Billiards

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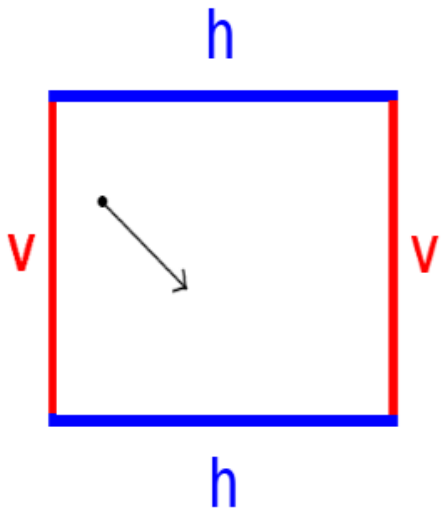
Introduction

- Billiard ball bouncing in a square
- Assume no gravity or friction

Basic Notation

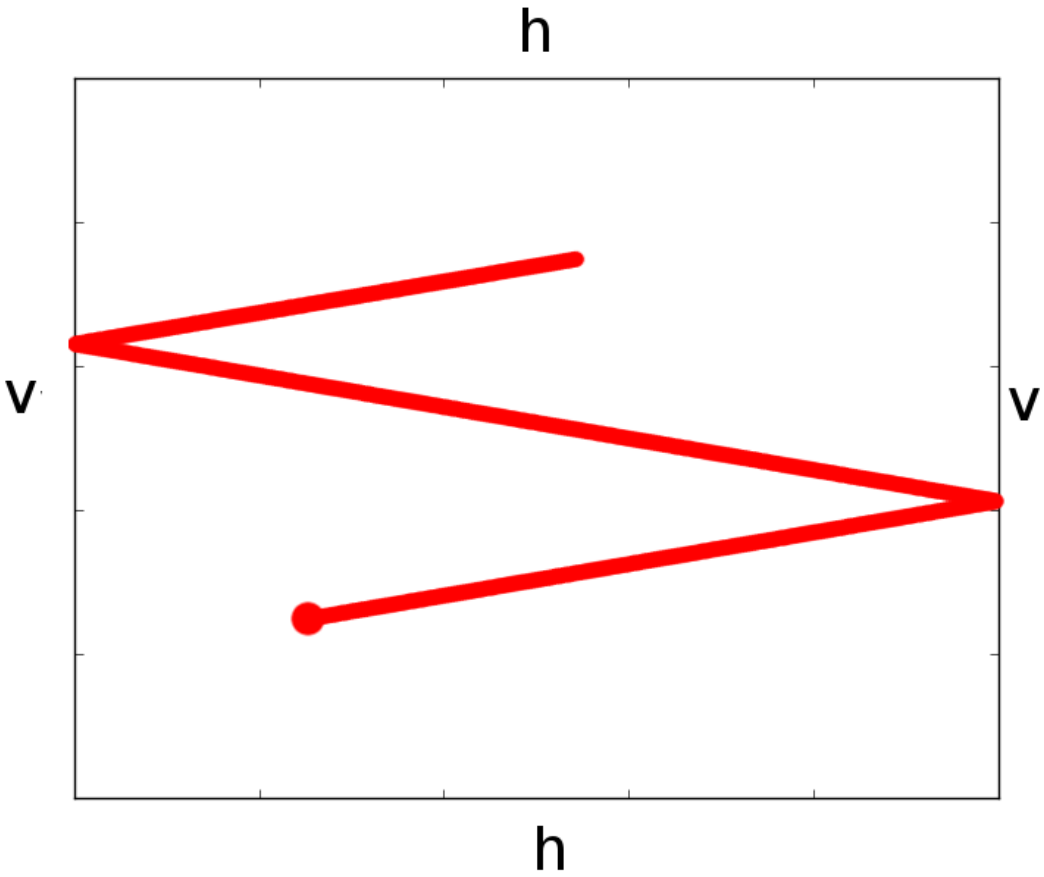
Definition

A table $T \subset \mathbb{R}^2$ is the unit square. Vertical sides are labelled with a v. Horizontal sides are labelled with an h.



