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MAE 6700
5/12/15
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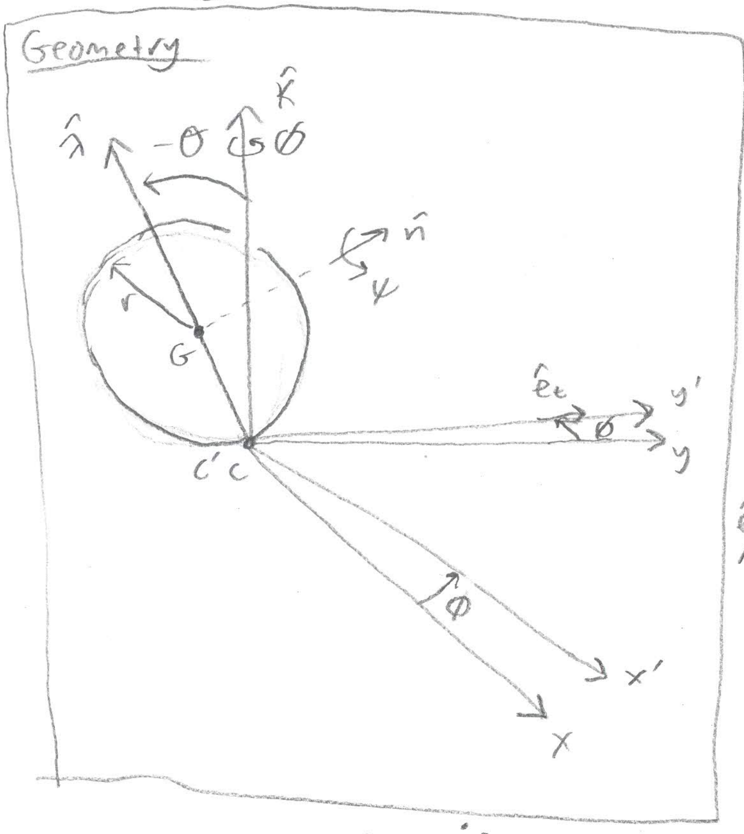
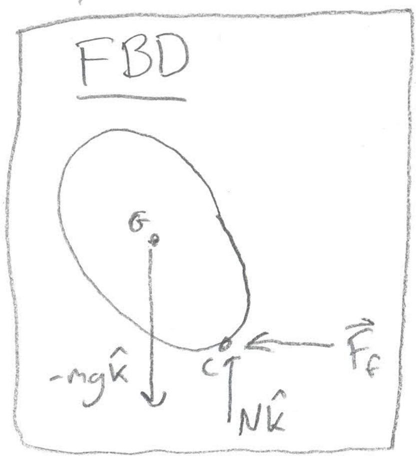
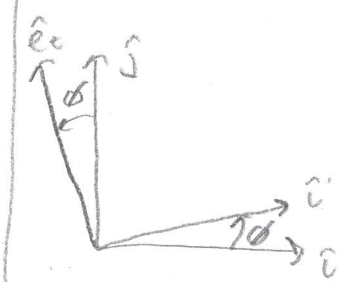
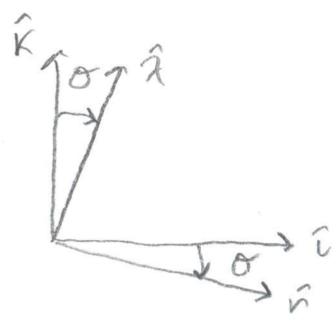
Final Project Questions MAE 6700

19) Rolling Disk in 3D. Find, Solve, Animate









- $AMB_{/C}$: $\sum \vec{M}_{/C} = \dot{\vec{H}}_{/C}$ C fixed in space

$$\sum \vec{M}_{/C} = \vec{r}_{G/C} \times m\vec{a}_G$$

$$\dot{\vec{H}}_{/C} = \vec{r}_{G/C} \times m\vec{a}_G + \underline{\underline{I}}^G \cdot \dot{\vec{\omega}}_{D/F} + \vec{\omega}_{D/F} \times (\underline{\underline{I}}^G \cdot \vec{\omega}_{D/F})$$

- Constraint $\vec{V}_{C'} = \vec{V}_C$

$$\vec{0} = \vec{V}_G + \vec{V}_{C'/G} = \vec{V}_G + \vec{\omega}_{D/F} \times \vec{r}_{C'/G} \qquad \vec{r}_{C'/G} = -r\hat{\lambda}$$

$$\vec{V}_G = \vec{\omega}_{D/F} \times r\hat{\lambda}$$

$$\vec{a}_G = \dot{\vec{V}}_G = \dot{\vec{\omega}}_{D/F} \times r\hat{\lambda} + \vec{\omega}_{D/F} \times r\dot{\hat{\lambda}} \qquad \dot{\vec{\omega}}_{D/F} = \vec{\alpha}$$

