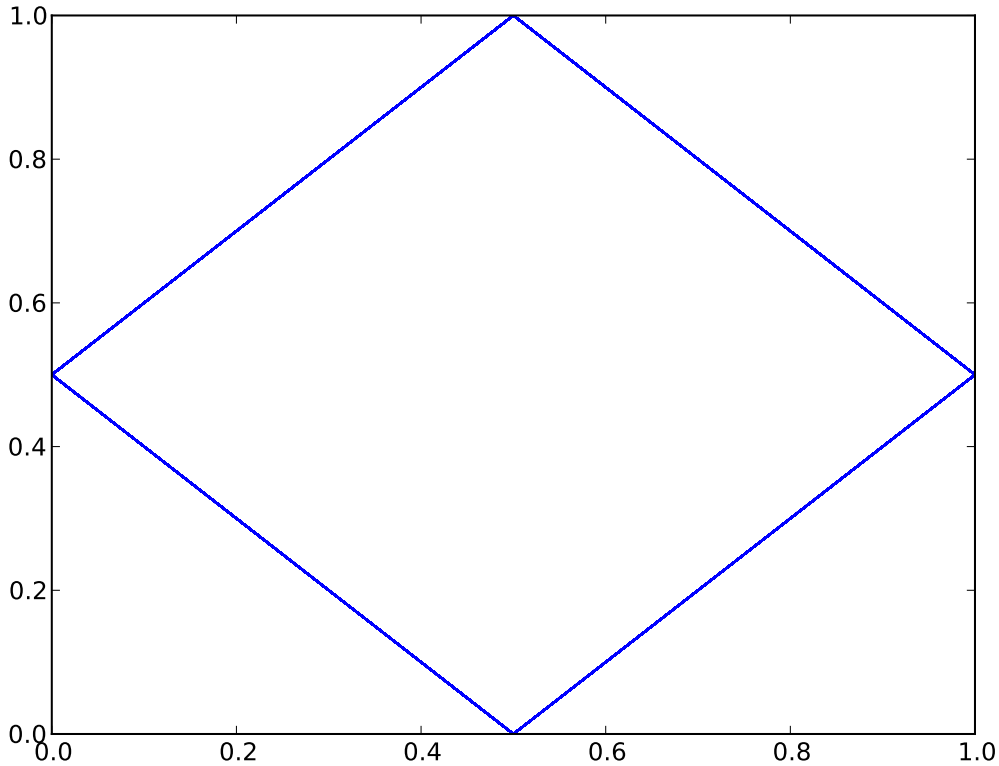
Introduction

- Billiard ball bouncing in a square
- Assume no gravity or friction
- Examine sequence of side collisions

