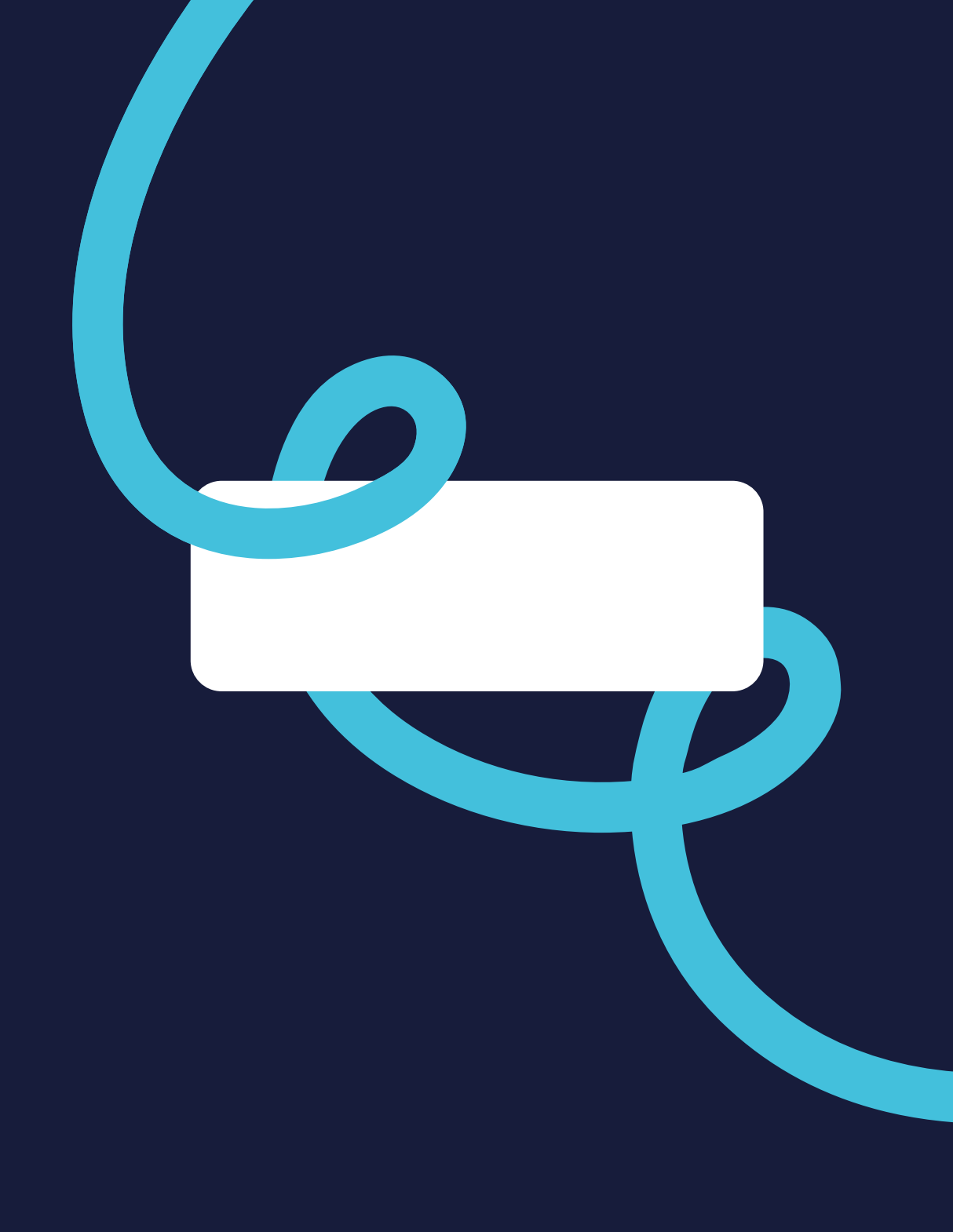


7.

a) $\alpha = 1 - 0.63 = 0.37$, $\overline{X_n} = 135.6$

$\rightarrow Z_{\frac{\alpha}{2}} = Z_{0.185} = 0.89$

$\overline{X_n} \pm 0.89 \frac{\sigma}{n} = 135.6 \pm 0.89 \frac{15}{12}$

$= 135.6 \pm 1.1125$

b) $\frac{\alpha}{2} = \frac{1 - 0.754}{2} = 0.123$

$Z_{0.123} = 1.16$

$\overline{X_n} \pm 1.16 \frac{5}{4} = 135.6 \pm 1.45$

c) $\frac{\alpha}{2} = \frac{1 - 0.91}{2} = 0.045$

$z_{0.045} = 1.70$

$\bar{x}_n \pm 1.70 \frac{5}{4} = 135.6 \pm 2.125$

d) It means that if we construct this interval for many sample we are 91% confident that the true p lies in the interval.

e) The CI becomes wider

f)

$z_{\frac{\alpha}{2}} = z_{0.01} = 2.33 \rightarrow 2.33 \cdot \frac{5}{4} = 2.913$

g) i) $2.33 \cdot \frac{15}{33} = 1.589$

ii) It becomes narrower as we increase n

h) $2.33 \cdot \frac{15}{x} = 5.66 \Rightarrow x = 6$

2.

