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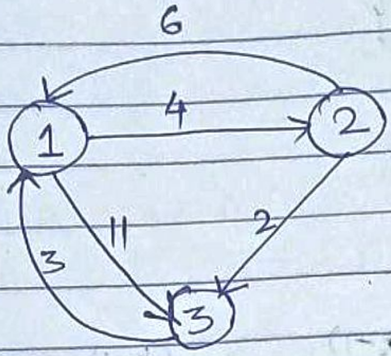
Floyd's Warshall Algorithm:-



Solution:-

$$D^0 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & \infty & 0 \end{array} \qquad \Pi[0] = \begin{bmatrix} NIL & 1 & 1 \\ 2 & NIL & 2 \\ 3 & NIL & NIL \end{bmatrix}$$

Now, D'

$$d_{ij} = \begin{cases} w_{ij} & \text{if } k=0 \\ \min(d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)}) & \text{if } k \geq 1 \end{cases}$$

// 1^{st} row and column will remain same as intermediate Vertex (1)

$$D' = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 11 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array} \qquad \Pi[1] = \begin{bmatrix} NIL & 1 & 1 \\ 2 & NIL & 2 \\ 3 & 1 & NIL \end{bmatrix}$$

$$\begin{aligned} D'[2][3] &= \min\{D^0[2][3],\ D^0[2][1] + D^0[1][3]\} \\ &= \min\{2,\ 6+11\} \\ &= \min\{2,\ 17\} \\ &= 2 \end{aligned}$$

$$D'[3][2] = \min\{D^0[3][2],\ D^0[3][1] + D^0[1][2]\}$$
$$= \min\{\infty,\ 3+4\}$$
$$= \min\{\infty, 7\}$$
$$= 7$$

Now, D^2

$$d_{ij} = \begin{cases} W_{ij} & \text{for } k=0 \\ \min\{d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\} & \text{for } k \geq 1 \end{cases}$$

// 2^{nd} row and 2^{nd} column will remain same as intermediate ~~is~~ vertex is 2.

$$D^2 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & \cancel{11}\,6 \\ 2 & 6 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array} \qquad \Pi[2] = \begin{bmatrix} NIL & 1 & 2 \\ 2 & NIL & 2 \\ 3 & 1 & NIL \end{bmatrix}$$

$$D^2[1][3] = \cancel{D}\min\{D'[1][3],\ D'[1][2] + D'[2][3]\}$$
$$= \min\{11,\ 4+2\}$$
$$= \min\{11, 6\}$$
$$= 6$$

$$D^2[3][1] = \min\{D'[3][1],\ D'[3][2] + D'[2][1]\}$$
$$= \min\{3,\ 7+6\}$$
$$= \min\{3, 13\}$$
$$= 3$$

Now, D^3

$$d_{ij} = \begin{cases} w_{ij} & \text{for } k=0 \\ \min\{d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\} & \text{for } k \geq 1 \end{cases}$$

// Now, 3rd row and 3rd column will remain same as intermediate vertex is 3.

$$D^3 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 6 \\ 2 & 5 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array} \quad ; \quad \Pi[3] = \begin{bmatrix} NIL & 1 & 2 \\ 3 & NIL & 2 \\ 3 & 1 & NIL \end{bmatrix}$$

$$\begin{aligned} D^3[1][2] &= \min\{D^2[1][2], D^2[1][3] + D^2[3][2]\} \\ &= \min\{4, 6+7\} \\ &= \min\{4, 13\} \\ &= 4 \end{aligned}$$

$$\begin{aligned} D^3[2][1] &= \min\{D^2[2][1], D^2[2][3] + D^2[3][1]\} \\ &= \min\{6, 2+3\} \\ &= \min\{6, 5\} \\ &= 5 \end{aligned}$$

$\therefore$ D^3 gives the shortest path between each pair of vertices.

$$D^3 = \begin{bmatrix} 0 & 4 & 6 \\ 5 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix} \quad \Pi[3] = \begin{bmatrix} NIL & 1 & 2 \\ 3 & NIL & 2 \\ 3 & 1 & NIL \end{bmatrix}$$