Example

Example

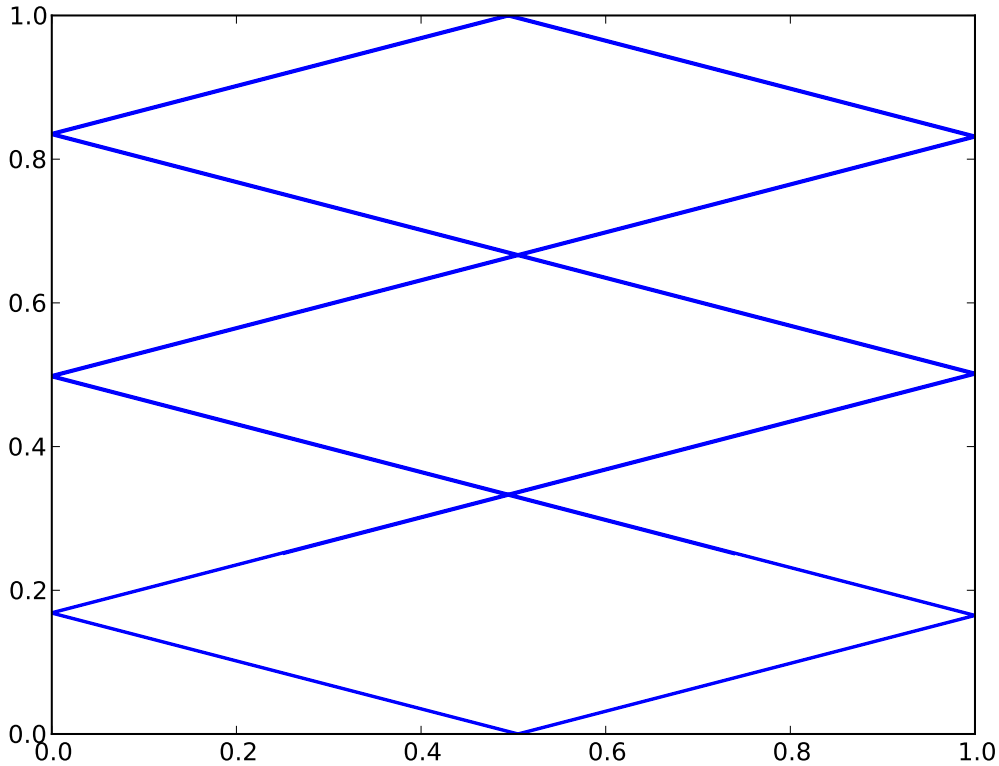
Examine the sequence: ‘abab‘



Another Example

Example

Examine the sequence: ‘aaabaaab‘

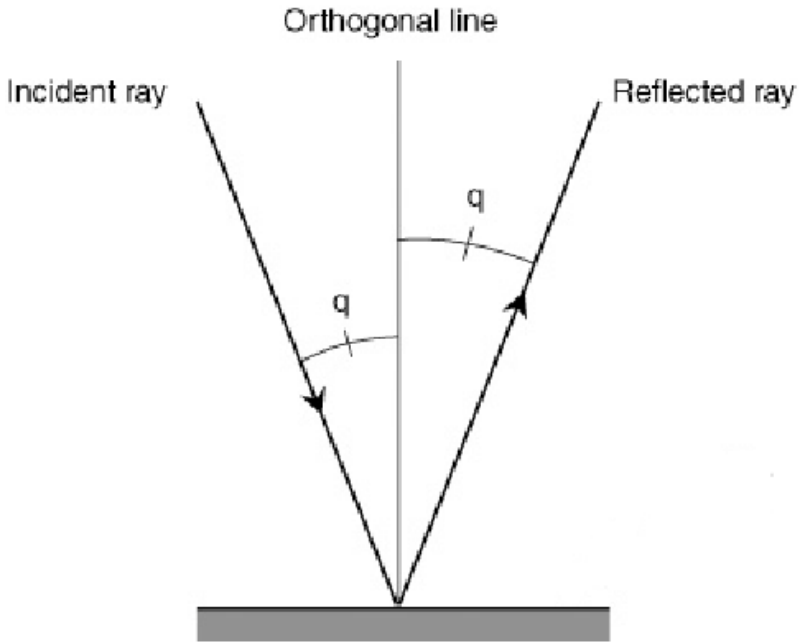


Presentation Outline

Notation

Definition

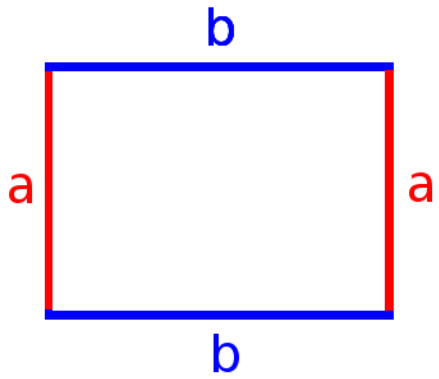
A table T is the unit square in R^2. A particle $p \in T$ begins at position $\bar{x}_0 \in T$ with velocity $\bar{v}$. When the particle reaches an edge of the table, velocity is reflected about the line perpendicular to the table's edge.



