Accessible Physics Concepts for Blind Students

By:
R.G. (Dick) Baldwin

Accessible Physics Concepts for Blind Students

By:
R.G. (Dick) Baldwin

Online:
< http://cnx.org/content/col11294/1.35/ >

OpenStax-CNX

This selection and arrangement of content as a collection is copyrighted by R.G. (Dick) Baldwin. It is licensed under the Creative Commons Attribution License 3.0 (http://creativecommons.org/licenses/by/3.0/).
Collection structure revised: August 12, 2011
PDF generated: October 2, 2015
For copyright and attribution information for the modules contained in this collection, see p. 772.

Table of Contents

1 Phy1000: Introduction to Accessible Physics Concepts 1
2 Phy1002: If You Can Imagine It, You Can Draw It using SVG 7
3 Phy1004: Manual Creation of Tactile Graphics 53
4 Phy1006: A small-scale demonstration of the IVEO Learning System 69
5 Phy1010: JavaScript 85
6 Brief Trigonometry Tutorial 105
7 Scale Factors, Ratios, and Proportions 155
8 Scientific Notation and Significant Figures 167
9 Units and Dimensional Analysis 181
10 Motion – Displacement and Vectors 209
11 Motion – Uniform and Relative Velocity 273
12 Motion – Variable Velocity and Acceleration 325
13 Force – Introduction to Statics, Equilibrium, and Forces 375
14 Force – Vector Solutions for Coplanar Forces Concurrent at a Point 383
15 Force – Moments, Torque, Couple, and Equilibrium 445
16 Force – Center of Gravity 497
17 Force and Motion – Introduction 525
18 Force and Motion – Units of Force 531
19 Force and Motion – Momentum, Impulse, and Conservation of Momentum 553
20 Energy – Work 577
21 Energy – Potential Energy 587
22 Energy – Kinetic and Mechanical Energy 595
23 Energy – Power 607
24 Energy – Internal and External Forces 613
25 Energy – Elastic and Inelastic Collisions in Two Dimensions 619
26 Relationships Among Kinematics, Newton's Laws, Vectors, 2D Motion, 2D Forces, Momentum, Work, Energy, and Power 649
27 Vector Subtraction 657
28 Circular Motion – Speed and Velocity 669
29 Circular Motion – Acceleration and Centripetal Force 675
30 Circular Motion – The Mathematics of Circular Motion 683
31 Angular Momentum – Rotational Kinetic Energy and Inertia 695
32 Vector Multiplication 711
33 Angular Momentum – The Mathematics of Torque 721
34 Angular Momentum – Torque, Work and Energy 739
35 Angular Momentum – Rotational Equilibrium 747
36 Angular Momentum – The Physics of Rolling Objects 759
Index 769
Attributions 772

Chapter 1

Phy1000: Introduction to Accessible Physics Concepts[1]

1.1 Table of Contents

- Preface (p. 1)
  - General (p. 1)
  - Impetus for this collection of modules (p. 2)
  - Prerequisite requirements (p. 2)
  - Supplemental material (p. 3)
- Discussion (p. 3)
  - Algebra (p. 3)
  - Using a graph board (p. 4)
  - Using a protractor (p. 4)
  - JavaScript programming (p. 4)
  - Trigonometry (p. 5)
  - Pictures and diagrams (p. 5)
  - Greek characters (p. 5)
- Resources (p. 5)
- Miscellaneous (p. 6)

1.2 Preface

1.2.1 General

This is the first module in a collection of modules designed to make physics accessible to blind students.

If you opened this page in the Book context, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the page in the context of the Book.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

[1] This content is available online at <http://cnx.org/content/m37432/1.5/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

1.2.2 Impetus for this collection of modules

I am a professor of Computer Information Technology at Austin Community College in Austin, TX. For the past couple of years, I have had the privilege of teaching object-oriented Java programming to a blind student. That has gone very well.

Assist a student in physics

During a recent semester, I volunteered to assist this student in her efforts to successfully complete a physics course that she was taking in another department of the college. I quickly learned that she faced significant barriers in this endeavor, not the least of which was the general lack of accessibility provided by the electronic versions of the textbook. (The student had access to both pdf and Word formats of the textbook.)

Not compatible with accessibility tools

I am not visually impaired, so for me the textbook was adequate. However, it contains numerous elements that don't work well when viewed by a blind student using accessibility tools such as an audible screen reader and an electronic line-by-line Braille display.

Pictures and diagrams

For example, each chapter contains numerous pictures and diagrams that are an integral part of the teaching material. In some cases, there is a reasonable verbal explanation for a picture or diagram and in some cases there is little or no explanation. Therefore, a blind student using this particular textbook is deprived of a significant portion of the teaching material.

Physics can be difficult under the best of conditions

Learning physics is difficult for many students even when they have full access to all of the available teaching resources. Learning physics is even more difficult when the student is deprived of some of the available teaching resources.

Greek characters

While it is not surprising that the textbook is full of Greek characters and other fancy typesetting elements such as subscripts, superscripts, vector symbols, etc., this also leads to difficulties relative to the use of a screen reader and Braille display.

Mathematics and equations

The language of physics is mathematics. Many blind students are fully capable of understanding mathematics and equations when they are presented in a format that is accessible to the student.

Hundreds of equations

Each chapter in the textbook contains dozens and in some cases hundreds of equations. The equations in the pdf version of the textbook look OK to a sighted person like myself. However, they mostly look like garbage when viewed by a blind student using an audible screen reader and a Braille display.

The Word version is worse

The equations are garbage when viewed by a sighted person using the Word version of the textbook. I don't know what process was used to convert the pdf version to a Word version, but I do know that the equations weren't properly converted from pdf format to Word format.

Overcoming the barriers

The situations described above are just a few of the barriers that I will attempt to overcome in the modules in this collection. I will have more to say about this later.

1.2.3 Prerequisite requirements

Accessibility tools

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).

[3] http://www.youtube.com/watch?v=c8plj9UsJbg

- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA) that is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer screen (http://www.userite.com/ecampus/lesson1/tools.php [6]). While I understand that these devices are fairly expensive, you may not be able to work through the exercises without one.

- The ability to create tactile graphics as described here [7] .

I will have more to say about the need for and the use of these tools later.

Prerequisite knowledge

The minimum prerequisites for understanding the material in these modules include the following.

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]). I will have more to say about this later.
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]). I will have more to say about this later as well.
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]). Once again, I will have more to say about this later.

- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]). Ditto on more to say about this later.
- An understanding of the creation and use of tactile graphics as described here [13] .

1.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

1.3 Discussion

The following sections expand on the discussion from above.

1.3.1 Algebra

As I mentioned earlier, the language of physics is mathematics. In order to understand the material in this collection, you will need a good foundation in algebra. If you have such a foundation, you will probably know that to be the case. If not, you might want to work on that before embarking on a course in physics.

[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[14] http://www.dickbaldwin.com/toc.htm

1.3.2 Using a graph board

There is a good chance that you already know how to use a graph board based on your earlier coursework in algebra. If not, there is a series of videos beginning at http://www.youtube.com/watch?v=c8plj9UsJbg [15] that will teach you what a graph board is and how to use it.

Why would I direct a blind student to a video?

Of course, you may be wondering why I would direct a blind student to an instructional video. My hope is that if you need this information on graph boards, you can find a sighted friend who will help you to work through the series of graph-board tutorials on the website referred to above.

You will need a graph board

You will need a graph board to complete many of the exercises in this collection. If you don't have access to one, it shouldn't be too difficult for you and a sighted friend to make one using a sheet of Styrofoam, corkboard, bulletin-board material or any flat surface into which you can push pins and expect them to stay until you pull them out.

Tactile grid lines

To create the tactile grid lines on your homemade graph board, ask your sighted friend to draw the grid lines on a sheet of heavy paper and then perforate them using a serrated tracing wheel (http://en.wikipedia.org/wiki/Tracing_wheel [16]). Then turn the paper over and pin it to the backing material so that you can feel the perforated grid lines.

Perforating with a tracing wheel

I find that it works well to place the paper on a sheet of Styrofoam to perforate the lines with the tracing wheel. Also, some tracing wheels work better than others for this purpose. The serrations are very sharp on some wheels and less sharp on others. The sharp ones work best.

1.3.3 Using a protractor

There is also a good chance that you already know how to use a protractor to measure angles based on your earlier coursework in algebra. If not, there is a series of videos beginning at http://www.youtube.com/watch?v=v-F06HgiUpw [17] that will teach you how to measure angles with a protractor.

Once again, you may be wondering why I would direct a blind student to an instructional video. As before, my hope is that if you need this information, you can find a sighted friend who will help you to work through the series of protractor video tutorials.

You will need a protractor

You will need a protractor to complete many of the exercises in this collection. While it may be possible to adapt a protractor normally used by sighted students for this purpose, this will probably be more difficult than making a graph board.

A protractor that looks like the one in the video can be purchased at http://shop.aph.org/webapp/wcs/stores/servlet/Product_Braille-Large%20Print%20Protractor_1-04115-00P_10001_11051 [18] . However, this is not a recommendation that you purchase this particular brand of protractor. You should do your own shopping and make your own decision.

1.3.4 JavaScript programming

As I stated earlier, you will need an introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [19] and http://www.w3schools.com/js/default.asp [20]) to work

[15] http://www.youtube.com/watch?v=c8plj9UsJbg
[16] http://en.wikipedia.org/wiki/Tracing_wheel
[17] http://www.youtube.com/watch?v=v-F06HgiUpw
[18] http://shop.aph.org/webapp/wcs/stores/servlet/Product_Braille-Large%20Print%20Protractor_1-04115-00P_10001_11051
[19] http://www.dickbaldwin.com/tocjscript1.htm
[20] http://www.w3schools.com/js/default.asp

through the exercises in this collection of modules. Why is that?

Incompatible typesetting elements

Typical physics textbooks designed for use by sighted students are full of typesetting elements that are often incompatible with accessibility tools such as screen reading software, a Braille display, or both.

In writing these modules, I will need a way to ensure that all of the text, including the text in equations and other mathematical expressions, is compatible with those tools. One way that I hope to do that is to ensure that all of the text consists only of the characters on a standard QWERTY keyboard (http://en.wikipedia.org/wiki/QWERTY [21]).

Restrict to QWERTY characters

One way to accomplish that goal is to write all equations and other mathematical expressions in a format that is compatible with a standard computer programming language that will only accept a subset of the characters on a QWERTY keyboard. I have selected the programming language named JavaScript for this purpose. While JavaScript is not my favorite programming language, it is adequate for this purpose and is probably the most accessible programming language for blind students.

Prior knowledge of JavaScript is not required

It will not be necessary for you to have prior knowledge of or to develop expertise in JavaScript programming to understand and work through these modules. One of the early modules in the collection will be a module on JavaScript programming. That module will be designed to teach you what you need to know about JavaScript programming to satisfy your needs. There will also be additional information about JavaScript programming scattered throughout the modules on an as-needed basis.

1.3.5 Trigonometry

While it would be advantageous for you to have prior knowledge of trigonometry, that is also not a requirement. One of the early modules in the collection will be an introduction to trigonometry. That module will be designed to teach you what you need to know to understand and work through the material in the modules. There may also be additional information about trigonometry scattered throughout the modules on an as-needed basis.

1.3.6 Pictures and diagrams

I will be very frugal with the use of pictures and diagrams.

In those cases where a picture or a diagram is needed, I will make such pictures and diagrams as simple as possible so that a blind student can print an enlarged version and ask a sighted friend to perforate the lines using a tracing wheel. Then the student can turn the paper over and explore the perforations while reading the text.

In many cases, I will provide detailed instructions for using a graph board to create the picture or diagram.

Finally, I will provide SVG graphics files for all pictures and diagrams so that you can convert them to tactile graphics as described here [22] .

1.3.7 Greek characters

I will not use any Greek characters in the modules. In those cases where the use of a Greek character is unavoidable or highly desirable, I will spell out the name of the character using QWERTY characters.

1.4 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as the modules in this collection are published.

[21] http://en.wikipedia.org/wiki/QWERTY
[22] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

1.5 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Introduction to Accessible Physics Concepts
- File: Phy1000.htm
- Revised: 09/29/15
- Keywords:
  - physics
  - accessible
  - blind
  - graph board
  - protractor
  - screen reader
  - Braille display
  - JavaScript
  - trigonometry

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 2

Phy1002: If You Can Imagine It, You Can Draw It using SVG[1]

2.1 Table of Contents

- Preface (p. 8)
  - General (p. 8)
  - Prerequisites (p. 8)
  - Viewing tip (p. 9)
    - Figures (p. 9)
    - Listings (p. 9)
  - Supplemental material (p. 10)
- Discussion (p. 10)
  - Creation of tactile graphics (p. 10)
  - What is SVG *(Scalable Vector Graphics)* ? (p. 10)
  - What does this mean to you? (p. 11)
  - Sample drawing (p. 12)
  - Two approaches (p. 20)
    - Writing raw SVG code (p. 20)
    - Using drawing tools (p. 22)
  - Sample program (p. 23)
    - Beginning of the program named Svg21a.java (p. 23)
    - Create a drawing canvas (p. 24)
    - Draw a rectangular border on the canvas (p. 26)
    - Draw the floor and the wall (p. 28)
    - Draw more rectangles (p. 31)
    - Draw a polygon (p. 33)
    - Draw the rectangular pulley support (p. 35)
    - Draw a circle (p. 35)
    - Draw more lines (p. 36)
    - Change line thicknesses (p. 38)
    - Draw text (p. 40)
    - Write the output file (p. 41)
    - The remaining Java code (p. 41)

[1] This content is available online at <http://cnx.org/content/m39607/1.6/>.

- The SVG graphics library (p. 42)
- Another sample program (p. 43)

- Writing, compiling, and running Java programs (p. 43)
  - Writing Java code (p. 43)
  - Preparing to compile and run Java code (p. 43)
    - The java development kit *(JDK)* (p. 43)
    - JDOM version 1.1.1 (p. 44)
  - Compiling and running Java code (p. 44)
- Resources (p. 45)
- Complete program listings (p. 45)
- Miscellaneous (p. 51)

2.2 Preface

2.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The study of physics is replete with requirements to create and analyze technical drawings. This is obviously more difficult for blind students than for sighted students. However, blind students can draw technical diagrams and the purpose of this module is to show you how. If you can imagine it, you can draw it using SVG.

2.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools *(as a minimum)* to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

2.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

2.2.3.1 Figures

- Figure 1 (p. 15) . Mirror image from the file named Svg21a1r.svg.
- Figure 2 (p. 19) . Non-mirror-image version of the image from the file named Svg21a1r.svg.

2.2.3.2 Listings

- Listing 1 (p. 21) . Raw SVG code for Figure 2.
- Listing 2 (p. 23) . Beginning of the program named Svg21a.java.
- Listing 3 (p. 25) . Create a drawing canvas.
- Listing 4 (p. 26) . SVG code to create a canvas.
- Listing 5 (p. 27) . Draw a rectangular border on the canvas.
- Listing 6 (p. 27) . SVG code to draw a rectangle.
- Listing 7 (p. 29) . Draw the floor and the wall.
- Listing 8 (p. 30) . SVG code to draw a line.
- Listing 9 (p. 32) . Draw more rectangles.
- Listing 10 (p. 33) . SVG code to draw more rectangles.
- Listing 11 (p. 33) . Draw a polygon.
- Listing 12 (p. 34) . SVG code to draw a polygon.
- Listing 13 (p. 35) . Draw the rectangular pulley support.
- Listing 14 (p. 35) . Draw a circle.
- Listing 15 (p. 36) . SVG code to draw a circle.
- Listing 16 (p. 37) . Draw more lines.
- Listing 17 (p. 38) . SVG code to draw more lines.
- Listing 18 (p. 39) . Set the stroke-width attribute value.
- Listing 19 (p. 39) . Modified stroke-width attribute value.
- Listing 20 (p. 40) . Draw text.
- Listing 21 (p. 41) . SVG code to draw text.
- Listing 22 (p. 41) . Write the output file.

[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

- Listing 23 (p. 42) . The remaining Java code.
- Listing 24 (p. 44) . Windows batch file.
- Listing 25 (p. 46) . The program named Svg21a.java.
- Listing 26 (p. 48) . The program named SvgLib21.java.
- Listing 27 (p. 50) . The program named Svg21.java.

2.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

2.3 Discussion

2.3.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explains how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, **you will need to** download the file named Phy1002.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

2.3.2 What is SVG *(Scalable Vector Graphics)* ?

The shortest answer that I can come up with is that SVG is a technology that makes it possible for a blind student to create technical drawings. If the student can imagine it, the student can draw it using SVG and drawing tools that I will provide in this module.

According to Wikipedia [18]

"Scalable Vector Graphics (SVG) is a family of specifications of an XML-based file format for describing two-dimensional vector graphics, both static and dynamic (i.e. interactive or animated).

The SVG specification is an open standard that has been under development by the World Wide Web Consortium (W3C) since 1999. SVG images and their behaviors are defined in XML text files. This means that they can be searched, indexed, scripted and, if required, compressed.

Since they are XML files *, SVG images can be created and edited with any text editor, but it is often more convenient to create these types of images with drawing programs such as Inkscape* [19] .

All major modern web browsers have at least some degree of support and render SVG markup directly, including Mozilla Firefox, Internet Explorer 9, Google Chrome and Safari. However, no earlier versions of Microsoft Internet Explorer (IE) support SVG natively."

The SVG home page

The SVG home page is located at http://www.w3.org/TR/SVG/ [20]

[14] http://www.dickbaldwin.com/toc.htm
[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Manual-Creation-of-Tactile-Gra
[16] http://cnx.org/content/m39607/latest/Phy1002.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[18] http://en.wikipedia.org/wiki/Scalable_Vector_Graphics
[19] http://inkscape.org/
[20] http://www.w3.org/TR/SVG/

That is where you will find technical specifications for the many capabilities that SVG has to offer. Those capabilities are vast. In this module, you will learn to create SVG files to draw the following **basic shapes** along with text:

- line
- rectangle
- circle
- ellipse
- polyline
- polygon

You will also learn how to manipulate certain aspects of the following **attributes** on those shapes and on the text that you create:

- stroke
- stroke-width
- stroke-opacity
- fill
- fill-opacity
- font-style
- font-weight

While this barely scratches the surface in terms of overall SVG capability, it does provide a set of tools that will put you in good stead relative to creating drawings for your science, technology, engineering, and mathematics courses.

2.3.3 What does this mean to you?

Let me refer back (p. 10) to the most important statement so far in this document:

" *Since they are XML files, SVG images can be created and edited with any text editor* "

What this means is that if you can imagine a technical drawing in terms of objects created from the basic shapes (p. 11) listed above along with their attributes (p. 11) , and you can mentally organize the sizes and positions of those objects in a drawing, you can use a text editor to create an SVG file, which, in turn can be used to render the drawing on the screen or on paper.

Using the SVG file

Once the drawing exists in the form of an SVG file, it can be printed and submitted as part of an assignment. Also, if you have access to the necessary equipment or assistance, it can be turned into a tactile drawing for you and other blind students to explore by touch.

You can also use the file format converter at http://www.online-utility.org/image_converter.jsp [21] to convert the file to other formats such as **png and jpeg** . This makes it possible for you to use the drawing for other purposes, such as conversion to sound using software that is available at http://www.seeingwithsound.com/winvoice.htm [22] .

And last but not least, if you happen to have access to the IVEO learning system [23] , the SVG files that you create can be used with that system to be explored by touch and sound.

Even though you may be blind or visually impaired and you may never have drawn anything in your life, don't let that stop you. If you can imagine it, you can draw it using SVG. My purpose in publishing this module is to help you develop that skill.

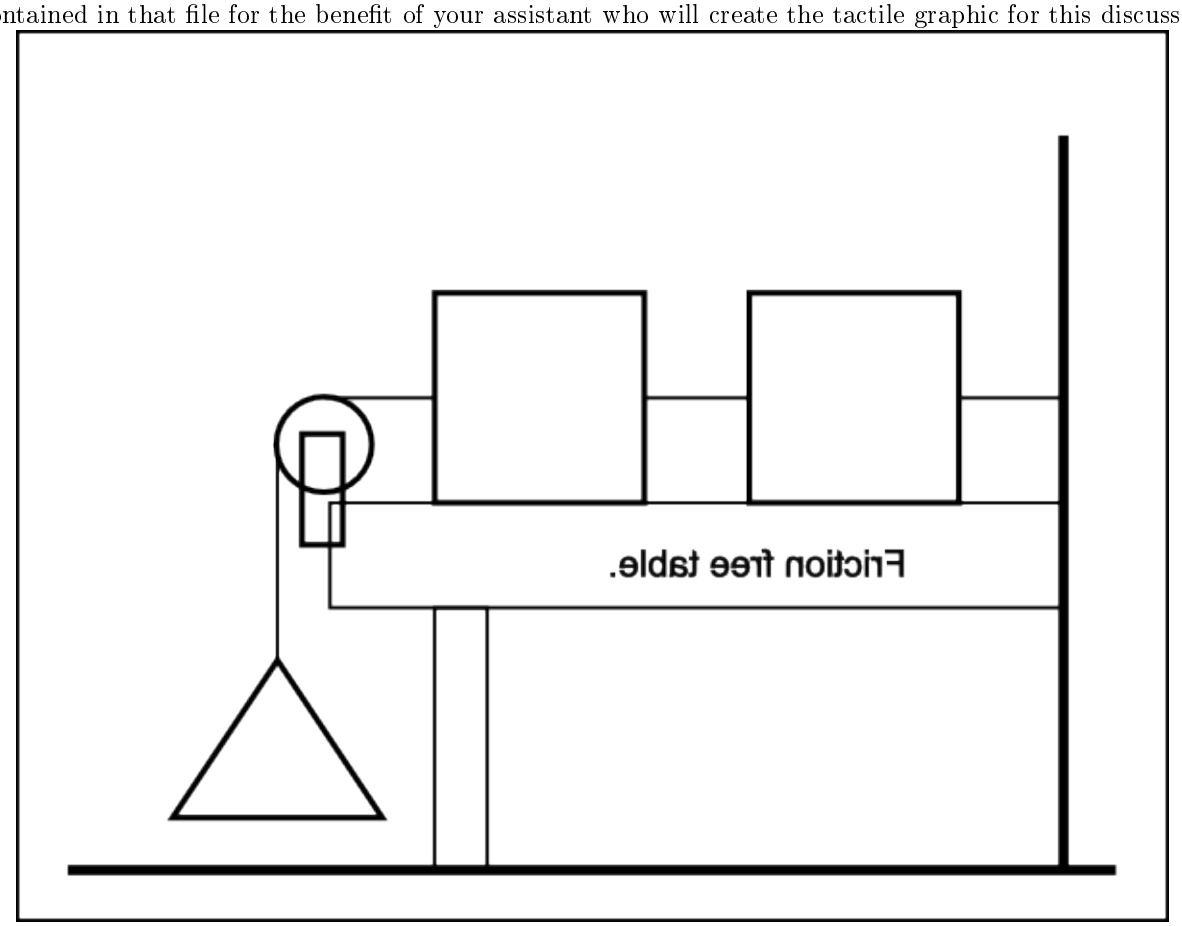
[21] http://www.online-utility.org/image_converter.jsp
[22] http://www.seeingwithsound.com/winvoice.htm
[23] http://www.viewplus.com/solutions/touch-audio-learning/

2.3.4 Sample drawing

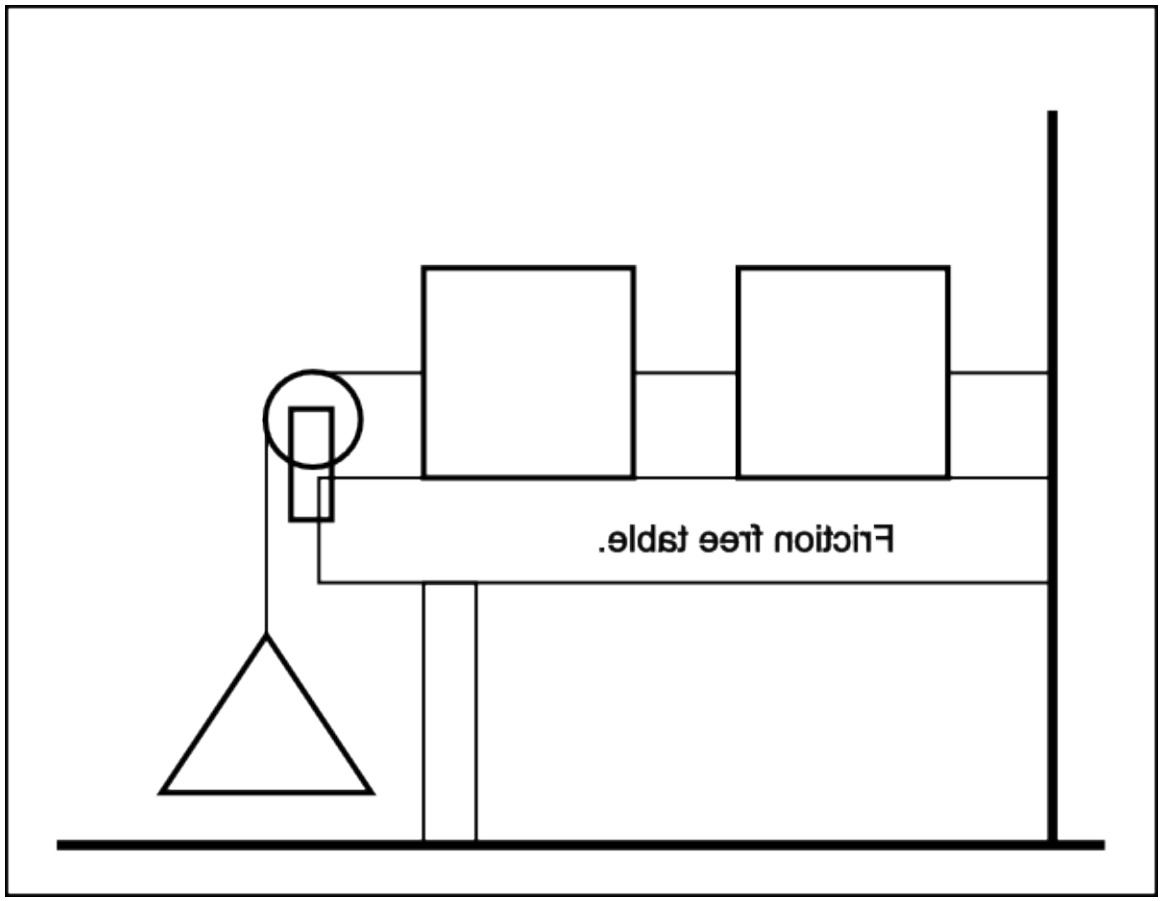
Before going any further, I am going to provide the SVG files for a sample drawing that I will discuss in detail later. Hopefully, you can ask your assistant to print the file named **Svg21a1r.svg** and create a tactile version of the drawing as described here [24] .

Tactile graphics

The file named **Svg21a1r.svg** contains a mirror image of the image that I created for this discussion. You should have downloaded that file earlier (p. 10) . Figure 1 (p. 15) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this discussion.



[24] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482


Friction free table.

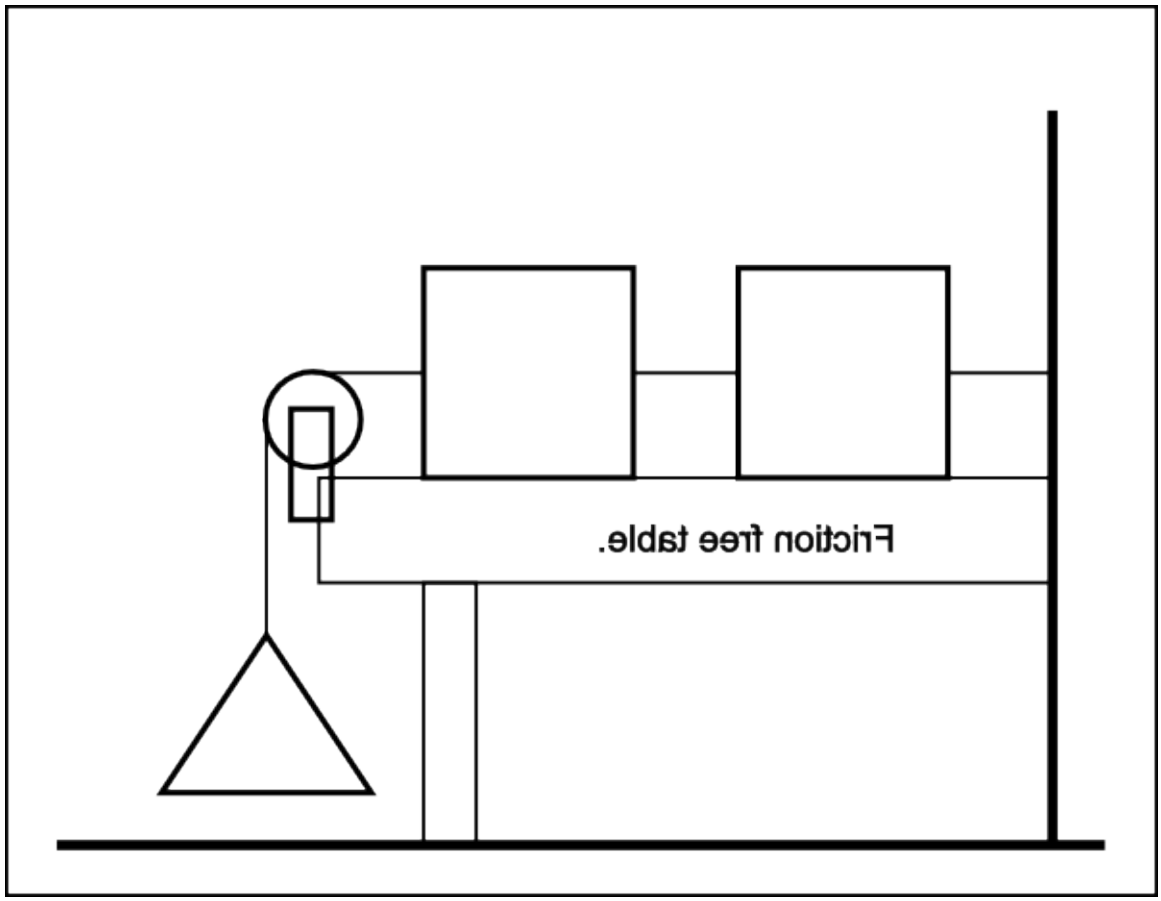

Friction free table.

Figure 1 . Mirror image from the file named Svg21a1r.svg.

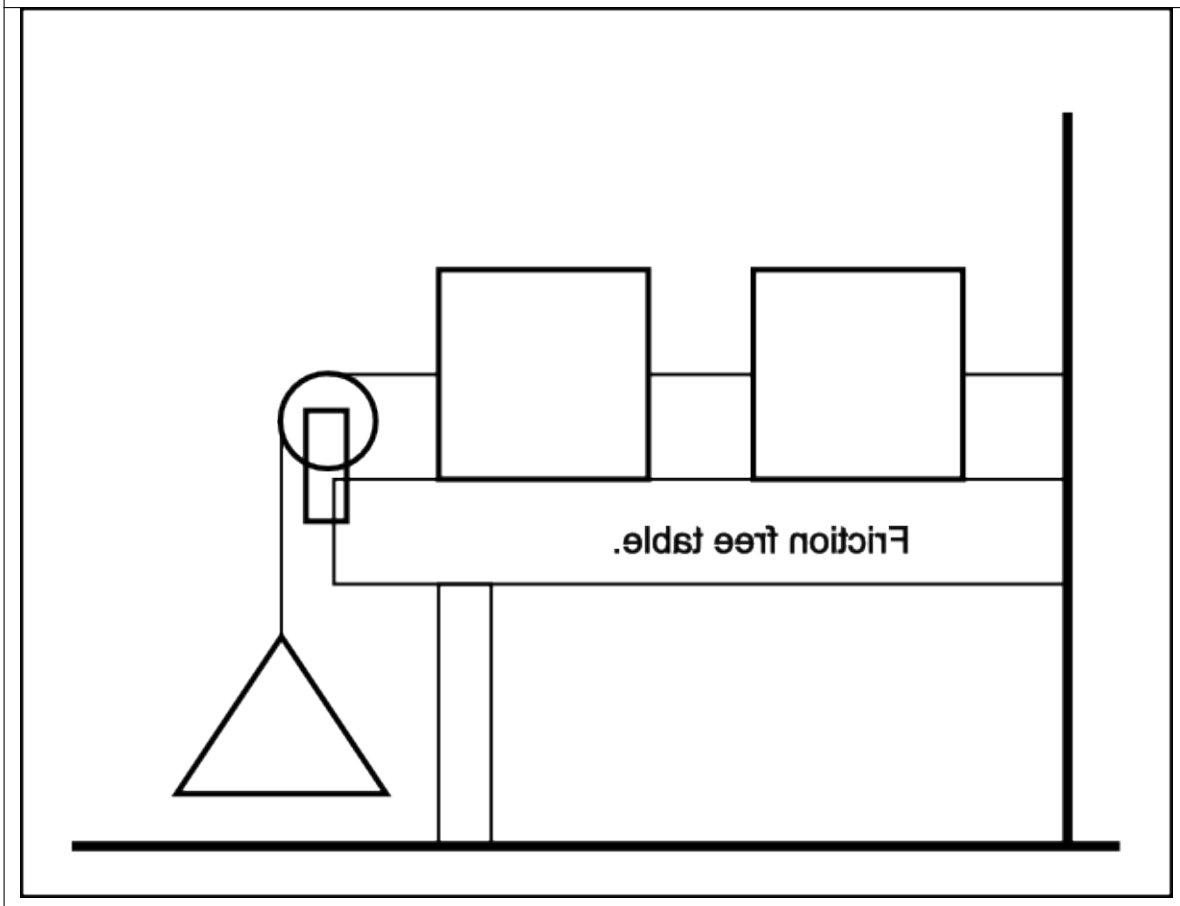
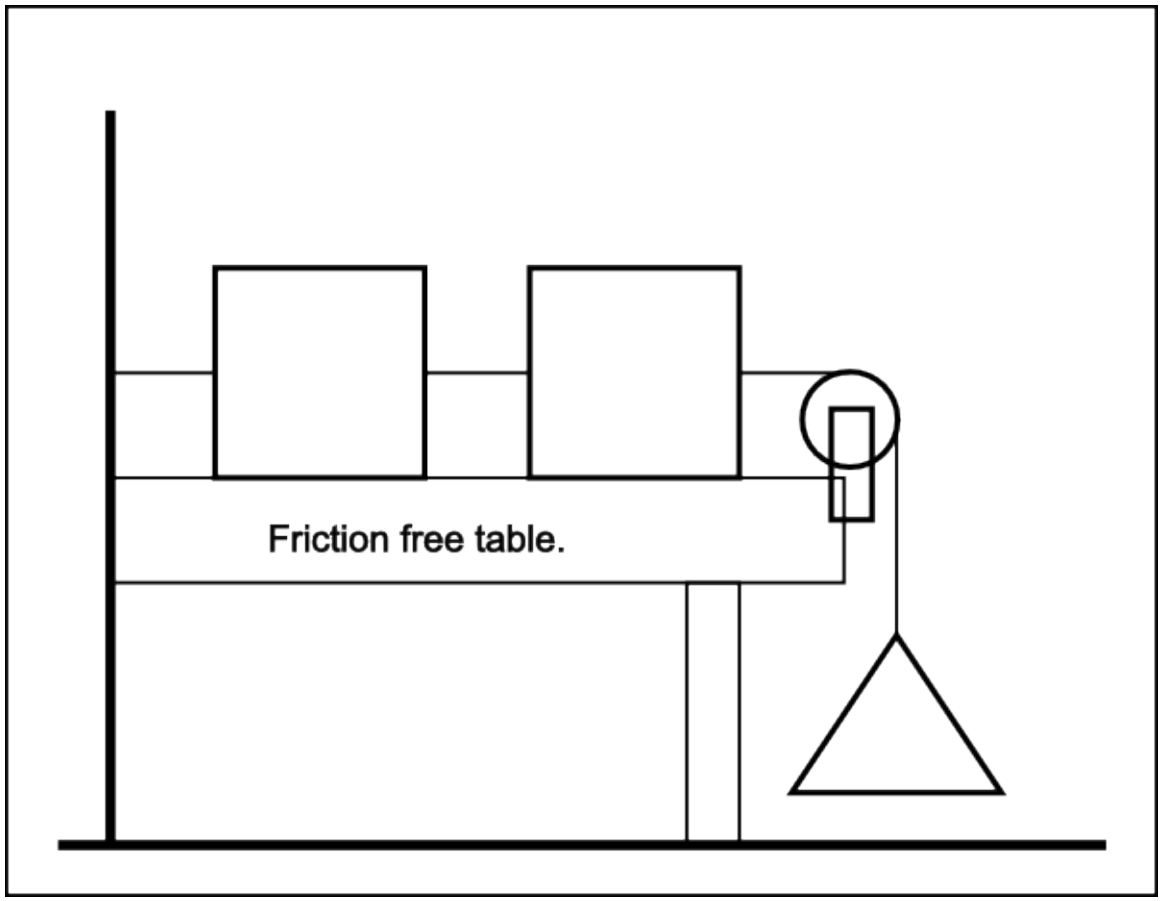
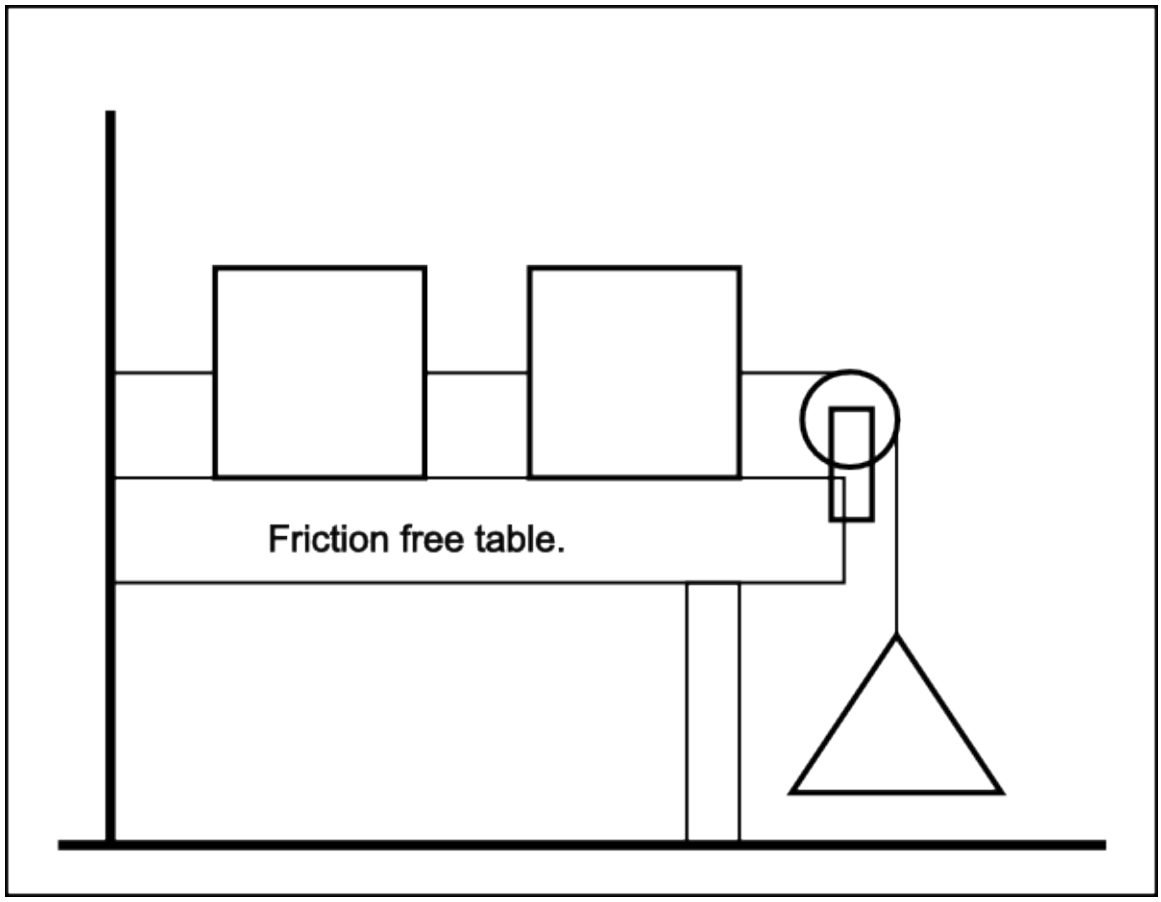


Table 2.1

Figure 2 (p. 19) shows a non-mirror-image version of the same image.


Friction free table.


Friction free table.

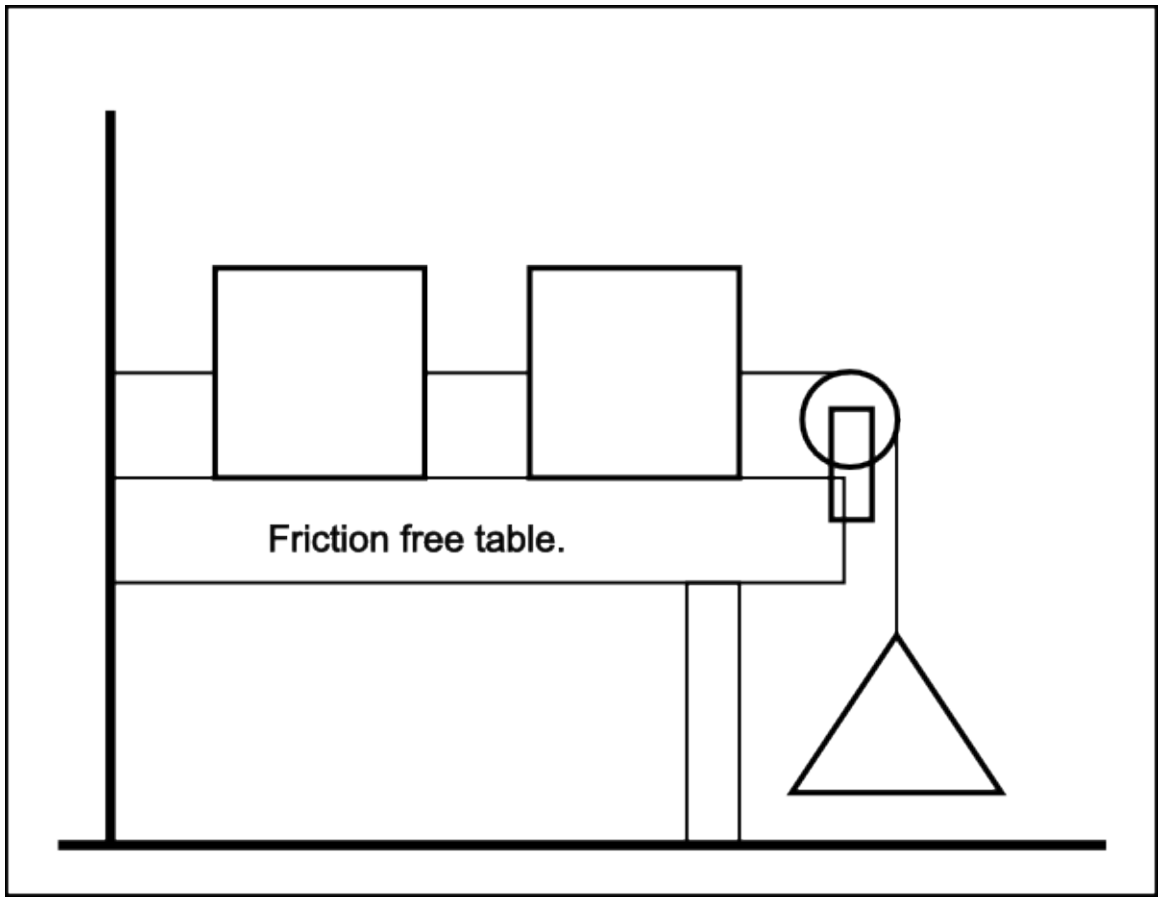

Friction free table.

Figure 2 . Non-mirror-image version of the image from the file named Svg21a1r.svg.

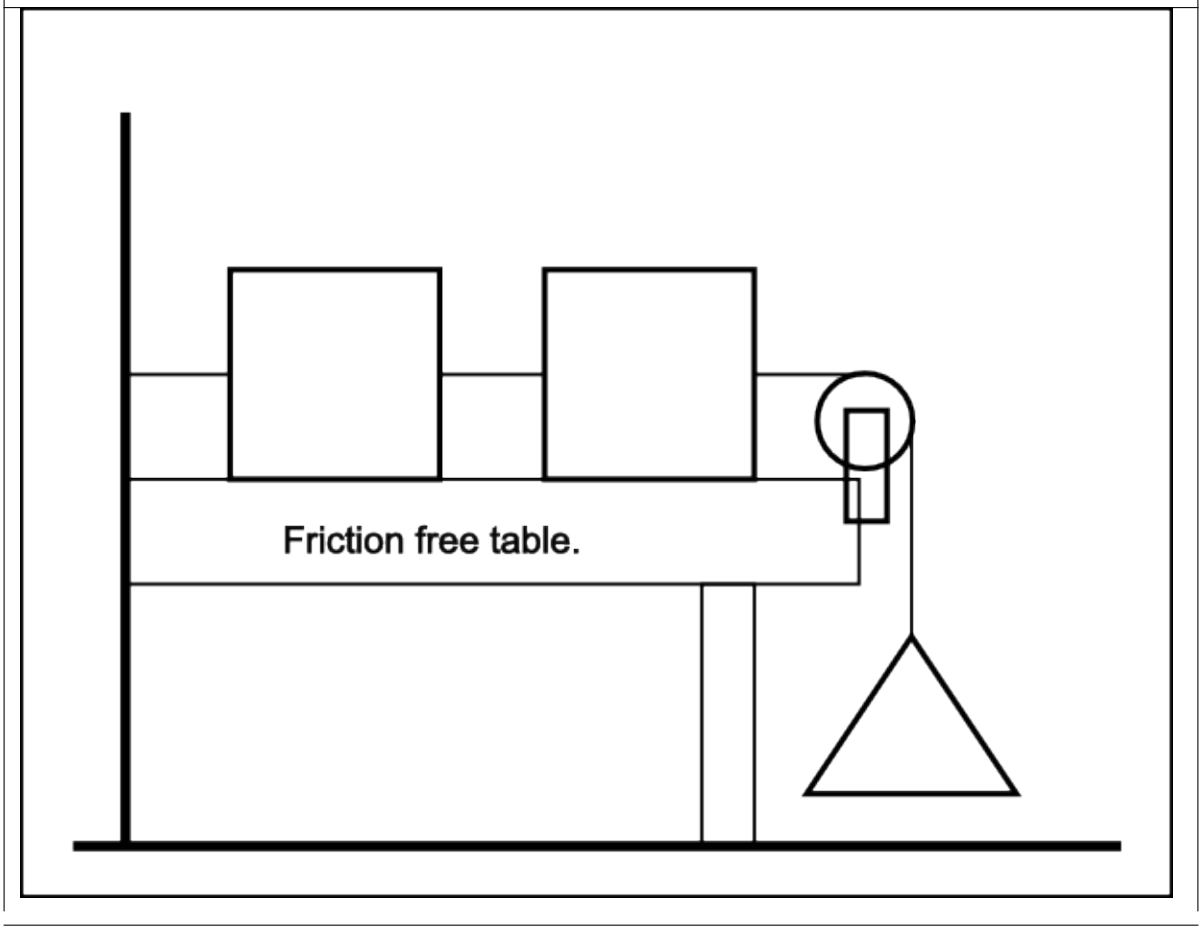


Table 2.2

This image contains only one line of text. It reads *"Friction free table"* and appears on the side of a table. Therefore, I didn't provide a key-value table as described here [25] .

What does the image show ?

Just in case you were unable to get a tactile version of the image, I will describe it to you. It isn't very complicated.

There are three objects connected together with a cord. Two of the objects, each of which has a square shape, are setting on top of a table. The left end of the table is attached to a wall. The right end of the table is supported by a table leg.

The leftmost object on the table is tied to the wall. The two objects on the table are tied to one another.

The third object is shaped like a triangle. It is connected to the rightmost square object with a cord, but it is not setting on the table. Instead, there is a pulley wheel connected to the rightmost corner of the table. The triangular object is hanging from the cord, which threads up and over the pulley wheel and connects to the rightmost square object, which is to the left of the pulley wheel.

A label on the table reads *"Friction free table."*

Straight lines, rectangles, circles, and polygons

[25] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

As you may have observed from the description, this drawing is made up entirely of straight lines, rectangles, a polygon for the triangle, and a circle for the pulley wheel. This is representative of many of the drawing used to illustrate physics concepts.

This drawing uses all of the basic shapes described earlier (p. 11) except for the ellipse and the polyline. Different line thicknesses were used to visually differentiate the objects from one another.

Processing an SVG file

An SVG file can be processed using an SVG processor, such as IE 9 or Firefox 5 to convert the commands contained in the SVG file into a drawing. If you are using a browser as your SVG processor, the drawing will appear in the browser window, from which it can be viewed and/or printed.

In addition, some products, such as Inkscape [26] and the IVEO learning system [27] can read the SVG file directly and use it to provide additional benefits such as converting text labels and shapes into spoken words and displaying the drawing in tactile form using an embossing graphic printer.

Also, as mentioned earlier (p. 11) , you can convert the SVG file to other formats, such as png and jpeg for use with other programs such as The vOICe Learning Edition [28] .

2.3.5 Two approaches

There are at least two approaches for using SVG to create a drawing like this:

- writing raw SVG code
- using drawing tools

2.3.5.1 Writing raw SVG code

As mentioned earlier, the contents of an SVG file are plain text. That text can be produced using any plain text editor, such as Windows Notepad.

If you are willing to study the specifications at http://www.w3.org/TR/SVG/ [29] , you can use your text editor to create raw SVG code and accomplish everything that is possible using SVG. However, that can be a daunting task.

Listing 1 (p. 21) shows the raw SVG code that produced the image shown in Figure 2 (p. 19) . You might conclude that you don't want to spend your time writing text like that when you should be studying physics concepts instead.

[26] http://inkscape.org/
[27] http://www.viewplus.com/solutions/touch-audio-learning/
[28] http://www.seeingwithsound.com/winvoice.htm
[29] http://www.w3.org/TR/SVG/

Listing 1 . Raw SVG code for Figure 2.

```
<?xml version="1.0" encoding="UTF-8"?>
<!DOCTYPE svg PUBLIC "-//W3C//DTD SVG 1.1//EN"
 "http://www.w3.org/Graphics/SVG/1.1/DTD/svg11.dtd">

<svg xmlns="http://www.w3.org/2000/svg" version="1.1"
 width="990" height="765">
  <title>Document Title</title>
  <rect fill="none" stroke="black" stroke-width="3"
   x="1" y="1" width="987" height="762">
    <title>rectangle</title>
  </rect>
  <line stroke="black" stroke-width="9"
   x1="45" y1="720" x2="945" y2="720">
    <title>line</title>
  </line>
  <line stroke="black" stroke-width="9"
   x1="90" y1="720" x2="90" y2="90">
    <title>line</title>
  </line>
  <rect fill="none" stroke="black" stroke-width="3"
   x="90" y="405" width="630" height="90">
    <title>rectangle</title>
  </rect>
  <rect fill="none" stroke="black" stroke-width="3"
   x="585" y="495" width="45" height="225">
    <title>rectangle</title>
  </rect>
  <rect fill="none" stroke="black" stroke-width="5"
   x="180" y="225" width="180" height="180">
    <title>rectangle</title>
  </rect>
  <rect fill="none" stroke="black" stroke-width="5"
   x="450" y="225" width="180" height="180">
    <title>rectangle</title>
  </rect>
  <polygon stroke="black" stroke-width="5" fill="none"
   points="675 675 855 675 765 540 ">
    <title>polygon</title>
  </polygon>
  <rect fill="none" stroke="black" stroke-width="5"
   x="709" y="346" width="35" height="95">
    <title>rectangle</title>
  </rect>
  <circle fill="none" stroke="black" stroke-width="5"
   cx="725" cy="355" r="41">
    <title>circle</title>
  </circle>
  <line stroke="black" stroke-width="3"
   x1="90" y1="315" x2="180" y2="315">
    <title>line</title>
  </line>
  <line stroke="black" stroke-width="3"
   x1="360" y1="315" x2="450" y2="315">
    <title>line</title>
```

Table 2.3

Although the concepts involved in manually writing SVG code aren't complicated, the process is very tedious and you are very likely to make errors in the coding process.

2.3.5.2 Using drawing tools

Fortunately for sighted students, drawing tools are readily available that make the creation of SVG drawings relatively easy. One of the best is the free open source SVG graphics editor named Inkscape [30] . Sighted students are able to use that editor with the mouse and the keyboard to create drawings in a visual drawing environment.

I use Inkscape [31] all of the time to create SVG files for the drawings that I need. However, I don't know of any blind students that have attempted to use Inkscape [32] . It doesn't look to me like it would be very accessible for blind students. However, it is free, so you should give it a try just so you will know for sure.

SVG drawing editors for blind students

I am unaware of any SVG drawing editors that are designed for use by blind students who are unable to use a mouse. *(If you know of any, please let me know.)* Therefore, I will provide an SVG graphics library that I have designed specifically for blind students in this module. It isn't Inkscape [33] ; far from it. However, it does not require the use of a mouse and it works well. It is my hope that the use of my library will make it possible for you to use SVG to draw the diagrams that you need to successfully pursue your coursework in physics and other technical areas.

Seeking improvements in the interface

There are a large number of excellent blind programmers scattered around the world. It is my hope that one or more of those programmers will pick up the challenge and develop an improved interface for the library that will make it even easier for blind students to draw using SVG.

I would like to see a JavaScript version of an SVG drawing editor designed for use by blind students. That's not because JavaScript is my favorite programming language, which it isn't. That is because JavaScript has the lowest barrier to entry of any programming environment that I am aware of. *(See my JavaScript module here* [34] .

How does it work?

Basically what I will provide in this module is an SVG graphics library written in the Java programming language along with a template and instructions for you to use in writing Java programs to produce the drawings that you need.

I don't have a fancy interface to go with the graphics library. Instead, I will provide a template that you can use to write a new Java program for each new drawing. The procedure will be to write a program that encapsulates the drawing that you have in your mind. When you run the program, it will produce the SVG file that describes your drawing.

If you determine that there are errors in your drawing, you can make corrections to your program code and run it again to get a new version of the SVG file.

Raw SVG code versus my SVG graphics library

Only you can decide whether you prefer to write raw SVG code or you prefer to use the graphics library. I will present examples of both in this module.

I will point out one major advantage of using the library, however. Once you learn how to write Java programs that incorporate the library to create drawings, there is nothing to prevent you from expanding those programs to also solve physics problem and draw graphs of the results.

For example, suppose you have a physics assignment to compute and draw the trajectory of a projectile. Using raw SVG code, you would first need to compute and save the coordinates of the projectile as a set of incremental data points. Then you could write raw SVG code incorporating that data to draw the trajectory.

[30] http://inkscape.org/
[31] http://inkscape.org/
[32] http://inkscape.org/
[33] http://inkscape.org/
[34] http://cnx.org/contents/9eb6aeee-1604-4bde-b81f-912622f036d1

Using the library, you could write a program that would compute and also draw the trajectory in a single operation. In my opinion, that would be a much cleaner solution to the assignment.

2.3.6 Sample program

A complete listing of the program named **Svg21a.java** , that was used to produce the drawing shown in Figure 2 (p. 19) is provided in Listing 25 (p. 46) near the end of the module.

This program requires access to the SVG graphics library in the file named **SvgLib21.java** . A complete listing of this program is shown in Listing 26 (p. 48) near the end of the module.

This program also requires access to the free Java Development Kit [35] , version 6 or later, which I will also discuss later.

Finally, this program also requires access to JDOM 1.1.1 [36] , which is free, and which I will also discuss later.

Purpose of the program

The primary purpose of this program is to demonstrate the use of my SVG graphics library in the file named **SvgLib21** . It uses that library to draw an abbreviated version of a mass-pulley system shown in Figure 4 of the module that you will find here [37] .

The drawing in that module contains several lines of text. However, this program draws only one line of text. Otherwise, the drawing produced by this program is the same as the drawing used in that module titled Force and Motion – Units of Force [38] .

I created the original drawing using Inkscape [39] . I created this drawing using my SVG graphics library and the program that I am about to discuss.

This program was tested using J2SE 6, JDOM 1.1.1, and Firefox 5 running under Windows Vista Home Premium Edition.

2.3.6.1 Beginning of the program named Svg21a.java

I will explain this program in fragments and explain how you can write similar programs to create the SVG drawing files that you need. The first fragment, which shows the beginning of the program, is shown in Listing 2 (p. 23) .

(Note that complete listings are provided in Listing 25 (p. 46) , Listing 26 (p. 48) , and Listing 27 (p. 50) . That code is ready to copy into your editor, save as Java source code files, compile, and run as explained under Writing, compiling, and running Java programs (p. 43) .)

Listing 2 . Beginning of the program named Svg21a.java.

```
import java.io.*;
import org.jdom.*;

public class Svg21a{
  public static void main(String[] args){

    //DO NOT MODIFY ANY OF THE CODE ABOVE THIS LINE.
    //////////////////////////////////////////////////////
```

[35] http://www.oracle.com/technetwork/java/javase/downloads/index.html
[36] http://www.jdom.org/
[37] http://cnx.org/contents/16d1b592-5126-458f-883e-3ae872cc18c0#Figure_4
[38] http://cnx.org/contents/16d1b592-5126-458f-883e-3ae872cc18c0
[39] http://inkscape.org/

Table 2.4

Java comments

Whenever you see the following character sequence, //, in a Java program, the text that follows to the end of the line is a comment. That is to say, that text is meant to provide information to a human reader and is ignored by the computer.

Listing 2 (p. 23) contains two comments. I will use many more comments in subsequent listings to help explain the code.

Java program files

Java programs are simply text files with the file name of your choice and an extension of .java. You can create those files using any plain text editor. I will explain later how you can "compile" those files to create executable programs.

If this were a module on computer programming, I would explain what is meant by the program code in Listing 2 (p. 23) . However, since this is not a module on computer program, I will simply tell you to replicate the text shown in Listing 2 (p. 23) at the beginning of your Java program file with one exception. That exception has to do with the name of the program and the name of the file.

The name of the program

The name of this program is **Svg21a** . You can see that name on the line following the word class in Listing 2 (p. 23) . You can use just about any name you want as long as the first character is a letter and the remainder of the name contains only letters and numbers. However, the name of the program, as shown in Listing 2 (p. 23) , must match the name of the file containing that program except that the file name must have an extension of .java.

For example, this program named **Svg21a** is stored in a file named **Svg21a.java** .

Also be aware that everything in Java, including program names and file names, is case sensitive. By that I mean that Joe is not the same as jOe, which is not the same as joE.

2.3.6.2 Create a drawing canvas

The next code fragment is shown at the top of Listing 3 (p. 25) . This fragment contains two Java programming statements.

(Usually Java program statements end with a semicolon.)

These must be the first two statements in your Java program and they must appear only once.

The first statement, down to the semicolon, creates the canvas on which the drawing will appear. You may modify this statement as explained below.

The second statement at the bottom of Listing 3 (p. 25) is a housekeeping statement and must not be modified.

Listing 3 . Create a drawing canvas.

```
//ONLY THE CODE BELOW THIS LINE CAN BE MODIFIED

//CREATE A DRAWING CANVAS
//This must be the first statement that you write in
// the program and it must appear only once.
//The following statement creates a canvas that is
// 8.5x11 inches in size in a landscape layout.
Element svg = SvgLib21.makeSvg(ns,
                               "Document Title",
                               11,  //width
                               8.5  //height
                               );


//DO NOT MODIFY THE FOLLOWING STATEMENT
//This statement must immediately follow the call to
// the makeSvg method above and this statement MUST
// NOT BE MODIFIED.
Document doc = new Document(svg,docType);
```

Table 2.5

What does this code mean?

The first statement shown in Listing 3 (p. 25) creates a canvas that is 8.5 x 11 inches in size in a landscape layout. In other words, the canvas has a width of 11 inches and a height of 8.5 inches. When you print the drawing produced on this canvas, it should fit perfectly on 8.5x11 inch paper provided that you tell the printer to print in landscape *(as opposed to portrait)* mode.

If your printer uses 8.5 x 11 inch paper, the only modification that you will want to make to this statement is to sometimes reverse the order of the width and height values *(see the comments)* to cause the canvas to accommodate portrait mode.

If your printer uses larger paper, you might want to modify the width and height values to accommodate the actual size of your printer paper.

When modifying the width and height values in the first statement, be careful not to delete the comma and DON'T MAKE ANY OTHER CHANGES to the statement with the possible exception of the *"Document Title"* parameter discussed below.

The Document Title

The **makeSvg** method, and most of the other **makeZzz** methods discussed below have a parameter that adds a title to the SVG element. These parameters have default values in this program such as "Document Title", "line", "rectangle", "circle", "ellipse", "polyline", and "polygon".

The purpose of these parameters is to provide compatibility with the speaking capability of the IVEO viewer [40] .

If the output SVG file is opened in the IVEO viewer [41] , the title for the **svg** element is spoken when the user opens the file.

The titles for the individual shapes are spoken by the IVEO viewer [42] when the user touches a corresponding shape on the touchpad or clicks on that shape on the screen.

[40] http://www.viewplus.com/solutions/touch-audio-learning/
[41] http://www.viewplus.com/solutions/touch-audio-learning/
[42] http://www.viewplus.com/solutions/touch-audio-learning/

If the SVG file won't be used with the IVEO viewer [43] , just leave the title strings unchanged. If the SVG file will be used with the the IVEO viewer [44] , you can modify those strings to cause the viewer to speak whatever titles you choose. *(Don't remove the quotation marks if you modify the title string.)*

You can read more about this capability under The SVG graphics library (p. 42) .

SVG code to create a canvas

If you were to delete all of the remaining code in Listing 25 (p. 46) down to but not including the statement that writes the output SVG file, the resulting SVG code would be that shown in Listing 4 (p. 26) .

Listing 4 . SVG code to create a canvas.

```
<?xml version="1.0" encoding="UTF-8"?>
<!DOCTYPE svg PUBLIC "-//W3C//DTD SVG 1.1//EN"
 "http://www.w3.org/Graphics/SVG/1.1/DTD/svg11.dtd">

<svg xmlns="http://www.w3.org/2000/svg" version="1.1"
 width="990" height="765">
  <title>Document Title</title>
```

Table 2.6

The width and the height of the canvas

The first three lines of text in Listing 4 (p. 26) constitute housekeeping information that you needn't worry about. It will always be the same.

The fourth and fifth lines of text in Listing 4 (p. 26) define the width and height of the canvas.

As you can see, the width is set to a value of 990 and the height is set to a value of 765.

The last line of text in Listing 4 (p. 26) is the **title** element discussed earlier (p. 25) .

SVG units

SVG doesn't inherently deal with dimensions in inches *(although it is possible to specify inches when you define the value for a size attribute.)* . Instead, it deals with dimensions in something that I will refer to as "SVG units", and for reasons that I am unable to explain, there appear to be 90 SVG units per inch or 35.43 SVG units per centimeter.

Thus, a width of 990 *(as shown in Listing 4* (p. 26)) corresponds to a width of 11 inches and a height of 765 corresponds to a height of 8.5 inches.

If you elect to write raw SVG code for you drawings, you will need to think in terms of SVG units instead of inches *(or write all of the size attribute values something like "8.5in")* . However, I designed my SVG graphics library so that you can think in terms of inches instead of SVG units without having to remember to specify the units for every size value.

2.3.6.3 Draw a rectangular border on the canvas

The Java code in Listing 5 (p. 27) draws a rectangular border on the canvas by creating an SVG element of type **rect** *(for rectangle)* .

[43] http://www.viewplus.com/solutions/touch-audio-learning/
[44] http://www.viewplus.com/solutions/touch-audio-learning/

Listing 5 . Draw a rectangular border on the canvas.

```
//Draw a rectangular border on the canvas.
Element border = SvgLib21.makeRect(svg,
        ns,
        "rectangle",//title
        0.015,/lower-left x-coordinate in inches
        0.015,//lower-left y-coordinate in inches
        10.97,//width in inches
        8.47  //height in inches
        );
```

Table 2.7

What are you allowed to change?

There are only six things that you are allowed to change in the code in Listing 5 (p. 27) *(pay attention to the comments)* :

1. The name of the **rect** element, which is **border** in Listing 5 (p. 27) . *(Later on, I will refer to this as an object instead of an element.)*
2. The title for the element, which is **"rectangle"** in Listing 5 (p. 27) .
3. The x-coordinate of the lower-left corner of the rectangle, which is 0.015 inches in Listing 5 (p. 27) .
4. The y-coordinate of the lower-left corner of the rectangle, which is 0.015 inches in Listing 5 (p. 27) .
5. The width of the rectangle, which is 10.97 inches in Listing 5 (p. 27) .
6. The height of the rectangle, which is 8.47 inches in Listing 5 (p. 27) .

Multiple rect elements

You can replicate this code to define as many **rect** elements as you need in your drawing so long as you provide a unique name for each element *(object)* .

The size of the rectangle

If you carefully examine the values that I specified for the coordinates of the lower-left corner, the width, and the height, you will see that I made the rectangle slightly smaller than the size of the paper so that it will fit just inside the edges of the paper.

SVG code to draw a rectangle

The use of the Java code in Listing 5 (p. 27) to draw a rectangle results in the SVG code shown in Listing 6 (p. 27) .

Listing 6 . SVG code to draw a rectangle.

```
<rect fill="none" stroke="black" stroke-width="1"
 x="1" y="1" width="987" height="762">
  <title>rectangle</title>
</rect>
```

Table 2.8

In order to force the SVG code to fit in this publication format, it was necessary for me to insert a line break following the "1". Those two lines were originally a single line in the SVG code.

View my tutorials

SVG is simply one flavor of something called XML. I have published hundreds of online tutorials on Java programming, XML, and SVG. If you are interested in reading what I have to say in those tutorials, just Google the following keywords:

- Richard Baldwin Java
- Richard Baldwin XML
- Richard Baldwin SVG

The rect element

The four lines in Listing 6 (p. 27) that begin with an angle bracket followed by **rect** and end with /> constitute what is called an XML element named **rect** .

The **rect** element has a **title** element as its content. The **title** element has the word **rectangle** as its content.

The attributes of the rect element

The following items are known as the **attributes** of the **rect** element:

- fill
- stroke
- stroke-width
- x
- y
- width
- height

The attribute values

The text that appears in quotation marks, such as "762" are known as the values of the attribute to which they are joined by an equals character "=".

How does it all work?

When an SVG processor, such as the one incorporated into Firefox 5, sees an SVG/XML element named **rect** in an SVG file, it knows that it needs to draw a rectangle. It then looks to the attributes and their values to determine other aspects of that rectangle.

For example, in this case, the SVG processor is told to draw a rectangle consisting of an outline only *(fill="none")* .

The color of the outline is to be black *(stroke="black")* .

The thickness of the outline is to be a single SVG unit *(stroke-width="1")* .

The lower-left corner of the outline is to be very close to the origin when described in SVG units *(x="1" and y="1")* .

The width of the rectangle is to be 987 SVG units *(width="987")* , and the height of the rectangle is to be 762 SVG units *(height="762")* .

If you were to run the program at this point, open the output SVG file in Firefox 5, and print the result, you would have a blank sheet of paper with a black outline barely inside the edges of the paper. *(Note, however, that on my system, there is a margin of approximately one-half inch at the bottom of the paper.)*

2.3.6.4 Draw the floor and the wall

In the image shown in Figure 2 (p. 19) , the floor consists of a line parallel to the horizontal axis near the bottom of the drawing. The wall consists of a line parallel to the vertical axis near the left side of the drawing. The Java code in Listing 7 (p. 29) causes those two lines to be drawn.

(All coordinate values and dimensions given in this and the following Java code are in inches, so I will stop mentioning that at this point.)

Listing 7 . Draw the floor and the wall.

```
//Draw the floor.
Element floor = SvgLib21.makeLine(svg,
        ns,
        "line",//title
        0.5, //x-coordinate of one end of line
        0.5, //y-coordinate of one end of line
        10.5,//x-coordinate of other end of line
        0.5  //y-coordinate of other end of line
        );

//Draw the wall.
Element wall = SvgLib21.makeLine(svg,
                                 ns,
                                 "line",
                                 1.0,
                                 0.5,
                                 1.0,
                                 7.5
                                 );
```

Table 2.9

Two SVG elements of type line

Each of the Java statements in Listing 7 (p. 29) causes a new SVG element of type line to be created. The first element is named **floor** in the Java code. *(**I will have more to say about element names later** .)* The second element is named **wall** . *(See Listing 8 (p. 30) later.)*

What can you change?

As before, there are only six things that you are allowed to change in each of the Java statements in Listing 7 (p. 29) :

- The names of the line elements *(floor and wall in Listing 7 (p. 29) .)*
- The values of the titles *("line" in Listing 7 (p. 29) .)*
- The x and y coordinate values for one end of each line.
- The x and y coordinate values for the other end of each line.

If you examine the x and y coordinate values for one end of the line named floor in Listing 7 (p. 29) *(0.5, 0.5)* , you will see that one end of the line is near the origin at the lower-left corner of the drawing.

If you examine the x and y coordinate values for the other end of the line named **floor** *(10.5, 0.5)* , you will see that the other end of the line is near the lower-right corner of the drawing. Furthermore, both y-coordinate values are 0.5, meaning that the line is parallel to the horizontal axis as desired.

A similar analysis of the line named **wall** will reveal that it intersects the floor near the left end at *(1.0, 0.5)* and is parallel to the vertical axis.

SVG code to draw a line

Listing 8 (p. 30) shows the two **line** elements created by the Java code shown in Listing 7 (p. 29) .

Listing 8 . SVG code to draw a line.

```
<line stroke="black" stroke-width="1"
 x1="45" y1="720" x2="945" y2="720">
  <title>line</title>
</line>

<line stroke="black" stroke-width="1"
 x1="90" y1="720" x2="90" y2="90">
  <title>line</title>
</line>
```

Table 2.10

An explanation of element names

I promised earlier (p. 29) that I would have more to say about element names later. That time has come.

SVG elements don't have names, other than the name that defines the type of element such as **rect** (*Listing 6* (p. 27)) and **line** (*Listing 8* (p. 30)). Names such as **border** (*Listing 5* (p. 27)), **floor** and **wall** (*Listing 7* (p. 29)) are purely Java mechanisms for keeping track of the different elements in the drawing.

(I should have been referring border, floor, and wall as objects instead of elements in the Java code earlier, but I didn't want to make things even more confusing than they may already be.)

The names disappear

By the time the Java code is converted into SVG code, those identifying names have disappeared and the SVG code consists of

- elements of specific types
- having attributes with
  - specific names
  - and specific values.

(There is also something in the SVG code called content, which is represented by the title element in Listing 8 (p. 30) *.)*

The line elements

The first SVG element named **line** in Listing 8 (p. 30) corresponds to the Java object named **floor** in Listing 7 (p. 29) . The second SVG element named **line** in Listing 8 (p. 30) corresponds to the Java object named **wall** in Listing 7 (p. 29) .

Attributes of the line elements

The stroke and stroke-width attributes in Listing 8 (p. 30) should already be familiar to you as should the coordinate attributes named x1, y1, x2, and y2.

Unlike the SVG code for the rect element in Listing 6 (p. 27) , where there was only one pair of coordinate attributes named x and y, a line element as shown in Listing 8 (p. 30) requires two sets of coordinate attributes. Therefore, the two sets are distinguished from one another by appending a 1 and a 2 to the basic attribute names of x and y *(x1, y1, x2, and y2)* .

If you compare the coordinate attributes in Listing 8 (p. 30) with the coordinate values in the Java code in Listing 7 (p. 29) , and convert from inches to SVG units, you should find that they match.

2.3.6.5 Draw more rectangles

The Java code in Listing 9 (p. 32) causes four more rectangles to be drawn that represent the following objects in the drawing:

- The top of the table on which two rectangular masses are setting.
- The table leg that supports the rightmost end of the table. *(The leftmost end is attached to the wall.)*

- One of the rectangular masses, referred to as Mass C. *(This mass sets on top of the table closest to the wall on the left end of the table.)*
- The other rectangular mass referred to as Mass B. *(This mass sets on top of the table closest to the rightmost end of the table away from the wall.)*

Listing 9 . Draw more rectangles.

```
//Draw the table top.
Element tableTop = SvgLib21.makeRect(svg,
                                     ns,
                                     "rectangle",
                                     1.0,
                                     3.0,
                                     7.0,
                                     1.0
                                     );


//Draw the table leg.
Element tableLeg = SvgLib21.makeRect(svg,
                                     ns,
                                     "rectangle",
                                     6.5,
                                     0.5,
                                     0.5,
                                     2.5
                                     );


//Draw Mass C
Element massC = SvgLib21.makeRect(svg,
                                     ns,
                                     "rectangle",
                                     2.0,
                                     4.0,
                                     2.0,
                                     2.0
                                     );


//Draw Mass B
Element massB = SvgLib21.makeRect(svg,
                                     ns,
                                     "rectangle",
                                     5.0,
                                     4.0,
                                     2.0,
                                     2.0
                                     );
```

Table 2.11

The corresponding SVG code

The Java code in Listing 9 (p. 32) causes the four **rect** elements shown in Listing 10 (p. 33) to be created in the output SVG code.

Listing 10 . SVG code to draw more rectangles.

```
<rect fill="none" stroke="black" stroke-width="1"
 x="90" y="405" width="630" height="90">
  <title>rectangle</title>
</rect>

<rect fill="none" stroke="black" stroke-width="1"
 x="585" y="495" width="45" height="225">
  <title>rectangle</title>
</rect>

<rect fill="none" stroke="black" stroke-width="1"
 x="180" y="225" width="180" height="180">
  <title>rectangle</title>
</rect>

<rect fill="none" stroke="black" stroke-width="1"
 x="450" y="225" width="180" height="180">
  <title>rectangle</title>
</rect>
```

Table 2.12

You already know all about drawing rectangles, so no further explanation of the code in Listing 9 (p. 32) and Listing 10 (p. 33) should be needed.

2.3.6.6 Draw a polygon

The next task is to draw a triangular object that represents Mass C in the drawing. This is accomplished by the Java code in Listing 11 (p. 33) that causes a polygon, in the shape of a triangle, to be drawn.

Listing 11 . Draw a polygon.

```
//Draw Mass A
Element massA = SvgLib21.makePolygon(
          svg,
          ns,
          "polygon",
          new double[]{
            7.5, 1.0,//x-y coordinate pair
            9.5, 1.0,//x-y coordinate pair
            8.5, 2.5 //x-y coordinate pair
          });
```

Table 2.13

What can you modify?

You can modify the following items in the Java code shown in Listing 11 (p. 33) .

- The name of the object *(**massA** in Listing 11* (p. 33)).
- The title *(**"polygon"** in Listing 11 (p. 33)).*
- The number of x-y coordinate pairs in the list of x-y coordinate pairs along with the values of the coordinates.

What is a polygon?

In SVG terminology, a polygon is a drawing that consists of a series of points in two-dimensional space connected by line segments. An additional line segment is automatically drawn from the last point to the first point. For example, triangles, pentagons, and hexagons are polygons.

What is a polyline?

In SVG terminology, a polyline is exactly like a polygon except that a line segment is not automatically drawn to connect the last point to the first point.

There are no polyline elements in this example drawing, but there is one in the program for a different drawing shown in Listing 27 (p. 50) . The Java code for drawing a polyline is the same as the Java code for drawing a polygon. The difference between the two occurs when the SVG processor draws the shapes and either does, or does not automatically connect the last point to the first point.

The most versatile shape

The polyline is the most versatile of all of the basic shapes. With enough patience, it can be used to draw any shape that can be drawn with curved lines. To draw a curved line using polyline elements, approximate it using a large number of short line segments. For example, polyline elements provide an ideal mechanism for drawing the kind of shapes that are commonly referred to as "curves" in math, physics, and engineering courses. By this, I mean a drawing that shows how a dependent variable behaves relative to an independent variable.

Drawing the polygon *(or the polyline)*

Getting back to the Java code in Listing 11 (p. 33) , you can insert any number *(two or more)* of x-y coordinate-pairs inside the curly brackets *(but you must insert them in pairs)* . Line segments will be drawn from the first coordinate location to the second, from the second to the third, and so on. *(Of course there need to be two or more coordinate pairs in order for things to make sense.)*

If you examine the coordinate values shown in Listing 11 (p. 33) , you will see that they define the vertices of a triangle whose base is parallel to the horizontal axis. Since this is a polygon, it will be drawn as a closed triangle with lines for all three sides. If it were a polyline, it would not be drawn as a closed triangle. Instead, only two lines would be drawn and the third side of the triangle would be open.

SVG code to draw a polygon

Listing 12 (p. 34) shows the SVG code produced by the Java code in Listing 11 (p. 33) .

Listing 12 . SVG code to draw a polygon.

```
<polygon stroke="black" stroke-width="1" fill="none"
 points="675 675 855 675 765 540 ">
  <title>polygon</title>
</polygon>
```

Table 2.14

The attribute named points

The polygon element in Listing 12 (p. 34) contains a new attribute name that you haven't seen before: **points** .

As you have probably figured out by now, the value of the attribute named points is a series of numeric values, separated by spaces, that represent the x-y coordinate pairs in Listing 11 (p. 33) , converted from inches to SVG units.

Hopefully by now you are beginning to see patterns that relate the Java code to the resulting SVG code.

2.3.6.7 Draw the rectangular pulley support

The drawing in Figure 2 (p. 19) shows a pulley connected to the rightmost end of the table. The drawing of the pulley consists of a rectangle as the support member and a circle as the pulley wheel. The Java code to draw the pulley support is shown in Listing 13 (p. 35) .

Listing 13 . Draw the rectangular pulley support.

```
    //Draw pulley support
Element pullySupport = SvgLib21.makeRect(svg,
                                         ns,
                                         "rectangle",
                                         7.883,
                                         3.595,
                                         0.392,
                                         1.06
                                         );
```

Table 2.15

The Java code in Listing 13 (p. 35) simply draws another rectangle, so it shouldn't need further explanation.

2.3.6.8 Draw a circle

The Java code in Listing 14 (p. 35) draws a circle to serve as the pulley wheel in the drawing.

Listing 14 . Draw a circle.

```
//Draw the pulley wheel.
Element pulleyWheel = SvgLib21.makeCircle(
          svg,
          ns,
          "circle",
          8.05, //x-coordinate of center of circle
          4.56, //y-coordinate of center of circle
          0.45  //radius of circle
          );
```

Table 2.16

What are you allowed to change?

There are only four things that you are allowed to change in the code in Listing 14 (p. 35) :

1. The name of the **circle** object, which is **pulleyWheel** in Listing 14 (p. 35) .
2. The title *(**"circle"** in Listing 14 (p. 35))* .
3. The value of the x-coordinate of the center of the circle.
4. The value of the y-coordinate of the center of the circle.
5. The radius of the circle.

You will need to examine the coordinate values for the center of the circle along with the radius of the circle and imagine how the position and size of the circle relates to the right end of the table top in Figure 2 (p. 19) . *(Or hopefully, get a tactile version of the drawing and explore it by touch.)*

SVG code to draw a circle

Listing 15 (p. 36) shows the SVG code produced by the Java code in Listing 14 (p. 35) .

Listing 15 . SVG code to draw a circle.

```
<circle fill="none" stroke="black" stroke-width="1"
 cx="725" cy="355" r="41">
  <title>circle</title>
</circle>
```

Table 2.17

By now, you should see the pattern and there should be no need to explain the relationship between the attributes of the circle element and the parameter values in the call to the **makeCircle** method in Listing 14 (p. 35) .

2.3.6.9 Draw more lines

The earlier section titled What does the image show (p. 19) describes how three cords are used to tie the masses to one another and to tie the leftmost mass to the wall. This is accomplished using the four calls to the Java **makeLine** method shown in Listing 16 (p. 37) .

