Simulation and Modeling Assignment
Testing Random Number Generators

Course No. CSE 412 Simulation and Modeling

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Uniformity test

***Parameters*:** $constant = 65539, seed = 1505095, \alpha = 0.1, X^2{}_{k-1,1-\alpha} = 27.20357$

In this test, we checked whether the U_i's generated appear to be uniformly distributed between 0 and 1 or not. We performed the chi-square test with all the given parameters.

We divide $[0, 1]$ into k sub-intervals of equal length and generate $U_1, U_2, \cdot \cdot \cdot, U_n$. For $j = 1, 2, ..., k$, we let f_j be the number of the Ui's that are in the jth sub-interval, and calculated the X^2. For large n, X^2 will have an approximate chi-square distribution with $k-1$ degrees of freedom under the null hypothesis ($H0 = U_i$'s are IID random variables).

Here in the uniformity test, the value of X^2 for different parameter values are smaller than $X^2{}_{k-1,1-\alpha}$. We know that we reject this hypothesis at level α if $X^2 > X^2{}_{k-1,1-\alpha}$ where $X^2{}_{k-1,1-\alpha}$ is the upper $1-\alpha$ critical point of the chi-square distribution with $k-1$ degrees of freedom. So, after conducting the test, we decided that this hypothesis is not rejected for different parameters. The results for the given parameters the shown in the Table 1

Table 1: Uniformity test results for different parameters

N	K	Chi	Rejected
20	10	2.0	No
20	20	14.0	No
500	10	13.08	No
500	20	25.28	No
4000	10	7.065	No
4000	20	15.84	No
10000	10	5.402	No
10000	20	16.32	No

Serial test

***Parameters*:** $constant = 65539, seed = 1505095, \alpha = 0.1$ The serial test is really just a generalization of the chi-square test to higher dimensions. If the individual U_i's are correlated, the distribution of the $d-$vectors U_i will deviate from $d-$dimensional uniformity; thus, the serial test provides an indirect check on the assumption that the individual U_i's are independent.

In this test, we calculated $d-$tuples. We then checked whether $d-$tuples are uniformly distributed on the $d-$dimensional unit hyper-cube $[0, 1]^d$. $X^2(d)$ will have an approximate chi-square distribution with $k^d - 1$ df. We used this hypothesis as a way to report our test results which differs for different parameters. The results for the given parameters the shown in the Table 2

Table 2: Serial test results for different parameters

N	k	d	chi	${X^2}_{k-1,1-\alpha}$	Rejected
20	4	2	16.0	22.307	No
20	8	2	44.8	77.745	No
20	4	3	27.2	77.745	No
20	8	3	161.6	552.374	No
500	4	2	134.272	22.307	Yes
500	8	2	155.392	77.745	Yes
500	4	3	250.176	77.745	Yes
500	8	3	403.52	552.374	No
4000	4	2	1008.112	22.307	Yes
4000	8	2	1035.52	77.745	Yes
4000	4	3	1799.776	77.745	Yes
4000	8	3	1958.0	552.374	Yes
10000	4	2	2508.384	22.307	Yes
10000	8	2	2529.4848	77.745	Yes
10000	4	3	4469.0976	77.745	Yes
10000	8	3	4621.526	552.374	Yes

Runs test

***Parameters*:** $constant = 65539, seed = 1505095, \alpha = 0.1, {X^2}_{6,1-\alpha} = 10.644$
In this runs test, we evaluated the independence assumption of the random number generator.

We examined the U_i sequence (or, equivalently, the Z_i sequence) for unbroken sub-sequences of maximal length within which the U_i's increase monotonically; such a sub-sequence is called a run up. For large n, R will have an approximate chi-square distribution with 6 df. We calculated if $R \geq {X^2}_{6,1-\alpha}$ and if so, then we rejected the null hypothesis as in previous test. The results for the given parameters the shown in the Table 3

Since runs tests look solely for independence (and not specifically for uniformity), it would probably be a good idea to apply a runs test before performing the chi-square or serial tests, since the last two tests implicitly assume independence.

Table 3: Runs test results for different parameters

N	$ri[1-6]$	Run(ri)	Rejected
20	[7, 5, 1, 0, 0, 0]	6.049	No
500	[84, 103, 50, 11, 2, 1]	1.57	No
4000	[652, 849, 369, 98, 18, 10]	8.58	No
10000	[1687, 2116, 913, 244, 49, 20]	11.461	Yes

Correlation test

***Parameters*:** $constant = 65539, seed = 1505095, \alpha = 0.1, Z_{1-\alpha/2} = 1.645$ In this test, we directly assessed whether the generated U_i's exhibit discernible correlation at lag j. Under the assumption that there is no correlation (that is $\rho_j = 0$) and assuming n is large A_j should have an approximate standard normal distribution.

After testing the null hypothesis ($H0$: U_i's have zero lag j correlation) at level α, we rejected the hypothesis if $|A_j| > Z_{1-\alpha/2}$. The results for the given parameters the shown in the Table 4

Table 4: Correlation test results for different parameters

N	j	Corr	Rejected
20	1	0.119	No
20	3	0.728	No
20	5	1.259	No
500	1	1.398	No
500	3	0.65	No
500	5	2.605	Yes
4000	1	0.27	No
4000	3	1.158	No
4000	5	0.0671	No
10000	1	0.383	No
10000	3	0.343	No
10000	5	0.251	No