Notation

Definition

Opposite sides of the table are named a and b. **Primary side** (most collisions) is a, **secondary side** is b.



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Collision string consists of the sides of the table that have been collided with for a given starting position and velocity.

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Example

Collision string: ‘aabaaabaabaaab‘, **Primary substrings**: ‘aa‘, ‘aaa‘

Problem Statement

Problem: Characterize the properties of collision sequences.

- Given a sequence of a's and b's, determine if it is a valid collision sequence.
- Given a valid collision sequence, determine a possible starting position and velocity.

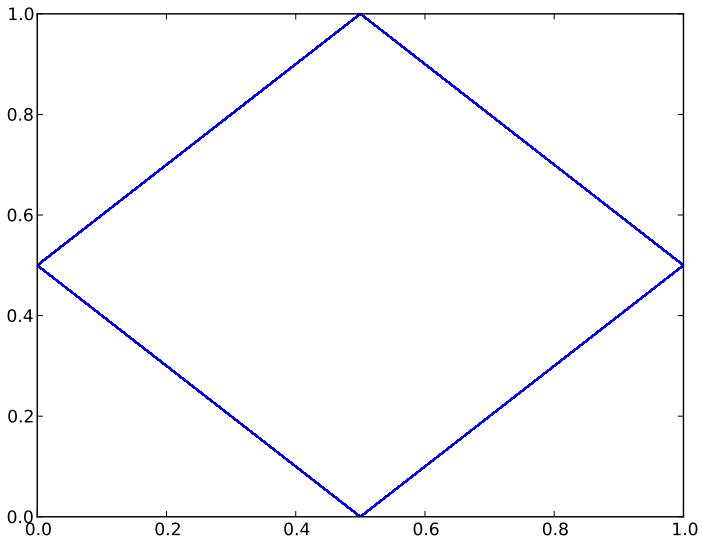
Representing Collision Strings

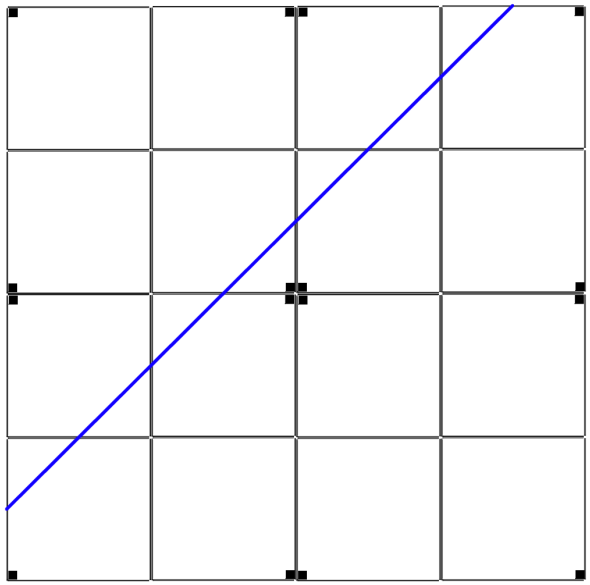
- Reflect squares about each side to create a tiling
- Solutions become lines in the plane
- Intersections become places where collisions occur

Representing Collision Strings

Example

Tiling of $\vec{x}_0 = (0, 0.5)$ and $\vec{v} = (0.25, 0.25)$.