$$\vec{a}_G = \vec{\alpha} \times r\hat{\lambda} + \vec{\omega}_{D/F} \times r\dot{\hat{\lambda}}$$

$$\vec{\omega}_{D/F} = \dot{\phi}\hat{k} + \dot{\theta}\hat{e}_t + \dot{\psi}\hat{n}$$

$$\vec{\alpha} = \ddot{\phi}\hat{k} + \dot{\phi}\cancel{\dot{\hat{k}}}^{\,0} + \ddot{\theta}\hat{e}_t + \dot{\theta}\dot{\hat{e}}_t + \ddot{\psi}\hat{n} + \dot{\psi}\dot{\hat{n}}$$

$$\dot{\hat{e}}_t = \dot{\phi}\hat{k} \times \hat{e}_t \qquad \dot{\hat{n}} = (\dot{\phi}\hat{k} + \dot{\theta}\hat{e}_t) \times \hat{n} \qquad \hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\hat{\lambda} = \cos\theta\,\hat{k} + \sin\theta\,\hat{i}' \qquad \hat{n} = \cos\theta\,\hat{i}' - \sin\theta\,\hat{k} \qquad \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{\underline{R}}_B = \underline{\underline{R}}(\hat{j}, \theta) \cdot \underline{\underline{R}}(\hat{k}, \phi)$$

Banked frame

$$\underline{\underline{R}}(\hat{j}, \theta) = \begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix} \qquad \underline{\underline{R}}(\hat{k}, \phi) = \begin{bmatrix} C_\phi & -S_\phi & 0 \\ S_\phi & C_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^F\underline{\underline{I}}^G = \underline{\underline{R}}_B \cdot {}^B\underline{\underline{I}}^G \cdot \underline{\underline{R}}_B^T \qquad {}^B\underline{\underline{I}}^G = \begin{bmatrix} 2I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

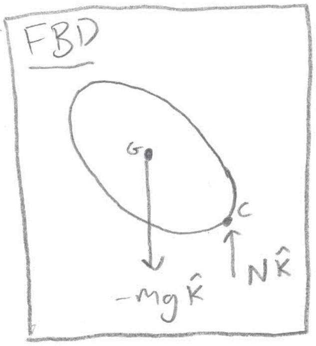
for uniform disk $I = \frac{mr^2}{4}$

Input above equations into MATLAB symbolic and solve AMB for $\ddot{\phi}$, $\ddot{\theta}$, and $\ddot{\psi}$

(Everything is represented in Fixed Frame)

Problem 20) Slidding Disk in 3D. Find, Solve, Animate

* see problem 19 for geometry picture



• AMB: $\sum \vec{M}_{/c} = \dot{\vec{H}}_{/c}$

$$\sum \vec{M}_{/c} = \vec{r}_{G/c} \times (-mg\hat{k})$$

$$\dot{\vec{H}}_{/c} = \dot{\vec{H}}_{G/c} + \dot{\vec{H}}_{/G}$$

$$\dot{\vec{H}}_{/c} = \vec{r}_{G/c} \times m\vec{a}_G + \underline{\underline{I}}^G \cdot \dot{\vec{w}}_{D/F} + \vec{w}_{D/F} \times (\underline{\underline{I}}^G \cdot \vec{w}_{D/F})$$

• LMB: $\{\sum \vec{F} = m\vec{a}_G\}$ $\qquad \sum \vec{F} = (N - mg)\hat{k}$

① $\{\} \cdot \hat{i} \Longrightarrow \ddot{x}_G = 0 \qquad \{\} \cdot \hat{j} \Longrightarrow \ddot{y}_G = 0$

• Constraint: $\quad z_c = 0\hat{k}$ (height of pnt of contact $=0$)

$$z_G = z_c + r\cos\theta\hat{k} = r\cos\theta\hat{k}$$

$$\tfrac{d}{dt}(z_G) = \dot{z}_G = -r\dot{\theta}\sin\theta\hat{k}$$

② $\tfrac{d}{dt}(\dot{z}_G) = \ddot{z}_G = -r\ddot{\theta}\sin\theta\hat{k} - r\dot{\theta}^2\cos\theta\hat{k}$

① & ② $\Longrightarrow \vec{a}_G = \ddot{x}_G\hat{i} + \ddot{y}_G\hat{j} + \ddot{z}_G$

$$\vec{a}_G = (-r\ddot{\theta}\sin\theta - r\dot{\theta}^2\cos\theta)\hat{k}$$

$$\vec{w}_{D/F} = \dot{\phi}\hat{k} + \dot{\theta}\hat{e}_t + \dot{\psi}\hat{n}$$

$$\vec{\alpha} = \dot{\vec{w}}_{D/F} = \ddot{\phi}\hat{k} + \cancel{\dot{\phi}\dot{\hat{k}}}^{\,0} + \ddot{\theta}\hat{e}_t + \dot{\theta}\dot{\hat{e}}_t + \ddot{\psi}\hat{n} + \dot{\psi}\dot{\hat{n}}$$

