Problem Set 3, Part a

Due: Thursday, October 22, 2015
Problem sets will be collected in class. **Please hand in each problem on a separate page to facilitate grading.**

Readings:

Chapters 7 and 8
For next week: Chapters 14, 15.

Problems:

1. Consider a different kind of process failure model for synchronous systems: a "transient failure" model. In this model, a process may fail at a particular round, which means that it sends an arbitrary subset of the messages it is supposed to send (perhaps all of them), and does not perform its state transition. A process that exhibits a transient failure at a round r continues as if nothing is wrong at the following round $r+1$. Permanent failure of a process at round r is modeled by transient failure at all rounds greater than or equal to r.

   For this problem, we assume that, at each round, at most one process exhibits a transient failure. We do not assume an overall bound on the number of processes that ever exhibit a transient failure during an execution.

   Now consider the agreement problem in this transient failure model: each process that does not fail permanently should eventually decide, subject to the usual (uniform) agreement condition for stopping agreement, and the strong validity condition (every process' decision is some process' initial value).

   Is this problem solvable in the given model? If so, describe an algorithm and sketch a proof that it works. If not, try to prove impossibility (carefully), using techniques like the ones in the Aguilera-Toueg paper.

2. This problem develops a proof of Sperner's Lemma for the special case of a 2-dimensional Bermuda Triangle.

   (a) Consider a path graph in which each vertex is colored with a "color" in $\{0, 1\}$. Suppose that the two endpoints are colored differently. Prove that there must be an odd number of edges in the path whose endpoints are colored differently.

   (b) Now consider a 2-dimensional Bermuda triangle graph of the sort depicted on p. 169, in which each vertex is colored with a "color" in $\{0, 1, 2.\}$. Suppose that the three corners are colored differently. Prove that there must be an odd number of basic triangles (i.e., triangles that are not decomposed any further) whose three corners are colored differently. It follows that there must be at least one basic triangle whose three corners are colored differently. **Hint:** Consider the parity of the number of 0-1 edges overall.

3. Exercise 7.17.

   Give a careful description of a modification to the *ThreePhaseCommit* algorithm that permits processes to decide and halt quickly in the failure-free case. Your algorithm should use a small constant number of rounds and $O(n)$ messages, in the failure-free case. Prove its correctness.

4. Exercise 8.5.

   (a) Define an I/O automaton A representing a reliable message channel that accepts and delivers messages from the union of two alphabets, M_1 and M_2. The message channel is supposed to preserve the order of messages from the same alphabet. Also, if a message from alphabet M_1 is sent prior to another message from alphabet M_2, then the corresponding deliveries must occur in the same order. However, if a message from M_1 is sent after a message from M_2, then the deliveries are permitted to occur in the opposite order. Your automaton should actually exhibit all of the allowable external behaviors. Be sure to give all components of A: the signature, states, start states, steps, and tasks, written in the pseudocode style used in the book, for example, on pages 204-205.

   (b) For your automaton, give an example of each of the following: a fair execution, a fair trace, an execution that is not fair, and a trace that is not fair.

5. Let A be any I/O automaton. Show how to construct another I/O automaton B with the same inputs and outputs as A, such that $fairtraces(B) \subseteq fairtraces(A)$, and such that B satisfies all the following restrictions:

   (a) B has only one task.

   (b) B has only one initial state.

   (c) In every state of B, at most one locally-controlled action is enabled.

   (d) For every state s and every action π, B has at most one transition of the form (s, π, s').