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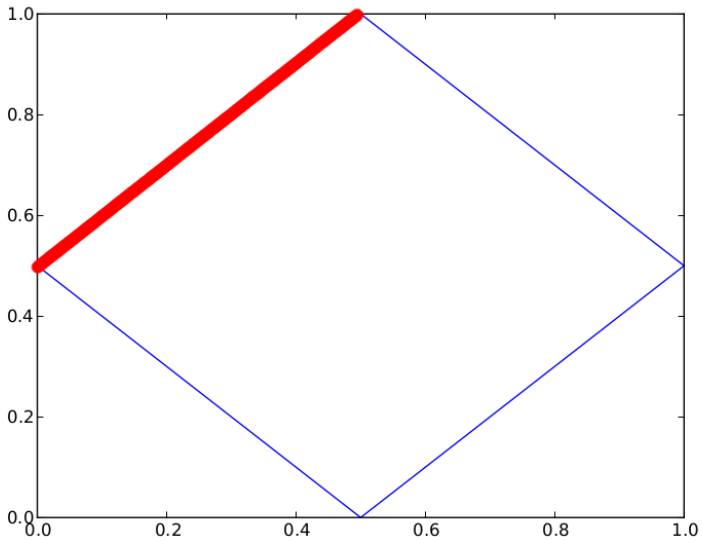

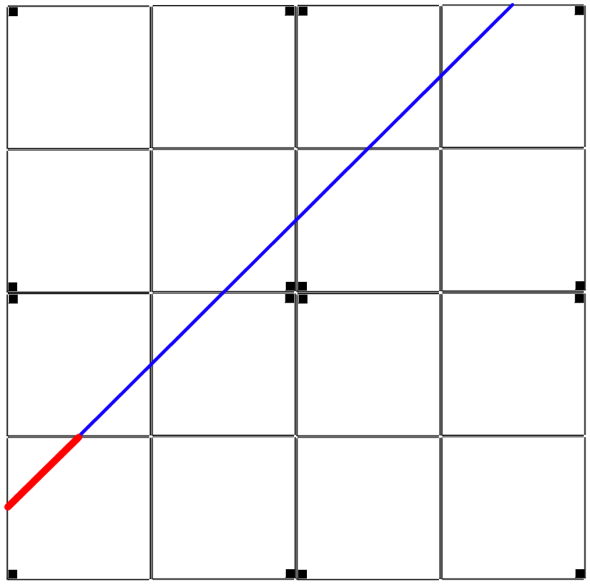

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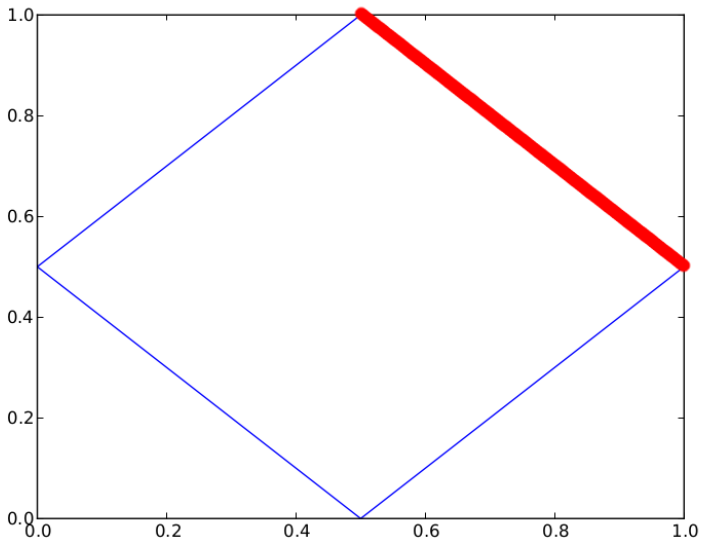