• unit vectors and $\frac{d}{dt}$(unit vectors)

$\dot{\hat{e}}_t = \dot{\phi}\hat{k} \times \hat{e}_t \qquad \dot{\hat{n}} = (\dot{\phi}\hat{k} + \dot{\theta}\hat{e}_t) \times \hat{n}$

$\hat{\lambda} = \cos\theta\hat{k} + \sin\theta\hat{i} \qquad \hat{n} = \cos\theta\hat{i} - \sin\theta\hat{k}$

$\hat{i} = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad \hat{j} = \begin{bmatrix}0\\1\\0\end{bmatrix} \qquad \hat{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$$\underset{\equiv}{{}^{F}I^{G}} = \underset{\equiv}{R}_B \cdot \underset{\equiv}{{}^{B}I^{G}} \cdot \underset{\equiv}{R}_B^T$$

Fixed — body — Banked Frame

$$\underset{\equiv}{{}^{B}I^{G}} = \begin{bmatrix} 2I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

For uniform disk $I = \frac{mr^2}{4}$

$$\underset{\equiv}{R}_B = \underset{\equiv}{R}(\hat{j}, \theta) \cdot \underset{\equiv}{R}(\hat{k}, \phi)$$

Banked

$$\underset{\equiv}{R}(\hat{j}, \theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} \qquad \underset{\equiv}{R}(\hat{k}, \phi) = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

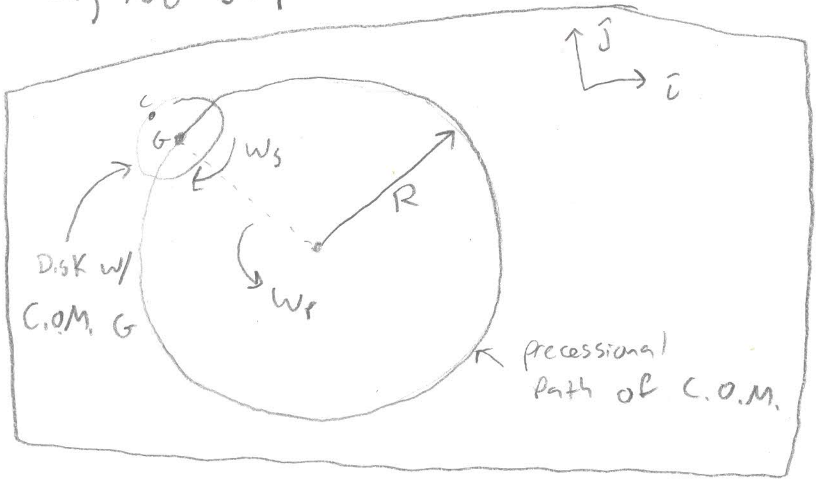
Input above equations into MATLAB symbolic & solve AMB & LMB for $\ddot{\phi}$, $\ddot{\theta}$, and $\ddot{\psi}$

(Everything is represented in Fixed Frame)

Problem 16/21) Disk on a plane. Constant lean θ constant pitch rate w_s. Find restrictions on radius of circular motion R, w_s, and w_p given r, m, g

* I assume uniform disk $I = \frac{mr^2}{4}$

a) No Slip



R = radius from center of precession to C.O.M.

• Constraint:

$$\vec{V}_G = \vec{V}_G$$

$$\vec{V}_G = (\dot{\phi}\hat{k} + \dot{\psi}\hat{n}) \times \hat{\lambda} r = r(w_p \sin\theta - w_s)\hat{e}_t$$

from rolling

$$\vec{V}_G = w_p R \hat{e}_t$$

from precession

$$\boxed{R = r\left(\sin\theta - \frac{w_s}{w_p}\right)}$$

radius from center of precession to C.O.M.

Using eqns from problem 19 but now:

$$\dot{\phi} = w_p \qquad \dot{\psi} = w_s \qquad \dot{\theta} = 0$$

- Using computer algebra solve for w_p. See noSlipSteady.m

$$\boxed{w_p = \pm\left[\underbrace{\sqrt{r\cos(\theta)[20g(\cos^2(\theta)-1) + 9rw_s^2\cos\theta}}_{*\text{ (under sqrt)}} + \frac{3}{5}\cdot\frac{w_s}{\sin\theta}\right]}$$

$$* \geq 0 \implies \boxed{w_s \geq \underbrace{\sqrt{\frac{20g}{9r}\frac{(1-\cos^2\theta)}{\cos\theta}}}_{\star\text{ (under sqrt)}}}$$

$$\star \geq 0 \implies \boxed{-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}}$$

b) No friction Disk. Circular Motion.

