

Billiards

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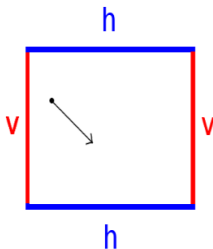
Introduction

- Billiard ball bouncing in a square
- Assume no gravity or friction

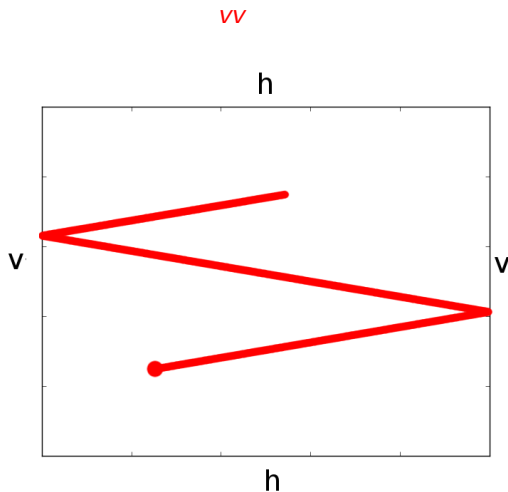
Basic Notation

Definition

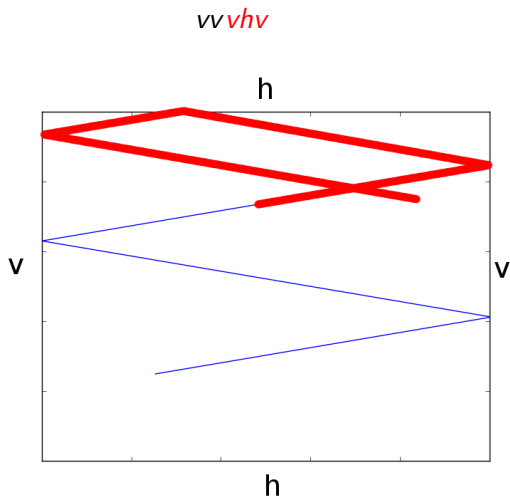
A table $T \subset \mathbb{R}^2$ is the unit square. Vertical sides are labelled with a v . Horizontal sides are labelled with an h .



