Image Denoising Based on Fuzzy and Intra-Scale Dependency in Wavelet Transform Domain

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Abstract

In this paper, we propose a new wavelet shrinkage algorithm based on fuzzy logic. Fuzzy logic is used for taking neighbor dependency and uncorrelated nature of noise into account in wavelet-based image denoising. For this reason, we use a fuzzy feature for enhancing wavelet coefficients information in the shrinkage step. Then a fuzzy membership function shrinks wavelet coefficients based on the fuzzy feature. We examine our image denoising algorithm in the dual-tree discrete wavelet transform, which is the new shiftable and modified version of discrete wavelet transform. Extensive comparisons with the state-of-the-art image denoising algorithm indicate that our image denoising algorithm has a better performance in noise suppression and edge preservation.

1. Introduction

Denoising has become an essential step in image analysis. Indeed, due to sensor imperfections, transmission channels defects, as well as physical constraints, noise weakens the quality of almost every acquired image.

Reducing the noise level is the main goal of an image denoising algorithm, while preserving the image features (such as edges, textures, etc.). The multi-resolution analysis performed by the wavelet transform has been shown to be a powerful tool to achieve these goals [1]. Indeed, in the wavelet domain, the noise is uniformly spread throughout the coefficients, while most of the image information is concentrated in the few largest ones. Classical wavelet-based denoising methods consist of three steps:

1. Compute the discrete wavelet transform (DWT).
2. Remove noise from the wavelet coefficients.
3. Reconstruct the enhanced image by using the inverse DWT.

Due to the linearity of the wavelet transform, additive noise in the image domain remains additive in the transform domain. If $y_{s,d}(i,j)$ and $x_{s,d}(i,j)$ denote the noisy and the noise-free wavelet coefficients of scale s and orientation d respectively, then we can model the additive noise in the transform domain as:

$$y_{s,d}(i,j) = x_{s.d}(i,j) + n_{s,d}(i,j) \tag{1}$$

where $n_{s,d}(i,j)$ is the corresponding noise component. We will only consider additive Gaussian white noise following a normal law defined by a zero mean and a known variance, that is $n \sim N(0,\sigma^2)$.

In image denoising, where a trade-off between noise suppression and the maintenance of actual image discontinuity must be made, solutions are required to detect important image details and accordingly adapt the degree of noise smoothing. We always postulate that noise is uncorrelated in the spatial domain; it is also uncorrelated in the wavelet domain. With respect to this principle, we use a fuzzy feature for single channel image denoising to enhance image information in wavelet sub-bands and then using a fuzzy membership function to shrink wavelet coefficients, accordingly. This feature space distinguishes between important coefficients, which belong to image discontinuity and noisy coefficients.

In addition, we examine our image denoising algorithm in the dual-tree DWT (DT-DWT), which provides both shiftable sub-bands and good directional selectivity and low redundancy [2].

This paper is structured as follows: Section 2 presents proposed image denoising algorithm. Section 3 gives results and the comparisons. Finally, we conclude with a brief summary in section 4.

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2. The Proposed Method

In this Section, we describe our image denoising algorithm. First, we decompose input noisy image using DT-DWT. After extracting fuzzy feature from different sub-bands at different scales, a fuzzy membership function shrinks wavelet coefficients based on the fuzzy feature. Finally, inverse DT-DWT of modified wavelet coefficients generates denoised image.

2.1. Fuzzy Feature

We want to give large weights to neighboring coefficients with similar magnitude, and vice versa. The larger coefficients, which produced by noise are always isolated or unconnected, but edge coefficients are clustering and persistence. It is well known that the adjacent points are more similar in magnitude. Therefore, we use a fuzzy function $m(l,k)$ of magnitude similarity and a fuzzy function $s(l,k)$ of spatial similarity, which is defined as:

$$m(l,k) = \exp\left(-\left(\frac{y_{s,d}(i,j) - y_{s,d}(i+l,j+k)}{Thr}\right)^2\right) \quad (2)$$

$$s(l,k) = \exp\left(-\left(\frac{l^2+k^2}{N}\right)\right) \quad (3)$$

where $y_{s,d}(i,j)$ and $y_{s,d}(i+l,j+k)$ are central coefficient and neighbor coefficients in wavelet sub-bands, respectively. $Thr = c \times \sigma_n, 3 \le c \le 4$, σ_n is estimated noise variance, and N is the number of coefficients in the local window $k \in [-K...K]$, and $l \in [-L...L]$.