Problem 20 showed $\ddot{x}_G = 0$ $\quad \ddot{y}_G = 0$

for circular precession $\dot{x}_G = 0$ $\quad \dot{y}_G = 0$

$\therefore$ Since C.O.M. is stationary, radius of center of precession to C.O.M. $R = 0$

$$\boxed{R = 0}$$

Using eqns from problem 20 but now:

$\dot{\theta} = 0 \quad \dot{\psi} = w_s \quad \dot{\phi} = w_p$

- Using computer algebra solve for w_p. See noFricSteady.m

$$\boxed{w_p = \pm \left[\underbrace{\sqrt{[r\cos\theta(4g\cos^2\theta - 4g + r w_s^2 \cos\theta]}}_{*} + \frac{w_s}{\sin\theta} \right]}$$

$$* \geq 0 \implies \underbrace{\boxed{w_s \geq \sqrt{\frac{4g(1-\cos^2\theta)}{r\cos\theta}}}}_{☆}$$

$$☆ \geq 0 \implies \cos\theta \geq 0 \quad \boxed{-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}}$$

c) Intersection between no slip and no friction

Look at no slip precession radius $R = r\left(\sin\theta - \frac{w_s}{w_p}\right)$

but now set it to no friction radius $R = 0$

$$0 = r\left(\sin\theta - \frac{w_s}{w_p}\right)$$

$$\boxed{\frac{w_s}{w_p} = \sin\theta}$$

Using computer algebra with this relationship

Solve w_s

$$\boxed{w_s = 2\sin\theta\underbrace{\sqrt{\frac{g}{r\cos\theta}}}_{*}}$$

$$* \geq 0 \Longrightarrow \boxed{-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}}$$

w_p is the same whether calculated using w_p eqn

from no slip or from no friction

d) Check Steady State Solutions with full Solution

Plots on next page

16a) This is a failed attempt to find w_s and w_p for no slip disk by looking at ${}^{P}\dot{\vec{H}}_{/C}$ relative to the Precession frame. the answer I got did not work so I used the results from using computer algebra

$\dot{\phi} = w_p \quad \dot{\psi} = w_s \quad \dot{\theta} = 0 \qquad \vec{r}_{G/C} = r\hat{\lambda}$

$\hat{\lambda}\cdot\hat{k} = \cos\theta$

$\hat{e}_t\cdot\hat{k} = 0$

$\hat{n}\cdot\hat{k} = -\sin\theta$

$\hat{k}\times\hat{n} = \cos\theta\,\hat{e}_t$

$\hat{k}\times\hat{\lambda} = \sin\theta\,\hat{e}_t$

$\hat{\lambda}\times\hat{e}_t = -\hat{\imath}'$

$\hat{\lambda}\times\hat{\imath}' = -\cos\theta\,\hat{e}_t$

$$\Sigma\vec{M}_{/C} = \dot{\vec{H}}_{/C} \qquad ①\ \Sigma\vec{M}_{/C} = \vec{r}_{G/C}\times(-mg\hat{k}) = mgr\sin\theta\,\hat{e}_t$$

$$\dot{\vec{H}}_{/C} = \dot{\vec{H}}_{/G} + \dot{\vec{H}}_{G/C} = \cancel{{}^{P}\dot{\vec{H}}_{/G}} + w_p\times\vec{H}_{/G} + \vec{r}_{G/C}\times m\vec{a}_G$$

$$= w_p\hat{k}\times\left[\left(I(\hat{\lambda}\hat{\lambda}+\hat{e}_t\hat{e}_t)+2I\,\hat{n}\hat{n}\right)\cdot(w_s\hat{n}+w_p\hat{k})\right] + \vec{r}_{G/C}\times m\vec{a}_G$$

$$= w_p\hat{k}\times\left[2Iw_s\hat{n} + Iw_p\cos\theta\,\hat{\lambda} - 2Iw_p\sin\theta\,\hat{n}\right] + \vec{r}_{G/C}\times m\vec{a}_G$$

$$② \quad = 2Iw_p(w_s - w_p\sin\theta)\cos\theta\,\hat{e}_t + Iw_p^2\cos\theta\sin\theta\,\hat{e}_t + \vec{r}_{G/C}\times m\vec{a}_G$$

$$\vec{a}_G = \vec{\alpha}\times\vec{r}_{G/C} + \vec{w}_{D/F}\times\dot{\vec{r}}_{G/C} = \vec{\alpha}\times\hat{\lambda}r + \vec{w}_{D/F}\times\dot{\hat{\lambda}}r$$

$$\vec{w}_{D/F} = w_p\hat{k} + w_s\hat{n}$$

$$\dot{\hat{n}} = w_p\hat{k}\times\hat{n} = w_p\cos\theta\,\hat{e}_t \qquad \dot{\hat{\lambda}} = w_p\hat{k}\times\hat{\lambda} = w_p\sin\theta\,\hat{e}_t$$

$$\vec{\alpha} = w_s w_p\cos\theta\,\hat{e}_t$$

$$\vec{a}_G = w_s w_p\cos\theta\, r(\hat{e}_t\times\hat{\lambda}) + (w_p\hat{k}+w_s\hat{n})\times r w_p\sin\theta\,\hat{e}_t$$