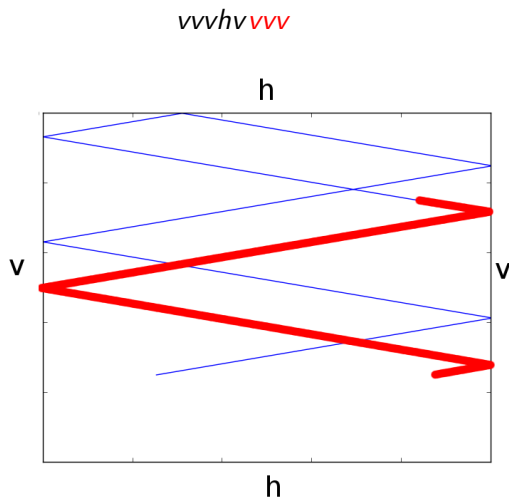
Example



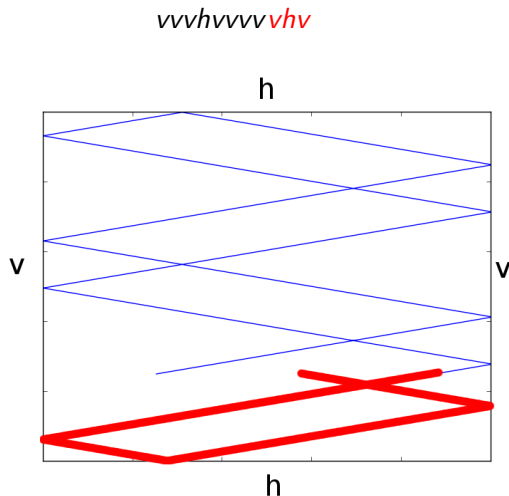
Example



Example

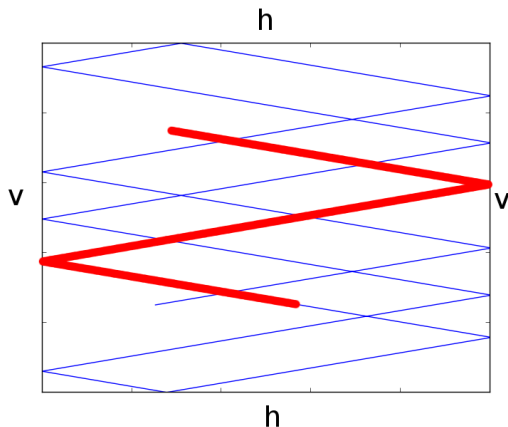


Example



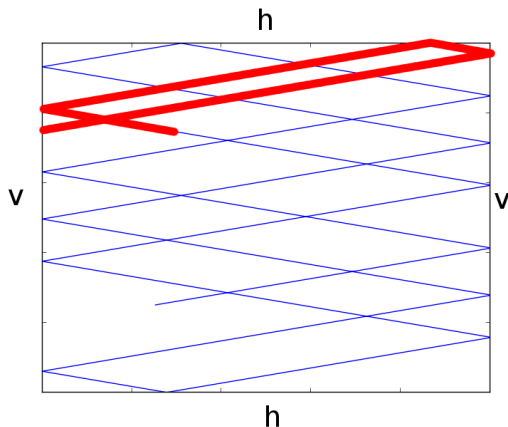
Example

$vvhvvvvvhv$ vv



Example

vvvhvvvvvhvvv *vhv*



Resulting Sequence

vvvhvvvvvvhvvvvhv

Presentation Outline

- 1 Introduction
- 2 Tiling
- 3 Theorems
- 4 Future Research

Problem Statement

Problem: Given a sequence of v and h collisions, determine if it is a valid collision sequence.

Basic Notation

Definition

v collision: when the ball collides with a v side

Definition

h collision: when the ball collides with an h side

Definition

Collision sequence (α): a sequence of v and h collisions which starts and ends with an h collision.

1 Introduction

2 Tiling

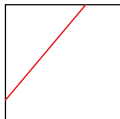
3 Theorems

4 Future Research

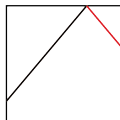
Representing Collision Sequences

- Tile the table in the plane for a more powerful representation of the problem
- Tiling will reflect the table about each side
- After tiling, we only need to deal with straight line trajectories in a tiled plane

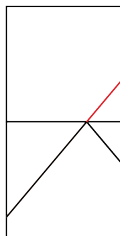
Tiling Tables



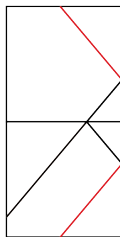
Tiling Tables



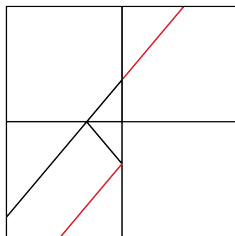
Tiling Tables



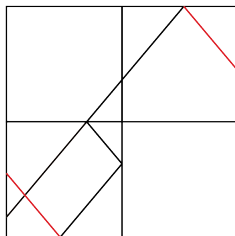
Tiling Tables



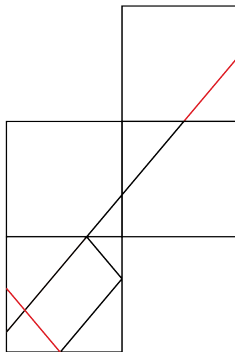
Tiling Tables



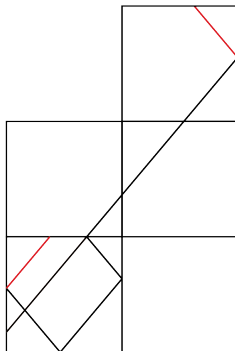
Tiling Tables



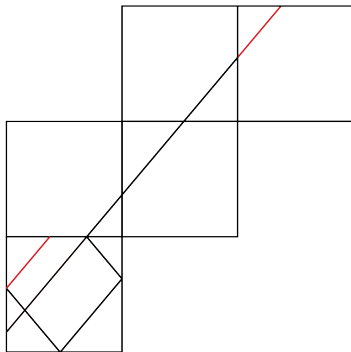
Tiling Tables



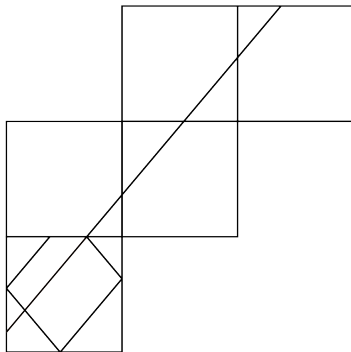
Tiling Tables



Tiling Tables



Tiling Tables



1 Introduction

2 Tiling

3 Theorems

4 Future Research

Indexing Definitions

$I(A, b)_k$: index of k^{TH} b in A

β_i : # of v 's between i^{TH} and $(i+1)^{\text{TH}}$ collisions
 $([I(\alpha, h)]_{i+1} - [I(\alpha, h)]_i)$

Theorem

$$\beta_i \geq 1 \quad i \in$$

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Extensions to Cubes

- Assign x , y , and z as the opposite pairs of faces of a cube.
- Characterize sequences of x , y , and z collisions.

Intuition

- Examine collisions in xy , yz , and xz planes.
- Movement in each plane is independent.
- Combine xy , yz , and xz collision sequences to get final sequence.

Example

Example

xy sequence: xxyyxx yz sequence: zzyzyz xz sequence: xxzzzx