Problem Set 1, Part a

Due: Thursday, September 24, 2015.
Problem sets will be collected in class. **Please hand in each problem on a separate page to facilitate grading.**
Students who agree to let us hand out their writeups can help us by writing elegant and concise solutions and formatting them using LaTeX.

Readings:

Chapters 1 and 2 of *Distributed Algorithms*; Sections 3.1-3.5.
For next week: Section 3.6; Chapter 4 (skip 4.5.3).

Problems:

Note: In all problems, you may cite theorems proved in the book without re-proving them.

1. (Based on Exercise 3.2) For the LeLann, Chang, Roberts algorithm,
   (a) Give a UID assignment for which $\Theta(n^2)$ messages are sent.
   (b) Give a UID assignment for which $\Theta(n)$ messages are sent.
   (c) Give a UID assignment for which $\Theta(n \log n)$ messages are sent.
   (d) Show that the average number of messages sent is $O(n \log n)$, where this average is taken over all possible orderings of the processes on the ring, each assumed to be equally likely.
2. We consider the problem of electing exactly *three* leaders in any ring of size at least 3 in which processes have UIDs. We assume that the algorithm can manipulate the UIDs using $(<, =, >)$. comparisons only.
   (a) Design a simple algorithm for this problem that uses $O(n^2)$ messages and just n rounds, in the worst case. Write your algorithm using pseudocode, as in the book.
   (b) State the key invariants that hold for your algorithm and are central to explaining why the algorithm works. Sketch a (preferably inductive) proof of why these invariants hold throughout the execution of your algorithm.
3. (Based on Exercise 3.8) Consider modifying the Hirschberg, Sinclair algorithm so that the processes explore by initiating the sending of tokens in one direction rather than both.
   (a) Show that the most straightforward modification to the algorithm in the text does not yield $O(n \log n)$ communication complexity. Give a good upper bound for the communication complexity.

(b) Add a little more cleverness to the algorithm in order to restore the $O(n \log n)$ complexity bound.

4. Consider the problem of electing a leader in a synchronous ring of known size n, where the processes have UIDs, but the only operations they can use for manipulating these UIDs are (equals, unequals) comparisons. Thus, the UIDs are abstract objects—you should not assume that they are a subset of the integers, or anything similar.

   Is this problem solvable or unsolvable? Prove carefully.