Listing 16 . Draw more lines.

```
//Draw cord from wall to Mass C
Element cordR = SvgLib21.makeLine(svg,
                                  ns,
                                  "line",
                                  1.0,
                                  5.0,
                                  2,
                                  5.0
                                  );


//Draw cord from Mass C to Mass B
Element cordQ = SvgLib21.makeLine(svg,
                                  ns,
                                  "line",
                                  4.0,
                                  5.0,
                                  5.0,
                                  5.0
                                  );


//Draw cord from Mass B to the top of the pulley.
Element cordP1 = SvgLib21.makeLine(svg,
                                   ns,
                                  "line",
                                   7.0,
                                   5.0,
                                   8.05,
                                   5.0
                                   );


//Draw the cord from the right side of the pulley to
// Mass A
Element cordP2 = SvgLib21.makeLine(svg,
                                   ns,
                                  "line",
                                   8.5,
                                   4.5,
                                   8.5,
                                   2.5
                                   );
```

Table 2.18

There is nothing new in Listing 16 (p. 37) .

SVG code to draw more lines

The Java code in Listing 16 (p. 37) produces the SVG code shown in Listing 17 (p. 38) .

Listing 17 . SVG code to draw more lines.

```
<line stroke="black" stroke-width="1"
 x1="90" y1="315" x2="180" y2="315">
  <title>line</title>
</line>

<line stroke="black" stroke-width="1"
 x1="360" y1="315" x2="450" y2="315">
  <title>line</title>
</line>

<line stroke="black" stroke-width="1"
 x1="630" y1="315" x2="725" y2="315">
  <title>line</title>
</line>

<line stroke="black" stroke-width="1"
 x1="765" y1="360" x2="765" y2="540">
  <title>line</title>
</line>
```

Table 2.19

There is also nothing new in Listing 17 (p. 38) .

2.3.6.10 Change line thicknesses

With the exception of text to be discussed shortly, we have now created an element for every object that we need in our drawing.

As you may have noticed, the value of the **stroke-width** attribute for every element created so far has been "1". That is the default value. We may not be satisfied with that default value in all cases. We may prefer that some of the lines that describe the geometrical objects be thicker than lines that describe other geometrical objects.

My SVG graphics library provides a method named **setStrokeWidth** that we can use to adjust the stroke-width attribute values for an element before the final output SVG file is written. *(As you will see if you examine Listing 27 (p. 50) , the library provides methods that let you adjust other attribute values as well.)*

Set the stroke-width

The line thickness is controlled by the value of the **stroke-width** attribute in the SVG element that causes the geometrical object to be drawn.

Listing 18 (p. 39) contains a series of Java statements that set new values for the stroke-width attribute for each of a variety of objects. Since the statements are all essentially the same, I will discuss only the first one.

Listing 18 . Set the stroke-width attribute value.

```
SvgLib21.setStrokeWidth(
     border,//name of the object of interest
     0.03   //new value for the stroke-width attribute
     );

SvgLib21.setStrokeWidth(floor,0.1);
SvgLib21.setStrokeWidth(wall,0.1);
SvgLib21.setStrokeWidth(tableTop,0.03);
SvgLib21.setStrokeWidth(tableLeg,0.03);
SvgLib21.setStrokeWidth(massC,0.05);
SvgLib21.setStrokeWidth(massB,0.05);
SvgLib21.setStrokeWidth(massA,0.05);
SvgLib21.setStrokeWidth(pullySupport,0.05);
SvgLib21.setStrokeWidth(pulleyWheel,0.05);
SvgLib21.setStrokeWidth(cordR,0.03);
SvgLib21.setStrokeWidth(cordQ,0.03);
SvgLib21.setStrokeWidth(cordP1,0.03);
SvgLib21.setStrokeWidth(cordP2,0.03);
```

Table 2.20

What are you allowed to change?

There are only two things that you can change in a call to the **setStrokeWidth** method as shown in Listing 18 (p. 39) :

- The name of the Java object of interest *(**border** for the first call in Listing 18* (p. 39)).
- The new value for the **stroke-width** attribute in the SVG element that corresponds to the specified Java object *(0.03 for the first call in Listing 18)* .

Recall that **border** is the name of the Java object that is used to draw a rectangular border on the canvas *(see Listing 5* (p. 27)).

Recall also that the initial default value of the stroke-width attribute for the rect element was equal to "1" *(see Listing 6* (p. 27)).

To see the effect of the first call to the **setStrokeWidth** method in Listing 18 (p. 39) , go back and take a look at the final SVG output code in Listing 1 (p. 21) . Pay particular attention to the first **rect** element.

Modified stroke-width attribute value

I have reproduced that **rect** element in Listing 19 (p. 39) for convenient viewing.

Listing 19 . Modified stroke-width attribute value.

continued on next page

```
<rect fill="none" stroke="black" stroke-width="3"
 x="1" y="1" width="987" height="762">
  <title>rectangle</title>
</rect>
```

Table 2.21

As mentioned earlier, the default value for the stroke-width attribute of the rect element shown in Listing 6 (p. 27) was "1". However, after I added the first call to the **setStrokeWidth** method in Listing 18 (p. 39) and re-ran the program, the attribute value was changed to "3" corresponding approximately to a line thickness of 0.03 inch.

If you compare the parameter values to the remaining calls to the **setStrokeWidth** method in Listing 18 (p. 39) with the final values of the stroke-width attribute values in Listing 1 (p. 21) , they should all correspond accordingly.

2.3.6.11 Draw text

All that we have left to do in this program is to draw some text and write the output SVG file.

The Java code in Listing 20 (p. 40) can be used to draw one line of text.

Listing 20 . Draw text.

```
//Draw text
Element textA = SvgLib21.makeText(
    svg,
    ns,
    2.5,    //x-coordinate of beginning of text
    3.3,    //y-coordinate of beginning of text
    "arial",//font-family (optionally "")
    32,     //font size in points
    "Friction free table." //text to be drawn
    );
```

Table 2.22

Usage

Begin by setting the name of the Java object *(**textA** in Listing 20* (p. 40) *)* to the name that you prefer.

Then set the x and y coordinate values for the location in the drawing where the text will be drawn. This specifies the location of the lower-left corner of the first character in the text string.

Then set the name of the font family *(" **arial** " in Listing 20* (p. 40)) or optionally leave that name blank. If no name is set *("")* or an invalid name is set, a default font family will be used.

Then set the font size to the desired font size in points *(**32** in Listing 20* (p. 40)).

Finally, set the last parameter to the string of text that is to be drawn.

Make sure that you include the quotation marks in both cases where they are used in Listing 20 (p. 40) .

Don't make any other changes to the code shown in Listing 20 (p. 40) .

Setting the font style and font weight

By default the text is **normal** *(not bold, not italic, etc.)* . My SVG graphics library provides methods by which you can change the weight and style of the text, such as making it bold and italic. *(See usage instructions for those methods in Listing 27 (p. 50) .)*

SVG code to draw text

Listing 21 (p. 41) shows the SVG code produced by the Java code in Listing 20 (p. 40) .

Listing 21 . SVG code to draw text.

```
<text fill="black" stroke="black" x="225" y="468"
  font-size="32" font-family="arial">
    Friction free table.
</text>
```

Table 2.23

You should be able to recognize all of the attributes and their values shown in Listing 21 (p. 41) .

The content of an element

There is something in Listing 21 (p. 41) that was not previously discussed in any detail – content. The actual text, *Friction free table* , is not an attribute. Instead, it is what is called **content** in XML/SVG.

In addition, many of the earlier SVG code fragments had elements whose content consisted of a **title** element, which in turn had text content with words like **line** , **polygon** , etc.

This isn't particularly important to you as a user of my SVG graphics library. However, if you elect to create drawings by writing raw SVG code, this is something that you will need to study a little more deeply.

2.3.6.12 Write the output file

If you elect to create your drawings by writing raw SVG code in a text editor, all you need to do to write the output file is to save the text file from inside the editor.

However, if you elect to use my SVG graphics library and create your drawings by writing Java code, you need to include the code shown in Listing 22 (p. 41) to cause the final output SVG file to be written.

(Don't include the .svg extension in the file name that you specify. It is added automatically.)

Listing 22 . Write the output file.

```
//WRITE OUTPUT FILE
SvgLib21.writePrettyFile("Svg21a",doc);
```

Table 2.24

This must be the last statement that you write in your program. Otherwise, you will get an incomplete file.

Set the value inside the quotation marks to the desired path and filename for the file.

(Don't include the .svg extension in the file name that you specify. It is added automatically.)

Don't make any other changes to the code in Listing 22 (p. 41) .

The Java code in Listing 22 (p. 41) writes the output file with the name **Svg21.svg** in the folder from which the program is being executed *(the current folder)* . Because it is being written into the current folder, it isn't necessary to provide a path.

2.3.6.13 The remaining Java code

The remaining code that you will need to include in your Java program file is shown in Listing 23 (p. 42) .

Simply copy this code, without modifications, and paste it at the end of your file.

Listing 23 . The remaining Java code.

```
    //ONLY THE CODE ABOVE THIS LINE CAN BE MODIFIED
    //################################################//
    //DO NOT MODIFY ANY OF THE FOLLOWING CODE.
  }//end main
  //-------------------------------------------------//

  //Create a String variable containing the namespace
  // URI to reduce the amount of typing that is required
  // later. Note that the variable name is short and
  // easy to type.
  static String ns = "http://www.w3.org/2000/svg";

  //For clarity, create strings containing the name of
  // the element that is constrained by the DTD (the
  // root element), the Public ID of the DTD, and the
  // System ID of the DTD.
  static String dtdConstrainedElement = "svg";
  static String dtdPublicID = "-//W3C//DTD SVG 1.1//EN";
  static String dtdSystemID =
     "http://www.w3.org/Graphics/SVG/1.1/DTD/svg11.dtd";


  static DocType docType = new DocType(
         dtdConstrainedElement,dtdPublicID,dtdSystemID);

}//end class
```

Table 2.25

2.3.7 The SVG graphics library

The Java code for my SVG graphics library is provided in Listing 26 (p. 48) . All you need to do is copy this code into a file named **SvgLib21.java** and place that file in the same folder with your program code. I will explain later what you need to do after that.

> The graphics library in Listing 26 was updated on 08/11/11 to add new features. However, the text in this module has not been updated to illustrate the use of those new features. The downloadable documentation has been updated to include those features.

IVEO compatibility

Note that the library was revised on 08/07/11 to add a **title** element to each shape element and to the **svg** element. As a result, when the output SVG file is opened in the IVEO viewer [45] and you click on a shape, information about that shape is spoken by IVEO. When you open the file in the IVEO viewer [46] , the text content of the **title** element belonging to the **svg** element is spoken.

Note that you must be careful about the order in which you add the shapes to the drawing. For example, a rectangle that is added after a line is added can cover the line and prevent the information about the line from being spoken by IVEO even though the rectangle may be transparent. Therefore, for IVEO compatibility, you must not allow one shape object to cover another shape object.

Graphics library documentation

Click here [47] to download a zip file named **SvgLib21Docs.zip** containing standard Java documentation for the graphics library.

To view the documentation, YOU MUST EXTRACT the contents of the zip file into an empty folder. *(Forgive me for shouting, but my students are constantly forgetting to extract material from a zip file before they try to use that material.)* Then open the file named **index.html** in your browser. If you are unfamiliar with the format of the documentation, the explanation at http://www.apl.jhu.edu/~hall/java/beginner/api.html [48] might be helpful.

2.3.8 Another sample program

Another sample program that produces a different drawing is provided in Listing 27 (p. 50) . This program contains extensive usage instructions in the form of comments for all of the capabilities of my SVG graphics library. If you encounter any difficulties using the library, you should consult the instructions in that program.

2.4 Writing, compiling, and running Java programs

2.4.1 Writing Java code

Fortunately, writing Java code is straightforward. You can write Java code using any plain text editor. You simply need to cause the output file to have an extension of .java.

There are a number of high-level Integrated Development Environments *(IDEs)* available, such as Eclipse and NetBeans, but they tend to be overkill for the relatively simple Java programs described in this module.

There are also some low-level IDEs available, such as JCreator and DrJava, which are very useful for sighted students. However, I don't know anything about their level of accessibility. I normally use a free version of JCreator, mainly because it contains a color-coded editor, but that feature wouldn't be useful for a blind student.

So, just find an editor that you are happy with and use it to write your Java code.

2.4.2 Preparing to compile and run Java code

Perhaps the most complicated thing is to get your computer set up for compiling and running Java code in the first place.

2.4.2.1 The java development kit *(JDK)*

You will need to download and install the free Java JDK from http://www.oracle.com/technetwork/java/javase/downloads/index.html [49]

[45] http://www.viewplus.com/solutions/touch-audio-learning/
[46] http://www.viewplus.com/solutions/touch-audio-learning/
[47] http://cnx.org/content/m39607/latest/SvgLib21Docs.zip
[48] http://www.apl.jhu.edu/~hall/java/beginner/api.html
[49] http://www.oracle.com/technetwork/java/javase/downloads/index.html

I notice that Java SE 7 has been released very recently. However, my SVG graphics library was tested using Java SE 6 Update 26, and that is what I would recommend. Also there is a 64-bit version, but my library has not been tested with the 64-bit version. If your operating system will accommodate it, I recommend that you stick with the 32 bit version just in case the 64-bit version is not compatible with my library.

You will find installation instructions on the download page shown above.

2.4.2.2 JDOM version 1.1.1

You will also need to download and install a class library named JDOM 1.1.1 at http://www.jdom.org/ [50]

When you do that download, you will receive a zip file that also contains some installation instructions. However, my experience is that those installation instructions are overkill, at least that is the case on a Windows machine.

All you need to do is to extract the file named **jdom.jar** from the zip file *(look for it in the build directory in the zip file)* , store it somewhere on your disk, and put it on the classpath at compile time and runtime. *(I will explain the bit about the classpath a little later.)*

In my case, I'm running Windows Vista Premium Home Edition and I elected to store the **jdom.jar** file in the **following folder** :

C:\Program Files (x86)\Java\jdom-1.1.1\build\jdom.jar

It doesn't really matter where you store it as long as you know how to specify that location in the classpath later.

2.4.3 Compiling and running Java code

There are a variety of ways to compile and run Java code. The way that I will describe here is the most basic and, in my opinion, the most reliable. These instructions apply to a Windows operating system. If you are using a different operating system, you will need to translate the instructions to your operating system.

Write your Java program

Begin by writing your Java program into a text file with an extension of .java. Save it in a folder somewhere on your disk. Make sure that you adhere to the earlier instructions regarding the name of the class and the name of the file, and remember that everything is case sensitive.

Create a file named **SvgLib21.java** that contains an exact replica of the Java code in Listing 26 (p. 48) . Store that file in the same folder as your Java program.

Create a batch file *(or whatever the equivalent is for your operating system)* containing the text shown in Listing 24 (p. 44) .

Then execute the batch file.

If everything is successful, a Firefox window should open showing your drawing ready to be printed.

[50] http://www.jdom.org/

Listing 24 . Windows batch file.

```
cls

del *.class
del Svg21a.svg
javac -cp ".;C:\Program Files (x86)\Java\jdom-1.1.1\build\jdom.jar" SvgLib21.java
javac -cp ".;C:\Program Files (x86)\Java\jdom-1.1.1\build\jdom.jar" Svg21a.java
java -cp ".;C:\Program Files (x86)\Java\jdom-1.1.1\build\jdom.jar"  Svg21a
start Firefox.exe Svg21a.svg
pause
```

Table 2.26

Comments regarding the batch file

Note that the text inside the quotation marks is the same as the location where I stored the file (p. 44) named **jdom.jar** . In fact, it is identical except that ".;" appears before that location in Listing 24 (p. 44) . You need to cause your batch file to identify the location of the file named **jdom.jar** on your system just like I did in Listing 24 (p. 44) .

Do not modify the text that reads **"SvgLib21.java"** in Listing 24 (p. 44) .

Replace the text that reads **"Svg21a"** in all three locations in Listing 24 (p. 44) with the name of your program. Note, however, that the first time it appears, it is specifying the name of the output SVG file. In case you elected to give your output SVG file a different name than the name of your program, you need to insert that name in place of **Svg21a.svg** .

Starting the browser automatically

Listing 24 (p. 44) also assumes that you have Firefox 5 or later installed on your system and starts it running automatically . *(It will probably also work with earlier versions of Firefox. However, I have been unable to cause either Google Chrome or IE 9 to start automatically using this approach.)*

In any event, the last line of text before the pause can be deleted from Listing 24 (p. 44) with no harmful effects. It simply won't start the browser automatically if you delete that text. In that case, you will have to manually open the output SVG file in the browser *(or in some other SVG processor program)* in order to print it. *(Opening the SVG file manually seems to work in Firefox 5, IE 9, and Google Chrome 12.)*

Don't delete the pause command

The **pause** command causes the command-line window to stay on the screen. You will need to examine the contents of the window if there are errors when you attempt to compile and run your program, so don't delete the pause command.

Translate to other operating systems

Remember, the format of the batch file in Listing 24 (p. 44) is a Windows format. If you are using a different operating system, you will need to translate the information in Listing 24 (p. 44) into the correct format for your operating system.

2.5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

2.6 Complete program listings

Complete listings of the three programs discussed in this module are provided in Listing 25 (p. 46) , Listing 26 (p. 48) , and Listing 27 (p. 50) .

Listing 25 . The program named Svg21a.java.

```
/*File Svg21a.java,
Copyright 2011, R.G.Baldwin

Revised 08/07/11 to support the addition of a title
parameter to each element for IVEO compatibility. If the
output SVG file is opened in IVEO, the title for the svg
element is spoken when the user opens the file, and the
titles for the individual elements are spoken when the
user touches a corresponding shape on the touchpad or
clicks on that shape on the screen. If the SVG file won't
be used with IVEO, just leave the title strings unchanged.

This program requires access to the file
named SvgLib21.java

This is a demonstration program.

This program uses JDOM 1.1.1 and an SVG graphics library
class of my own design named SvgLib21 to draw an
abbreviated version of the mass-pulley system shown in Figure 4
of the module that you will find at
http://cnx.org/contents/16d1b592-5126-458f-883e-3ae872cc18c0.

Only one line of text is drawn by this program. Otherwise,
the drawing produced by this program is the same as
the drawing in the file named Phy1150a1.svg used in
that module titled Force and Motion -- Units of Force. The
original drawing was produced by a sighted person using
Inkscape. This drawing was produced by a sighted person
using Baldwin's svg drawing library.

Tested using J2SE 6, JDOM 1.1.1, and Firefox 5
running under Windows Vist Home Premium Edition.
**********************************************************/
import java.io.*;
import org.jdom.*;

public class Svg21a{
  public static void main(String[] args){

    //DO NOT MODIFY ANY OF THE CODE ABOVE THIS LINE.
    //################################################//
    //ONLY THE CODE BELOW THIS LINE CAN BE MODIFIED

    //CREATE A DRAWING CANVAS
    //This must be the first statement that you write in
    // the program and it must appear only once.
    //The following statement creates a canvas that is
    // 8.5x11 inches in size in a landscape layout.
    Element svg = SvgLib21.makeSvg(ns,
                                   "Document Title",
                                   11,//width
                                   8.5  //height
                                   );
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 2.27

Note that the strange comments in Listing 26 (p. 48) were placed there so that the program named **javadoc** could be used to produce standard Java documentation [51] for the graphic library.

[51] http://cnx.org/content/m39607/latest/SvgLib21Docs.zip

Listing 26 . The program named SvgLib21.java.

```
import java.io.*;
import org.jdom.*;
import org.jdom.output.XMLOutputter;
import org.jdom.output.Format;

/**
File SvgLib21.java
<p>
Copyright 2011, R.G.Baldwin
<p>
Revised 08/11/11 to add a method named setDescription
that can be called to add a <desc> element to any other
element. The description can be spoken in IVEO by first
selecting the element an pressing Ctrl+d.
<p>
Revised 08/07/11 to add a parameter for the title to
allow the user to pass in a string for the title element.
<p>
Revised 08/06/11 to add a title element to each shape
element. As a result, when the SVG file is
opened in IVEO and you click on the shape, the name of
the shape is spoken by IVEO. Note, however, that you
must be careful about the order in which you add the
shapes to the drawing. For example, a rectangle that
is added after a line is added can cover the line
and prevent the information about the line from being
spoken even though the rectangle may be transparent.
Therefore, for IVEO compatibility, you must not allow
one shape object to cover another shape object.
<p>
DESCRIPTION
<p>
This is a graphics library that is designed to eliminate
much of the pain involved in writing JDOM code to
create SVG output. The library contains individual
static methods that are used to construct and return
all of the SVG basic shape elements, text elements,
description elements and comment elements.
<p>
In addition there are methods to set various attributes
on shape elements and text elements.

This library provides methods to instantiate, return,
and manipulate the following types of SVG elements:
<ul>
<li>line</li>
<li>rect</li>
<li>circle</li>
<li>ellipse</li>
<li>polyline</li>
<li>polygon</li>
<li>desc</li>
<li>comment</li>
<li>text</li>
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 2.28

Listing 27 . The program named Svg21.java.

```
/*File Svg21.java,
Copyright 2011, R.G.Baldwin

Revised 08/07/11 to support the addition of a title
parameter to each element for IVEO compatibility. If the
output SVG file is opened in IVEO, the title for the svg
element is spoken when the user opens the file, and the
titles for the individual elements are spoken when the
user touches a corresponding shape on the touchpad or
clicks on that shape on the screen. If the SVG file won't
be used with IVEO, just leave the title strings unchanged.

This program requires access to the file
named SvgLib21.java

This is a demonstration program.

This program uses JDOM 1.1.1 and an SVG graphics library
class of my own design named SvgLib21 to create an XML
file named Svg21.svg that draws at least one of each of
the following six basic SVG shapes when rendered in an
SVG graphics engine such as Firefox 5.

    * rectangle
    * circle
    * ellipse
    * line
    * polyline
    * polygon

In addition, the program illustrates the creation of the
following two types of elements in the output SVG file.

    * description
    * comment

The main purpose is to demonstrate how to create an SVG
file using the JDOM SVG graphics library that can be
displayed using Firefox 5 or IE 9. The file can also
be opened in other programs such as InkScape and IVEO.

All coordinate values are in inches and fractions of
inches.

One svg unit equals approximately 0.011 inch. An svg
unit is not necessarily the same size as a pixel on your
monitor or your printer. However, dimensions specified
in inches should be very close when you print the image.

By default, all lines that define the geometric shapes
are black and are one pixel wide. This can be changed
by calling appropriate methods to change attribute values.

By default, the fill attribute for all geometric shapes
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 2.29

2.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: If You Can Imagine It, You Can Draw It using SVG
- File: Phy1002.htm
- Revised: 09/29/15
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · SVG
  - · scalable vector graphics

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 3

Phy1004: Manual Creation of Tactile Graphics[1]

3.1 Table of Contents

- Preface (p. 53)
  - General (p. 53)
  - Prerequisites (p. 54)
  - Viewing tip (p. 54)
    - Figures (p. 54)
  - Supplemental material (p. 54)
- Discussion (p. 55)
  - Scalable vector graphics (p. 55)
  - Download the svg file (p. 55)
  - The need for graphics in physics (p. 55)
  - A range of options (p. 55)
  - Manual embossing (p. 56)
- Resources (p. 67)
- Miscellaneous (p. 67)

3.2 Preface

3.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

This module describes one of the ways for creating tactile graphics for the modules in this collection.

[1] This content is available online at <http://cnx.org/content/m38546/1.9/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

3.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

As you will see in this module, there are some additional requirements for creating and exploring tactile graphics.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

3.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

3.2.3.1 Figures

- Figure 1 (p. 60) . Mirror image from file 1.svg.
- Figure 2 (p. 64) . Normal image from file 1.svg.
- Figure 3 (p. 65) . Text values for Braille keys in file 1.svg.

3.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

3.3 Discussion

I will begin this discussion with a discussion of a type of graphics file known as Scalable Vector Graphics file.

3.3.1 Scalable vector graphics

There are a variety of different formats for storing graphics information in disk files. One of those formats, and the one that I have adopted for this collection of modules, is called Scalable Vector Graphics [13] . Files containing Scalable Vector Graphics information typically have an extension of svg, and are often referred to simply as svg files.

You will find a good explanation of Scalable Vector Graphics in the Wikipedia article titled Scalable Vector Graphics [14] .

Can be scaled without corruption

For my purposes, the major advantage of using svg files is that they can be enlarged or reduced in size without corrupting the image. For example, the images in the svg files that I will provide are designed to be printed on 8.5x11 inch paper stock. However, if you have access to a printer that can handle larger paper, you can use a program such as the free svg editing program named Inkscape [15] to enlarge the images without corrupting them. You can then print the larger images on larger paper stock.

3.3.2 Download the svg file

You will need to download an svg graphics file named 1.svg to complete the work in this module. Click this link to download a zip file named 1004.zip [16] containing the file named 1.svg.

3.3.3 The need for graphics in physics

It is very difficult to learn introductory physics without having access to a variety of pictures, charts, and diagrams. Many of you are likely to need those materials in tactile form. While we can transmit words, sounds, and pictures via the Internet, we still don't have the ability to transmit tactile graphics via the Internet.

This means that it will be necessary for you to use the svg files that I provide and to make your own arrangements for having those files converted into tactile graphics.

3.3.4 A range of options

Depending on the resources that you have available, you have a range of options for the creation and exploration of tactile graphics.

My plan is to provide the necessary svg graphics files to produce tactile graphics for a range of different resources.

Machine embossing of tactile graphics

One of the available options for creating (and exploring) machine-embossed tactile graphics (on paper) is the IVEO Hands on Learning System [17] from ViewPlus Technologies [18] . (See the disclaimer (p. 67) below.)

The svg files that I will provide are intended to be compatible with the IVEO Learning System.

The opposite end of the budgetary spectrum

At the opposite end of the budgetary spectrum is the student whose only resource for tactile graphics is a human embosser using various tools from a tactile graphics kit [19] to manually emboss printed versions

[13] http://www.w3.org/TR/SVG/
[14] http://en.wikipedia.org/wiki/Scalable_Vector_Graphics
[15] http://inkscape.org/
[16] http://cnx.org/content/m38546/latest/1004.zip
[17] http://www.viewplus.com/solutions/touch-audio-learning/
[18] http://www.viewplus.com/products/braille-printers/
[19] http://www.fredshead.info/2006/01/tactile-graphics-kit-replacement-parts.html

of the svg files. The files (and the supplementary information that I will provide) are designed to support manual embossing as well as machine embossing. (The file that I will provide for this module is intended for manual embossing only. Future modules will include the necessary files for machine embossing.)

Steps

There are two steps involved in first creating and then using tactile graphics:

1. Embossing a paper copy of the graphic
2. Exploring the embossed copy of the graphic using touch, and in some cases, sound.

Embossing the graphic

There are at least two ways to emboss the paper copy of the graphic:

1. Manual embossing
2. Machine embossing using a graphics-compatible Braille printer such as those from ViewPlus Technologies [20] .

Exploring the graphic

There are at least two possibilities for exploring the graphic:

- Exploration using touch alone.
- Exploration using the IVEO Hands on Learning System with a ViewPlus touchpad, which adds sound to the mix.

Several combinations available

Depending on available resources, individual students might find themselves combining either of the two embossing methods with either of the two exploration methods. The files that I will provide for many of the modules will be designed to satisfy all four possible combinations.

However, my guess is that most students will find themselves in a situation where they are limited to manual embossing and exploration by touch only when they first begin studying the modules in this collection. Therefore, I will limit the discussion in this module to the use of the svg files for manual embossing. I will explain how to use the svg files for the other three combinations in future modules

3.3.5 Manual embossing

If there is at least one sighted person who is willing to assist you, you should be able to use the svg files that I will provide to create manually-embossed tactile graphics for the images in this collection. I will refer to that sighted person as "your assistant" in the following discussion.

Description of the scenario

This scenario assumes that you don't have access to an embossing printer and you don't have access to the computer and touchpad resources necessary to support the IVEO system. Therefore, you will need to arrange for an assistant to manually emboss the images for you. You will also need to explore the embossed image by touch alone, using the supplementary information that I will provide in this module.

The file named 1.svg

For this scenario, you will need to extract and print the file named 1.svg from the zip file mentioned earlier (p. 55) . You could use the free IVEO Viewer software to print it on an ordinary non-embossing printer. However, since this scenario has no IVEO involvement, it isn't necessary to use the IVEO Viewer software to print it. A simple alternative approach is to

- Ask the your assistant to open the file named 1.svg in either Firefox 5 (or later) or Internet Explorer 9 (or later).
- Set Page Setup on the File menu to Portrait or Landscape as appropriate.
- Select Print Preview

[20] http://www.viewplus.com/products/braille-printers/

- Use the print scaling capabilities of the browser to make the image as large as will fit on a single page.
- Print the file.

Another alternative

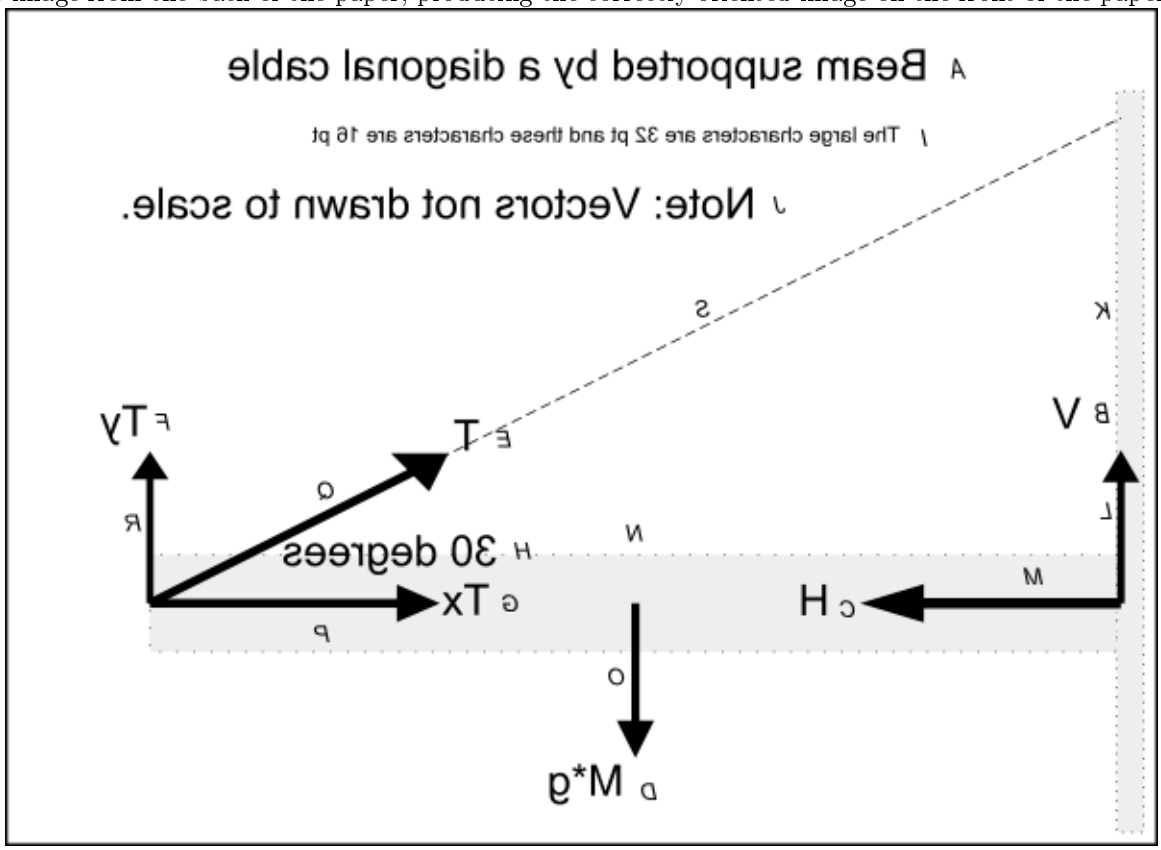
Another alternative, (which may do a better job of maintaining the actual size of the graphic than either browser mentioned above), is to print the file using a free svg drawing program named Inkscape (see http://inkscape.org/download/ [21]).

Inkscape can appear to be rather daunting when it first appears on the screen. However, the process of opening a file in Inkscape and printing the file can all be handled by making simple selections from the **File** menu.

More importantly, as I mentioned earlier, if your printer can accommodate paper that is wider than 8.5 inches, the Inkscape program can also be used to enlarge the image in the svg file to provide you with a larger tactile graphic image.

A scaled version of the graphic

Figure 1 (p. 60) shows a scaled version of the graphic contained in the file named 1.svg for the benefit of your assistant who will emboss the image. Note that this is a mirror image of the image that is to be presented to the student after embossing. Having a mirror image makes it possible for the assistant to emboss the image from the back of the paper, producing the correctly-oriented image on the front of the paper.



[21] http://inkscape.org/download/

A Beam supported by a diagonal cable
J The large characters are 32 pt and these characters are 16 pt
J Note: Vectors not drawn to scale.
S
K
B V
F Ty
E T
Q
R
N
H 30 degrees
L
M
G Tx
C H
P
O
D M*g

A Beam supported by a diagonal cable
I The large characters are 32 pt and these characters are 16 pt
J Note: Vectors not drawn to scale.
S
K
B V
E T
F Ty
Q
R
L
N
H 30 degrees
M
G Tx
C H
P
O
D M*g

Figure 1 . Mirror image from file 1.svg.

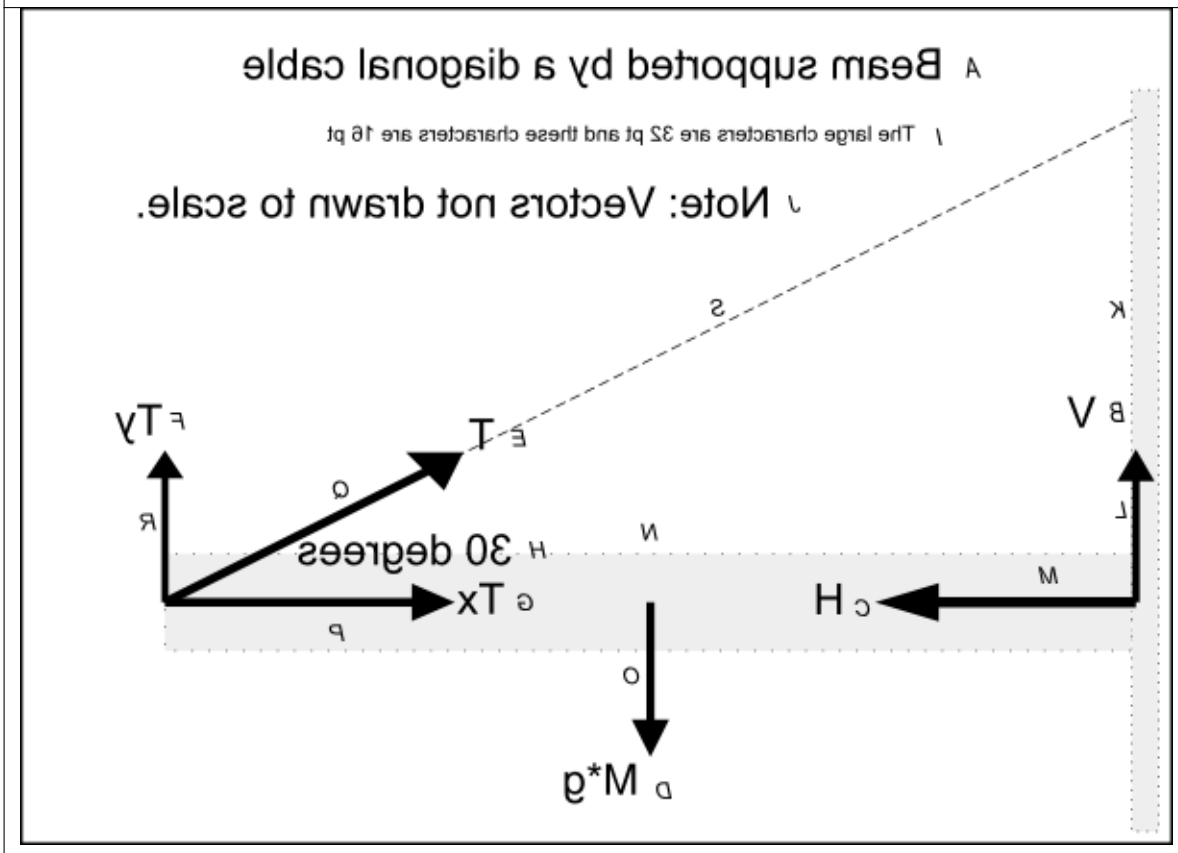
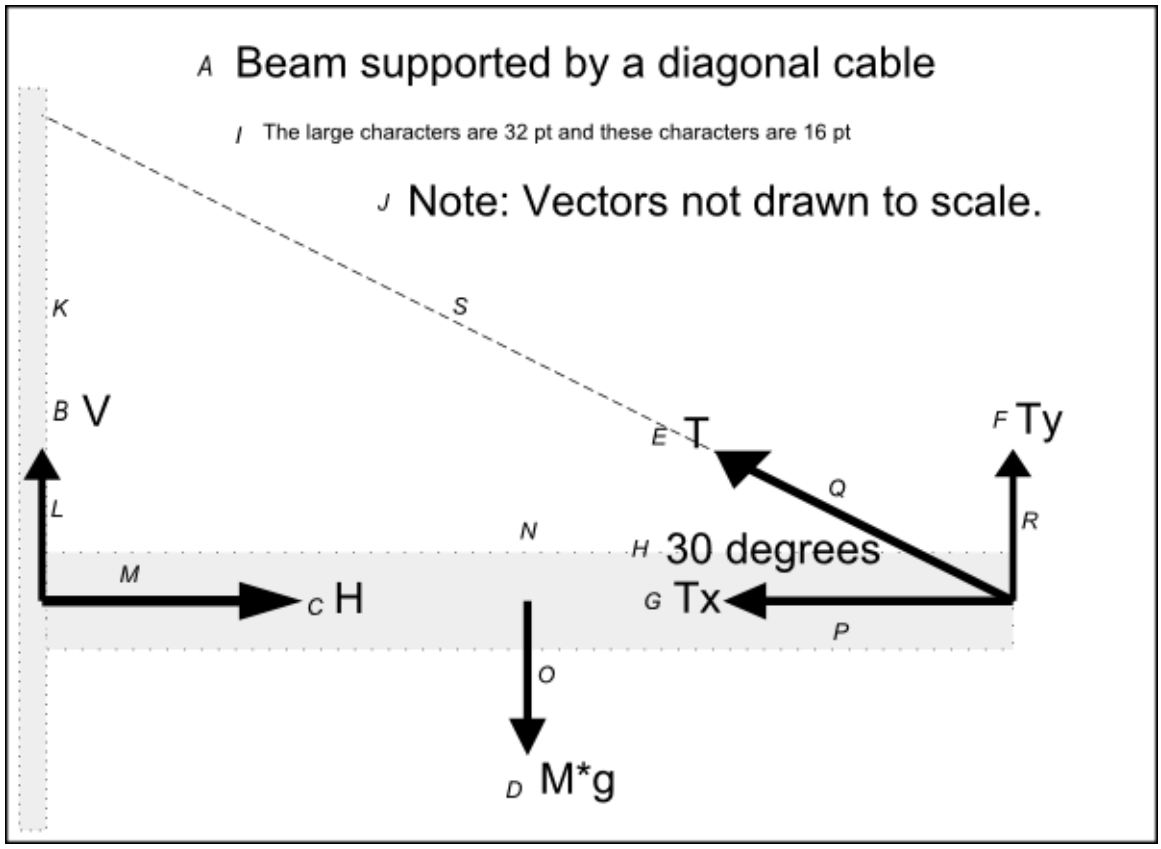
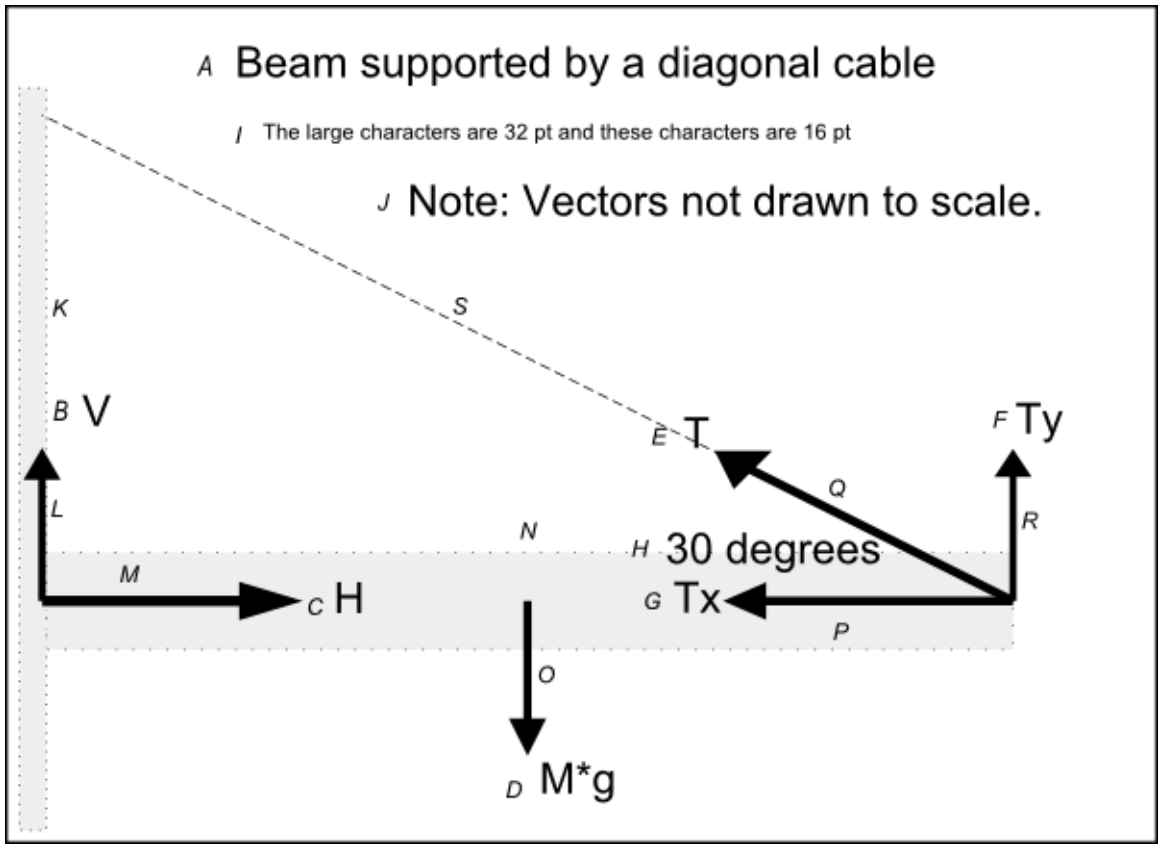


Table 3.1

Figure 2 (p. 64) shows the same image in normal (not mirror image) orientation.


A Beam supported by a diagonal cable
I The large characters are 32 pt and these characters are 16 pt
J Note: Vectors not drawn to scale.
K
S
B V
E T
F Ty
Q
L
N
H 30 degrees
R
M
C H
G Tx
P
O
D M*g


A Beam supported by a diagonal cable
I The large characters are 32 pt and these characters are 16 pt
J Note: Vectors not drawn to scale.
K
S
B V
E T
F Ty
Q
L
R
N
H 30 degrees
M
C H
G Tx
P
O
D M*g

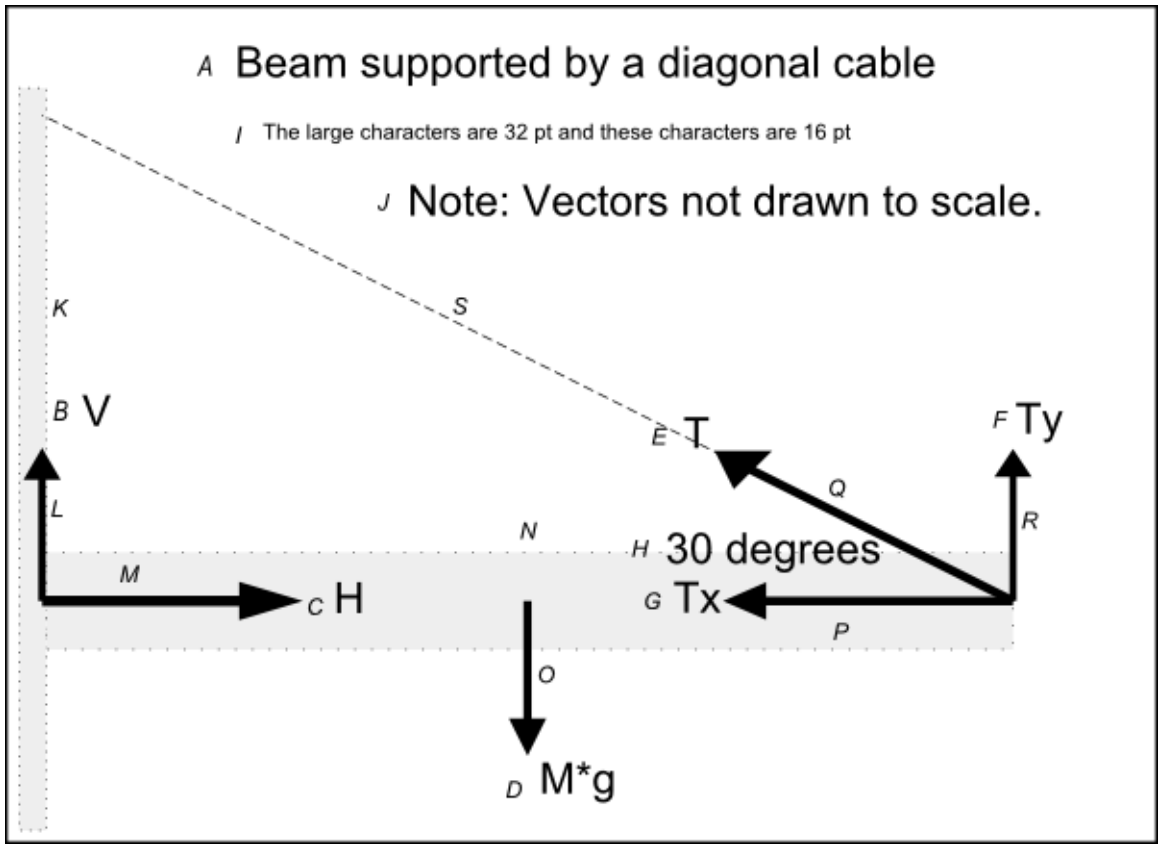

A Beam supported by a diagonal cable
I The large characters are 32 pt and these characters are 16 pt
J Note: Vectors not drawn to scale.
K
S
B V
E T
F Ty
L
Q
R
N
H 30 degrees
M
C H
G Tx
P
O
D M*g

Figure 2 . Normal image from file 1.svg.

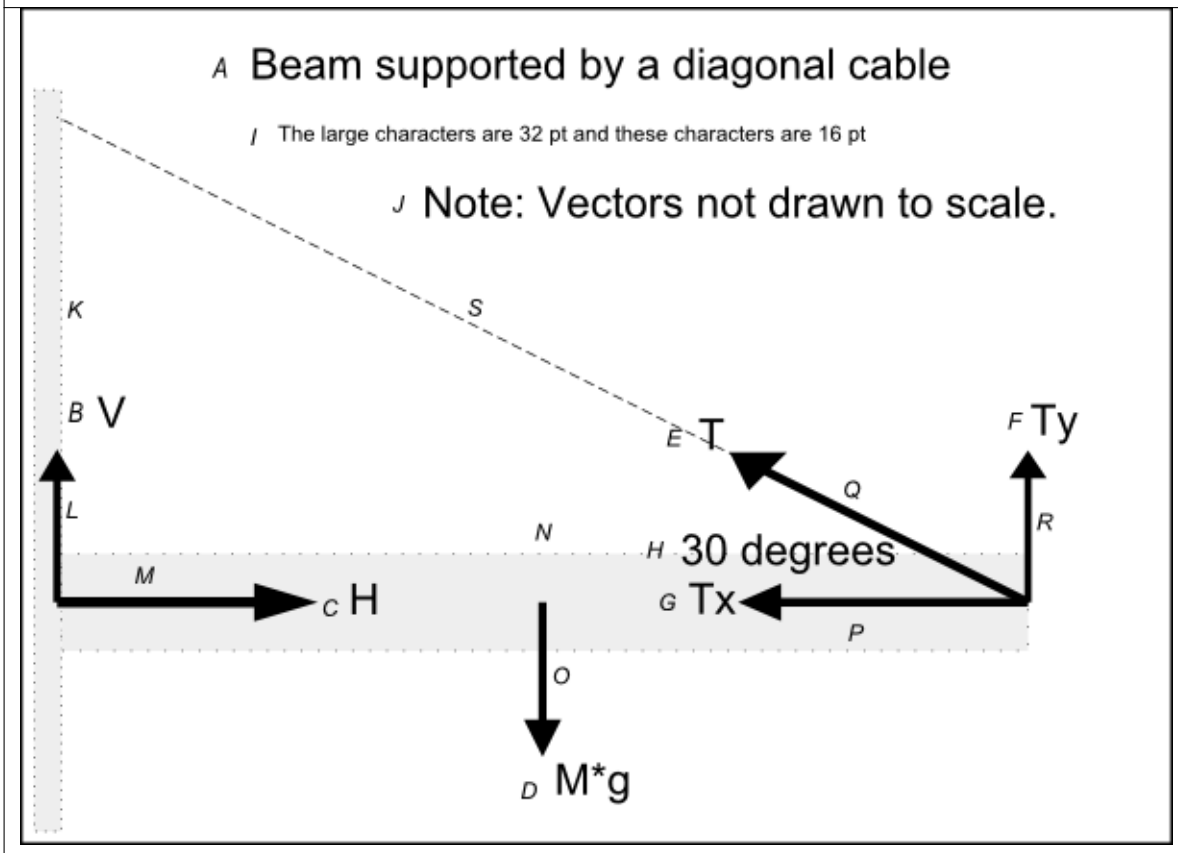


Table 3.2

The image

This image is provided as a test case to allow you and your assistant to experiment and to determine what works best for you. Much of the information and many of the objects in the image have to do with things that you haven't learned yet, so you shouldn't expect to understand why they are there.

Your assistant will probably notice that all of the text is printed backwards. This is because the svg file contains a mirror image of the actual image. In effect, your assistant will emboss the image from the back side of the paper. When you turn it over and explore the front side using touch, you will be exploring the image in the orientation that it is intended to be viewed.

Your assistant will also notice that there is a (reversed) letter in a small oblique font to the right (from her viewpoint) of each of the major text elements in the image, plus a few other letters in that same font scattered throughout the image. These are key characters, which are to be embossed in Braille. I will have more to say about this later.

Manual embossing

If your assistant has experience with manual embossing, the two of you probably know more about manually embossing tactile graphics than I do. However, manual embossing experience is not a requirement. Just about any sighted person should be able to emboss the images with your help.

Mostly straight lines

Most of the lines for the images in these modules will either be straight lines or gently curving lines that can be embossed using a serrated tracing wheel. (Serrated tracing wheels can be purchased at fabric, hobby, or craft stores. If there is a choice, ask your assistant to purchase the wheel with the sharpest serrations.)

Just ask your assistant to emboss narrow lines once in the center of the line, and to emboss wide lines twice, once on each side of the line. Very wide lines can be embossed three times, once on each side and once in the center. The use of a straight edge as a guide works very well for straight lines. If the line is not straight, your assistant should do her best to follow the line with the tracing wheel on a freehand basis.

A backing pad is required

Don't attempt to emboss the image with the paper on a hard surface. You will need to place it on a backing pad of some sort so that the serrations will penetrate the paper. A block of Styrofoam works pretty well for this purpose, as does a piece of corrugated cardboard from a cardboard packing box. Many tactile graphics kits [22] include a backing pad, but those kits are pretty expensive and may be overkill for your needs.

You may be able to identify another inexpensive material for a backing pad that works even better. If you do, I would like to hear about it so that I can pass that information along to other students.

Don't emboss the English text characters

The most difficult thing about manually embossing the image in Figure 1 (p. 60) is the task of embossing the English text labels in a form that is accessible to a blind student. Therefore, I don't intend for your assistant to emboss that text, unless she elects to do so using Braille as described below (p. 66) .

Key characters

The image in the file named 1.svg contains 19 strategically placed key characters consisting of the characters from A through S. (Other images in other modules will have different numbers of key characters.) As mentioned earlier, the key characters are printed in a smaller oblique font to make them easily distinguishable from the regular text. (They are also printed as a mirror image of the actual English character.)

Emboss the lines and Braille the key characters

Your assistant should emboss all of the lines in the image, and should replace the smaller, oblique key characters of "A" through "S" with corresponding Braille characters using a slate and stylus. This may be the point where you will need to help. If your assistant doesn't know Braille, have her place the Braille template over the character and tell you what the character is so that you can emboss it yourself.

Alternatively, your assistant can find a visual chart showing Braille characters for the alphabet at Wikipedia. (See http://en.wikipedia.org/wiki/Braille#Letters_and_numbers [23] .)

Make sure the orientation is correct

Each Braille character should be embossed in reversed orientation relative to that chart. For example, when you turn the paper over and touch it, you should recognize the Braille character for an "A" where the key value "A" appears (reversed) in English text on the printed mirror image.

In addition to the key characters, you or your assistant should emboss a Braille label of your own choosing on each image so that you can identify it later.

You may need a flag

Some of the key characters, such as the letter "A", with a small number of dots may be difficult for you to locate on the embossed image. Therefore, you and your assistant may need to emboss some sort of a flag near the Braille character to alert you of its presence. One possibility would be to use the tracing wheel to emboss a small X next to the Braille key character. If the two of you come up with a flag that is both effective and easy to create, I would like to hear about it so that I can pass the information along to other students.

Key-value pairs

Figure 3 (p. 65) contains the text values associated with each of the Braille key characters shown in Figure 1 (p. 60) .

Figure 3 . Text values for Braille keys in file 1.svg.

continued on next page

[22] http://www.fredshead.info/2006/01/tactile-graphics-kit-replacement-parts.html
[23] http://en.wikipedia.org/wiki/Braille#Letters_and_numbers

```
A: Beam supported by a diagonal cable
B: V
C: H
D: M*g
E: T
F: Ty
G: Tx
H: 30 degrees
I: The large characters are 32 pt and these characters are 16 pt
J: Note: Vectors not drawn to scale
K: Wall supporting beam
L: Vertical support vector at wall
M: Horizontal support vector at wall
N: Beam
O: Weight vector for beam
P: Horizontal component of tension vector
Q: Tension vector
R: Vertical component of tension vector
S: Physical support cable
```

Table 3.3

The text values in the right-hand column in Figure 3 (p. 65) are the text values that you would read if all of the text on the image were embossed in Braille. However, embossing all of that text in Braille would make your assistant's job much more difficult. Therefore, in the interest of simplicity, my approach will be to present the text for an individual image as shown in Figure 3, (p. 65) and to provide Braille key characters on the images that you can use to tie the text to the image.

On the other hand

If your assistant is good at manually embossing with Braille and can spare the time to do so, there is no reason that she can't simply emboss Braille right over the printed text. Then, except for keys that refer to objects such as the key labeled "M" in Figure 2 (p. 64) , you and your assistant can simply ignore the keys.

The intended operational mode

The intended operational mode is for you to locate an object of interest on the embossed image, locate the Braille key associated with that object, and then come back to Figure 3 (p. 65) to read the text associated with that object.

A vector diagram

Once you begin exploring the embossed image from the file named 1.svg by touch, you will discover that there are several objects on the image that consist of heavy straight lines with arrow heads. Those objects are what we will refer to as vectors in subsequent modules.

This diagram includes a vertical wall on the left side of the image. A rectangular beam protrudes horizontally from the wall towards the right a little below the vertical center of the image. A supporting cable is attached to the right end of the beam at an angle of 30 degrees and attaches back to the wall above the point where the beam is attached to the wall.

The image shows the vectors associated with various forces in the wall-beam-cable configuration along with the beam and the cable in the background. This will be a common theme throughout this collection. A picture of something will be presented in the background and vectors will be shown in the foreground.

Very light gray shading

Your assistant will note that I represented the wall and the beam with a very light gray shading and a few widely-spaced dots along the edges. I also represented the cable as a dashed line.

I'm not sure of the best way to emboss the beam and the wall. One approach would be to simply use the tracing wheel and emboss the outline of the beam and the wall. However, I'm concerned that the addition of the horizontal and vertical lines required to do that would make it more difficult for you to discern the more important information, which is the location and direction of each vector.

You and your assistant will probably need to discuss this issue and determine what works best for you in terms of identifying the location and shape of the beam and the wall. Ideally, you will come up with a solution that can be applied to the background pictures in other images in future modules.

One option might be to print two copies of the file and ask your assistant to emboss only the outer frame and the background picture in one, and to emboss everything but the background picture in the other.

If you come up with a really good idea in this regard, I would like to hear what it is so that I can pass it along to other students.

3.4 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

3.5 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Manual Creation of Tactile Graphics
- File: Phy1004.htm
- Revised: 09/30/15
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - tactile graphics
  - embossing
  - IVEO

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does

receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 4

Phy1006: A small-scale demonstration of the IVEO Learning System[1]

4.1 Table of Contents

- Preface (p. 70)
  - General (p. 70)
  - Prerequisites (p. 70)
  - Viewing tip (p. 71)
    - Figures (p. 71)
    - Listings (p. 71)
  - Supplemental material (p. 71)
- Discussion (p. 71)
  - Creation of tactile graphics (p. 71)
  - What is the IVEO Hands-on-Learning System? (p. 71)
    - A disclaimer (p. 71)
    - A description by the manufacturer (p. 72)
    - The component parts (p. 72)
    - Brief operational description (p. 72)
  - A program to create a drawing (p. 73)
  - Instructions (p. 76)
    - Acquire a USB digitizer tablet (p. 76)
    - Create the SVG file (p. 76)
    - Install Inkscape (p. 77)
    - Print a mirror image (p. 77)
    - Manually emboss the image (p. 77)
    - Load the SVG file into the IVEO Viewer (p. 78)
    - Explore the image with your fingers (p. 78)
  - Tactile, audio, and visual information (p. 79)
- Resources (p. 80)
- Complete program listings (p. 80)
- Miscellaneous (p. 82)

[1] This content is available online at <http://cnx.org/content/m40516/1.2/>.

4.2 Preface

4.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

According to the manufacturer [3] , *"The IVEO Hands-on-Learning System* [4] *teaches using touch, sound, and sight combined. This method utilizes any or all three learning modalities to help students learn faster and retain information longer, meanwhile, making learning more fun and interactive."*

Unfortunately, it may not be practical for you to get a hands-on demonstration. This module explains how you might be able to create your own small-scale demonstration at little or no cost.

4.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [5]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [6]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [7] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [8]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [9] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [10]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [11]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [12]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [13] and http://www.w3schools.com/js/default.asp [14]).

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.viewplus.com/
[4] http://www.viewplus.com/solutions/touch-audio-learning/
[5] http://www.youtube.com/watch?v=c8plj9UsJbg
[6] http://www.youtube.com/watch?v=v-F06HgiUpw
[7] http://www.nvda-project.org/
[8] http://www.userite.com/ecampus/lesson1/tools.php
[9] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[10] http://www.youtube.com/watch?v=c8plj9UsJbg
[11] http://www.youtube.com/watch?v=v-F06HgiUpw
[12] http://www.clarku.edu/~djoyce/trig/
[13] http://www.dickbaldwin.com/tocjscript1.htm
[14] http://www.w3schools.com/js/default.asp

- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [15] .
- An understanding of and the ability to use my SVG graphics library to draw diagrams as described here [16] .

4.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

4.2.3.1 Figures

- Figure 1 (p. 75) . Image from the file named Svg23.svg.

4.2.3.2 Listings

- Listing 1 (p. 73) . Code requiring changes.
- Listing 2 (p. 81) . Complete listing of the program named Svg23.java.
- Listing 3 (p. 82) . Batch file to compile and run the program named Svg23.java.

4.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [17] .

4.3 Discussion

4.3.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [18] explains how to create tactile graphics from svg files that I will provide.

Normally in these modules, I provide the SVG files that you need to create tactile graphic. However, in this module, you will create your own SVG file by editing, compiling and running a program that I will provide named **Svg23.java** .

4.3.2 What is the IVEO Hands-on-Learning System ?

4.3.2.1 A disclaimer

I have no affiliation with ViewPlus Technologies Inc. [19] , the manufacturer of the IVEO [20] system, other than being an acquaintance and admirer of Dr. John Gardner, President and Chief Technical Officer of the company. However, as an independent observer, I believe that IVEO [21] is a great system and I wish it were possible for every student to have access to such a system.

The words that you see here are my words, except for a few words that I copied from the ViewPlus [22] website and presented in quotation marks earlier (p. 70) .

[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[16] http://cnx.org/contents/0803a1bf-9c6d-435a-a5f1-e21b1dbd978c
[17] http://www.dickbaldwin.com/toc.htm
[18] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[19] http://www.viewplus.com/solutions/touch-audio-learning/
[20] http://www.viewplus.com/solutions/touch-audio-learning/
[21] http://www.viewplus.com/solutions/touch-audio-learning/
[22] http://www.viewplus.com/solutions/touch-audio-learning/

4.3.2.2 A description by the manufacturer

I will begin by referring you to an article written by John Gardner titled *Accessing Graphical Information with IVEO SVG* at http://www.access2science.com/jagqn/iveo.html [23] . I recommend that you read that article first and then come back and finish reading this module.

4.3.2.3 The component parts

If I understand it correctly, a complete IVEO System [24] consists of the following components:

- A computer
- An embossing machine capable of producing embossed or tactile graphics
- A free program called the **IVEO Viewer** that can be downloaded from http://downloads.viewplus.com/software/IVEOViewer/ [25]
- A large 11x14 touchpad

In this module, I will assume that you have the necessary computer. I will also assume that you don't have a suitable graphics embossing machine but instead you have a friendly sighted assistant who is willing to manually emboss the required printed image for you.

I will assume that you can download the IVEO Viewer [26] software.

I will assume that you do not have an 11x14 touchpad, but instead you are able to come up with a suitable USB digitizing tablet. I will write this module around the use of a rather antiquated Wacom 5.0 x 3.62-inch digitizing tablet that I have owned for quite a few years. Your digitizing tablet will probably be a different size, so I will explain how to accommodate the difference is size.

I will also assume that you can download and install the free **Inkscape** program from http://inkscape.org/download/ [27] . That program will be needed to flip and print an SVG file to produce a mirror image that your assistant can manually emboss from the back side.

4.3.2.4 Brief operational description

The operation of the IVEO System [28] essentially consists of the following steps. These steps assume that your hardware and software system has been installed and tested.

- Open the SVG file in the IVEO Viewer [29]
- Make adjustments, if necessary, in the **Page Setup** item on the **File** menu to specify the size of the page containing the drawing.
- Print the drawing from the **File** menu on an embossing graphic printer.
- Place the embossed graphic image on the touchpad and clamp it in position.
- Press **Ctrl+U** and step through a spoken process of calibrating the touchpad with the current image.

- Explore the embossed graphic image with your fingers, pressing with a single finger on those features of the embossed image that interest you. If you press on a feature and you hear spoken information about that feature, press **Ctrl+d** and you may or may not hear additional information about the feature. This will depend on whether additional information was provided by the author of the SVG file.
- Exercise other features, such as a feature that allows you to zoom in on specific areas of the image.

[23] http://www.access2science.com/jagqn/iveo.html
[24] http://www.viewplus.com/solutions/touch-audio-learning/
[25] http://downloads.viewplus.com/software/IVEOViewer/
[26] http://downloads.viewplus.com/software/IVEOViewer/
[27] http://inkscape.org/download/
[28] http://www.viewplus.com/solutions/touch-audio-learning/
[29] http://downloads.viewplus.com/software/IVEOViewer/

Since I am assuming that you don't have access to either a touchpad or an embossing graphics printer, I will show you how to work around some of these steps. However, even with the workarounds, you should still be able to get a feel for the behavior of the system.

4.3.3 A program to create a drawing

A complete Java program named **Svg23.java** is provided in Listing 2 (p. 81) .

The purpose of this program is to demonstrate how to use my graphics library named **SvgLib21** *(see Listing 26 here* [30] *)* to create an SVG file containing a simple drawing that can be used with a common USB digitizer tablet and the IVEO System [31] .

All dimensions in the program are written in terms of the variables named **width** and **height** . Therefore, the only change that that you should need to make is to replace the dimensions of my digitizer tablet with the dimensions of your digitizer tablet in the code fragment shown in Listing 1 (p. 73) .

Having done that, you should be able to compile and run the program and it should produce an output file named **Svg23.svg** .

Listing 1 . Code requiring changes.

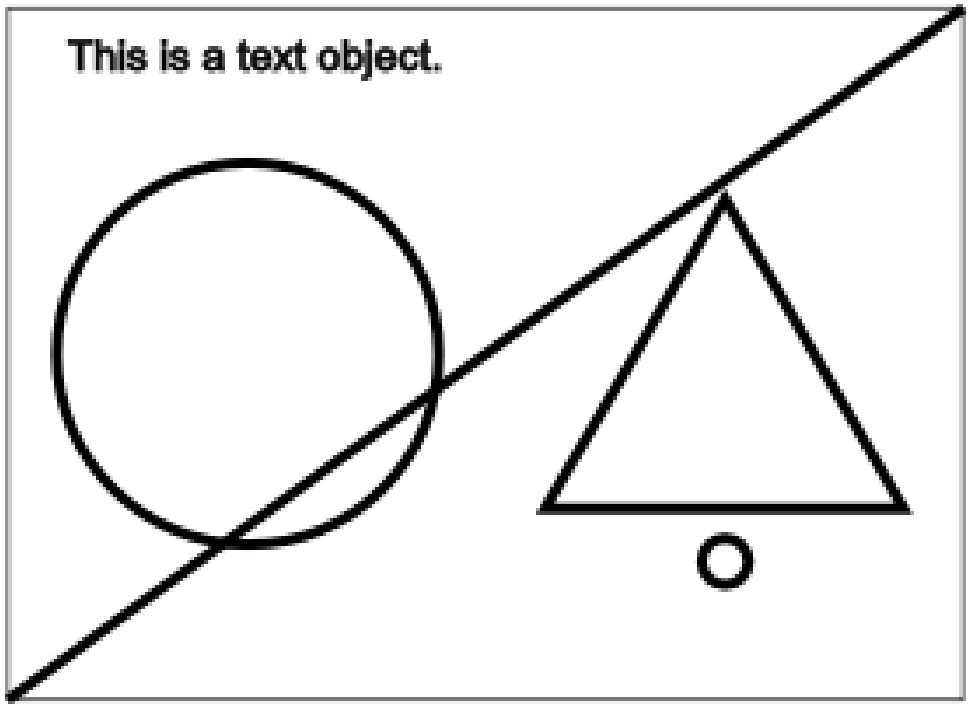
```
    //Change the following two values to the width and
// the height of your digitizing pad in inches.
double width = 5.0;
double height = 3.62;
```

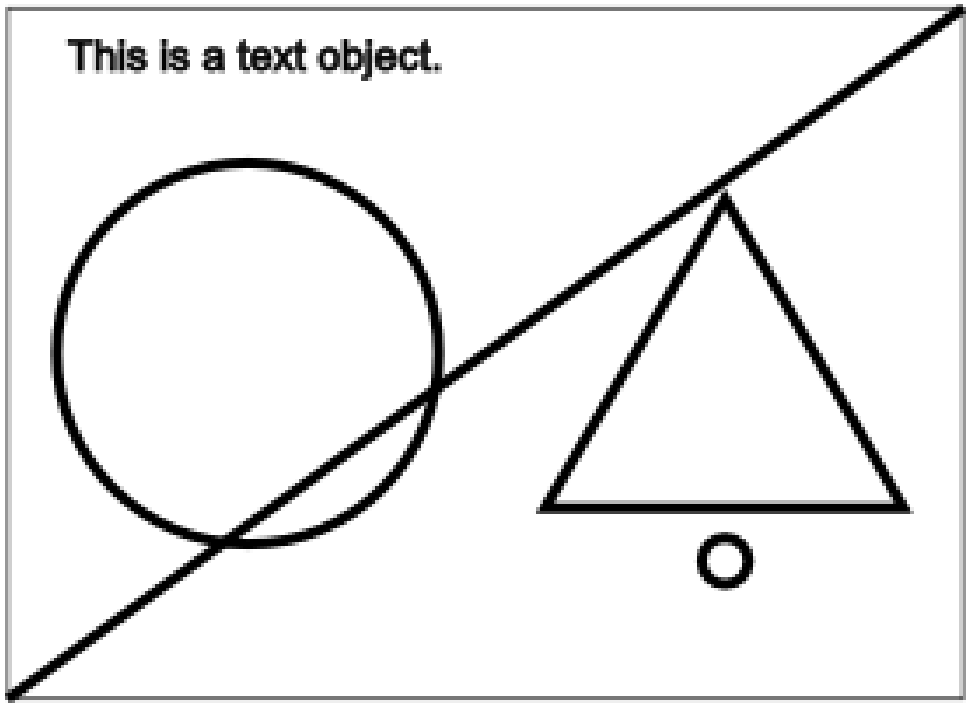
Table 4.1

The image

For the benefit of your assistant who will emboss your image, Figure 1 (p. 75) shows a reduced-size version of the image in the output SVG file for the dimensions of my digitizer tablet. The image should be very similar for the dimensions of your digitizer tablet.

[30] http://cnx.org/contents/0803a1bf-9c6d-435a-a5f1-e21b1dbd978c
[31] http://www.viewplus.com/solutions/touch-audio-learning/


This is a text object.


This is a text object.

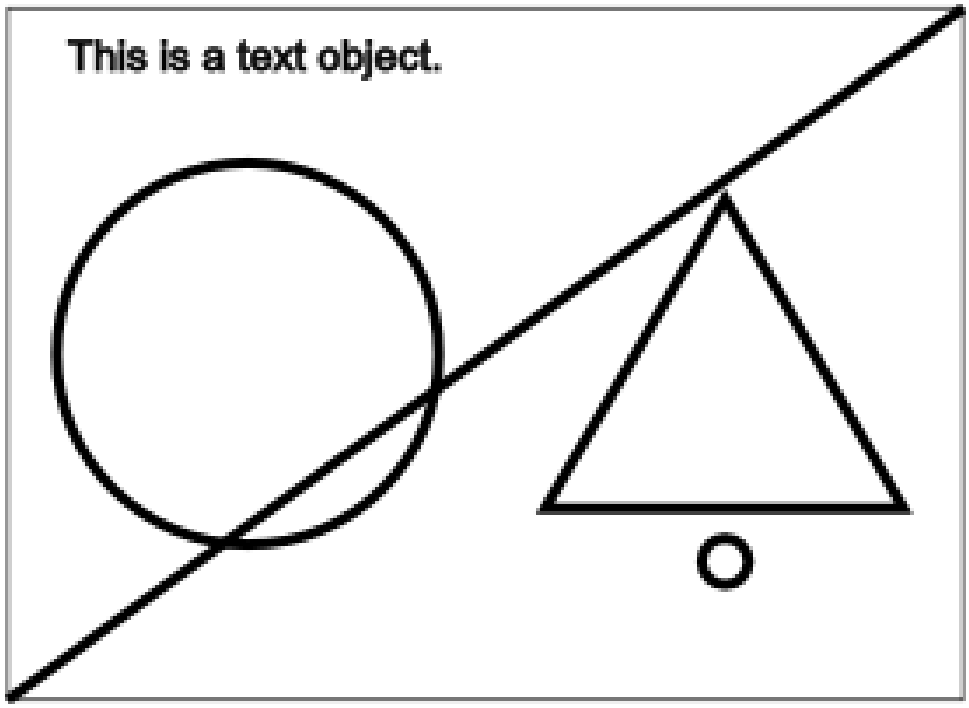


Figure 1 . Image from the file named Svg23.svg.

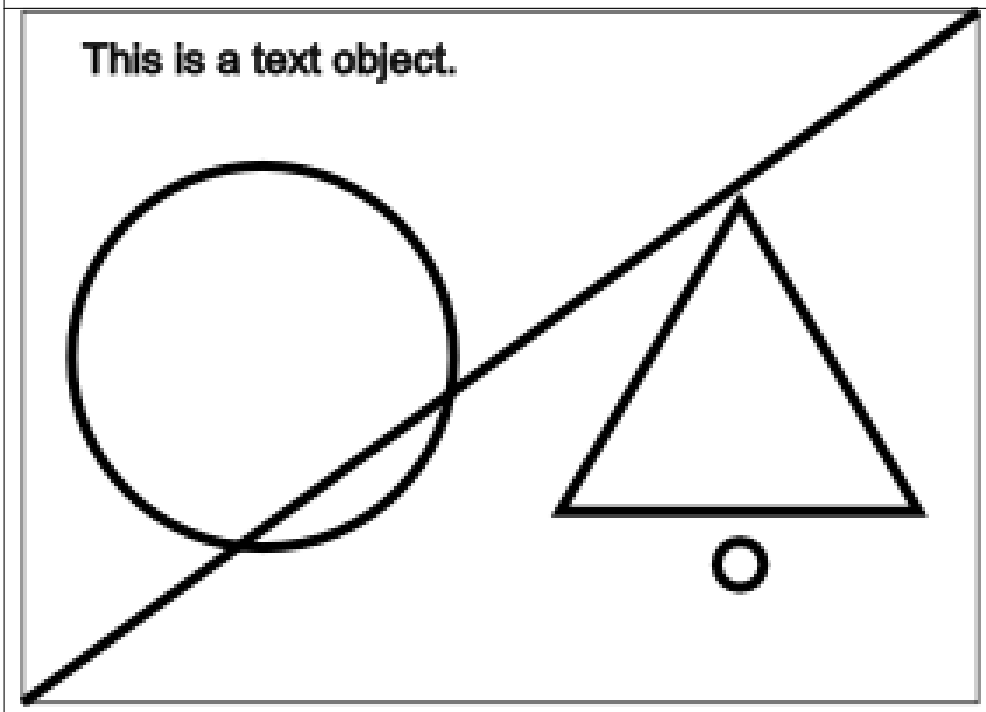


Table 4.2

Description of the image

For your benefit, the image consists of a rectangle drawn with a thin line that is the same size as the active area on my digitizer tablet *(5.0 x 3.62 inches)* .

A circle with a radius of one inch is centered in the left half of the drawing.

A triangle, with its base parallel to the horizontal axis, is horizontally centered in the right half of the drawing.

A diagonal line extends from the lower left corner to the upper right corner of the drawing.

Some text shows in the upper-left corner of the drawing.

There is a small circle below the triangle, which I will refer to as a ***pseudo-text*** *marker* . I will explain the purpose of the pseudo-text marker later.

Although the dimensions of your digitizer tablet will probably be different than mine, the overall layout of the shapes in your SVG file should be very similar to mine.

4.3.4 Instructions

In this section, I will provide instructions and explain the steps that you and your assistant will need to take in order to create your own small-scale demonstration of the IVEO System [32] .

4.3.4.1 Acquire a USB digitizer tablet

I put this instruction at the beginning of the list because without a USB digitizer tablet *(or something to suitably replace the touchpad)* , there can be no demonstration. Assuming that you don't have such a tablet, you should check with your friends, take a look on Ebay, etc.

(I will point out, however, that if you have low vision but can still see the computer screen and can use a mouse, you may be able to create this demonstration without the need for a digitizer tablet.)

The brand and size of the tablet isn't terribly important, although the larger the tablet, the closer your small-scale demonstration will come to representing the real thing.

It is critical that the tablet connects to the computer via a USB port. It is also critical that the operation of the tablet emulates a mouse. By this, I mean that when you touch the tactile image on the tablet in a given location, the effect must be the same as if you were to use a mouse and click the corresponding location on the image on the screen.

Some tablets work only with a stylus, some work from either a finger touch or a stylus, and possibly some only work from a finger touch. Once again, this isn't too important, but the use of a tablet that works from a finger touch will be closer to the real thing than one that requires a stylus. *(Mine requires a stylus.)*

4.3.4.2 Create the SVG file

Once you have acquired your digitizer tablet and you know the dimensions of the active area on your tablet, it is time to create an SVG file that you can use for testing.

Copy the Java code from Listing 2 (p. 81) into a text file with the name **Svg23.java** . Make the changes shown in Figure 1 (p. 75) to replace the dimensions of my tablet with the dimensions of your tablet. Note that dimensions are in inches.

Copy the text from Listing 3 (p. 82) into a text file with the name **Svg23.bat** .

Using the two files listed above, along with the instructions in the module titled If You Can Imagine T, You Can Draw It using SVG [33] , and *Listing 26 here* [34] *)* , compile and run the program named **Svg23.java** , producing an output file named **Svg23.svg** .

[32] http://www.viewplus.com/solutions/touch-audio-learning/
[33] http://cnx.org/contents/0803a1bf-9c6d-435a-a5f1-e21b1dbd978c
[34] http://cnx.org/contents/0803a1bf-9c6d-435a-a5f1-e21b1dbd978c

4.3.4.3 Install Inkscape

This would be a good time for you to download and install the Inkscape [35] program. You will be needing it shortly so that your assistant can

- create a mirror image of the image in the SVG file, and
- print the mirror image for manual embossing.

4.3.4.4 Print a mirror image

> The Inkscape program is a relatively complex program, and an inexperienced user could easily damage or modify an SVG file. Therefore, the instructions in this section should be performed using a copy of the file named **Svg23.svg** .

Ask your assistant to start the **Inkscape** program and to open a copy of the file named **Svg23.svg** in that program.

- Press **Ctrl+Alt+A** to prepare the image for a horizontal flip. Your assistant should see a bunch of little arrows on the screen indicating that all of the objects in the image have been selected.
- Pull down the **Object** menu and select **Flip Horizontal** . Your assistant should see the image flipped from its normal state to a mirror image of its normal state.
- Press **Ctrl+P** to cause the flipped image to be printed.
- Press **Ctrl+Q** to quit. When the opportunity presents itself, click the button labeled **Close without saving** ..

The printed output

Your assistant should see the mirror image of an image that is very similar to Figure 1 (p. 75) . However, the overall size of the image will be different from Figure 1 (p. 75) due to the differing sizes of our respective tablets.

All of the shapes in the image should be contained inside a rectangle that is very close to the size of the active area on your tablet. If that is not the case, something is wrong and you will need to backtrack and fix the problem.

4.3.4.5 Manually emboss the image

Ask your assistant to use a tracing wheel or something similar to manually emboss the printed mirror image. Ask him or her to try to emboss the heavy lines as double lines to make them easier for you to locate and identify with your fingers.

A text block

There will be some text in the upper-right corner of the mirror image. It won't be necessary to emboss the characters in that text. Simply emboss some marks that will alert you to the location of the text.

A pseudo-text marker

There is also a small circle underneath the triangle. *(Depending on the size of your digitizer tablet, the circle may actually appear on top of the triangle.)* Once again, it won't be necessary to emboss that circle as a circle. Simply emboss an X there so that you will be able to discern the location of that circle. *(This is the pseudo-text marker that I referred to earlier (p. 76) . I will explain its purpose later.)*

Attach the image to the digitizer tablet

Once the image has been embossed, trim the image very carefully at the rectangular border. Using masking tape or something similar turn the image over with the ink side down and affix it to the active area of your tablet. Be very careful because proper alignment is critical at this point.

Ask your assistant to somehow mark the upper-left and lower-right corners of the rectangle so that you can discern them later when times comes to calibrate the touchpad.

[35] http://inkscape.org/download/

4.3.4.6 Load the SVG file into the IVEO Viewer

Start the IVEO Viewer [36] program running. When it is running and ready for action, you should hear a welcome message.

Open the SVG file

Press **Ctrl+O** to open the SVG file. Navigate the disk and open the file named **Svg23.svg** . When the file opens in IVEO, you should hear something like the following:

"Document title description of the document"

The page setup operation

Open the **File** menu and select the item named **Page Setup** .

Select **Letter (Landscape)** if your rectangle is wider than it is tall. Select **Letter (Portrait)** if your rectangle is taller than it is wide.

Then click the **OK** button.

You may be asked *"Do you want to fit the image to the new page size?"* If so, click the **Yes** button.

Calibrate the touchpad

Press **Ctrl+U** to calibrate the touchpad to your image.

You will be asked to *"Touch top left corner of the image."*

At this point, you should touch the top left corner of the image that is taped onto your tablet. Follow the verbal instruction and press the space bar to confirm.

You will then be asked to *"Touch bottom right corner of the image."* Touch the bottom right corner of the image that is taped onto your tablet, and press the space bar to confirm.

You should hear *"calibration finished."*

4.3.4.7 Explore the image with your fingers

You are now ready to begin the exploration of the image using your fingers, or a combination of your fingers and a stylus depending on the kind of tablet that you are using.

> Even though you are probably accustomed to exploring with multiple fingers, when time comes to touch an object and elicit a spoken response, you should touch with only one finger. Otherwise the system won't know which touch to pay attention to and you probably won't get the desired result.

Touch the text object

If you touch the area near the upper-left corner that your assistant has identified as containing text, you should hear *"This is a text object."* If you are interested in doing so, you can locate the code in Listing 2 (p. 81) responsible for this behavior by searching that listing for the matching text.

Touch the circle

Touch anywhere inside the circle in the left half of your image and you should hear *"Circle."*

Then press **Ctrl+d** and you should hear *"Circle Description of the circle."*

In other words, once you touch one of the objects and hear a voice that identifies the object, you can press **Ctrl+d** to hear more information about the object.

Once again, if you are interested in doing so, you can search for this text in the Java code in Listing 2 (p. 81) and see the code that is responsible for this behavior.

Touch the triangle

If you touch anywhere inside the triangle in the right half of your image, you should hear *"Triangle."* Pressing **Ctrl+d** at that point will provide additional information about the triangle.

Touch the pseudo-text marker

If you touch the marker underneath *(or perhaps on)* the triangle, you should hear *"This is pseudo text identified by a text marker."*

This is an alternative to actually putting text in the drawing as was done above (p. 78) . This approach can be used to include text in the drawing without allowing the text to cause the drawing to become messy.

[36] http://downloads.viewplus.com/software/IVEOViewer/

It is useful when there is a lot text that needs to be included in the drawing. It is also useful when two ordinary text objects would overlap.

The downside is that a sighted teacher or a low-vision student can't see the text. It only becomes exposed when the SVG file is processed using the IVEO system [37] .

Touch the line

Try to touch the line that extends from the lower left to the upper right corner of the rectangle. This can sometimes be difficult, particularly if the printed and embossed version of the drawing isn't well registered on the touchpad.

If you succeed in touching the line, you should hear *"Diagonal line."* Pressing **Ctrl+d** should then result in a voice saying *"Diagonal line Description of diagonal line."*

The difficulty of touching the line is one illustration of the need for a large touchpad for use with IVEO. If this same drawing were enlarged to fit on the 11x14 inch IVEO touchpad, the line would be much thicker and easier to touch.

The border rectangle

There is a good possibility that when you attempted to touch the line, you heard *"Border rectangle"* instead of *"Diagonal line."* This is because all of the area within the rectangular border not covered by one of the other objects belongs to the rectangular object that forms the border.

If you did hear *"Border rectangle"* and you were to press **Ctrl+d** , you should hear *"Border rectangle Description of border rectangle."*

The document title

If you press the d key at any time, you should hear *"Document title Description of the document."* This is the same thing that you should have heard when the SVG file was opened in the IVEO Viewer [38] .

4.3.5 Tactile, audio, and visual information

The words that are spoken when you touch an object or press **Ctrl+d** are defined by the author of the SVG file when the file is created. The purpose is to provide information that can't easily be provided solely through tactile means.

In other words, there is a limit to the amount of information about a drawing or an image that can be provided solely through touch. For the blind student, IVEO makes it possible to supplement the tactile information with additional information that is delivered by voice. Thus, a blind student has access to both tactile and audible information when exploring the drawing.

If a student has low vision but still has some ability to see the screen, visual information is also presented in the form of the drawing on the screen. The low-vision student may be able to explore the drawing using a mouse and receive the same audible information and may not need a touchpad. Thus, a low-vision student has access to tactile, audible, and visual information.

Not much information in this demonstration file

I was the author of this demonstration SVG file. Because it is a demonstration, I didn't include much real information in the audible realm. My objective was simply to give your an opportunity to see how the system works.

As you read earlier in the article titled Accessing Graphical Information with IVEO SVG [39] , the IVEO System [40] provides some other features as well. However, now that you have reached this point, you can explore those other features on your own. Just remember that some features, such as zooming for example, probably can't be demonstrated with a small digitizing tablet.

[37] http://www.viewplus.com/solutions/touch-audio-learning/
[38] http://downloads.viewplus.com/software/IVEOViewer/
[39] http://www.access2science.com/jagqn/iveo.html
[40] http://www.viewplus.com/solutions/touch-audio-learning/

4.4 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

4.5 Complete program listings

Listing 2 (p. 81) contains the Java code to produce the SVG file needed for this demonstration.

Listing 2 . Complete listing of the program named Svg23.java.