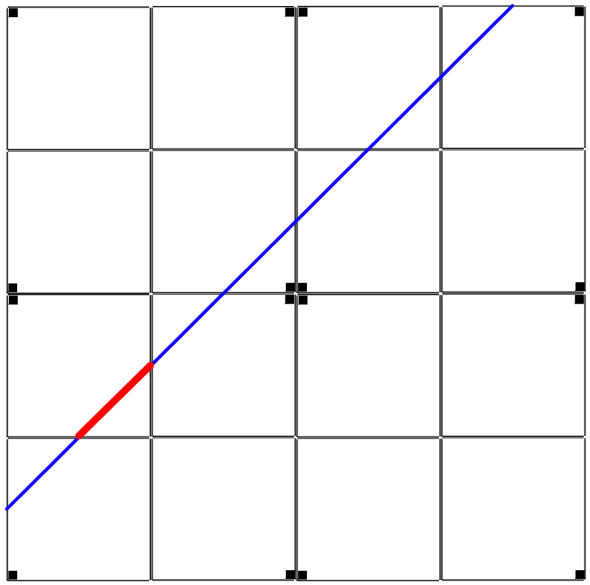

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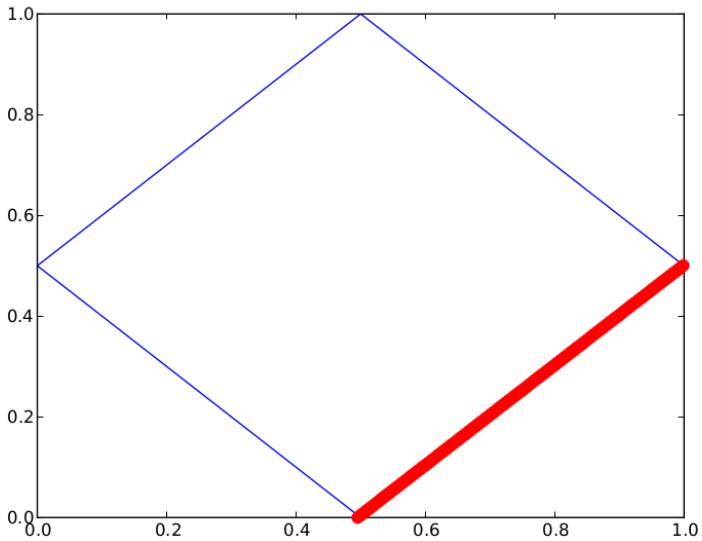

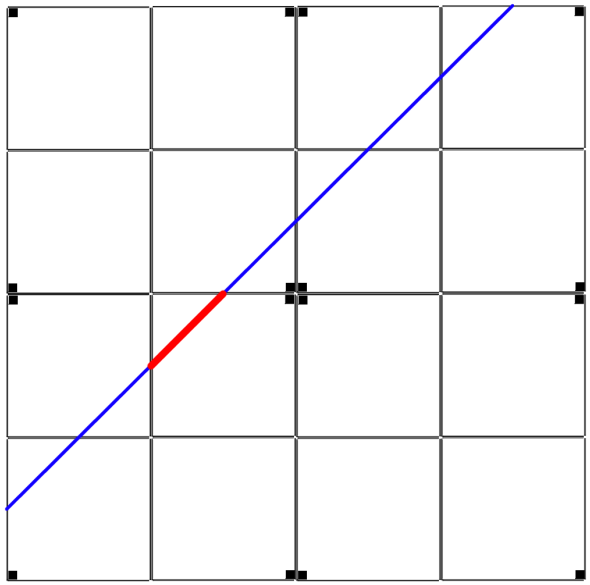

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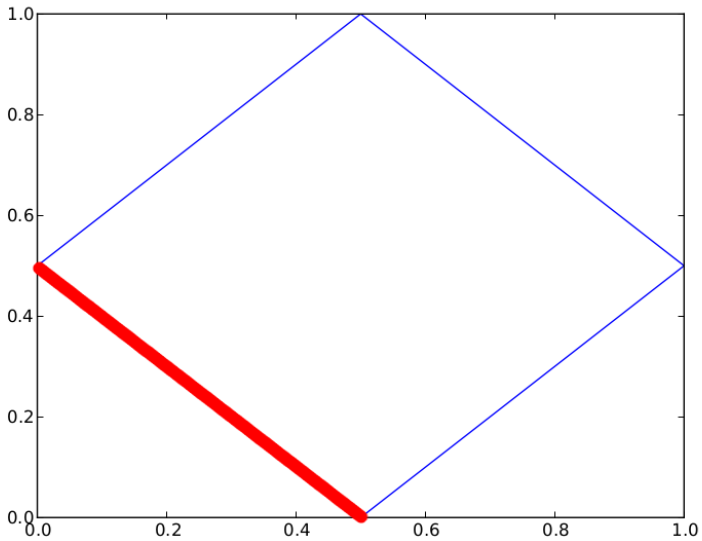