$\frac{mr^2}{4}$

$$\vec{a}_G = w_s w_p\cos\theta\, r\,\hat{n} - w_p^2 r\sin\theta\,\hat{\imath}' + w_p w_s r\sin\theta\,\hat{\lambda}$$

$$\vec{r}_{G/C}\times m\vec{a}_G = r^2 m w_s w_p\cos\theta(\hat{\lambda}\times\hat{n}) - r^2 m w_p^2\sin\theta(\hat{\lambda}\times\hat{\imath}') + \cancel{\hat{\lambda}\times\hat{\lambda}}$$

$$③ \quad = r^2 m w_s w_p\cos\theta\,\hat{e}_t + r^2 m w_p^2\sin\theta\cos\theta\,\hat{e}_t$$

$$\hat{e}_t\cdot\{① = ② + ③\} \quad mgr\sin\theta = 2Iw_p w_s\cos\theta - 2Iw_p^2\sin\theta\cos\theta + Iw_p^2\sin\theta\cos\theta + r^2 m w_s w_p\cos\theta + r^2 m w_p^2\sin\theta\cos\theta$$

$$0 = w_p^2(-I\sin\theta\cos\theta + r^2 m\sin\theta\cos\theta) + w_p(2Iw_s\cos\theta + r^2 m w_s\cos\theta) - mgr\sin\theta$$

let $I = \frac{mr^2}{4}$ uniform disk

$$④ \quad 0 = w_p^2\left(\tfrac{3}{4}mr^2\sin\theta\cos\theta\right) + w_p\left(\tfrac{3}{2}mr^2 w_s\cos\theta\right) - mgr\sin\theta$$

Quadratic formula $\rightarrow$ $b^2 - 4ac \geq 0$

$$\tfrac{9}{4} m^2 r^4 w_s^2 \cos^2(\theta) + 3 m^2 r^3 g \sin^2\theta \cos\theta \geq 0$$

$$w_s \geq \sqrt{\frac{-3m^2 r^3 g \sin^2\theta \cos\theta}{(\frac{9}{4}) m^2 r^4 \cos^2(\theta)}} \qquad w_s \geq \sqrt{\frac{-4\ g \sin^2(\theta)}{3\ r \cos(\theta)}} \Bigg\} \text{⑤}$$

⑤ ≥ 0 $\quad \cos(\theta) \leq 0 \quad$ $\boxed{\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}}$

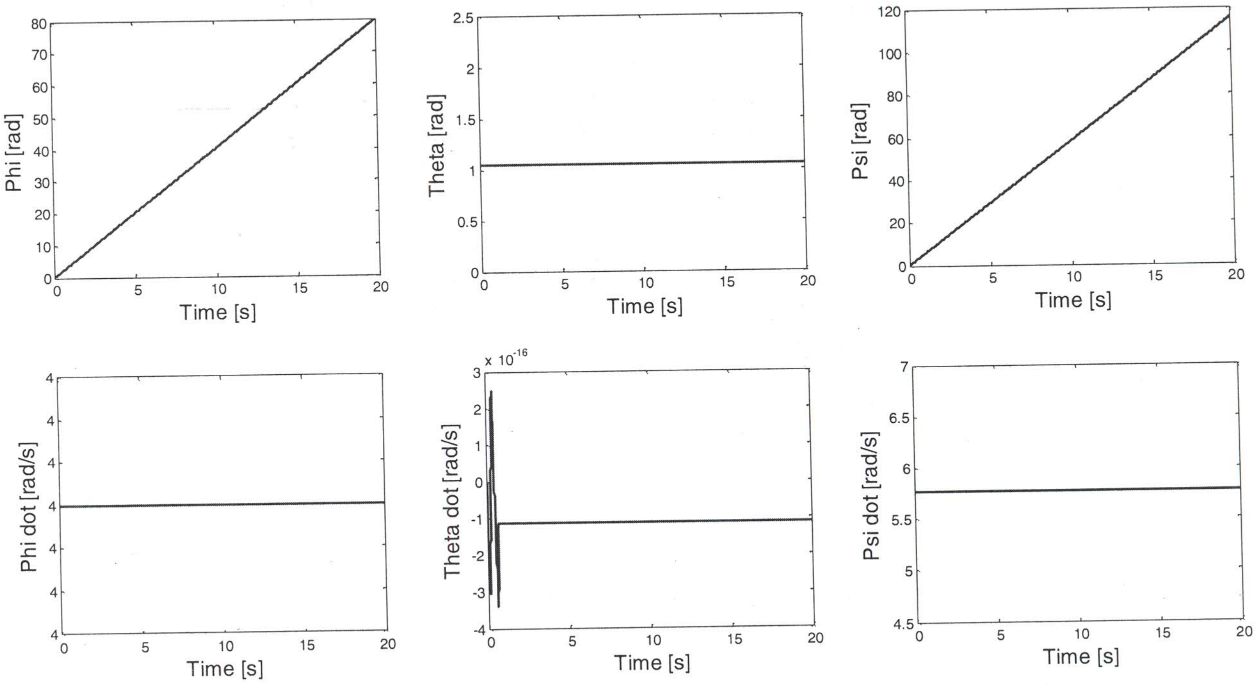
$$\boxed{w_s \geq \sqrt{\frac{-4\ g \sin(\theta)}{3\ r \cos(\theta)}}}$$

$$w_p = \frac{-\frac{3}{2} m r^2 w_s \cos\theta \pm \sqrt{m^2 r^2 r \cos\theta (\frac{9}{4} r w_s^2 \cos\theta + 3 g \sin^2\theta)}}{\frac{3}{2} m r^2 \sin\theta \cos\theta}$$