```
    /*File Svg23.java,
The purpose of this program is to demonstrate how to use
my graphics library named SvgLib21 to create a simple
SVG file that can be used with a small Wacom digitizer
tablet with the IVEO learning system.

All dimensions are written in terms of width and height.
Therefore, the only change that should be needed is to
change the hard-coded values for width and height in
inches where indicated.
*******************************************************/
import java.io.*;
import org.jdom.*;

public class Svg23{
  public static void main(String[] args){

    //DO NOT MODIFY ANY OF THE CODE ABOVE THIS LINE.
    //################################################//
    //ONLY THE CODE BELOW THIS LINE CAN BE MODIFIED

    /**
     *CREATE A DRAWING CANVAS
     *This must be the first statement that you write in
     *the program and it must appear only once.
     *The following statement creates a canvas that is
     *8.5x11 inches in size in a portrait layout. The
     *size was chosen to match my printer page size.
     *I will actually restrict the size of my drawing
     *to 5.0x3.62 inches by drawing a border rectangle of
     *that size and then drawing everything inside that
     *rectangle. This size was chosen to match the size
     *of my Wacom digitizer pad. The title is "Document
     *Title", which will be spoken by IVEO when the
     *SVG file is loaded into IVEO.
     */
     Element svg = SvgLib21.makeSvg(ns,
                                    "Document Title",
                                    8.5,
                                    11
                                    );


    //DO NOT MODIFY THE FOLLOWING STATEMENT
    //This statement must immediately follow the call to
    // the makeSvg method above and this statement MUST
    // NOT BE MODIFIED.
    Document doc = new Document(svg,docType);

    /**
     *Add a description element to the document.
     *The description can be spoken in IVEO by pressing
     *the d key.
     */
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 4.3

Listing 3 (p. 82) contains the commands for a batch file that can be used, with modifications, to compile and run the program named **Svg23.java** . See If You Can Imagine It, You Can Draw It using SVG [41] for instructions on how to use the batch file.

Listing 3 . Batch file to compile and run the program named Svg23.java.

```
    cls

set classpath=.;C:\Program Files (x86)\Java\jdom-1.1.1\build\jdom.jar
del *.class
del Svg23.svg
javac SvgLib21.java
javac Svg23.java
java Svg23
start Firefox.exe Svg23.svg
pause
```

Table 4.4

4.6 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: A small-scale demonstration of the IVEO Learning System
- File: Phy1006.htm
- Revised: 09/30/15
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - touchpad
  - digitizer tablet

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it

[41] http://cnx.org/contents/0803a1bf-9c6d-435a-a5f1-e21b1dbd978c

possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 5

Phy1010: JavaScript[1]

5.1 Table of Contents

- Preface (p. 86)
  - General (p. 86)
  - Prerequisites (p. 86)
  - Viewing tip (p. 87)
    - Figures (p. 87)
    - Listings (p. 87)
  - Supplemental material (p. 87)
- General background information (p. 87)
- Discussion (p. 87)
  - Why I chose JavaScript (p. 88)
  - Facilities required (p. 88)
  - A minimal JavaScript script (p. 88)
  - Strings (p. 90)
  - Screen output (p. 90)
  - Structured programming (p. 90)
    - Functions (p. 91)
    - Arithmetic operators (p. 93)
    - Sequence (p. 94)
    - Selection (p. 94)
    - Relational and logical operators (p. 97)
    - Variables (p. 97)
    - The string concatenation operator (p. 99)
    - Repetition (p. 99)
  - Programming errors (p. 101)
    - Assistance using Google Chrome (p. 101)
    - Assistance using Firefox (p. 102)
- Run the scripts (p. 102)
- Resources (p. 103)
- Miscellaneous (p. 103)

[1] This content is available online at <http://cnx.org/content/m37433/1.4/>.

5.2 Preface

5.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

5.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]). (This information will be provided in a later module.)

- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]). (The purpose of this module is to help you gain that understanding.)
- An understanding of the creation and use of tactile graphics as described here [13] .

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

5.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

5.2.3.1 Figures

- Figure 1 (p. 89) . Output from script in Listing 1.
- Figure 2 (p. 93) . Output from script in Listing 2.
- Figure 3 (p. 94) . Binary arithmetic operators.
- Figure 4 (p. 94) . General syntax for selection statement.
- Figure 5 (p. 95) . Real-world analogy of a selection statement.
- Figure 6 (p. 96) . Output from script in Listing 3.
- Figure 7 (p. 97) . Relational operators.
- Figure 8 (p. 99) . Output from script in Listing 4.
- Figure 9 (p. 100) . General syntax for a while loop.
- Figure 10 (p. 101) . Output from script in Listing 5.

5.2.3.2 Listings

- Listing 1 (p. 89) . A minimal JavaScript script.
- Listing 2 (p. 92) . An example function named getHalf.
- Listing 3 (p. 96) . A selection script example.
- Listing 4 (p. 98) . A sample script that uses variables.
- Listing 5 (p. 100) . A simple while loop.

5.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

5.3 General background information

I explained in the earlier module titled Introduction to Accessible Physics Concepts [15] why you will need an introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [16] and http://www.w3schools.com/js/default.asp [17]) to understand the modules in this collection. I won't repeat that explanation here. Instead, I will simply launch into the material to help you gain that understanding.

5.4 Discussion

The goal of this module is to provide an introductory JavaScript programming tutorial that is accessible to blind students with no programming experience.

[14] http://www.dickbaldwin.com/toc.htm
[15] http://cnx.org/contents/aba5529d-dddd-43de-8748-48bb84f818c0/Phy1000-Introduction-to-Access
[16] http://www.dickbaldwin.com/tocjscript1.htm
[17] http://www.w3schools.com/js/default.asp

5.4.1 Why I chose JavaScript

I chose JavaScript for use in these modules for several reasons.

Free

First, JavaScript is free. The capability to program in JavaScript is available to anyone who has a modern browser installed on their computer. Therefore, cost is not an excuse for not learning to program with JavaScript.

If you are reading this module using a modern browser, you have the ability to program using JavaScript immediately. You don't have to go out and buy anything, so that isn't an excuse for putting it off until tomorrow.

If you don't have a modern browser, you can download a free copy of the Firefox browser at http://www.mozilla.com/en-US/firefox/firefox.html [18] .

Fun

Also, programming with JavaScript can be fun. There are a lot of really interesting things that you can do with JavaScript such as playing sound files (see http://www.javascripter.net/faq/sound/play.htm [19]).

OOP

JavaScript encompasses modern programming concepts. For example, JavaScript is based on the concept of objects. Object-Oriented Programming (OOP) is here to stay. (For an extensive discussion of OOP, see the early lessons in my online programming tutorials at http://www.dickbaldwin.com/toc.htm [20] .)

A free audible, tactile scientific calculator

Most importantly, I chose JavaScript because you will be able to use it, along with a screen reader and a Braille display to create your own scientific calculator. You can use JavaScript to create solutions to many of the exercises in the modules in this collection of physics concepts.

5.4.2 Facilities required

To use JavaScript for its intended purpose in the modules in this collection, you will need the following:

- Access to the Internet, a screen reader, and a Braille display.
- A modern browser such as Firefox 3.6 (see http://www.mozilla.com/en-US/firefox/firefox.html [21]).
- A plain text editor such as Windows notepad, or my favorite, Arachnophilia, which can be downloaded for free at http://www.arachnoid.com/arachnophilia/ [22] .

5.4.3 A minimal JavaScript script

Listing 1 (p. 89) shows a minimal JavaScript script.

[18] http://www.mozilla.com/en-US/firefox/firefox.html
[19] http://www.javascripter.net/faq/sound/play.htm
[20] http://www.dickbaldwin.com/toc.htm
[21] http://www.mozilla.com/en-US/firefox/firefox.html
[22] http://www.arachnoid.com/arachnophilia/

Listing 1 . A minimal JavaScript script.

```
    <!-- File JavaScript01.html -->
<html><body>
<script language="JavaScript1.3">

document.write("Insert JavaScript between script tags.","</br>")
document.write("Hello from JavaScript")

</script>
</body></html>
```

Table 5.1

Run the script

To run this JavaScript script:

- Copy all of the text from the body of Listing 1 (p. 89) into your plain text editor and save the file with an extension of .htm (test.html for example).
- Open that file in your browser. (In most cases, you should be able to simply drag the file and drop it onto an open browser page to open it. If that doesn't work, open it from the browser's **File** menu.)

And the result is...

When you do that, the text shown in Figure 1 (p. 89) should appear in the browser window.

Figure 1 . Output from script in Listing 1.

```
    Insert JavaScript between script tags.
Hello from JavaScript
```

Table 5.2

That's all there is to it

All you have to do to write and run a JavaScript script is to:

- Copy the text (often referred to as code or source code) from Listing 1 (p. 89) into your plain text editor.
- Replace the code between the two lines containing the word **script** with the code for your new script, leaving the two lines containing the word **script** intact.
- Save the file with an extension of .html. (The file can have any legal file name so long as the extension is .html.)
- Open the new html file in your browser.

When you do that, the script code will be executed.

Display results in the browser window

If your script code contains statements that begin with **document.write** , followed by a pair of matching parentheses, (as shown in Listing 1 (p. 89)), the code in the parentheses will be evaluated and the results of that evaluation will appear in your browser window.

At that point, you will have access to your script code as well as the results of running your script using your screen reader and your Braille display.

5.4.4 Strings

In programming, we often refer to a group of sequential characters (such as your first name) as a **string** . In JavaScript format (often called syntax), such groups of characters are surrounded by matching quotation marks to cause the group to be recognized as a single string.

The following strings appear in the JavaScript code in Listing 1 (p. 89) :

1. "Insert JavaScript between script tags."
2. "</br>"
3. "Hello from JavaScript"

The first two strings appear, separated by a comma, inside the matching parentheses following the first occurrence of **document.write** . (We often call the items inside the matching parentheses the argument list.)

The last item in the above list appears in the argument list following the second occurrence of **document.write** in Listing 1 (p. 89) .

5.4.5 Screen output

If you examine Figure 1 (p. 89) , you will see that the first and third items from the above list (p. 90) appear, without their quotation marks, in the browser window. However, the second item does not appear in the browser window.

Purpose of document.write

Although this is a simplification, for purposes of the modules in this collection, it will suffice to say that the purpose of the **document.write** command is to cause the items in its argument list to be displayed in the browser window. You can place one or more items in the argument list. If there is more than one item in the argument list, the items must be separated by a comma as shown in Listing 1 (p. 89) .

Displaying strings

With some exceptions, items that appear in the argument list surrounded by matching quotation marks (strings) will be displayed in the browser window.

The exceptions include the second item in the above list (p. 90) . This item is a special command to the browser, commonly known as a **break** tag. The occurrence of a **break** tag tells the browser to go down to the next line before displaying any additional material.

Such browser commands usually begin with a left angle bracket as shown in Listing 1 (p. 89) . Because of this, it is usually wise to avoid displaying strings that begin with left angle brackets unless you know for sure that your string won't be interpreted as a command to the browser.

Displaying material other than strings

I will show you how to display material other than strings in a later section titled The string concatenation operator (p. 99) . Before getting into that, however, I will discuss several other topics.

5.4.6 Structured programming

When a student enrolls in the Object-Oriented Programming course that I teach at my college, I expect that student to have knowledge of something called structured programming, which is generally defined as including the following topics:

1. Functions with parameter passing
2. Sequence
3. Selection (if-else)
4. Repetition (for, while, and do-while loops)

I also expect them to know how to use variables and how to use operators (add, subtract, multiply, divide, etc.).

You will need to have an introductory knowledge of these topics to understand the JavaScript scripts that I will use to explain physics concepts in later modules. I will briefly explain these topics in this module and will discuss them further in later modules where they are used.

5.4.6.1 Functions

Functions, or procedures as they are called in some languages, provide a fundamental building block for virtually every programming language. The purpose of a function is to encapsulate the ability to perform a task into a single set of code and to be able to execute that code from a variety of different locations within the script. This can eliminate the requirement to repeat the same code over and over when the same task is required at multiple points in the script.

The surface area of a sphere

For example, if you write a script that frequently needs to calculate the surface area of a sphere, you can encapsulate those calculations in a function. Then, whenever your script needs to know the surface area of a sphere, it can simply call the function and provide the radius of the sphere as a parameter. The function will perform the calculation and return the answer to be used by the script at that point.

A JavaScript function definition

A JavaScript function definition has the following basic parts as shown in Listing 2 (p. 92) :

1. The **function** keyword.
2. The function name.
3. A comma-separated list of arguments enclosed in parentheses. (If there are no arguments, which is perfectly legal, the parentheses must still follow the function name but they will be empty.)
4. The statements in the body of the function enclosed in curly brackets.

Two sides to every function

There are two sides to the use of every function:

1. The function definition.
2. The function call.

The definition names the function and specifies how it will behave when it is called.

The call to the function temporarily passes control to the statements in the function causing them to behave as previously defined.

Once the statements have been executed, control is returned to the point in the script where the call was made. The function may, or may not return a value when it returns control.

The argument list

As in the sphere example discussed above, it is often useful to pass information to the function for it to use in doing whatever it is supposed to do (but this is not always required). When we call the function, we include parameters in the call to the function that match up with the argument list mentioned above. That is the mechanism used to pass information to a function. (This will probably make more sense when you see an example. Again, in some cases, no arguments are required.)

The purpose of a function

Usually (but not always), the purpose of a function is to calculate or otherwise determine some value and return it. In the sphere example mentioned earlier, the purpose of the function would be to calculate and return the surface area of the sphere. Returning a value is accomplished using the **return** keyword in the body of the function.

Sometimes, the purpose of a function is not to return a value, but instead to cause some action to occur, such as displaying information in the browser window. In that case, a **return** statement is not required. However, it doesn't cause any problem to put a **return** statement at the end of the function's body with nothing to the right of the word return.

An example function named getHalf

The code in Listing 2 (p. 92) defines a function named **getHalf** and then calls that function from two different locations in a script, passing a different parameter value with each call.

Listing 2 . An example function named getHalf.

```
    <!-- File JavaScript02.html -->
<html><body>
<script language="JavaScript1.3">

//This is the syntax for a comment.

//Define the function named getHalf()
function getHalf(incomingParameter) {
  return incomingParameter/2;
}//end function getHalf()

document.write("Call getHalf for 10.6","</br>")
document.write("Half is: ", getHalf(10.6),"</br>");

document.write("Call getHalf again for 12.3","</br>")
document.write("Half is: ", getHalf(12.3));

</script>
</body></html>
```

Table 5.3

A note about comments

Note the line of code immediately following the first **script** tag that begins with //. Whenever JavaScript code contains such a pair of slash marks (that are not inside of a quoted string), everything from that point to the end of the line is treated as a comment. A comment is intended for human consumption only and is completely ignored when the script is run.

The function definition

The function definition in Listing 1 (p. 89) consists of the three lines of code following the comment that begins with "//Define the function..."

As explained earlier, this function definition contains:

- The **function** keyword
- The function name: **getHalf**
- An argument list containing a single parameter named **incomingParameter**
- A function body enclosed in a pair of matching curly brackets

Simpler than the surface area of a sphere

The function named **getHalf** is somewhat simpler than one that could be used to calculate the surface area of a sphere, but the basic concept is the same. This function expects to receive one parameter.

The code in the body of the function uses the division operator, "/", to divide the value of the incoming parameter by 2. Then it returns the result of that calculation. When the function returns, the return value will replace the call to the function in the calling script.

Two calls to the function

The function is called twice in the body of the script in Listing 2 (p. 92) , passing a different value for the parameter during each call.

Each call to the function named **getHalf** is embedded as one of the elements in the argument list following **document.write** .

write is also a function

Although I didn't mention this earlier because you weren't ready for it yet, **write** is also the name of a function. However, the **write** function is predefined in JavaScript and we can use it to display information in the browser window without having to define it first.

(Actually, **write** is a special kind of a function that we call a **method** , but you don't need to worry about that unless you want to dig much deeper into the object-oriented aspects of JavaScript.)

Two calls to the getHalf function

The script in Listing 2 (p. 92) calls the function named **getHalf** twice in two different locations, each of which is embedded in the argument list of a call to the **write** method. Each call passes a different parameter value to the **getHalf** function.

The order of operations

When a call to a function or method (such as the call to the **write** method) includes a call to another function or method in its argument list, the call to the function in the argument list must be completed before the call can be made to the function having the argument list. Therefore in each of these two cases, the call to the **getHalf** function must be completed before the call to the **write** method can be executed.

Output from script in Listing 2 (p. 92)

Each call to the **getHalf** function returns a value that is half the value of its incoming parameter.

As I mentioned earlier, when the function returns, the returned value replaces the call to the function. Therefore, in each case, the returned value from the call to the **getHalf** function becomes part of the argument list for the **write** method before that method is called. This causes the two calls to the **write** method in Listing 2 (p. 92) to display the values returned from the calls to the **getHalf** function as shown in Figure 2 (p. 93) .

Figure 2 . Output from script in Listing 2.

```
    Call getHalf for 10.6
Half is: 5.3
Call getHalf again for 12.3
Half is: 6.15
```

Table 5.4

Good programming design

It is a principle of good programming design that each function should perform only one task, and should perform it well. The task performed by the function named **getHalf** is to calculate and return half the value that it receives, and it does that task very well.

5.4.6.2 Arithmetic operators

The division operation in Listing 2 (p. 92) introduced the use of arithmetic operators.

In computer programming jargon, we speak of operators and operands. Operators operate on operands.

As a real-world example, if you were to go to the hospital for knee surgery, the surgeon would be the **operator** and you would be the **operand** . The surgeon would operate on you.

Binary operators

The operators that we will use in the modules in this collection will usually be restricted to those that have two operands, a **left operand** and a **right operand** . (Operators with two operands are commonly called **binary operators** .)

In the function named **getHalf** in Listing 2 (p. 92) , the "/" character is the division operator. The left operand is **incomingParameter** and the right operand is **2** . The result is that the incoming parameter is divided by the right operand (2).

Binary arithmetic operators

The binary arithmetic operators supported by JavaScript are shown in Figure 3 (p. 94) .

Figure 3 . Binary arithmetic operators.

```
+ Addition:        Adds the operands
- Subtraction:     Subtracts the right operand from the left operand
* Multiplication:  Multiplies the operands
/ Division:        Divides the left operand by the right operand
% Modulus:         Returns integer remainder of dividing the left operand
                   by the right operand
```

Table 5.5

We will use these operators extensively as we work through the physics exercises in future modules.

5.4.6.3 Sequence

Of the four items listed under Structured programming (p. 90) earlier, the simplest one is **sequence** .

The concept of sequence in structured programming simply means that code statements can be executed in sequential order. Listing 2 (p. 92) provides a good example of the sequential execution of statements. The code in Listing 2 (p. 92) shows four sequential statements that begin with **document.write.**

5.4.6.4 Selection

The next item that we will discuss from the list under Structured programming (p. 90) is **selection** . While not as simple as **sequence** , selection is something that you probably do many times each day without even thinking about it. Therefore, once you understand it, it isn't complicated.

General syntax for selection statement

The general syntax for a selection statement (often called an **if-else** statement) is shown in Figure 4 (p. 94) .

Figure 4 . General syntax for selection statement.

```
if(conditional expression is true){
  execute code
}else{//optional
  execute alternative code
}//end selection statement
```

Table 5.6

A selection statement performs a logical test that returns either true or false. Depending on the result, specific code is executed to control the behavior of the script.

A real-world analogy

Figure 5 (p. 95) shows a real-world analogy of a selection statement that you might make on your day off.

Figure 5 . Real-world analogy of a selection statement.

```
if(it is not raining){
  Play tennis
  Go for a walk
  Relax on the beach
}else{//optional
  Make popcorn
  Watch TV
}//end selection statement
```

Table 5.7

In this analogy, you would check outside to confirm that it is not raining. If the condition is true (meaning that it isn't raining), you would play tennis, go for a walk, and then relax on the beach. If it is raining, (meaning that the test condition is false), you would make some popcorn and relax in front of the TV.

The else clause is optional

Note that when writing JavaScript code, the **else** clause shown in Figure 5 (p. 95) is optional. In other words, you might choose to direct the script to take some specific action if the condition is true, but simply transfer control to the next sequential statement in the script if the condition is false.

A selection script example

Listing 3 (p. 96) shows a sample script containing two selection statements (commonly called **if-else** statements) in sequence.

Listing 3 . A selection script example.

```
    <!-- File JavaScript03.html -->
<html><body>
<script language="JavaScript1.3">

if(3 > 2){
document.write("3 is greater than 2.","</br>")
}else{
document.write("3 is not greater than 2.")
}//end if

if(3 < 2){
document.write("3 is less than 2.","</br>")
}else{
document.write("3 is not less than 2.")
}//end if

</script>
</body></html>
```

Table 5.8

If 3 is greater than 2...

The conditional clause in the first **if** statement in Listing 3 uses the "greater-than" relational operator ">" to determine if the literal value 3 is greater than the literal value 2, producing the first line of output shown in Figure 6 (p. 96) .

Figure 6 . Output from script in Listing 3 (p. 96) .

```
    3 is greater than 2.
3 is not less than 2.
```

Table 5.9

The test returns true

Since 3 is always greater than 2, the statement in the line immediately following the first test in Listing 3 (p. 96) is executed and the code in the line following the word **else** is skipped producing the first line of output text shown in Figure 6 (p. 96) .

If 3 is less than 2...

The conditional clause in the second **if** statement in Listing 3 (p. 96) uses the "less-than" relational operator (see Figure 7 (p. 97)) to determine if the literal value 3 is less than the literal value 2.

The test returns false

Since 3 isn't less than 2, the statement in the line immediately following the second test in Listing 3 (p. 96) is skipped and the statement in the line immediately following the second word **else** is executed producing the second line of output text shown in Figure 6 (p. 96) .

5.4.6.5 Relational and logical operators

I doubt that I will need to use logical operators in the modules in this collection. If I do, I will explain them at the time. However, I will use relational operators.

The relational operators that are supported by JavaScript are shown in Figure 7 (p. 97) .

Figure 7 . Relational operators.

```
>  Left operand is greater than right operand
>= Left operand is greater than or equal to right operand
<  Left operand is less than right operand
<= Left operand is less than or equal to right operand
== Left operand is equal to right operand
!= Left operand is not equal to right operand
```

Table 5.10

As with the arithmetic operators discussed earlier, we will use these operators extensively as we work through the physics exercises in future modules.

5.4.6.6 Variables

The next item in the list under Structured programming (p. 90) is **repetition** . Before I can explain repetition, however, I need to explain the use of variables.

What is a variable?

You can think of a variable as the symbolic name for a pigeonhole in memory where the script can store a value. The script can change the values stored in that pigeonhole during the execution of the script. Once a value has been stored in a variable, that value can be accessed by calling out the name of the variable.

Variable names

Variable names must conform to the naming rules for identifiers. A JavaScript identifier must start with a letter or underscore "_". Following this, you can use either letters or the symbols for the digits (0-9) in the variable name.

JavaScript is case sensitive. Therefore letters include the characters "A" through "Z" (uppercase) and the characters "a" through "z" (lowercase).

Declaring a variable

In many languages, including JavaScript, you must *declare* a variable before you can use it. However, JavaScript is very loose in this regard. There are two ways to declare a variable in JavaScript:

- By simply assigning it a value; for example, x = 10
- By using the keyword **var** ; for example, **var** x = 10

Scope

When working with variables in JavaScript and other languages as well, you must always be concerned about an issue known as scope. Among other things, scope determines which statements in a script have access to a variable.

Two kinds of variables

JavaScript recognizes two kinds of variables:

- local
- global

Local variables are variables that are declared inside a function. Global variables are variables that are declared outside a function.

Scope

Local variables may only be accessed by other code within the same function following the declaration of the variable. Hence, the scope of local variables is limited to the function or method in which it is declared.

Global variables may be accessed by any code within the script following the declaration of the variable. Hence, the scope of global variables is the entire script.

Use of the keyword var

Using **var** to declare a global variable is optional. However, you must use **var** to declare a variable inside a function (a local variable).

A sample script that uses variables

A sample script that uses variables to store data is shown in Listing 4 (p. 98) .

Listing 4 . A sample script that uses variables.

```
    <!-- File JavaScript04.html -->
<html><body>
<script language="JavaScript1.3">

//Define the function named getArea()
function getArea(theRadius) {
  //declare a local variable
  var theArea

  //calculate the area
  theArea = Math.PI * theRadius * theRadius
  return theArea
}//end function getArea()
//=======================================//

//declare a global variable without keyword var
area = getArea(3.2)//call the function
document.write("Area is: " + area)

</script>
</body></html>
```

Table 5.11

The area of a circle

Listing 4 (p. 98) begins by defining a function that will compute and return the area of a circle given the radius of the circle as an incoming parameter.

The code in the function begins by declaring a variable named **theArea** . Effectively, this declaration sets aside a pigeon hole in memory and gives it the name **theArea** . Once the variable is declared, it can be accessed by calling out its name. It can be used to store data, or it can be used to retrieve data previously stored there.

Behavior of the function named getArea

You may recall that the area of a circle is calculated by multiplying the mathematical constant PI by the radius squared. There is no squaring operator in JavaScript. Therefore, you can square a value by multiplying it by itself.

PI times the square of the radius

The code in the function named **getArea** in Listing 4 (p. 98) computes the area and uses the **assignment** operator "=" to store the result in the variable named **theArea** . This statement represents a case where a value is being stored in a variable (assigned to the variable) for safekeeping.

Then the code in the function extracts the value stored in the variable named **theArea** and returns that value. After that, the function terminates and returns control to the place in the calling script from which it was originally called.

The variable named area

Further down the page in Listing 4 (p. 98) , the script declares a variable named **area** without using the keyword **var** . (Note, however, that the keyword **var** could have been used here. I prefer that approach for reasons that I won't get into here.)

The script calls the **getArea** function, passing a radius value of 3.2 as a parameter. As you learned earlier, the value returned by the function replaces the call to the function, which is then assigned to the variable named **area** .

Display the results

Then the script calls the **write** method to display some text followed by the value stored in the variable named **area** , producing the output shown in Figure 8 (p. 99) in the browser window.

Figure 8 . Output from script in Listing 4 (p. 98) .

```
Area is: 32.169908772759484
```

Table 5.12

5.4.6.7 The string concatenation operator

The code in Listing 4 (p. 98) exposes another operator that I will refer to as the *string concatenation operator* .

Note the argument list for the call to the **write** method in Listing 4 (p. 98) . In addition to being used to perform numeric addition, the plus operator "+" can be used to concatenate (join) two strings.

If two strings are joined by the + operator, the two strings will produce a new string that replaces the combination of the two original strings and the + operator.

If the left operand to the + operator is a string and the right operand is a numeric value (or the name of a variable containing a numeric value), the numeric value will be replaced by a string of characters that represent that numeric value and the two strings will be concatenated into a single string.

5.4.6.8 Repetition

The last item in the list under Structured programming (p. 90) is **repetition** , and that will be the topic of this section.

Repetition (also referred to as looping) means the act of causing something to repeat.

A repetition or loop is a set of commands that executes repeatedly until a specified condition is met.

JavaScript supports three loop statements:

- while
- for
- do-while

The while loop

The **while** loop is not only the simplest of the three, it is also the most fundamental. It can be used to satisfy any requirement for repetition in a JavaScript script. The other two exist solely for added convenience in some situations. Therefore, I will concentrate on the **while** loop, and leave the other two be be discussed in a future module, if at all.

Loop while a condition is true

A **while** loop executes the statements in its body for as long as a specified condition evaluates to true. The general syntax of a **while** loop is shown in Figure 9 (p. 100) .

Figure 9 . General syntax for a while loop.

```
while(condition is true){
  //Execute statements in body of loop.
}//end while statement
```

Table 5.13

When the condition is no longer true...

When the conditional expression evaluates to false, control passes to the next statement following the **while** loop.

while loops can be nested inside of other statements, including other **while** loops.

An infinite loop

As with all loop statements, you must be careful to make certain that the conditional expression eventually evaluates to false. Otherwise, control will be trapped inside the **while** loop in what is commonly called an infinite loop.

A simple while loop

The script in Listing 5 (p. 100) illustrates the use of a simple **while** loop.

Listing 5 . A simple while loop.

```
    <!-- File JavaScript05.html -->
<html><body>
<script language="JavaScript1.3">

cnt = 4//initialize a counter variable
while(cnt >= 0){//begin while loop
  //display value of counter
  document.write("cnt = " + cnt + "</br>")
  cnt = cnt - 1//decrement counter
}//end while loop

document.write("The End.");

</script>
</body></html>
```

Table 5.14

A counting loop

This script initializes the value of a counter variable named **cnt** to 4. Control continues to loop (iterate) for as long as the value of the counter is greater than or equal to zero. During each iteration of the loop, the current value of **cnt** is displayed and then the value of **cnt** is reduced by a value of 1.

Repeat the test

At that point, control returns to the top of the loop where the test is repeated. This process produces the first five lines of output text shown in Figure 10 (p. 101) .

Figure 10 . Output from script in Listing 5 (p. 100) .

```
    cnt = 4
cnt = 3
cnt = 2
cnt = 1
cnt = 0
The End.
```

Table 5.15

When the test returns false...

When the test in Listing 5 (p. 100) returns false, meaning that **cnt** is no longer greater than or equal to zero, control exits the **while** loop and goes to the next statement following the **while** loop. This is the statement that calls the **write** method to display the last line of text in Figure 10 (p. 101) .

5.4.7 Programming errors

From time to time, we all make errors when writing scripts or programs. Typical errors include typing a period instead of a comma, failing to include a matching right parenthesis, etc. Usually, when you make a programming error using JavaScript, some or all of the script simply doesn't execute.

Finding and fixing the errors

The worst thing about programming errors is the need to find and fix them. The Firefox and Google Chrome browsers have easy-to-use mechanisms to help you identify the cause of the problem so that you can fix it. Internet Explorer probably has similar capability, but so far, I haven't figured out how to access it.

My recommendation is to simply open the files containing your JavaScript code in either Firefox or Google Chrome. Or, you can open the file in Internet Explorer, but if it doesn't do what you expect it to do, open it again in Google Chrome or Firefox for assistance in finding and fixing the problem.

5.4.7.1 Assistance using Google Chrome

You can open the JavaScript console in the Chrome browser by holding down the Ctrl key and the Shift key and pressing the J key. The console will open at the bottom of the Chrome browser window. You can also close the console with the same keystroke.

The format of the console is a little messy and may be difficult for blind students to navigate. However, it can be useful in locating errors if you can navigate it.

An error message in the console

If you open an html file containing a JavaScript error in the browser while the console is open, an error message will appear in the console. For example, I am looking at such an error as I type this document. It consists of a round red circle with a white x followed by the following text:

"Uncaught SyntaxError: Unexpected number"

The file name and line number

On the far right side of the same line is text that reads junk.html:23. That is the name of the file and the line number in that file containing the error. That text is a hyperlink. If the hyperlink is selected, another part of the console opens showing the offending line of JavaScript code. I'm not sure that would be useful to a blind student, because navigation within the console would probably be a problem.

The description is unreliable

Also, in this particular case the description of the error isn't very useful in determining the cause of the error although sometimes it may be useful. My advice is not to put too much faith in that description. The error was actually a missing relational operator in a comparison clause.

The line number is very important

Probably the most useful information is the line number that you can use to go back and examine your source code, looking for an error in that line of code.

5.4.7.2 Assistance using Firefox

You can open an error console when using the Firefox browser by holding down the Ctrl key and the Shift key and pressing the J key. The console will open in a separate window. Unlike with Chrome, repeating the keystroke won't close the error console.

An error message in the console

If you open an html file containing a JavaScript error in the browser while the error console is open, an error message will appear in the console. For example, I am looking at such an error as I type this document. It consists of a round red circle with a white x and the following text:

missing) after condition

file: –html file name and path here– Line: 23

while(h 0){

The middle line is a hyperlink

The middle line of text that contains the file name to the left of the line number is a hyperlink. In this case, the hyperlink may be useful, depending on how things are treated by your screen reader and your Braille display. If you select the link, a window will open showing the source code with the problem line highlighted. Pressing the right arrow key will cause a blinking cursor to appear between the first and second characters in that line. If a blind student can identify the highlighted line with the blinking cursor, that would be useful.

The description is unreliable

As with Chrome, in this particular case the description of the error isn't very useful in determining the cause of the error although sometimes it may be useful. My advice is not to put too much faith in that description. The error was actually a missing relational operator in a comparison clause.

The line number is very important

Probably the most useful information is the line number that you can use to go back and examine your source code, looking for an error in that line of code.

5.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

5.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

5.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: JavaScript
- File: Phy1010.htm
- Revised: 09/30/15
- Keywords:
  - physics
  - accessible
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - function
  - argument
  - parameter
  - sequence
  - selection
  - repetition
  - structured programming
  - string
  - arithmetic operators
  - relational operators
  - string concatenation
  - variable

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 6

Brief Trigonometry Tutorial[1]

6.1 Table of Contents

- Preface (p. 105)
  - General (p. 105)
  - Prerequisites (p. 106)
  - Viewing tip (p. 106)
    - Figures (p. 106)
    - Listings (p. 107)
  - Supplemental material (p. 107)
- General background information (p. 107)
- Discussion (p. 108)
  - Degrees versus radians (p. 108)
  - Sine, cosine, and tangent (p. 110)
    - The sine and arcsine of an angle (p. 117)
    - The cosine and arccosine of an angle (p. 122)
    - The tangent and arctangent of an angle (p. 126)
  - Dealing with quadrants (p. 130)
  - Normal (non-mirror-image) graphics (p. 145)
- Run the scripts (p. 153)
- Resources (p. 153)
- Miscellaneous (p. 153)

6.2 Preface

6.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

[1] This content is available online at <http://cnx.org/content/m37435/1.8/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

Many of the computational requirements for an introductory physics course involve trigonometry. This module provides a brief tutorial on trigonometry fundamentals that is designed to be accessible to blind students.

6.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer screen (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]). *(The purpose of this module is to help you to gain such an understanding.)*
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the manual creation and use of tactile graphics as described here [13] .

6.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

6.2.3.1 Figures

- Figure 1 (p. 109) . Output for script in Listing 1.
- Figure 2 (p. 115) . Mirror image from file Phy1020b1.svg.
- Figure 3 (p. 116) . Page Setup for file Phy1020b1.svg.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

- Figure 4 (p. 116) . Text values for Braille keys in file Phy1020b2svg.
- Figure 5 (p. 119) . Output for script in Listing 2.
- Figure 6 (p. 120) . Output for script in Listing 3.
- Figure 7 (p. 121) . Interesting sine equations.
- Figure 8 (p. 124) . Interesting cosine equations.
- Figure 9 (p. 125) . Output for script in Listing 5
- Figure 10 (p. 126) . Two very important equations.
- Figure 11 (p. 129) . Interesting tangent equations.
- Figure 12 (p. 130) . Output for script in Listing 7.
- Figure 13 (p. 134) . Sinusoidal values at 90-degree increments.
- Figure 14 (p. 135) . Sinusoidal values at 45-degree increments.
- Figure 15 (p. 136) . Sinusoidal values at 22.5-degree increments.
- Figure 16 (p. 140) . Mirror image plot of cosine and sine curves from the file named Phy1020a1svg.
- Figure 17 (p. 142) . Algebraic signs versus quadrants.
- Figure 18 (p. 144) . Output from the code in Listing 9.
- Figure 19 (p. 149) . Normal image from file Phy1020b1.svg.
- Figure 20 (p. 153) . Normal image plot of cosine and sine curves from the file named Phy1020a1svg.

6.2.3.2 Listings

- Listing 1 (p. 109) . Conversions between radians and degrees.
- Listing 2 (p. 118) . Arcsin of 3-4-5 triangle.
- Listing 3 (p. 120) . Finding length of the opposite side.
- Listing 4 (p. 123) . Arccosine of 3-4-5 triangle.
- Listing 5 (p. 125) . Finding the length of the adjacent side.
- Listing 6 (p. 128) . Arctan of 3-4-5 triangle.
- Listing 7 (p. 129) . Finding the length of the opposite side.
- Listing 8 (p. 132) . Sinusoidal amplitude versus angle.
- Listing 9 (p. 143) . A function to deal with quadrants.

6.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

6.3 General background information

Many of the computational requirements for an introductory physics course involve trigonometry. This module provides a brief tutorial on trigonometry fundamentals that is designed to be accessible to blind students.

Sine, cosine, and tangent

There are many topics, such as identities, that are covered in an introductory trigonometry course that won't be covered in this module. Instead, this module will concentrate mainly on performing computations on right angles using the sine, cosine, and tangent of an angle.

If I find it necessary to deal with identities in a later module, I will come back and update this module accordingly.

[14] http://www.dickbaldwin.com/toc.htm

6.4 Discussion

Download files

You will need to download two svg graphics files to complete the work in this module. Click this link to download a zip file named Phy1020.zip [15] containing those svg files.

If you don't already have it, you may also need to download and install the free IVEO Viewer software. As of this writing, the Viewer is available for downloading at http://www.viewplus.com/products/software/hands-on-learning/ [16] .

Graph board and protractor

Unless you can create tactile graphics on paper, you will need your graph board and your protractor to perform the exercises in this module. Please prepare your graph board with perpendicular horizontal and vertical axes with the origin located near the center of the graph board.

6.4.1 Degrees versus radians

The most common unit of angular measurement used by the general public is the degree. As you are probably aware, there are 360 degrees in a circle.

The most common unit of angular measurement used by scientists and engineers is the radian.

(If you would like more background on radians, go to http://www.clarku.edu/~djoyce/trig/ [17] .)

Conversions between radians and degrees

You may or may not be aware that one radian is equal to approximately 57.3 degrees. It is easier to remember, however, that 180 degrees is equal to PI radians where PI is the mathematical constant having an approximate value of 3.14159. We will use this latter relationship extensively to convert from degrees to radians and to convert from radians to degrees while working through the exercises in these modules.

An exercise involving degrees and radians

Let's do a short exercise involving degrees and radians. Please create an html file containing the code shown in Listing 1 (p. 109) and open it in your browser.

[15] http://cnx.org/content/m37435/latest/Phy1020.zip
[16] http://www.viewplus.com/products/software/hands-on-learning/
[17] http://www.clarku.edu/~djoyce/trig/

Listing 1 . Conversions between radians and degrees.

```
    <!-- File JavaScript01.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var degrees = 90
var radians = toRadians(degrees)
document.write("degrees = " + degrees +
               " : radians = " + radians + "</br>")
radians = 1
degrees = toDegrees(radians)
document.write("radians = " + radians +
               " : degrees = " + degrees + "</br>")

radians = Math.PI
degrees = toDegrees(radians)
document.write("radians = " + radians +
               " : degrees = " + degrees)

</script>
</body></html>
```

Table 6.1

Output for script in Listing 1

When you open the file in your browser, the text shown in Figure 1 (p. 109) should be displayed in the browser window.

Figure 1 . Output for script in Listing 1.

```
    degrees = 90 : radians = 1.5707963267948965
radians = 1 : degrees = 57.29577951308232
radians = 3.141592653589793 : degrees = 180
```

Table 6.2

The toRadians and toDegrees functions

Because it will frequently be necessary for us to convert between degrees and radians, I decided to write two functions that we will use to make those conversions. That will eliminate the need for us to stop and think about the conversion (and possibly get it backwards) when writing code. We will simply call the function that performs the conversion in the required direction.

The **toRadians** function expects to receive an input parameter describing an angle in degrees and returns the value for that same angle in radians.

The **toDegrees** function expects to receive an input parameter describing an angle in radians and returns the value for that same angle in degrees.

Global variables named degrees and radians

The code in Listing 1 (p. 109) begins by declaring global variables named **degrees** and **radians** . The variable named **degrees** is initialized to 90 degrees.

The **toRadians** function is called to convert that value of degrees to radians. The returned value in radians is stored in the variable named **radians** .

Display contents of both variables

Then the **document.write** method is called to display the values contained in both variables, producing the first line of output text shown in Figure 1 (p. 109) .

Modify variable contents, convert, and display again

Following that, a value of 1 is assigned to the variable named **radians** . The **toDegrees** function is called to convert that value to degrees, and the result is stored in the variable named **degrees** .

Once again, the **document.write** method is called to display the current contents of both variables, producing the second line of output text shown in Figure 1 (p. 109) .

One more time

Finally, the mathematical constant, PI, is stored in the variable named **radians** . Then that value is converted to degrees and stored in the variable named **degrees** . The current values in both variables are displayed, producing the last line of output text shown in Figure 1.

And the results were...

As you can see from Figure 1 (p. 109) ,

- Ninety degrees is equal to 1.57 radians
- One radian is equal to 57.296 degrees
- 3.14 (PI) radians is equal to 180 degrees

A template

You might want to save your html file as a template for use with future exercises that require conversions between radians and degrees. This will be particularly useful when we write scripts that use JavaScript's built-in trigonometric methods. Those methods deal with angles almost exclusively in radians while we tend to think of angles in degrees. We will use these two functions to perform conversions between degrees and radians when required.

6.4.2 Sine, cosine, and tangent

An exercise involving a right triangle

For the next exercise, I would like for you to create a right triangle on your graph board by placing pushpins at the following coordinates:

- The origin
- x=3, y=0
- x=3, y=4

(If you are able to use the svg file mentioned below (p. 111) to create tactile graphics, it probably won't be necessary for you to do the graph-board exercise. You can explore the tactile graphics instead.)

The vertices of a right triangle

Each pushpin represents a vertex of a right triangle. If you enclose all three pushpins with a rubber band, you will have "drawn" a right triangle with its base on the horizontal axis.

The base of the triangle will have a length of three units. Another side of the triangle will have a length of four units. The hypotenuse of the triangle will have still another length.

The angle at the origin

The base and the hypotenuse will form an angle at the origin opening outward to the right. As mentioned before, the base will be on the horizontal axis. The side that connects the base and the far end of the hypotenuse will be parallel to the vertical axis.

Names for the three sides of the right triangle

Lets establish some names for the three sides of the right triangle when located on the graph board in this manner.

We will continue to refer to the hypotenuse as the hypotenuse and abbreviate it **hyp** . We will refer to the base as the *adjacent* side relative to the angle at the origin and abbreviate it **adj** . We will refer to the third side as the *opposite* side relative to the angle at the origin and abbreviate it **opp** .

Tactile graphics – the file named Phy1020b1.svg

Much of the material in the next several paragraphs will only make sense to you if you are very familiar with the module named Manual Creation of Tactile Graphics [18] .

When you downloaded the zip file named Phy1020.zip using the link given above (p. 108) , you should have found that the zip file contains a file named Phy1020b1.svg.

The purpose of this file is to make it possible for you to create tactile graphics for the right triangle. The procedure for creating the tactile graphics is explained in the earlier module named Manual Creation of Tactile Graphics [19] .

Image from file Phy1020b1.svg

For the benefit of any sighted persons that may be assisting you, Figure 2 (p. 115) shows a reduced version of the graphic contained in the file named Phy1020b1.svg. This is a mirror image of the image that is to be presented to the student after embossing. A non-mirror-image version is presented in Figure 19 (p. 149) .

[18] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[19] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

A 3-4-5 Triangle
4
Not drawn to scale
Hypotenuse
opp
hyp
Opposite side
53.13 degrees
adj
0
Adjacent side
3
0

A 3-4-5 Triangle
Not drawn to scale
Hypotenuse
opp
hyp
Opposite side
53.13 degrees
adj
Adjacent side
0
3
4

A 3-4-5 Triangle
4
Not drawn to scale
Hypotenuse
opp
hyp
Opposite side
53.13 degrees
adj
0
Adjacent side
0
3

Figure 2 . Mirror image from file Phy1020b1.svg.

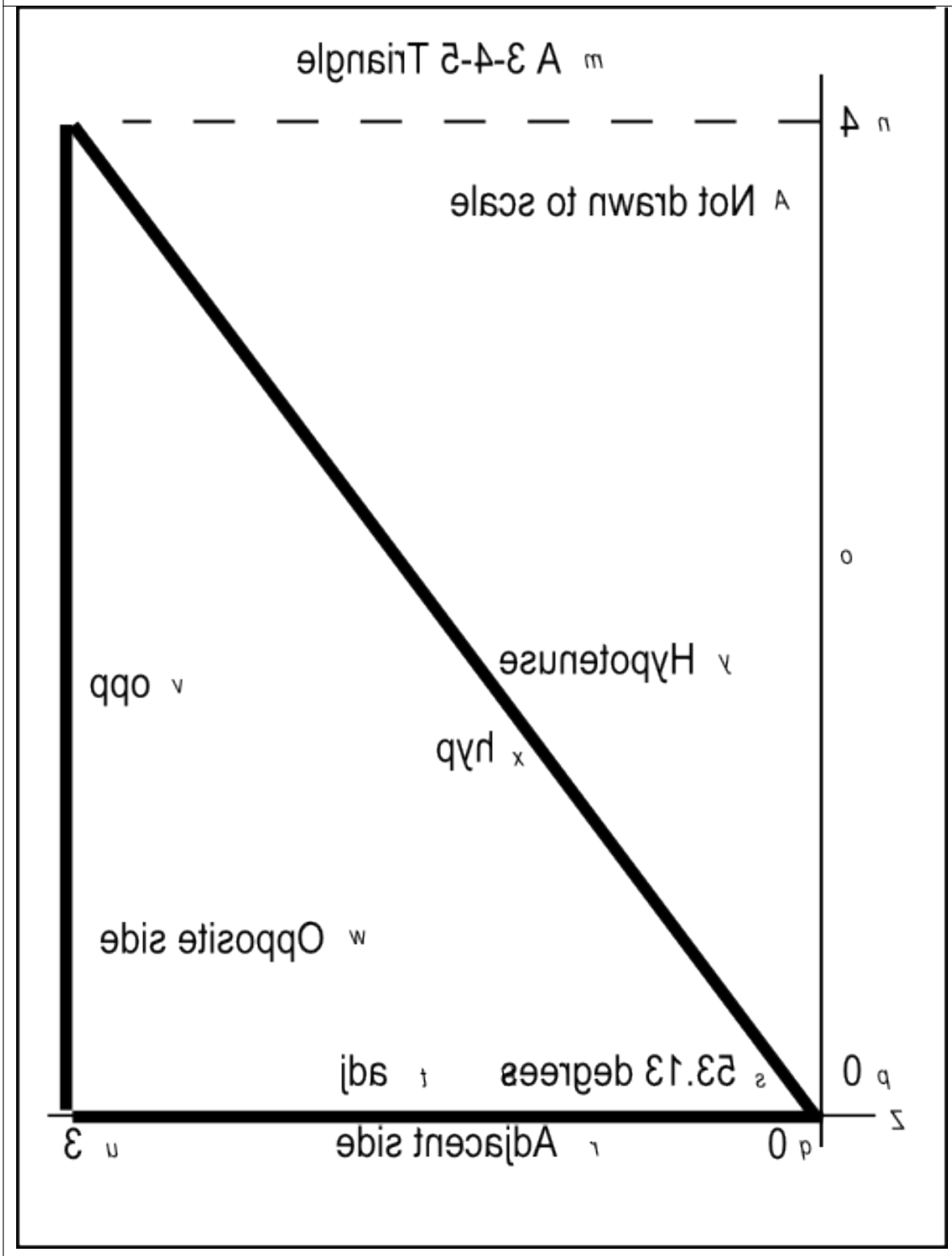


Table 6.3

Page Setup information

If you use the IVEO Viewer software to print this svg file, you will need to set up the page by selecting Page Setup on the IVEO File menu. The Page Setup information for this file is shown in Figure 3 (p. 116) . These are the default settings for Letter (Portrait) in IVEO.

Figure 3 . Page Setup for file Phy1020b1.svg.

```
    Paper List: Letter (Portrait)
Paper Width: 8.5 inch
Paper Height: 11.0 inch
Orientation: portrait
Left Margin: 0.75 inch
Right Margin: 0.3 inch
Top Margin: 0.75 inch
Bottom Margin: 0.3 inch
```

Table 6.4

Braille keys

Note that the file named Phy1020b1.svg and Figure 2 (p. 115) contain keys using the characters from "m" through "z" and "A" that can be used to create Braille characters during manual embossing. The key characters are shown in an oblique font that is smaller than the normal text. The purpose of these keys is explained in the earlier module titled Manual Creation of Tactile Graphics [20] .

Key-value pairs

Figure 4 (p. 116) contains the text values associated with each of the Braille keys.

Figure 4 . Text values for Braille keys in file Phy1020b2svg.

```
m: A 3-4-5 Triangle
n: 4
o: Vertical axis
p: 0
q: 0
r: Adjacent side
s: 53.13 Degrees
t: adj
u: 3
v: opp
w: Opposite side
x: hyp
y: Hypotenuse
z: Horizontal axis
A: Not drawn to scale
```

[20] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

Table 6.5

The length of the hypotenuse

Now that you have your right triangle on the graph board, or you have access to tactile graphics created from the svg file, and you know the lengths of the adjacent and opposite sides, do you remember how to calculate the length of the hypotenuse?

The Pythagorean theorem

Hopefully you know that for a right triangle, the square of the hypotenuse is equal to the sum of the squares of the two other sides. Thus, the length of the hypotenuse is equal to the square root of the sum of the squares of the other two sides.

In this case we can do the arithmetic in our heads to compute the length of the hypotenuse. (I planned it that way.)

The square of the adjacent side is 9. The square of the opposite side is 16. The sum of the squares is 25, and the square root of 25 is 5. Thus, the length of the hypotenuse is 5.

A 3-4-5 triangle

You have created a rather unique triangle. You have created a right triangle in which the sides are either equal to, or proportional to the integer values 3, 4, and 5.

I chose this triangle on purpose for its simplicity. We will use it to investigate some aspects of trigonometry.

6.4.2.1 The sine and arcsine of an angle

You will often hear people talk about the sine of an angle or the cosine of an angle. Just what is the sine of an angle anyway?

Although the sine of an angle is based on very specific geometric considerations involving circles (see http://www.clarku.edu/~djoyce/trig/ [21]), for our purposes, the sine of an angle is simply a ratio between the lengths of two different sides of a right triangle.

A ratio of two sides

For our purposes, we will say that the sine of an angle is equal to the ratio of the opposite side and the hypotenuse. Therefore, in the case of the 3-4-5 triangle that you have on your graph board, the **sine of the angle** at the origin is equal to 4/5 or 0.8.

If we know the lengths of the hypotenuse and the opposite side, we can compute the sine and use it to determine the value of the angle. (We will do this later using the arcsine.)

Conversely, if we know the value of the angle but don't know the lengths of the hypotenuse and/or the opposite side, we can obtain the value of the sine of the angle using a scientific calculator (such as the Google calculator) or lookup table.

The sine of an angle – sample computation

Enter the following into the Google search box:

sin(53.13010235415598 degrees)

The following will appear immediately below the search box:

sin(53.13010235415598 degrees) = 0.8

This matches the value that we computed above (p. 117) as the ratio of the opposite side and the hypotenuse.

The arcsine (inverse sine) of an angle

The arcsine of an angle is the value of the angle having a given sine value. In other words, if you know the value of the sine of an unknown angle, you can use a scientific calculator or lookup table to find the value of the angle.

[21] http://www.clarku.edu/~djoyce/trig/

For example, we know that the sine of the angle at the origin on your graph board is 4/5. From that, we can determine the value of the angle. However, we probably can't do this calculation in our heads so we will use the Google calculator to compute the value of the angle.

The arcsine of an angle – sample computation

Enter the following into the Google search box:

arcsin(4/5) in degrees

The following will appear immediately below the search box:

arcsin(4/5) = 53.1301024 degrees

This is the angle that corresponds to a ratio of the opposite side to the hypotenuse of 4/5.

We can also write a JavaScript script to perform the calculation, which we will do shortly.

Getting the angle for a known sine value

Please use your protractor to measure and record the angle at the origin on your graph board, or measure it on your tactile graphic. Then create an html file containing the code shown in Listing 2 (p. 118) and open it in your browser.

Listing 2 . Arcsin of 3-4-5 triangle.

```
    <!-- File JavaScript02.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var opp = 4
var hyp = 5
var ratio = opp/hyp
var angRad = Math.asin(ratio)
var angDeg = toDegrees(angRad)

document.write("radians = " + angRad + "</br>")
document.write("degrees = " + angDeg)

</script>
</body></html>
```

Table 6.6

The output for the angle

When you open your html file in your browser, the output shown in Figure 5 (p. 119) should appear in the browser window.

Figure 5 . Output for script in Listing 2.

```
   radians = 0.9272952180016123
degrees = 53.13010235415598
```

Table 6.7

Did you measure the angle to be 53 degrees with your protractor. If so, congratulations. If not, you should probably take another look at it.

Define conversion functions

The code in Listing 2 (p. 118) begins by defining the functions named **toRadians** and **toDegrees** that we developed earlier in Listing 1 (p. 109) . (In this case, we will only need the function named **toDegrees** so I could have omitted the code for the function named **toRadians** .)

Declare and initialize variables

Then the code in Listing 2 (p. 118) declares and initializes variables to represent the lengths of the opposite side and the hypotenuse for the triangle on your graph board (**opp** and **hyp**). Then it computes and saves the ratio of the two. (We learned earlier that the ratio is the value of the sine of the angle at the origin even though we don't know the value of the angle.)

The built-in Math.asin method

JavaScript has a built-in method named **Math.asin** that receives the sine value for an unknown angle and returns the value of the corresponding angle in radians. (The **Math.asin** method has the same purpose at the word arcsin in the Google calculator.)

The returned value is an angle between -PI/2 and PI/2 radians. (I will have more to say about this later.)

Listing 2 (p. 118) calls the **Math.asin** method, passing the ratio (sine of the angle) as a parameter, and stores the returned value in a variable named **angRad** .

Then Listing 2 (p. 118) calls the **toDegrees** method, passing the value of **angRad** as a parameter and stores the returned value in a variable named **angDeg** .

Finally, Listing 2 (p. 118) calls the **document.write** method twice in success to display the angle values shown in Figure 5 (p. 119) .

Another exercise with a different viewpoint

Now let's approach things from a different viewpoint. Assume that

- You know the value of the angle in degrees.
- You know the length of the hypotenuse.
- You need to find the length of the opposite side.

Assume also that for some reason you can't simply measure the length of the opposite side. Therefore, you must calculate it. This is a common situation in physics, so let's see if we can write a script that will perform that calculation for us.

Please create an html file containing the code shown in Listing 3 (p. 120) and open the file in your browser.

Listing 3 . Finding length of the opposite side.

```
    <!-- File JavaScript03.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var hyp = 5
var angDeg = 53.13
var angRad = toRadians(angDeg)
var sine = Math.sin(angRad)

var opp = hyp * sine

document.write("opposite = " + opp + "</br>")

hyp = opp/sine

document.write("hypotenuse = " + hyp + "</br>")

</script>
</body></html>
```

Table 6.8

The output for the opposite side

When you open your html file in your browser, the output shown in Figure 3 should appear in your browser window.

Figure 6 . Output for script in Listing 3.

```
opposite = 3.999994640742543
hypotenuse = 5
```

Table 6.9

Computing length of opposite side with the Google calculator

We could also compute the length of the opposite side using the Google calculator.

The length of the opposite side – sample computation

Enter the following into the Google search box:

5*sin(53.1301024 degrees)

The following will appear immediately below the search box:

5 * sin(53.1301024 degrees) = 4

This is the length of the opposite side for the given angle and the given length of the hypotenuse.

Interesting equations

We learned earlier that the sine of the angle is equal to the ratio of the opposite side and the hypotenuse. We also learned that the angle is the arcsine of that ratio.

If we know any two of those values (**angle** , **opp** , **hyp**), we can find the third (with a little algebraic manipulation) as shown in Figure 7 (p. 121) .

Figure 7 . Interesting sine equations.

```
sine(angle) = opp/hyp

angle = arcsine(opp/hyp)
opp = hyp * sine(angle)
hyp = opp/sine(angle)
```

Table 6.10

Getting back to Listing 3

After defining the radian/degree conversion functions, Listing 3 (p. 120) declares and initializes variables representing the length of the hypotenuse and the angle in degrees. (Note that the angle in degrees was truncated to four significant digits, which may introduce a slight inaccuracy into the computations.)

Get and use the sine of the angle

That angle is converted to radians and passed as a parameter to the **Math.sin** method, which returns the value of the sine of the angle.

The value for the sine of the angle is then used in an algebraic equation to compute the length of the opposite side, which is displayed in Figure 6 (p. 120) . (This equation is one of the equations shown in Figure 7 (p. 121) .)

Looks very close to me

As you can see, the computed value for the opposite side shown in Figure 6 (p. 120) is extremely close to the known value of 4 units.

Re-compute the length of the hypotenuse

After that, the value of the hypotenuse is re-computed (as though it were the unknown in the problem) using the value of the sine and the recently computed value of the opposite side. (Once again, one of the equations from Figure 7 (p. 121) is used to perform the computation.) The output length for the hypotenuse is shown in Figure 6 (p. 120) , and it matches the known value.

Example usage of Math.asin and Math.sin methods

Listing 2 (p. 118) and Listing 3 (p. 120) provide examples of how to use the JavaScript **Math.asin** and **Math.sin** methods to find the angle, the opposite side, or the hypotenuse of a right triangle when the other two are known as shown by the equations in Figure 7 (p. 121) .

6.4.2.2 The cosine and arccosine of an angle

You are going to find the discussion in this section to be very similar to the discussion in the previous section on the sine and the arcsine of an angle.

Once again, although the cosine of an angle is based on very specific geometric considerations involving circles (see http://www.clarku.edu/~djoyce/trig/ [22]), for our purposes, the cosine of an angle is simply a ratio between the lengths of two different sides of a right triangle.

A ratio of two sides

For our purposes, we will say that the cosine of an angle is equal to the ratio of the adjacent side and the hypotenuse. Therefore, in the case of the 3-4-5 triangle that you have on your graph board, the cosine of the angle at the origin is equal to 3/5 or 0.6.

As before, if we know the lengths of the hypotenuse and the adjacent side, we can compute the cosine and use it to determine the value of the angle. (We will do this later.)

Conversely, if we know the value of the angle but don't know the lengths of the hypotenuse and/or the adjacent side, we can obtain the cosine value (the ratio of the adjacent side and the hypotenuse) using a scientific calculator or lookup table and use it for other purposes later.

The cosine of an angle – sample computation

Enter the following into the Google search box:

cos(53.13010235415598 degrees)

The following will appear immediately below the search box:

cos(53.13010235415598 degrees) = 0.6

This matches the ratio of the adjacent side to the hypotenuse for a 3-4-5 triangle.

The arccosine (inverse cosine) of an angle

The arccosine of an angle is the value of the angle having a given cosine value. In other words, if you know the value of the cosine of an unknown angle, you can use a scientific calculator or lookup table to find the value of the angle.

Getting the angle for a known cosine value

For example, we know that the cosine of the angle at the origin on your graph board is 0.6. From that, we can determine the value of the angle using either the Google calculator or JavaScript.

The arccosine of an angle – sample computation

Enter the following into the Google search box:

arccos(3/5) in degrees

The following will appear immediately below the search box:

arccos(3/5) = 53.1301024 degrees

This is the angle that corresponds to a ratio of the adjacent side to the hypotenuse of 3/5.

As you should expect. the computed angle is the same as before. We didn't change the angle, we simply computed it using a different approach.

Getting the angle using JavaScript

Please create an html file containing the code shown in Listing 4 (p. 123) and open it in your browser.

[22] http://www.clarku.edu/~djoyce/trig/

Listing 4 . Arccosine of 3-4-5 triangle.

```
    <!-- File JavaScript04.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var adj = 3
var hyp = 5
var ratio = adj/hyp
var angRad = Math.acos(ratio)
var angDeg = toDegrees(angRad)

document.write("radians = " + angRad + "</br>")
document.write("degrees = " + angDeg)

</script>
</body></html>
```

Table 6.11

Similar to a previous script

If you examine the code in Listing 4 (p. 123) carefully, you will see that it is very similar to the code in Listing 2 (p. 118) with a couple of exceptions:

- The variable **opp** having a value of 4 was replaced by the variable **adj** having a value of 3.
- The call to the **Math.asin** method was replaced by a call to the **Math.acos** method.

The output

When you load your html file into your browser, it should produce the output shown earlier in Figure 5 (p. 119) . In other words, we know that the angle at the origin didn't change. What changed was the manner in which we computed the value of that angle.

Different approaches to the same solution

In Listing 2 (p. 118) , we used the length of the hypotenuse and the length of the opposite side, along with the arcsine method to compute the angle.

In Listing 4 (p. 123) , we used the length of the hypotenuse and the length of the adjacent side, along with the arccosine method to compute the angle.

Which approach should you use?

As would be expected, since the angle didn't change, both approaches produced the same result. Deciding which approach to use often depends on the values that are available to use in the computation.

Sometimes you only have the lengths of the hypotenuse and the opposite side available, in which case you could use the arcsine. Sometimes you only have the lengths of the hypotenuse and the adjacent side

available, in which case you could use the arccosine. Sometimes you have the lengths of both the opposite side and the adjacent side in addition to the length of the hypotenuse, in which case you can use either approach.

Both approaches use the length of the hypotenuse

It is important to note however that both of these approaches require you to have the length of the hypotenuse. Later in this module we will discuss the tangent and arctangent for an angle, which allows us to work with the opposite side and the adjacent side devoid of the length of the hypotenuse. (Of course, if you have the lengths of the opposite side and the adjacent side, you can always find the length of the hypotenuse using the Pythagorean theorem.)

Interesting cosine equations

The equations in Figure 8 (p. 124) are similar to equations in Figure 7 (p. 121) . The difference is that the equations in Figure 7 (p. 121) are based on the use of the sine of the angle and the opposite side whereas the equations in Figure 8 (p. 124) are based on the use of the cosine of the angle and the adjacent side.

As you can see in Figure 8 (p. 124) , if you know any two of the values for **angle** , **adj** , and **hyp** , you can find the other value. This is illustrated in the script shown in Listing 5 (p. 125) , which produces the output shown in Figure 9 (p. 125) .

Figure 8 . Interesting cosine equations.

```
cosine(angle) = adj/hyp

angle = arccosine(adj/hyp)
adj = hyp * cosine(angle)
hyp = adj/cosine(angle)
```

Table 6.12

Finding the length of the adjacent side

The code in Listing 5 (p. 125) is very similar to the code in Listing 2 (p. 118) . The main difference is that Listing 2 (p. 118) is based on the use of the sine of the angle and the length of the opposite side whereas Listing 5 (p. 125) is based on the use of the cosine of the angle and the length of the adjacent side.

Listing 5 . Finding the length of the adjacent side.

```
    <!-- File JavaScript05.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var hyp = 5
var angDeg = 53.13
var angRad = toRadians(angDeg)
var cosine = Math.cos(angRad)

var adj = hyp * cosine

document.write("adjacent = " + adj + "</br>")

hyp = adj/cosine

document.write("hypotenuse = " + hyp + "</br>")

</script>
</body></html>
```

Table 6.13

No further explanation needed

Because of the similarity of Listing 5 (p. 125) and Listing 2 (p. 118) , no further explanation of the code in Listing 5 (p. 125) should be needed. As you can see from Figure 9 (p. 125) , the output values match the known lengths for the hypotenuse and the adjacent side for the triangle on your plot board.

Figure 9 . Output for script in Listing 5.

```
adjacent = 3.0000071456633126
hypotenuse = 5
```

Table 6.14

Computing length of adjacent side with the Google calculator

We could also compute the length of the adjacent side using the Google calculator.

The length of the adjacent side – sample computation

Enter the following into the Google search box:

5*cos(53.1301024 degrees)

The following will appear immediately below the search box:

5 * cos(53.1301024 degrees) = 3

This is the length of the adjacent side for the given angle and the given length of the hypotenuse.

Two very important equations

From an introductory physics viewpoint, two of the most important and perhaps most frequently used equations from Figure 7 (p. 121) and Figure 8 (p. 124) are shown in Figure 10 (p. 126) .

Figure 10 . Two very important equations.

```
opp = hyp * sine(angle)
adj = hyp * cosine(angle)
```

Table 6.15

These two equations are so important that it might be worth your while to memorize them. Of course, you will occasionally need most of the equations in Figure 7 (p. 121) and Figure 8 (p. 124) , so you should try to remember them, or at least know where to find them when you need them.

Vectors

As you will see later in the module that deals with vectors, you are often presented with something that resembles the hypotenuse of a right triangle whose adjacent side is on the horizontal axis and whose opposite side is parallel to the vertical axis.

The thing that looks like the hypotenuse of a right triangle is called a **vector** . It has a length and it has a direction. Typically, the direction is stated as the angle between the vector and the horizontal axis. Thus, the direction is analogous to the angle at the origin in the triangle on your graph board.

Horizontal and vertical components

For reasons that I won't explain until we get to that module, you will often need to compute the horizontal and vertical components of the vector. The horizontal component is essentially the adjacent side of our current right triangle. Thus, the value of the horizontal component can be computed using the second equation in Figure 10 (p. 126) .

The vertical component is essentially the opposite side of our current right triangle, and its value can be computed using the first equation in Figure 10 (p. 126) .

6.4.2.3 The tangent and arctangent of an angle

Once again, although the tangent of an angle is based on very specific geometric considerations involving circles (see http://www.clarku.edu/~djoyce/trig/ [23]), for our purposes, the tangent of an angle is simply a ratio between the lengths of two different sides of a right triangle.

A ratio of two sides

For our purposes, we will say that the tangent of an angle is equal to the ratio of the opposite side and the adjacent side. Therefore, in the case of the 3-4-5 triangle that you have on your graph board, the **tangent of the angle at the origin is equal to** 4/3 or 1.333.

Not limited to 1.0

[23] http://www.clarku.edu/~djoyce/trig/

Note that the absolute value for the sine and the cosine of an angle is limited to a maximum value of 1.0. However, the tangent of an angle is not so limited. In fact, the tangent of 45 degrees is 1.0 and the tangent of 90 degrees is infinity. This results from the length of the adjacent side, which is the denominator in the ratio, going to zero at 90 degrees.

Dividing by zero in a script is usually not a good thing. This is a pitfall that you must watch out for when working with tangents. I will provide code later on that shows you how deal with this issue.

Computing the tangent

If we know the lengths of the opposite side and the adjacent side, we can compute the tangent and use it for other purposes later without having to know the value of the angle.

Conversely, if we know the value of the angle but don't know the lengths of the adjacent side and/or the opposite side, we can obtain the tangent value using a scientific calculator or lookup table and use it for other purposes later.

The tangent of an angle – sample computation

Enter the following into the Google search box:

tan(53.13010235415598 degrees)

The following will appear immediately below the search box:

tan(53.13010235415598 degrees) = 1.33333333

This agrees with the ratio that we computed earlier (p. 126) .

The arctangent (inverse tangent) of an angle

The arctangent of an angle is the value of the angle having a given tangent value. (For example, as mentioned above, the arctangent of infinity is 90 degrees and the arctangent of 1.0 is 45 degrees.) In other words, if you know the value of the tangent of an unknown angle, you can use a scientific calculator or lookup table to find the value of the angle.

For example, we know that the tangent of the angle at the origin on your graph board is 1.333. From that, we can determine the value of the angle.

The arctangent of an angle – sample computation

Enter the following into the Google search box:

arctan(4/3) in degrees

The following will appear immediately below the search box:

arctan(4/3) = 53.1301024 degrees

We can also write a JavaScript script to perform the calculation.

Getting the angle for a known tangent value using JavaScript

Please create an html file containing the code shown in Listing 6 (p. 128) and open it in your browser.

Listing 6 . Arctan of 3-4-5 triangle.

```
    <!-- File JavaScript06.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var opp = 4
var adj = 3
var ratio = opp/adj
var angRad = Math.atan(ratio)
var angDeg = toDegrees(angRad)

document.write("radians = " + angRad + "</br>")
document.write("degrees = " + angDeg)

</script>
</body></html>
```

Table 6.16

The output from the script

Once again, when you open this file in your browser, the output shown in Figure 5 (p. 119) should appear in your browser window.

The code in Listing 6 (p. 128) is very similar to the code in Listing 2 (p. 118) . They both describe the same right triangle, so the output should be the same in both cases.

The code in Listing 2 (p. 118) uses the opposite side and the hypotenuse along with the arcsine to compute the angle. The code in Listing 6 (p. 128) uses the opposite side and the adjacent side along with the arctangent to compute the angle. Otherwise, no further explanation should be required.

Interesting tangent equations

In the spirit of Figure 7 (p. 121) and Figure 8 (p. 124) , Figure 11 (p. 129) provides some interesting equations that deal with the angle, the opposite side, and the adjacent side. Given any two, you can find the third using either the tangent or arctangent.

Figure 11 . Interesting tangent equations.

continued on next page

```
tangent(angle) = opp/adj

angle = arctangent(opp/adj)
opp = tangent(angle) * adj
adj = opp/tangent(angle)
```

Table 6.17

An exercise involving the tangent

Please copy the code from Listing 7 (p. 129) into an html file and open it in your browser.

Listing 7 . Finding the length of the opposite side.

```
    <!-- File JavaScript07.html -->
<html><body>
<script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//===========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//===========================================//

var adj = 3
var angDeg = 53.13
var angRad = toRadians(angDeg)
var tangent = Math.tan(angRad)

var opp = adj * tangent

document.write("opposite = " + opp + "</br>")

adj = opp/tangent

document.write("adjacent = " + adj + "</br>")

</script>
</body></html>
```

Table 6.18

When you open your html file in your browser, the output shown in Figure 12 (p. 130) should appear in your browser window. We can see that the values in Figure 12 (p. 130) are correct for our 3-4-5 triangle.

Figure 12 . Output for script in Listing 7.

```
    opposite = 3.9999851132269173
adjacent = 3
```

Table 6.19

Very similar code

The code in Listing 7 (p. 129) is very similar to the code in Listing 3 (p. 120) and Listing 5 (p. 125) . The essential differences are that

- Listing 3 (p. 120) uses the sine along with the opposite side and the hypotenuse.
- Listing 5 (p. 125) uses the cosine along with the adjacent side and the hypotenuse.
- Listing 7 (p. 129) uses the tangent along with the opposite side and the adjacent side.

You should be able to work through those differences without further explanation from me.

The cotangent of an angle

There is also something called the cotangent of an angle, which is simply the ratio of the adjacent side to the opposite side. If you know how to work with the tangent, you don't ordinarily need to use the cotangent, so I won't discuss it further.

Computing length of opposite side with the Google calculator

We could also compute the length of the opposite side using the Google calculator.

The length of the opposite side – sample computation

Enter the following into the Google search box:

3*tan(53.1301024 degrees)

The following will appear immediately below the search box:

3 * tan(53.1301024 degrees) = 4.00000001

6.4.3 Dealing with different quadrants

Up to this point, we have dealt exclusively with angles in the range of 0 to 90 degrees (the first quadrant). As long as you stay in the first quadrant, things are relatively straightforward.

As you are probably aware, however, angles can range anywhere from 0 to 360 degrees (or more). Once you begin working with angles that are greater then 90 degrees, things become a little less straightforward.

Another svg file

When you downloaded the zip file named Phy1020.zip using the link given above (p. 108) , you should also have found that the zip file contains a file named Phy1020a1.svg.

The purpose of this file is to make it possible for you to create tactile graphics for sine and cosine curves using the procedure explained in the earlier module named Manual Creation of Tactile Graphics [24] .

If you have the ability to create tactile graphics, you don't need to perform the work in the following graph board exercise. However, you should read about it anyway because that will probably help you to better understand the sine and the cosine of an angle.

I will get back to tactile graphics after I describe the graph board exercise.

Another graph board exercise

In this graph board exercise, we will plot a graph of the amplitude of the sine of an angle on the vertical axis versus the angle itself on the horizontal.

[24] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

We will also do the same thing for the cosine. It would be very good if you could plot one curve on the top half of your graph board and the other curve on the bottom half of your graph board so that you can easily compare the two.

Interpreting gridline values

Since I don't know how many tactile grid lines there are on your graph board, I can't tell you exactly how to interpret the grid lines so as to make maximum use of the space on the board. All that I can tell you is that the vertical amplitude values for each curve will range from -1.0 to +1.0. We would like to plot values from -360 degrees to + 360 degrees on the horizontal. You should interpret the values of the gridlines on your graph board accordingly.

Sinusoidal amplitude versus angle

Please copy the code from Listing 8 (p. 132) into an html file and open the file in your browser.

Listing 8 . Sinusoidal amplitude versus angle.

```
    <;!-- File JavaScriptZZ.html -->
<;html><;body>
<;script language="JavaScript1.3">

function toRadians(degrees){
  return degrees*Math.PI/180
}//end function toRadians
//=========================================//

function toDegrees(radians){
  return radians*180/Math.PI
}//end function toDegrees
//=========================================//

var angInc = 90
var angStart = -360
var ang = angStart
var angEnd = 360
var sine
var cosine

while(ang <;= angEnd){
  //Compute sine and cosine of angle
  sine = Math.sin(toRadians(ang))
  cosine = Math.cos(toRadians(ang))

  //Reduce the number of digits in the output
  sine = (Math.round(100*sine))/100
  cosine = (Math.round(100*cosine))/100

  //Display the results
  document.write("Angle: " + ang +
                 " Sine: " + sine +
                 " Cosine: " + cosine +
                 "<;/br>")

  //Increase the angle for next iteration
  ang = ang + angInc
}//end while loop

<;/script>
<;/body><;/html>
```

Table 6.20

Output from the script

When you open your html file in your browser, the output shown in Figure 13 (p. 134) should appear in your browser window.

Figure 13 . Sinusoidal values at 90-degree increments.

```
    Angle: -360 Sine: 0 Cosine: 1
Angle: -270 Sine: 1 Cosine: 0
Angle: -180 Sine: 0 Cosine: -1
Angle: -90 Sine: -1 Cosine: 0
Angle: 0 Sine: 0 Cosine: 1
Angle: 90 Sine: 1 Cosine: 0
Angle: 180 Sine: 0 Cosine: -1
Angle: 270 Sine: -1 Cosine: 0
Angle: 360 Sine: 0 Cosine: 1
```

Table 6.21

Figure 13 (p. 134) contains the data for two different curves. One is a sine curve and the other is a cosine curve.

Plot the points using pushpins

You should be able to plots these data values as two separate curves on your graph board by inserting pushpins at the coordinate values shown and then connecting the pushpins with rubber bands, pipe cleaners, yarn, flexible wire, or something similar. (Rubber bands might not work if you are using a homemade plot board constructed from Styrofoam, because the pins pull too easily.)

Remember, the angle values from -360 degrees (-2*PI radians) to +360 degrees (+2*PI radians) are horizontal coordinates while the corresponding values for the sine and cosine are vertical coordinates.

Saw tooth curves

Once you have plotted the points, you should be able to discern two curves, each of which is a saw tooth.

The two curves have exactly the same shape, but one is shifted horizontally relative to the other. For example, the sine curve has a value of zero at an angle of zero (the origin) and it is asymmetric about the vertical axis.

The cosine curve, on the other hand has a value of 1 at an angle of zero and it is symmetric about the vertical axis.

Periodic curves

These are periodic curves. For example, the shape of the sine curve between -360 and 0 is the same as the shape of the sine curve between 0 and +360. Each of those ranges represents one *cycle* of the periodic curve.

We only computed the values from -360 to +360. However, if we had computed the values from -3600 to + 3600, the overall shape of the curve would not differ from what we have here. The shape of each cycle of the curve would be identical to the shape of the cycle to the left and the cycle to the right.

Not really a saw tooth

The sine and cosine curves don't really have a saw tooth shape. That is an artifact of the fact that we didn't compute enough points to reliably describe the shape of the curves. Let's improve on that.

Modify the script

Modify the code in your script to initialize the value of the variable named **angInc** to 45 degrees instead of 90 degrees and then load the revised version into your browser. This will cause the script to fill in data points between the points that we already have producing the output shown in Figure 16 (p. 135) .