The following example shows spatial similarity for a 5×5 local window (i.e. $L=2, K=2$). The central weight is set to zero that is very useful for removing isolated noise:

$$s(l,k) = \begin{pmatrix} 0.73 & 0.82 & 0.85 & 0.82 & 0.73 \\ 0.82 & 0.92 & 0.96 & 0.92 & 0.82 \\ 0.85 & 0.96 & 0 & 0.96 & 0.85 \\ 0.82 & 0.92 & 0.96 & 0.92 & 0.82 \\ 0.73 & 0.82 & 0.85 & 0.82 & 0.73 \end{pmatrix} \quad (4)$$

According the two fuzzy functions, we can get adaptive weight $w(l,k)$ for each neighboring coefficient:

$$w(l,k) = m(l,k) \times s(l,k) \quad (5)$$

Using the adaptive weights $w(l,k)$, we can obtain the fuzzy feature for each coefficient in wavelet sub-bands as follows:

$$f(i,j) = \frac{\sum_{l=-L}^{L} \sum_{k=-K}^{K} w(l,k) \times \left| y_{s,d}(i+l, j+k) \right|}{\sum_{l=-L}^{L} \sum_{k=-K}^{K} w(l,k)} \quad (6)$$

In the literature, the local mean is also used for improving the wavelet coefficients information in the shrinkage step [3-5]. Figure 1 shows the difference between the fuzzy feature and local mean for distinguishing between important coefficients, which belong to edge structure and noisy coefficient in wavelet transform domain. As it can be seen in Figure 1, the new fuzzy feature can effectively distinguish between edge structure and noise compare to the local mean.

2.2. Fuzzy shrinkage rule

The second step in the wavelet denoising procedure usually consists of shrinking the wavelet coefficients: the coefficients that contain primarily noise should be reduced to negligible values, while the ones containing a significant noise-free component should be reduced less. Here, we use a fuzzy rule based on the fuzzy feature for shrinking the wavelet coefficients.

Fuzzy rules are linguistic IF-THEN constructions that have the general form "IF A THEN B," where A and B are (collections of) propositions containing linguistic variables. A is called the premise or antecedent and B is the consequence of the rule [6].

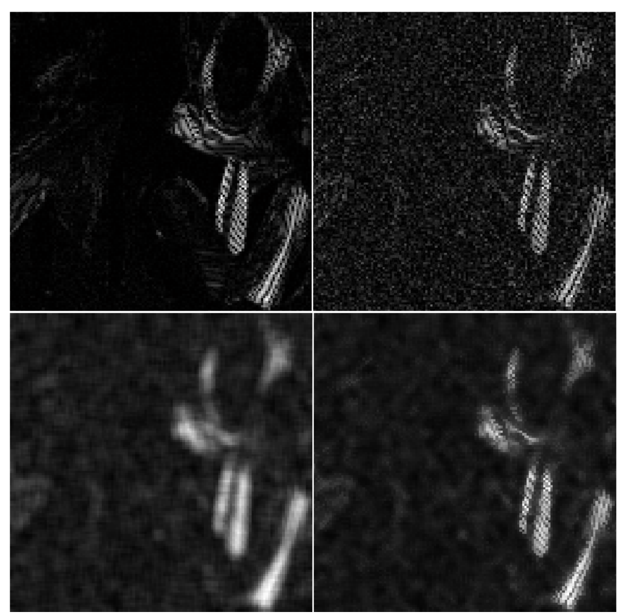

Figure 1. From top left, clockwise: A sub-band of DT-DWT for "Barbara" image, noisy version of it, the fuzzy feature, and Local mean.

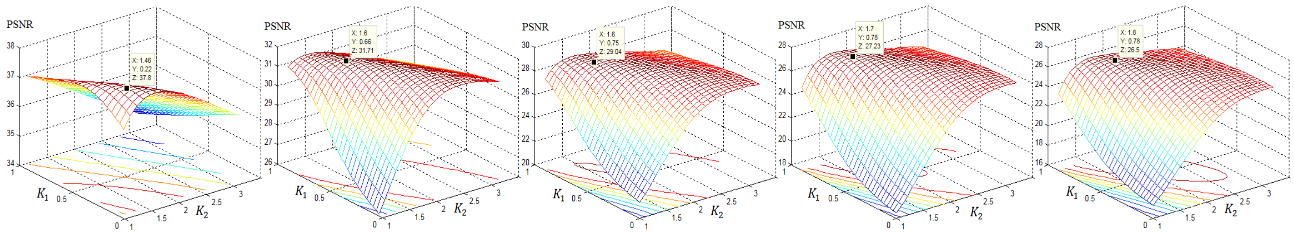

Figure 2. K_1 and K_2 versus PSNR for the different noise variances: left to right 5, 15, 25, 35, and 40.