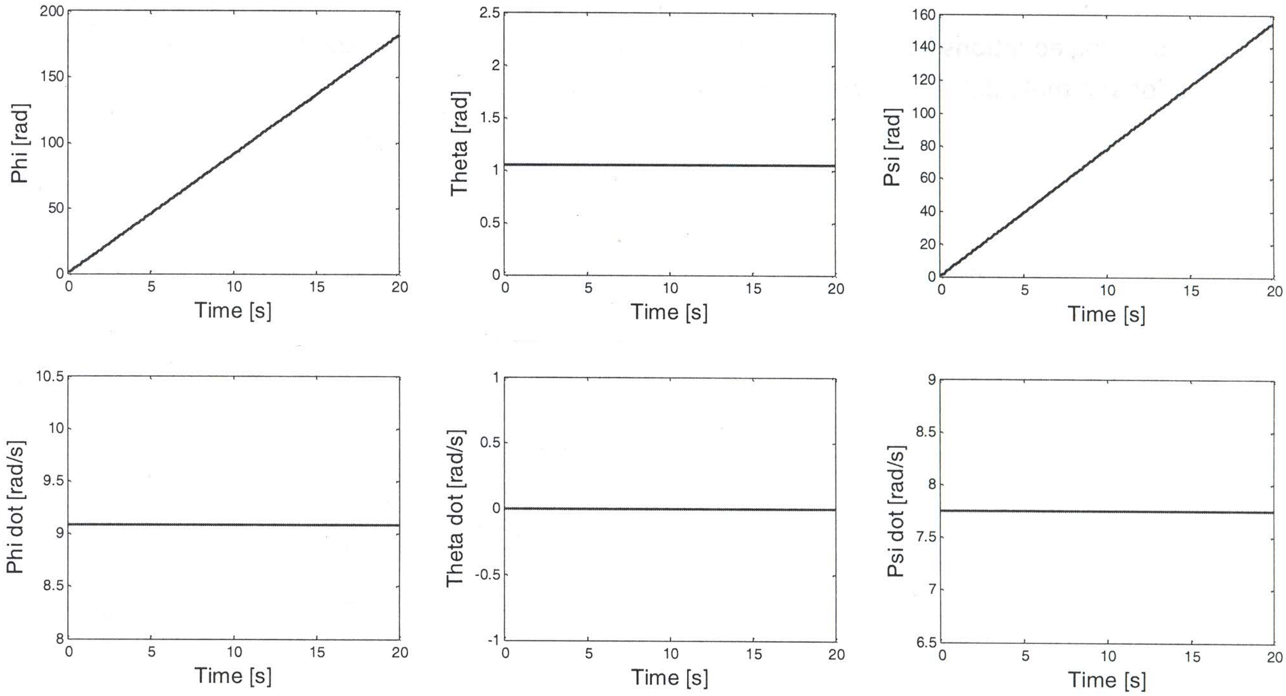
$$\boxed{w_p = \frac{-w_s}{\sin\theta} \pm \frac{2\sqrt{r\cos\theta(\frac{9}{4} r w_s^2 \cos\theta + 3 g \sin^2\theta}}{3\ r \sin\theta \cos\theta}}$$

22) Agreement between full solution and steady state.

a) No slip. Using equations from question 16 for wp and ws in full solution equations. Constant phi dot and psi dot and theta dot=0 indicates circular motion.