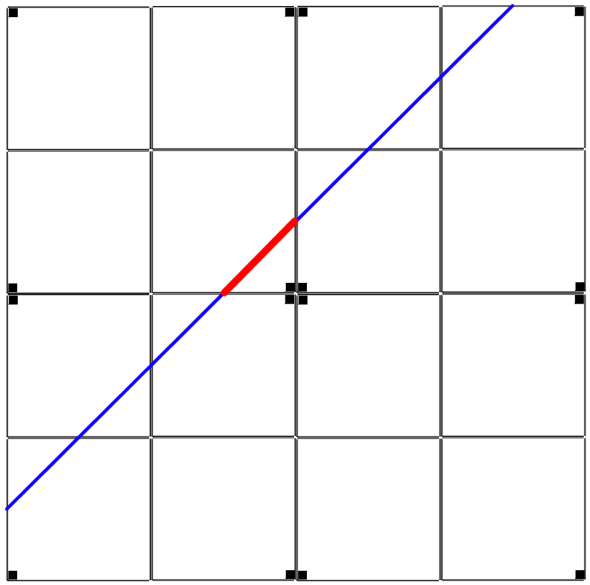

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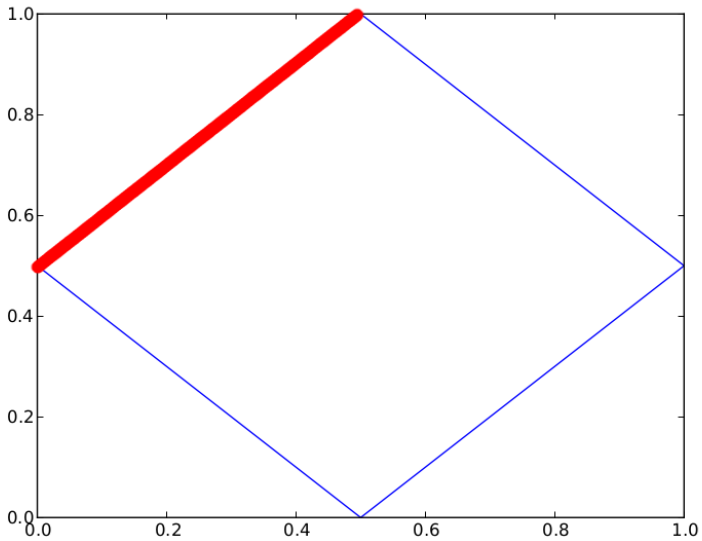