After finding the fuzzy feature, we will define Linguistic IF-THEN rules for shrinking wavelet coefficients as follows:

IF the fuzzy feature $f(i,j)$ is *large* **THEN** shrinkage of wavelet coefficients $y_{s,d}(i,j)$ is *small*.

In fact, the fuzzy feature indicates how coefficients in the noisy sub-band should be shrunk. Fuzzy membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between zero and one. MF is often given the designation of μ. MF is built from several basic functions such as: piece-wise linear functions, the Gaussian distribution function, the sigmoid curve, quadratic and cubic polynomial curves.

Here, we use the spline-based curve, which is a mapping on the vector x, and is named because of its S- shape. The parameters T_1 and T_2 locate the extremes of the sloped portion of the curve as given by:

$$\mu(x)=\begin{cases}0 & x\le T_1\\ 2\times\left(\dfrac{x-T_1}{T_2-T_1}\right)^2 & T_1\le x\le \dfrac{T_1+T_2}{2}\\ 1-2\times\left(T_2-\dfrac{x}{T_2-T_1}\right)^2 & \dfrac{T_1+T_2}{2}\le x\le T_2\\ 1 & x\ge T_2\end{cases} \qquad (7)$$

Finally, the estimated noise free signal is obtained using the following formula:

$$\hat{x}_{s,d}(i,j)=\mu(f(i,j))\times y_{s,d}(i,j) \qquad (8)$$

where s and d are scale and orientation of DT_DWT sub-bands, and $f(i,j)$ is obtained using (6).

For building fuzzy membership function, the two thresholds (T_1 and T_2) must be determined. We found out that T_1 and T_2 are related to the $\hat{\sigma}_n$, which is the noise variance using median estimator. For achieving the nonlinear relation, we have tested our noise reduction algorithm with different noisy images. In each different noise variance, we obtained best values for the T_1 and T_2. It can be seen in Figure 2, best values for K_1 and K_2 are obtained based on following equations for different noise variances:

$$T_1=K_1\times\hat{\sigma}_n \quad (9), \qquad T_2=K_2\times\hat{\sigma}_n \quad (10)$$

We use polynomial curve fitting in MATLAB toolbox and fit a fifth order polynomial on the points of K_1 and K_2 versus estimated noise variance. We use polynomial coefficients to obtain best values for T_1 and T_2 at different noise variances:

$$T_1=\left(\sum_{i=1}^{5} p_{1i}\times\left(\hat{\sigma}_n\right)^i\right)\times\hat{\sigma}_n \qquad (11)$$

$$T_2=\left(\sum_{i=1}^{5} p_{2i}\times\left(\hat{\sigma}_n\right)^i\right)\times\hat{\sigma}_n \qquad (12)$$

where p_1 and p_2 are the polynomials coefficients:

$$p_1=10^3\times[37.87,-27.44,7.756,-1.068,0.0731,-0.0012]$$
$$p_2=10^3\times[19.25,-13.12,3.450,-0.429,0.447,0.0009]$$

When a noisy image passes low frequency filter banks of wavelet transform, at each level, it will be smoother and therefore, noise and high frequency regions in image (i.e. edges) are reduced. For this reason, we add a descending term to the T_2. In other words, wavelet coefficients in lower levels are less shrunk:

$$T_2=\left(\sum_{i=1}^{5} p_{2i}\times\left(\hat{\sigma}_n\right)^i\right)\times\hat{\sigma}_n\times\exp\left(-\frac{l-1}{T_3}\right) \qquad (13)$$

where l is the level of decomposition, and $8<T_3<10$.

3. Experimental Results

In this section, we compare our image denoising algorithm with some of the best state-of-the-art techniques: Sendur's et al. bivariate MAP estimator with local variance estimation [7], Pizurica's Prob-Shrink [4], Luisier's et al. Sure-Let [8], and Chen's et al. Neigh-Shrink [3].