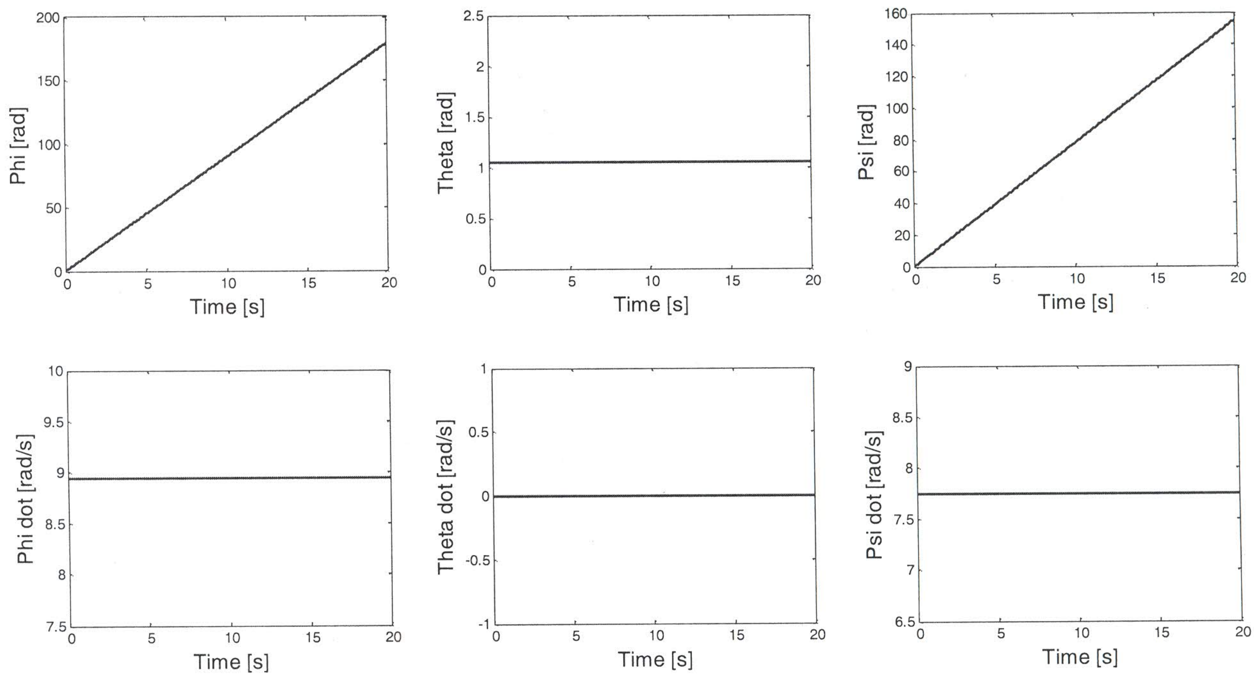


b) No friction. Using equations from question 16 for wp and ws in full solution equations. Constant phi dot and psi dot and theta dot=0 indicates circular motion.

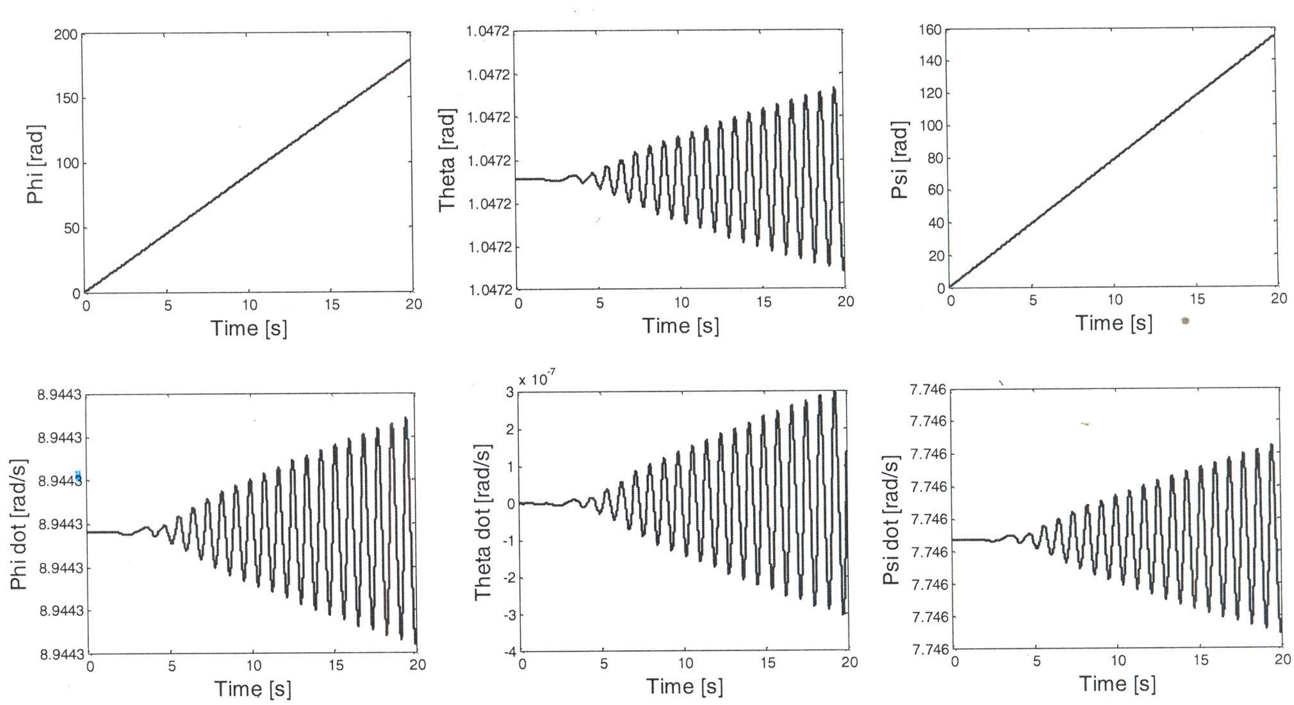


2) Intersection between no slip and no friction. Using equations from question 16 for wp and ws in full solution equations. The same, constant wp and ws values for no slip and no friction indicate that the intersection equations work using the full equations.

