#2 (Jan 11)

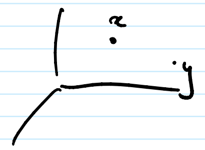
Thursday, 11 January 2024 11:16 am

Geodesic flow

→ doesn't have problem of v. field (only depends on position

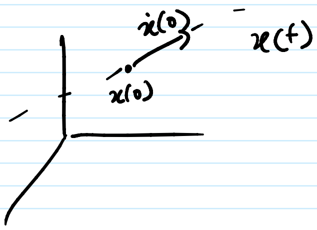
Ingredients: M mfld, g Riemmanian metric

e.g. $M = \mathbb{R}^3$, $g = \delta$ $\quad g(x,y) = x_1 y_1 + x_2 y_2 + x_3 y_3$



$$d_{Euc}(x,y) = \sqrt{(y_1 - x_1)^2 + (y_2 - x_2)^2 + (y_3 - x_3)^2}$$

Geodesics in $\mathbb{R}^3$: "straight lines"



Initial data: $(x(0), \dot{x}(0))$ @ $t=0$

geodesic $\quad x(t) = x(0) + t\dot{x}(0)$

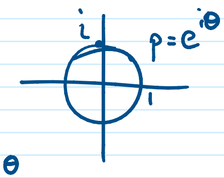
Two main approaches: 1) "shortest path" (vague)

→ parametrized by length

→ "variational principle"

2) "acceleration $= 0$" "$\frac{d}{dt}$(velocity)" need connection ∇ because two velocities not in same v. space.

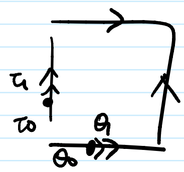
eg. S^1

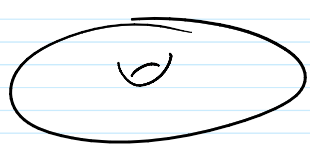


→ describe $x = e^{i\theta}$

→ geodesics $x(t) = e^{i(\theta_0 + t\theta_1)}$, $\dot{x}(t) = i\theta_1 e^{i(\theta_0 + t\theta_1)}$

eg. $S^1 \times S^1 = T^2$





→ geodesic in the product $(x(t), y(t)) = (e^{i(\theta_0 + \theta_1 t)}, e^{i(\tau_0 + \tau_1 t)})$

→ emergence of non-periodic trajectories

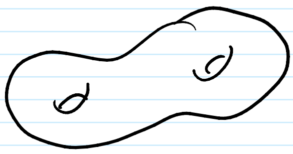
S^2

→ divergence to non periodic trajectories

eg. S^2 (g_{Euc})

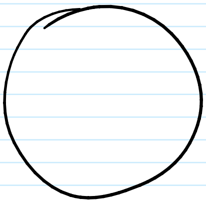
all periodic, w/ varying freq.

eg Σ_2



Uniformization theorem

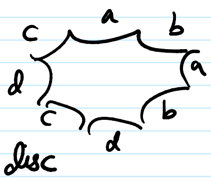
→ $\exists$ constant curvature $= -1$ metric on any $\Sigma_{g\geq 2}$ compact oriented surface



D_2 hyperbolic disc

geodesics = circular arcs w/ ideal boundary at $\perp$

can cut T^2 to get



→ can be embedded in Poincare disc