Figure 14 . Sinusoidal values at 45-degree increments.

```
    Angle: -360 Sine: 0 Cosine: 1
Angle: -315 Sine: 0.71 Cosine: 0.71
Angle: -270 Sine: 1 Cosine: 0
Angle: -225 Sine: 0.71 Cosine: -0.71
Angle: -180 Sine: 0 Cosine: -1
Angle: -135 Sine: -0.71 Cosine: -0.71
Angle: -90 Sine: -1 Cosine: 0
Angle: -45 Sine: -0.71 Cosine: 0.71
Angle: 0 Sine: 0 Cosine: 1
Angle: 45 Sine: 0.71 Cosine: 0.71
Angle: 90 Sine: 1 Cosine: 0
Angle: 135 Sine: 0.71 Cosine: -0.71
Angle: 180 Sine: 0 Cosine: -1
Angle: 225 Sine: -0.71 Cosine: -0.71
Angle: 270 Sine: -1 Cosine: 0
Angle: 315 Sine: -0.71 Cosine: 0.71
Angle: 360 Sine: 0 Cosine: 1
```

Table 6.22

Plot the new points

Every other line of text in Figure 14 (p. 135) should contain sine and cosine values for angles that are half way between the points that you already have plotted. Use pushpins to plot the new points and connect all of the points in each curve using rubber bands, pipe cleaners, or whatever you find most useful for this purpose.

Same shape but shifted horizontally

The two curves still have the same shape, although shifted horizontally relative to one another and they are still periodic. However, they no longer have a saw tooth shape. They tend to be a little more rounded near the peaks and they are beginning to provide a better representation of the actual shapes of the sine and cosine curves.

Let's do it again

Change the value of the variable named **angInc** from 45 degrees to 22.5 degrees and load the new version of the html file into your browser. Now the output should look like Figure 15 (p. 136) .

Figure 15 . Sinusoidal values at 22.5-degree increments.

```
    Angle: -360 Sine: 0 Cosine: 1
Angle: -337.5 Sine: 0.38 Cosine: 0.92
Angle: -315 Sine: 0.71 Cosine: 0.71
Angle: -292.5 Sine: 0.92 Cosine: 0.38
Angle: -270 Sine: 1 Cosine: 0
Angle: -247.5 Sine: 0.92 Cosine: -0.38
Angle: -225 Sine: 0.71 Cosine: -0.71
Angle: -202.5 Sine: 0.38 Cosine: -0.92
Angle: -180 Sine: 0 Cosine: -1
Angle: -157.5 Sine: -0.38 Cosine: -0.92
Angle: -135 Sine: -0.71 Cosine: -0.71
Angle: -112.5 Sine: -0.92 Cosine: -0.38
Angle: -90 Sine: -1 Cosine: 0
Angle: -67.5 Sine: -0.92 Cosine: 0.38
Angle: -45 Sine: -0.71 Cosine: 0.71
Angle: -22.5 Sine: -0.38 Cosine: 0.92
Angle: 0 Sine: 0 Cosine: 1
Angle: 22.5 Sine: 0.38 Cosine: 0.92
Angle: 45 Sine: 0.71 Cosine: 0.71
Angle: 67.5 Sine: 0.92 Cosine: 0.38
Angle: 90 Sine: 1 Cosine: 0
Angle: 112.5 Sine: 0.92 Cosine: -0.38
Angle: 135 Sine: 0.71 Cosine: -0.71
Angle: 157.5 Sine: 0.38 Cosine: -0.92
Angle: 180 Sine: 0 Cosine: -1
Angle: 202.5 Sine: -0.38 Cosine: -0.92
Angle: 225 Sine: -0.71 Cosine: -0.71
Angle: 247.5 Sine: -0.92 Cosine: -0.38
Angle: 270 Sine: -1 Cosine: 0
Angle: 292.5 Sine: -0.92 Cosine: 0.38
Angle: 315 Sine: -0.71 Cosine: 0.71
Angle: 337.5 Sine: -0.38 Cosine: 0.92
Angle: 360 Sine: 0 Cosine: 1
```

Table 6.23

A lot of data points

Once again, every other line of text in Figure 15 (p. 136) contains new sine and cosine values for angles that you don't have plotted yet.

Plotting all of these point is going to require a lot of pushpins and a lot of effort. Before you do that, let's think about it.

Two full cycles

You should have been able to discern by now that your plots for the sine and cosine graphs each contain two full cycles. An important thing about periodic functions is that once you know the shape of the curve for any one cycle, you know the shape of the curve for every cycle from minus infinity to infinity. The shape of every cycle is exactly the same as the shape of every other cycle.

Saving pushpins and effort

If you are running out of pushpins or running out of patience, you might consider updating your plots for only one cycle.

You should be able to discern that your curves no longer have a saw tooth shape. Each time we have run the script, we have sampled the amplitude values of each curve at twice as many points as before. Therefore, the curves should be taking on a smoother rounded shape that is better representation of the actual shape of the curves.

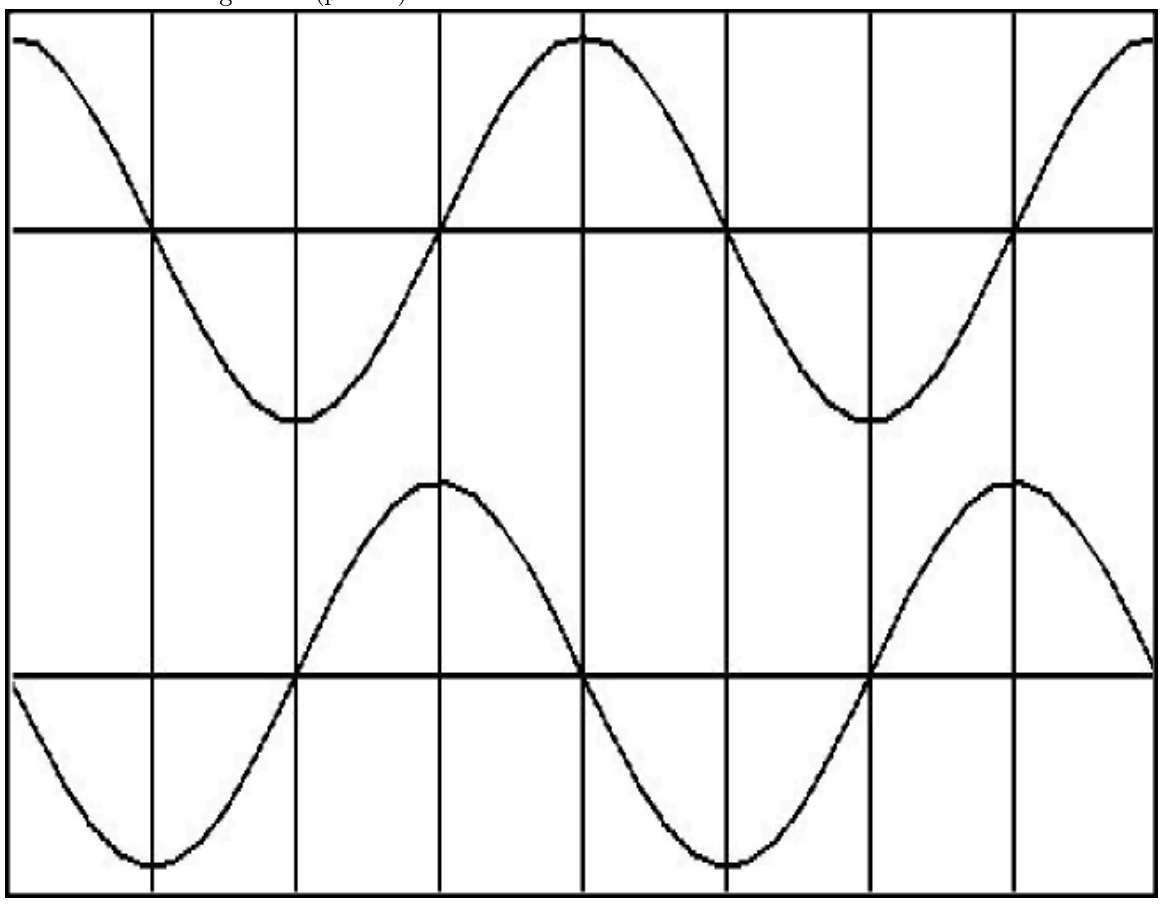
Continue the process

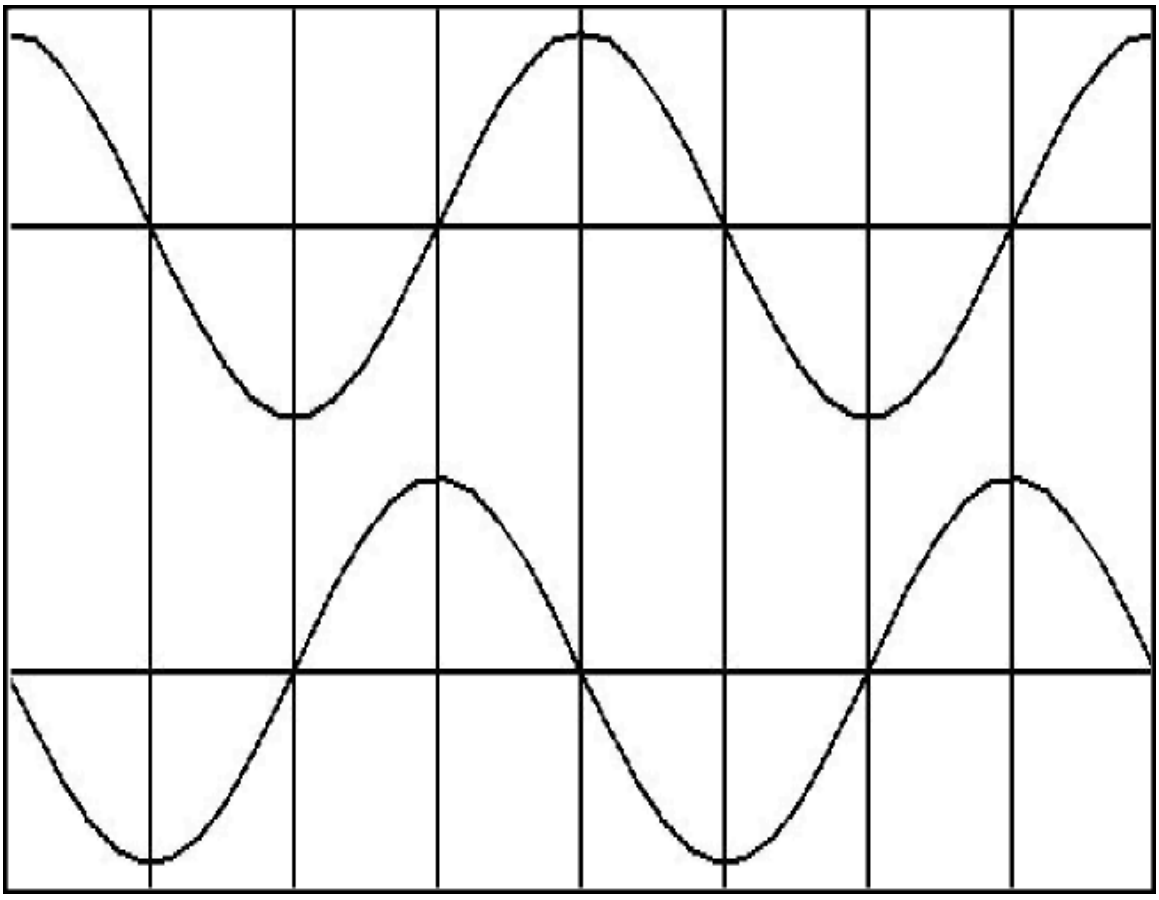
You can continue this process of improving the curves for as long as you have the graph board space, pushpins, and patience to do so. Just divide the value of the variable named **angInc** by a factor of two and rerun the script. That will produce twice as many data points that are only half as far apart on the horizontal axis.

If you choose to do so, you can plot only the new points in one-half of a cycle to get an idea of the shape. By now you should have discerned that each half of a cycle has the same shape, only one half is above the horizontal axis and the other half is a mirror image below the axis.

Plot of cosine and sine curves

Getting back to tactile graphics, for the benefit of any sighted person that may be assisting you, Figure 16 (p. 140) shows a cosine curve plotted above a sine curve very similar to the curves that you have plotted on your graph board. (Figure 16 (p. 140) shows a mirror image of the actual curves because the file named Phy1020a1.svg is intended to be used for manual embossing from the back of the paper.) A non-mirror-image version is shown in Figure 20 (p. 153) .





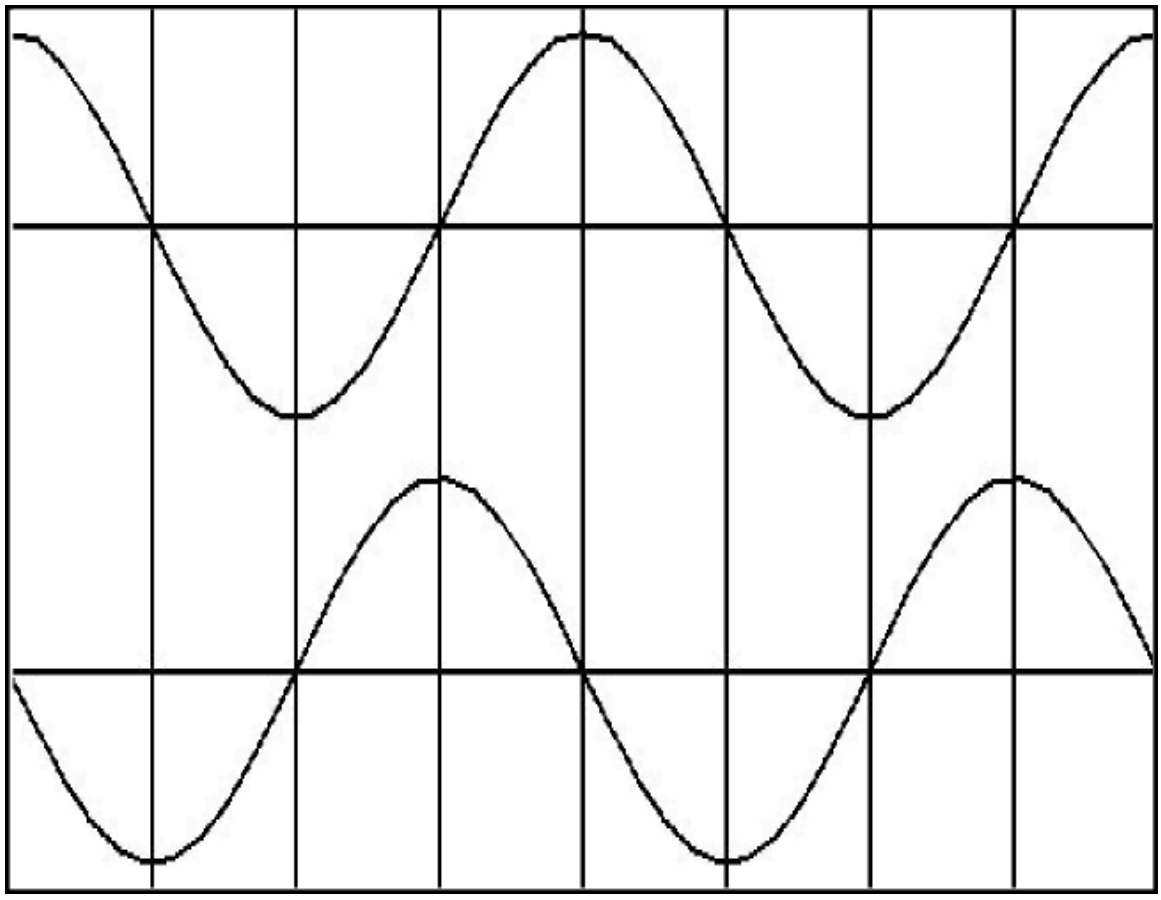


Figure 16 . Mirror image plot of cosine and sine curves from the file named Phy1020a1svg.

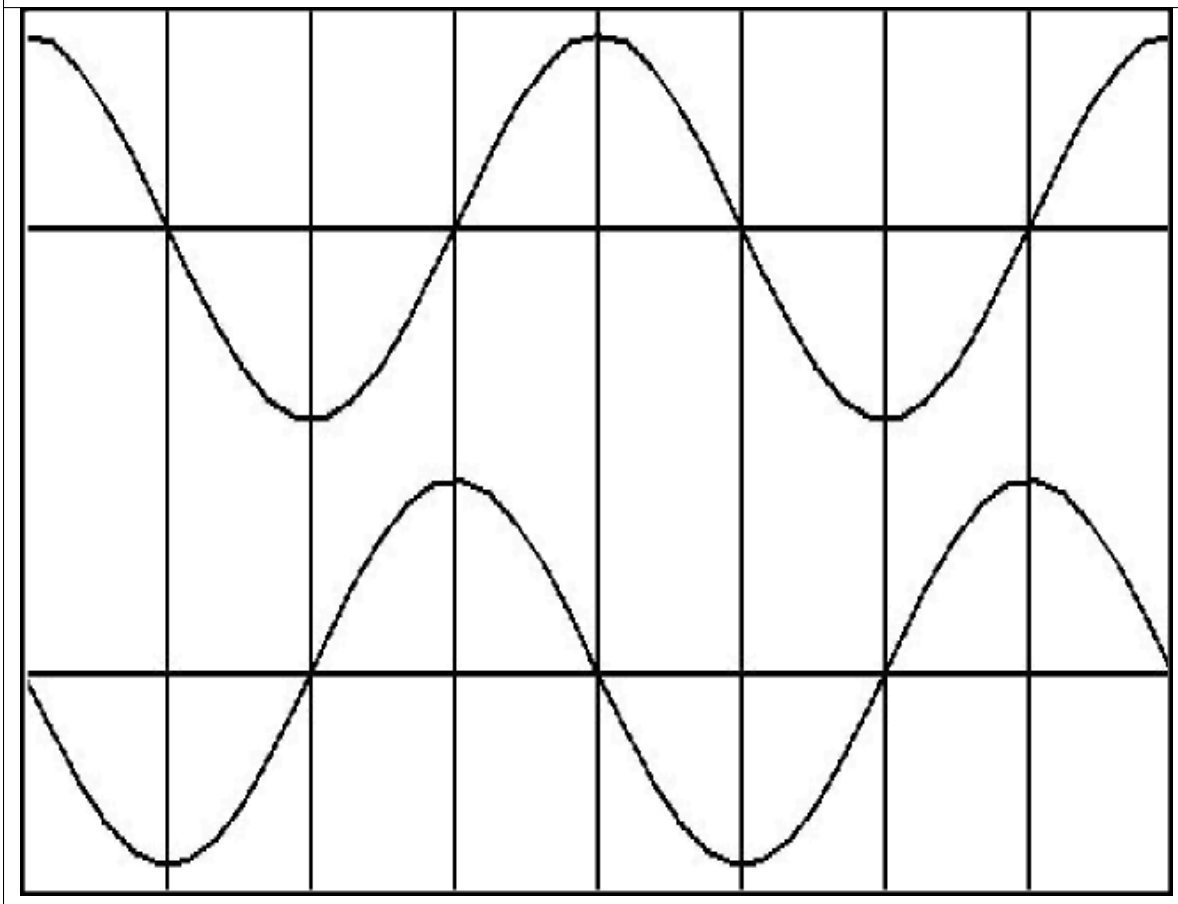


Table 6.24

This image contains lines that aren't straight, but with a little care, your assistant should be able to do a reasonably good job of embossing them when the image is printed as the full 8.5x11 inch version.

Page setup

If you use the IVEO Viewer to print the file named Phy1020a1.svg, you should use the default Page Setup selection for Letter (Landscape).

Grid lines

The image in Figure 16 (p. 140) contains 7 vertical grid lines. The vertical grid line in the center represents an angle of zero degrees. The space between each grid line on either side of the center represents an angle of 90 degrees or PI/2 radians.

There are two horizontal grid lines. One is one-fourth of the way down from the top. The other is one-fourth of the way up from the bottom.

The curves

A cosine curve is plotted with vertical values relative to the top grid line. It extends from -360 degrees on the left to +360 degrees on the right.

A sine curve is plotted with vertical values relative to the bottom grid line. It also extends from -360 degrees on the left to +360 degrees on the right. (Note once again that Figure 16 (p. 140) was flipped horizontally to crate a mirror image.)

Return values for the Math.asin, Math.acos, and Math.atan methods

I told you earlier that the **Math.asin** method returns a value between -PI/2 and PI/2. However, I didn't tell you that the **Math.acos** method returns a value between 0 and PI, or that the **Math.atan** method returns a value between -PI/2 and PI/2. You now have enough information to understand why this is true.

Smooth curves

If you examine the two curves that you have just plotted, you can surmise that the sine and cosine functions are smooth curves whose values range between -1 and +1 inclusive. For every possible value between -1 and +1, there is an angle in the range -PI/2 and PI/2 whose sine value matches that value. There is also an angle in the range 0 and PI whose cosine value matches that value.

(Although you haven't plotted the curve for the tangent, a similar situation holds there also.)

An infinite number of angles

Therefore, given a specific numeric value between -1 and +1, there are an infinite number of angles whose sine and cosine values match that numeric value and the method has no way of distinguishing between them. Therefore, the **Math.asin** method returns the matching angle that is closest to zero and the **Math.acos** method returns the matching positive angle that is closest to zero.

What can we learn from this?

One important thing that we can learn is there is no difference between the sine or cosine of an angle and the sine or cosine of a different angle that differs from the original angle by 360 degrees. Thus, the **Math.asin** and **Math.acos** methods cannot be used to distinguish between angles that differ by 360 degrees. (As you learned above, the situation involving the **Math.asin** and **Math.acos** methods is even more stringent than that.)

One-quarter cycle contains all of the information

Another thing that we can learn is that once you know the shape of the cosine curve from 0 degrees to 90 degrees, you have enough information to construct the entire cosine curve and the entire sine curve across any range of angles. Every possible value or the negative of every possible value that can occur in a sine or cosine curve occurs in the cosine curve between 0 degrees and 90 degrees. Furthermore, the order of those values is also well defined.

Think about these relationships

You should think about these kinds of relationships. As I mentioned earlier, as long as we are working with angles between 0 and 90 degrees, everything is relatively straightforward. However, once we start working with angles between 90 degrees and 360 degrees (or greater), things become a little less straightforward.

If you have a good picture in your mind of the shape of the two curves between -360 degrees and +360 degrees, you may be able to avoid errors once you start working on physics problems that involve angles outside the range of 0 to 90 degrees.

Quadrants

We often think of a two-dimensional space with horizontal and vertical axes and the origin at the center in quadrants. Each quadrant is bounded by half the horizontal axis and half the vertical axis.

It is common practice to number the quadrants in counter-clockwise order with the upper-right quadrant being quadrant 1, the upper-left quadrant being quadrant 2, the bottom-left quadrant being quadrant 3, and the bottom-right quadrant being quadrant 4.

Angles fall in quadrants

If you measure the angle between the positive horizontal axis and a line segment that emanates from the origin, quadrant 1 contains angles between 0 and PI/2, quadrant 2 contains the angles between PI/2 and PI, quadrant 3 contains the angles between PI and 3*PI/2, and quadrant 4 contains the angles between 3*PI/2 and 2*PI (or zero). (Note that I didn't attempt to reconcile the inclusion of each axis in the two quadrants on either side of the axis.)

Algebraic signs versus quadrant number

It is sometimes useful to consider how the algebraic sign of the sine, cosine, and tangent values varies among the four quadrants. Figure 17 (p. 142) contains a table that shows the sign of the sine, cosine, and tangent values for each of the four quadrants

Figure 17 . Algebraic signs versus quadrants.

```
        1 2 3 4
sine    + + - -
cosine  + - - +
tangent + - + -
```

Table 6.25

Working with arctangents is more difficult than arcsine or arccosine

Working with arctangent is somewhat more difficult than working with arcsine or arccosine, if for no other reason than the possibility of dividing by zero when working with the arctangent.

Listing 9 (p. 143) shows a JavaScript function named getAngle that deals with this issue.

Listing 9 . A function to deal with quadrants.

```
    <!---------------- File JavaScript09.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(adjacent,opposite){
  if((adjacent == 0) && (opposite == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((adjacent == 0) && (opposite > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((adjacent == 0) && (opposite < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((adjacent < 0) && (opposite >= 0)){
    //Correct to second quadrant
    return Math.atan(opposite/adjacent)*180/Math.PI + 180;
  }else if((adjacent < 0) && (opposite <= 0)){
    //Correct to third quadrant
    return Math.atan(opposite/adjacent)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(opposite/adjacent)*180/Math.PI;
  }//end else
}//end function getAngle


//Modify these values and run for different cases.
var adj = 3;
var opp = 4;

document.write("adj = " + adj.toFixed(2) +
" opp = " + opp.toFixed(2) + " units</br>");

document.write("angle = " + getAngle(adj,opp).toFixed(2)
  + " units</br>");

var adj = -3;
var opp = 4;

document.write("adj = " + adj.toFixed(2) +
" opp = " + opp.toFixed(2) + " units</br>");

document.write("angle = " + getAngle(adj,opp).toFixed(2)
  + " units</br>");
```



```
var adj = -3;
var opp = -4;
```

Table 6.26

The code in Listing 9 (p. 143) begins by defining a function named getAngle that accepts the signed values of the adjacent side and the opposite side of the right triangle and returns the angle that the hypotenuse makes with the positive horizontal axis.

Then the code in Listing 9 (p. 143) tests the result for four different triangles situated in each of the four quadrants.

Figure 18 (p. 144) shows the output produced by this script.

Figure 18 . Output from the code in Listing 9.

```
   Start Script
adj = 3.00 opp = 4.00 units
angle = 53.13 units
adj = -3.00 opp = 4.00 units
angle = 126.87 units
adj = -3.00 opp = -4.00 units
angle = 233.13 units
adj = 3.00 opp = -4.00 units
angle = -53.13 units
```

Table 6.27

This is an issue that will become important when we reach the module that deals with vectors in all four quadrants.

Structure of the script

The script shown in Listing 9 (p. 143) begins by defining a function named **getAngle** . The purpose of this function is to return an angle in degrees in the correct quadrant based on the lengths of the adjacent and opposite sides of an enclosing right triangle. As mentioned above, the returned angle is the angle that the hypotenuse makes with the positive horizontal axis.

An indeterminate result

The getAngle function calls the Math.atan method to compute the angle whose tangent is the ratio of the opposite side to the adjacent side of a right triangle.

If the lengths of both the opposite and adjacent sides are zero, the ratio opposite/adjacent is indeterminate and the value of the angle cannot be computed. In fact there is no angle corresponding to the ratio 0/0. However, the function must either return the value of an angle, or must return some sort of flag indicating that computation of the angle is not possible.

In this case, the function simply returns the value zero for the angle.

Avoiding division by zero

If the length of adjacent side is zero and the length of opposite side is not zero, the ratio opposite/adjacent is infinite. Therefore, the value of the angle cannot be computed. However, in this case, the angle is known to be 90 degrees (for opposite greater than zero) or 270 degrees (-90 degrees, for opposite less than zero). The getAngle function traps both of those cases and returns the correct angle in each case.

Correcting for the quadrant

The Math.atan method receives one parameter and it is either a positive or negative value. If the value is positive, the method returns an angle between 0 and 90 degrees. If the value is negative, the method returns an angle between 0 and -90 degrees. Thus, the angles returned by the Math.atan method always lie in the first or fourth quadrants.

(Actually, as I mentioned earlier, +90 degrees and -90 degrees are not possible because the tangent of +90 degrees or -90 degrees is an infinitely large positive or negative value. However, the method can handle angles that are very close to +90 or -90 degrees.)

A negative opposite/adjacent ratio

If the opposite/adjacent ratio is negative, this doesn't necessarily mean that the angle lies in the fourth quadrant. That negative ratio could result from a positive value for opposite and a negative value for adjacent. In that case, the angle would lie in the second quadrant between 90 degrees and 180 degrees.

The getAngle function tests the signs of the values for opposite and adjacent. If the signs indicate that the angle lies in the second quadrant, the value returned from the Math.atan method is corrected to place the angle in the second quadrant. The corrected angle is returned by the getAngle function.

A positive opposite/adjacent ratio

Similarly, if the opposite/adjacent ratio is positive, this doesn't necessarily mean that the angle lies in the first quadrant. That positive ratio could result from a negative opposite value and a negative adjacent value. In that case, the angle would lie in the third quadrant between 180 degrees and 270 degrees.

Again, the getAngle function tests the signs of the values for opposite and adjacent. If both values are negative, the value returned from the Math.atan method is corrected to place the angle in the third quadrant.

No corrections required...

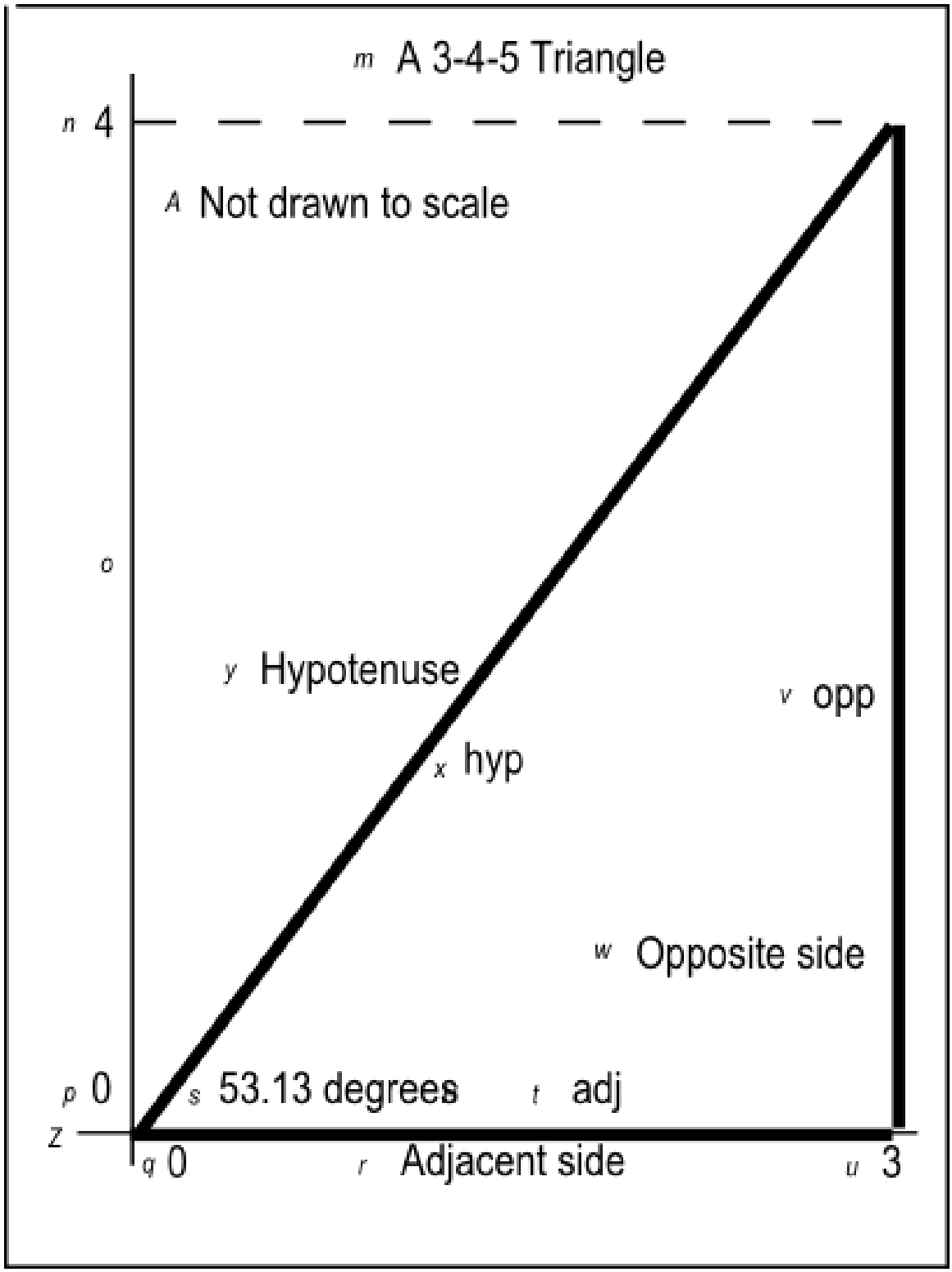
Finally, if no corrections are required for the quadrant, the getAngle function returns the value returned by the Math.atan method. Note however, that in all cases, the Math.atan method returns the angle in radians. That value is converted to degrees by the getAngle function and the returned value is in degrees.

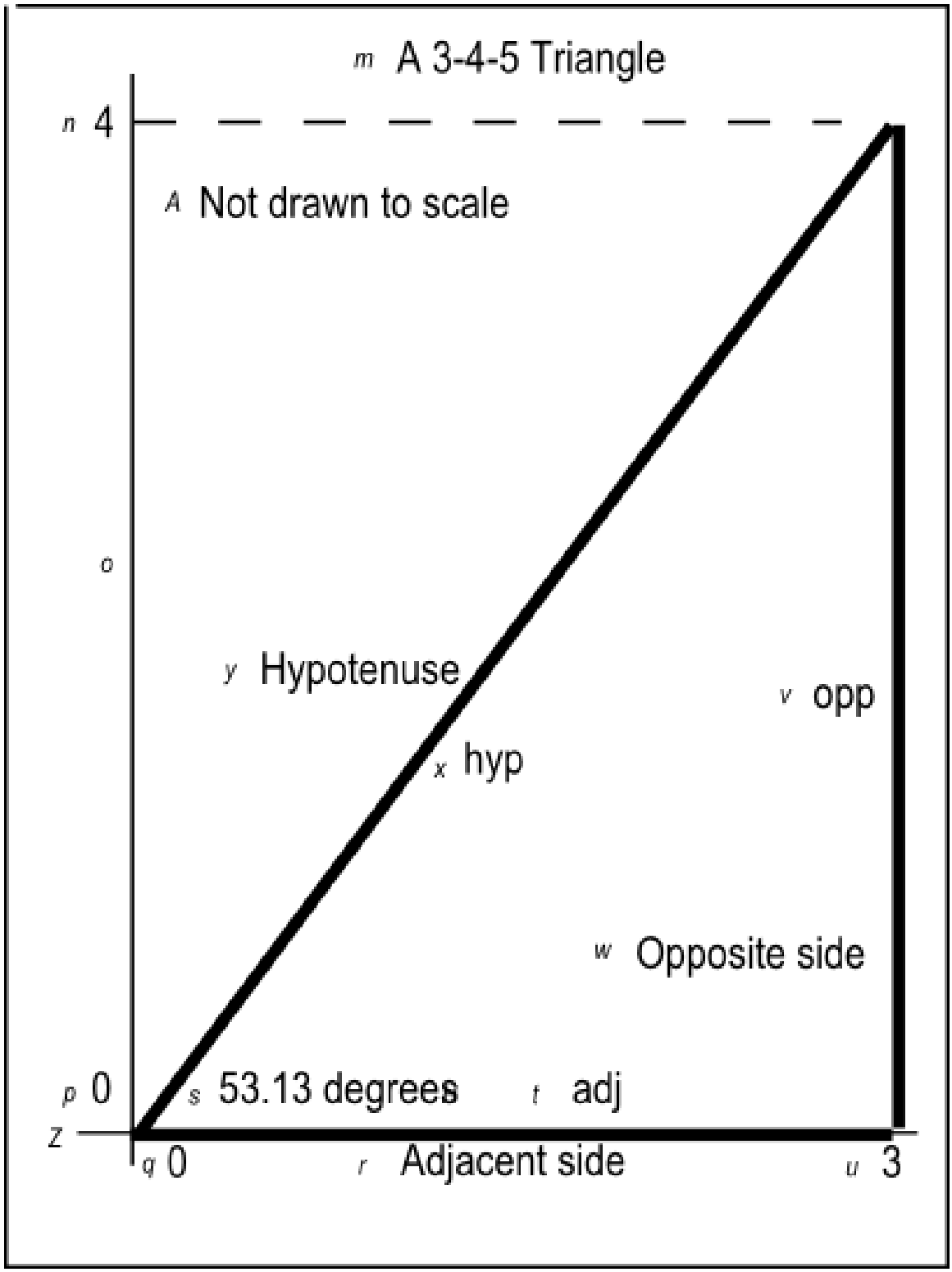
Positive and negative angles

As you can see from the results of the test shown in Figure 18 (p. 144) , angles in the first, second, and third quadrants are returned as positive angles in degrees. However, angles in the fourth quadrant are returned as negative angles in degrees.

6.4.4 Normal (non-mirror-image) graphics

Figure 2 (p. 115) and Figure 16 (p. 140) show the actual mirror image versions of the images contained in the svg files. Figure 19 (p. 149) and Figure 20 (p. 153) show the same images in their non-mirror-image orientation.


m A 3-4-5 Triangle
n 4
A Not drawn to scale
o
y Hypotenuse
v opp
x hyp
w Opposite side
p 0
s 53.13 degrees
t adj
Z
q 0
r Adjacent side
u 3


m A 3-4-5 Triangle
n 4
A Not drawn to scale
o
y Hypotenuse
v opp
x hyp
w Opposite side
p 0
s 53.13 degrees
t adj
Z
q 0
r Adjacent side
u 3

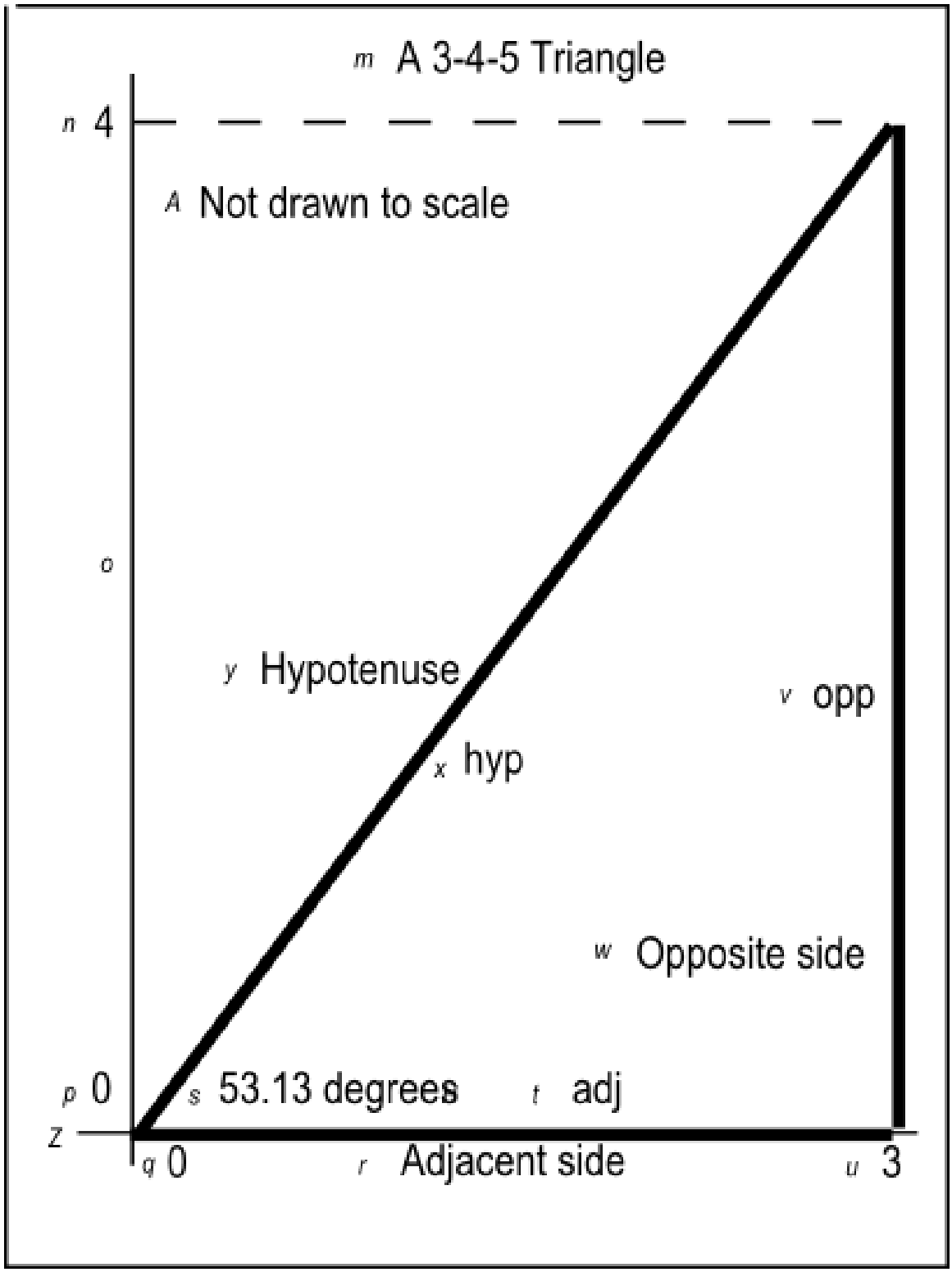

m A 3-4-5 Triangle
n 4
A Not drawn to scale
o
y Hypotenuse
v opp
x hyp
w Opposite side
p 0
s 53.13 degrees
t adj
Z
q 0
r Adjacent side
u 3

Figure 19 . Normal image from file Phy1020b1.svg.

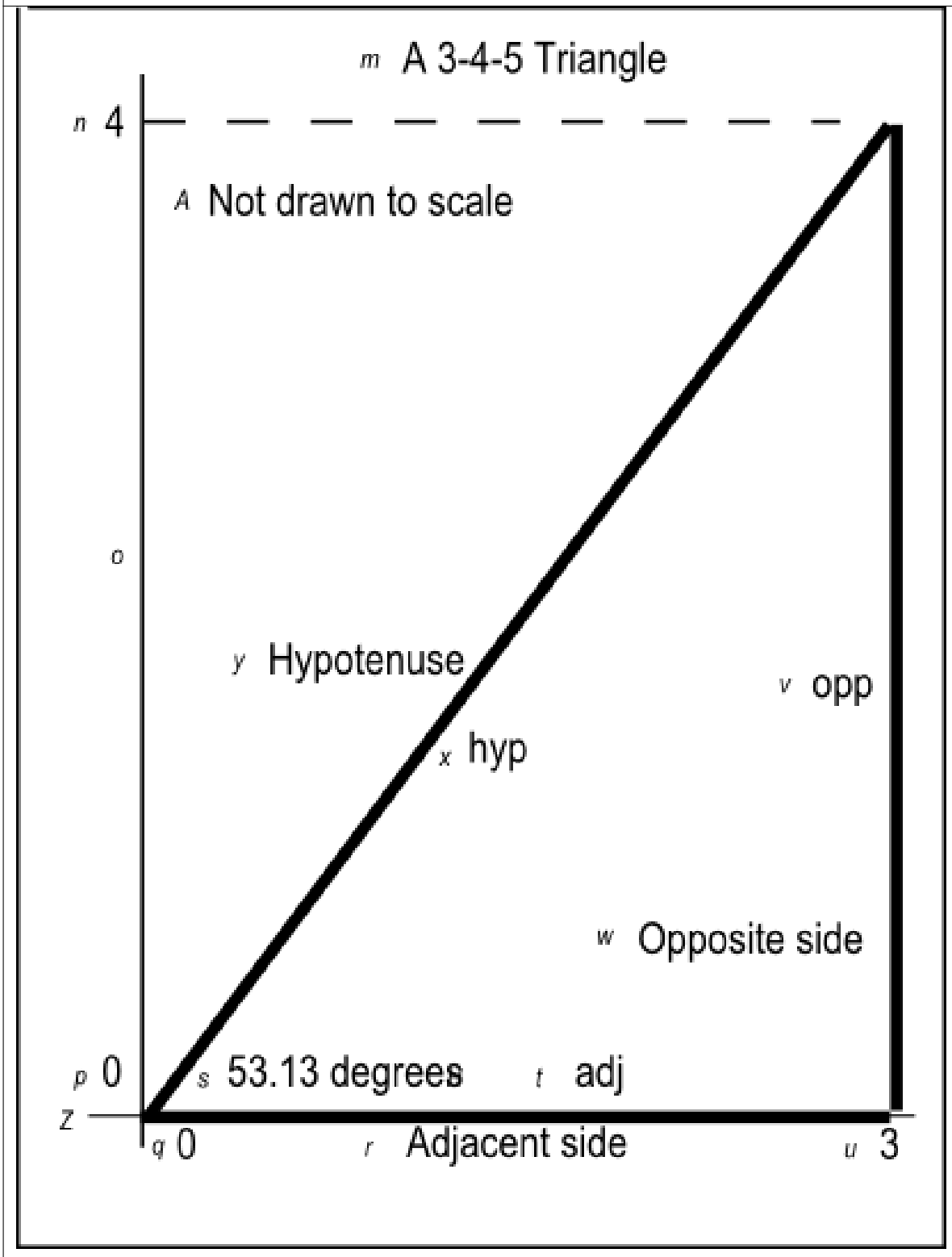
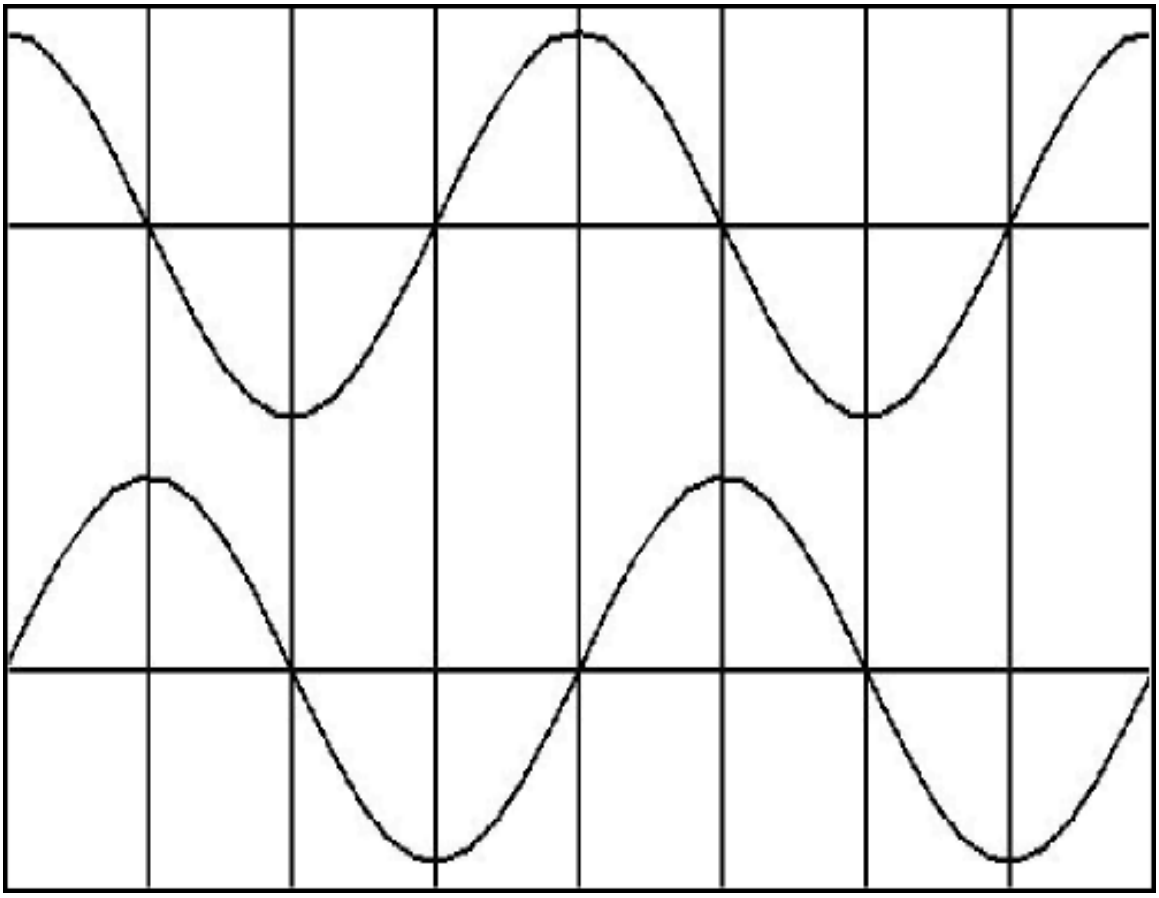
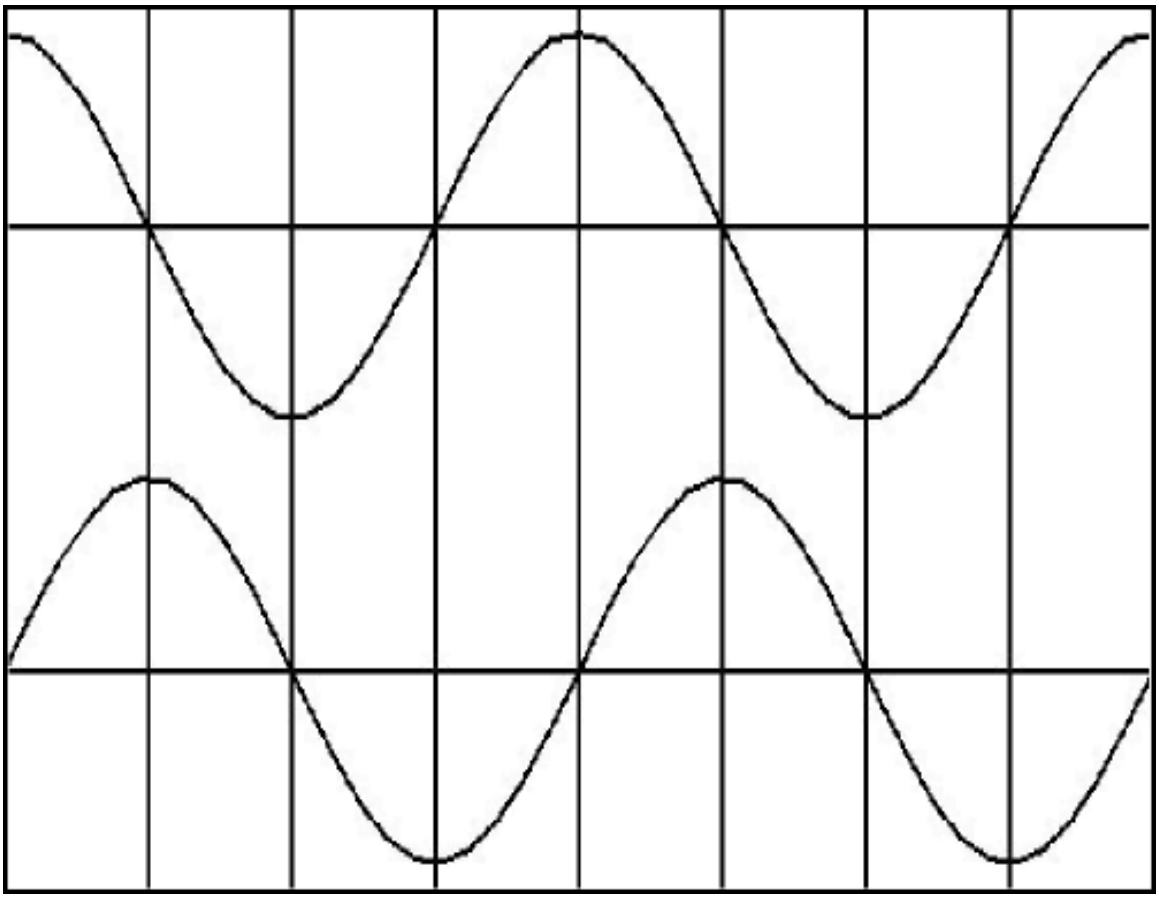


Table 6.28





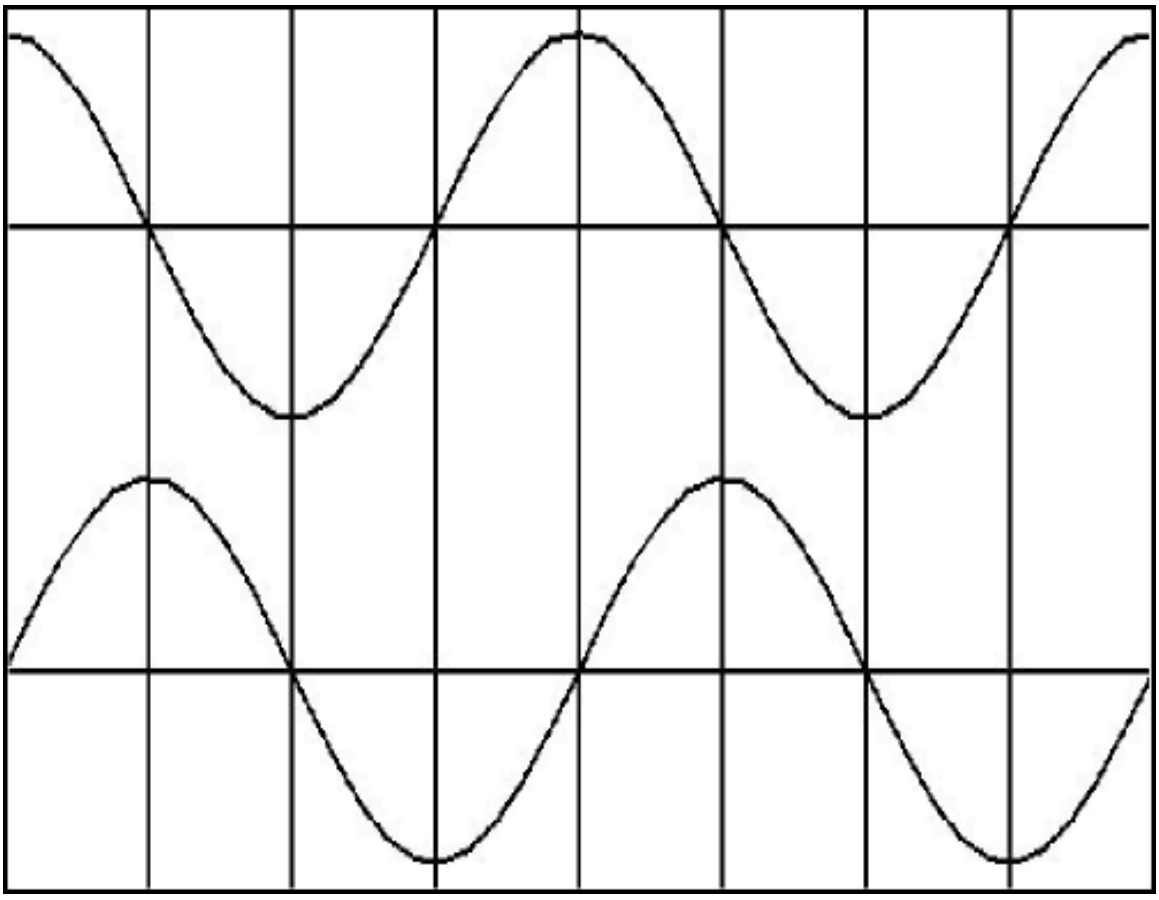


Figure 20 . Normal image plot of cosine and sine curves from the file named Phy1020a1svg.

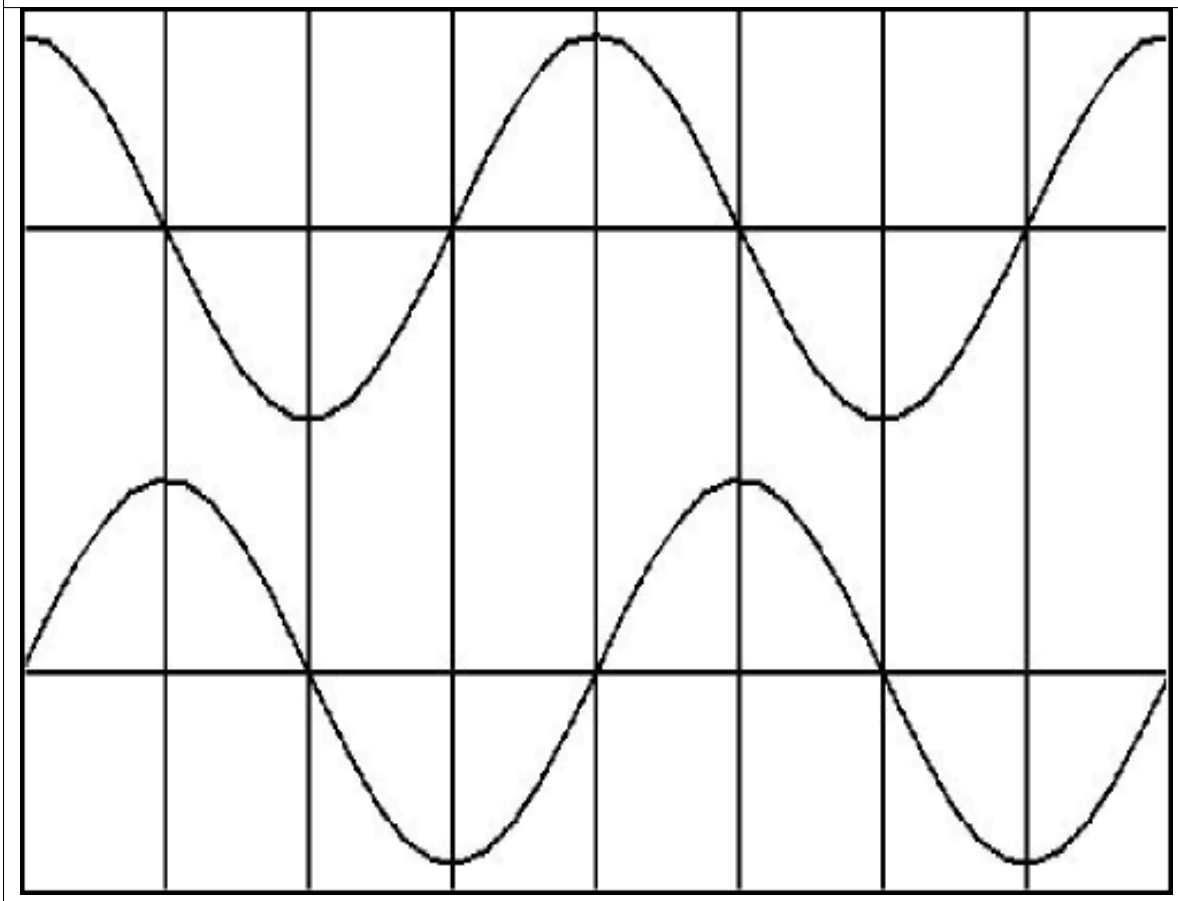


Table 6.29

6.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

6.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

6.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Brief Trigonometry Tutorial
- Revised: 09/30/15
- File: Phy1020.htm
- Keywords:
  - physics
  - accessible
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - angle
  - sine
  - cosine
  - tangent
  - arcsine
  - arccosine
  - arctangent
  - quadrant

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 7

Scale Factors, Ratios, and Proportions[1]

7.1 Table of Contents

- Preface (p. 155)
  - General (p. 155)
  - Prerequisites (p. 156)
  - Viewing tip (p. 156)
    - Figures (p. 156)
    - Listings (p. 157)
  - Supplemental material (p. 157)
- General background information (p. 157)
- Discussion (p. 157)
  - Scale factors (p. 157)
  - Ratios (p. 157)
  - Proportions (p. 159)
- Run the scripts (p. 164)
- Resources (p. 164)
- Miscellaneous (p. 164)

7.2 Preface

7.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

This module provides a brief tutorial on scale factors, ratios, and proportions that is designed to be accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m37436/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

7.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

7.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

7.2.3.1 Figures

- Figure 1 (p. 159) . Screen output for Listing #1.
- Figure 2 (p. 161) . Screen output for Listing #2.
- Figure 3 (p. 164) . Screen output for Listing #3.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

7.2.3.2 Listings

- Listing 1 (p. 158) . Exercise on scale factors
- Listing 2 (p. 161) . Circumference is proportional to radius.
- Listing 3 (p. 163) . Area is proportional to radius squared.

7.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

7.3 General background information

Mathematical expressions are used in physics to describe relationships that are difficult to express in words. The expressions use algebraic symbols to represent quantities that consist of numbers and units.

Measurements are important

Conclusions that are drawn in physics and other sciences ranging from chemistry to the social sciences are often based on measurements such as length, width, weight, salinity, population, density, etc.

Each number in an equation often represents the results of a measurement, which is made in terms of a standard. The units indicate which standard was used to make the measurements.

Knowledge of units is critical

A number that is used to indicate the result of a measurement is of little value unless we know the units in which the measurement was made. For example, it isn't very useful to know that the length of an object is 125 unless we know whether the units are meters, centimeters, millimeters, or miles.

The Google calculator

Although I won't go into detail in this module, I will tell you that the Google calculator is very good at helping you to keep track of units. For example, if you enter the following expression in the Google search box,

3 ft + 1 yd + 36 inches

The following result will be displayed immediately below the search box:

(3 ft) + (1 yd) + (36 inches) = 2.7432 meters

7.4 Discussion

We often express the relationship between two items using a scale factor.

7.4.1 Scale factors

For example, we might say that that a colt doubled its weight in one year. This means that the colt's weight after one year was equal to the colt's weight at birth multiplied by a factor of two. The factor is the number by which a quantity is multiplied or divided when it is changed from one value to another.

7.4.2 Ratios

The factor is the ratio of the new value to the old value. Regarding the colt mentioned above, we might write:

NewWeight / BirthWeight = 2

where the slash character "/" indicates division.

Percentage increases

[14] http://www.dickbaldwin.com/toc.htm

It is also common for us to talk about something increasing or decreasing by a given percentage value. For example, we might say that a colt increased its weight by 50-percent in one year. This is equivalent to saying:

NewWeight = BirthWeight * (1 + 50/100)

This assumes that when evaluating a mathematical expression:

- The asterisk character " * " indicates multiplication.
- Computations begin inside of the inner-most pair of matching parentheses and work outward from there.
- Multiplication and division are performed before addition and subtraction are performed.

This is typical behavior for computer programs and spreadsheets, but not necessarily for hand calculators.

Percentage decreases

In my dreams, I might say that I went on a diet and my weight decreased by 25-percent in one year. This would be equivalent to saying:

NewWeight = OldWeight * (1 - 25/100)

Exercise on scale factors

Write a script that has the following behavior. Given a chalk line that is 100 inches long, draw other chalk lines that are:

- A. Twice the length of the original line.
- B. Twenty-five percent of the length of the original line.
- C. Twenty-five percent greater than the length of the original line.
- D. Twenty-five percent less than the length of the original line.

My version of the script is shown in Listing 1.

Listing 1 . Exercise on scale factors

```
    <!-- File JavaScript01.html -->
<html><body>
<script language="JavaScript1.3">

//Do the computations
var origLine = 100
var lineA = 2 * origLine
var lineB = (25/100) * origLine
var lineC = (1 + 25/100) * origLine
var lineD = (1 - 25/100) * origLine

//Display the results
document.write("Original line = "
               + origLine + "</br>")
document.write("A = " + lineA + "</br>")
document.write("B = " + lineB + "</br>")
document.write("C = " + lineC + "</br>")
document.write("D = " + lineD + "</br>")

</script>
</body></html>
```

Table 7.1

Screen output

When you copy the code from Listing 1 into an html file and open the file in your web browser, the output text shown in Figure 1 should appear in your browser window.

Figure 1 . Screen output for Listing #1.

```
    Original line = 100
A = 200
B = 25
C = 125
D = 75
```

Table 7.2

Note that although they sound similar, specifications B and D above (p. 158) don't mean the same thing.

7.4.3 Proportions

We talk about increasing or changing a value by some factor because we can often simplify a problem by thinking in terms of proportions.

A symbol for proportionality

Physics textbooks often use a character that doesn't appear on a QWERTY keyboard to indicate "is proportional to." That character is probably not in the vocabulary of typical screen readers and is probably not compatible with Braille displays.

I will use a "$" character for that purpose because:

- It does appear on a QWERTY keyboard.
- It isn't typically used in mathematical expressions unless American currency is involved.
- It is not a JavaScript operator.

For example, I will write A $ B to indicate that A is proportional to B.

When we say that A is proportional to B, or

A $ B

we mean that if B increases by some factor, then A must increase by the same factor.

Circumference of a circle

Let's illustrate what we mean with a couple of examples. For the first example, we will consider the circumference of a circle. Hopefully, you know that the circumference of a circle is given by the expression:

C = 2 * PI * r

where:

- C is the circumference of the circle
- PI is the mathematical constant 3.14159...
- r is the radius of the circle

From this expression, we can conclude that

C $ r

If we modify the radius...

If we double the radius, the circumference will also double. If we reduce the radius by 25-percent, the circumference will also be reduced by 25-percent. This is illustrated by the script in Listing 2.

Listing 2 . Circumference is proportional to radius.

```
    <!-- File JavaScript02.html -->
<html><body>
<script language="JavaScript1.3">

var r = 10
var C = 2 * Math.PI * r
document.write("r =" + r +
               ", C = " + C + "</br>")

//Multiply r by 2. Then display r and C
r = r * 2
C = 2 * Math.PI * r
document.write("r =" + r +
               ", C = " + C + "</br>")

//Reduce r by 25%, Then display r and C
r = r * (1 - 25/100)
C = 2 * Math.PI * r
document.write("r =" + r +
               ", C = " + C + "</br>")

</script>
</body></html>
```

Table 7.3

Output from the script

When you open the script shown in Listing 2 in your browser, the text shown in Figure 2 should appear in your browser window.

Figure 2 . Screen output for Listing #2.

```
    r =10, C = 62.83185307179586
r =20, C = 125.66370614359172
r =15, C = 94.24777960769379
```

Table 7.4

Figure 2 shows the value of the circumference for three different values for the radius. You should be able to confirm that the combination of the three lines of output text satisfy the proportionality rules stated earlier (p. 160) .

For example, you can confirm these results by entering the following three expressions in the Google search box and recording the results that appear immediately below the search box:

2*pi*10

2*pi*20

2*pi*15

Area of a circle

Before I can discuss the area of a circle, I need to define a symbol that we can use to indicate exponentiation.

A symbol for exponentiation

Physics textbooks typically use a superscript character to indicate that a value is raised to a power, such as the radius of a circle squared.

Superscripts may or may not be in the vocabulary of screen readers, but probably are not compatible with Braille displays. Therefore, we need a symbol that is compatible with both.

When I need to indicate that a value is raised to a power, I will use the "^" character, such as in the following text that indicates radius squared, or radius raised to the second power.

radius^2

In those cases where the exponent is a fraction, or is negative, I will surround it with parentheses, such as in

radius^(1/2), and

distance^(-2)

The first term indicates the square root of the radius. The second term indicates that distance is being raised to the -2 power.

I chose to use the "^" character to indicate exponentiation because it is used as the exponentiation operator in some programming languages, such as BASIC. The "^" character is also recognized by the Google calculator as an exponentiation operator.

Unfortunately, there is no exponentiation operator in JavaScript, so we will need a different approach to raise a value to a power in our JavaScript scripts. As you will see later, we will use the built-in **Math.pow** method for that purpose.

An expression for the area of a circle

From earlier courses, you should know that the area of a circle is given by

A = PI * r^2

where

- A is the area.
- PI is the mathematical constant 3.14159...
- r is the radius of the circle.

Proportional to the square of the radius

From this, we can conclude that the area of a circle is not proportional to the radius. Instead, it is proportional to the square of the radius as in

A $ r^2

If you change the radius...

If you change the value of the radius, the area changes in proportion to the square of the radius. If the radius doubles, the area increases by four. If the radius is decreased by 25-percent, the area decreases by more than 25-percent. This is illustrated by the script in Listing 3.

Listing 3 . Area is proportional to radius squared.

```
    <!-- File JavaScript03.html -->
<html><body>
<script language="JavaScript1.3">

var r = 10
var A = Math.PI * Math.pow(r,2)
document.write("r =" + r +
               ", A = " + A + "</br>")

//Multiply r by 2. Then display r and C
r = r * 2
var A = Math.PI * Math.pow(r,2)
document.write("r =" + r +
               ", A = " + A + "</br>")

//Reduce r by 25%, Then display r and C
r = r * (1 - 25/100)
var A = Math.PI * Math.pow(r,2)
document.write("r =" + r +
               ", A = " + A + "</br>")

//Compute and the display the cube root
// of a number.
var X = Math.pow(8,1/3)
document.write("Cube root of 8 = " + X)

</script>
</body></html>
```

Table 7.5

The JavaScript Math.pow method

Listing 3 calls a built-in JavaScript method that I have not used before: **Math.pow** . This method is called to raise a value to a power. It requires two parameters. The first parameter is the value that is to be raised to a power and the second parameter is the power to which the value is to be raised.

The method returns the method raised to the power.

Fractional exponents

Although this topic is not directly related to the discussion on proportionality, as long as I am introducing the method named **Math.pow** , I will point out the it is legal for the exponent to be a fraction. The last little bit of code in Listing 3 raises the value 8 to the 1/3 power. This actually computes the cube root of the value 8. As you should be able to confirm in your head, the cube root of 8 is 2, because two raised to the third power is 8.

Output from the script

When you open the script shown in Listing 3 in your browser, the text shown in Figure 3 should appear in your browser window.

Figure 3 . Screen output for Listing #3.

```
    r =10, A = 314.1592653589793
r =20, A = 1256.6370614359173
r =15, A = 706.8583470577034
Cube root of 8 = 2
```

Table 7.6

An examination of the first three lines of text in Figure 3 should confirm that they satisfy the proportionality rules for the square of the radius described earlier (p. 162) .

The last line of text in Figure 3 confirms that the **Math.pow** method can be used to compute roots by specifying fractional exponents as the second parameter.

7.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

7.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

7.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Scale Factors, Ratios, and Proportions
- Revised: 09/30/15
- File: Phy1030.htm
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · scale
  - · factor
  - · ratio

· proportion
· Google calculator

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 8

Scientific Notation and Significant Figures[1]

8.1 Table of Contents

- Preface (p. 167)
  - General (p. 167)
  - Prerequisites (p. 168)
  - Viewing tip (p. 168)
    - Figures (p. 168)
    - Listings (p. 169)
  - Supplemental material (p. 169)
- General background information (p. 169)
  - Accuracy and precision (p. 169)
  - Scientific notation (p. 170)
  - Significant figures (p. 171)
- Discussion and sample code (p. 172)
- Run the scripts (p. 179)
- Resources (p. 179)
- Miscellaneous (p. 179)

8.2 Preface

8.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The purpose of this module is to explain the use of scientific notation and significant figures in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m37445/1.5/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

8.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

8.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

8.2.3.1 Figures

- Figure 1 (p. 172) . Examples of significant figures.
- Figure 2 (p. 173) . Screen output from Listing #1.
- Figure 3 (p. 175) . Screen output from Listing #2.
- Figure 4 (p. 176) . Behavior of the toPrecision method.
- Figure 5 (p. 179) . Screen output from Listing #3.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

8.2.3.2 Listings

- Listing 1 (p. 173) . An exercise involving addition.
- Listing 2 (p. 175) . An exercise involving multiplication.
- Listing 3 (p. 178) . An exercise involving combined operations.

8.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

8.3 General background information

This section will contain a discussion of accuracy, precision, scientific notation, and significant figures.

8.3.1 Accuracy and precision

Let's begin with a brief discussion of accuracy and precision. These two terms are often confused in everyday conversation, but they have very different meanings in the world of science and engineering.

Accuracy

In science and engineering, the accuracy of a measurement system is the degree of closeness of measurements of a quantity to its actual (true) value.

Precision

The precision of a measurement system (also called reproducibility or repeatability) is the degree to which repeated measurements under unchanged conditions show the same result.

Four possibilities

A measurement system can be:

- Both accurate and precise.
- Accurate but not precise.
- Precise but not accurate.
- Neither accurate nor precise.

A hypothetical experiment

Consider an experiment where a firearm is clamped into a fixture, very carefully aimed at a bulls eye on a downrange target, and fired six times. (Although you may never have seen or touched a firearm, you probably have a pretty good idea of how they behave.)

If the six holes produced by the bullets in the target fall in a tight cluster in the bulls eye, the system can be considered to be both accurate and precise.

If all of the holes fall in the general area of the bulls eye but the cluster is not very tight, the system can be considered to be accurate but not precise.

If all of the holes fall in a tight cluster but the cluster is some distance from the bulls eye, the system can be considered to be precise but not accurate.

If the holes are scattered across a wide area of the target, the system can be considered to be neither accurate nor precise.

Another use of the word precision

Another use of the word precision, which will be important in this module, is based on the concept that the precision of a measurement describes the units that you use to measure something.

How tall are you?

[14] http://www.dickbaldwin.com/toc.htm

For example, if you tell someone that you are about five feet tall, that wouldn't be very precise. If you told someone that you are 62 inches tall, that would be more precise. If you told someone that you are 62.3 inches tall, that would be even more precise, and if you told someone that you are 62.37 inches tall, that would be very precise for a measurement of that nature.

The smaller the unit...

The smaller the unit you use to measure with, the more precise the measurement can be. For example, assume that you measure someone's height with a tactile measuring stick that is longer than the person is tall. Assume also that the measuring stick is graduated only in feet. In that case, the best that you could hope for would be to get the measurement correct to the nearest foot and perhaps estimate a second digit to the nearest tenth of a foot.

One-inch graduations

On the other hand, if the measuring stick is graduated in inches and you are careful, you should be able to get the measurement correct to the nearest inch and perhaps estimate another digit to the nearest tenth of an inch. The second measurement using the one-inch graduations would be more precise than the first using the one-foot graduations.

Diminishing returns

If the measuring stick were graduated in tenth-inch units, however, that may or may not lead to a more precise measurement. That would be approaching the point of diminishing returns where your inability to take full advantage of the more precise graduations and the inability of the subject to always stand with the same degree of rigidity might come into play.

8.3.2 Scientific notation

According to Wikipedia [15] , scientific notation is a way of writing numbers that accommodates values too large or small to be conveniently written in standard decimal notation. The notation has a number of useful properties and is commonly used by scientists and engineers. A variation of scientific notation is also used internally by computers.

Scientific notation format

Numbers in scientific notation are written using the following format:

x * 10^y

which can be read as the value **x** multiplied by ten raised to the **y** power where **y** is an integer and **x** is any real number. (Constraints are placed on the value of **x** when using the **normalized** form of scientific notation which I will explain below.)

The values for **x** and **y** can be either positive or negative.

The term referred to as **x** is often called the **significand** or the **mantissa** (not to be confused with the term mantissa used with common logarithms).

The computer display version of scientific notation

Because of difficulties involved in displaying superscripts in the output of computer programs, a typical display of a number in scientific notation by a computer program might look something like the following example:

-3.141592653589793e+1

where

- Either numeric value can be positive, negative, or zero.
- The number of digits in the numeric value to the left of the **e** (the mantissa) may range from a few to many.
- The **e** may be either upper-case or lower-case depending on the computer and the program.

A power of ten is understood

In this format, it is understood that the number consists of the value to the left of the **e** (the mantissa) multiplied by ten raised to a power given by the value to the right of the **e** (the exponent).

[15] http://en.wikipedia.org/wiki/Scientific_notation

For example, in JavaScript exponential format, the value -10*Math.PI is displayed as
-3.141592653589793e+1
The value Math.PI/10 is displayed as
3.141592653589793e-1
The value Math.PI is displayed as
3.141592653589793e+0
The value 0 is displayed as
0e+0

The normalized form of scientific notation

Using general scientific notation, the number -65700 could be written in several different ways including the following:

- -6.57 * 10^4
- -65.7 * 10^3
- -657 * 10^2

In normalized scientific notation, the exponent is chosen such that the absolute value of the mantissa is at least one but less than ten. For example, -65700 is written:
-6.57 * 10^4
In normalized notation the exponent is negative for a number with absolute value between 0 and 1. For example, the value 0.00657 would be written:
6.57 * 10^(-3)
The 10 and the exponent are usually omitted when the exponent is 0.

8.3.3 Significant figures

According to Wikipedia [16] , The significant figures of a number are those digits that carry meaning contributing to its precision. This includes all digits except:

- Leading zeros where they serve merely as placeholders to indicate the scale of the number (.00356 for example).
- Spurious digits introduced, for example, by calculations carried out to greater accuracy than that of the original data, or measurements reported to a greater precision than the equipment supports.

A popular physics textbook provides a more complete set of rules for identifying the significant figures in a number:

1. Nonzero digits are always significant.
2. Final or ending zeros written to the right of the decimal point are significant.
3. Zeros written to the right of the decimal point for the purpose of spacing the decimal point are not significant.
4. Zeros written to the left of the decimal point may be significant, or they may only be there to space the decimal point. For example, 200 cm could have one, two, or three significant figures; it's not clear whether the distance was measured to the nearest 1 cm, to the nearest 10 cm, or to the nearest 100 cm. On the other hand, 200.0 cm has four significant figures (see rule 5). Rewriting the number in scientific notation is one way to remove the ambiguity.
5. Zeros written between significant figures are significant.

Ambiguity of the last digit in scientific notation

Again, according to Wikipedia [17] , it is customary in scientific measurements to record all the significant digits from the measurements, and to guess one additional digit if there is any information at all available to

[16] http://en.wikipedia.org/wiki/Significant_figures
[17] http://en.wikipedia.org/wiki/Scientific_notation

the observer to make a guess. The resulting number is considered more valuable than it would be without that extra digit, and it is considered a significant digit because it contains some information leading to greater precision in measurements and in aggregations of measurements (adding them or multiplying them together).

Examples of significant digits

Referring back to the physics textbook mentioned earlier, Figure 1 (p. 172) shows:

- Four different numbers
- The number of significant figures in each number.
- The default JavaScript exponential representation of each number.

Figure 1 . Examples of significant figures.

```
1.  409.8          4          4.098e+2
2.  0.058700       5          5.87e-2
3.  9500           ambiguous  9.5e+3
4.  950.0 * 10^1   4          9.5e+3
```

Table 8.1

Note that the default JavaScript exponential representation fails to display the significant trailing zeros for the numbers on row 2 and row 5. I will show you some ways that you may be able to deal with this issue later but you may not find them to be very straightforward.

8.4 Discussion and sample code

Beyond knowing about scientific notation and significant figures from a formatting viewpoint, you need to know how to perform arithmetic while satisfying the rules for scientific notation and significant figures.

Performing arithmetic involves **three main rules** :

1. For addition and subtraction, the final result should be rounded so as to have the same number of decimal places as the number that was included in the sum that has the smallest number of decimal places. In accordance with the discussion early in this module, this is the least precise number.
2. For multiplication and division, the final result should be rounded to have the same number of significant figures as the number that was included in the product with the smallest number of significant figures.
3. The two rules listed above should not be applied to intermediate calculations. For intermediate calculations, keep as many digits as practical. Round to the correct number of significant figures or the correct number of decimal places in the final result.

An exercise involving addition

Please copy the JavaScript code shown in Listing 1 (p. 173) into an html file and open the file in your browser.

Listing 1 . An exercise involving addition.

```
    <!-- File JavaScript01.html -->
<html><body>
<script language="JavaScript1.3">

//Compute and display the sum of three
// numbers
var a = 169.01
var b = 0.00356
var c = 385.293
var sum = a + b + c
document.write("sum = " + sum + "</br>")

//Round the sum to the correct number
// of digits to the right of the decimal
// point.
var round = sum.toFixed(2)
document.write("round = " + round + "</br>")

//Display a final line as a hedge against
// unidentified coding errors.
document.write("The End")

</script>
</body></html>
```

Table 8.2

Screen output

When you open the html file in your browser, the text shown in Figure 2 (p. 173) should appear in your browser.

Figure 2 . Screen output from Listing #1.

```
    sum = 554.30656
round = 554.31
The End
```

Table 8.3

The code in Listing 1 (p. 173) begins by declaring three variables named **a** , **b** , and **c** , adding them together, and displaying the sum in the JavaScript default format in the browser window.

Too many decimal digits

As you can see from the first line in Figure 2 (p. 173) , the result is computed and displayed with eight decimal digits, five of which are to the right of the decimal point. We know, however, from rule #1 (p. 172) , that we should present the result rounded to a precision of two digits to the right of the decimal point in

order to match the least precise of the numbers included in the sum. In this case, the value stored in the variable named **a** is the least precise.

Correct the problem

The code in Listing 1 (p. 173) calls a method named **toFixed** on the value stored in the variable **sum** passing a value of 2 as a parameter to the method. This method returns the value from **sum** rounded to two decimal digits. The returned value is stored in the variable named **round** . Then the script displays that value as the second line of text in Figure 2 (p. 173) .