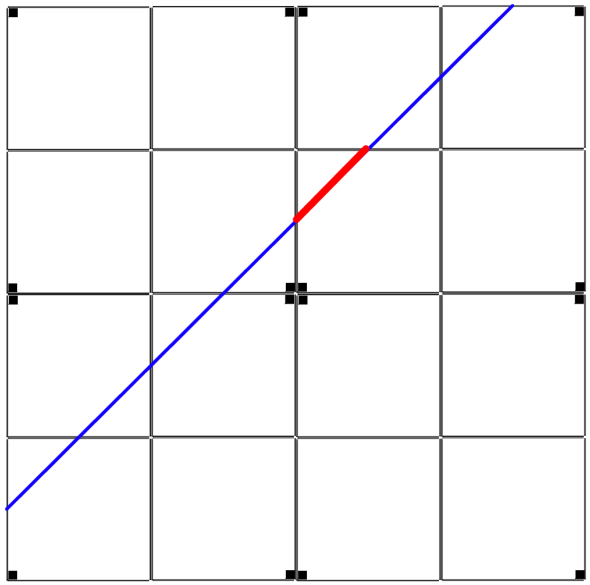

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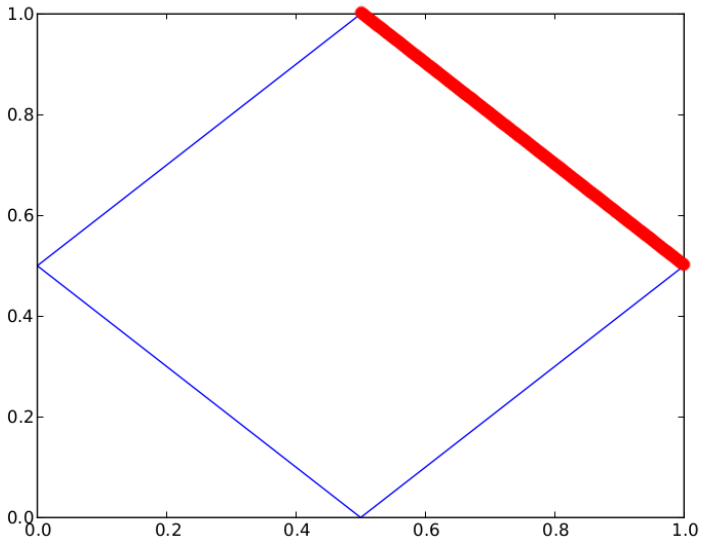

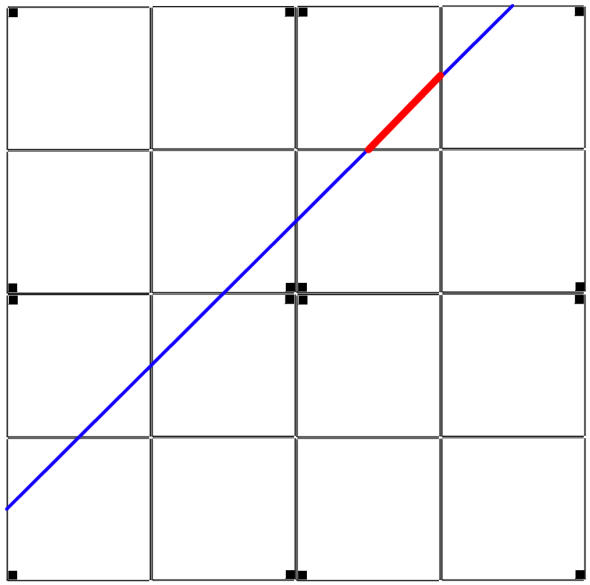

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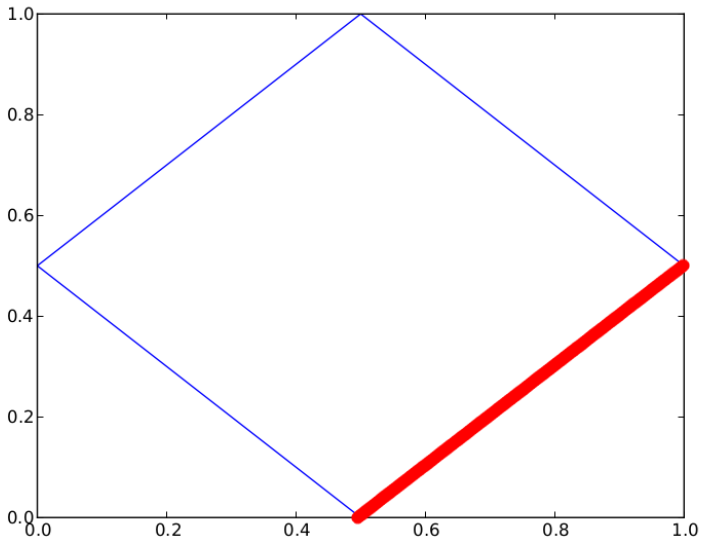

