Problem Set 1, Part b

Due: Thursday, September 24, 2015
Problem sets will be collected in class. **Please hand in each problem on a separate page to facilitate grading.**
Students who agree to let us hand out their writeups can help us by writing elegant and concise solutions and formatting them using LaTeX.

Readings:

Sections 3.6, 4.1-4.5 of *Distributed Algorithms.*
Gallager, Humblet, Spira paper, listed in the on-line course reading list
Luby paper
Metivier et al. paper

For next week:
Peleg book, Chapters 7 and 8; Section 24.2.

Problems:

5. In this problem, consider comparison-based algorithms in bidirectional rings with UIDs.
   (a) Design an algorithm for the *Mod-3 Counting Problem*, in which each process is required to output n mod 3, where n is the total number of processes in the ring. Prove an upper bound on the number of messages used in your algorithm. Try to get the smallest value you can for this measure.
   (b) Prove the best *lower bound* you can on the number of messages required to solve the Mod-3 Counting Problem in the worst case.

6. Assume a network based on a connected undirected graph G with a distinguished leader i_0. Processes have no knowledge about G, except that the leader knows that it is the leader and the other processes know that they aren't. Assume that processes have UIDs.

   Suppose that each process i starts with an integer-valued *temperature*, $temp_i$. The problem is for the leader process i_0 to determine the *average temperature*, that is, the mean of all the individual *temp* values.
   (a) Describe informally an efficient algorithm that allows process i_0 to output the average temperature.
   (b) Give pseudocode in the style in the book for your algorithm.
   (c) Prove that your algorithm is correct.
   (d) Analyze the algorithm's time and message complexity.

7. Exercise 4.17 in Distributed Algorithms:

   In the *SynchGHS* algorithm, show that it is not the case that $O(diam)$ rounds are always sufficient to complete each level of the computation.

8. Suppose that Luby's Maximal Independent Set algorithm (the version covered in class) is executed in a ring of large size n. Suppose, for simplicity, that at each round, all the chosen ids are distinct.

(a) Prove that, in the first phase of the algorithm, the probability that any particular edge is removed from the graph is at least 2/3.
Optional: What is the exact probability?

(b) Use your answer to part (a) to prove an upper bound on the expected number of phases to complete Luby's algorithm in a ring of (large) size n. Use a constant coefficient instead of big-O notation.

9. Exercise 4.22 in Distributed Algorithms:

Consider a *line network*, that is, a linear collection of n processes $1, \ldots, n$, where each process is bidirectionally connected to its neighbors. Assume that each process i can distinguish its left from its right and knows whether or not it is an endpoint.

Assume that each process i initially has a large integer value v_i and that it can hold in memory only a constant number of such values at any time. Design an algorithm to sort the values among the processes, that is, to cause each process i to return one output value o_i, where the multiset of outputs is equal to the multiset of inputs and $o_1 \leq \ldots \leq o_n$. Try to design the most efficient algorithm you can, both in terms of the number of messages and the number of rounds. Prove your claims.