The output text that reads "The End"

There is a downside to using JavaScript (as opposed to other programming languages such as Java). By default, if there is a coding error in your script, there is no indication of the error in the output in the main browser window. Instead, the browser simply refuses to display some or all of the output that you are expecting to see. (Remember, I told you that JavaScript is not my favorite programming language, but it is probably the most accessible for blind students who have no programming experience.)

Put a marker at the end

Writing the script in such a way that a known line of text, such as "The End" will appear following all of the other output won't solve coding errors. However, if it doesn't appear, you will know that there is a coding error and some or all of the output text may be missing.

JavaScript and error consoles

I explained how you can open a JavaScript console in the Google Chrome browser or an error console in the Firefox browser in an earlier module titled JavaScript for Blind Students [18] . While the diagnostic information provided in those consoles is limited, it will usually indicate the line number in the source code where the programming error was detected. Knowing the line number will help you examine the code and fix the error.

An exercise involving multiplication

Please copy the code shown in Listing 2 (p. 175) into an html file and open it in your browser.

[18] http://cnx.org/contents/9eb6aeee-1604-4bde-b81f-912622f036d1

Listing 2 . An exercise involving multiplication.

```
    <!-- File JavaScript02.html -->
<html><body>
<script language="JavaScript1.3">

//Compute and display the product of three
// numbers, each having a different number
// of significant figures.
var a = 169.01
var b = 0.00356
var c = 386.253
var product = a * b * c
document.write("product = " + product + "</br>")

//Round the product to the correct number
// of significant figures
var rounded = product.toPrecision(5)
document.write("rounded = " + rounded + "</br>")

//Display a final line as a hedge against
// unidentified coding errors.
document.write("The End")

</script>
</body></html>
```

Table 8.4

The screen output

When you open your html file in your browser, the text shown in Figure 3 (p. 175) should appear in your browser window.

Figure 3 . Screen output from Listing #2.

```
    product = 232.39900552679998
rounded = 232.40
The End
```

Table 8.5

The code in Listing 2 (p. 175) begins by declaring three variables named **a** , **b** , and **c** , multiplying them together, and displaying the product in the browser window. Each of the factors in the product have a different number of significant figures, with the factor of value 169.01 having the least number (5) of significant figures. We know from rule #2 (p. 172) , therefore, that we need to present the result rounded to five significant figures.

The toPrecision method

Listing 2 (p. 175) calls a method named **toPrecision** on the variable named **product** , passing the desired number of significant figures (5) as a parameter. The method rounds the value stored in **product** to the desired number of digits and returns the result, which is stored in the variable named **rounded** . Then the contents of the variable named **rounded** are displayed, producing the second line of text in Figure 3 (p. 175) .

What about other parameter values

Note that the method named **toPrecision** knows nothing about significant figures. It was up to me to figure out the desired number of significant figures in advance and to pass that value as a parameter to the method.

Although this has nothing to do with significant figures, it may be instructive to examine the behavior of the method named **toPrecision** for several different parameter values.

Figure 4 (p. 176) shows the result of replacing the parameter value of 5 in the call to the **toPrecision** method with the values in the first column of Figure 4 (p. 176) and displaying the value returned by the method.

Figure 4 . Behavior of the toPrecision method.

```
1  rounded = 2e+2
2  rounded = 2.3e+2
3  rounded = 232
4  rounded = 232.4
5  rounded = 232.40
6  rounded = 232.399
7  rounded = 232.3990
10 rounded = 232.3990055
15 rounded = 232.399005526800
20 rounded = 232.39900552679998214
```

Table 8.6

And the point is...

The point to this is to emphasize that the method named **toPrecision** is not a method that knows how to compute and display the required number of significant figures. Instead, according to the JavaScript documentation:

"The toPrecision() method formats a number to a specified length. A decimal point and nulls are added (if needed), to create the specified length."

It is up to you, the author of the script, to determine what that length should be and to provide that information as a parameter to the **toPrecision** method.

Combined operations

This is where things become a little hazy. I have been unable to find definitive information as to how to treat the precision and the number of significant figures when doing computations that combine addition and/or subtraction with multiplication and/or division.

Two contradictory procedures

I have found two procedures documented on the web that seem to be somewhat contradictory. Both sources seem to say that you should perform the addition and/or subtraction first and that you should apply rule #1 (p. 172) to the results. However, they differ with regard to how stringently you apply that rule before moving on to the multiplication and/or division.

The more stringent procedure

One source seems to suggest that you should round the results of the addition and/or subtraction according to rule #1 (p. 172) and replace the addition or subtraction expression in your overall expression with the rounded result. Using that approach, you simply create one the factors that will be used later in the multiplication and/or division. That factor has a well-defined number of significant figures.

Then you proceed with the multiplication and/or division and adjust the number of significant figures in the final result according to rule #2 (p. 172) .

The less stringent procedure

The other source seems to suggest that you mentally round the results of the addition and/or subtraction according to rule #1 (p. 172) and make a note of the number of significant figures that would result if you were to actually round the result. However, you should not actually round the result at that point in time. In other words, you should use the raw result of the addition and/or subtraction as a factor in the upcoming multiplication and/or division knowing that you may be carrying excess precision according to rule #1 (p. 172) .

Then you proceed with the multiplication and/or division and adjust the number of significant figures in the final result according to rule #2 (p. 172) . However, when you adjust the number of significant figures, you should include the number of significant figures from your note in the decision process. If that is the smallest number of significant figures of all the factors, you should use it as the number of significant figures for the final result.

Consult with your instructor

Before accepting either of these procedures as the correct procedure, I recommend that you consult with your physics instructor to confirm which, if either of the procedures is correct for combined operations.

An exercise involving combined operations

Evaluate the following expression and display the final result with the correct number of significant figures.

(169.01 + 3294.6372) * (0.00365 - 29.333)

Please copy the code from Listing 3 (p. 178) into an html file and open it in your browser.

Listing 3 . An exercise involving combined operations.

```
    <!-- File JavaScript03.html -->
<html><body>
<script language="JavaScript1.3">

//Compute, fix the number of decimal places,
// and display the sum of two numbers.
var a1 = 169.01
var a2 = 3294.6372
var aSum = (a1 + a2).toFixed(2)
document.write("aSum = " + aSum + "</br>")

//Compute, fix the number of decimal places,
// and display the difference between two
// other numbers.
var b1 = 0.00356
var b2 = 29.333
var bDiff = (b1 - b2).toFixed(3)
document.write("bDiff = " + bDiff + "</br>")

//Compute and display the product of the
// sum and the difference.
var product = aSum * bDiff
document.write("product = " + product + "</br>")

//Round the product to the correct number
// of significant figures based on the least
// number of significant figures in the
// factors.
var final = product.toPrecision(5)
document.write("final = " + final + "</br>")

//Display a final line as a hedge against
// unidentified coding errors.
document.write("The End")

</script>
</body></html>
```

Table 8.7

When you open your html file in your browser, the text shown in Figure 5 (p. 179) should appear in the browser window.

Figure 5 . Screen output from Listing #3.

```
    aSum = 3463.65
bDiff = -29.329
product = -101585.39085000001
final = -1.0159e+5
The End
```

Table 8.8

The more stringent procedure

The code in Listing 3 (p. 178) implements the more stringent procedure (p. 176) , not because it is necessarily the correct one. Rather, it is simpler to implement in a script.

Do addition and subtraction first

Listing 3 (p. 178) begins by adding two numbers, adjusting the precision to the least precise of the two numbers, and saving the result in the variable named **aSum** .

Then Listing 3 (p. 178) subtracts one number from another number, adjusts the precision to the least precise of the two numbers, and saves the result in the variable named **bDiff** .

Display to get information on significant figures

Both results are displayed immediately after they are obtained. This is necessary for me to know which one has the least number of significant figures. I need to know that to be able to properly adjust the number of significant figures in the final product.

In other words, it was necessary for me to write and execute the addition/subtraction portion of the script in order to get the information required to write the remainder of the script.

Do the multiplication

Then Listing 3 (p. 178) multiplies the sum and difference values and displays the result in the default format with far too many significant figures as shown by the third line of text in Figure 5 (p. 179) .

Finally Listing 3 (p. 178) adjusts the number of significant figures in the product based on the number of significant figures in **bDiff** and displays the final result with five significant figures in normalized scientific (exponential) notation.

8.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

8.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

8.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Scientific Notation and Significant Figures
- File: Phy1040.htm
- Revised: 10/01/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · screen reader
  - · Braille display
  - · JavaScript
  - · scientific notation
  - · significant figures
  - · accuracy
  - · precision

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 9

Units and Dimensional Analysis[1]

9.1 Table of Contents

- Preface (p. 181)
  - General (p. 181)
  - Prerequisites (p. 182)
  - Viewing tip (p. 182)
    - Figures (p. 183)
    - Listings (p. 183)
  - Supplemental material (p. 183)
- General background information (p. 183)
  - Formatting mathematical expressions (p. 183)
  - What do we mean by units? (p. 184)
  - SI units (p. 184)
  - Prefixes for powers of ten (p. 186)
  - Dimensional analysis (p. 186)
- Discussion and sample code (p. 186)
  - Exercise on manually converting units (p. 187)
  - Using JavaScript to convert units (p. 189)
  - More substantive JavaScript examples (p. 193)
    - Free fall exercise (p. 193)
    - A plotting exercise (p. 195)
  - Non-mirror-image graphics (p. 202)
- Run the scripts (p. 207)
- Resources (p. 207)
- Miscellaneous (p. 207)

9.2 Preface

9.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m37480/1.6/>.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The purpose of this module is to explain units and dimensional analysis in a format that is accessible to blind students.

9.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12])).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

9.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

9.2.3.1 Figures

- Figure 1 (p. 184) . Single-line format.
- Figure 2 (p. 185) . SI base units.
- Figure 3 (p. 185) . Examples of SI derived units.
- Figure 4 (p. 186) . A sampling of SI prefixes.
- Figure 5 (p. 191) . Screen output for Listing #1.
- Figure 6 (p. 194) . Screen output for Listing #2.
- Figure 7 (p. 197) . Screen output for Listing #3.
- Figure 8 (p. 201) . Mirror image from the file named Phy1050a1.svg.
- Figure 9 (p. 202) . Key-value table for the image shown in Figure 8.
- Figure 10 (p. 206) . Non-mirror-image version of the image from the file named Phy1050a1.svg.

9.2.3.2 Listings

- Listing 1 (p. 190) . Convert from paces to miles.
- Listing 2 (p. 194) . Free fall exercise.
- Listing 3 (p. 196) . A plotting exercise.

9.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

9.3 General background information

As a young engineering student many years ago, I was told that when evaluating mathematical expressions, I should always embed the units in the expression, manipulate them algebraically, and confirm that the units that survive until the end of the evaluation are correct relative to the problem specifications.

Three good reasons

There are at least three good reasons for following this procedure:

- It shows what the units of the result are. A common mistake is to get the correct numerical result of a calculation but to write it with the wrong units.
- It shows where unit conversions must be performed. If units that should have canceled do not cancel, we need to go back and perform the necessary conversions. For example, when a speed in miles per hour is being calculated and the result comes out with units of miles per second, we should convert seconds to hours.
- It helps to locate mistakes. If a velocity (meters per second) is being calculated and the units come out as seconds per meter, we know to look for an error.

9.3.1 Formatting mathematical expressions

A significant disadvantage

This is an area where blind students may be at a significant disadvantage relative to sighted students due mainly to the fact that blind students using screen readers and Braille displays can only see one line of text at a time. It is much easier (for me anyway) to correctly manipulate the units when the expression is written on two or more lines.

[14] http://www.dickbaldwin.com/toc.htm

Multiple-line format

What I mean by this is that a sighted student would typically write a fraction as a horizontal line with the numerator above the line and the denominator below the line using superscripts to indicate exponents. The product of two fractions would typically be written in the same format with a multiplication indicator joining the two fractions.

Single-line format

A blind student constrained to seeing only one line of text at a time would probably need to write and evaluate the product of the two fractions using a format something like that shown in Figure 1 (p. 184) .

Figure 1 . Single-line format.

```
((3*x^2)/(4*x) * (6/(4*x) = (18*x^2)/(16*x^2) = 18/16
```

Table 9.1

With enough time and practice, I might become almost as proficient in evaluating expressions as shown in Figure 1 (p. 184) as in evaluating expressions in a multiple-line format, but I'm afraid that I would need a lot of time and a lot of practice to achieve that proficiency.

We must work with what we are given

However, we must work with what we are given so the mathematical expressions in this module (and most of the other modules as well) will all be in a single-line format. The good news is that the single-line format can usually be easily translated into a format that is suitable for evaluation using JavaScript.

9.3.2 What do we mean by units?

Let me try to answer this important question with an example.

If you were to walk from your home to your school and count your steps, you might report that the distance from your home to your school is 2112 steps or paces. If you are a male student, this would probably be approximately one mile, using assumptions that I will explain later. If you are a female, it would probably be somewhat less than a mile because your pace would probably be a little shorter than your male friend's pace.

A pace is a unit of measurement

In this case, the pace would be a unit of measurement. However, it would not be a standard unit because the length of a pace for one student is often different from the length of a pace for a different student. A male student, for example, typically has a longer pace length than a female student.

Standard units

Standard units are units whose value has been set by some official agency that has the authority to set standards. One such standard unit is the mile, which as you may know is equal to 5280 foots (more correctly pronounced feet).

A foot is also a standard unit, which you may also know is equal to 12 inches. An inch is another standard unit.

9.3.3 SI units

The discrepancy between the pronunciation of the unit **foot** and its plural **feet** is an example of the need for more consistency in the use of units. Many physics books use a system of units called **SI units** . SI is an abbreviation for a French name, which I am unable to pronounce, and which is probably also not compatible with your screen reader and your Braille display.

I won't attempt to explain much about SI units. You can probably find a good explanation in your textbook, and if not, you can Google SI units and find hundreds of web pages that explain the system in varying levels of detail.

Tables of SI units

Most of those references will probably also provide tables for the units, but those tables may be partially incompatible with your screen reader and Braille display due to the extensive use of superscripts. Therefore, I will attempt to provide accessible tables in Figure 2 (p. 185) and Figure 3 (p. 185) .

Don't ignore the details

Note, however, that you should not ignore the online SI-unit tables altogether. They are likely to contain important information that I won't reproduce here, such as the fact that the meter is a unit of length and its value is the length of the path travelled by light in a vacuum during a time interval of 1/299792458 of a second.

Base units and derived units

When reading about SI units, you will find that they are often divided into base units and derived units. I will put the base units in Figure 2 (p. 185) and some sample derived units in Figure 3 (p. 185) .

Figure 2 . SI base units.

Base Quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 9.2

Note that the list of derived units in Figure 3 (p. 185) is only a sampling of different units that can be derived from the base units.

The exponentiation indicator

As is the case throughout these modules, the character "^" that you see used extensively in Figure 3 (p. 185) indicates that the character following the ^ is an exponent. Note also that when the exponent is negative, it is enclosed along with its minus sign in parentheses for clarity.

Figure 3 . Examples of SI derived units.

area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	m/s
acceleration	meter per second squared	m/s^2
wave number	reciprocal meter	m^(-1)
mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m^3/kg
current density	ampere per square meter	A/m^2

Table 9.3

9.3.4 Prefixes for powers of ten

As an alternative to explicitly writing powers of ten, SI uses prefixes for units to indicate power of ten factors. Figure 4 (p. 186) shows some of the powers of ten and the SI prefixes used for them.

Figure 4 . A sampling of SI prefixes.

Prefix-(abbreviation)	Power of Ten
peta-(P)	10^15
tera-(T)	10^12
giga-(G)	10^9
mega-(M)	10^6
kilo-(k)	10^3
deci-(d)	10^(-1)
centi-(c)	10^(-2)
milli-(m)	10^(-3)
micro-(Greek letter mu)	10^(-6)
nano-(n)	10^(-9)
pico-(p)	10^(-12)
femto-(f)	10^(-15)

Table 9.4

9.3.5 Dimensional analysis

As typically used in physics, the word *dimensions* means basic types of units such as time, length, and mass (see Figure 2 (p. 185)).

There are many different units that are used to express length, for example, such as foot, mile, meter, inch, etc.

Because they all have dimensions of length, you can convert from one to another. For example one mile is equal to 5280 feet.

Cannot convert between units of different dimensions

When evaluating mathematical expressions, we can add, subtract, or equate quantities only if they have the same dimensions (although they may not necessarily be given in the same units). For example, it is possible to add 3 meters to 2 inches (after converting units), but it is not possible to add 3 meters to 2 kilograms.

To analyze dimensions, you should treat them as algebraic quantities using the same procedures that we will use in the upcoming exercise on manually converting units.

9.4 Discussion and sample code

Let's manually work through an exercise to determine the distance from your home to your school in miles given that we know the distance in paces. (I will convert this to a JavaScript script later in the module.)

As we discussed earlier, you have already determined that the distance from your home to your school is 2112 paces. We have also recognized that the length of one of your paces is likely to be different from the length of someone else's paces.

9.4.1 Exercise on manually converting units

This is a somewhat long and tedious explanation, but unless you already have quite a lot of experience in this area, you should probably bear with me as I walk you through it.

Convert your pace length to standard units

First we must convert the length of your pace to standard units, which we can then convert to miles.

A calibration exercise

Assume that you go into an empty parking lot, mark your starting position, walk 100 paces, and then mark your ending position on the pavement. Assume that you then use a tactile measuring tape to measure the distance from the starting position to the ending position and you find that distance to be 3000 inches.

From that, we could conclude that for your pace length,

100 paces = 3000 inches

Thus, we have calibrated your pace length in terms of the standard unit inch.

The plural versus the singular

Note that the use of the plural such as **inches** and the singular such as **inch** has no impact on the results of our computations. we can switch back and forth from the plural to the singular at will as long as it makes sense to do so.

A conversion factor

If we divide both sides of the above equation (p. 187) by 100 paces, we get

1 = 3000 inches/100 paces = 30 inches/pace

Note that we now have a 1 on the left side and a fraction on the right side.

Multiplication by 1

Multiplying a value by 1 doesn't change the value. This means that if we multiply a term in an algebraic expression by

30*inch/pace

we won't change the intrinsic value of the term, although we might change the units in which that value is expressed.

The term

30*inch/pace

can be used to convert a quantity in units of paces to the same quantity in units of inches.

Move your body forward

So now we know that on the average, each time you execute one pace, you have moved your body forward by 30 inches. The next task is to determine how far you have moved your body when you have executed 2112 paces (the distance from home to school measured in paces).

Convert from paces to inches

We can begin by determining the distance from home to school in inches as follows:

distance = (2112*pace) * (30*inch/pace)

A review of algebra

At this point, let's review a little algebra:

Given the expression: ((w/x)*(y/z))

we can rewrite this as

(w*y)/(x*z)

Given this expression, we can reverse the process by rearranging and factoring out the terms producing our original expression:

((w/x)*(y/z))

An algebraic manipulation

We determined earlier that

distance = (2112*pace) * (30*inch/pace)

Applying the algebraic process shown above to the problem at hand, we can rearrange terms and factor our expression to produce

distance = (2112 * 30 * pace * inch)/pace

The distance in inches

At this point, we have a fraction that contains the unit pace in both the numerator and the denominator. We can cancel those two terms leaving us with

distance = (2112 * 30 * inch) = 63360 * inch

So good so far. We now know the distance from home to school as expressed in units of inches (even though the original measurement was made in paces and not in inches). The distance hasn't changed. What has changed is the units in which we are expressing that distance.

Still not what we are looking for

While this is interesting, it still isn't what we are looking for. We want to know the distance in miles. Therefore, we need to convert the distance in inches to the distance in miles.

How many inches are in a mile?

At this point, we don't know how many inches are in a mile (but we could calculate it if we wanted to). However, we do know how many inches are in a foot and we know how many feet are in a mile.

A conversion factor for inches to feet

For starters, lets calculate a factor that we can use to convert from inches to feet. We know that

12*inches = 1*foot

If we divide each side by 12*inches, we get

1 = 1*foot/12*inches

Let's refer to this as a conversion factor

Multiplication by 1

Since the expression on the right side is equal to 1, we can multiply any expression in an equation with the conversion factor

1*foot/12*inches

without changing the intrinsic value of that expression. (Once again, although we won't be changing the intrinsic value of the expression, we may cause that intrinsic value to be expressed in different units.)

We can use this conversion factor to convert a distance value expressed in inches into the value for the same distance expressed in feet.

A conversion factor for feet to inches

We could also use the reciprocal of the expression to convert a distance value expressed in feet into the value for the same distance expressed in inches. I will have more to say on this later.

A conversion factor for feet to miles

Similarly, we could use the same procedure to show that

1 = 1*mile/5280*foot

Again, since the expression on the right side is equal to 1, we can multiply any expression in an equation with the expression on the right without changing the intrinsic value of the original expression. This conversion expression can be used to convert from feet to miles. Its reciprocal could be used to convert from miles to feet.

Back to the problem at hand

We determined earlier that the distance expressed in inches from home to school is

distance = 63360 * inch

We can probably agree that multiplying the expression on the right as follows would not change the intrinsic value for the distance.

distance = 63360 * inch * 1 * 1

In fact, we could probably agree that it wouldn't change anything at all.

Substitute the two conversion factors

Having agreed on that, we can substitute the two conversion factors derived earlier, each of which has a value of 1, for the last two factors in our distance equation.

distance = 63360 * inch * (1*foot/12*inch) * (1*mile/5280*foot)

Rearranging the terms

We can rearrange the terms and rewrite this equation as

distance = (63360 * inch * 1*foot * 1 * mile)/(12*inch*5280*foot)

Canceling like terms

Canceling out like terms in the numerator and denominator leaves us with

distance = (63360*mile)/(12*5280)

Doing the arithmetic, we get

distance = 1*mile

(Obviously, I started out with a set of numbers that were designed to cause the final answer to be one mile, but that was simply for convenience.)

There you have it

You have seen a detailed procedure for converting from a distance expressed in paces (for a specific individual) to the same distance expressed in miles. Obviously, once you understand the overall procedure, you could omit and/or combine some of the steps to simplify the process.

9.4.2 Using JavaScript to convert units

Let's write, execute, and analyze a script that solves the same problem.

The objective of the script is to convert a distance of 2112 paces to a distance in miles where it is given that

- 100 paces = 3000 inches
- 12 inches = 1 foot
- 5280 feet = 1 mile

Please copy the code from Listing 1 (p. 190) into an html file and open the file in your browser.

Listing 1 . Convert from paces to miles.

```
    >!-- File JavaScript01.html ------------------------------ >
>html ≫body >
>script language="JavaScript1.3" >

var d  = 2112   //distance, units = paces

document.write(
"d  = " + d  + " paces" + ">/br >")

var d2 = 3000   //distance, units = inches
var d3 = 100    //distance, units = paces

var f1 = d2/d3  //factor, units = inches/pace
document.write(
"f1 =" + f1 + " inches/pace" + ">/br >")

//pace*inch/pace = inch
var d  = d *f1  //distance, units = inches
document.write(
"d  = " + d  + " inches" + ">/br >")

var f2 = 1/12   //factor, units = feet/inch
document.write(
"f2 = " + f2 + " feet/inch" + ">/br >")

//inch*feet/inch = feet
var d  = d *f2  //distance, units = feet
document.write(
"d  = " + d  + " feet" + ">/br >")

var f3 = 1/5280 //factor, units = miles/foot
document.write(
"f3 = " + f3 + " miles/foot" + ">/br >")

//feet*mile/feet = mile
var d  = d *f3  //distance, units = miles
document.write(
"d  = " + d  + " mile" + ">/br >")

//Now reverse the process using reciprocals
//(pace/inch)*(inch/foot)*(foot/mile) = pace/mile
var f4 =(1/f1)*(1/f2)*(1/f3) //factor, units = paces/mile
document.write(
"f4 = " + f4 + " paces/mile" + ">/br >")

//miles*paces/mile = paces
var d  = d *f4  //distance, units = paces
document.write(
"d  = " + d  + " paces" + ">/br >")

document.write("The End")

>/script >
>/body ≫/html >
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 9.5

Screen output

When you open the html file in your browser, the text shown in Figure 5 (p. 191) should appear in your browser window.

Figure 5 . Screen output for Listing #1.

```
    d = 2112 paces
f1 =30 inches/pace
d = 63360 inches
f2 = 0.08333333333333333 feet/inch
d = 5280 feet
f3 = 0.0001893939393939394 miles/foot
d = 1 mile
f4 = 2112 paces/mile
d = 2112 paces
The End
```

Table 9.6

Discuss the output first

Let's begin our discussion with the screen output shown in Figure 5 (p. 191) and then go back and examine the code that produced that output.

Figure 5 (p. 191) begins with a distance (labeled **d**) expressed in units of paces. The value is 2112 paces, which is the distance from your home to your school discussed earlier.

Convert paces to inches

Then Figure 5 (p. 191) shows the conversion factor (labeled **f1**) that can be used to convert a value expressed in paces to a value expressed in inches.

That conversion factor is applied to the original distance producing an output consisting of the same distance expressed in inches instead of paces. The distance is now 63360 inches.

Convert from inches to feet

Following that, **f2** shows the conversion factor that can be used to convert a value expressed in inches to the same value expressed in feet.

That conversion factor is applied to the current distance value producing an output consisting of the same distance expressed in feet instead of inches. The distance is now 5280 feet.

Convert from feet to miles

Finally, Figure 5 (p. 191) shows the conversion factor (labeled **f3**) that can be used to convert a value expressed in feet to the same value expressed in miles.

That conversion factor is applied to the current distance value producing a final output consisting of the same distance expressed in miles instead of feet. That distance is now one mile (obviously I chose the numbers to make it come out that way).

That is the number that we were looking for.

Reverse the process

If a conversion factor X can be used to convert from A-units to B-units, the reciprocal of X can be used to convert from B-units back to A-units.

As you will see later, the conversion factor **f4** shown in Figure 5 (p. 191) was computed as the product of the reciprocals of **f1** , **f2** , and **f3** . The result is that the conversion factor labeled **f4** can be used to convert a value expressed in miles to the same value expressed in paces (according to the pace-length for one individual).

That conversion factor was applied to the distance expressed in miles to produce the same distance expressed in paces. As you might have guessed, this value of 2112 paces shown near the end of Figure 5 (p. 191) matches the value shown at the beginning of Figure 5 (p. 191) .

Analysis of the script

Now lets analyze the script code in Listing 1 (p. 190) that produced the text output shown in Figure 5 (p. 191) .

Listing 1 (p. 190) begins by declaring and initializing the variables **d** , **d2** , and **d3** , which are respectively,

- The home to school distance in paces.
- The two values that represent the relationship between paces and inches for one individual.

Various values are displayed as the script executes using calls to the **document.write** method. You should have no difficulty identify the code that displays the values, so I will skip over that code in this discussion.

Compute, save, and apply the conversion factor f1

Then Listing 1 (p. 190) declares a variable named **f1** and populates it with the value of a conversion factor that can be used to convert paces to inches. This conversion factor is computed from the known relationship between paces and inches mentioned earlier.

The conversion factor named **f1** is applied to the distance in paces converting it to a value that represents the same distance in inches (63360 inches).

Analysis of the units

The comment that reads

//pace*inch/pace = inch

is an analysis that shows the units that will result from applying the conversion factor to the distance at this point. As you can see, the pace terms will cancel and the result will be in inches.

Compute, save, and apply the conversion factor f2

Following that, a conversion factor named **f2** is computed that can be used convert a value expressed in inches to the same value expressed in feet. This conversion factor is based on the known fact that there are 12 inches in a foot.

The factor named **f2** is applied to the distance that is now expressed in inches converting it to a new value that expresses the same distance in feet (5280 feet).

Analysis of the units

The comment that reads

//inch*feet/inch = feet

is an analysis that shows the units that will result from applying the conversion factor to the distance at this point. As you can see, the inch terms will cancel and the result will be in feet.

Compute, save, and apply the conversion factor f3

Following that, a conversion factor named **f3** is computed that can be used to convert a value expressed in feet to the same value expressed in miles. This conversion factor is based on the known fact that there are 5280 feet in a mile.

The factor named **f3** is applied to the distance that is now expressed in feet converting it to a new value that expresses the same distance in miles (1 mile).

Analysis of the units

The comment that reads

//feet*mile/feet = mile

is an analysis that shows the units that will result from applying the conversion factor to the distance at this point. As you can see, the feet terms will cancel and the result will be in miles.

That satisfies the specifications

That satisfies the original program specification. However, I mentioned earlier that if a conversion factor X can be used to convert from A-units to B-units, the reciprocal of X can be used to convert from B-units back to A-units.

Reversing the process

Continuing with Listing 1 (p. 190) , the comment that reads
//(pace/inch)*(inch/foot)*(foot/mile) = pace/mile
shows the units that survive from a product of the reciprocals of **f1** , **f2** , and **f3** . As you can see, after canceling out inches and feet, the result of multiplying the reciprocals of those three conversion factors is a new conversion factor that can be used to convert a value expressed in miles to a new value that represents the same distance expressed in paces.

That conversion factor is applied to the distance in miles producing an output of 2112, which unsurprisingly, is the distance in paces that we started off with.

9.4.3 More substantive JavaScript examples

Assume that you drop a rock from a balloon at a height of 10000 feet above the ground, Further assume that the positive direction is up and the negative direction is down.

What would be the height of the rock above the ground at the end of ten seconds? What would be the height of the rock at the end of twenty seconds?

An equation relating distance, acceleration, and time

As you will learn in a future module, the following equation gives the distance that the rock will travel as a function of time assuming that the **initial velocity is zero** . (The assumption is that the rock was simply dropped **and not thrown** .)

d = 0.5 * g * t^2

where

- d is the distance traveled
- g is the acceleration of gravity (approximately -32.2 ft/s^2)
- t is time is seconds

9.4.3.1 Free fall exercise

Please copy the code from Listing 2 (p. 194) into an html file and open it in your browser.

Listing 2 . Free fall exercise.

```
    >!-- File JavaScript02.html --<
>html<>body<
>script language="JavaScript1.3"<

//Function to compute free-fall distance for a given time
// in seconds.
function distance(time){
  var g = -32.174; //acceleration of gravity in feet/sec^2
  var d = 0.5 * g * time * time;//(feet/sec^2)*sec^2 = feet
   return new Number(d.toFixed(0));
}//end function

//Compute and display height at ten seconds.
var t1 = 10;//time in seconds
var d1 = distance(t1);//distance traveled in feet
var h1 = 10000 + d1;//height in feet

document.write("At " + t1 + " seconds:" +">/br<");
document.write("distance traveled = " + d1 +" feet>/br<")
document.write("height = " + h1 +" feet>/br<")

//Compute and display height at twenty seconds.
var t2 = 20;//time in seconds
var d2 = distance(t2);//distance traveled in feet
var h2 = 10000 + d2;//height in feet

document.write(">/br<At " + t2 + " seconds:" +">/br<");
document.write("distance traveled = " + d2 +" feet>/br<")
document.write("height = " + h2 +" feet>/br<")

document.write(">/br<The End")

>/script<>/body<>/html<
```

Table 9.7

Screen output

When you open the html file in your browser, the text shown in Figure 6 (p. 194) should appear in your browser window.

Figure 6 . Screen output for Listing #2.

continued on next page

```
    At 10 seconds:
distance traveled = -1609 feet
height = 8391 feet

At 20 seconds:
distance traveled = -6435 feet
height = 3565 feet

The End
```

Table 9.8

As you can see from Figure 6 (p. 194) , after ten seconds, the rock will have traveled a distance of -1609 feet. (The minus sign indicates downward motion.)

Adding this value to the initial height of 10000 feet produces a value of 8391 for the height of the rock at the end of 10 seconds.

Similarly, Figure 6 (p. 194) gives us 3565 for the height of the rock at 20 seconds.

Analysis of the code

The code in Listing 2 (p. 194) begins by defining a function that computes and returns the distance traveled for a rock falling toward the earth for a given time interval since release assuming that the initial velocity was zero (the rock was simply dropped).

This function implements the equation (p. 193) shown earlier, and expects to receive the time as an input parameter. It returns the distance traveled during that time interval.

Note that the interaction of units is shown in the comments with the result that the returned value is in feet.

Call the function twice in succession

Then the code calls that function twice in succession for two different time intervals (10 seconds and 20 seconds) to determine the distance traveled during each time interval.

In both cases, the distance traveled is added to the initial height of 10000 feet to determine the height of the rock at the end of the time interval.

Also, in both cases, the time interval, the distance traveled, and the resulting height of the rock above the ground are displayed as shown in Figure 6 (p. 194) .

Note again that in all cases, the interactions of the various units are shown in the comments.

9.4.3.2 A plotting exercise

Let's do another exercise and this time plot the results.

Please prepare your plot board to plot a curve defined by several points. Interpret the grid lines such that you can plot values ranging from 0 to 10000 feet on the vertical axis and you can plot values ranging from 0 to 30 seconds on the horizontal axis.

A tactile graph

If you have an assistant who will download the zip file named Phy1050.zip [15] , extract the file named Phy1050a1.svg, and use that file to create a tactile graph as described in the module named Manual Creation of Tactile Graphics [16] . You can explore the tactile graph instead. Regardless, you should write the script and examine the printed output as described in the following paragraphs.

JavaScript code

Copy the code from Listing 3 (p. 196) into an html file and open it in your browser.

[15] http://cnx.org/content/m37480/latest/Phy1050.zip
[16] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac

Listing 3 . A plotting exercise.

```
    <!-- File JavaScript03.html --<
<html≪body<
<script language="JavaScript1.3"<

//Function to compute free-fall distance for a given time
// interval in seconds.
function distance(time){
  var g = -32.174; //acceleration of gravity in feet/sec^2
  var d = 0.5 * g * time * time;//(feet/sec^2)*sec^2 = feet
    return new Number(d.toFixed(0));
}//end function

//Compute the height of the rock every two seconds from
// release until the rock hits the ground.
var t = 0;//seconds
while(t <= 30){
  d = distance(t);//distance traveled in feet
  h = 10000 + d;//height in feet

  //Don't allow the height to go negative (underground)
  if(h < 0){
    h = 0;
  }//end if

  //Display time and height at current time.
  document.write("time = " + t +
                 " height = " + h + "<br/<");
  t = t + 2;
}//end while

document.write("</br<The End")

</script≪/body≪/html<
```

Table 9.9

The screen output

When you open the html file in your browser, the text shown in Figure 7 (p. 197) should appear in your browser window.

Figure 7 . Screen output for Listing #3.

continued on next page

```
    time = 0 height = 10000
time = 2 height = 9936
time = 4 height = 9743
time = 6 height = 9421
time = 8 height = 8970
time = 10 height = 8391
time = 12 height = 7683
time = 14 height = 6847
time = 16 height = 5882
time = 18 height = 4788
time = 20 height = 3565
time = 22 height = 2214
time = 24 height = 734
time = 26 height = 0
time = 28 height = 0
time = 30 height = 0

The End
```

Table 9.10

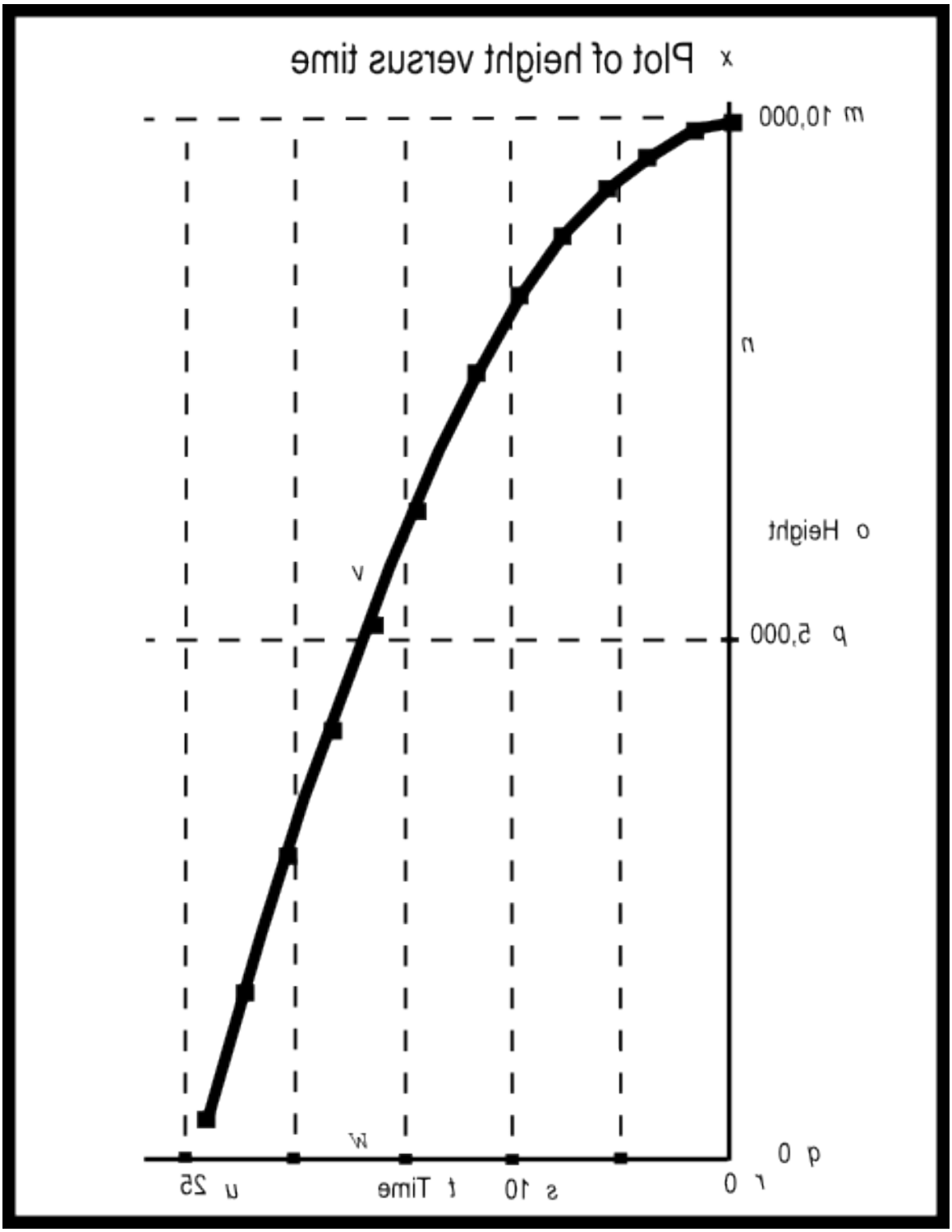
Plot the data

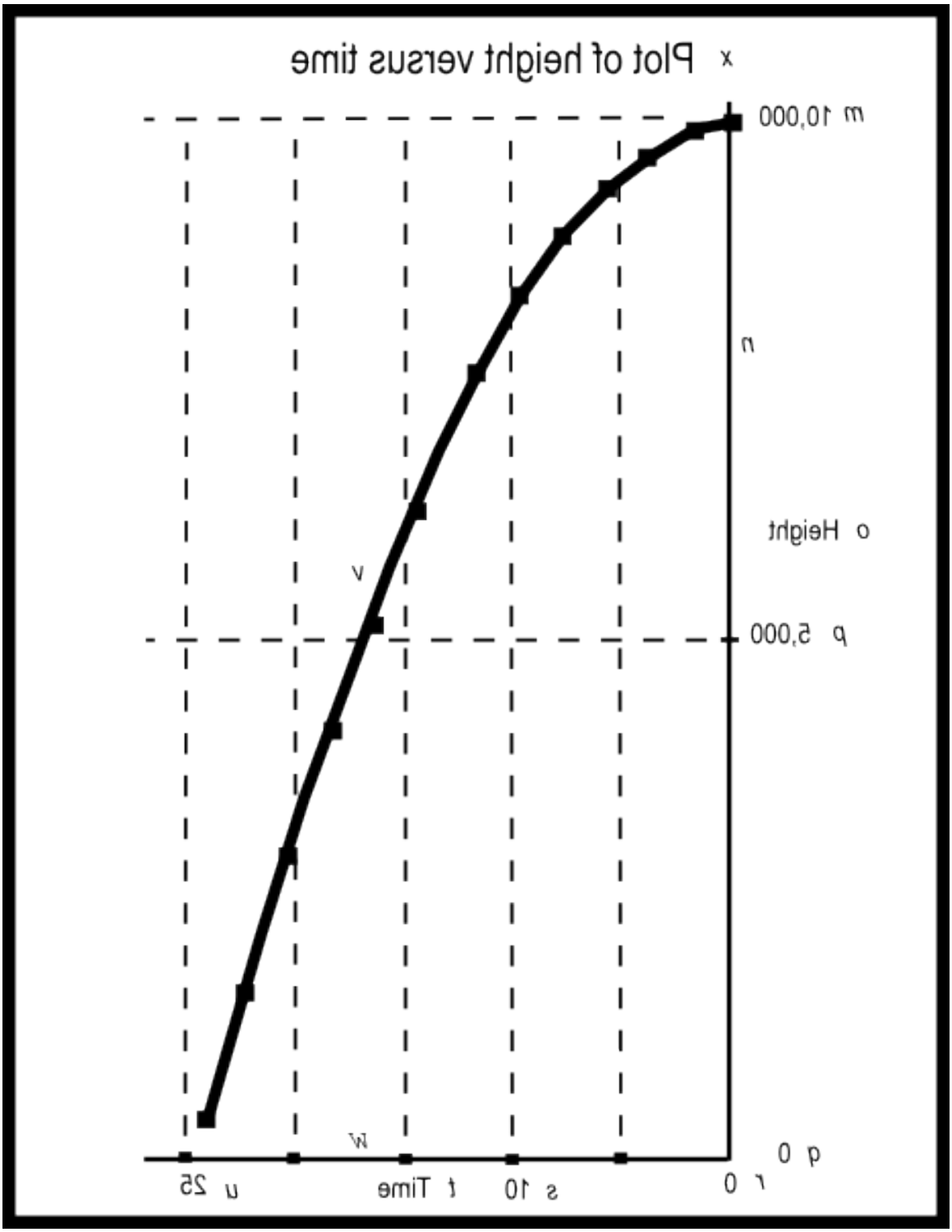
Using pushpins, rubber bands, pipe cleaners, or whatever works best for you, plot the 16 points shown in Figure 7 (p. 197) on your graph board. Your horizontal axis should be the time axis and your vertical axis should be the height axis.

If we have both done everything correctly, the shape of your curve should be an upside-down parabola with its high point at a value of 10000. (If the word parabola is new to you, don't worry about it. It's not important for the purpose of this module.)

The tactile graph

For the benefit of your assistant who will create the tactile graph, Figure 8 (p. 201) shows the image that is contained in the file named Phy1050.svg. As usual, this is a mirror image of the actual image in anticipation of the fact that your assistant will emboss the image from the back side of the paper. A non-mirror-image version of this same image is shown in Figure 10 (p. 206) .


Plot of height versus time
x
m 10,000
n
o Height
p 5,000
q 0
r 0
s 10
t Time
u 25


Plot of height versus time
x
m 10,000
n
o Height
v
p 5,000
w
q 0
r 0
s 10
t Time
u 25

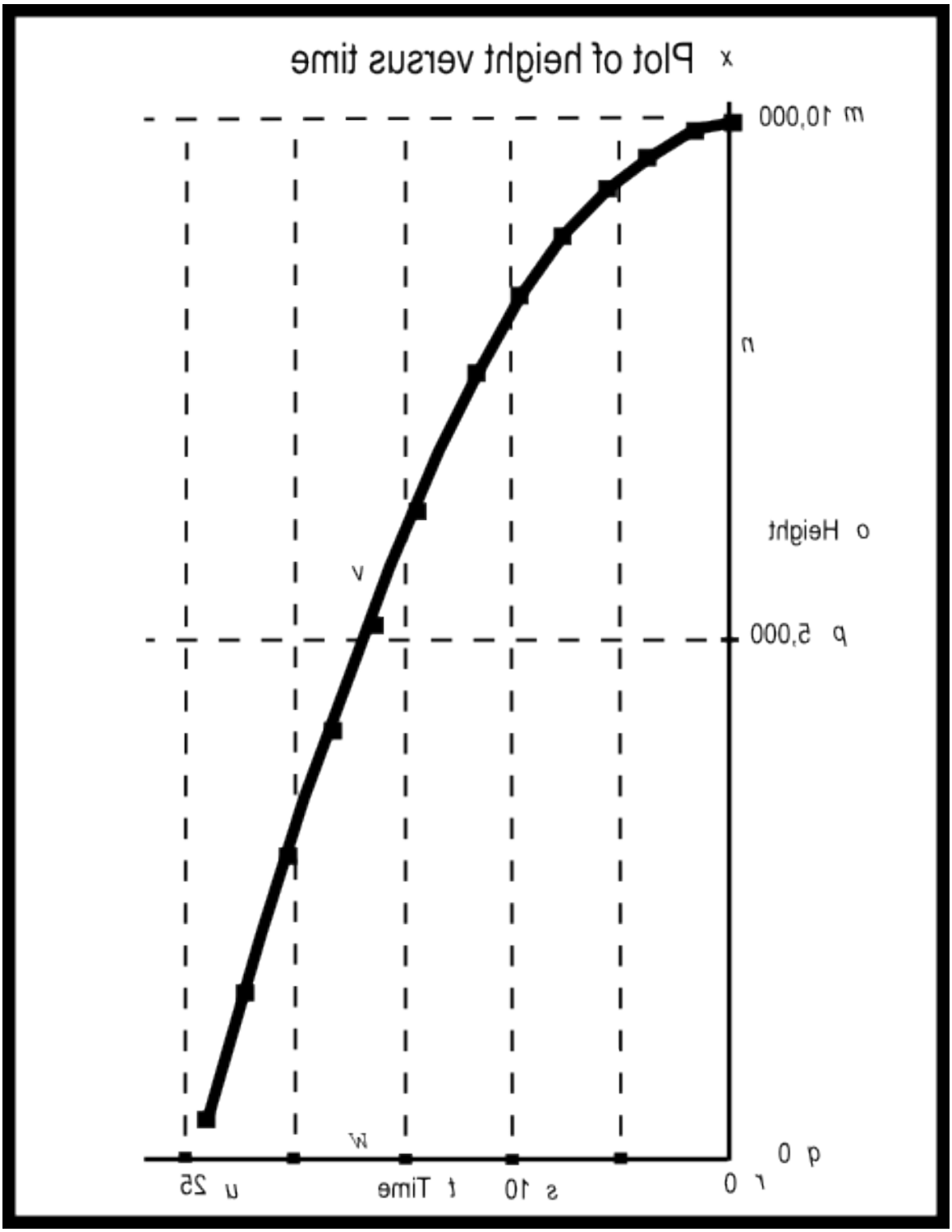

Plot of height versus time
x
m 10,000
n
o Height
v
p 5,000
w
q 0
r 0
s 10
t Time
u 25

Figure 8 . Image from the file named Phy1050a1.svg.

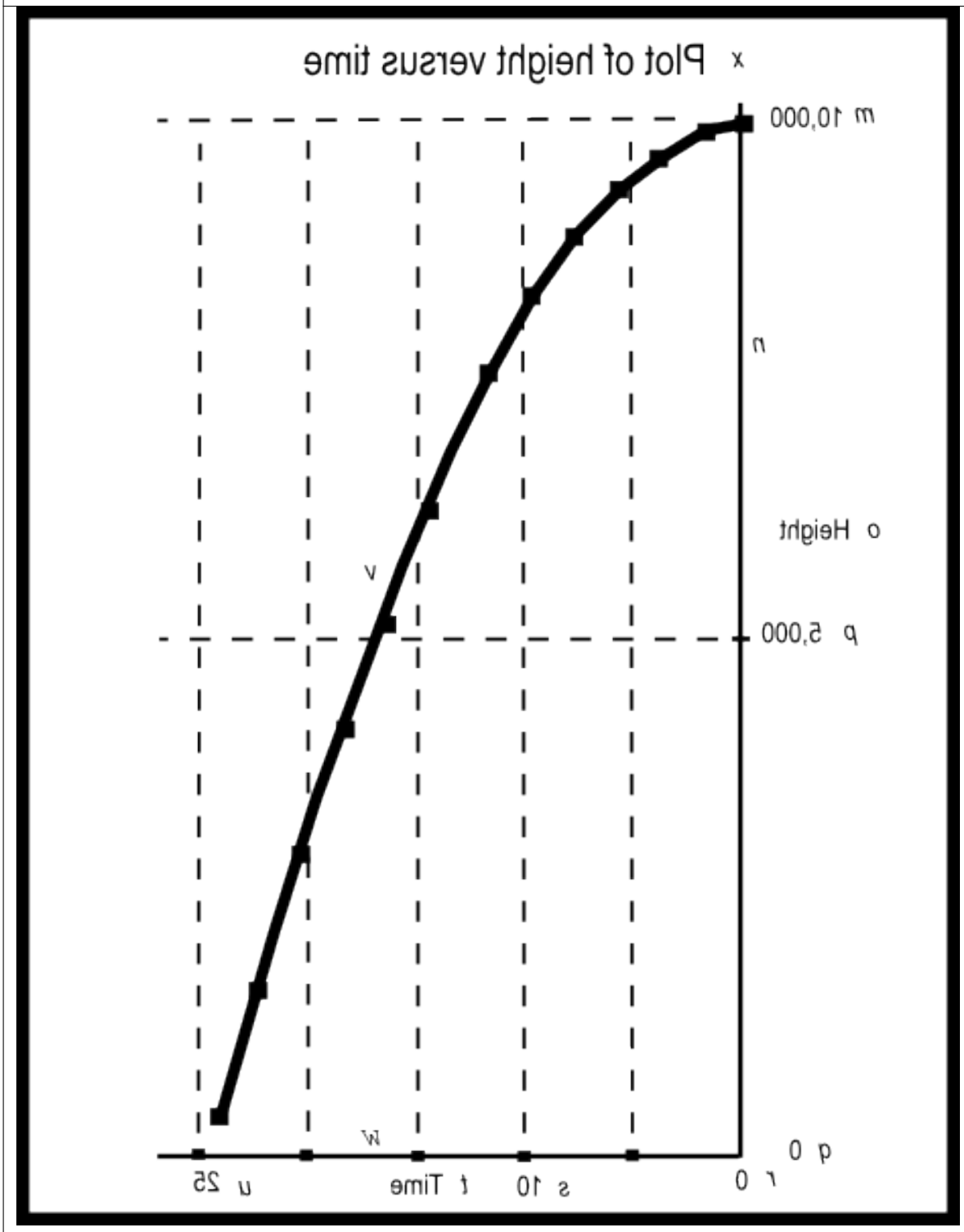


Table 9.11

Key-value pairs

Hopefully, by now you are familiar with the concept of key-value pairs as used in these modules. If not, please go back and study the section on Key-value pairs in conjunction with Figure 2 of the module named Manual Creation of Tactile Graphics [17] .

Figure 9 (p. 202) below contains the key-value table for the image shown in Figure 8 (p. 201) .

Figure 9 . Key-value table for the image shown in Figure 8.

```
m: 10,000
n: Vertical axis
o: Height
p: 5,000
q: 0
r: 0
s: 10
t: Time
u: 25
v: Height versus time curve
w: Horizontal axis
x: Plot of height versus time
```

Table 9.12

Change in height increases with time

Getting back to the height versus time values shown in Figure 7 (p. 197) and plotted on your graph board or tactile graph, the value for height should decrease for each successive time value.

However, the change in height should increase for each successive time interval until the height goes to zero. The height goes to zero when the rock has landed on the ground somewhere between 24 and 26 seconds. It continues at zero after that.

(Note that I put code into the script to prevent the value of height from going negative.)

The increase in the change in the height value during each successive time interval is the result of the time being squared in the equation given earlier (p. 193) . It is also the result of the acceleration of gravity that causes the downward speed of the rock to increase as the rock falls.

Analysis of the code

Listing 3 (p. 196) calls a function copied from the previous exercise inside a **while** loop to compute the distance traveled and the resulting height for a series of time values ranging from 0 to 30 seconds in two-second intervals.

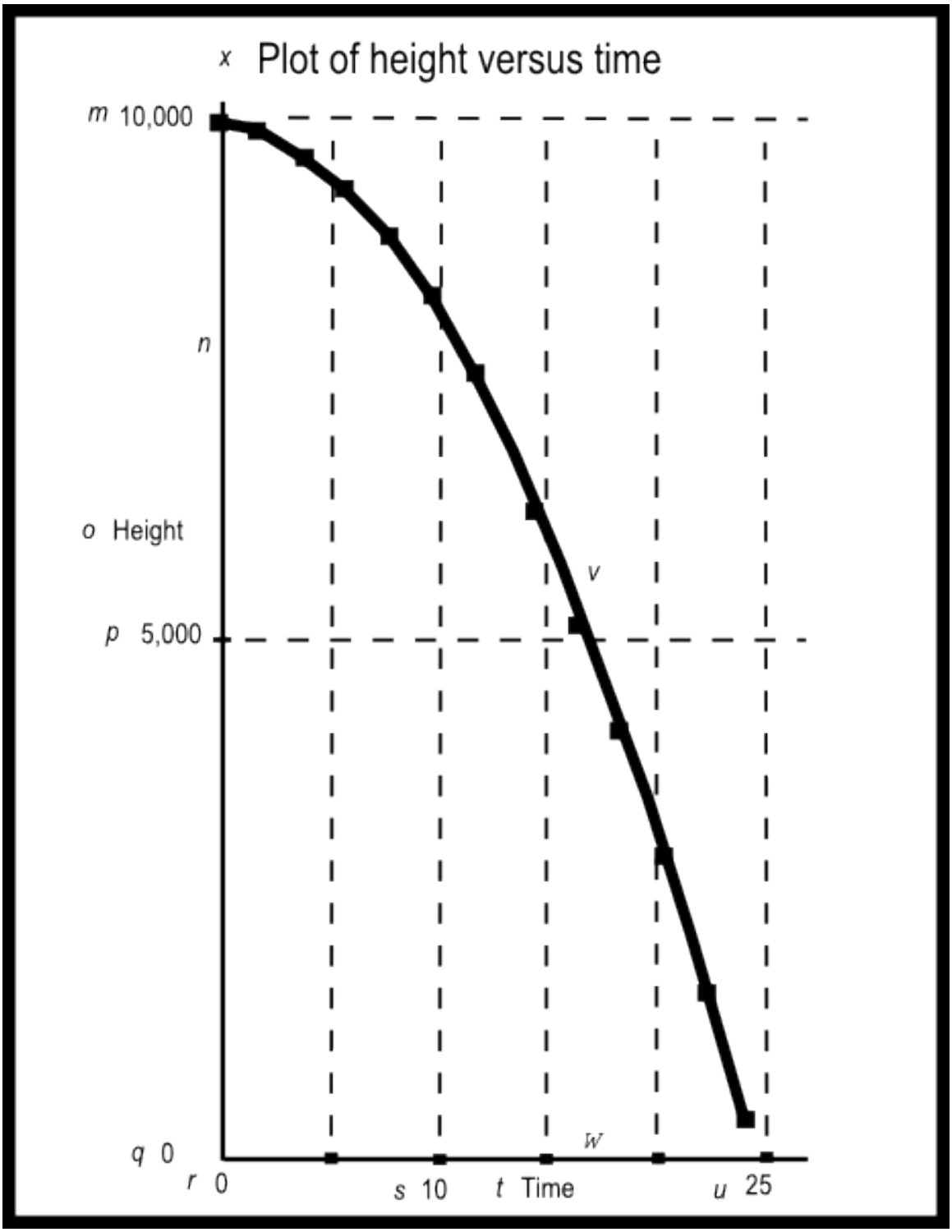
Those values are displayed as shown in Figure 7 (p. 197) .

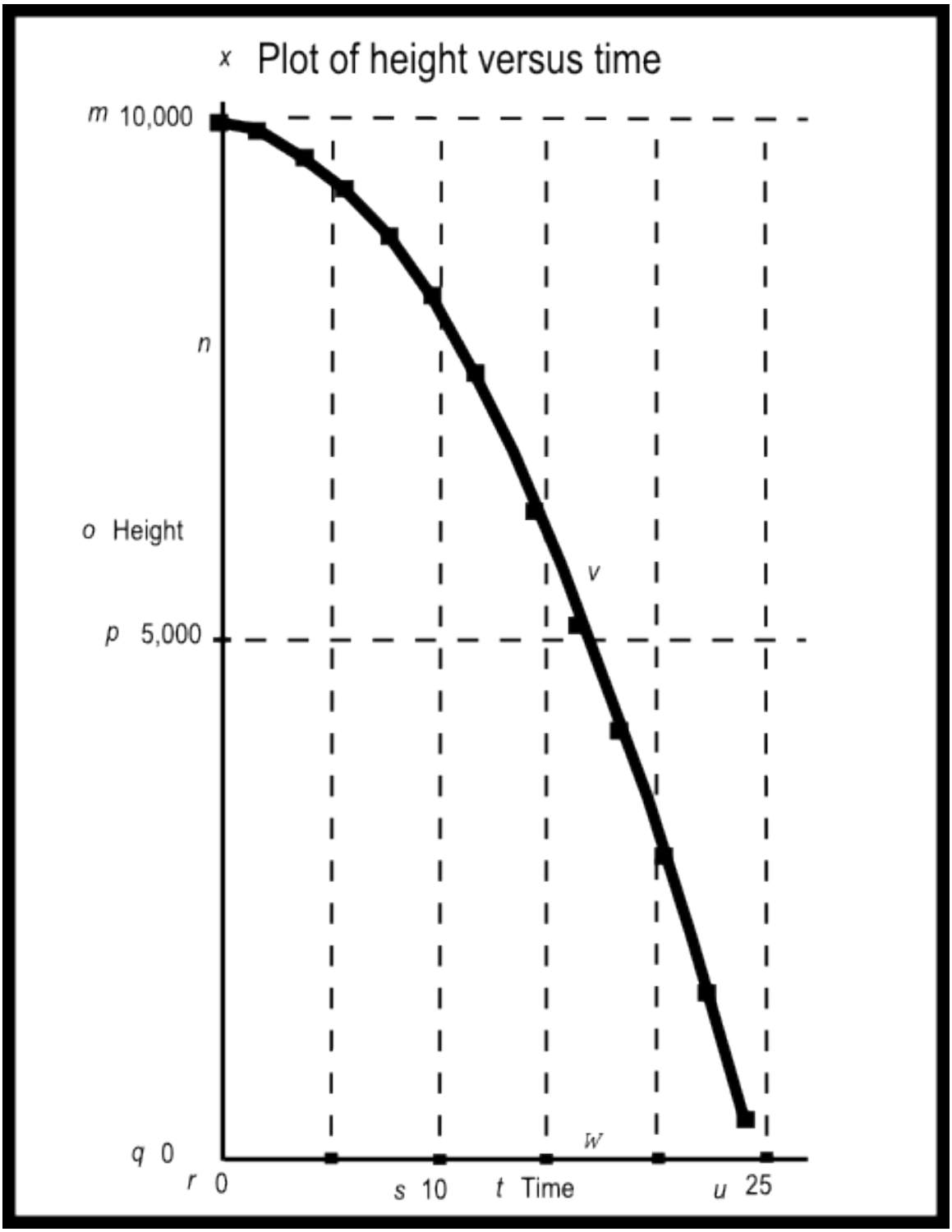
Understanding the code in Listing 2 (p. 194) and Listing 3 (p. 196) is important. However, it is more important at this point that you understand the handling of units as shown in the comments in the code.

9.4.4 Non-mirror-image graphics

Figure 8 (p. 201) shows the actual mirror image contained in the file named Phy1050a1.svg. Figure 10 (p. 206) shows a non-mirror-image version of that same image.

[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac


x Plot of height versus time
m 10,000
n
o Height
v
p 5,000
q 0
w
r 0
s 10
t Time
u 25


x Plot of height versus time
m 10,000
n
o Height
v
p 5,000
w
q 0
r 0
s 10
t Time
u 25

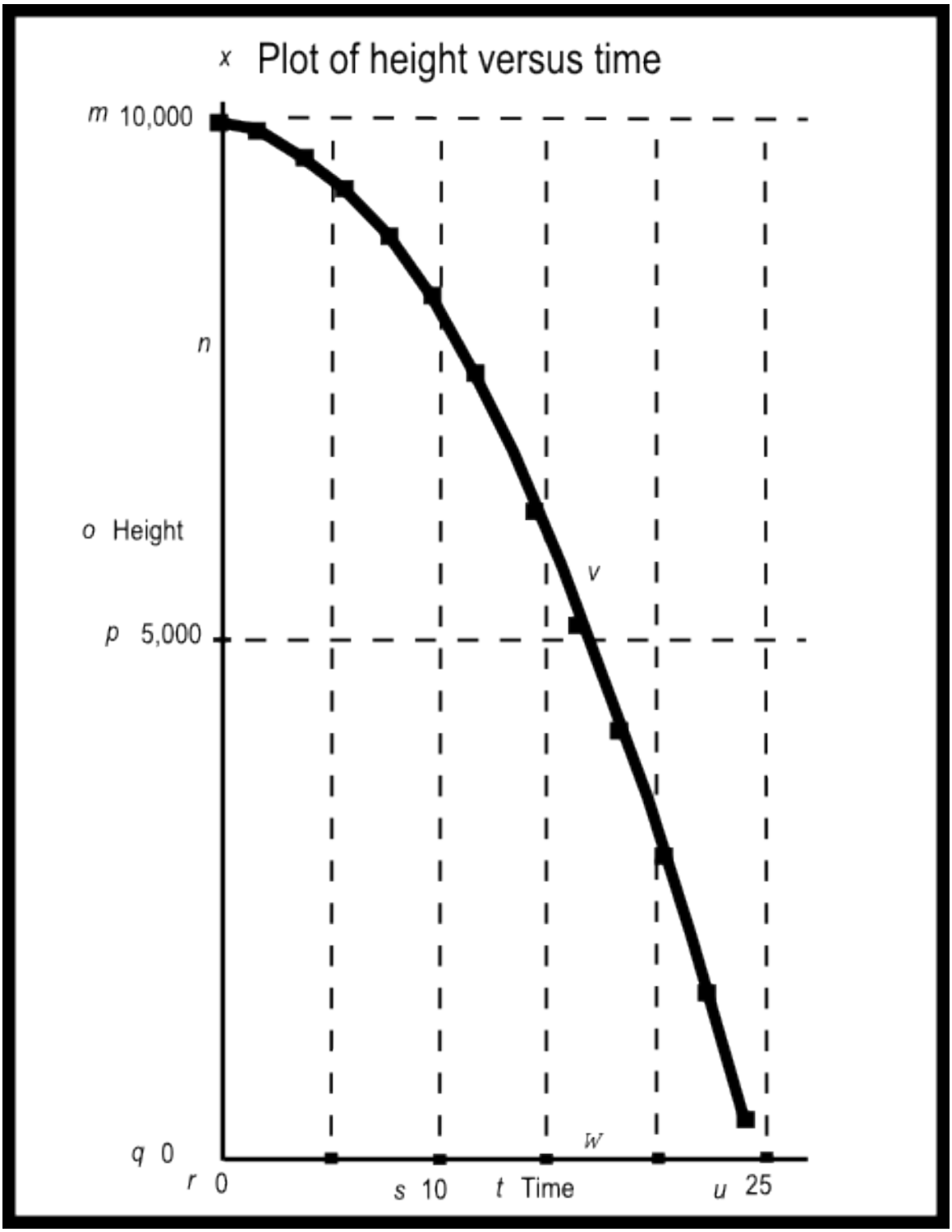

x Plot of height versus time
m 10,000
n
o Height
v
p 5,000
w
q 0
r 0
s 10
t Time
u 25

Figure 10 . Non-mirror-image version of the image from the file named Phy1050a1.svg.

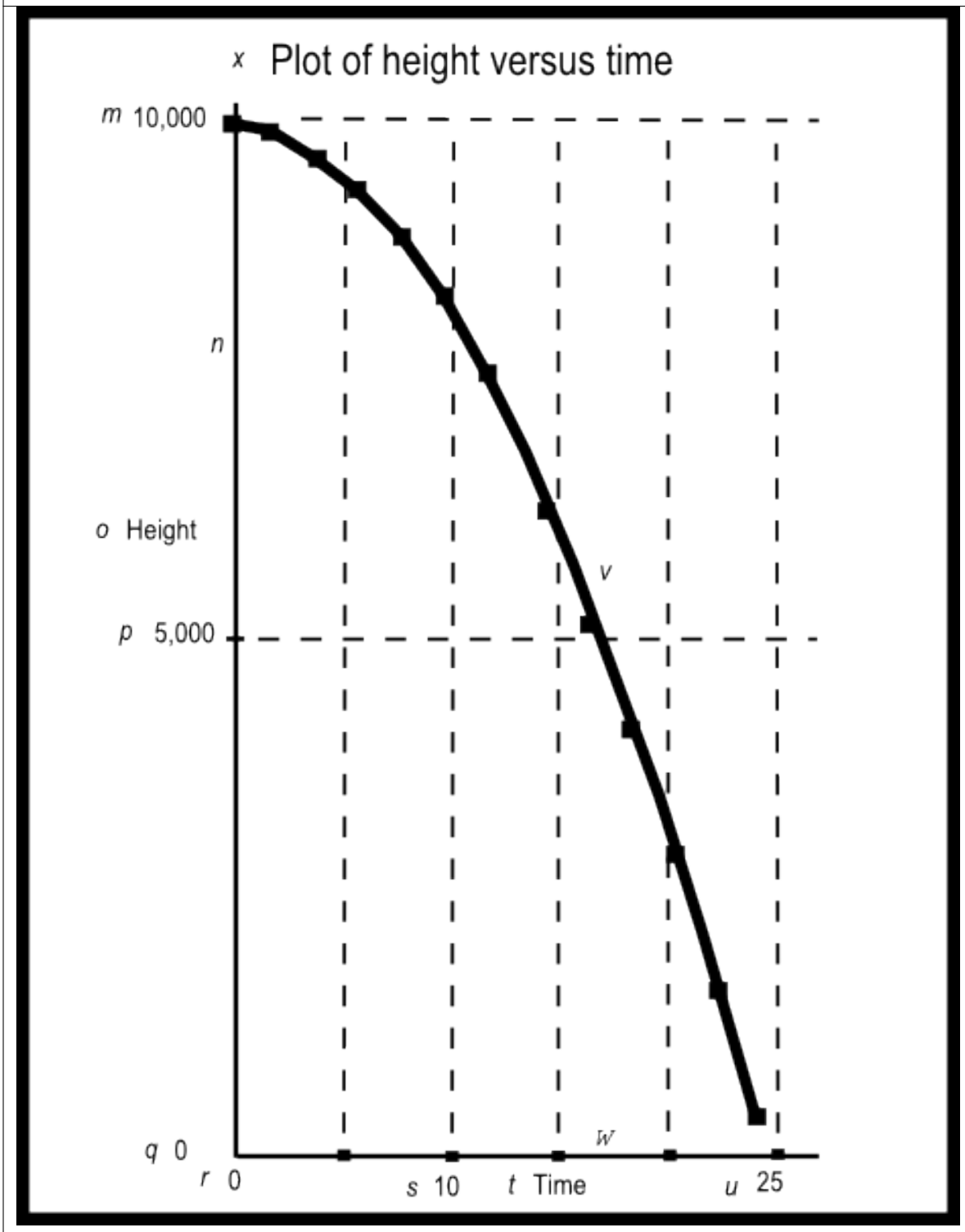


Table 9.13

9.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

9.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

9.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Units and Dimensional Analysis
- File: Phy1050.htm
- Revised: 10/01/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · screen reader
  - · Braille display
  - · JavaScript
  - · units
  - · dimensional analysis
  - · SI units
  - · powers of ten
  - · acceleration of gravity

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Chapter 10

Motion – Displacement and Vectors[1]

10.1 Table of Contents

- Preface (p. 209)
  - General (p. 209)
  - Prerequisites (p. 210)
  - Viewing tip (p. 210)
    - Figures (p. 210)
    - Listings (p. 211)
  - Supplemental material (p. 211)
- General background information (p. 211)
- Discussion and sample code (p. 212)
  - Creation of tactile graphics (p. 212)
  - Displacement (p. 212)
  - Addition and subtraction of vectors (p. 223)
  - Mathematical solutions (p. 240)
    - Using the Math.atan method (p. 241)
    - Mathematical solution to an earlier problem (p. 244)
    - Subtracting vectors (p. 247)
    - Adding more than two vectors (p. 247)
  - Non-mirror-image graphics (p. 249)
- Run the scripts (p. 270)
- Resources (p. 270)
- Miscellaneous (p. 270)

10.2 Preface

10.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

[1] This content is available online at <http://cnx.org/content/m37660/1.4/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The purpose of this module is to introduce motion and to explain displacement and vector analysis in a format that is accessible to blind students.

10.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

10.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

10.2.3.1 Figures

- Figure 1 (p. 213) . Key-value pairs for the image in Phy1060a1.svg.
- Figure 2 (p. 217) . Mirror image contained in the file named Phy1060a1.svg.
- Figure 3 (p. 218) . Key-value pairs for the image in Phy1060b1.svg.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

- Figure 4 (p. 222) . Mirror image contained in the file named Phy1060b1.svg.
- Figure 5 (p. 224) . Key-value pairs for the image in Phy1060c1.svg.
- Figure 6 (p. 228) . Mirror image contained in the file named Phy1060c1.svg.
- Figure 7 (p. 229) . Key-value pairs for the image in Phy1060d1.svg.
- Figure 8 (p. 233) . Mirror image contained in the file named Phy1060d1.svg.
- Figure 9 (p. 235) . Key-value pairs for the image in Phy1060e1.svg.
- Figure 10 (p. 239) . Mirror image contained in the file named Phy1060e1.svg.
- Figure 11 (p. 243) . Screen output for Listing #1.
- Figure 12 (p. 246) . Screen output for Listing #2.
- Figure 13 (p. 249) . Screen output for Listing #3.
- Figure 14 (p. 253) . Non-mirror-image version of the image contained in the file named Phy1060a1.svg.
- Figure 15 (p. 257) . Non-mirror-image version of the image contained in the file named Phy1060b1.svg.
- Figure 16 (p. 261) . Non-mirror-image version of the image contained in the file named Phy1060c1.svg.
- Figure 17 (p. 265) . Non-mirror-image version of the image contained in the file named Phy1060d1.svg.
- Figure 18 (p. 269) . Non-mirror-image version of the mage contained in the file named Phy1060e1.svg.

10.2.3.2 Listings

- Listing 1 (p. 242) . Using the Math.atan method.
- Listing 2 (p. 245) . The sum of two vectors.
- Listing 3 (p. 247) . The sum of four vectors.

10.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

10.3 General background information

Bodies - not a reference to people

To begin with, physics textbooks often refer to something called *bodies* . By this, they are not referring to living bodies or the carcasses of once living creatures. Instead, they are referring to what people might ordinarily call objects.

For example, somewhere in this series of modules, we will discuss something call *ed a free body diagram* . A free body diagram can be drawn relative to any object, such as a child sitting motionless in a swing on the playground or a piano hanging on a rope.

Rest and motion

While nothing in the universe is truly motionless (at rest), some things around us such as buildings seem to be at rest, while other things such as planes, trains, and busses appear to be in motion.

There is no such thing as absolute rest. However, two bodies moving together at the same rate (with reference to some third body) may be deemed to be at rest relative to one another.

Therefore, if you are standing motionless on the Earth, you are at rest relative to the Earth with reference to the Sun.

Kinematics and kinetics

[14] http://www.dickbaldwin.com/toc.htm

The fundamental properties of motion are often referred to as **kinematics** . The relationships between motion and force are often referred to as **kinetics** . These are the topics that will be covered in this and the next several modules in this collection.

Displacement

If one body is moved with respect to another body, either one of them may be said to have had a **displacement** . Displacement, along with vector analysis will be the primary topics covered in this module.

10.4 Discussion and sample code

10.4.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explained how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1060.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

10.4.2 Displacement

As mentioned above, if one body is moved with respect to another body, either one of them may be said to have had a **displacement** .

Displacement is not the same as distance

Assume, for example, that you go to the local high school and run five laps around the track, stopping exactly where you started. The distance that you have run will be the length of the path that you followed while running. However, while you may be completely out of breath by that point in time, the magnitude of your displacement will have been zero.

Scalars versus vectors

Distance is a **scalar** quantity, while displacement is a **vector** quantity. Scalar quantities have only magnitude, such as three kilometers.

Vector quantities have two parts: **magnitude** and **direction** . For example, you might describe a vector quantity as having a magnitude of three kilometers and a direction of northwest.

Magnitude and direction

A displacement vector must have both magnitude and direction. For example, if you were to run about three-fourths of the way around the track, the distance that you ran would be the length of your path.

However, the magnitude of your displacement would be the straight-line distance from your starting point to your stopping point.

The direction of your displacement would be an angle measured between some reference (such as due east, for example) and an imaginary line connecting your starting point and your stopping point.

[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[16] http://cnx.org/content/m37660/latest/Phy1060.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

If you stop where you start...

Getting back to the original proposition, if your stopping point is exactly the same as your starting point, the magnitude of your displacement vector will be zero and the angle of your displacement vector will be indeterminate.

Vector notation

Assume that there are three spots marked on the top of a table as A, B, and C. If an object is moved from A to B and then from B to C, using vectors, it may be stated that:

vecAC = vecAB + vecBC

where vecAB, vecBC, and vecAC represent vectors.

Typical notation

A typical notation for a vector in a physics textbook is something like AB, possibly in boldface and/or italics, with a small arrow drawn above the two letters. However, there is no such small arrow on a QWERTY keyboard, and even if there were, it might not be compatible with your screen reader and/or your Braille display.

My alternative notation

Therefore, I will use the following notation

vecAB

to indicate a vector extending from the spot labeled A to the spot labeled B.

In order to determine direction in an unambiguous way, you often need to know where the vector starts (the tail) and where it ends (the head). The notation used above indicates that the tail of the vector is at A and the head is at B.

In some cases, particularly in JavaScript code where a vector is defined by magnitude and direction instead of by a starting point and an ending point, I will refer to a vector simply as

vecA, vecB, etc.

A graph board exercise

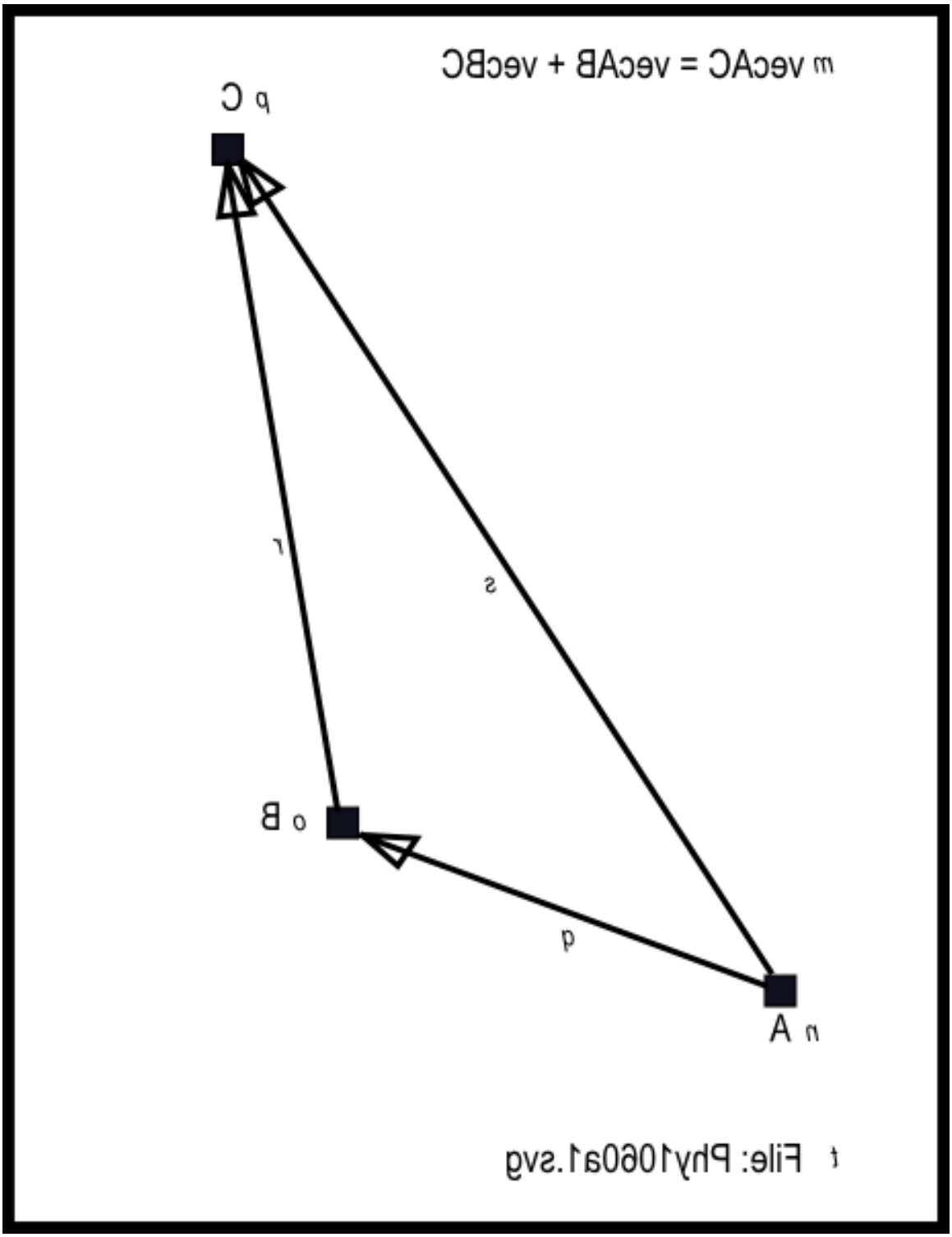
The svg file for this exercise is named Phy1060a1.svg. The table of key-value pairs for this file is provided in Figure 1 (p. 213) .

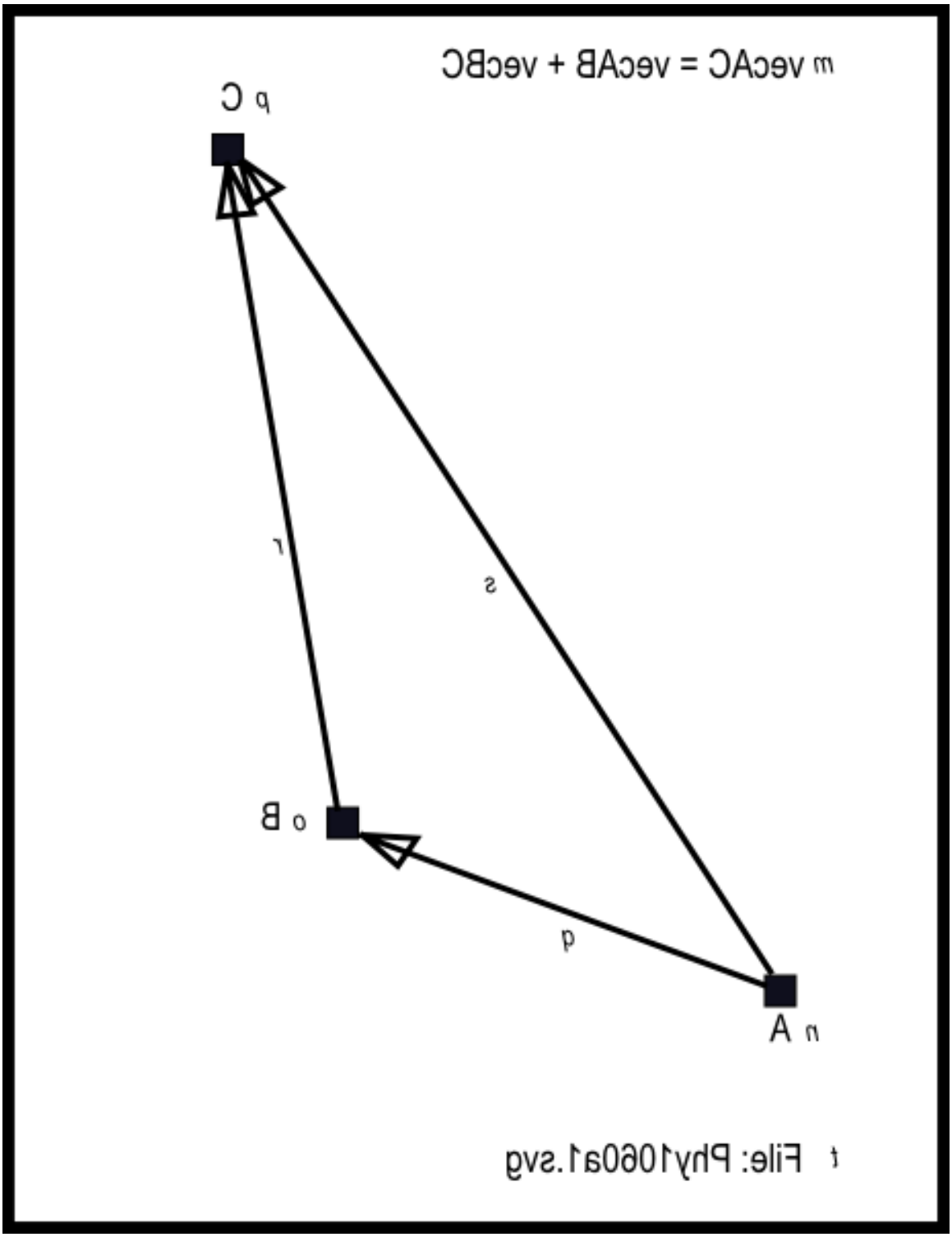
Figure 1 . Key-value pairs for the image in Phy1060a1.svg.