The standard grayscale test images: “Barbara,” “Boat,” and “Goldhill” were chosen as the experimental dataset, which capture by simulated additive Gaussian white noise at three different power levels. In addition, we objectively measured the experimental results by the peak signal-to-noise ratio (PSNR) index.

The parameters of each method have been set according to the values given their respective authors in the corresponding referred papers. Table 1 summarizes the results obtained. As it can be observed in Table 1, our method, which uses a simple fuzzy rule, has better results in all cases.

For subjective evaluation, there are two important criteria: the visibility of processing artifacts and preserving image edges. Figures 3 illustrate the results of noisy “Barbara” image using different methods. Additionally, we would like to stress that our new fuzzy method exhibits the fewest number of artifacts and preserves most of edges compared to other methods.

4. Conclusions

In this paper, we propose a new wavelet-based image denoising using a fuzzy rule. We use the DT-DWT for decomposing noisy image, because it is shift invariant, and has more directional sub-bands compared to the DWT. In other words, proposing a new method for shrinking wavelet coefficients in the second step of the wavelet-based image denoising is the main novelty of this paper. The comparison of the denoising results obtained with our algorithm, and with the best state-of-the-art methods, demonstrate the performance of our fuzzy approach, which gave the best output PSNRs for most of the images. In addition, the visual quality of our denoised images exhibits the fewest number of artifacts, and preserves most of edges compared to other methods.

Table 1. Comparison of some of the most efficient denoising methods ($PSNR_{dB}$)

Metho d	*Neigh-Shrink* [3]	*Bi-Shrink* [7]	*Sure-let* [8]	*Prob-Shrink* [4]	*Proposed*
Barbara 512×512					
σ_n=10	30.43	33.17	32.18	33.05	**33.96**
σ_n=20	25.92	29.13	27.98	29.21	**30.22**
σ_n=30	23.74	26.47	25.83	26.99	**28.04**
Goldhill 512×512					
σ_n=10	29.88	32.20	32.54	32.62	**33.14**
σ_n=20	27.48	28.45	29.51	29.55	**29.96**
σ_n=30	26.30	26.07	27.89	27.93	**28.35**
Boat 512×512					
σ_n=10	30.22	32.97	32.55	33.02	**33.49**
σ_n=20	27.30	29.03	29.46	29.73	**30.13**
σ_n=30	25.79	26.27	27.63	27.75	**28.21**

Figure 3: A part of noise free “Barbara” image and noisy version of it, $_n$=30 (top row, left to right). Bi-Shrink and Sure-Let (middle row). Prob-Shrink and proposed method (bottom-row)

References

[1] S. Mallat, “A theory for multiresolution signal decomposition: The wavelet representation,” IEEE Trans. Pattern Anal. Mach. Intell., vol. 11, no. 7, Jul. 1989, 674–693.

[2] N. G. Kingsbury, “A Dual-Tree Complex Wavelet Transform with Improved Orthogonality and Symmetry Properties,” Proc. IE*EE Conf. on Image Processing*, 2000, 375-378

[3] G.Y. Chen, T.D. Bui, A.Krzyzak, “Image denoising with neighbour dependency and customized wavelet and threshold,”*Pattern Recognition* 38, 2005, 115–124.

[4] A. Pizurica, W. Philips, “Estimating the probability of the presence of a signal of interest in multiresolution single- and multiband image denoising,” *IEEE Transactions on Image Process*. 2006, 654-665.

[5] S. Schulte, B. Huysmans, A. Pizurica, E. E. Kerre1 and W. Philips, “A New Fuzzy-based Wavelet Shrinkage Image Denoising Technique.” *Springer Verlag*, 2006, 12-23.

[6] W. Siler and J. J. Buckley, Fuzzy Expert Systems and Fuzzy Reasoning, By ISBN 0-471-38859-9, John Wiley & Sons, Inc, 2005

[7] L. Şendur and I. W. Selesnick, ”Bivariate shrinkage with local variance estimation, *IEEE Signal Proc. Letters*, vol. 9, no. 12, Dec. 2002, pp. 438-441.

[8] T. Blu, F.Luisier, “The SURE-LET approach to image denoising. IEEE Trans,” Image Process 16, 2007, pp. 2778–2786.