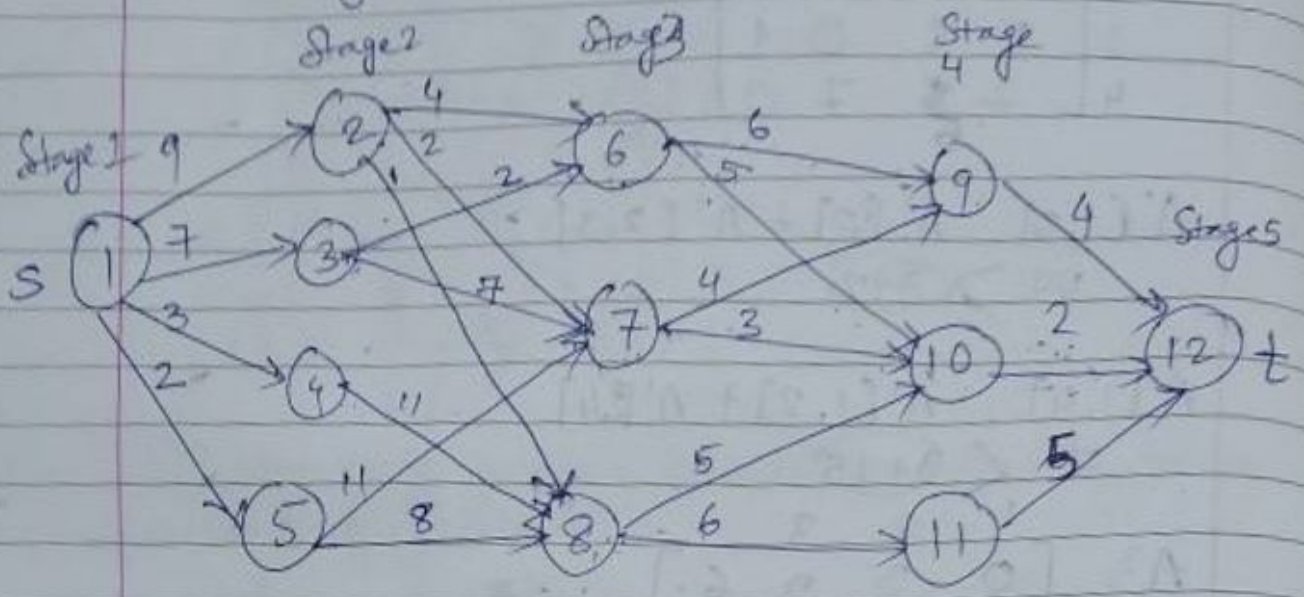
Multistage Graph



V	1	2	3	4	5	6	7	8	9	10	11	12
Cost	16	7	9	18	15	7	5	7	4	2	5	0
d	2/3	7	6	8	8	10	10	10	12	12	12	12

Stage 5 :-

cost (5,12) = 0 (d = 12)

↓ ↓

Stage Vertex

Stage 4:-

cost (4,9) = 4 (d = 12) | cost (4,10) = 2 (d = 12) | cost (4,11) = 5 (d = 12)

Stage 3:-

cost (3,6) = min { c(6,9) + c(4,9) , c(6,10) + cost (4,10) }

= min { 6+4, 5+2 }

= min {10, 7} = 7 (d = 10)

cost (3,7) = min { c(7,9) + cost(4,9) , c(7,10) + cost (4,10) }

= min { 4+4, 3+2 }

= min {8, 5}

= 5 (d = 10)

cost(3,8) = min { c(8,10) + cost(4,10), c(8,11) + cost(4,11) }
= min { 5+2, 6+5 }
= min { 7, 11 }
= 7 dot (d=10)

For Stage 2:

cost(2,2) = min { c(2,6) + cost(3,6), c(2,7) + cost(3,7), c(2,8) + cost(3,8) }
= min { 4+7, 2+5, 1+7 }
= min { 11, 7, 8 }
= 7 (d=7)

cost(2,3) = min { c(3,6) + cost(3,6), c(3,7) + cost(3,7) }
= min { 2+7, 7+5 }
= min { 9, 12 }
= 9 (d=6)

cost(2,4) = min { c(4,8) + cost(3,8) }
= 11+7
= 18 (d=8)

cost(2,5) = min { c(5,7) + cost(3,7), c(5,8) + cost(3,8) }
= min { 11+5, 8+7 }
= min { 16, 15 }
= 15 (d=8)

Stage 1

cost(1,1) = min{ c(1,2) + cost(2,2),
c(1,3) + cost(2,3),
c(1,4) + cost(2,4),
c(1,5) + cost(2,5)}
= min{9+7, 7+9, 3+18, 2+15}
= min{16, 16, 21, 17}
= 16 (d = 2 or 3)

For finding shortest path, [Consider table]

stage ↓ Vertex No ↓
d(1,1) = 2, d(2,2) = 7, d(3,7) = 10, d(4,10) = 12

Path 1

∴ Path = 1 - 2 - 7 - 10 - 12
= 16

stage ↓ Vertex No ↓
d(1,1) = 3, d(2,3) = 6, d(3,6) = 10, d(4,10) = 12

Path 2

∴ Path = 1 - 3 - 6 - 10 - 12
= 16

Here, there are 2 optimal solutions.

FORMULA:-

stage ↓ Vertex No ↓

$$cost(i, j) = \min \{ c(j, l) + cost(i+1, l) \}$$
$$\langle j, l \rangle \in E$$
$$l \in V_{i+1}$$