```
m: vecAC = vecAB + vecBC
n: A
o: B
p: C
q: This is vector AB
r: This is vector BC
s: This is vector AC, which is the sum of the other two vectors
t: File: Phy1060a1.svg
```

Table 10.1

The image contained in this file is shown in Figure 2 (p. 217) . A non-mirror-image version of the file is shown in Figure 14 (p. 253) .


vecA
vecB
vecC
r
s
q


C
B
A
r
s
q

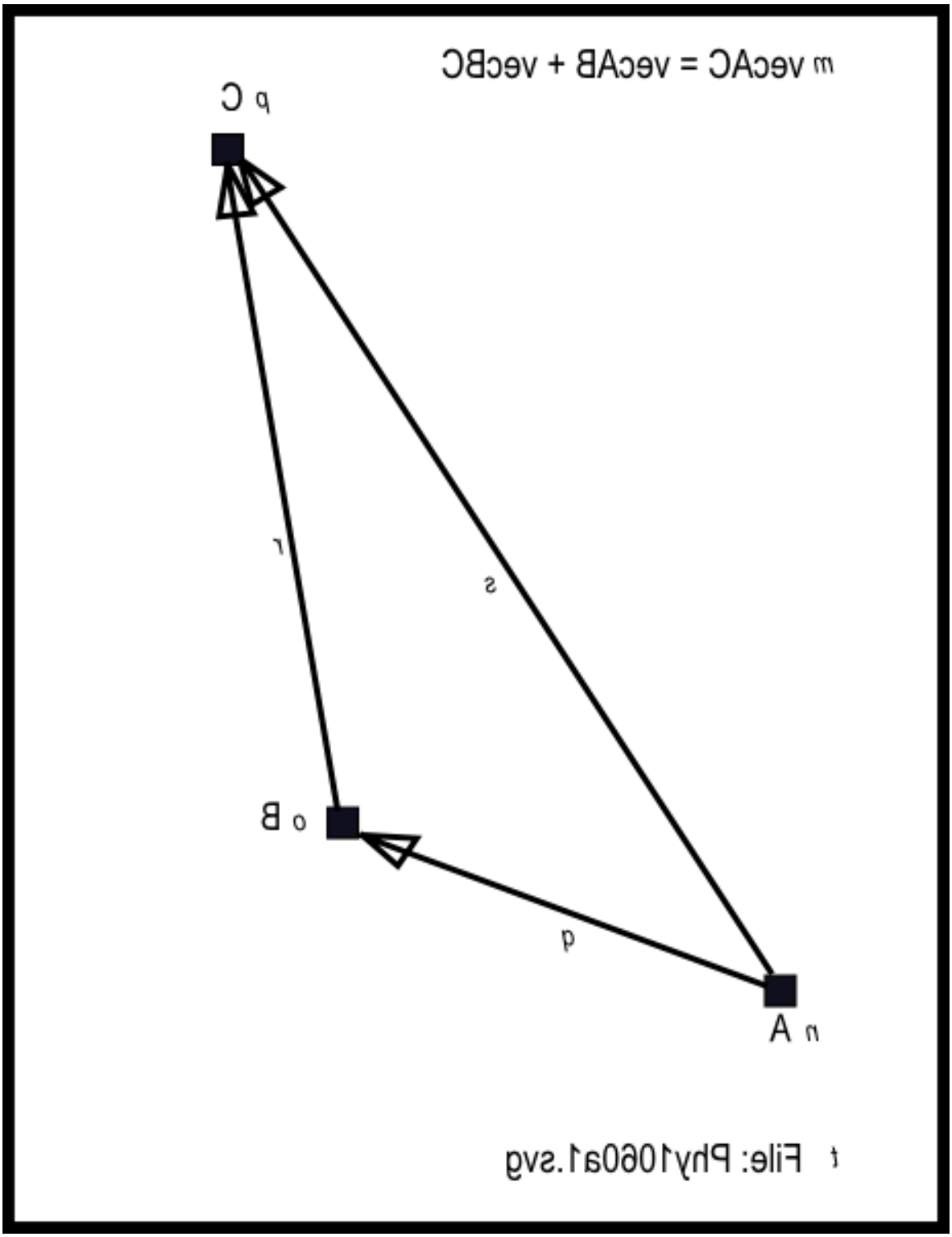

vecA
vecB
vecC
r
s
q
A
B
C

Figure 2 . Mirror image contained in the file named Phy1060a1.svg.

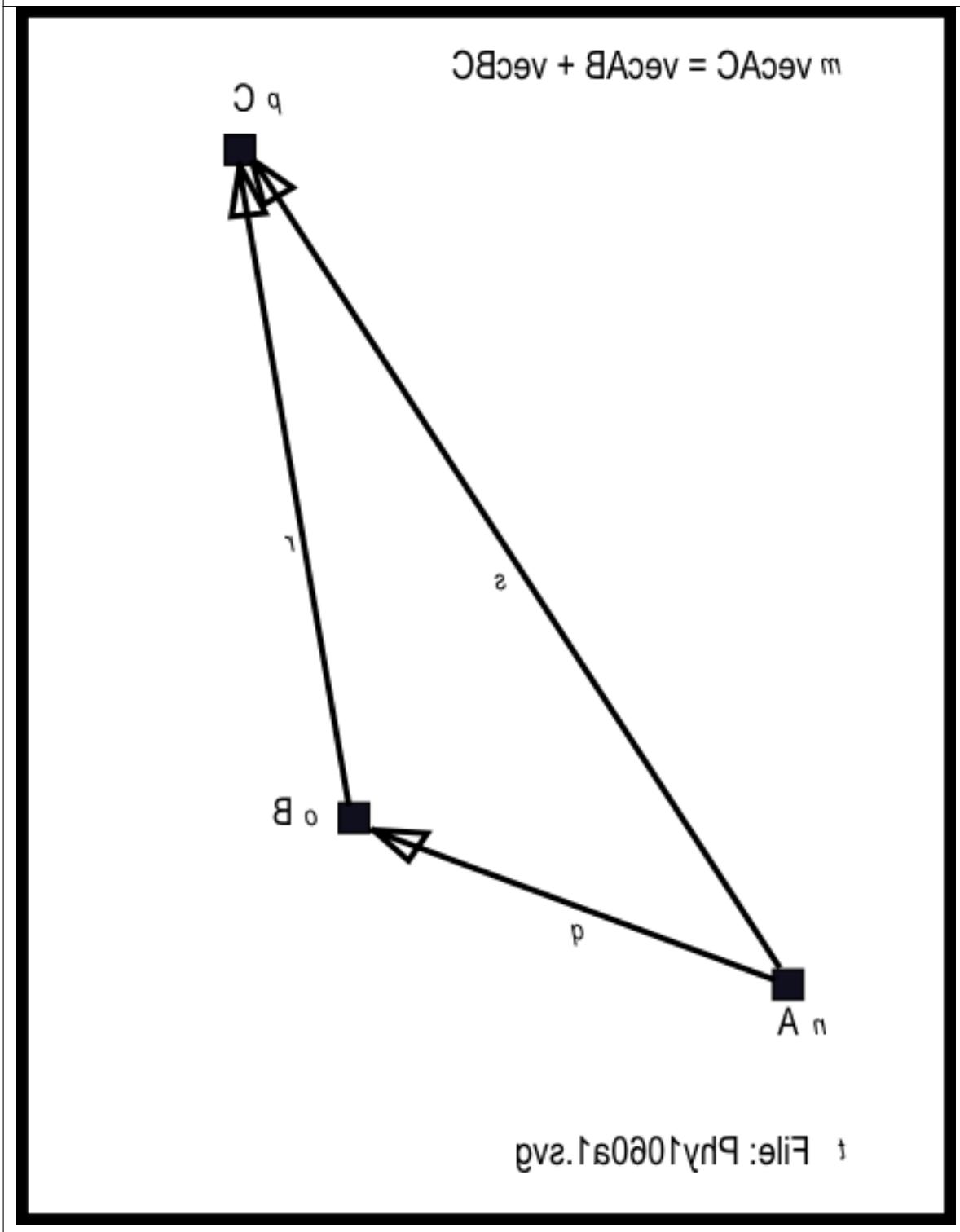


Table 10.2

Graph board instructions

Please mark three points on your graph board using pushpins and mentally label them A, B, and C. Then use rubber bands, pipe cleaners, or whatever works best for you to connect them in such a way that they form a triangle. That triangle is a physical representation of the vector equation

vecAC = vecAB + vecBC

What does this mean?

This vector equation does not mean that the algebraic sum of two sides of the triangle are equal to the third side. However, it does mean that the displacement vecAB together with the displacement vecBC, have an effect that is equivalent to the single displacement vector vecAC.

Scalar addition and subtraction

When adding or subtracting scalars, you only need to take the magnitude values of the scalars into account and perform normal arithmetic using those magnitude values. An example would be to add 6 birds and 5 birds to produce a sum of 11 birds.

Vector addition and subtraction

However, when adding or subtracting vectors, you not only need to take the magnitude of the vectors into account, you also need to take the directions of the vectors into account.

Another graph board exercise – a familiar example

The svg file for this exercise is named Phy1060b1.svg. The table of key-value pairs for this file is provided in Figure 3 (p. 218) .

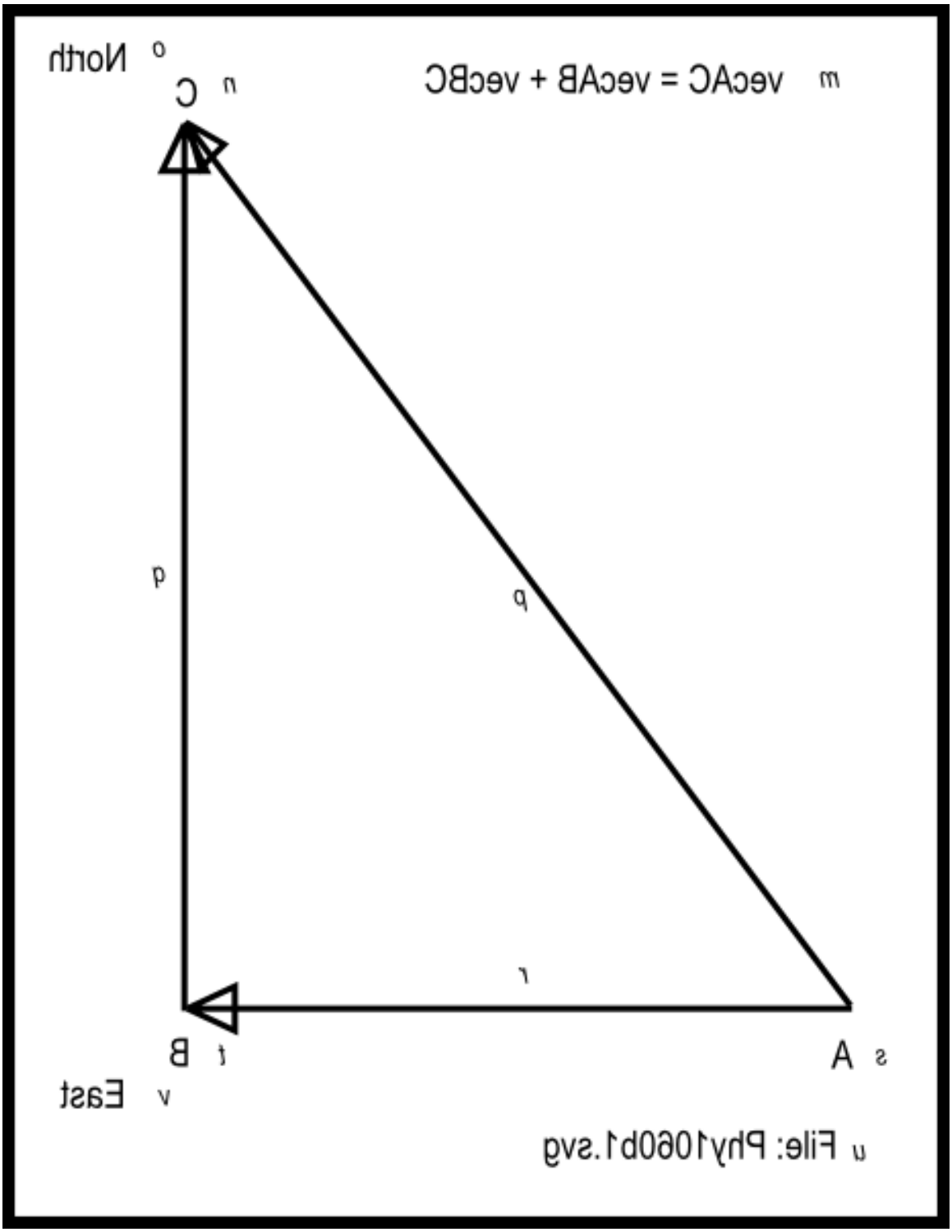
Figure 3 . Key-value pairs for the image in Phy1060b1.svg.

```
m: vecAC = vecAB + vecBC
n: C
o: North
p: Resultant vector A C
q: Displacement vector B C
r: Displacement vector A B
s: A
t: B
u: File: Phy1060b1.svg
v: East
```

Table 10.3

The image contained in this file is shown in Figure 4 (p. 222) . A non-mirror-image version of the file is shown in Figure 15 (p. 257) .

North
C
vecAC = vecAB + vecBC
q
p
r
B
A
East
File: Phy1060b1.svg


North
C
vecAC = vecAB + vecBC
q
p
r
B
A
East
File: Phy1060b1.svg

North
C
vecAC = vecAB + vecBC
q
p
r
B
A
East
File: Phy1060b1.svg

Figure 4 . Mirror image contained in the file named Phy1060b1.svg.

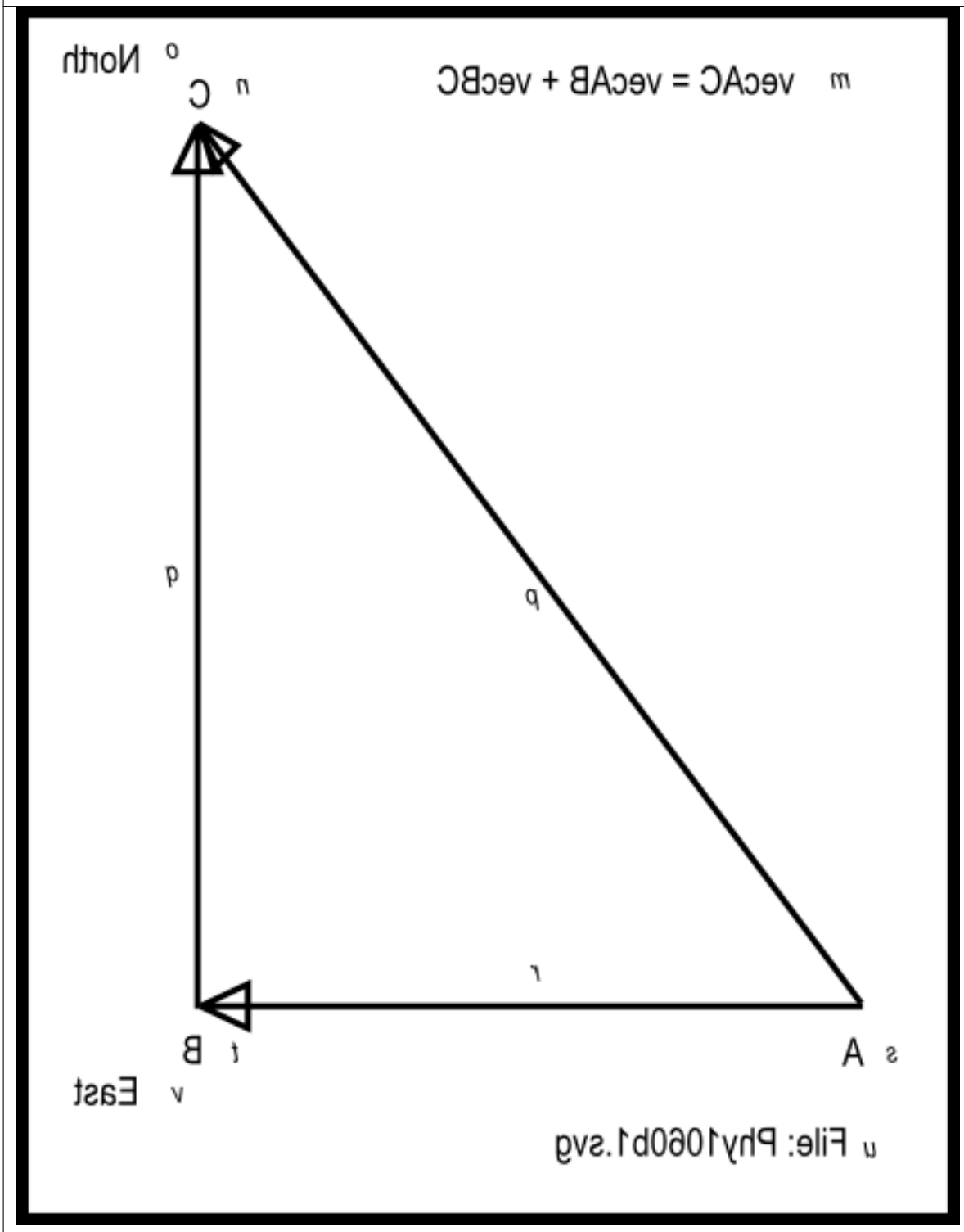


Table 10.4

Graph board instructions

Place a pin in the graph board and mentally designate that point as point A. Now pretend that you walk 30 meters due east and place a pin at that point. Mentally designate that point as point B.

Up to this point, your displacement can be represented by vecAB with a magnitude of 30 and an angle of zero (assuming that due east represents an angle of zero).

Walk 40 meters due north

Now pretend that you walk 40 meters due north, mentally designate the stopping point as C, and place a pin there. Your displacement relative to point B can be represented as vecBC with a magnitude of 40 meters and a direction of north or 90 degrees.

Displacement relative to point A

The interesting question is, what is your displacement relative to your starting point at A?

The displacement vector vecAC is the straight line from point A to point C, pointing in a direction from A to C. This vector is shown in the vector equation

vecAC = vecAB + vecBC

as the sum of the two vectors that describe the two segments of your journey from A to C.

Graphical addition of vectors

This process of connecting two or more vectors tail-to-head in sequence and then measuring the distance and direction from the tail of the first vector to the head of the last vector is known as the graphical addition of vectors. The resulting vector (in this case vecAC) is known as the sum of the vectors that make up the sequence. (The resulting vector is often called the resultant.)

A 3-4-5 triangle

Recalling what we learned in an earlier module where I discussed a 3-4-5 triangle, the magnitude of the vector from A to C (the hypotenuse of a right triangle with sides of 30 and 40) is 50 meters. I also recall that the angle is approximately 53 degrees relative to the horizontal (east-west) line.

You should be able to measure the length of vecAC as well as the angle between that vector and the horizontal and get reasonable agreement with the above.

No requirement for a right triangle

While this example is based on a right triangle, that is not a requirement for adding vectors. I chose a 3-4-5 right triangle for the example because I knew what the answer would be in advance and also because it should have been familiar to you.

Graphic representation of vectors

Many problems in kinematics and kinetics can be solved graphically using a graphical representation of vectors.

Vectors are typically drawn as a heavy line between two points with an arrow head at one end and nothing in particular at the other end. The end with the arrow head is commonly called the head of the vector and the other end is commonly called the tail.

Vectors on a graph board

As a blind student, you can't easily draw vectors, but you can represent vectors on your graph board by connecting two pushpins with something straight (like a stretched rubber band) and using something tactile to mark the head.

(Perhaps you could use a pushpin with a cylindrical top to indicate the tail and a pushpin with a spherical top to represent the head. Another option might be to get some small buttons and place the pin at the head through one of the holes in the button.)

Somewhat inconvenient

Constructing vectors on a graph board is less convenient for blind students than drawing them on graph paper is for sighted students. Fortunately, as you will soon learn, it isn't necessary to use graphics to solve vector problems. You can also solve such problems mathematically, which will often be the better choice for blind students.

10.4.3 Addition and subtraction of vectors

There are at least two kinds of quantities in physics:

- scalars, having magnitude only
- vectors, having both magnitude and direction

You already know how to do scalar arithmetic. Otherwise, you probably wouldn't be interested in physics.

One textbook describes vectors as "straight lines, with proper directions, indicated by arrowheads, and with lengths drawn to scale designated by vecAB (my notation), etc.

Another example

Let's go back to our original equation

vecAC = vecAB + vecBC

and assume that the magnitude of vecAB is 30 and the magnitude of vecBC is 40. The sum of those two vectors (vecAC) can have a magnitude anywhere from 10 to 70 depending on the relative angles of vecAB and vecBC.

A triangle with sides of 30, 4 0, and ?

The svg file for this exercise is named Phy1060c1.svg. The table of key-value pairs for this file is provided in Figure 5 (p. 224) .

Figure 5 . Key-value pairs for the image in Phy1060c1.svg.

```
m: vecAC = vecAB + vecBC
n: C
o: North
p: Resultant vector A C
q: Displacement vector B C
r: Displacement vector A B
s: A
t: B
u: File: Phy1060c1.svg
v: East
w: 135 degrees
```

Table 10.5

The image contained in this file is shown in Figure 6 (p. 228) . A non-mirror-image version of the file is shown in Figure 16 (p. 261) .

m vecAC = vecAB + vecBC
o North
n C
q
p
w 135 degrees
r
B
t
A
s
v East
u File: Phy1060c1.svg

vecAC = vecAB + vecBC
North
C
A
B
135 degrees
East
File: Phy1060c1.svg

m vecAC = vecAB + vecBC
o North
n C
q
p
w 135 degrees
r
B
t
A
s
v East
u File: Phy1060c1.svg

Figure 6 . Mirror image contained in the file named Phy1060c1.svg.

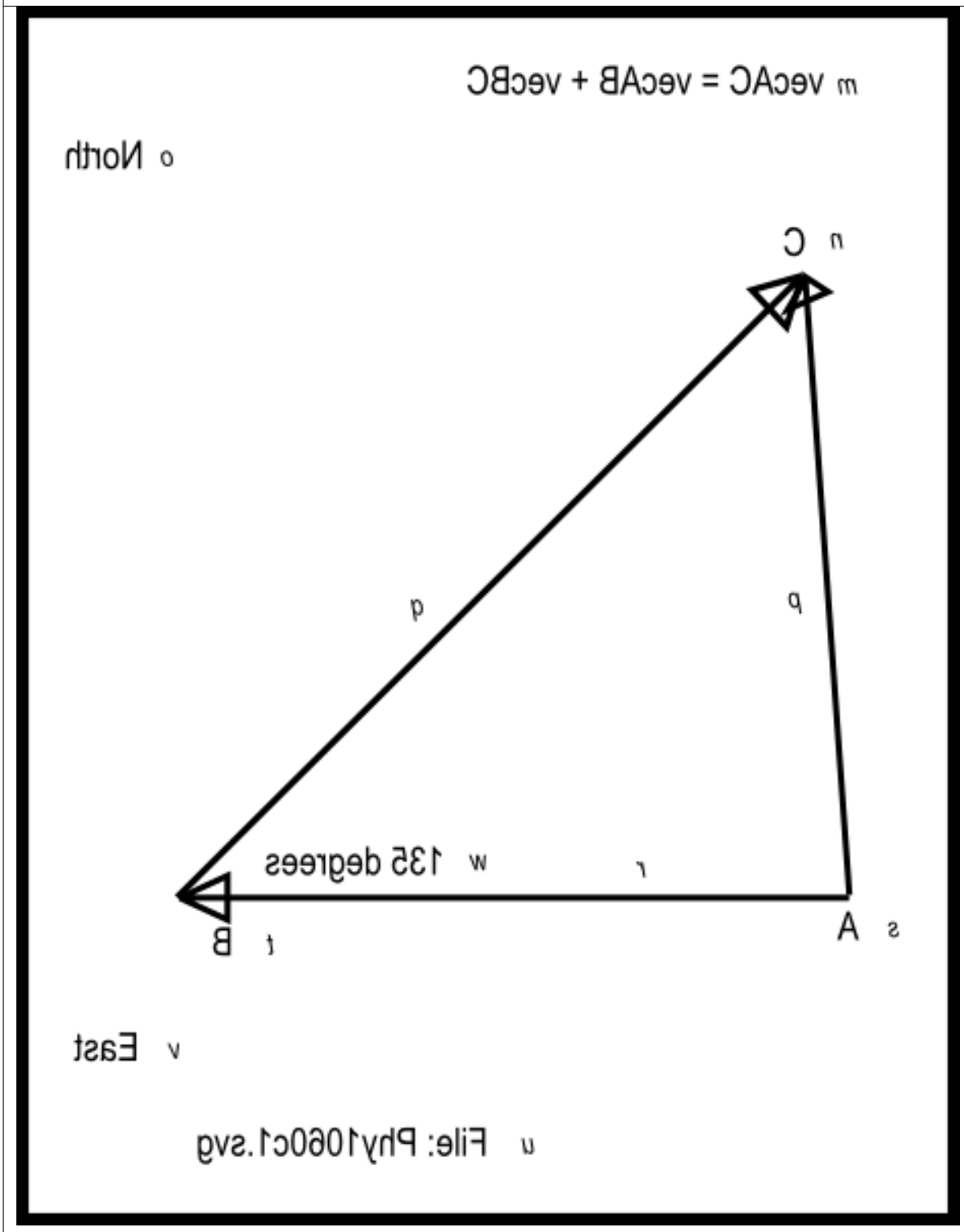


Table 10.6

Graph board instructions

Go back to the graph board and change the direction of vecBC to northwest (135 degrees relative to the east-west horizontal line with east being zero degrees) keeping the length at 40 meters.

What happened to the displacement?

Now what is the displacement of the point C relative to the point A? I can't do the arithmetic in my head for this problem, but I can measure the length of vecAC to be about 28 meters and the angle of vecAC to be a little less than 90 degrees. (I will show you how to write a script to solve this problem mathematically later.)

Any number of displacements can be added

There is no limit to the number of displacement vectors that can be added in this manner. For example, pretend that you walk from point A,

- 10 meters east to point B,
- 12 meters southwest to point C,
- 30 meters north to point D,
- 15 meters east to point E,
- 35 meters southwest to point F, and
- 44 meters north to point G where you stop.

Your displacement vecAG will be

vecAG = vecAB + vecBC + vecCD + vecDE + vecEF + vecFG

even if your zigzag path crosses back over itself one or more times.

Tactile graphics for the sum of six vectors

The svg file for this exercise is named Phy1060d1.svg. The table of key-value pairs for this file is provided in Figure 7 (p. 229) .

Figure 7 . Key-value pairs for the image in Phy1060d1.svg.

```
m: A
n: B
o: C
p: D
q: E
r: F
s: G
t: The sum of six vectors
u: Not drawn to scale
v: Resultant vector A G
w: File: Phy1060d1.svg
```

Table 10.7

The image contained in this file is shown in Figure 8 (p. 233) . A non-mirror-image version of the file is shown in Figure 17 (p. 265) .

The sum of six vectors
Not drawn to scale
G
s
v
D
p
E
q
B
n
A
m
F
r
C
o
File: Phy1060d1.svg
w

The sum of six vectors
Not drawn to scale
G
v
D
E
A
B
F
C
File: Phy1060d1.svg

The sum of six vectors
Not drawn to scale
G
v
D
E
B
A
F
C
File: Phy1060d1.svg

Figure 8 . Mirror image contained in the file named Phy1060d1.svg.

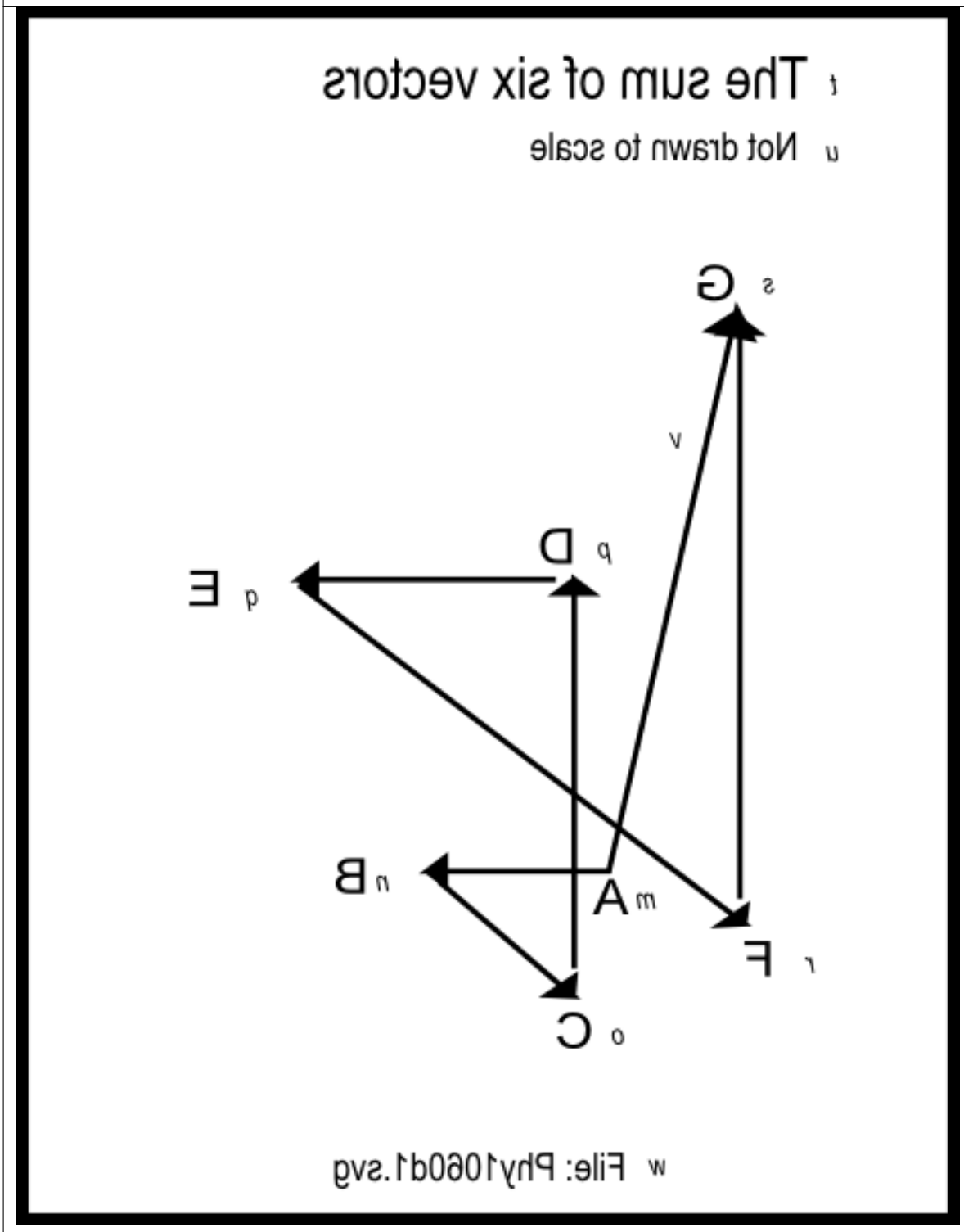


Table 10.8

Can be extended to three (or more) dimensions

The physics textbook titled College Physics by Mendenhall, Eve, Keys, and Sutton contains the following example.

"If a man climbs up the mast of a ship sailing east while the tide caries it south, then that sailor may have three displacements at right angles, vecAB 30 feet upward, vecBC 100 feet eastward, vecCD 20 feet southward, all with respect to the bed of the ocean. The three displacements vecAB, vecBC, and vecCD are equivalent to the single displacement vecAD; and in the same way, any number of displacements may be added together."

Difficult to draw

Of course, we can't easily draw that vector diagram on a flat sheet of paper or construct it on a flat graph board, but we can solve for the displacement vecAD if we are familiar with computations in three dimensions. (An explanation of 3D computations is beyond the scope of this module.)

The parallelogram law

There is a law that states:

The sum of two vectors in a plane is represented by the diagonal of a parallelogram whose adjacent sides represent the two vector quantities.

The resultant

As I mentioned earlier, the sum of two or more vectors is called their **resultant** .

In general, the resultant is not simply the algebraic or arithmetic sum of the vector magnitudes. Instead, in our original vector equation

vecAC = vecAB + vecBC

vecAC is the resultant of the sum of vecAB and vecBC in the sense that a single vector vecAC would have the same effect as vecAB and vecAD acting jointly.

Geometrical addition

The parallelogram law is a form of geometrical addition. In physics, it is often used to find graphical solutions to problems involving forces, velocities, displacements, accelerations, electric fields, and other *directed* quantities. (Directed quantities have both magnitude and direction.)

Solving a vector problem with a parallelogram

Pretend that you walk from point A

- 5 meters east to point B, followed by
- 6 meters at an angle of 30 degrees (north of east) to point C.

Let's use your graph board and the parallelogram law to find the resultant displacement vector vecAC.

Tactile graphics

The svg file for this exercise is named Phy1060e1.svg. The table of key-value pairs for this file is provided in Figure 9 (p. 235) .

Figure 9 . Key-value pairs for the image in Phy1060e1.svg.

```
m: Adding vectors using the parallelogram law.
n: B
o: C
p: A
q: Vector B C
r: Resultant vector
s: Vector A B
t: Line parallel to vector A B
u: Line parallel to vector B C
v: File: Phy1060e1.svg
```

Table 10.9

The image contained in this file is shown in Figure 10 (p. 239) . A non-mirror-image version of the file is shown in Figure 18 (p. 269) .

Adding vectors using the parallelogram law.
C
t
r
q
u
s
B
A
File: Phy10606e1.svg

m Adding vectors using the parallelogram law.
o C
t
r
q
u
s
n B
p A
v File: Phy1060e1.svg

Adding vectors using the parallelogram law.
C
t
r
q
u
s
B
A
File: Phy10606e1.svg

Figure 10 . Mirror image contained in the file named Phy1060e1.svg.

Table 10.10

Put both vector tails at point A

Mentally designate the starting point on your graph board as point A. Construct a vector with a length of 5 meters and an angle of 0 degrees relative to the horizontal with its tail at point A. Mentally designate this vector as vecAB.

Construct a second vector with a length of 6 meters at an angle of 30 degrees with its tail at point A. Mentally designate this vector as vecBC.

Parallel lines

Now comes the tricky part. Using a rubber band and a push pin, construct a line at least 6 meters long starting at the head of vecBC. Make this line parallel to vecAB.

(There are drawing tools that make it easy for sighted students to draw a line that is parallel to another line. This is one reason why this is a popular technique for graphically adding vectors.)

Construct another line, at least 7 meters long, starting at the head of vecAB. Make this line parallel to vecBC.

A parallelogram

Push a pin into the graph board at the intersection of the two lines.

The two lines and the two vectors should now form a parallelogram. In other words, you should now have four pins located at the four corners of a parallelogram.

Solving for the resultant vector

According to the parallelogram law, the resultant vector, vecAC, is represented by a directed line that extends from point A to the intersection of the two lines that you constructed earlier. In other words, the sum of the two vectors is equal to a diagonal line that extends from the starting point to the opposite corner on the parallelogram that you constructed.

Subtracting vectors

In normal arithmetic subtraction, a **subtrahend** is subtracted from a **minuend** to find a **difference**.

To subtract a subtrahend vector from a minuend vector (in a plane), add 180 degrees to the direction of the subtrahend vector, causing it to point in the opposite direction, and add the modified subtrahend vector to the minuend vector.

If you subtract vecAB in the above example from vecBC, you should be able to show that the difference vector is represented by the other diagonal in the parallelogram. However, you will probably find it easier to do this mathematically rather than doing it graphically.

10.4.4 Mathematical solutions

The horizontal component of the sum of two or more vectors is the sum of the horizontal components of the vectors.

The vertical component of the sum of two or more vectors is the sum of the vertical components of the vectors.

The horizontal and vertical components

The horizontal and vertical components respectively of a vector vecAB are equal to

- vecABh = vecABmag * cos(angle)
- vecABv = vecABmag * sin(angle)

where

- angle is the angle that the vector makes relative to the horizontal axis.
- vecABh, vecABv, and vecABmag are respectively the horizontal component, the vertical component, and the magnitude of the vector vecAB

The magnitude of the resultant vector

Given the horizontal and vertical components of the resultant vector, the magnitude of the resultant vector can be found using the Pythagorean theorem as the square root of the sum of the squares of the horizontal and vertical components.

The angle of the resultant vector

The angle that the resultant vector makes with the horizontal axis is the arctangent (corrected for quadrant) of the ratio of the vertical component to the horizontal component.

10.4.4.1 Using the Math.atan method

We will need to deal with several issues that arise when using the Math.atan method. Therefore, I will write a script that contains a function to deal with those issues. Then we can simply copy that function into future scripts without having to consider those issues at that time.

Please copy the code shown in Listing 1 (p. 242) into an html file and open the file in your browser.

Listing 1 . Using the Math.atan method.

```
    <!-- File JavaScript01.html -->
<html><body>
<script language="JavaScript1.3">

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle

//Test for a range of known angles.
var angIn = 0;
var angOut = 0;
var x = 0;
var y = 0;
while(angIn <= 360){
  x = Math.cos(angIn*Math.PI/180);
  y = Math.sin(angIn*Math.PI/180);
  angOut = getAngle(x,y).toFixed(1);
  document.write("angle = " + angOut + "</br>");
  angIn = angIn + 30;
}//end while loop

document.write("The End")

</script>
</body></html>
```

Table 10.11

The screen output

The text shown in Figure 11 should appear in your browser window when you open the html file in your browser.

Figure 11 . Screen output for Listing #1.

```
    angle = 0.0
angle = 30.0
angle = 60.0
angle = 90.0
angle = 120.0
angle = 150.0
angle = 180.0
angle = 210.0
angle = 240.0
angle = 270.0
angle = -60.0
angle = -30.0
angle = -0.0
The End
```

Table 10.12

Structure of the script

The script shown in Listing 1 (p. 242) begins by defining a function named **getAngle** . The purpose of this function is to return an angle in degrees in the correct quadrant based on the lengths of the adjacent and opposite sides of an enclosing right triangle.

Then the script uses a **while** loop to test the function for a series of known angles ranging from 0 to 360 degrees inclusive.

An indeterminate result

The getAngle function calls the Math.atan method to compute the angle whose tangent is the ratio of the opposite side (y) to the adjacent side (x) of a right triangle.

If both x and y are zero, the ratio y/x is indeterminate and the value of the angle cannot be computed. In fact there is no angle corresponding to the ratio 0/0. However, the function must either return the value of an angle, or must return some sort of flag indicating that computation of the angle is not possible.

In this case, the function simply returns the value zero for the angle.

Avoiding division by zero

If the value of x is zero and the value of y is not zero, the ratio y/x is infinite. Therefore, the value of the angle cannot be computed. However, in this case, the angle is known to be 90 degrees (for y greater than zero) or 270 degrees (-90 degrees, for y less than zero). The getAngle function traps both of those cases and returns the correct angle in each case.

Correcting for the quadrant

The Math.atan method receives one parameter and it is either a positive or negative value. If the value is positive, the method returns an angle between 0 and 90 degrees. If the value is negative, the method returns an angle between 0 and -90 degrees. Thus, the angles returned by the Math.atan method always lie in the first or fourth quadrants.

(Actually, as I mentioned earlier, +90 degrees and -90 degrees are not possible because the tangent of +90 degrees or -90 degrees is an infinitely large positive or negative value. However, the method can handle angles that are very close to +90 or -90 degrees.)

A negative y/x ratio

If the y/x ratio is negative, this doesn't necessarily mean that the angle lies in the fourth quadrant. That negative ratio could result from a positive value for y and a negative value for x. In that case, the angle would lie in the second quadrant between 90 degrees and 180 degrees.

The getAngle function tests the signs of the values for y and x. If the signs indicate that the angle lies in the second quadrant, the value returned from the Math.atan method is corrected to place the angle in the second quadrant. The corrected angle is returned by the getAngle function.

A positive y/x ratio

Similarly, if the y/x ratio is positive, this doesn't necessarily mean that the angle lies in the first quadrant. That positive ratio could result from a negative y value and a negative x value. In that case, the angle would lie in the third quadrant between 180 degrees and 270 degrees.

Again, the getAngle function tests the signs of the values for y and x. If both values are negative, the value returned from the Math.atan method is corrected to place the angle in the third quadrant.

No corrections required...

Finally, if no corrections are required for the quadrant, the getAngle function returns the value returned by the Math.atan method. Note however, that in all cases, the Math.atan method returns the angle in radians. That value is converted to degrees by the getAngle function and the returned value is in degrees.

Positive and negative angles

As you can see from the results of the test shown in Figure 11 , angles in the first, second, and third quadrants are returned as positive angles in degrees. However, angles in the fourth quadrant are returned as negative angles in degrees.

10.4.4.2 Mathematical solution to an earlier problem

Earlier (p. 224) in this module, I described a problem involving the sum of two displacement vectors. One vector had a magnitude of 30 meters with an angle of 0. The other vector had a magnitude of 40 meters with an angle of 135 degrees.

On the basis of a graphic solution, I estimated the length of the resultant vector to be about 28 meters and the angle of the resultant vector to be a little less than 90 degrees. I promised to provide a mathematical solution for that problem later, and that time has come.

Please copy the code shown in Listing 2 (p. 245) into an html file and open that file in your browser.

Listing 2 . The sum of two vectors.

```
    <!-- File JavaScript02.html -->
<html><body>
<script language="JavaScript1.3">

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle


//Specify the magnitudes and angles of two vectors.
//Convert the angles from degrees to radians.
var vecABmag = 30;//at an angle of 0
var vecABang = 0*Math.PI/180;//degrees*radians/degrees=radians

var vecBCmag = 40;//at an angle of 135
var vecBCang = 135*Math.PI/180;//degrees*radians/degrees=radians


//Compute the horizontal and vertical components of each vector.
var vecABh = vecABmag*Math.cos(vecABang);
var vecABv = vecABmag*Math.sin(vecABang);

var vecBCh = vecBCmag*Math.cos(vecBCang);
var vecBCv = vecBCmag*Math.sin(vecBCang);

//Compute the sums of the horizontal and vertical components from
// the two vectors to get the horizontal and vertical component
// of the resultant vector.
var vecResultH = vecABh + vecBCh;
var vecResultV = vecABv + vecBCv;

//Use the Pythagorean theorem to compute the magnitude of the
// resultant vector.
```

Table 10.13

Screen output

When you open the html file in your browser, the text shown in Figure 12 should appear in your browser window.

Figure 12 . Screen output for Listing #2.

```
   Hello from JavaScript
vecABv = 0
vecABh = 30
vecBCv = 28.284271247461902
vecBCh = -28.2842712474619
vecResultMag = 28.336261665087125
vecResultAng = 86.5286817554714
The End
```

Table 10.14

Explanation of the code

Listing 2 (p. 245) begins with a copy of the getAngle method that I explained in conjunction with Listing 1 (p. 242) . I won't repeat that explanation here.

Define magnitudes and angles of vectors

Then Listing 2 (p. 245) declares and initializes four variables that represent the magnitude and angle of each of the two vectors. The variable names that end with "mag" contain magnitude values and the variable names that end with "ang" contain angle values, which are converted from degrees to radians.

Compute horizontal and vertical components

Then Listing 2 (p. 245) calls the Math.cos and Math.sin methods to compute the horizontal and vertical components of each vector. The variable names that end with "h" contain horizontal component values and the variable names that end with "v" contain vertical component values.

Compute the sums of the components

The procedure for adding two or more vectors is to

- Add the horizontal components of all the vectors to get the horizontal component of the resultant vector.
- Add the vertical components of all the vectors to get the vertical component of the resultant vector.
- Compute the magnitude (if needed) of the resultant vector as the square root of the **sum of the squares** of the horizontal and vertical components.
- Compute the angle (if needed) of the resultant vector as the arctangent of the ratio of the vertical component and the horizontal component.

Listing 2 (p. 245) computes the sum of the horizontal components from each of the vectors and saves the result in the variable named **vecResultH** . Then it computes the sum of the vertical components and saves the result in the variable named **vecResultV** .

Compute the magnitude of the resultant

In many cases, simply knowing the horizontal and vertical components is the solution to the problem. However, in some cases, it is necessary to convert those components into a magnitude and an angle.

There are a couple of ways to compute the magnitude. One way is to use the Pythagorean theorem as described above (p. 246) to compute the square root of the sum of the squares of the horizontal and vertical components.

The Math.sqrt and Math.pow methods

The method named Math.sqrt can be called to return the square root of its positive argument. (If the argument is negative, the method returns NaN, which is an abbreviation for "Not a Number".)

The method named Math.pow can be called to raise the value specified by the first argument to the power specified by the second argument.

Listing 2 (p. 245) calls those two methods to compute the magnitude of the resultant and to save that value in the variable named **vecResultMag** .

Compute the angle of the resultant

Then Listing 2 (p. 245) calls the getAngle method that we developed earlier to get the angle of the resultant and to save the angle in degrees in the variable named **vecResultAng** .

Display the results

Finally Listing 2 (p. 245) calls the document.write method several times in succession to display the values stored in some of the variables mentioned above in the browser window as shown in Figure 12 . As you can see, the magnitude and angle of the resultant agrees with the estimate that I made using the graphic method earlier.

10.4.4.3 Subtracting vectors

To subtract one vector from another using the mathematical approach, simply reverse the algebraic sign on the horizontal and vertical components for the subtrahend vector and perform the addition shown in Listing 2 (p. 245) .

10.4.4.4 Adding more than two vectors

Let's work through a problem that requires the addition of four vectors.

A graphical solution

First I would like for you to lay out the vectors, tail-to-head on your graph board to create a solution to this problem.

Pretend that you take a walk consisting of the following sequential segments. Angles are specified relative to east with east being zero degrees. Positive angles are counter-clockwise and negative angles are clockwise.

The segments are as follows:

- 12 meters at 20 degrees
- 8 meters at 120 degrees
- 10.5 meters at -74 degrees
- 12.5 meters at 145 degrees

What is your final displacement?

When you reach the end of the walk, what is your displacement (distance and direction) relative to the location at which you began the walk. You should be able to do a reasonable job of measuring the displacement using your graph board.

A mathematical solution

Please copy the code shown in Listing 3 (p. 247) into an html file and open the html file in your browser.

Listing 3 . The sum of four vectors.

continued on next page

```
    <!-- File JavaScript03.html -->
<html><body>
<script language="JavaScript1.3">

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle


//Specify the magnitudes and angles of four vectors.
//Convert the angles from degrees to radians.
var vecAmag = 12;
var vecAang = 20*Math.PI/180;//degrees*radians/degrees=radians

var vecBmag = 8;
var vecBang = 120*Math.PI/180;//degrees*radians/degrees=radians

var vecCmag = 10.5;
var vecCang = -74*Math.PI/180;//degrees*radians/degrees=radians

var vecDmag = 12.5;
var vecDang = 145*Math.PI/180;//degrees*radians/degrees=radians


//Compute the horizontal and vertical components of each vector.
var vecAh = vecAmag*Math.cos(vecAang);
var vecAv = vecAmag*Math.sin(vecAang);

var vecBh = vecBmag*Math.cos(vecBang);
var vecBv = vecBmag*Math.sin(vecBang);

var vecCh = vecCmag*Math.cos(vecCang);
var vecCv = vecCmag*Math.sin(vecCang);

var vecDh = vecDmag*Math.cos(vecDang);
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 10.15

Screen output

The text shown in Figure 13 should appear in your browser when you open the html file in your browser.

Figure 13 . Screen output for Listing #3.

```
   Hello from JavaScript
vecResultMag = 8.11
vecResultAng = 90.49
The End
```

Table 10.16

As you can see, the displacement vector is just a little over eight meters at an angle of about 90.5 degrees. In other words, your ending position was approximately 8 meters north of your starting position. Hopefully you were able to arrive at a very similar answer using your graph board.

Explanation of the code

The code in Listing 3 (p. 247) is very similar to the code in Listing 2 (p. 245) . However, there is more of it because the code is adding four vectors instead of only 2.

As before, Listing 2 (p. 245) begins with the getAngle method that we developed earlier.

Magnitude and angle for each vector

Then Listing 2 (p. 245) defines eight variables containing the magnitudes and angles of the four segments of the walk.

(Note that the names that I used for the variables in this script are somewhat simpler than the names that I used in Listing 2.)

Horizontal and vertical components

Following that, Listing 2 (p. 245) computes the horizontal and vertical components for each of the four vectors and saves those values in eight different variables.

Sums of components

Then Listing 2 (p. 245) computes and saves the sum of all four horizontal components and computes and saves the sum of all four vertical components.

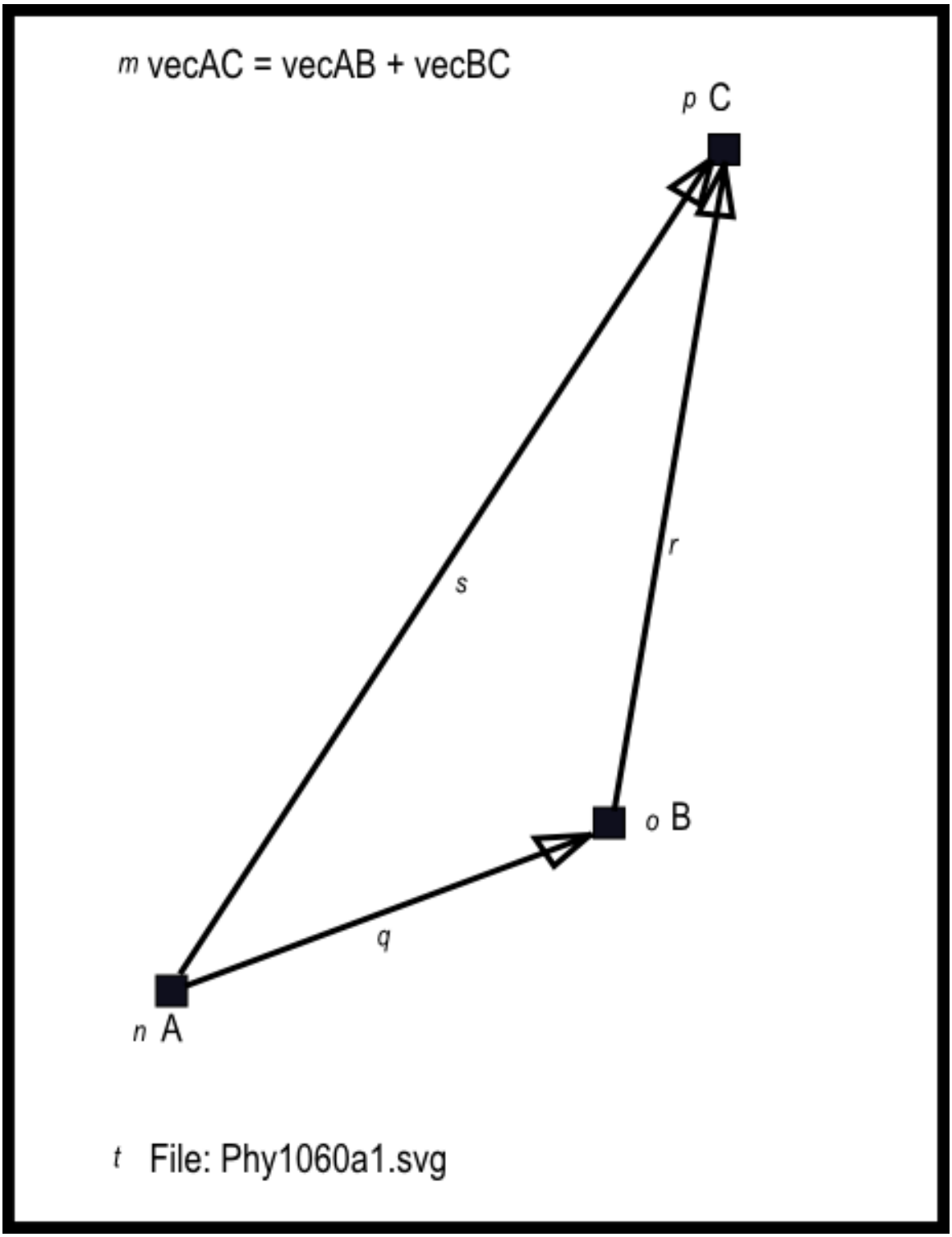
After that, there is essentially no difference between the script in Listing 3 (p. 247) and the script in Listing 2 (p. 245) .

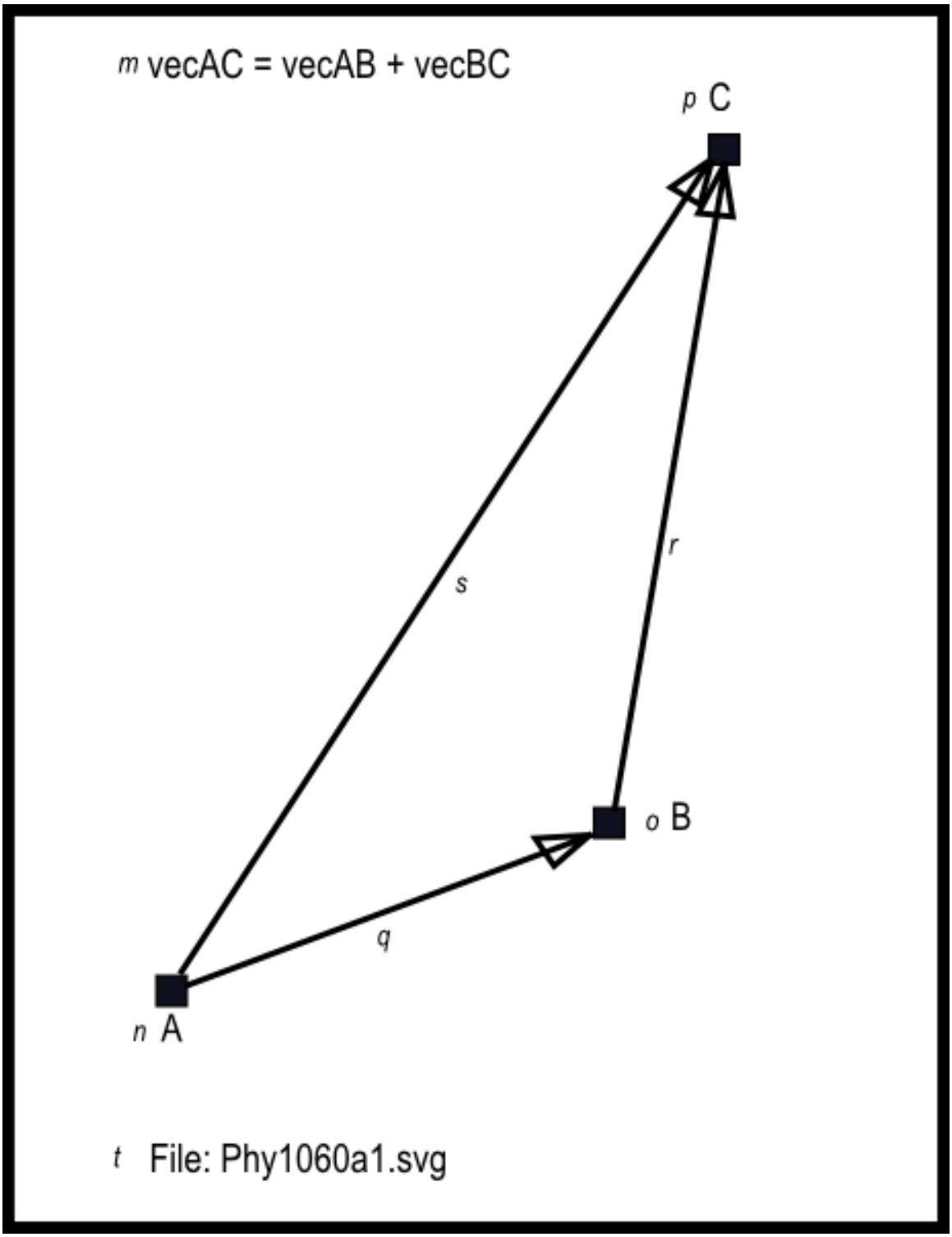
Other vector operations

In addition to the addition and subtraction of vectors, there are other operations that can be performed using vectors, such as the dot product and the cross product. However, those operations are beyond the scope of this module.

10.4.5 Non-mirror-image graphics

Figure 14 (p. 253) through Figure 18 (p. 269) below are non-mirror-image versions of Figure 2 (p. 217) , Figure 4 (p. 222) , Figure 6 (p. 228) , Figure 8 (p. 233) , and Figure 10 (p. 239) .


m vecAC = vecAB + vecBC
p C
r
s
o B
q
n A
t File: Phy1060a1.svg


m vecAC = vecAB + vecBC
p C
r
s
o B
q
n A
t File: Phy1060a1.svg

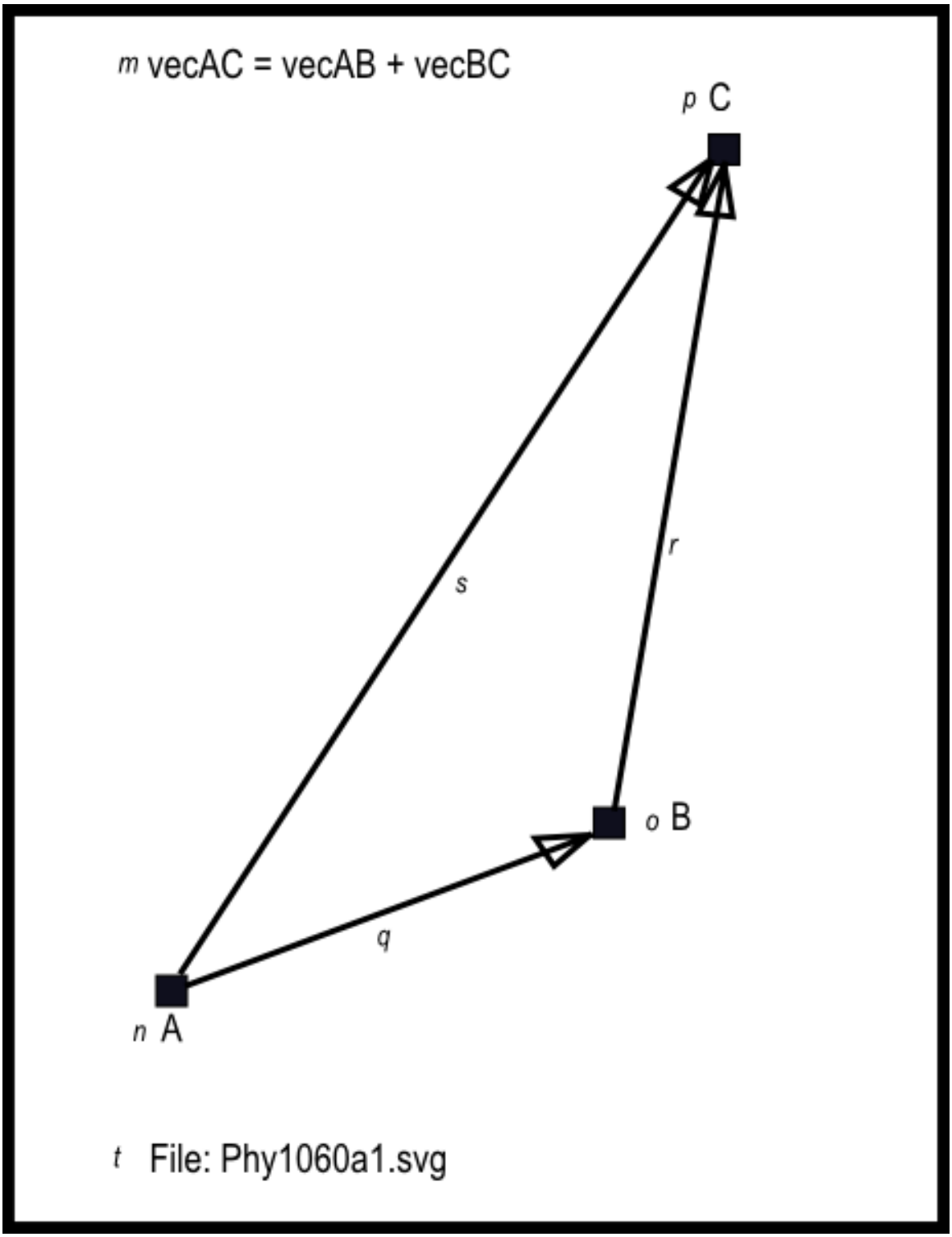

m vecAC = vecAB + vecBC
p C
r
s
o B
q
n A
t File: Phy1060a1.svg

Figure 14 . Non-mirror-image version of the image contained in the file named Phy1060a1.svg.

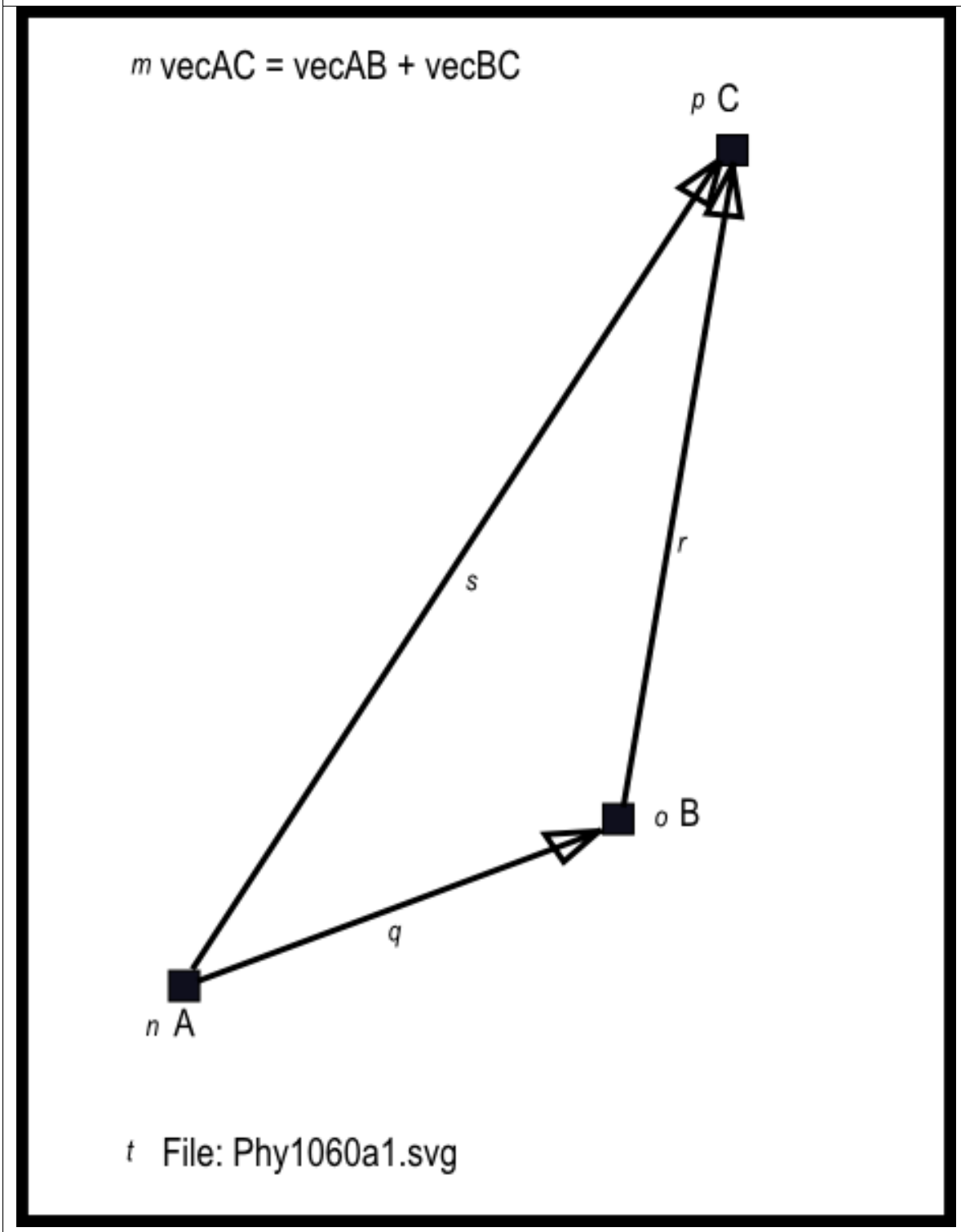
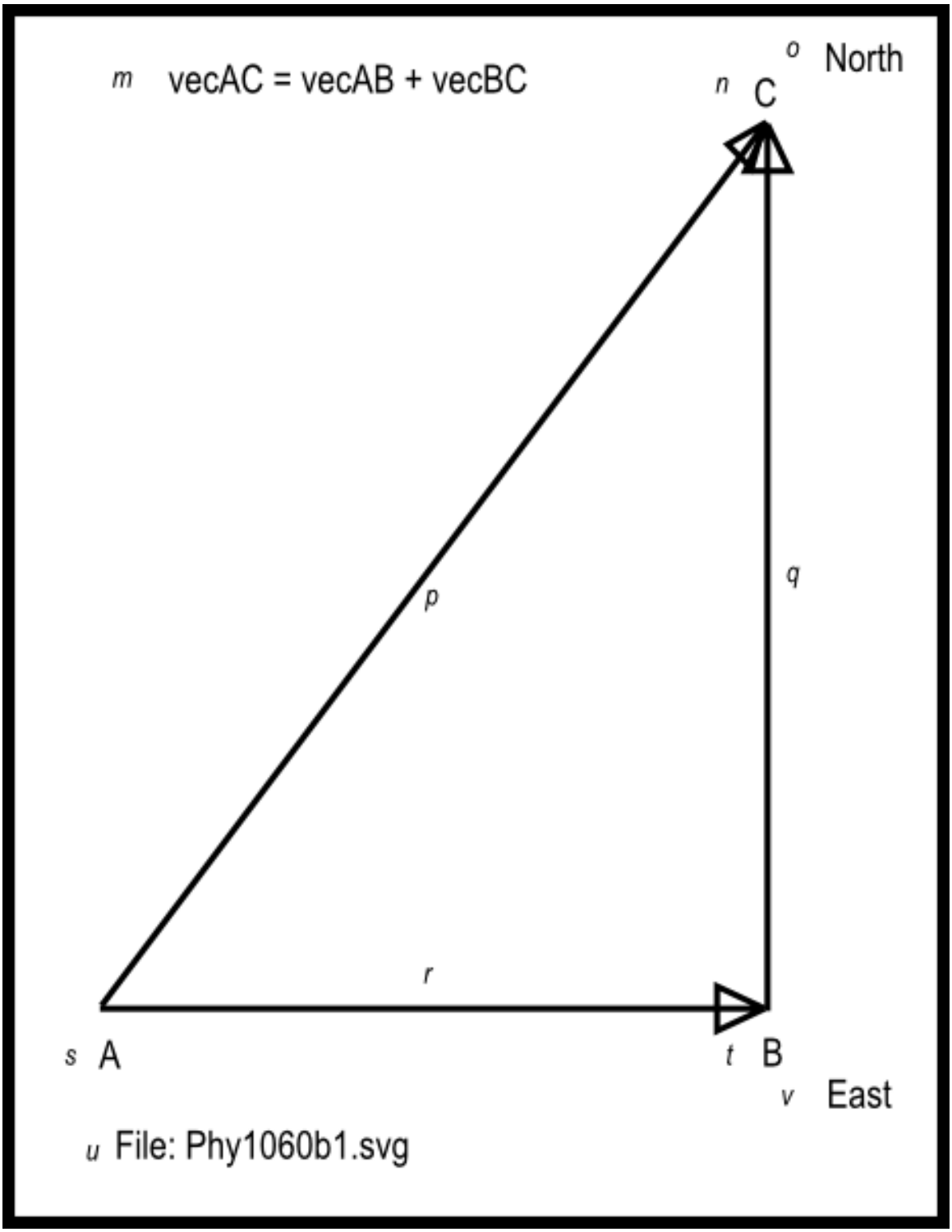
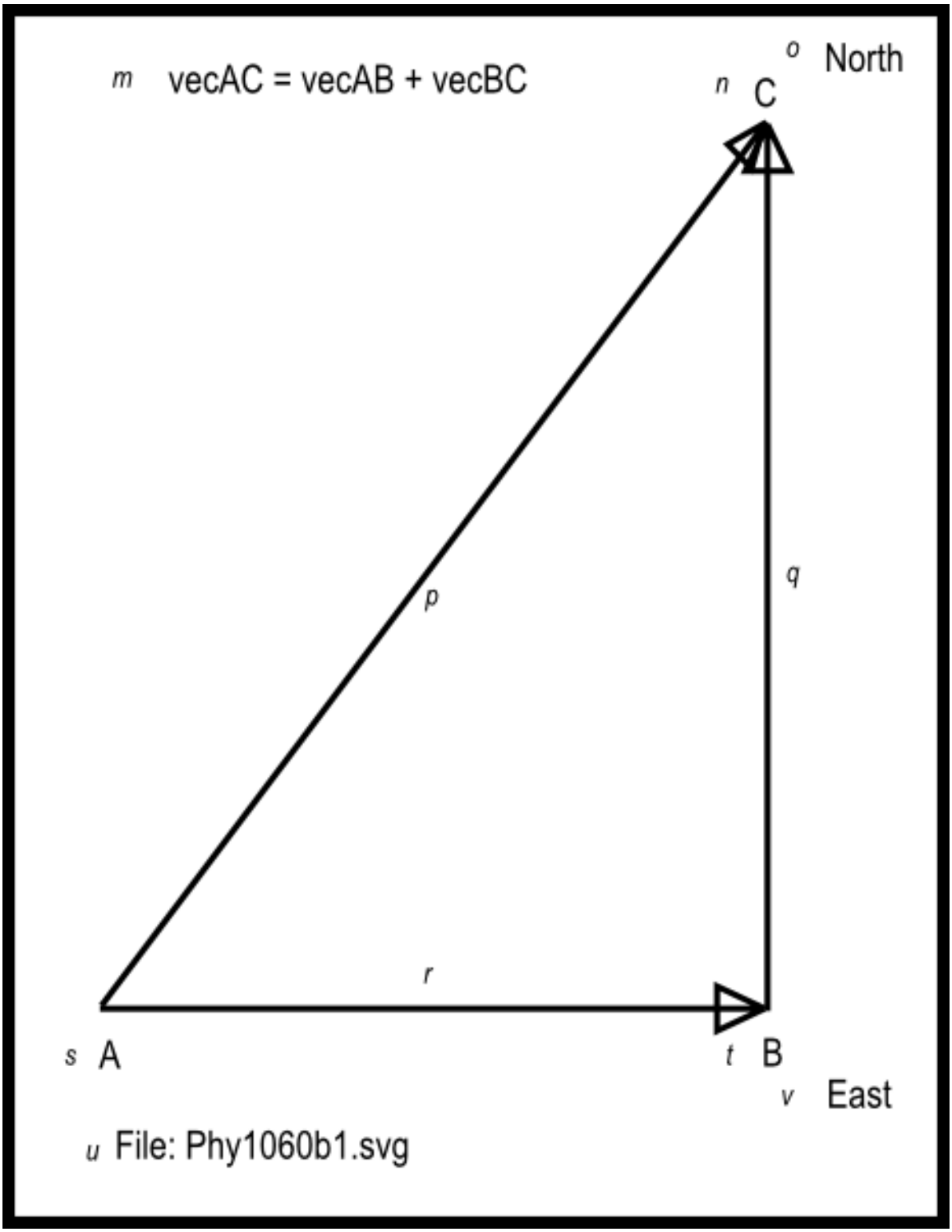


Table 10.17


m vecAC = vecAB + vecBC
o North
n C
p
q
r
s A
t B
v East
u File: Phy1060b1.svg


m vecAC = vecAB + vecBC
o North
n C
p
q
r
s A
t B
v East
u File: Phy1060b1.svg

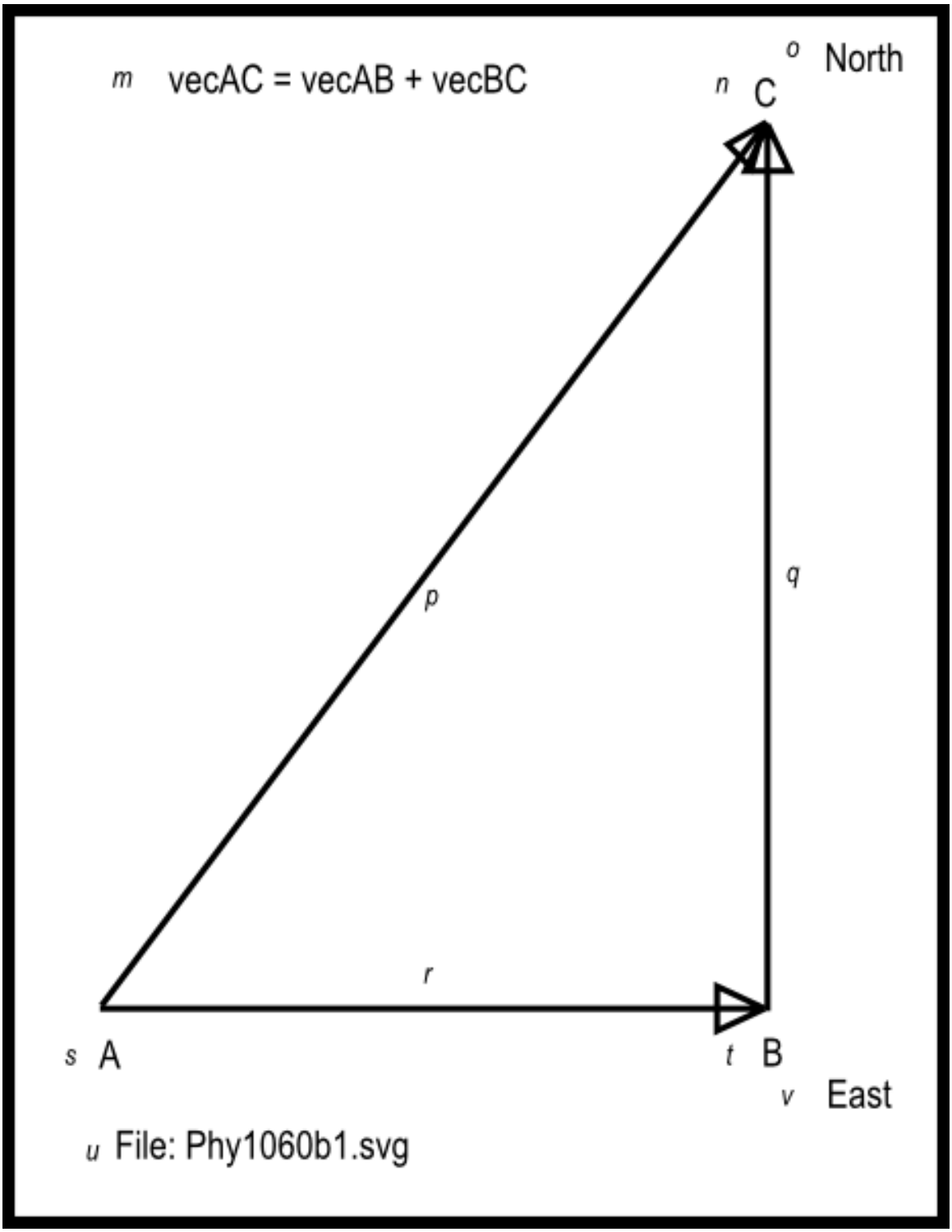

m vecAC = vecAB + vecBC
o North
n C
p
q
r
s A
t B
v East
u File: Phy1060b1.svg

Figure 15 . Non-mirror-image version of the image contained in the file named Phy1060b1.svg.