$\bar{X}_n \pm Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{n}$

$\sigma = 7$, $n = 50$, $\bar{X}_n = 44.3$

$Z_{0.90} = 1.28$

a. $44.3 \pm 1.28 \frac{7}{50} = 44.3 \pm 0.1792$

b. We are 90% confident that if we take lots of samples the true p is in the 90% of the intervals

c.

$z_{0.95} = 1.64$

$\hookrightarrow 44.3 \pm 1.64 \cdot \frac{7}{50} = 44.3 \pm 0.2296$

d. $44.3 \pm 2.33 \frac{7}{50} = 44.3 \pm 0.3262$

$z_{0.99} = 2.33$

e. $\frac{99}{100} \cdot 5000 = 4950$

f. $44.3 \pm 2.33 \frac{7}{820} = 44.3 \pm 0.0199$

g. $(-\infty, \infty)$

3. $n=48$, 36 said yes $\Longrightarrow \hat{p}_n = \frac{3}{4}$

a. $\hat{p}_n = \frac{36}{48}$

Margin of error $= z * \sqrt{\frac{\hat{p}_n(1-\hat{p}_n)}{n}}$

$z_{0.75} = 0.77 \rightarrow 0.77 * \sqrt{\frac{\frac{1}{4} \cdot \frac{3}{4}}{48}} = 0.048$

$\rightarrow 0.75 \pm 0.048$

b. $\hat{p}_n = \frac{3}{4}$, $z_{0.95} = 1.64$

$\rightarrow 1.64 \cdot \sqrt{\frac{3}{16 \cdot 48}} = 0.103$

$\rightarrow 0.75 \pm 0.103$

c. It means that if we take many samples, 95% of them contain the true p within their interval. The interval is bigger for C because the confidence is higher

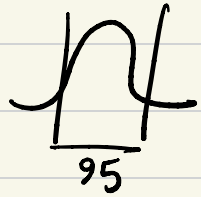
d. $8000 \times \frac{95}{100} = 7600$

e. $(-\infty, \infty)$

4. $n = 242$, $\hat{p}_n = \frac{85}{242}$

$\alpha = 0.05$

$\alpha = 0.025$



a. Margin of error $= z_{0.625} \cdot \sqrt{\frac{p(1-p)}{242}}$

$= 1.96\sqrt{\frac{85}{242}} = 0.06$

$$\begin{array}{r} 13 \\ 3\,12 \\ 242 \\ -\ 85 \\ \hline 157 \end{array}$$

$\Rightarrow 0.35 \pm 0.06$

b. $1 - 0.826 = 0.087 \rightarrow z_{0.087} = 1.36$

$\Rightarrow 0.35 \pm 1.36\sqrt{\frac{(85)(157)}{242^3}} = 0.35 \pm 0.042$

c. 82.6% of intervals among the many samples that we get include the true p

d. $1 - 0.99 = 0.01/2 = 0.005$

$\rightarrow z_{0.005} = 2.58$

$\rightarrow 0.35 \pm 2.58\sqrt{\frac{(85)(157)}{242^3}} = 0.35 \pm 0.079$

e. $n = 1210, \ \hat{p} = \frac{415}{1210} = 0.343$

$0.343 \pm 2.58\sqrt{\frac{(415)(1210-415)}{1210^3}} = 0.343 \pm 0.035$

5.

a. We can assume the data is normal as it closely follows the line

b $S_n^2 = 80.11$, $\alpha = 0.10$, $\frac{\alpha}{2} = 0.05$

$$\hookrightarrow L, R = \left(\frac{(n-1)S_n^2}{\chi^2_{n-1, \frac{\alpha}{2}}}, \frac{(n-1)S_n^2}{\chi^2_{n-1, 1-\frac{\alpha}{2}}} \right)$$

$$= L, R = \left(\frac{1121.54}{\chi^2_{n-1, \frac{\alpha}{2}}}, \frac{1121.54}{\chi^2_{n-1, \frac{\alpha}{2}}} \right)$$

$$= (L, R) = \left(\frac{1121.54}{23.68}, \frac{1121.54}{6.571} \right) = (47.36, 170.68)$$

c.

$$L, R = \left(\frac{1121.54}{26.12}, \frac{1121.54}{5.63} \right)$$

$$= (42.94, 199.21)$$

d. $L, R = \left(\frac{1121.54}{29.14}, \frac{1121.54}{4.66} \right)$

$= (38.49, 240.67)$

e. We are 98% confident that p is within this interval

f.

i) $\bar{x}_n \sim N(162.5, 9.3^2)$

ii) Approximate because it's not exactly a normal distribution

6. $n = 15$, $\mu = 100$, $\sigma = 12$ where

σ is known and μ is unknown

d. It's 0.9

e. It becomes 0.925

f. Becomes 0.953

g. The probability increases