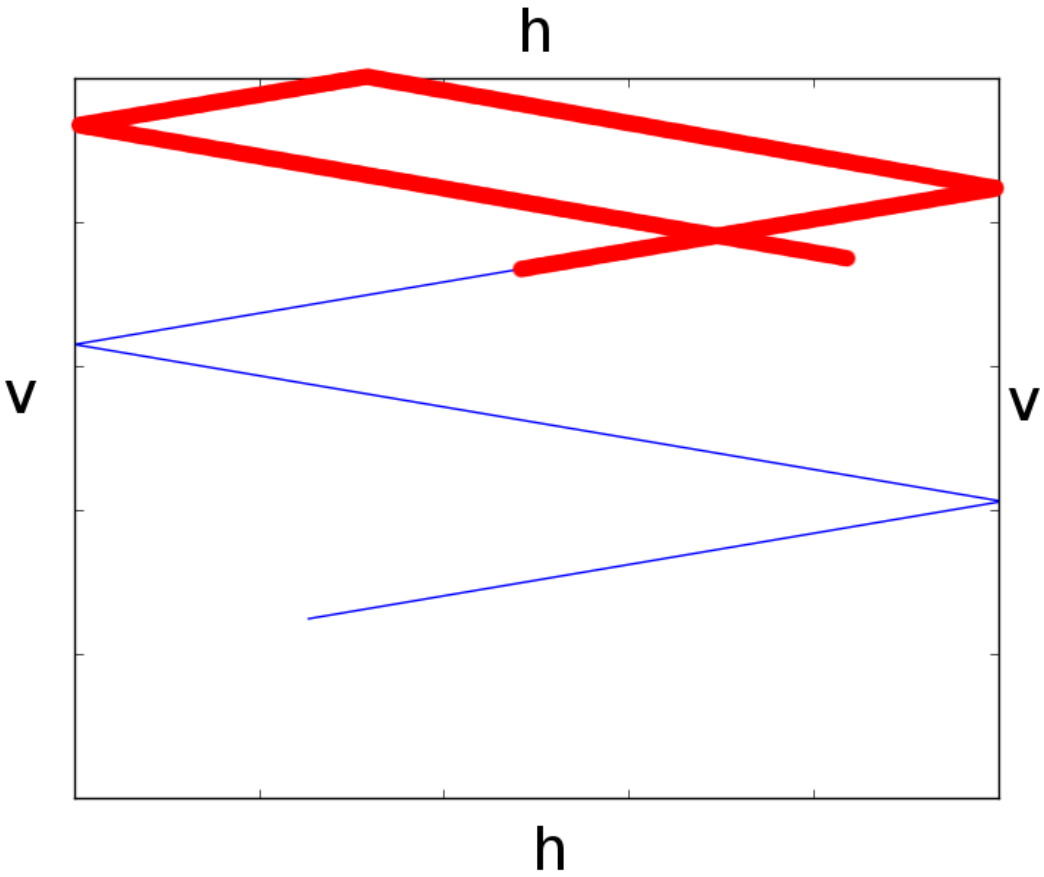
Example

vv



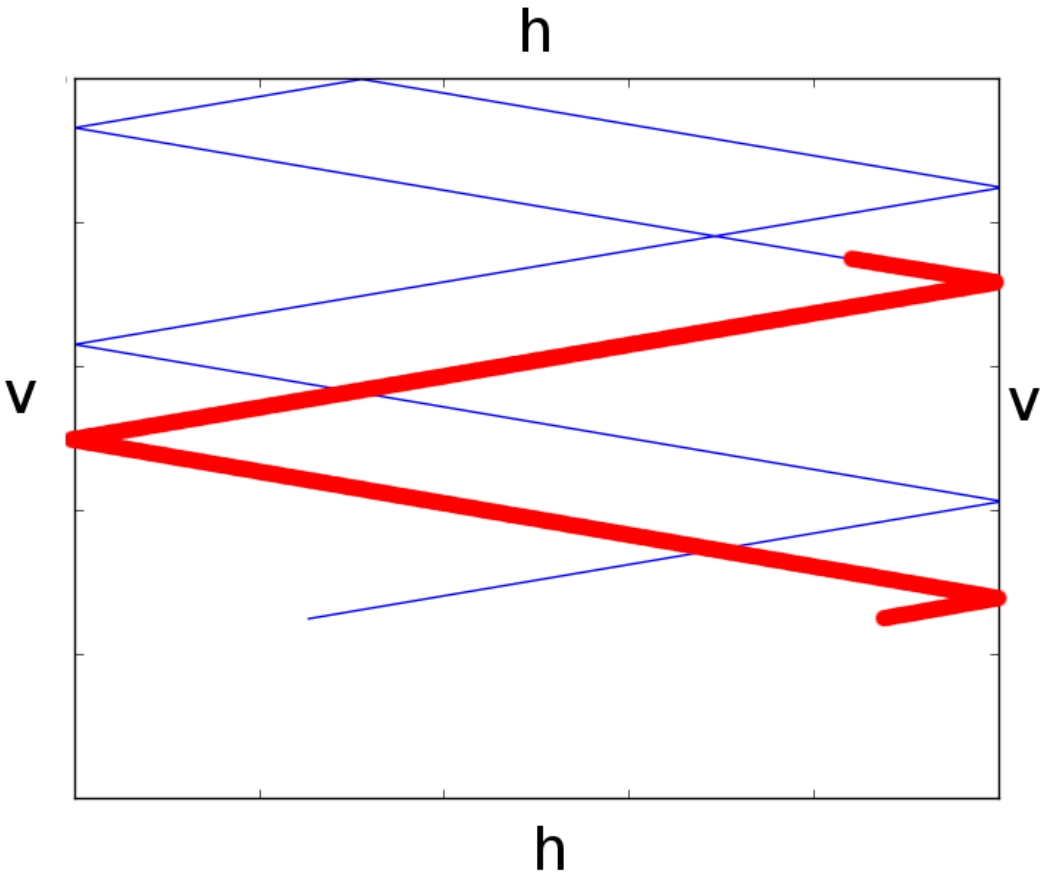
Example

vvvhv



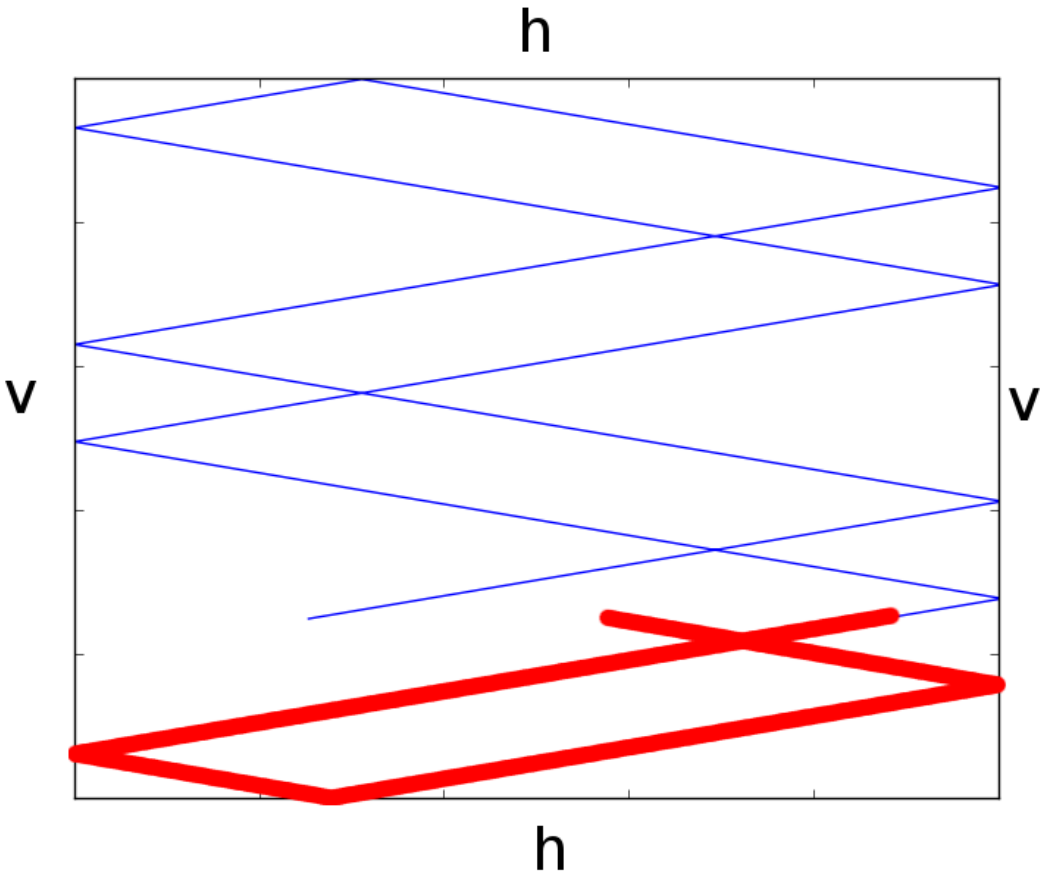
Example

vvvhv *vvv*



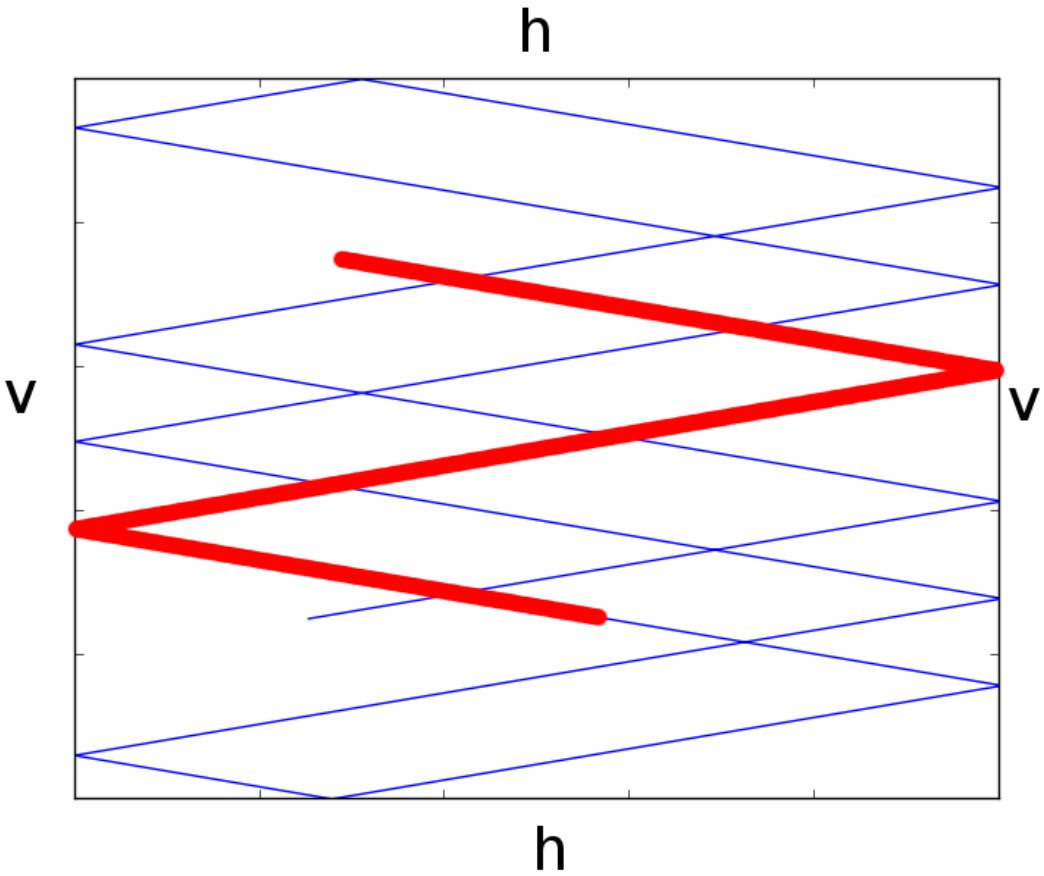
Example

vvvhvvvvvhv



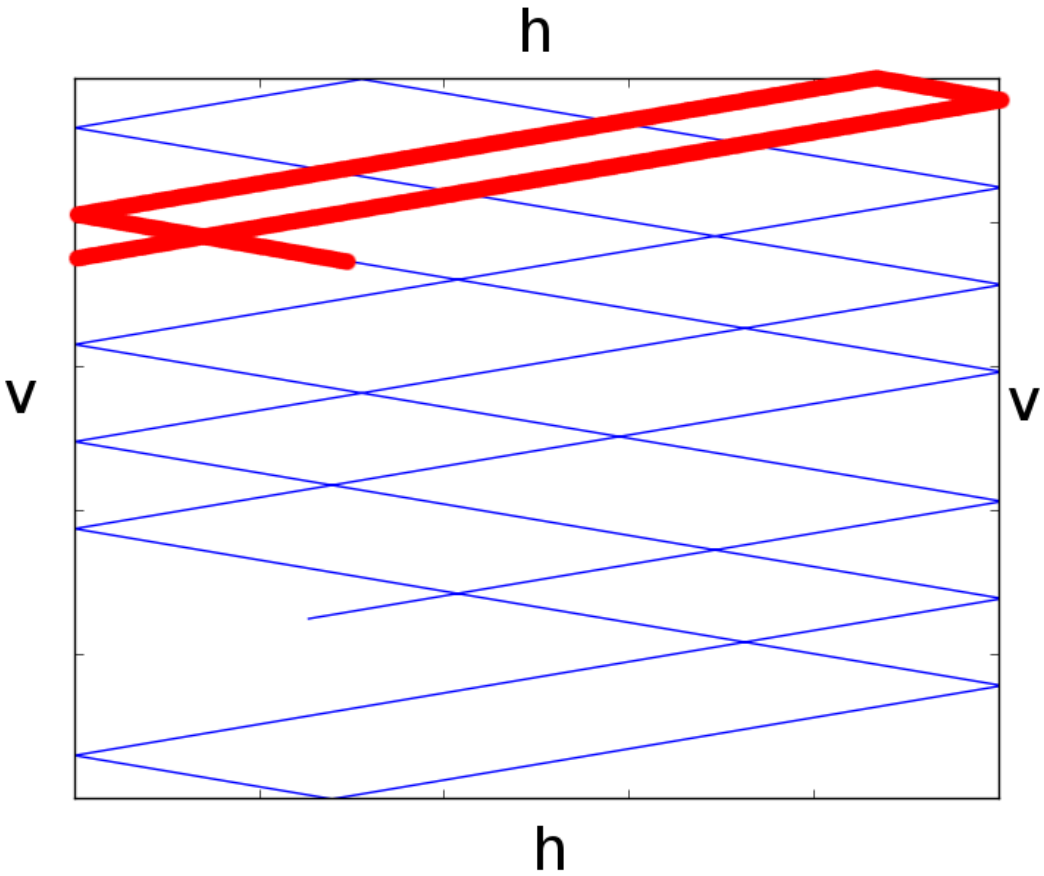
Example

vvvhvvvvvhv vv



Example

vvvhvvvvvhvvv vhv



Resulting Sequence

vvvhvvvvvhvvvvhv

Presentation Outline

Problem Statement

Problem: Given a sequence of v and h collisions, determine if it is a valid collision sequence.

Basic Notation

Definition

v collision: when the ball collides with a *v* side

Definition

h collision: when the ball collides with an *h* side

Definition

Collision sequence (α): a sequence of *v* and *h* collisions which starts and ends with an *h* collision.

Representing Collision Sequences

- Tile the table in the plane for a more powerful representation of the problem
- Tiling will reflect the table about each side
- After tiling, we only need to deal with straight line trajectories in a tiled plane

Tiling Tables

Extensions to Cubes

- Assign x, y, and z as the opposite pairs of faces of a cube.
- Characterize sequences of x, y, and z collisions.

Intuition

- Examine collisions in xy, yz, and xz planes.
- Movement in each plane is independent.
- Combine xy, yz, and xz collision sequences to get final sequence.

Example

Example

xy sequence: *xxyyxx* *yz* sequence: *zzyzyz* *xz* sequence: *xxzzzx*