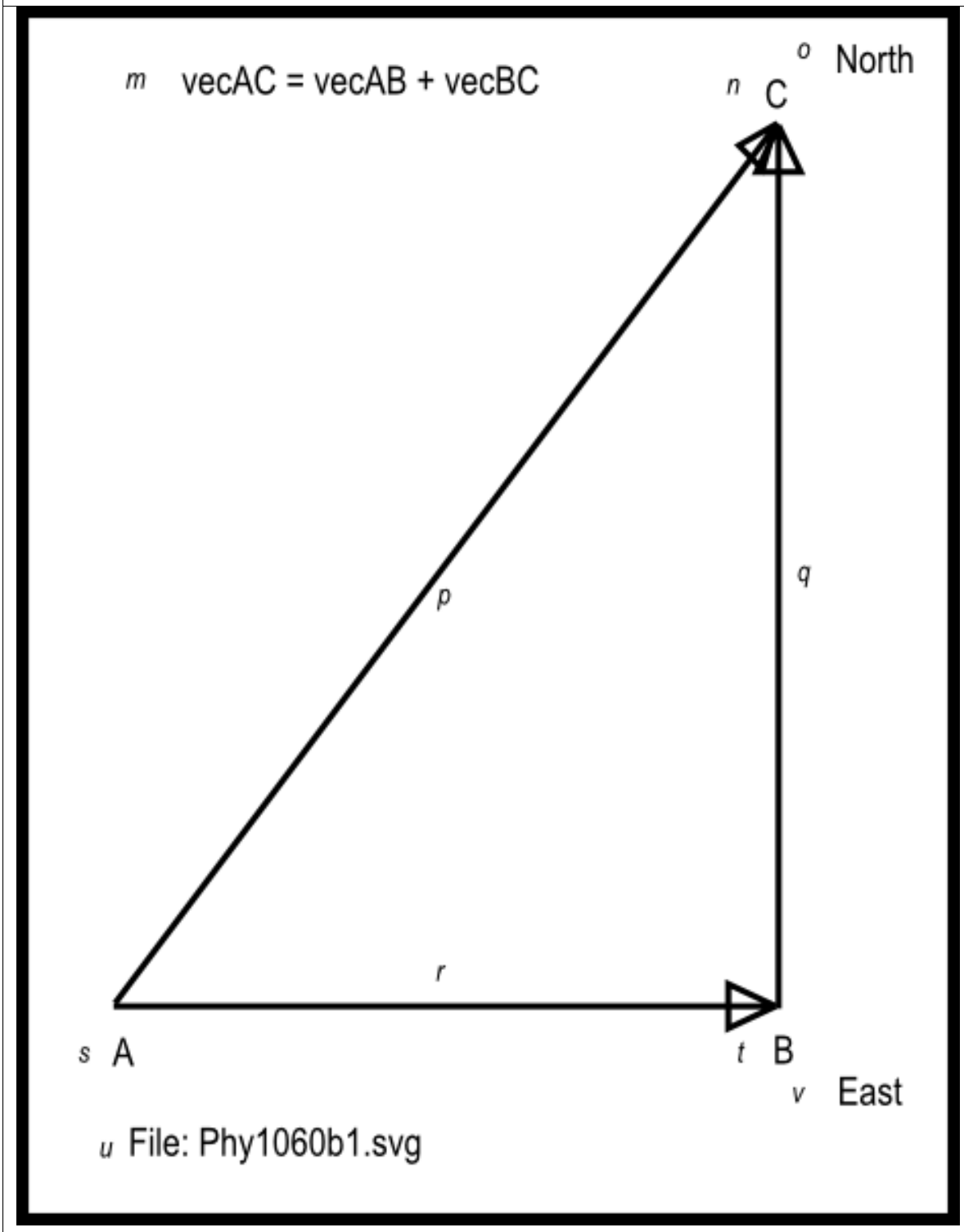
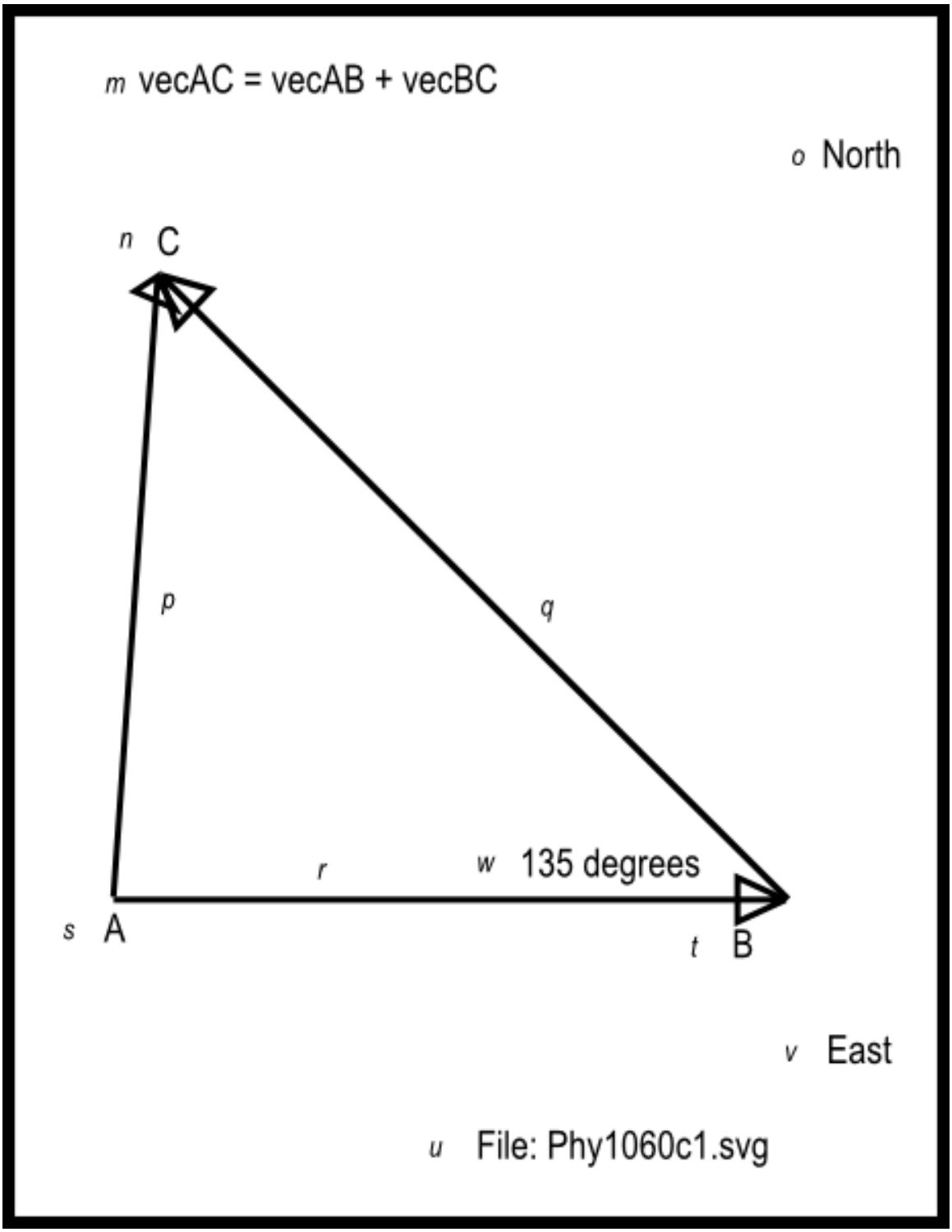
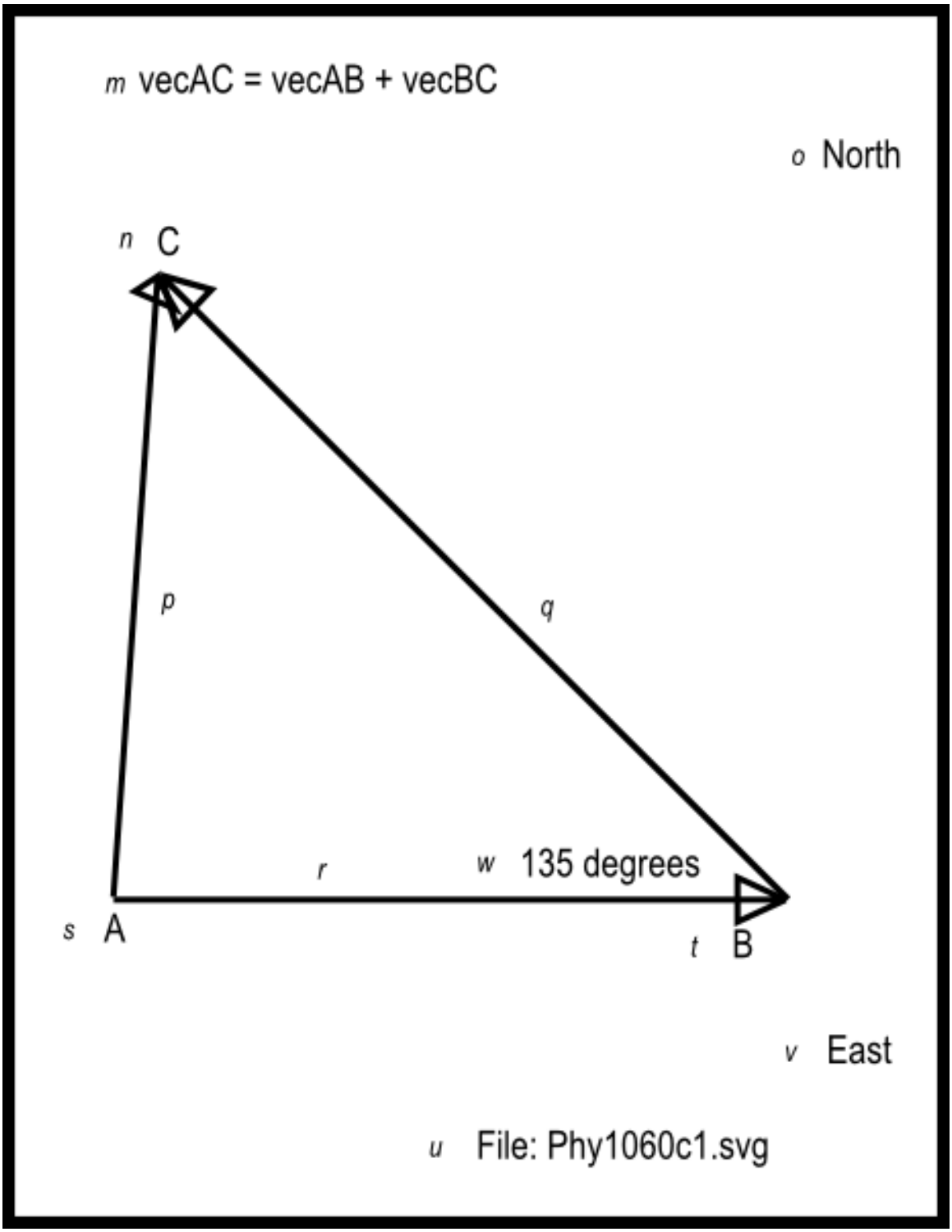


Table 10.18


m vecAC = vecAB + vecBC
o North
n C
p
q
r
w 135 degrees
s A
t B
v East
u File: Phy1060c1.svg


m vecAC = vecAB + vecBC
o North
n C
p
q
r
w 135 degrees
s A
t B
v East
u File: Phy1060c1.svg

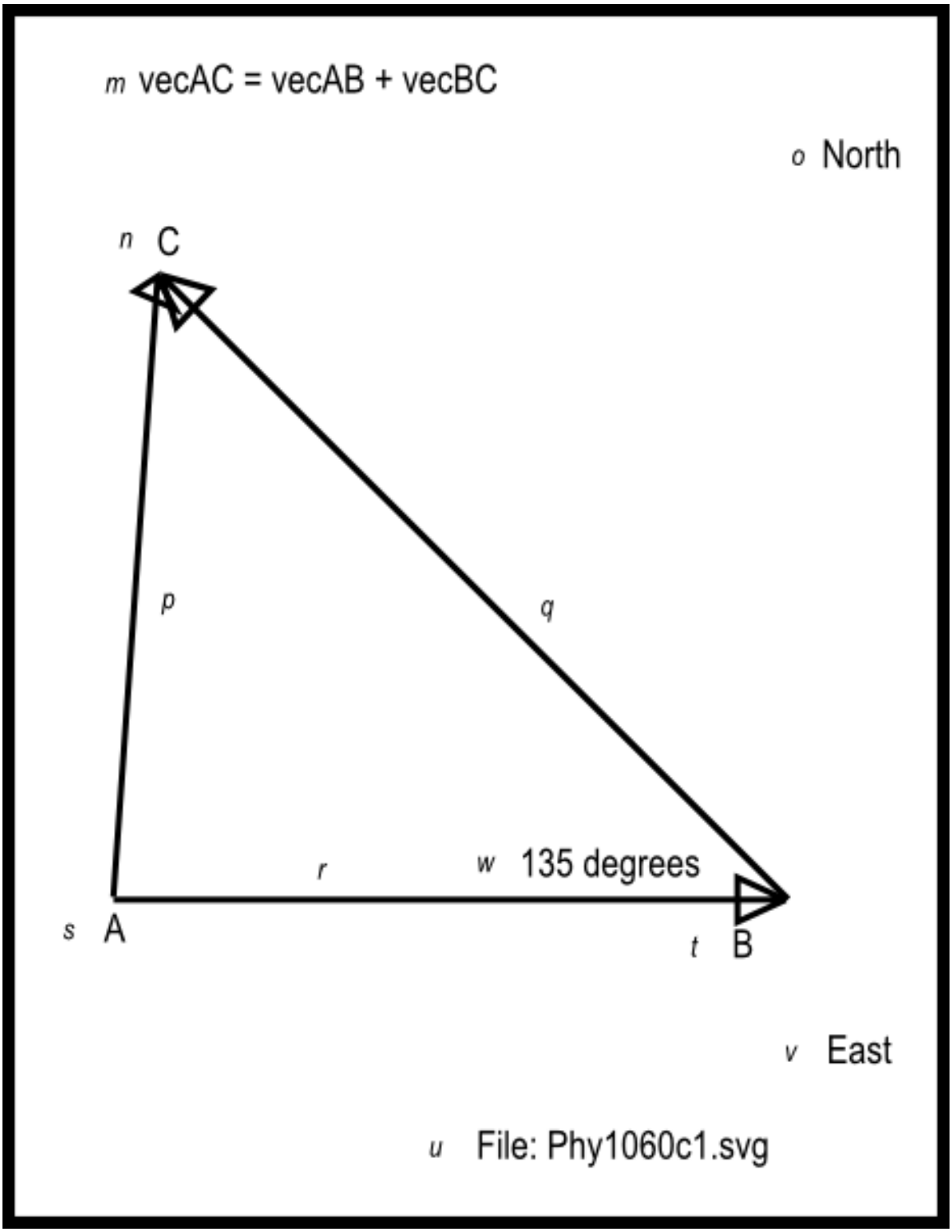

m vecAC = vecAB + vecBC
o North
n C
p
q
r
w 135 degrees
s A
t B
v East
u File: Phy1060c1.svg

Figure 16 . Non-mirror-image version of the image contained in the file named Phy1060c1.svg.

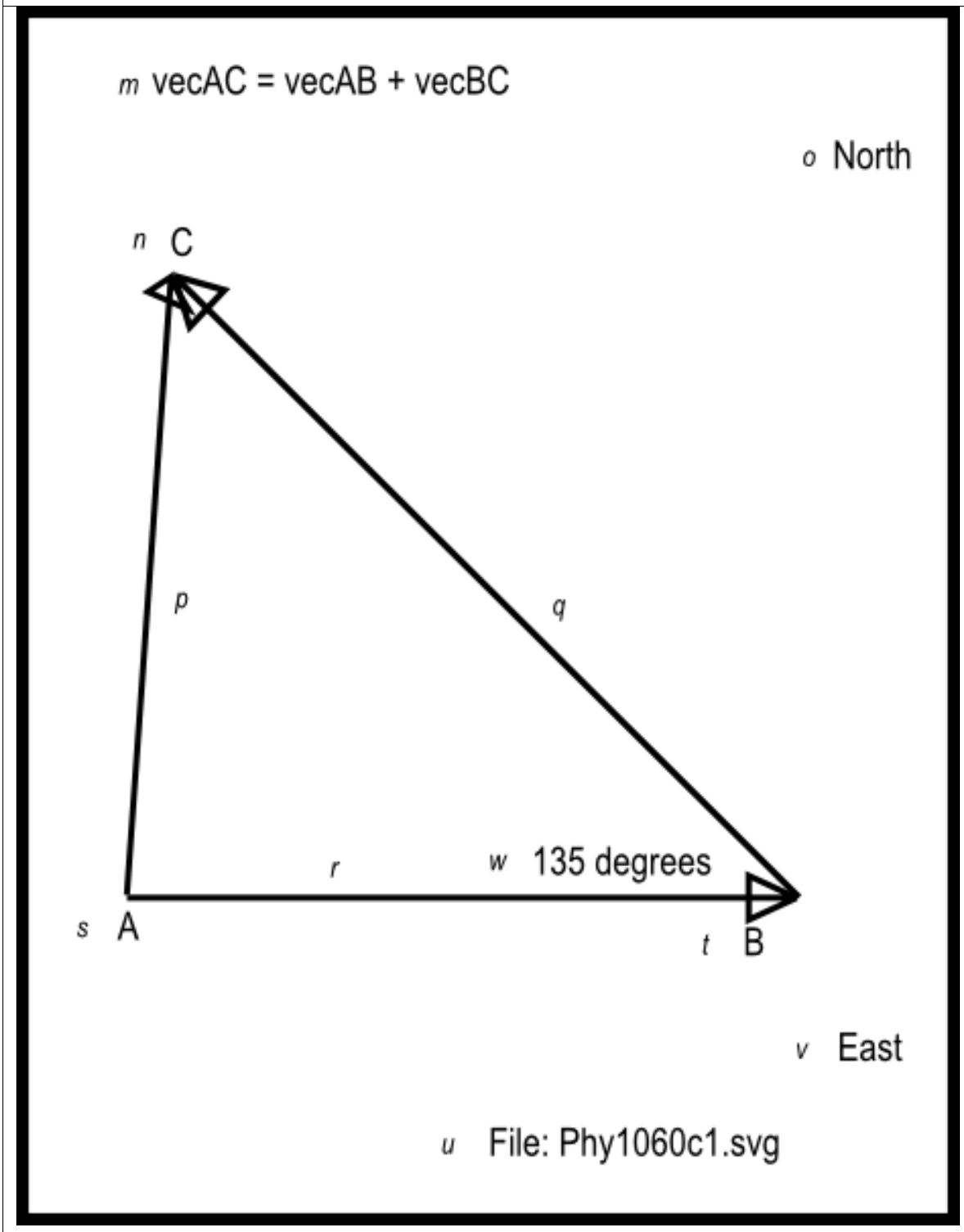
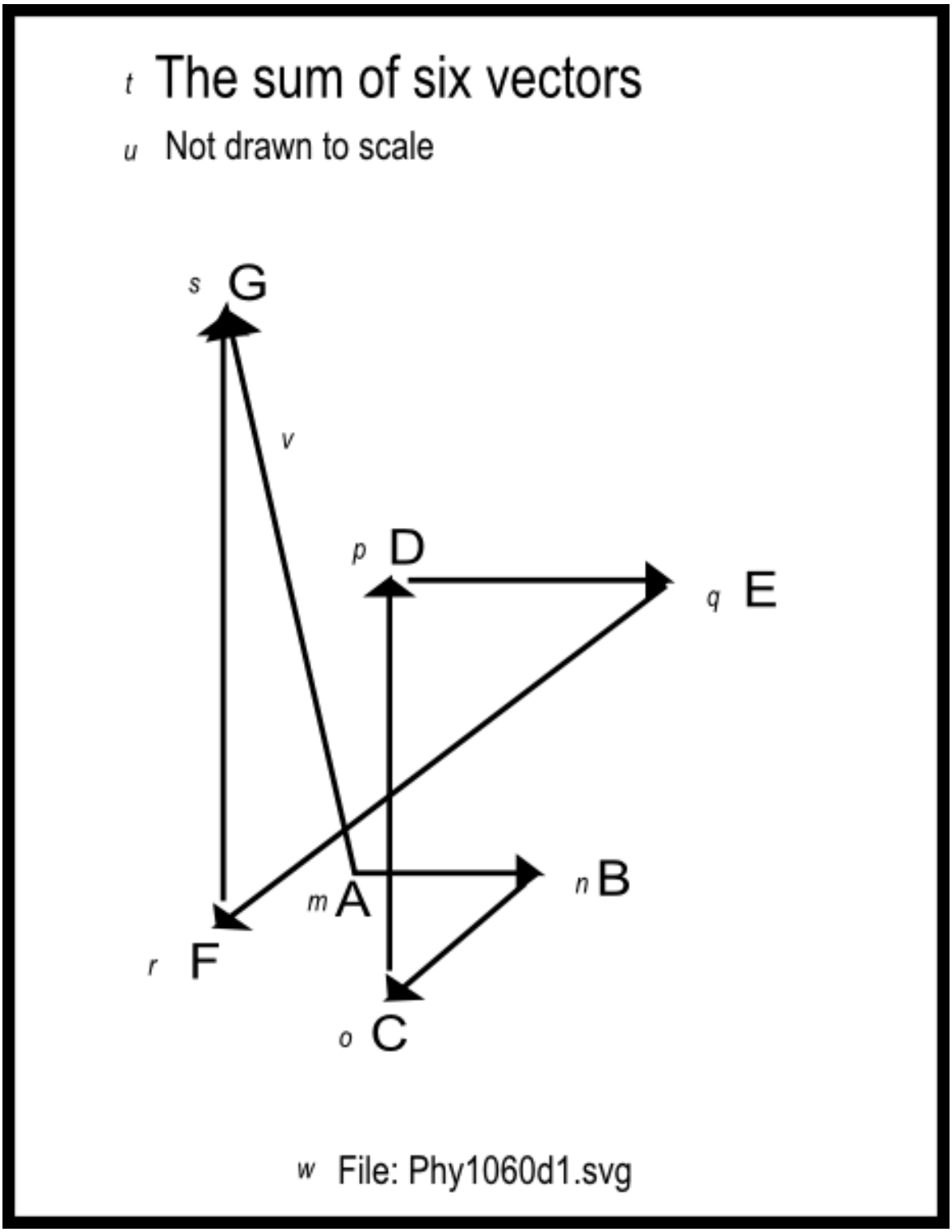
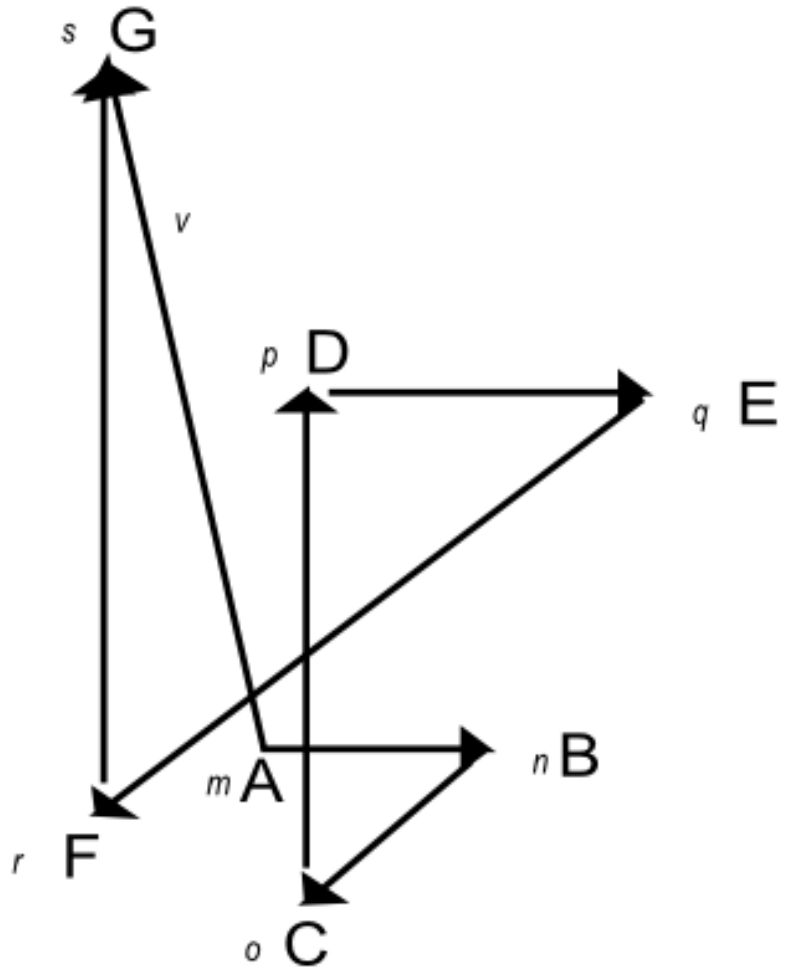


Table 10.19


The sum of six vectors
Not drawn to scale
G
v
D
E
A
B
F
C
File: Phy1060d1.svg

t The sum of six vectors

u Not drawn to scale



w File: Phy1060d1.svg

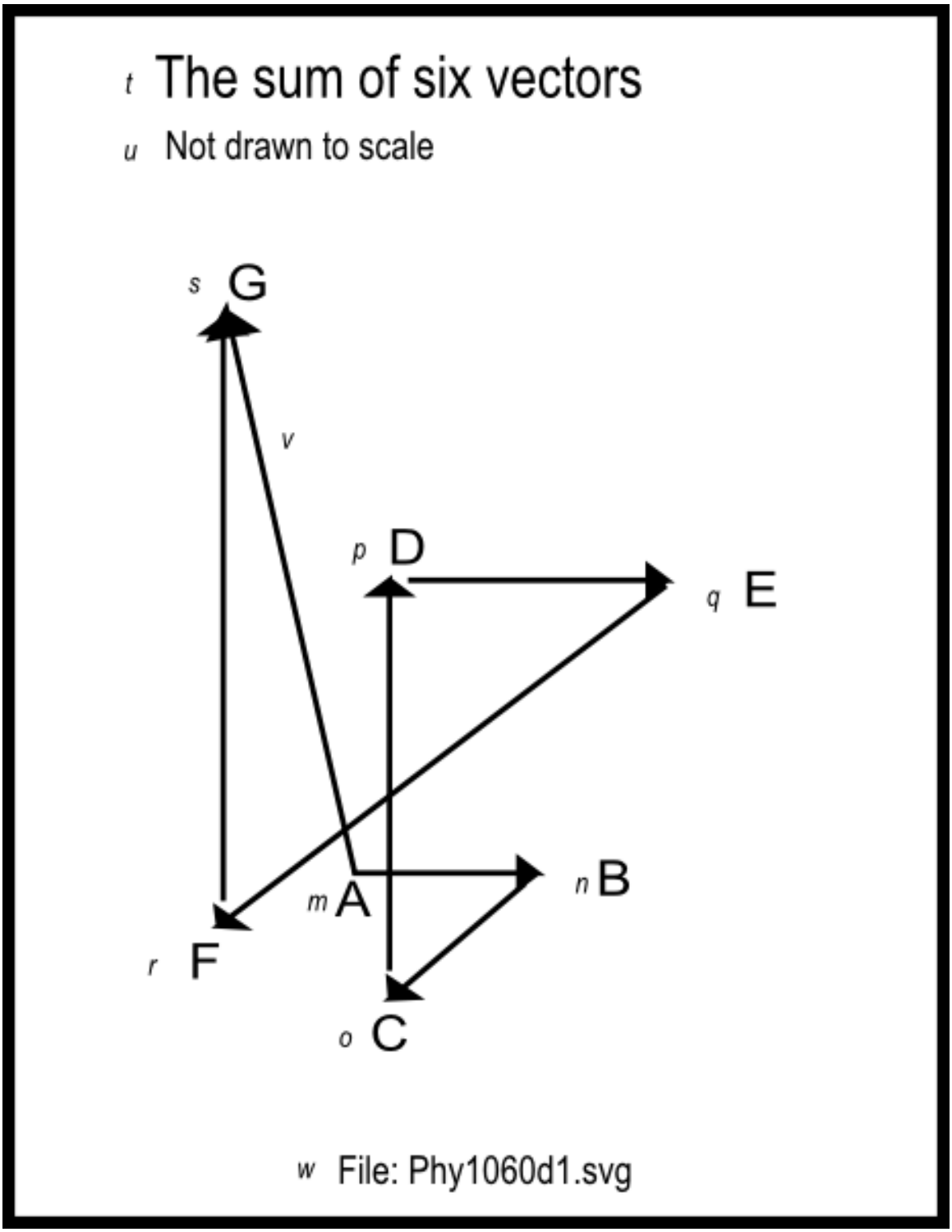

The sum of six vectors
Not drawn to scale
G
v
D
E
A
B
F
C
File: Phy1060d1.svg

Figure 17 . Non-mirror-image version of the image contained in the file named Phy1060d1.svg.

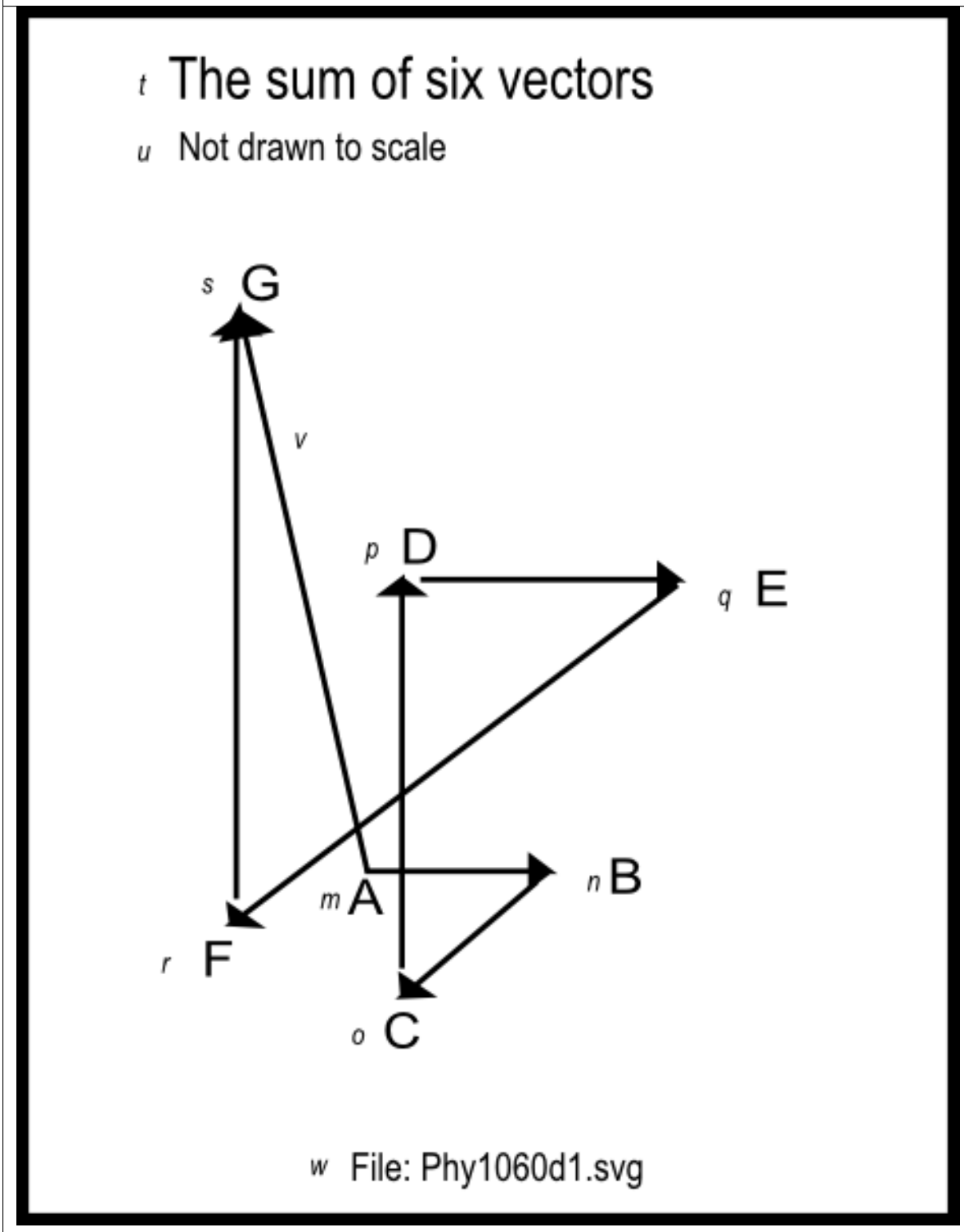
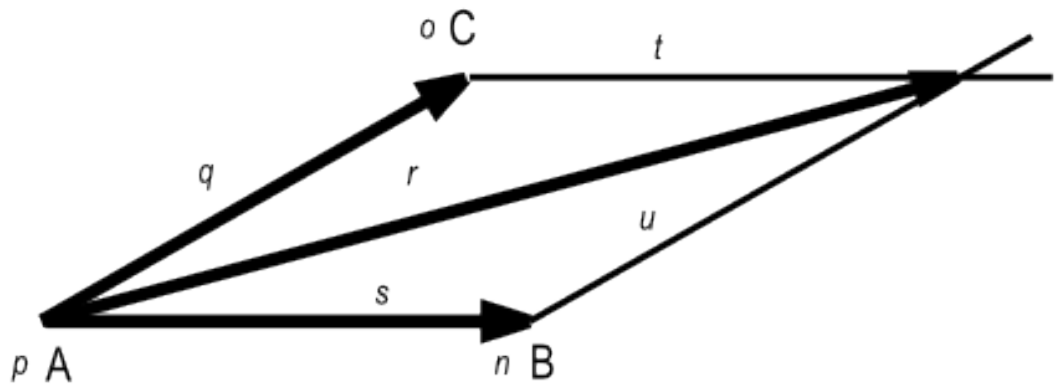


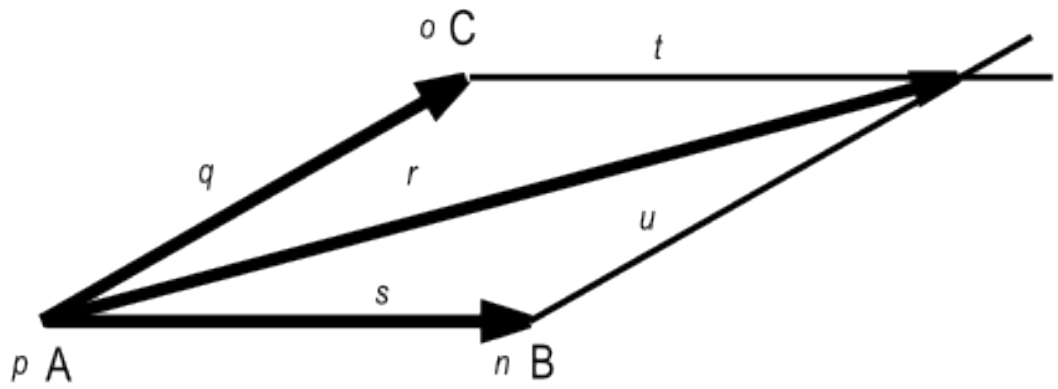
Table 10.20

m Adding vectors using the parallelogram law.



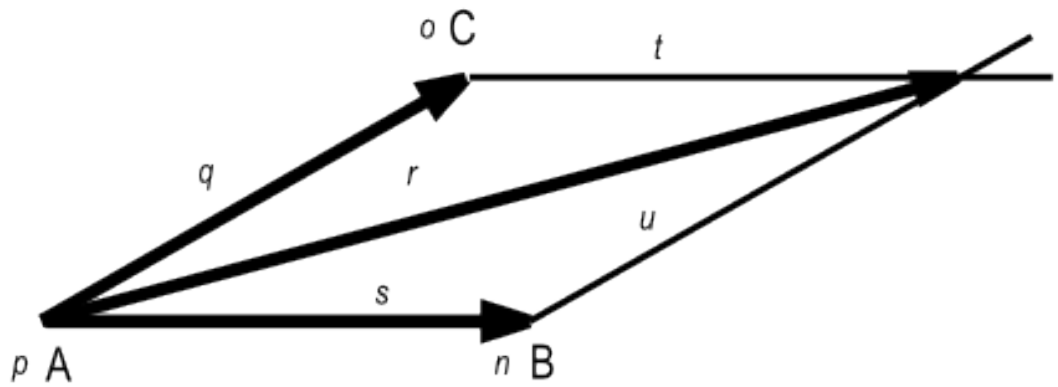
v File: Phy1060e1.svg

m Adding vectors using the parallelogram law.



v File: Phy1060e1.svg

m Adding vectors using the parallelogram law.



v File: Phy1060e1.svg

Figure 18 . Non-mirror-image version of the mage contained in the file named Phy1060e1.svg.

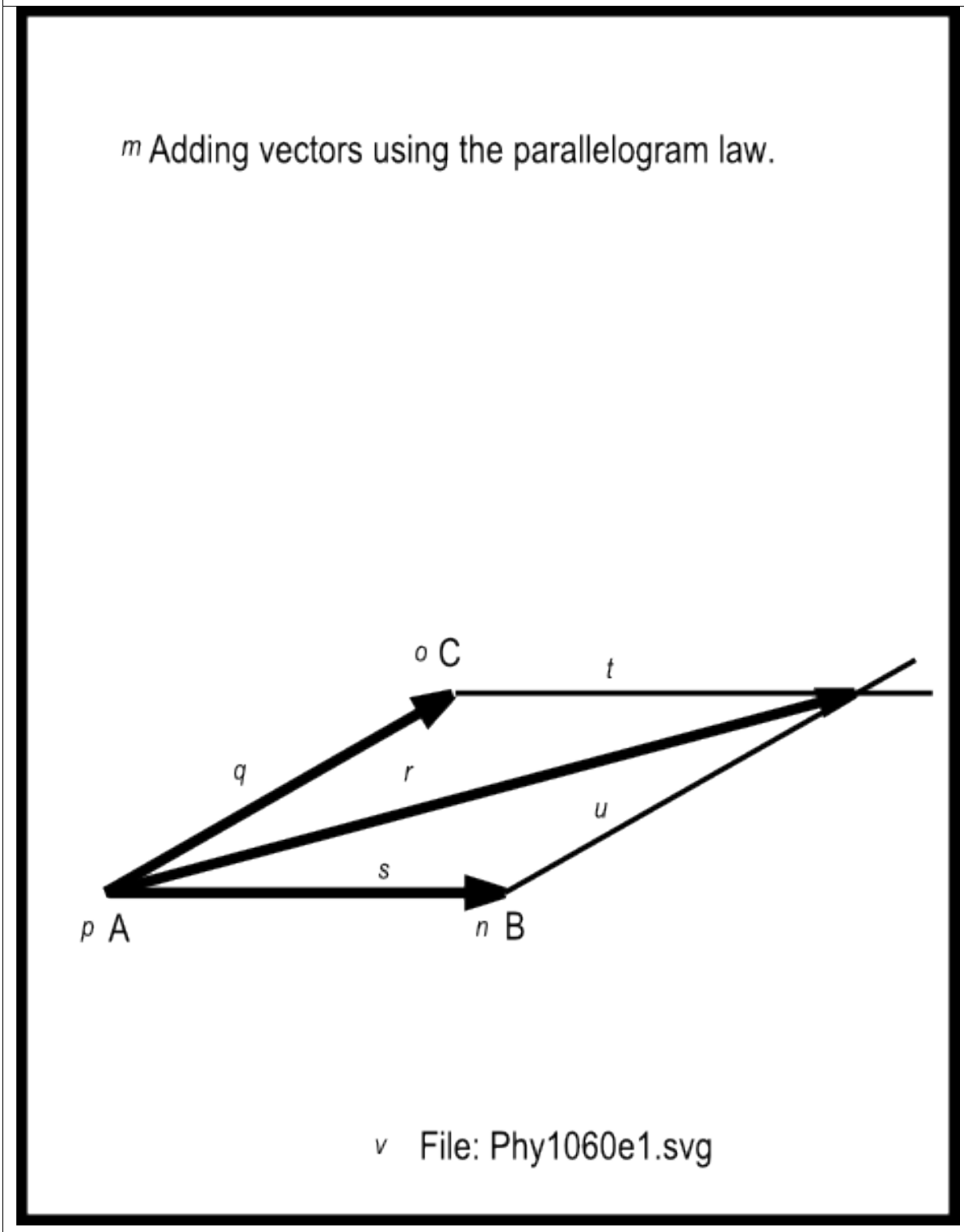


Table 10.21

10.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

10.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

10.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Motion – Displacement and Vectors for Blind Students
- File: Phy1060.htm
- Revised: 10/01/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · motion
  - · rest
  - · body
  - · displacement
  - · velocity
  - · acceleration
  - · kinematics
  - · kinetics
  - · resultant
  - · subtrahend
  - · minuend

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 11

Motion – Uniform and Relative Velocity[1]

11.1 Table of Contents

- Preface (p. 273)
  - General (p. 273)
  - Prerequisites (p. 274)
  - Viewing tip (p. 274)
    - Figures (p. 274)
    - Listings (p. 275)
  - Supplemental material (p. 275)
- General background information (p. 275)
- Discussion and sample code (p. 276)
  - Creation of tactile graphics (p. 276)
  - A simple exercise on uniform velocity (p. 276)
  - An exercise on average velocity (p. 279)
  - Multiple concurrent velocities (p. 287)
    - Exercise #1 for a man on a train (p. 287)
    - Exercise #2 for a man on a train (p. 296)
    - An exercise with three vectors in a plane (p. 301)
  - Non-mirror-image graphics (p. 307)
- Run the scripts (p. 323)
- Resources (p. 323)
- Miscellaneous (p. 323)

11.2 Preface

11.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

[1] This content is available online at <http://cnx.org/content/m38042/1.4/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The purpose of this module is to explain uniform velocity and relative velocity in a format that is accessible to blind students.

11.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

11.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

11.2.3.1 Figures

- Figure 1 (p. 278) . Screen output for Listing #1.
- Figure 2 (p. 279) . Key-value pairs for the image in Phy1070a1.svg.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

- Figure 3 (p. 283) . Mirror image contained in the file named Phy1070a1.svg.
- Figure 4 (p. 286) . Screen output for Listing #2.
- Figure 5 (p. 288) . Key-value pairs for the image in Phy1070b1.svg.
- Figure 6 (p. 291) . Mirror image contained in the file named Phy1070b1.svg.
- Figure 7 (p. 293) . Screen output for Listing #3.
- Figure 8 (p. 296) . Key-value pairs for the image in Phy1070c1.svg.
- Figure 9 (p. 300) . Mirror image contained in the file named Phy1070c1.svg.
- Figure 10 (p. 301) . Screen output for Exercise #2 for a man on a train.
- Figure 11 (p. 301) .Key-value pairs for the image in Phy1070d1.svg.
- Figure 12 (p. 305) .Mirror image contained in the file named Phy1070d1.svg.
- Figure 13 (p. 307) . Screen output for Listing #4.
- Figure 14 (p. 311) . Non-mirror-image contained in the file named Phy1070a1.svg.
- Figure 15 (p. 315) . Non-mirror-image contained in the file named Phy1070b1.svg.
- Figure 16 (p. 319) . Non-mirror-image contained in the file named Phy1070c1.svg.
- Figure 17 (p. 323) . Non-mirror-image contained in the file named Phy1070d1.svg.

11.2.3.2 Listings

- Listing 1 (p. 278) . A simple exercise on uniform velocity.
- Listing 2 (p. 285) . An exercise on average velocity.
- Listing 3 (p. 292) . Exercise #1 for a man on a train.
- Listing 4 (p. 306) . An exercise with three vectors in a plane.

11.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

11.3 General background information

What is velocity?

Velocity is the time rate of change of position. Since displacement is the change of position, velocity is also the rate of displacement. Since displacement is a vector quantity, velocity is also a vector quantity – it has magnitude and direction.

An airplane may traverse 350 miles in a northeasterly direction in one hour. This statement describes a vector quantity (velocity) because it has both magnitude and direction.

The same airplane may traverse 350 miles in a southwesterly direction in one hour. This statement describes a different velocity. Even though the magnitude is the same in both cases, the direction is different. Hence, the velocity is different.

What is speed?

Speed is distance covered in a unit of time without reference to direction. Since there is no reference to direction, speed is a scalar quantity.

A car may traverse 60 miles in one hour. An airplane may traverse 350 miles in one hour. An excellent runner may traverse one mile in three minutes. All of these statements describe scalar quantities (speed) since there is no reference to direction.

A person in a seat on a Ferris wheel may travel around the axle in the center of the wheel at a constant rotational speed. However, that person's velocity is not constant. The velocity is continually changing because the direction of motion is continually changing.

[14] http://www.dickbaldwin.com/toc.htm

Uniform velocity

Velocity is uniform when equal distances along a straight line are traversed in equal intervals of time. In this case

$\mathbf{v = s/t}$

where

- v is uniform velocity
- s is the distance traveled (displacement)
- t is an interval of time

What about the direction?

Because uniform velocity occurs along a straight line, the computations involving uniform velocity, distance, and time may not necessarily include the direction of that straight line. However, since both displacement and velocity are vector quantities, it is understood that the direction of velocity is the same as the direction of the displacement.

On the other hand, as you will see later, when combining two or more uniform velocities into a single resultant velocity, the directions of the different uniform velocities becomes extremely important.

Speed, distance, and time

Note also that this same equation can be applied to computations involving speed, distance, and time where

- v represents speed and not velocity
- s is the distance traveled (not necessarily in a straight line)
- t is an interval of time

Units of velocity

The units of velocity are units of distance (such as meters) divided by units of time (such as seconds).

11.4 Discussion and sample code

11.4.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explains how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1070.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

11.4.2 A simple exercise on uniform velocity

Let's begin this section with a short exercise involving uniform velocity.

How long does it take an airplane traveling at a uniform velocity of 350 miles per hour to traverse a distance of 700 miles?

Please copy the code shown in Listing 1 (p. 278) into an html file and open the file in your browser.

[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[16] http://cnx.org/content/m38042/latest/Phy1070.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

Listing 1 . A simple exercise on uniform velocity.

```
    <!---------------- File JavaScript01.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script" + "</br>")

var velocity = 350;//uniform velocity in miles/hour
var distance = 700;//miles

var time = distance/velocity;//miles/(miles/hour) = hours

document.write("uniform velocity = " + velocity +
               " miles/hour</br>");
document.write("distance = " + distance + " miles</br>");
document.write("time = " + time + " hours</br>");
document.write("End Script")

</script>
</body></html>
```

Table 11.1

Screen output

The text shown in Figure 1 (p. 278) should appear in your browser window when you open the html file in your browser.

Figure 1 . Screen output for Listing #1.

```
    Start Script
speed = 350 miles/hour
distance = 700 miles
time = 2 hours
End Script
```

Table 11.2

Analysis of the code

The equation given earlier (p. 276) that shows the relationship among uniform velocity, distance, and time can be manipulated algebraically to find any one of the three terms when the values of the other two terms are known. The objective in this exercise is to find the time required to traverse a known distance with a known uniform velocity.

Algebraic manipulation of the equation

Multiplying both sides of the equation by t and dividing both sides by v allows us to rewrite the equation as

t = s/ v

where

- v is uniform velocity
- s is the distance traveled
- t is an interval of time

The solution to the problem is...

This version of the equation is used in Listing 1 (p. 278) to solve for the time required to traverse 700 miles at a uniform velocity of 350 miles/hour. The answer is 2 hours as shown in Figure 1 (p. 278) .

Declare and initialize variables

The code in Listing 1 (p. 278) begins by declaring and initializing variables named **velocity** and **distance** . Note that the units are shown in the comments. I recommend that you always do that in order to keep the units straight.

Perform the arithmetic

Then Listing 1 (p. 278) declares a variable named **time** and sets its value equal to **distance** divided by **velocity** . Once again, note the treatment of the units in the comments, showing that the units for the result of the division are hours.

Display the results

Finally, Listing 1 (p. 278) calls the document.write method several times in succession to display the results shown in Figure 1 (p. 278) .

11.4.3 An exercise on average velocity

A hockey puck is struck by a player causing it to travel northeast at a uniform velocity of 50 feet per second for 15 feet, at which point it is struck by another player.

When struck by the second player, the direction of motion is changed to northwest with a uniform velocity of 60 feet per second.

It is stopped by a third player after traveling 20 feet in the northwest direction.

- What is the time required for the puck to complete each leg of its journey?
- What is the average velocity of the puck from the start to the end of its journey?

Tactile graphics

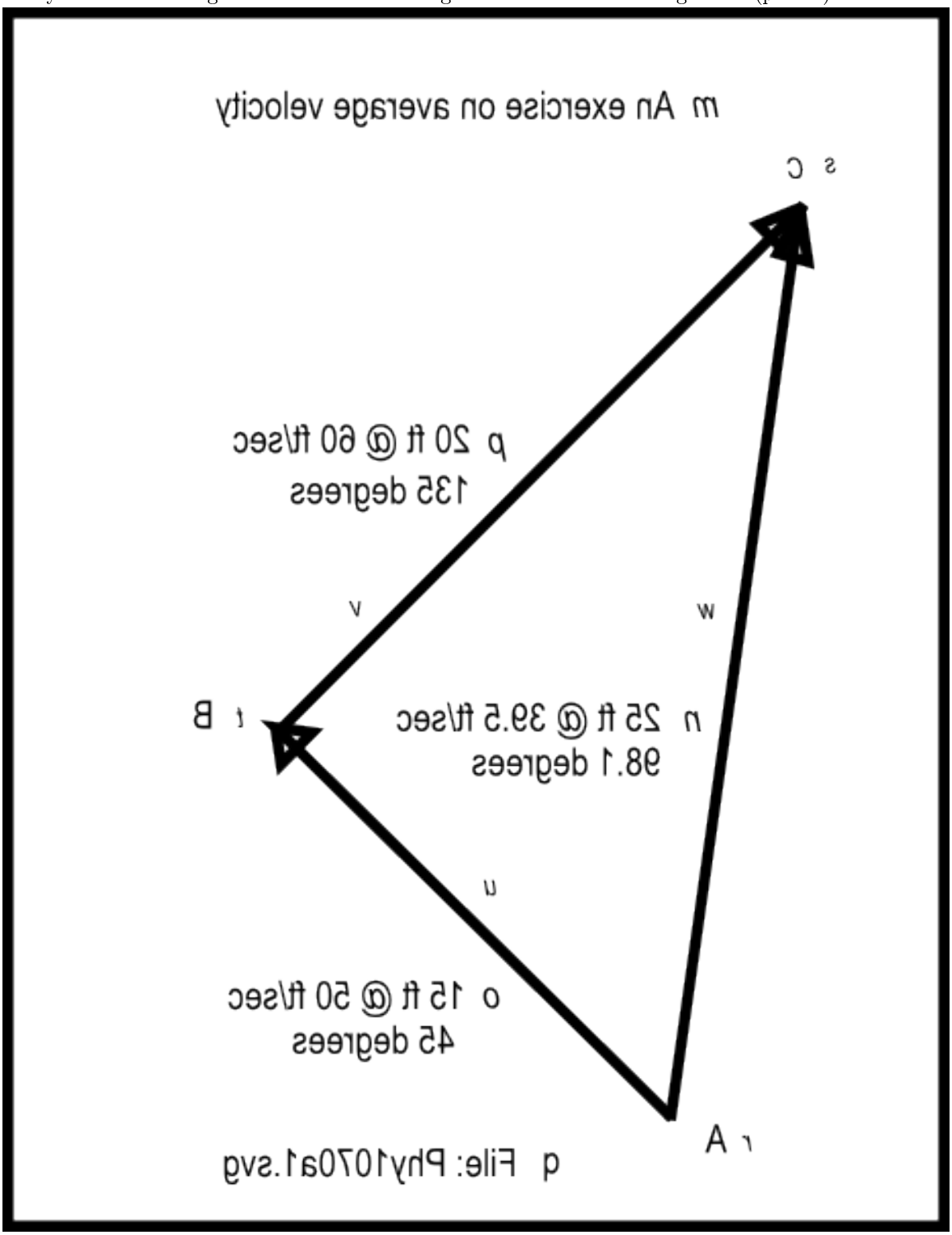
An svg file named Phy1070a1.svg is provided for the creation of a tactile displacement vector diagram for this scenario. The table of key-value pairs for this file is provided in Figure 2 (p. 279) . Some of the values shown are based on a JavaScript solution to the problem shown later in Figure 4 (p. 286) .

Figure 2 . Key-value pairs for the image in Phy1070a1.svg.

```
m: An exercise on average velocity
n: 25 ft @ 39.5 ft/sec 98.1 degrees
o: 15 ft @ 50 ft/sec 45 degrees
p: 20 ft @ 60 ft/sec 135 degrees
q: File: Phy1070a1.svg
r: A
s: C
t: B
u: Displacement vector A B
v: Displacement vector B C
w: Resultant vector A C
```

Table 11.3

The image contained in this file is shown in Figure 3 (p. 283) for the benefit of your assistant who will manually emboss the diagram. A non-mirror-image version is shown in Figure 14 (p. 311) .



An exercise on average velocity
C
p 20 ft @ 60 ft/sec
135 degrees
v
w
B
n 25 ft @ 39.5 ft/sec
98.1 degrees
u
o 15 ft @ 50 ft/sec
45 degrees
A
File: Phy1070a1.svg

m An exercise on average velocity
s C
p 20 ft @ 60 ft/sec
135 degrees
v
w
t B
n 25 ft @ 39.5 ft/sec
98.1 degrees
u
o 15 ft @ 50 ft/sec
45 degrees
r A
q File: Phy1070a1.svg

Figure 3 . Mirror image contained in the file named Phy1070a1.svg.

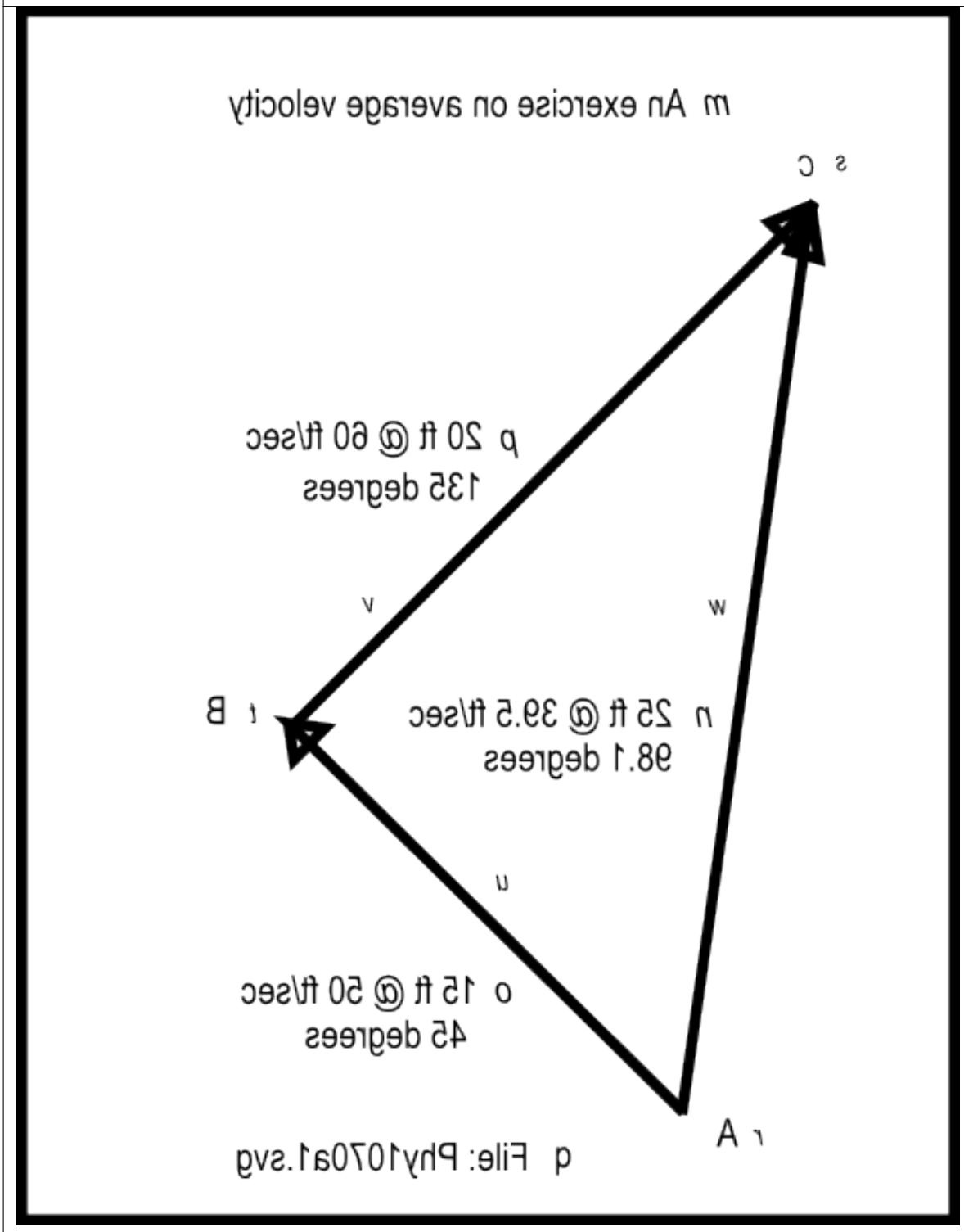


Table 11.4

Let's write a script to solve the problem

Please copy the code from Listing 2 (p. 285) into an html file and open it in your browser.

Listing 2 . An exercise on average velocity.

```
    <!---------------- File JavaScript02.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle

//Compute the time required to traverse each leg. Identify
// the two legs as A and B.
var vecAmag = 15;//displacement in feet
var vecAang = 45;//displacement angle in degrees
var vecAvel = 50;//velocity magnitude in feet per second
var timeA = vecAmag/vecAvel;//feet/(feet/second) = seconds

var vecBmag = 20;//displacement in feet
var vecBang = 135;//displacement angle in degrees
var vecBvel = 60;//velocity magnitude in feet per second
var timeB = vecBmag/vecBvel;//feet/(feet/second) = seconds

//Compute the overall displacement
//Compute the horizontal and vertical components
// of each vector.
var vecAh = vecAmag*Math.cos(vecAang*Math.PI/180);
var vecAv = vecAmag*Math.sin(vecAang*Math.PI/180);

var vecBh = vecBmag*Math.cos(vecBang*Math.PI/180);
var vecBv = vecBmag*Math.sin(vecBang*Math.PI/180);

//Compute the sums of the horizontal and vertical
// components from the two vectors to get the
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 11.5

Screen output

The text shown in Figure 4 (p. 286) should appear in your browser window when the html file is opened in your browser.

Figure 4 . Screen output for Listing #2.

```
   Start Script
Time for leg A = 0.30 seconds
Time for leg B = 0.33 seconds
Displacement magnitude = 25.00 feet
Displacement angle = 98.13 degrees
Total time = 0.63 seconds
Average velocity magnitude = 39.47 feet/second
Average velocity angle = 98.13 degrees
End Script
```

Table 11.6

Analysis of the code

In this exercise, the puck makes a journey across the ice consisting of two sequential legs in different directions with different magnitudes of velocity. Although each leg involves motion along a line, the total trip is not along a line.

The objective of the script is to

- Compute and display the time required to traverse each leg of the trip based on the given information.
- Compute and display the magnitude and the angle of the displacement from start to finish.
- Compute and display the time required to complete the entire trip, which is the sum of the times from both legs.
- Compute and display the magnitude of the average velocity as the magnitude of the displacement divided by the total time.
- Recognize that the angle of the average velocity is the same as the angle of the displacement.

The getAngle function

The code in Listing 2 (p. 285) begins with the **getAngle** function that we developed and used an earlier module. The purpose of the function is to receive the adjacent and opposite side values for a right triangle as parameters and to return the angle in degrees in the correct quadrant.

I explained this function in an earlier module and won't repeat that explanation in this module.

Compute the time for each leg

Following the definition of the getAngle function, Listing 2 (p. 285) computes the time required to traverse each leg of the trip. For variable naming purposes, the two legs are identified as A and B.

Variable names that end with "mag" contain vector magnitudes. Variable names that end with "ang" contain vector angles. Variable names that end with "vel" contain the magnitudes of straight-line, uniform velocities.

The time to complete each leg of the trip is computed by dividing the magnitude of the displacement vector for that leg by the magnitude of the straight-line velocity for that leg. The resulting times have units of seconds, and are saved in the variables named **timeA** and **timeB** .

The overall displacement

The magnitude and angle of the overall displacement for the two-leg trip (identified by the variables named **vecResultMag** and **vecResultAng** respectively) are computed using procedures that were explained in an earlier module. Therefore, I won't repeat that explanation in this module.

The magnitude of the overall displacement is stored in the variable named **vecResultMag** , and the angle for the overall displacement is stored in the variable named **vecResultAng** .

The total time

The total time for the trip, identified by the variable named **totalTime** , is computed as the sum of the times for the individual legs.

Magnitude of the average velocity vector

The magnitude of the average velocity vector, identified by the variable named **vecVelMag** , is compute by dividing the magnitude of the displacement vector by the total time. This results in a value having units of feet/second as shown by the comments.

The direction of the average velocity vector

The direction of the average velocity vector, identified by the variable named **vecVelAng** , is recognized as being the same as the direction of the overall displacement vector with units of degrees.

Display the results

Finally, the **document.write** method is called several times in succession to display the output text shown in Figure 4 (p. 286) .

11.4.4 Multiple concurrent velocities

When a body has multiple concurrent velocities, the overall velocity of the body is equal to the vector sum of the individual velocities. You can compute that vector sum in any way that works for you, including the parallelogram rule, a tail-to-head vector diagram, or the mathematical techniques that we will use here.

Relative velocity

In this section, we will also be dealing with a topic called **relative velocity** . For example, suppose we see a man walking down the aisle in a passenger car of a train that is moving slowly at a uniform velocity along a straight track. How fast is the man moving?

The frame of reference

The answer to that question depends on the frame of reference of the observer. For example, to another passenger in the same rail car, it may appear that the man is moving at about 3 feet per second, which is a reasonably comfortable walking speed.

However, to someone standing on the ground outside of the passenger car (pretend that the side of the car is transparent), it may appear that the man is moving much faster or much slower than 3 feet per second, depending on which direction the man is moving relative to the motion of the train.

It could even turn out that insofar as the outside observer is concerned, the man isn't moving at all, or is moving backwards.

11.4.4.1 Exercise #1 for a man on a train

A man is walking along the aisle in a passenger car of a train that is moving at 1 mile per hour toward the east. The man is walking in the same direction that the train is moving. The man is walking at a uniform velocity of 2.933 feet per second. (You will see shortly why I chose such a strange walking speed for the man.) What is the man's overall velocity with reference to the ground?

Tactile graphics

An svg file named Phy1070b1.svg is provided for the creation of a tactile velocity vector diagram for this scenario. The table of key-value pairs for this file is provided in Figure 5 (p. 288) .

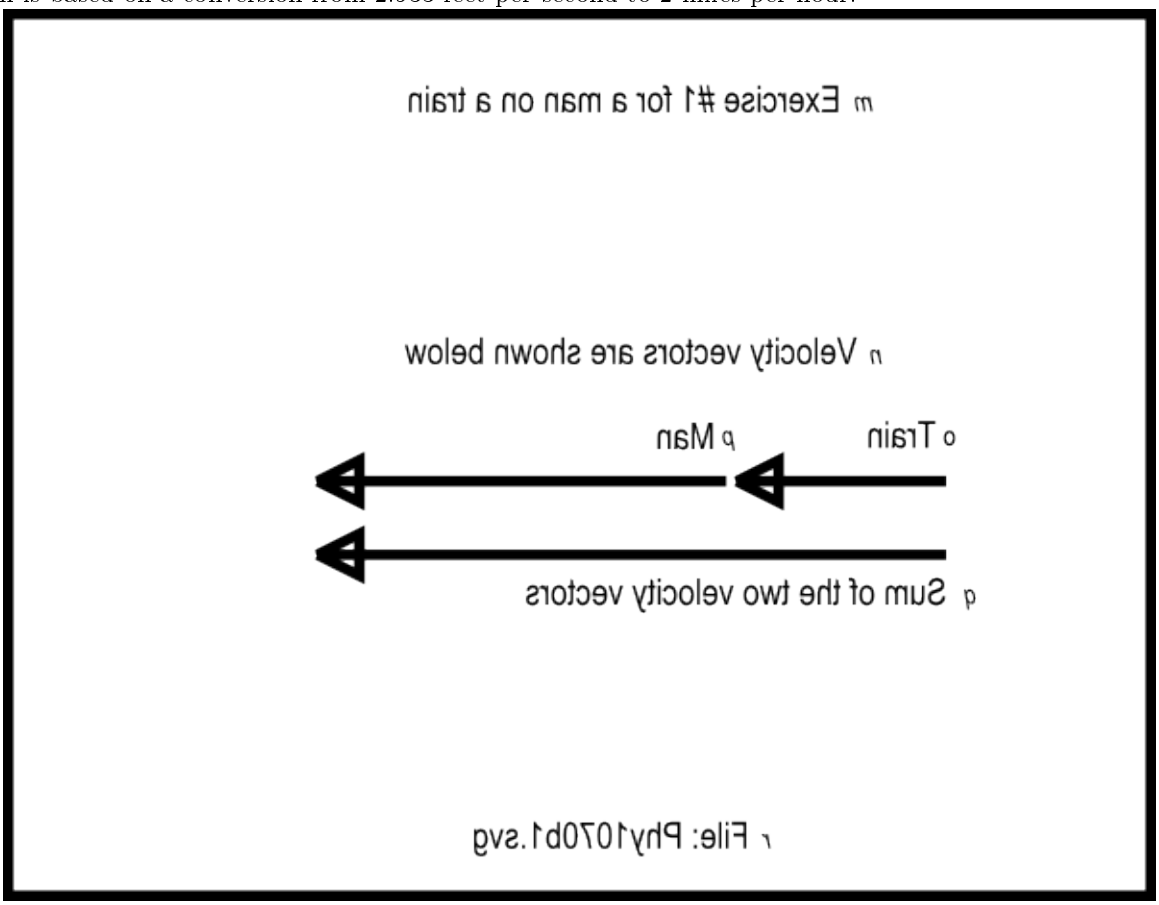
Figure 5 . Key-value pairs for the image in Phy1070b1.svg.

```
m: Exercise #1 for a man on a train
n: Velocity vectors are shown below
o: Train
p: Man
q: Sum of the two velocity vectors
r: File: Phy1070b1.svg
```

Table 11.7

The image contained in this file is shown in Figure 6 (p. 291) for the benefit of your assistant who will manually emboss the diagram. A non-mirror-image version is shown in Figure 15 (p. 315) .

The units for both velocity vectors must be the same. Therefore, the length of the velocity vector for the man is based on a conversion from 2.933 feet per second to 2 miles per hour.



m Exercise #1 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070b1.svg

m Exercise #1 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070b1.svg

Figure 6 . Mirror image contained in the file named Phy1070b1.svg

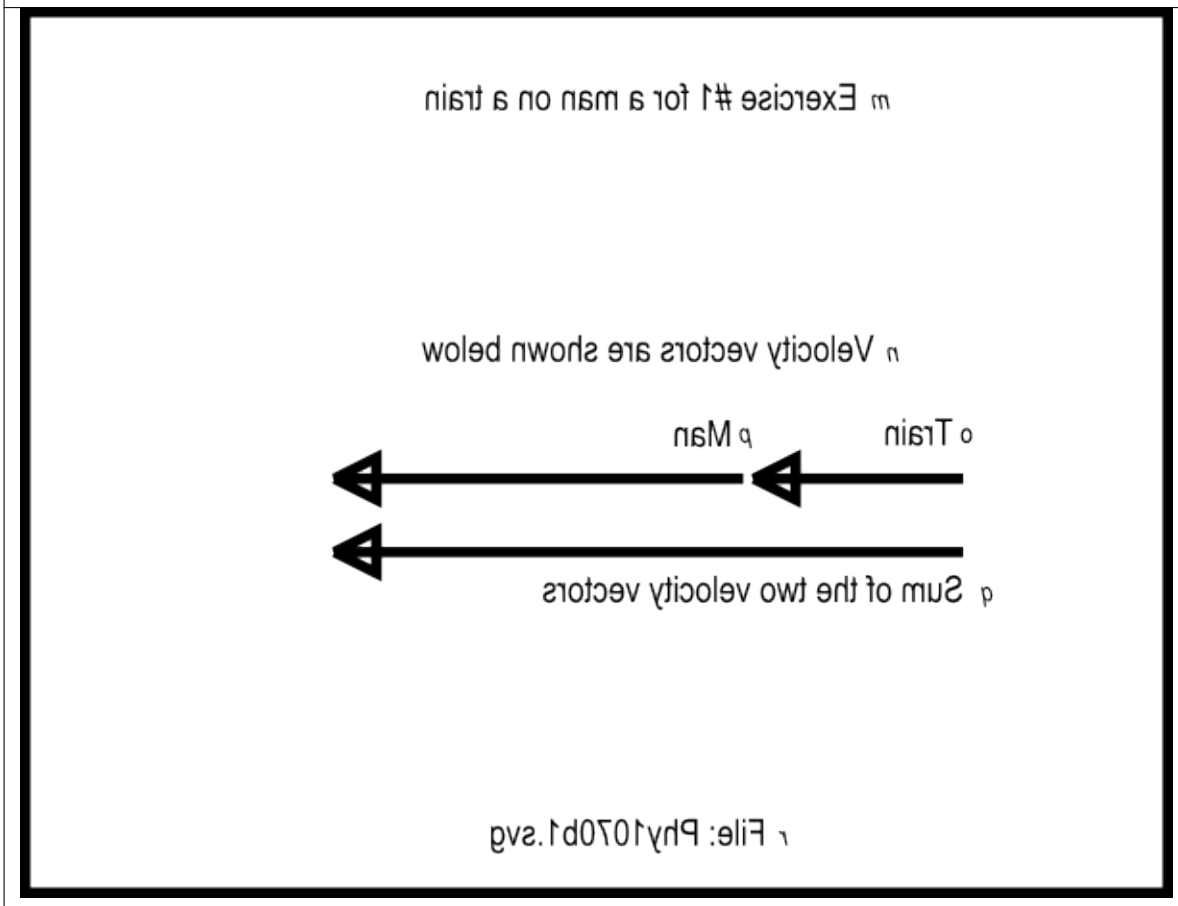


Table 11.8

JavaScript code

Please copy the code shown in Listing 3 (p. 292) into an html file and open the file in your browser.

Listing 3 . Exercise #1 for a man on a train.

```
    <!---------------- File JavaScript03.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){

  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle
//------------------------------------------------------------//

//The purpose of this function is to add two vectors, given
// the magnitude and angle (in degrees) for each vector.
// The magnitude and angle (in degrees) of the resultant
// vector is returned in a two-element array, with the
// magnitude in the element at index 0 and the angle in the
// element at index 1. Note that this function calls the
// getAngle function, which must also be provided in the
// script. To use this function to subtract one vector from
// another, add 180 degrees to the angle for the subtrahend
// vector before passing the angle to the function. To use
// this function to add more than two vectors, add the first
// two vectors as normal, then add the output from this
// function to the third vector, etc., until all of the
// vectors have been included in the sum.
function vectorSum(vecAmag,vecAang,vecBmag,vecBang){
  var vecResult = new Array(2);
  //Compute the horizontal and vertical components
  // of each vector.
  var vecAh = vecAmag*Math.cos(vecAang*Math.PI/180);
  var vecAv = vecAmag*Math.sin(vecAang*Math.PI/180);
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 11.9

Screen output

The text shown in Figure 7 (p. 293) should appear in your browser window when you open the html file in your browser.

Figure 7 . Screen output for Listing #3.

```
   Start Script
Velocity magnitude = 3.00 miles/hour
Velocity angle = 0.00 degrees
End Script
```

Table 11.10

Analysis of the code

As before, the script begins by defining the getAngle function, which doesn't require further explanation.

The vectorSum function

As you can see from Listing 3 (p. 292) , this script contains a new function named **vectorSum** . The purpose of this function is to add two vectors, given the magnitude and angle (in degrees) for each vector as incoming parameters. (The function assumes that both incoming magnitude parameters are expressed in the same units.)

The need to add two vectors occurs so often in physics that I decided to encapsulate the capability into a function that we can copy into future scripts. That will relieve us of the need to rewrite the code each time we need that functionality.

Return an array object

The magnitude and the angle (in degrees) of the resultant vector is returned in a two-element array object, with the magnitude in the element at index 0 and the angle in the element at index 1. I will have a more to say about arrays shortly.

Note that this function calls the getAngle function, which must also be provided in the script.

Subtraction of vectors

The function can also be used to subtract one vector from another. To subtract one vector from another, simply add 180 degrees to the angle for the subtrahend vector before passing the angle to the function.

Adding more than two vectors

Although the function can only be used to add two vectors, it can be called repeatedly to add any number of vectors two at a time.

To use this function to add more than two vectors, add the first two vectors as normal. Then add the vector output from the function to the third vector, etc., until all of the vectors have been included in the sum.

The code in the vectorSum function

Most of the code in the function is the same as code that I have explained in earlier modules. However, there are a couple of lines of code that are different.

The function begins with the following statement:

var vecResult = new Array(2);

The purpose of this statement is to create a two-element array object named **vecResult** .

What is an array object?

You can think of an array object as a software object containing pigeon holes into which you can store data and from which you can later retrieve that data. This particular array object has two pigeon holes, and they are numbered 0 and 1.

This array object will be used as a container to return the magnitude and angle values for the resultant vector.

Can only return one item

A JavaScript function can only return one item. Up to this point in these modules, that item has consisted of a single numeric value, such as the angle value that is returned from the getAngle function.

However, the vectorSum function needs to return two different values. One way to do that is to put those two values in the pigeon holes of an array object and return that array object as the one item that can be returned.

Store the resulting magnitude in the array object

Once the array object is created, the code in the function shown in Listing 3 (p. 292) is essentially the same as the code used earlier to add two vectors, down to the statement that begins with:

vecResult[0] = Math.sqrt(Math.pow(vecResultH,2) +...

That statement computes the magnitude of the resultant vector the same way as before, but instead of storing the value of the magnitude in a variable, it is stored in the first pigeon hole of the array object.

The value is directed into the first pigeon hole (technically called element) by the "[0]" that you see following the name of the array object in that statement.

Store the resulting angle in the array object

Immediately thereafter, the statement in Listing 3 (p. 292) that reads

vecResult[1] = getAngle(vecResultH,vecResultV);

computes the angle for the resultant vector and stores it in the second element in the array object (the element identified by the index value 1).

Return the array object for use later

These same index values along with the square brackets "[ ]" will be used later to retrieve the magnitude and angle values from the array object.

Finally, the function returns the array object to the calling script by executing the statement that reads

return vecResult;

The main script body

The main script body begins where indicated by the comment in Listing 3 (p. 292) .

The first four statements establish values for the magnitude and direction of the two vectors that represent the train and the man. There is nothing in those statements that you haven't seen before.

Note the treatment of the units

However, I encourage you to pay attention to the treatment of units in the comments for the value stored in the variable named **vecManMag** . The arithmetic in that statement uses two different conversion factors to convert the value from feet per second to miles per hour.

Once again, working through the units in this fashion can help you to organize your arithmetic correctly.

Call the vectorSum function

The statement that begins

var resultant = vectorSum(vecTrainMag...

calls the new vectorSum function to add the two vectors, passing the magnitude and angle for each vector as parameters.

Store the returned array object in a variable

This statement also declares a new variable named **resultant** . The array object that is returned from the vectorSum function is stored in this variable.

Once this statement finishes executing, the magnitude and angle of the resultant vector have been computed and saved for later use. (In this script, the only use of those two values is to display them later. However, they will be used in a more significant way in another exercise later.)

Code to display the results

The call to the vectorSum function is followed by three calls to the document.write method. The first two calls display magnitude and angle values, and the third call simply displays some text to indicate that the script has finished executing.

Displaying the magnitude value

If you examine the statement containing the first call to the document.write method, you will see that the argument list contains the following expression:

resultant[0].toFixed(2)

In this case, the "[0]" means to retrieve the value stored in the array element identified by the index value 0.

Once that value has been retrieved, the built-in toFixed method is called, passing the literal value 2 as a parameter, to format the value so that it will be displayed with two digits to the right of the decimal point.

Figure 7 (p. 293) shows that value displayed in the line of text that reads:

Velocity magnitude = 3.00 miles/hour

(Note that very few of the values that I display in these modules comply with the rules for decimal digits and significant figures that I explained in an earlier module titled *Scientific Notation and Significant Figures for Blind Students* .)

A partial solution to the problem

A partial answer to the question posed for this exercise is that the magnitude of the man's overall velocity with respect to the ground is 3 miles per hour as shown in Figure 7 (p. 293) .

The rest of the solution

The next statement in Listing 3 (p. 292) uses a similar syntax to retrieve the value from the second element in the array object (the element with an index value of 1) and to display it as

Velocity angle = 0.00 degrees

as shown in Figure 7 (p. 293) .

This means that the man is moving due east (the same direction that the train is moving) with a velocity of 3 miles per hour with reference to the ground.

Analysis of the results

Now I will tell you why I used a strange walking speed (2.933 feet per second) for the man. As it turns out, this is very close to 2 miles per hour, and I wanted the man to have a walking speed that would result in simple numeric results.

Let's review the problem

The train is moving east with a uniform velocity of 1 mile per hour relative to the ground.

The man is moving east with a uniform velocity of 2 miles per hour (2.933 feet per second) relative to the train.

What is the man's velocity relative to the ground?

The algebraic sum of the magnitudes

Because the man and the train are both moving along the same straight line, and the man's velocity is stated relative to the train, the magnitude of the man's velocity relative to the ground is the algebraic sum of the magnitudes of the two velocities. Because they are both moving in the same direction, that algebraic sum is additive.

An analysis of the velocities

The train is moving 1 mile per hour with reference to the ground, and the man is moving 2 miles per hour with reference to the train (along the same line and in the same direction as the train). Therefore, the man is moving 1+2=3 miles per hour with reference to the ground.

This means that an observer standing on the ground at a point on a line perpendicular to the train would perceive the man to be moving to the to the right with a velocity of 3 miles per hour (assuming that the train is moving to the right).

A zero-degree angle is not required

Note that it isn't necessary that the train and the man be moving at an angle of zero degrees (east). The magnitude of the result would be the same regardless of the direction that they are moving provided they are moving along the same straight line.

We will specify a different common direction for the train and the man in the next exercise.

11.4.4.2 Exercise #2 for a man on a train

A man is walking in a passenger car of a train that is moving at 1 mile per hour toward the northeast (45 degrees). The man is walking in the opposite direction than the train is moving at 2.933 feet per second. (This means that the angle for the man's velocity vector is 225 degrees.)

What is the man's velocity with reference to the ground?

Tactile graphics

An svg file named Phy1070c1.svg is provided for the creation of a tactile velocity vector diagram for this scenario. The table of key-value pairs for this file is provided in Figure 8 (p. 296) .

Figure 8 . Key-value pairs for the image in Phy1070c1.svg.

```
m: Exercise #2 for a man on a train
n: Velocity vectors are shown below
o: Train
p: Man
q: Sum of the two velocity vectors
r: File: Phy1070c1.svg
```

Table 11.11

The image contained in this file is shown in Figure 9 (p. 300) for the benefit of your assistant who will manually emboss the diagram. A non-mirror-image version is shown in Figure 16 (p. 319) .

The units for both velocity vectors must be the same. Therefore, the length of the velocity vector for the man is based on a conversion from 2.933 feet per second to 2 miles per hour.

m Exercise #2 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy107Oc1.svg

m Exercise #2 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy107Oc1.svg

m Exercise #2 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070c1.svg

Figure 9 . Mirror image contained in the file named Phy1070c1.svg

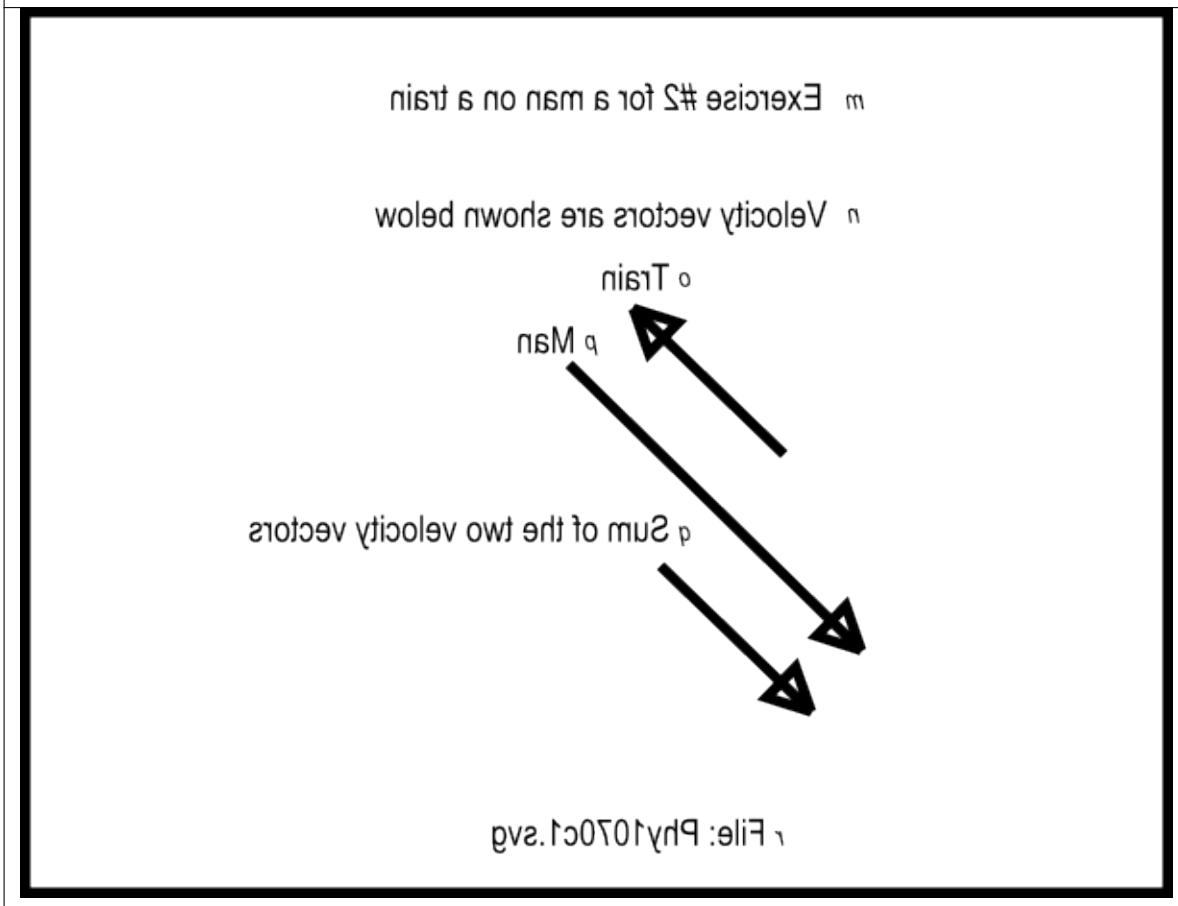


Table 11.12

Make a new html file

Please copy the code from the previous exercise into a new html file, make the changes described below, and open the html file in your browser.

- Change the value of vecTrainAng from 0 to 45.
- Change the value of vecManAng from 0 to 225.

(Because of the small differences between this script and the previous script, I won't publish a copy of this new script here.)

Screen output

The text shown in Figure 10 (p. 301) should appear in your browser window when you open the file in your browser.

Figure 10 . Screen output for Exercise #2 for a man on a train.

```
   Start Script
Velocity magnitude = 1.00 miles/hour
Velocity angle = 225.00 degrees
End Script
```

Table 11.13

Analysis of the results

The man and the train are still moving along the same straight line even though the angle is no longer 0. Therefore, the magnitudes of the two vectors still add algebraically. In this case, however, the man and the train are moving in opposite directions, so they are no longer additive.

The man has a greater velocity magnitude

The magnitude of the man's velocity in the direction of 225 degrees is greater than the magnitude of the train's velocity in the opposite direction of 45 degrees.