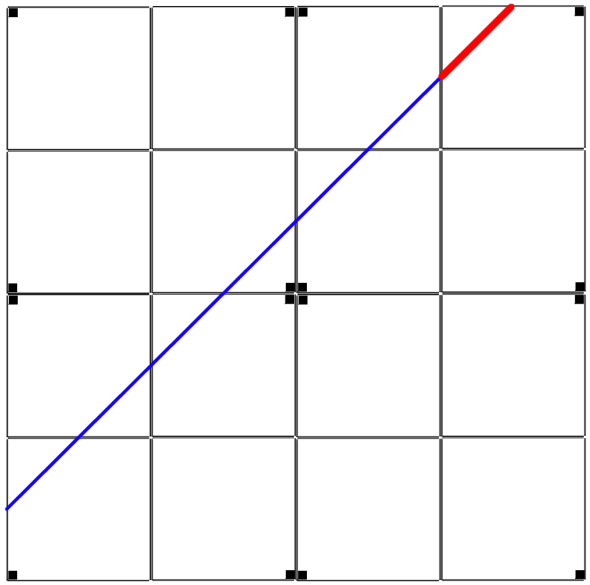

Extensions to Tileable Polygons

Other Tileable Polygons:

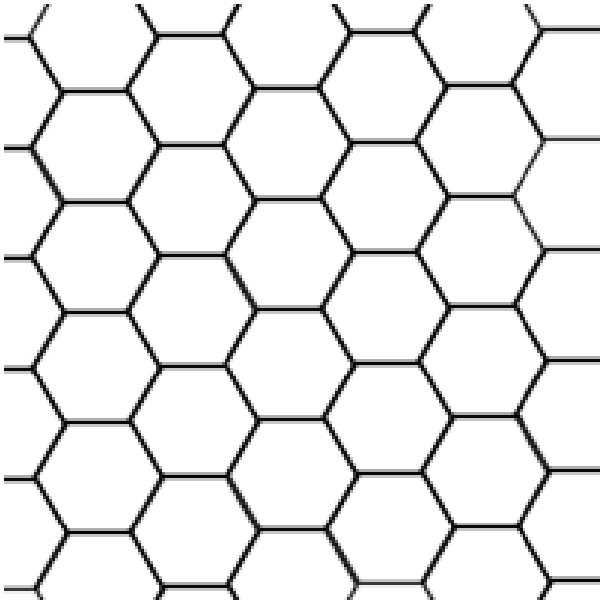

Figure: Regular Hexagons

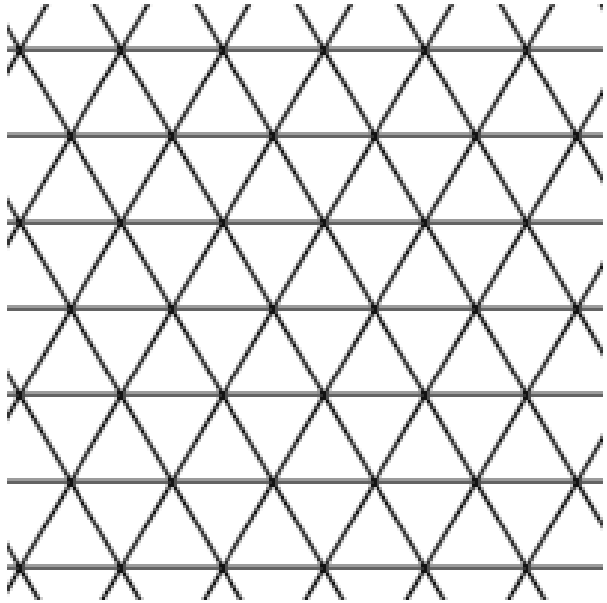

Figure: Equilateral Triangles

Extensions to Non-Tileable Polygons

- Irregular triangles
- Pentagons
- Octagons

Extensions to Circles

- Characterize how particle bounces around circle
- Analog to a, b might be sequence of collision points as you move around circle.