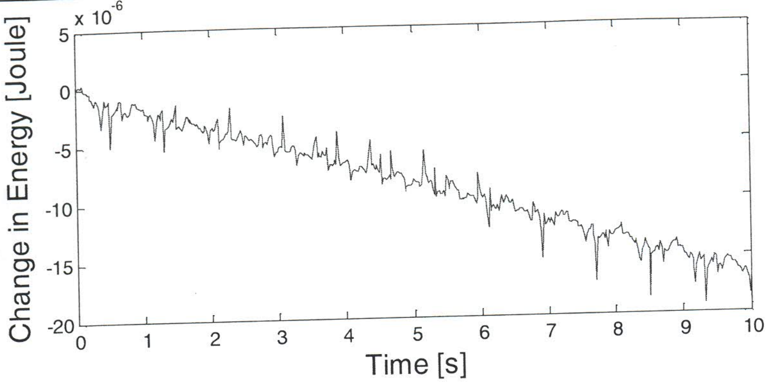
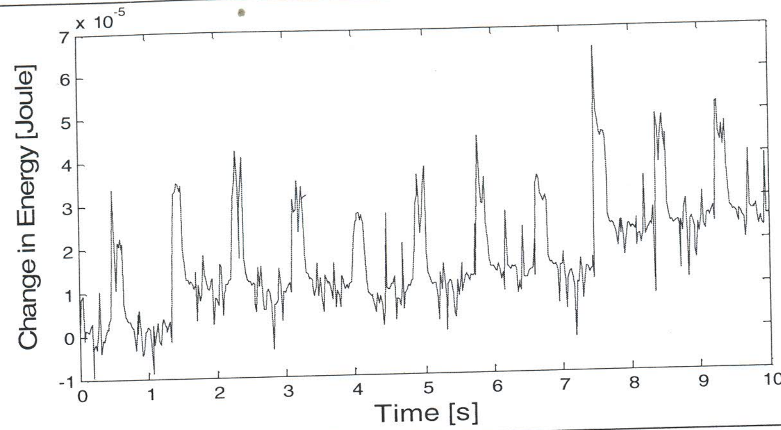
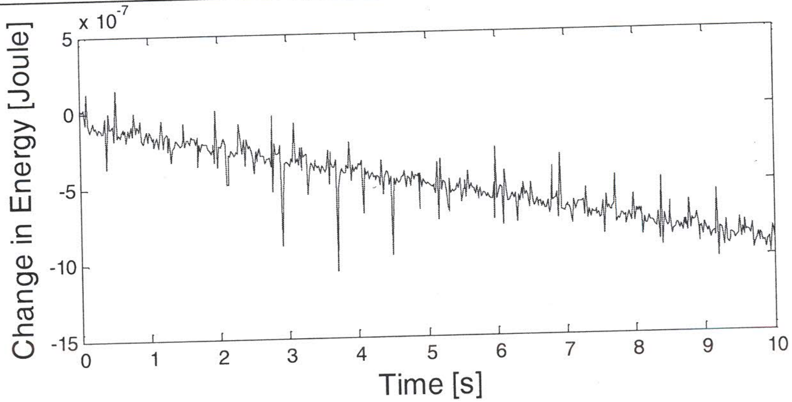
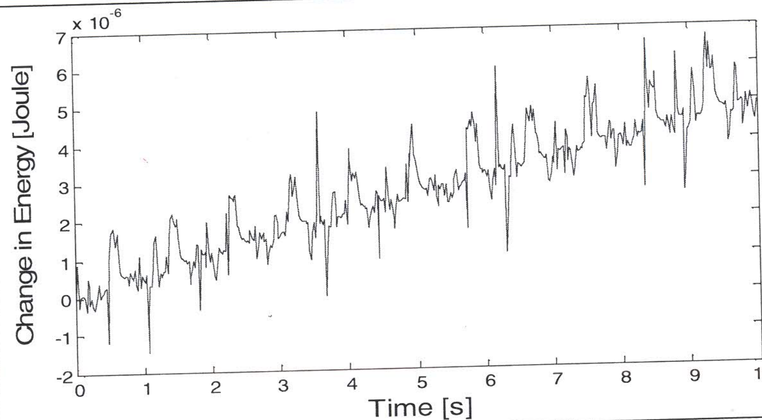
No friction:



No slip:



Convergence of energy as abstol and reltol is decreased helps validate the code

ODE abstol/ reltol tolerance	No Friction	No Slip
1e-6		
1e-7		
1e-8