As you can see from Figure 10 (p. 301) , the man's velocity with reference to the ground is 1 mile per hour at an angle of 225 degrees. This means that an observer standing on the ground at a point on a line perpendicular to the train would perceive the man to be moving to the left at 1 mile per hour (assuming that the train is moving to the right).

11.4.4.3 An exercise with three vectors in a plane

Now we are going to take a look at an exercise involving three vectors in a plane, which are not in a line.

An aircraft carrier is steaming east with a uniform velocity of 2 miles per hour relative to the ground beneath the ocean. A large platform on the deck is sliding northeast with a uniform velocity of 2 miles per hour relative to the ship. A man is walking north on the platform with a uniform velocity of 2 miles per hour relative to the platform.

What is the velocity of the platform with reference to the ground below the ocean?

What is the velocity of the man with reference to the ground below the ocean?

Tactile graphics

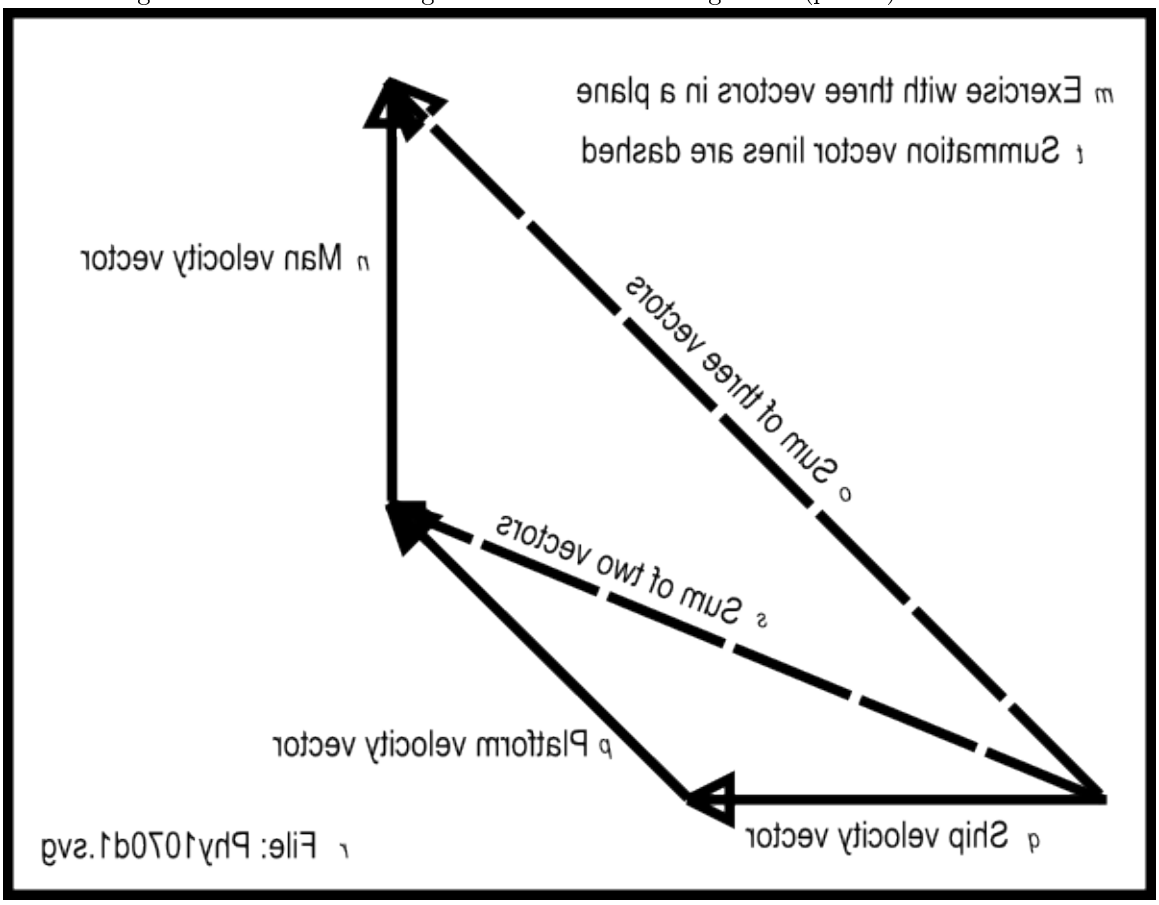
An svg file named Phy1070d1.svg is provided for the creation of a tactile velocity vector diagram for this scenario. The table of key-value pairs for this file is provided in Figure 11 .

Figure 11 . Key-value pairs for the image in Phy1070d1.svg.

```
m: Exercise with three vectors in a plane
n: Man velocity vector
o: Sum of three vectors
p: Platform velocity vector
q: Ship velocity vector
r: File: Phy1070d1.svg
s: Sum of two vectors
t: Summation vector lines are dashed
```

Table 11.14

The image contained in this file is shown in Figure 12 for the benefit of your assistant who will manually emboss the diagram. A non-mirror-image version is shown in Figure 17 (p. 323) .



m Exercise with three vectors in a plane
t Summation vector lines are dashed
n Man velocity vector
o Sum of three vectors
s Sum of two vectors
p Platform velocity vector
q Ship velocity vector
r File: Phy1070d1.svg

m Exercise with three vectors in a plane
t Summation vector lines are dashed
n Man velocity vector
o Sum of three vectors
s Sum of two vectors
p Platform velocity vector
q Ship velocity vector
r File: Phy1070d1.svg

Figure 12 . Mirror image contained in the file named Phy1070d1.svg

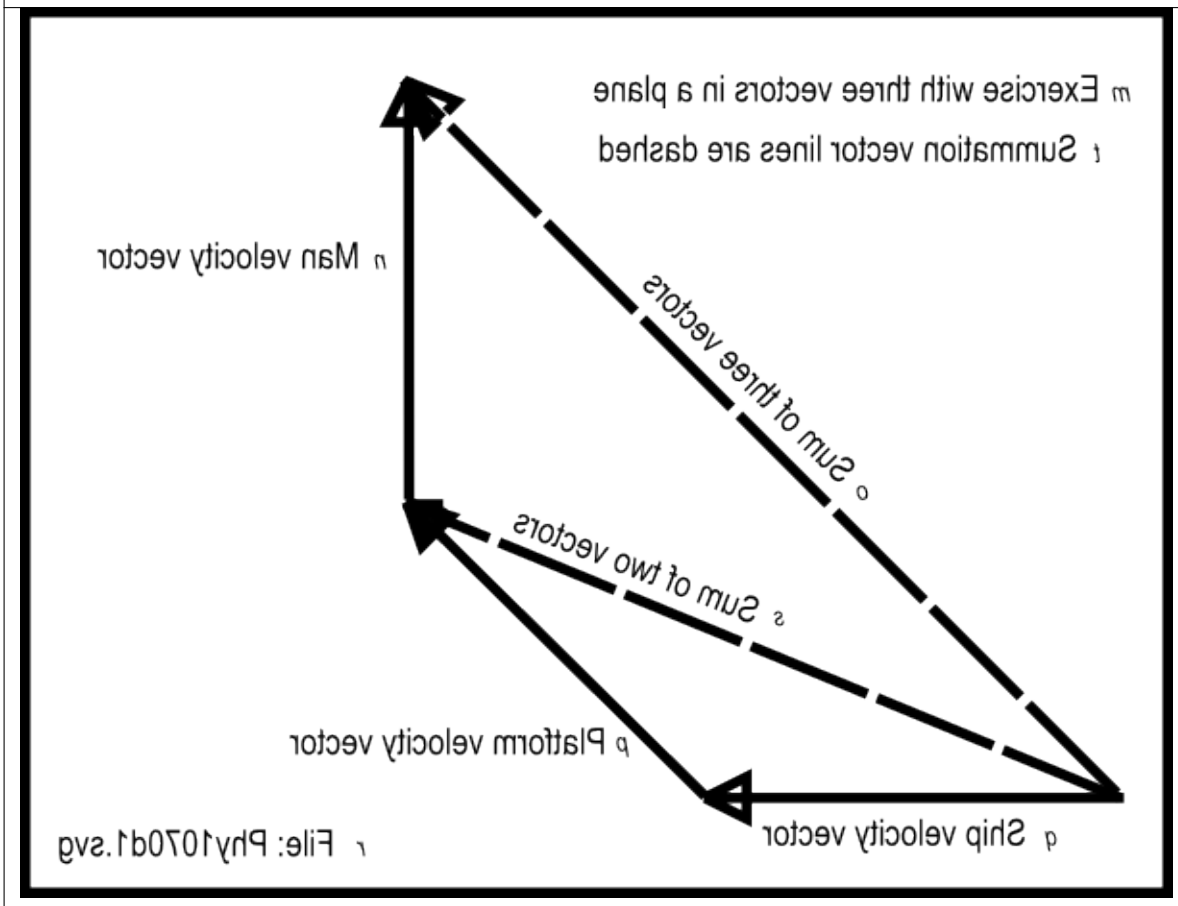


Table 11.15

Create the script

Please copy the code from Listing 4 (p. 306) into an html file and open the html file in your browser.

Listing 4 . An exercise with three vectors in a plane.

```
<!---------------- File JavaScript04.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){

  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle
//-------------------------------------------------------------//

//The purpose of this function is to add two vectors, given
// the magnitude and angle (in degrees) for each vector.
// The magnitude and angle (in degrees) of the resultant
// vector is returned in a two-element array, with the
// magnitude in the element at index 0 and the angle in the
// element at index 1. Note that this function calls the
// getAngle function, which must also be provided in the
// script. To use this function to subtract one vector from
// another, add 180 degrees to the angle for the subtrahend
// vector before passing the angle to the function. To use
// this function to add more than two vectors, add the first
// two vectors as normal, then add the output from this
// function to the third vector, etc., until all of the
// vectors have been included in the sum.
function vectorSum(vecAmag,vecAang,vecBmag,vecBang){
  var vecResult = new Array(2);
  //Compute the horizontal and vertical components
  // of each vector.
  var vecAh = vecAmag*Math.cos(vecAang*Math.PI/180);
  var vecAv = vecAmag*Math.sin(vecAang*Math.PI/180);
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 11.16

Screen output

The text shown in Figure 13 (p. 307) should appear in your browser window when the html file is opened in your browser.

Figure 13 . Screen output for Listing #4.

```
   Start Script
Platform velocity magnitude = 3.70 miles/hour
Platform velocity angle = 22.50 degrees
Man velocity magnitude = 4.83 miles/hour
Man velocity angle = 45.00 degrees
End Script
```

Table 11.17

Analysis of the code

The velocity of the platform relative to the ground is the vector sum of the ship's velocity and the velocity of the platform relative to the ship. The man's velocity is the vector sum of the platform's velocity relative to the ground and the man's velocity relative to the platform.

Listing 4 (p. 306) begins with copies of the getAngle function and the vectorSum function, which I have already explained. This discussion begins at the comment that reads "Main body of script begins here."

The main body of the script

The main body begins with the declaration and initialization of six variables whose contents represent the magnitude and the angle of the ship, the platform, and the man.

Add the first two vectors

The vectorSum function is called to add the ship's velocity vector (relative to the ground) and the platform's velocity vector (relative to the ship). The resultant vector produced by adding those two vectors is stored in the array-object variable named **platformVel** for use later. This is the platform's velocity vector relative to the ground.

Add the man's vector to the sum

Then the vectorSum function is called again to add the platform's velocity vector (relative to the ground) and the man's velocity vector (relative to the platform). Note that the two values stored in the **platformVel** array object are extracted and passed as parameters to the vectorSum function.

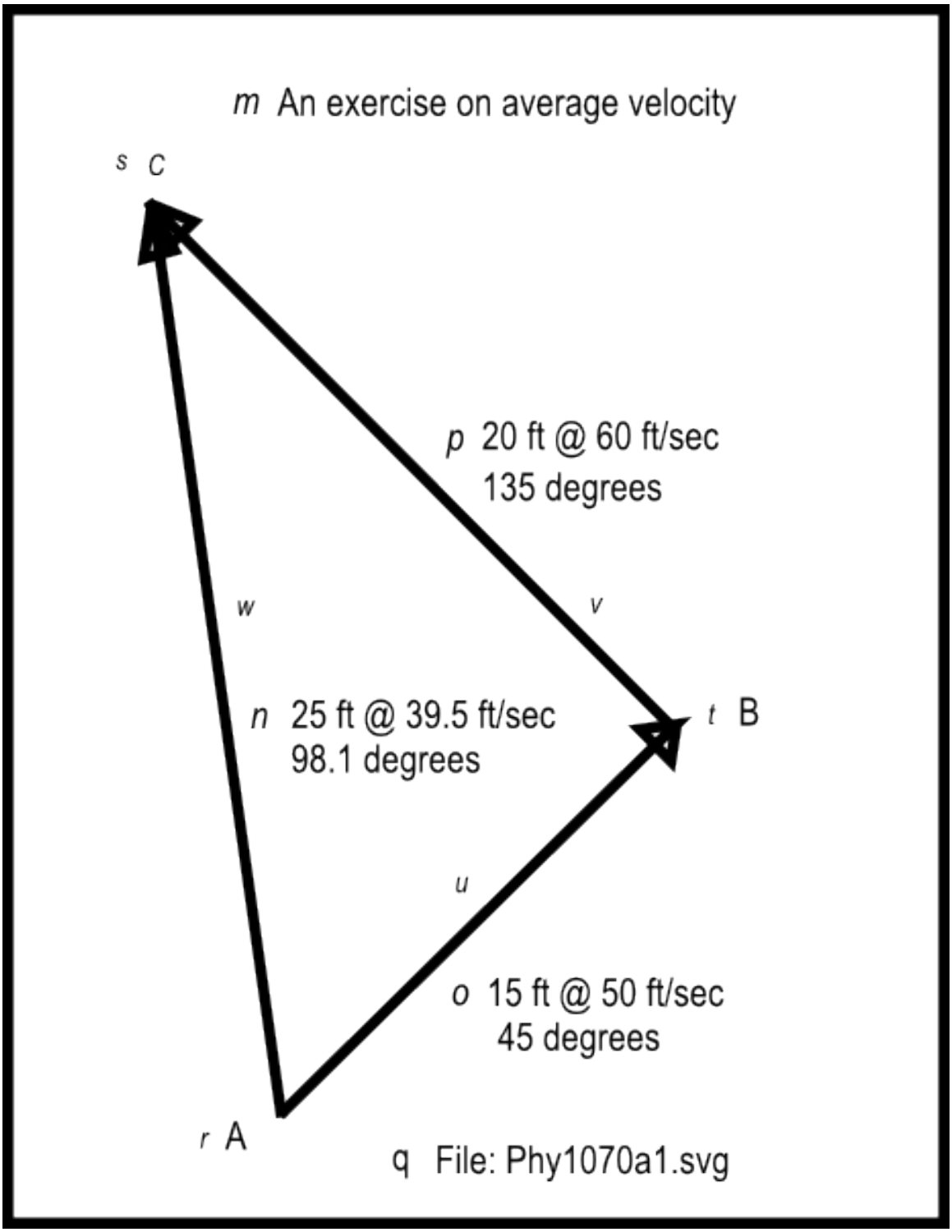
The resultant vector produced by that addition is saved in the array-object variable named **manVel** . This is the man's velocity vector relative to the ground.

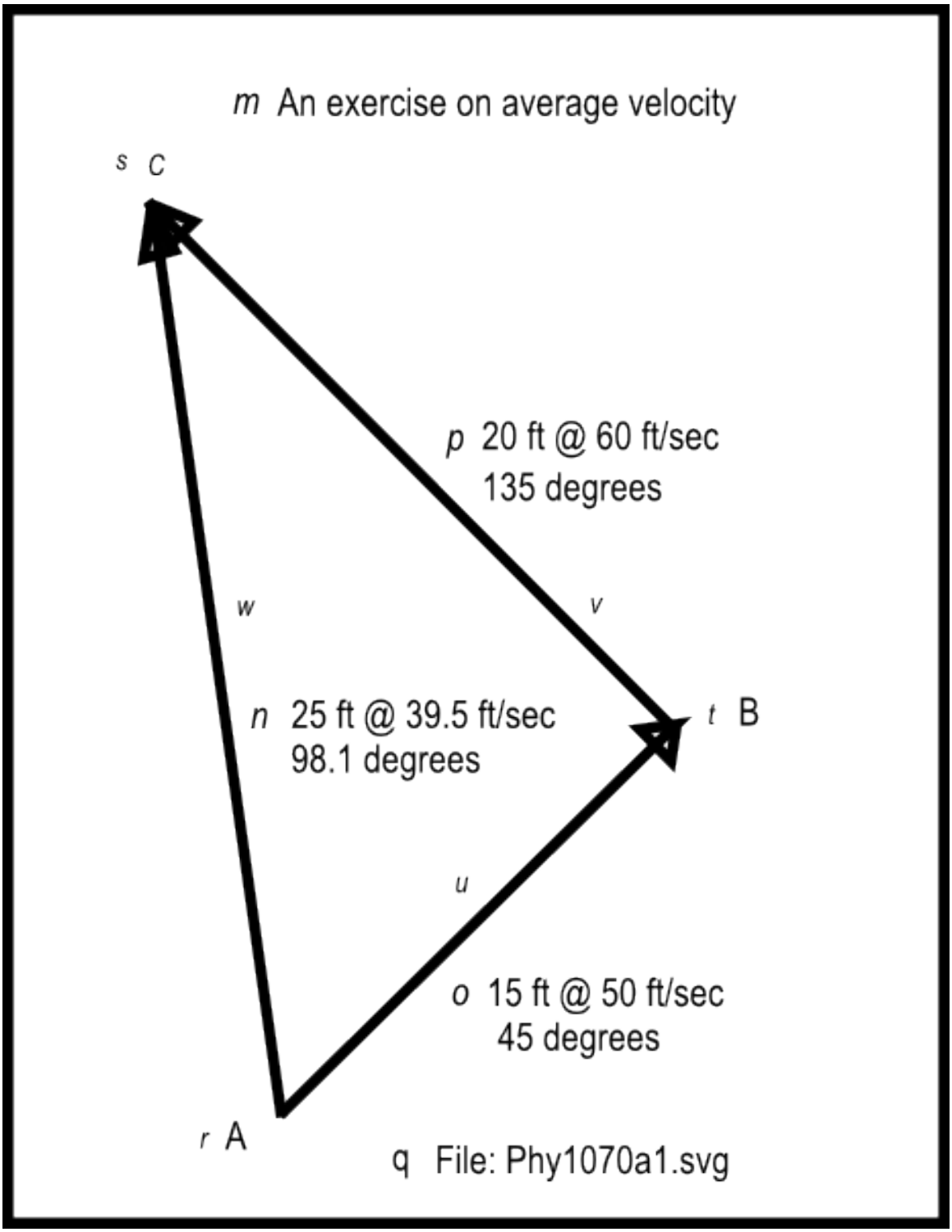
Display the results

Finally, the method named document.write is called four times in succession to extract and print the four values stored in the two array objects, producing the text output shown in Figure 13 (p. 307) .

11.4.5 Non-mirror-image graphics

Figure 14 (p. 311) through Figure 17 (p. 323) below are non-mirror-image versions of Figure 3 (p. 283) , Figure 6 (p. 291) , Figure 9 (p. 300) , and Figure 12 (p. 305) respectively.


m An exercise on average velocity
s C
p 20 ft @ 60 ft/sec
135 degrees
w
v
n 25 ft @ 39.5 ft/sec
98.1 degrees
t B
u
o 15 ft @ 50 ft/sec
45 degrees
r A
q File: Phy1070a1.svg


m An exercise on average velocity
s C
p 20 ft @ 60 ft/sec
135 degrees
w
v
n 25 ft @ 39.5 ft/sec
98.1 degrees
t B
u
o 15 ft @ 50 ft/sec
45 degrees
r A
q File: Phy1070a1.svg

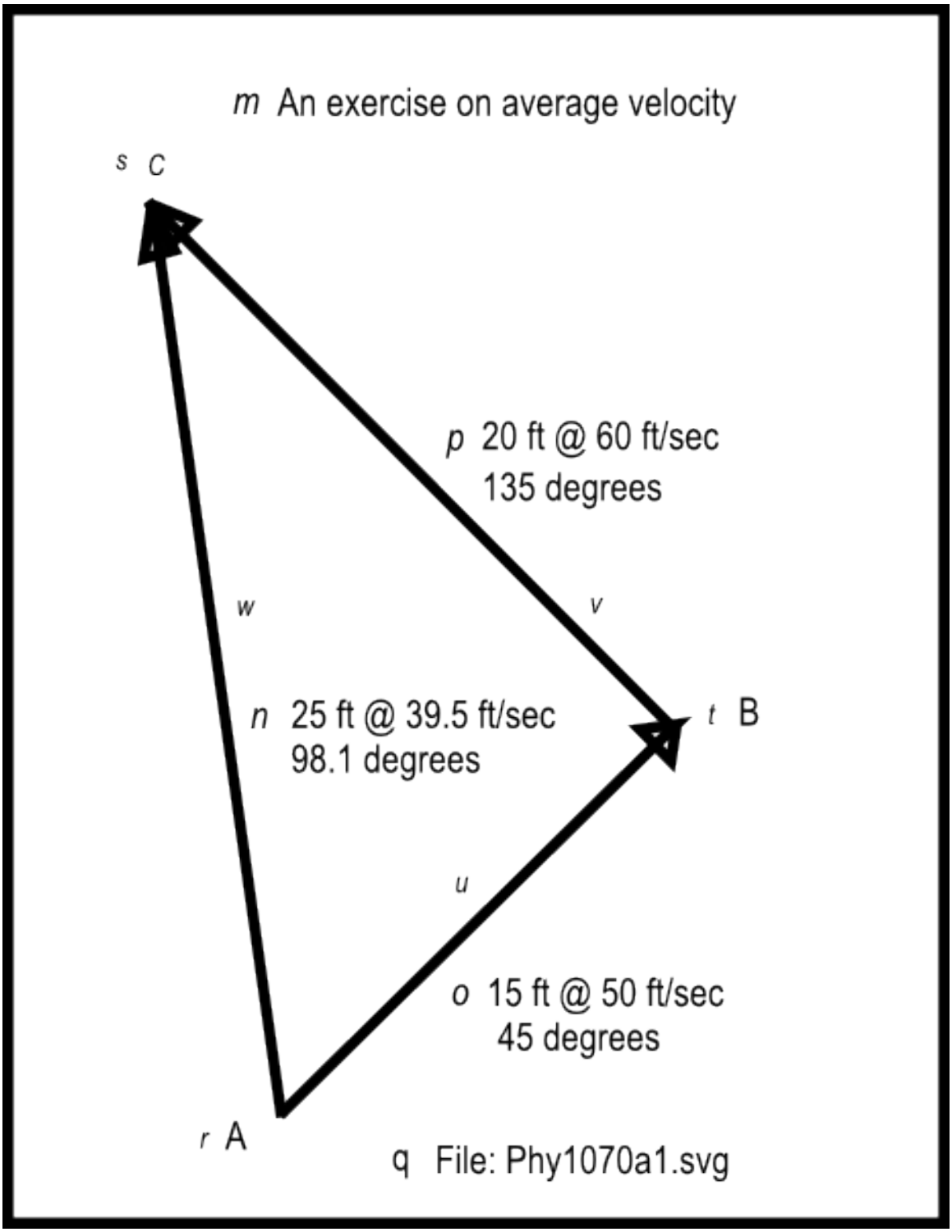

m An exercise on average velocity
s C
p 20 ft @ 60 ft/sec
135 degrees
w
v
n 25 ft @ 39.5 ft/sec
98.1 degrees
t B
u
o 15 ft @ 50 ft/sec
45 degrees
r A
q File: Phy1070a1.svg

Figure 14 . Non-mirror-image contained in the file named Phy1070a1.svg.

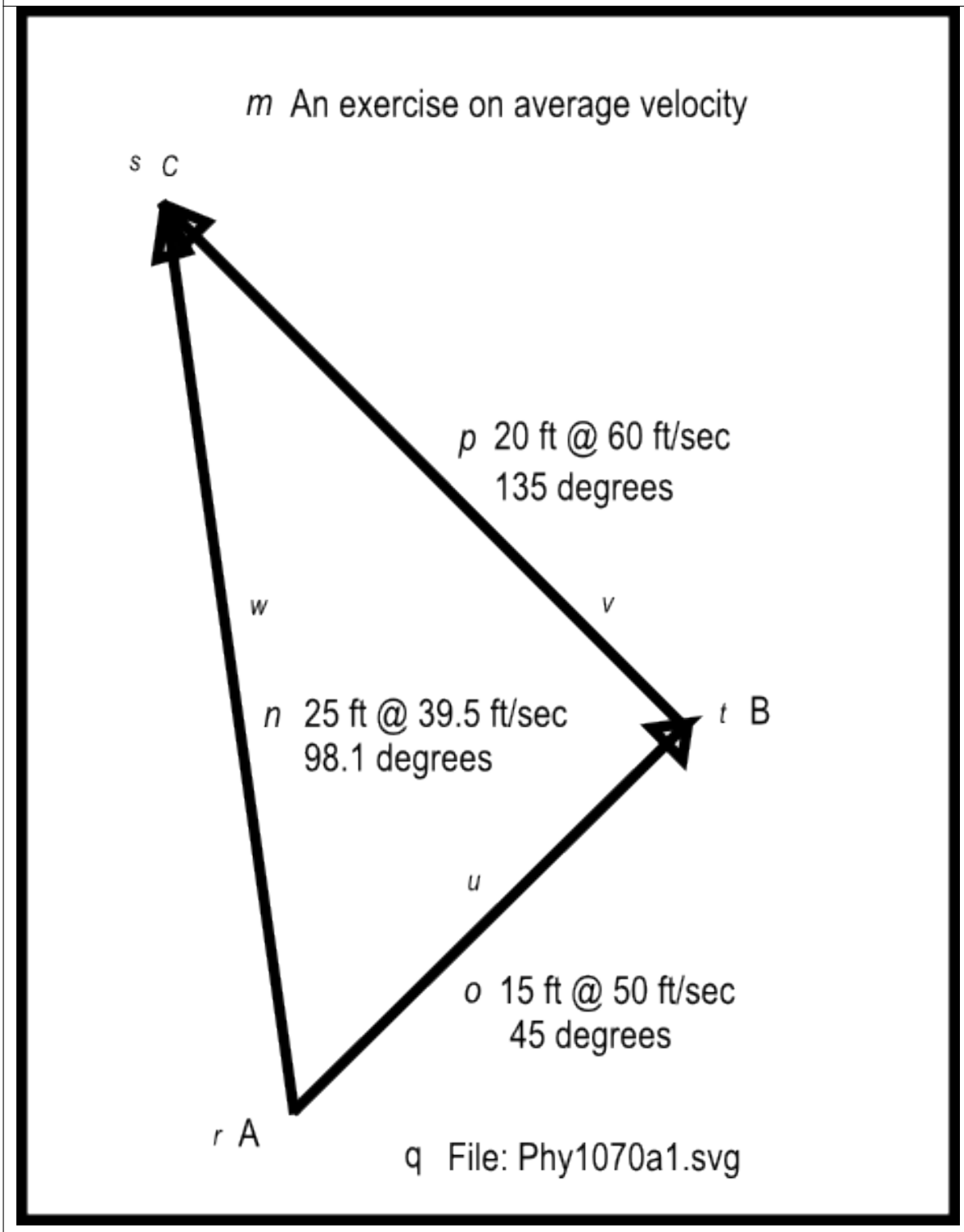
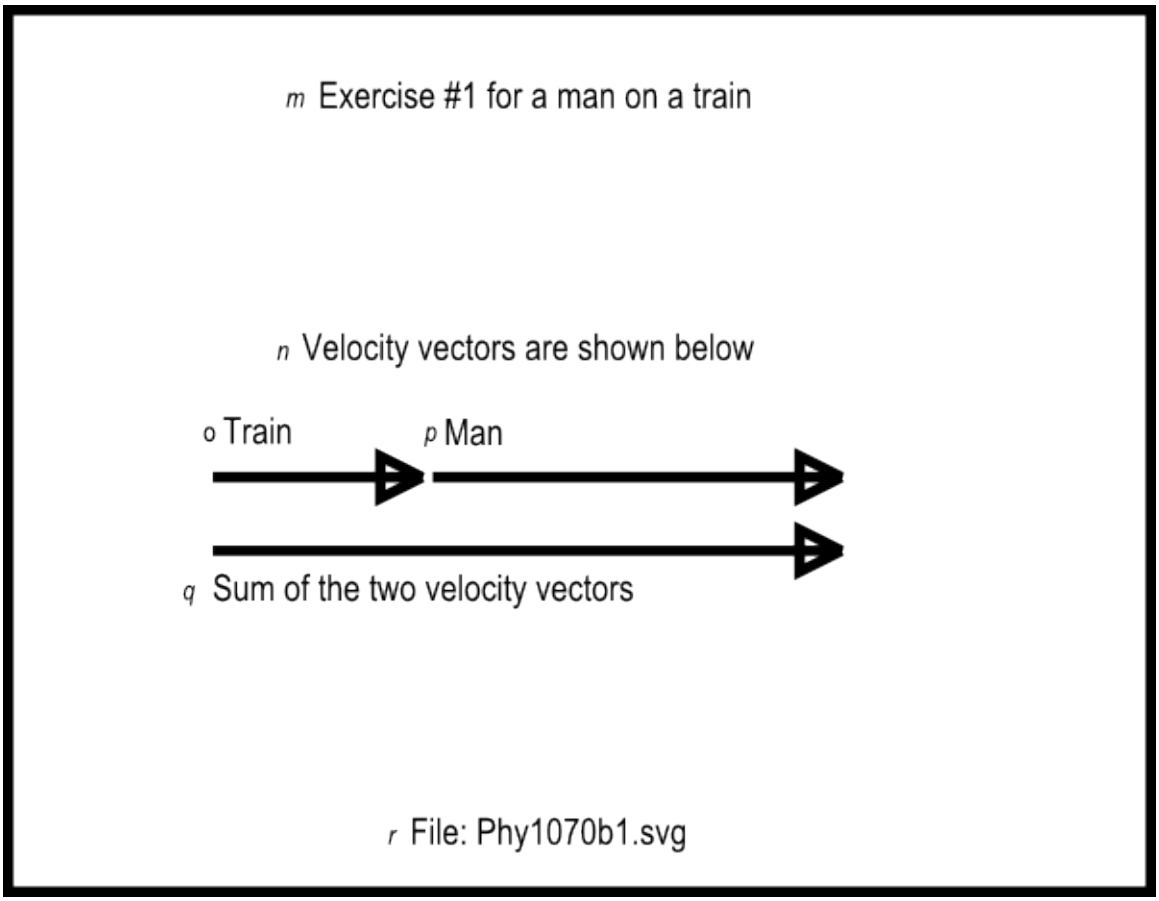
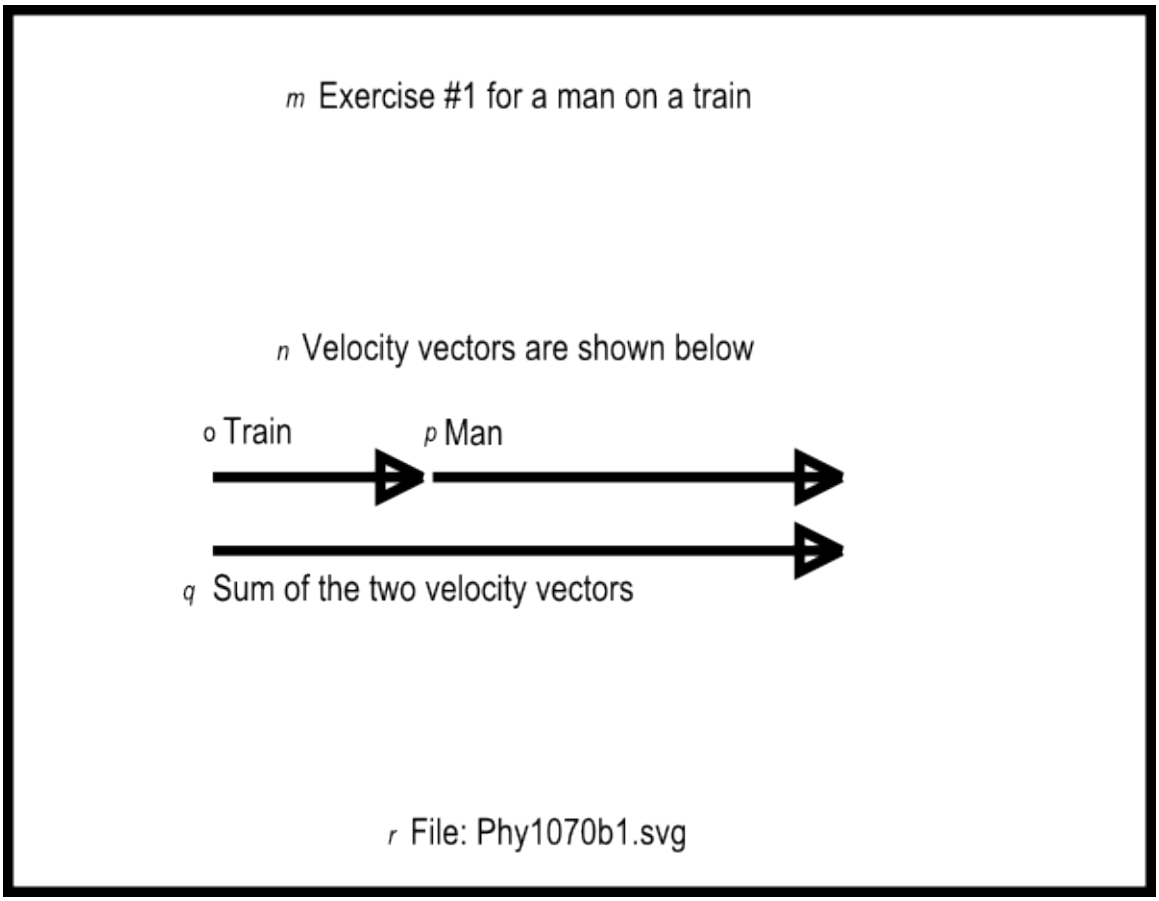


Table 11.18


m Exercise #1 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070b1.svg


m Exercise #1 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070b1.svg

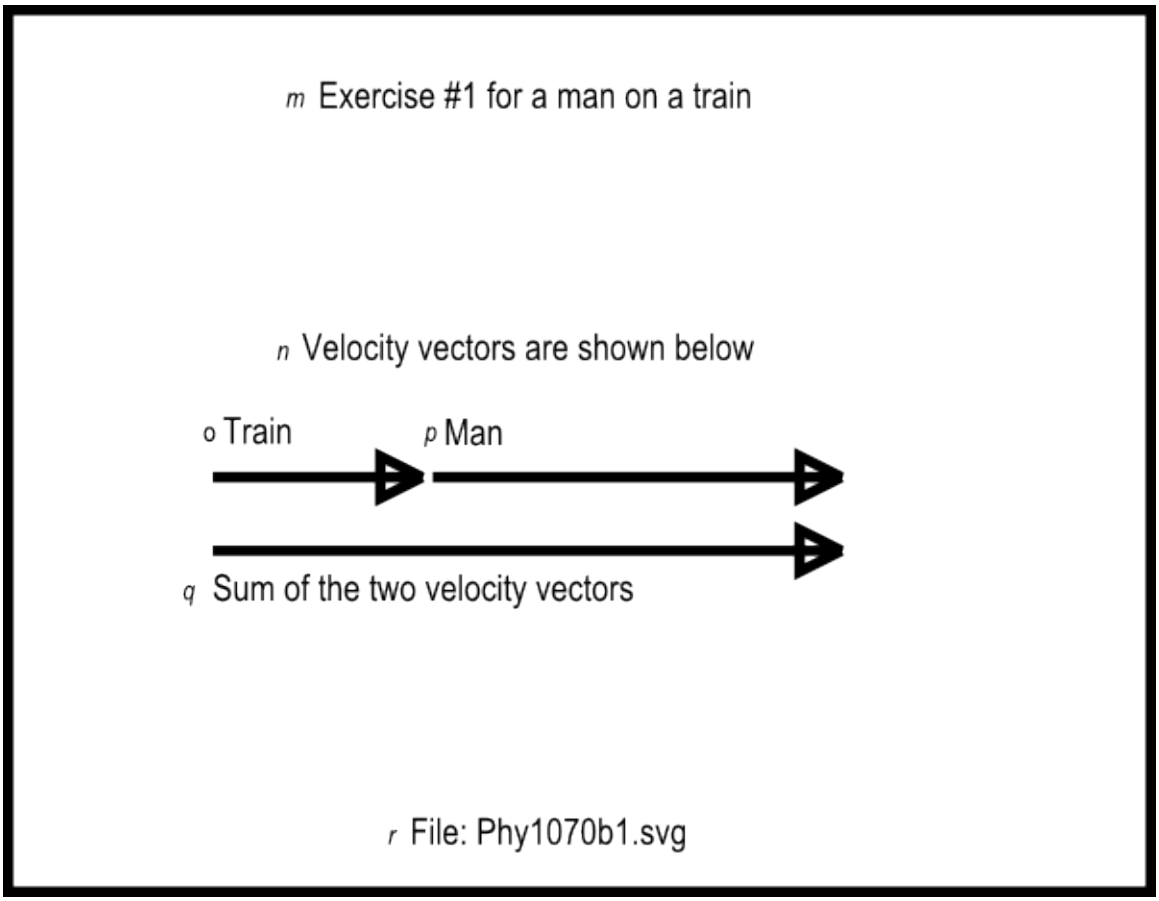

m Exercise #1 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070b1.svg

Figure 15 . Non-mirror-image contained in the file named Phy1070b1.svg.

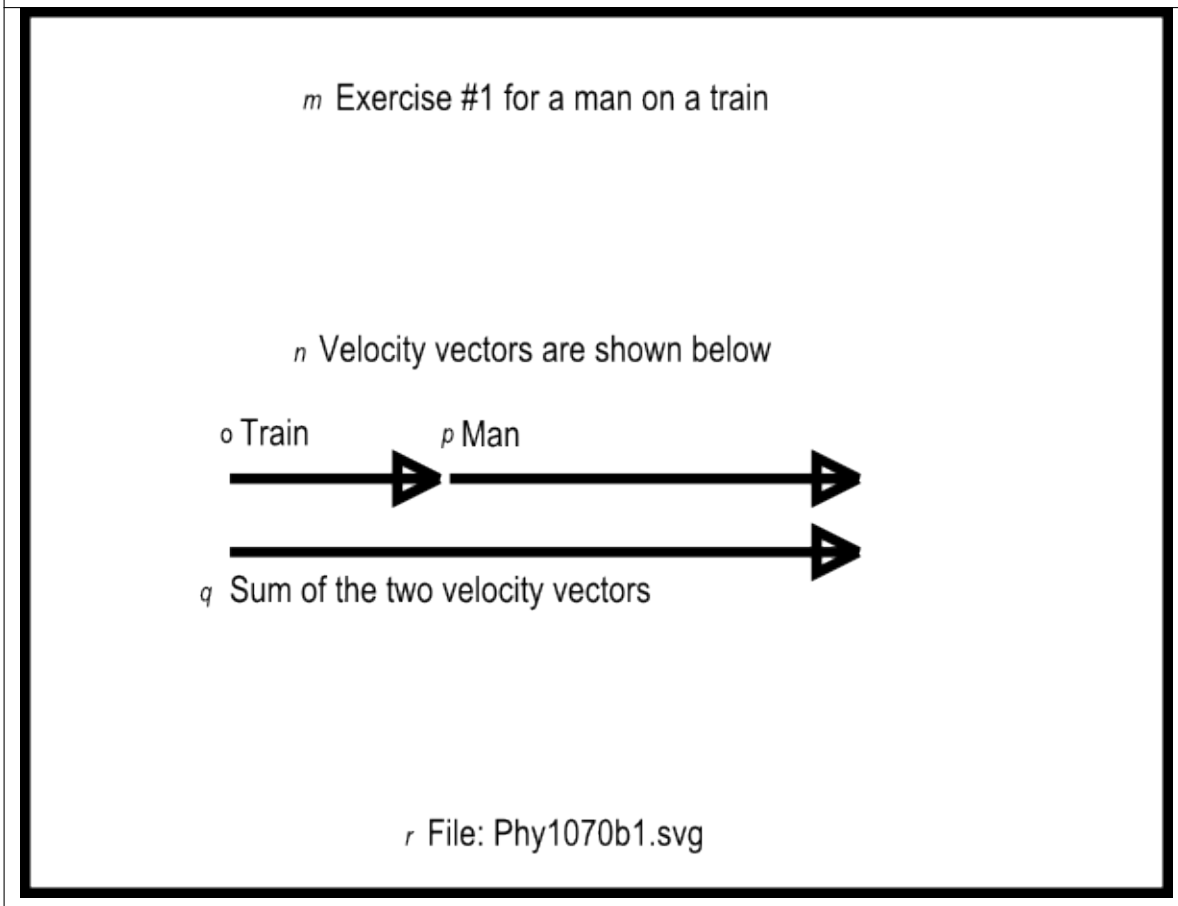
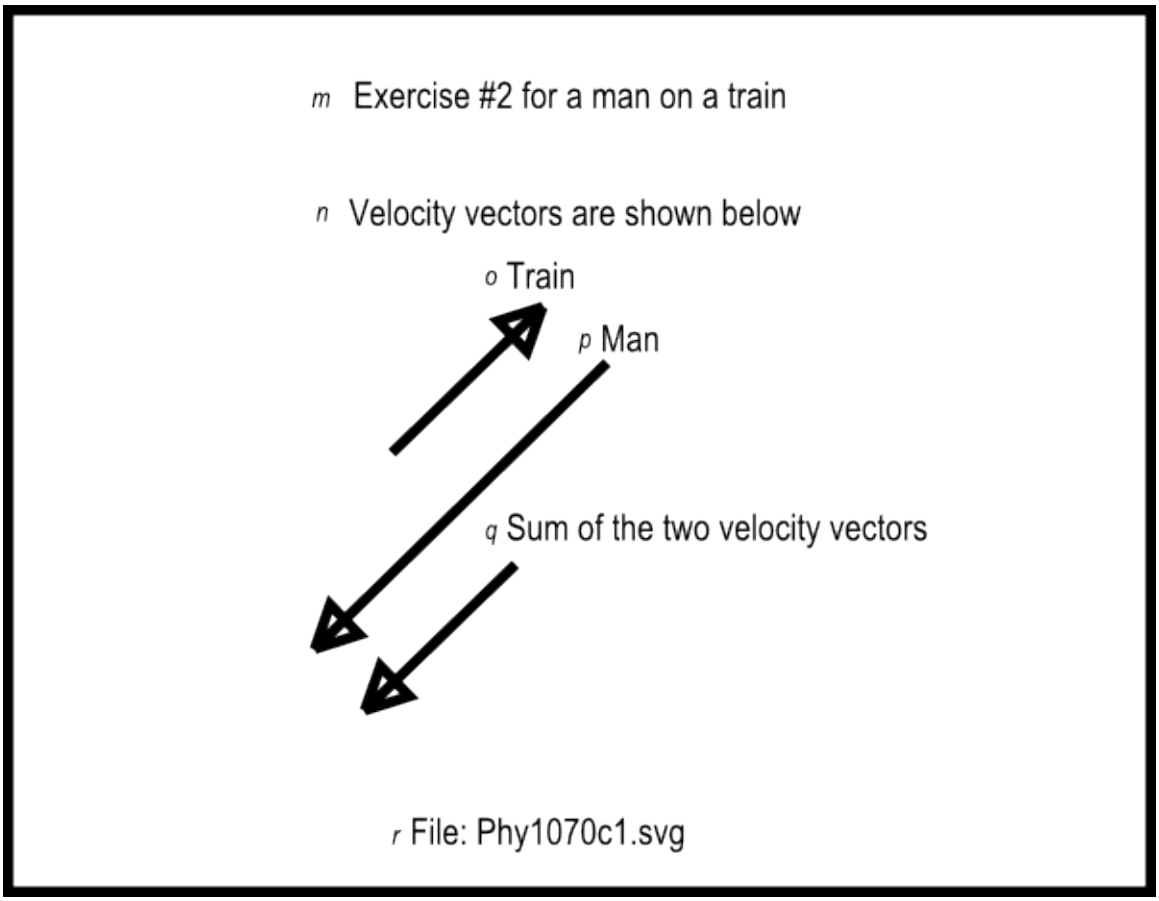
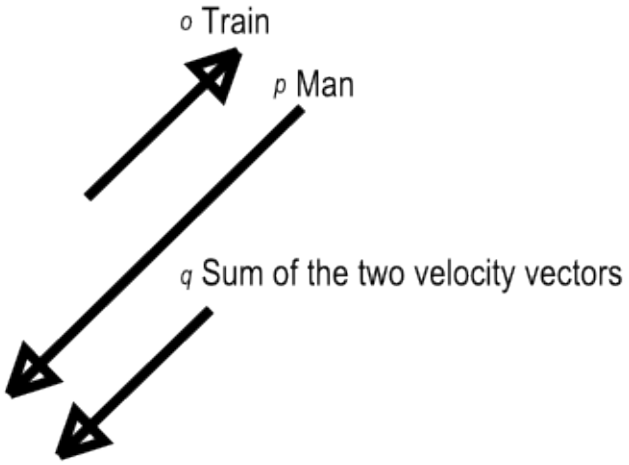


Table 11.19


m Exercise #2 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070c1.svg

m Exercise #2 for a man on a train

n Velocity vectors are shown below



r File: Phy1070c1.svg

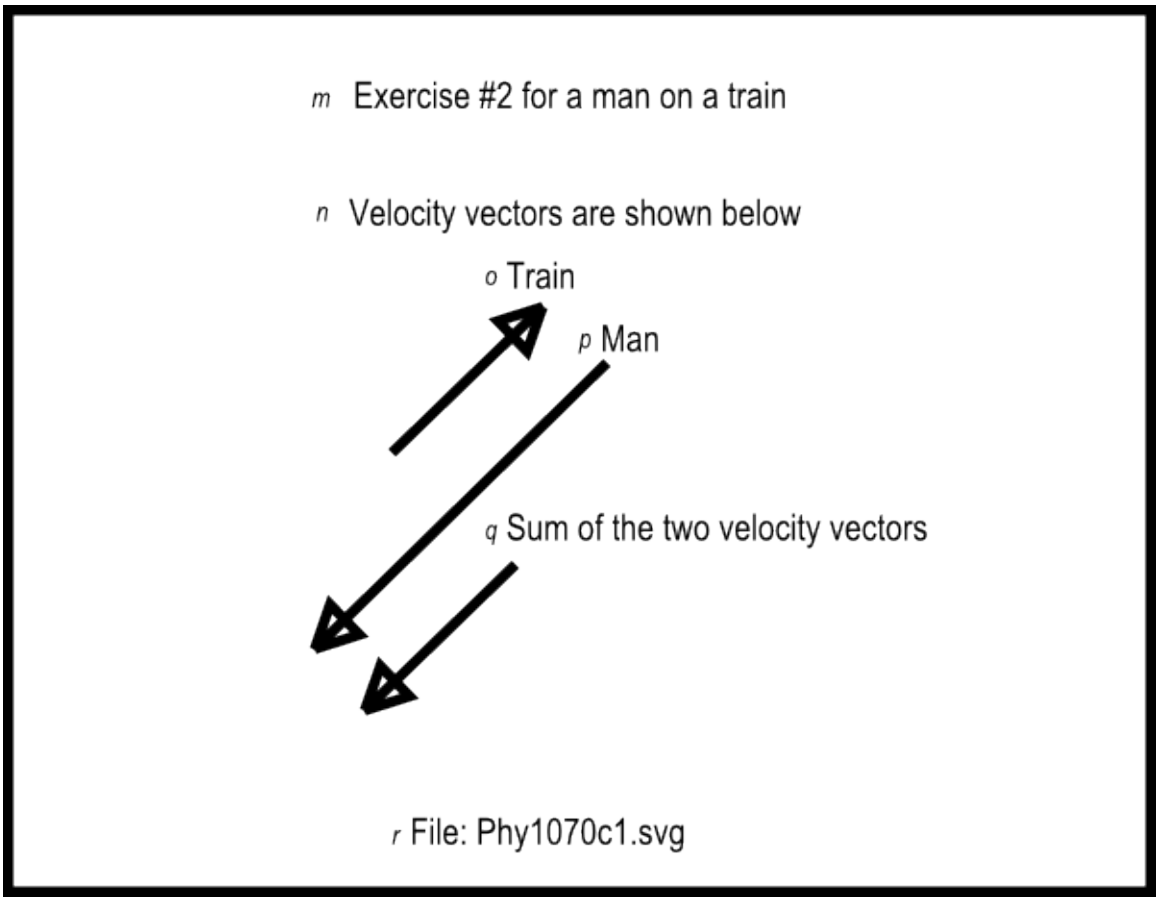

m Exercise #2 for a man on a train
n Velocity vectors are shown below
o Train
p Man
q Sum of the two velocity vectors
r File: Phy1070c1.svg

Figure 16 . Non-mirror-image contained in the file named Phy1070c1.svg.

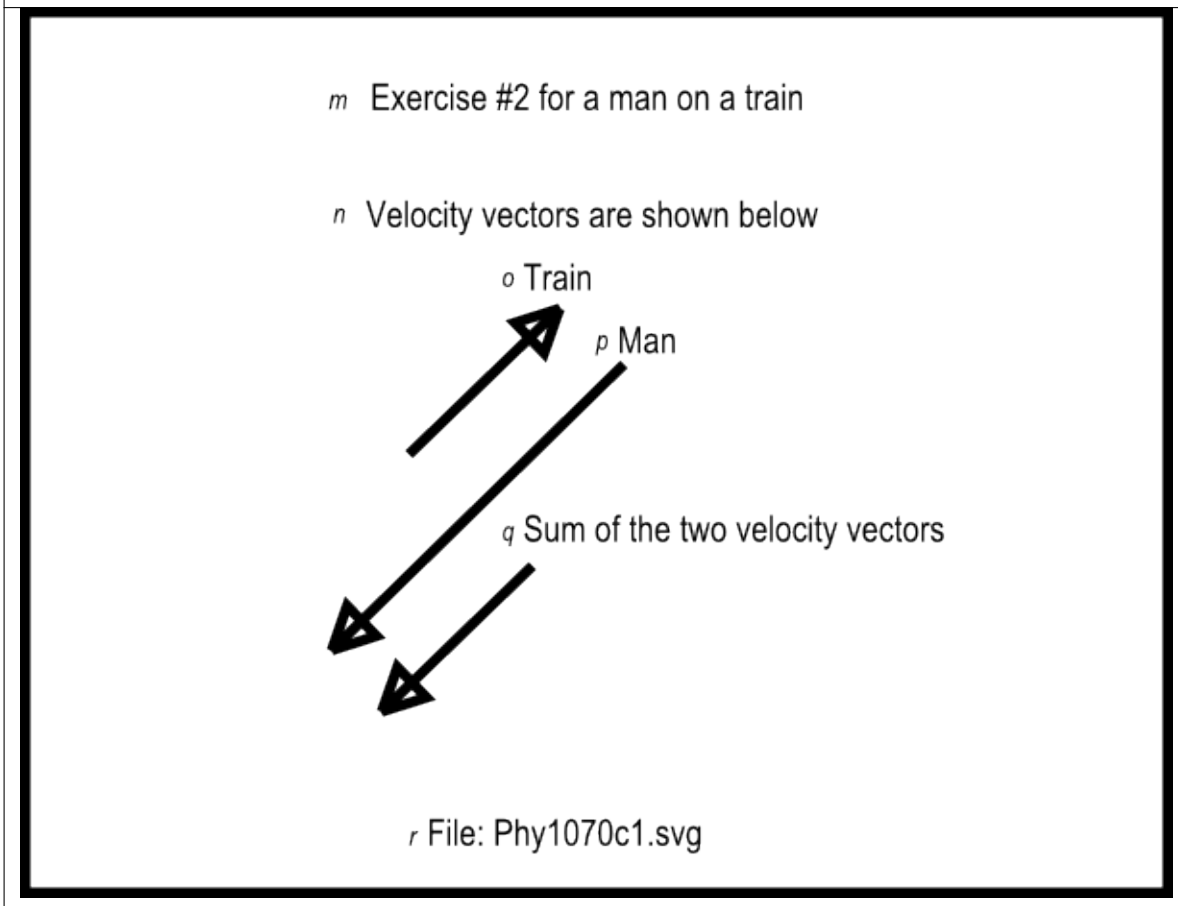
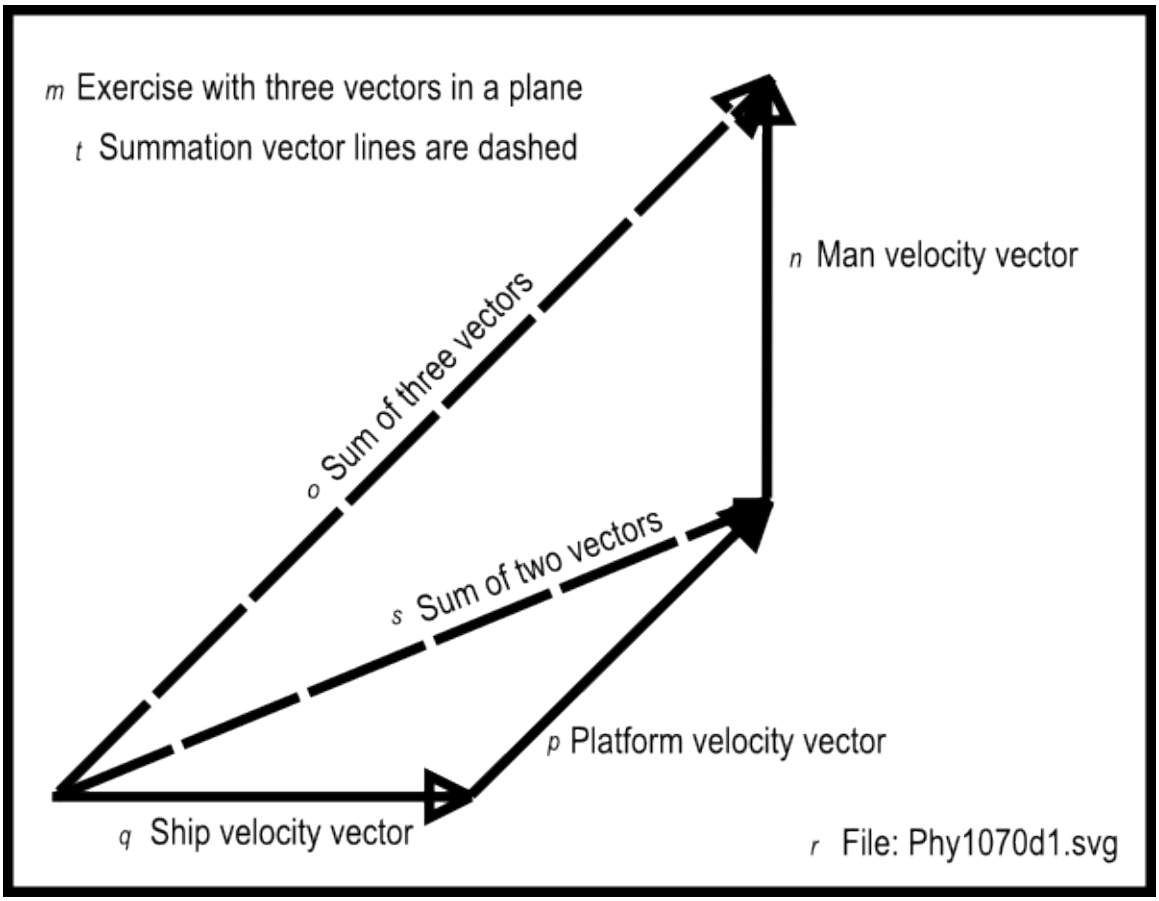
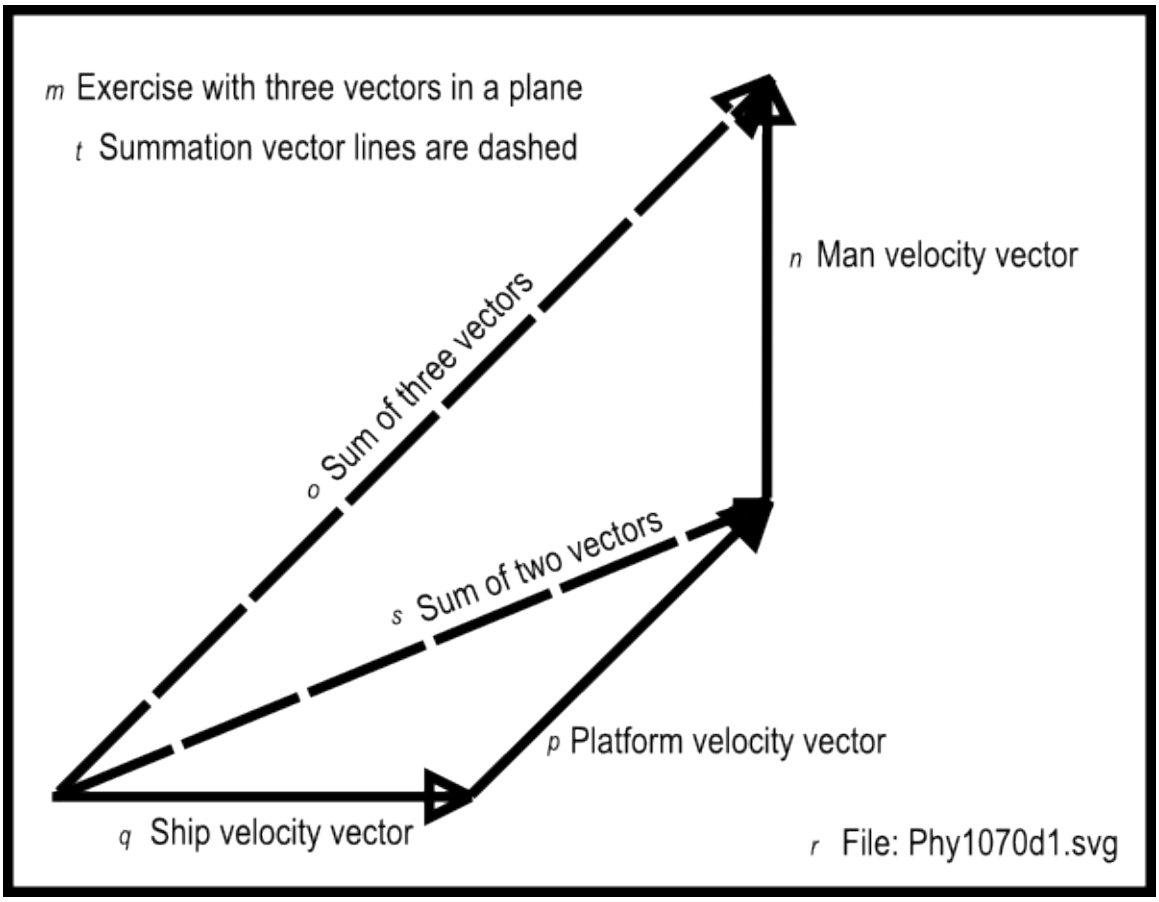


Table 11.20


m Exercise with three vectors in a plane
t Summation vector lines are dashed
n Man velocity vector
o Sum of three vectors
s Sum of two vectors
p Platform velocity vector
q Ship velocity vector
r File: Phy1070d1.svg


m Exercise with three vectors in a plane
t Summation vector lines are dashed
n Man velocity vector
o Sum of three vectors
s Sum of two vectors
p Platform velocity vector
q Ship velocity vector
r File: Phy1070d1.svg

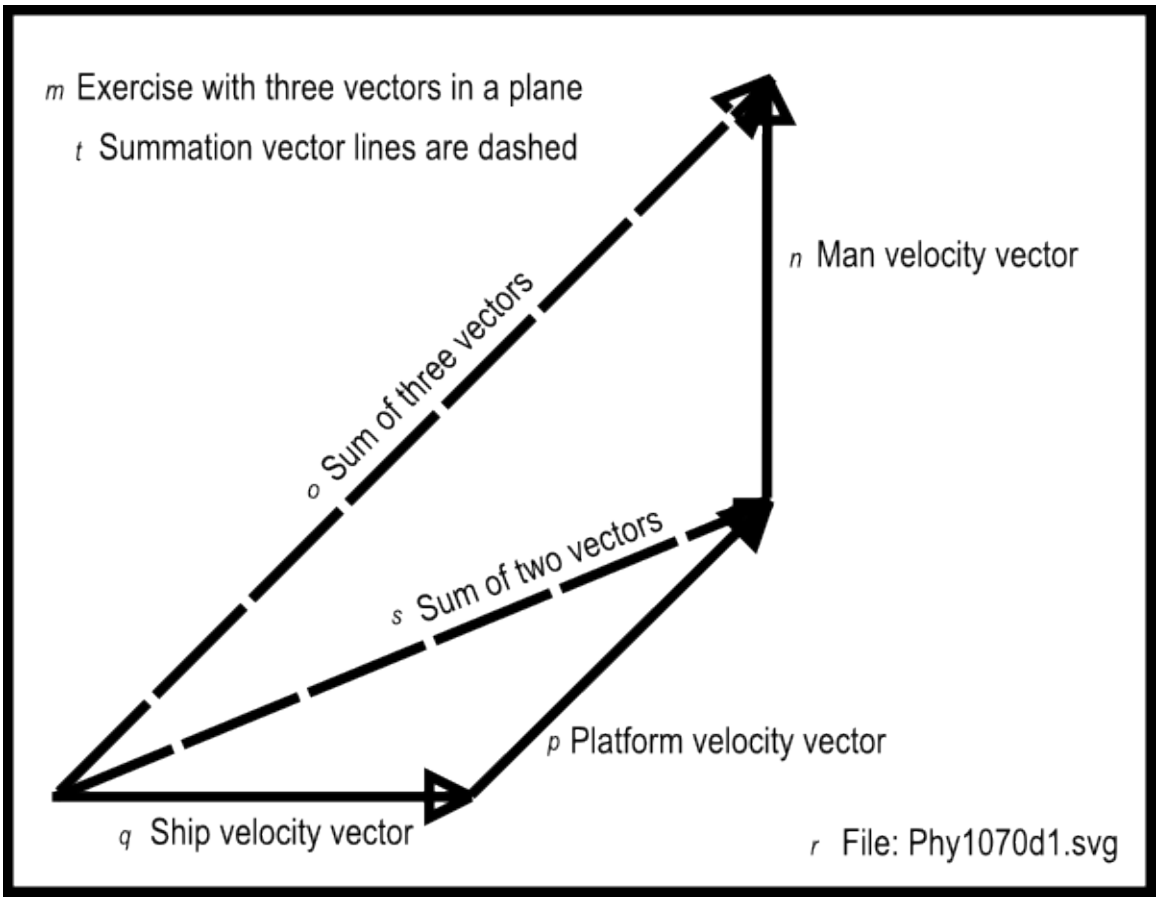

m Exercise with three vectors in a plane
t Summation vector lines are dashed
n Man velocity vector
o Sum of three vectors
s Sum of two vectors
p Platform velocity vector
q Ship velocity vector
r File: Phy1070d1.svg

Figure 17 . Non-mirror-image contained in the file named Phy1070d1.svg.

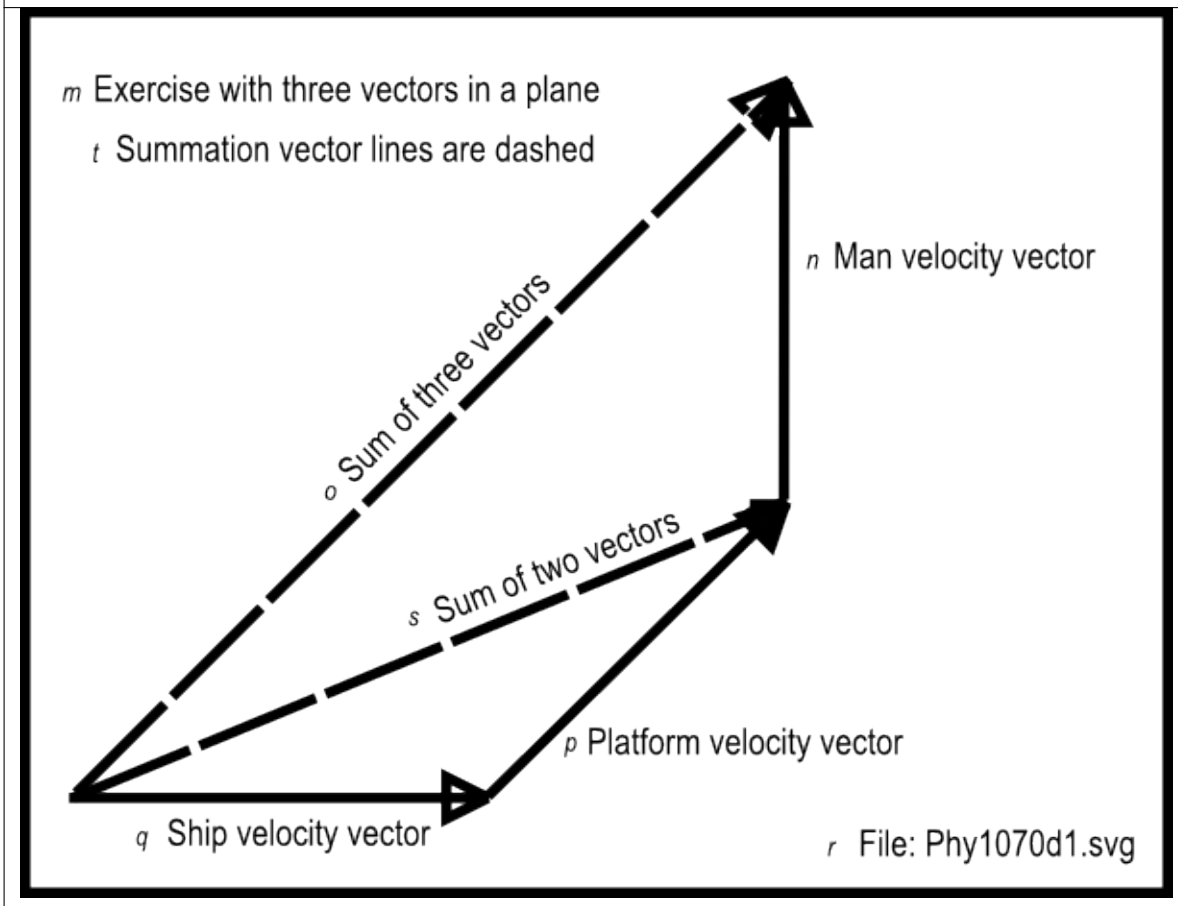


Table 11.21

11.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

11.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

11.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Motion – Uniform and Relative Velocity
- File: Phy1070.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · vector
  - · velocity
  - · uniform velocity
  - · relative velocity
  - · resultant
  - · motion
  - · displacement
  - · subtrahend
  - · array object

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 12

Motion – Variable Velocity and Acceleration[1]

12.1 Table of Contents

- Preface (p. 326)
  - General (p. 326)
  - Prerequisites (p. 326)
  - Viewing tip (p. 327)
    - Figures (p. 327)
    - Listings (p. 327)
  - Supplemental material (p. 327)
- General background information (p. 327)
  - Variable velocity (p. 327)
  - Acceleration (p. 328)
  - The acceleration of gravity (p. 329)
- Discussion and sample code (p. 330)
  - Creation of tactile graphics (p. 330)
  - Variable velocity exercise #1 (p. 331)
  - Variable velocity exercise #2 (p. 345)
  - Acceleration of gravity exercise #1 (p. 348)
  - Acceleration of gravity exercise #2 (p. 352)
  - Acceleration of gravity exercise #3 (p. 365)
  - Other useful equations (p. 369)
    - Exercise to find the velocity (p. 369)
    - Exercise to find the height (p. 371)
- Run the scripts (p. 372)
- Resources (p. 372)
- Miscellaneous (p. 372)

[1] This content is available online at <http://cnx.org/content/m38074/1.4/>.

12.2 Preface

12.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The purpose of this module is to explain variable velocity and acceleration in a format that is accessible to blind students.

12.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

12.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

12.2.3.1 Figures

- Figure 1 (p. 335) . Mirror image contained in the file named Phy1080a1.svg.
- Figure 2 (p. 339) . Non-mirror-image contained in the file named Phy1080a1.svg.
- Figure 3 (p. 340) . Key-value pairs for the image in Phy1080a1.svg.
- Figure 4 (p. 343) . Screen output for Listing #1.
- Figure 5 (p. 344) . Displacement versus time for first five time intervals.
- Figure 6 (p. 347) . Screen output for Listing #2.
- Figure 7 (p. 351) . Screen output for Listing #3.
- Figure 8 (p. 355) . Screen output for Listing #4 at 45 degrees.
- Figure 9 (p. 359) . Mirror image contained in the file named Phy1080b1.svg.
- Figure 10 (p. 363) . Non-mirror-image contained in the file named Phy1080b1.svg.
- Figure 11 (p. 363) . Key-value pairs for the image in Phy1080b1.svg.
- Figure 12 (p. 364) . Screen output for Listing #4 at 60 degrees.
- Figure 13 (p. 368) . Screen output for Listing #5.
- Figure 14 (p. 370) . Screen output for Listing #6.
- Figure 15 (p. 372) . Screen output for Listing #7.

12.2.3.2 Listings

- Listing 1 (p. 341) . Variable velocity exercise #1.
- Listing 2 (p. 346) . Variable velocity exercise #2.
- Listing 3 (p. 350) . Acceleration of gravity exercise #1.
- Listing 4 (p. 354) . Acceleration of gravity exercise #2.
- Listing 5 (p. 367) . Acceleration of gravity exercise #3.
- Listing 6 (p. 370) . Exercise to find the velocity.
- Listing 7 (p. 371) . Exercise to find the height.

12.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

12.3 General background information

I will provide an introduction to variable velocity, acceleration, and the acceleration of gravity in this section. I will provide exercises on those topics in the next section.

12.3.1 Variable velocity

What is velocity?

To review, you learned in an earlier module that velocity is the rate of change of position. Since displacement is a change in position, velocity is also the rate of displacement.

Uniform versus variable velocity

[14] http://www.dickbaldwin.com/toc.htm

Previous modules have dealt exclusively with uniform velocity. However, many situations in life involve variable velocity and acceleration. For example, the simple act of stopping an automobile at a traffic light and then resuming the trip once the light turns green involves variable velocity and acceleration.

Three equations

I will explain both of these topics in this module using three well-known physics equations that deal with the displacement of a projectile in a vacuum **under the influence of gravity** .

The equation that we will spend the most time on is

h = v0*t + 0.5*g*t^2

where

- h is the distance of the projectile above the surface of the earth
- v0 is the initial velocity of the projectile
- t is time in seconds
- g is the acceleration of gravity, approximately 9.8 meters per second squared, or approximately 32.2 feet per second squared at the surface of the earth. (We will also do an exercise involving the acceleration of gravity on the moon.)

(I will provide the other two equations later (p. 369) .)

12.3.2 Acceleration

Everyone is familiar with the acceleration that occurs when a motor vehicle speeds up or slows down. When the vehicle speeds up very rapidly, the positive acceleration forces us against the back of the seat. (This involves the relationship among force, mass, and acceleration, which will be the subject of a future module.)

If the vehicle slows down very rapidly or stops suddenly, the negative acceleration may cause us to crash into the windshield, the dashboard, or a deployed airbag.

The accelerator pedal

A common name for the pedal that causes gasoline to be fed to the engine is often called the accelerator pedal because it causes the vehicle to speed up. (However, I have never heard anyone refer to the pedal that causes the vehicle to slow down as the deceleration pedal. Instead, it is commonly called the brake pedal.)

Definitions

Displacement is a change in position.

Velocity is the rate of change of position or the rate of displacement.

Acceleration is the rate of change of velocity.

Jerk is the rate of change of acceleration (not covered in this module).

According to this author [15] , there is no universally accepted name for the rate of change of jerk.

The algebraic sign of acceleration

When the velocity of a moving object increases, that is viewed as positive acceleration. When the velocity of the object decreases, that is viewed as negative acceleration.

Uniform or variable acceleration

Acceleration may be uniform or variable. It is uniform only if equal changes in velocity occur in equal intervals of time.

A vector quantity

Acceleration has both direction and magnitude. Therefore, acceleration is a vector quantity.

The units for acceleration

The above definition (p. 328) for acceleration leads to some interesting units for acceleration. For example, consider a situation in which the velocity of an object changes by 5 feet/second in a one-second time interval. Writing this as an algebraic expression gives us

(5 feet/second)/second

Multiplying the numerator and the denominator of the fraction by 1/second gives us

5 feet/(second*second)

[15] http://math.ucr.edu/home/baez/physics/General/jerk.html

This is often written as
5 feet/second^2
which is pronounced five feet per second squared.

12.3.3 The acceleration of gravity

The exercises in the remainder of this module are based on the following two assumptions:

- For practical purposes, the effect of the acceleration of gravity is the same regardless of the height of an object above the surface of the earth, provided that the distance above the surface of the earth is small relative to the radius of the earth.
- In the absence of an atmosphere, all objects fall toward the earth with the same acceleration regardless of their masses.

Let's examine the validity of these two assumptions.

Newton's law of universal gravitation

This law [16] states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. (Separately it has been shown that large spherically symmetrical masses attract and are attracted as if all their mass were concentrated at their centers.)

Effect of gravity versus altitude

An object at the surface of the earth is approximately 3963 miles from the center of the earth. An object that is six miles above the surface of the earth (typical flying altitude for a passenger plane) is approximately 3969 miles from the center of the earth.

Therefore, (if I didn't make an arithmetic error) the gravitational attraction between the earth and that object changes by less than 0.5 percent when the object is transported from the surface of the earth to a position that is six miles above the surface of the earth.

For practical purposes, therefore, we can assume that the acceleration of gravity is constant up to at least 32000 feet above the surface of the earth.

Acceleration is independent of mass

Newton's law, as applied to the gravitational attraction of the earth, expressed in algebraic terms, looks something like **the following** :

f =E*m/d^2
where

- f is the attractive force between the earth and another object
- E is the mass of the earth
- m is the mass of the other object
- d is the distance between the center of mass of the earth and the center of mass of the other object

Force equals mass times acceleration

In a future module, we will learn that an object that is free to move will move and will accelerate when subjected to a force. The acceleration of the object will be proportional to the force and inversely proportional to its mass. In other words,

a = f/m
where

- a is the acceleration in units such as meters/sec^2
- f is force in units such as kilograms*meters/sec^2 (newtons)
- m is mass in units such as kilograms

[16] http://en.wikipedia.org/wiki/Newton's_law_of_universal_gravitation

By multiplying both sides of the equation by m, we get a more **common presentation** of this relationship, which is

f = m*a

where the symbols mean the same as listed above (p. 329) .

A ratio of two different forces of gravity

Now let's use the equation from above (p. 329) to form a ratio of the forces exerted on two different masses by the earth, assuming that both masses are the same distance from the center of the earth.

f1/f2 =(E*m1/d^2)/(E*m2/d^2)

Cancel like terms to simplify

If we cancel like terms from the numerator and denominator of the expression on the right, we can simplify the ratio to

f1/f2 = m1/m2

Replacing the forces on the left by the expression from above (p. 330) , we get

m1*a1/m2*a2 = m1/m2

Multiplying both sides by m2/m1 we get

a1/a2 = 1

or

a1 = a2

This shows that the acceleration resulting from the gravitational force exerted on two objects that are equally distant from the center of mass of the earth is the same regardless of the differences in mass of the two objects.

12.4 Discussion and sample code

I will present and explain several different scenarios based on the above assumptions in this section.

12.4.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [17] explained how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1080.zip [18] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [19] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[18] http://cnx.org/content/m38074/latest/Phy1080.zip
[19] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

12.4.2 Variable velocity exercise #1

An archer that is six feet tall shoots an arrow directly upward with a velocity of 100 feet per second. Assume the arrow is at a height of 6 feet when it leaves the bow. Also ignore the effects of air resistance.

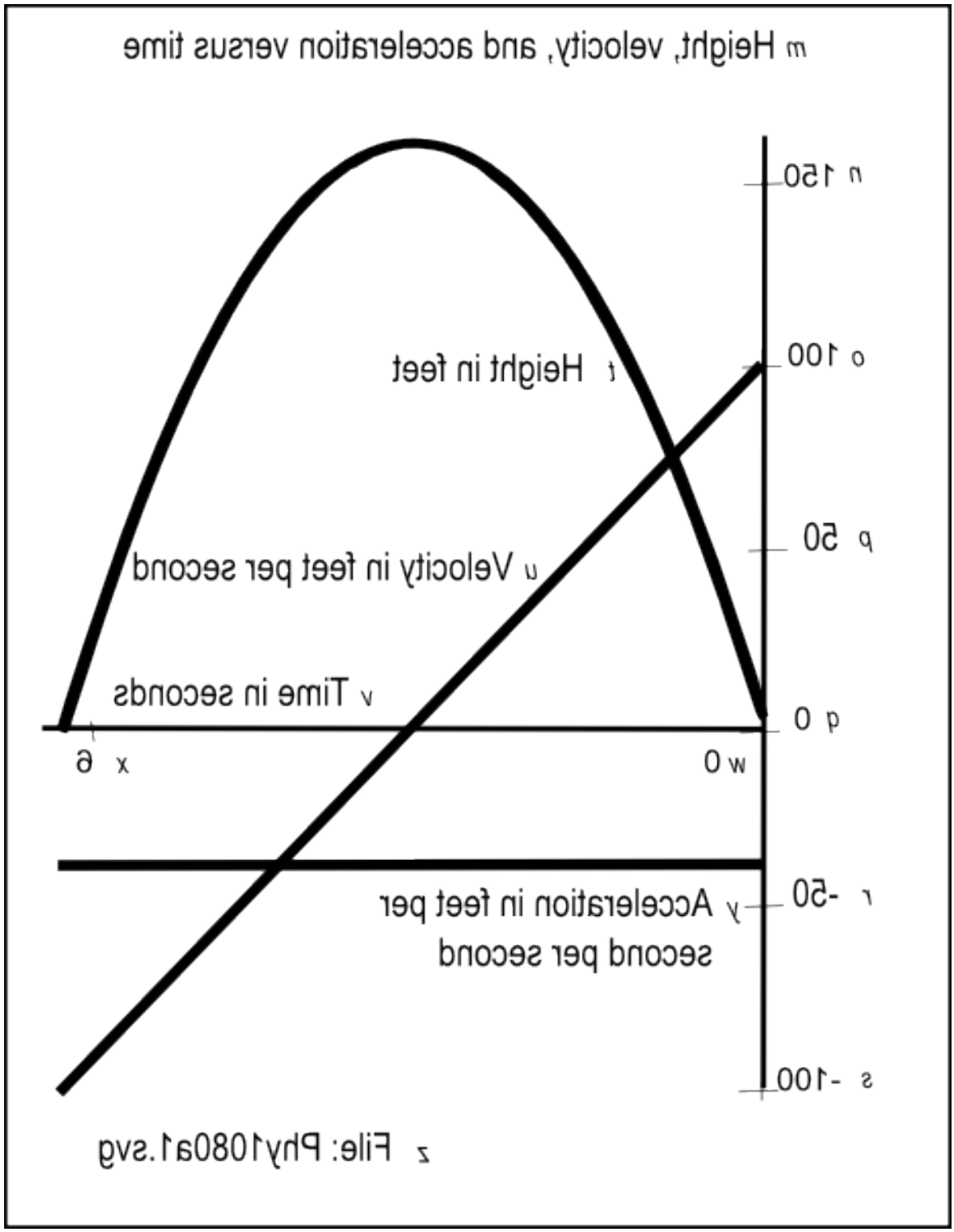
Plot height versus time

Use your graph board and about 30 pushpins to plot the height of the arrow on the vertical axis versus time on the horizontal axis. Because of the large number of points, you shouldn't need to connect the pins to get a good feel for the shape of the graph. Plot one point each 0.25 seconds. Allow for a maximum time value of about 7 seconds and a maximum height value of about 165 feet.

Tactile graphics

The svg file named Phy1080a1 contains a graph that serves for this exercise plus the next couple of exercises. This graph plots the height of the arrow, the velocity of the arrow, and the acceleration of the arrow versus time from the time that it is released until it strikes the ground approximately six seconds later.

The image contained in this file is shown in Figure 1 (p. 335) for the benefit of your assistant who will manually emboss the diagram. Note that as usual, this is a mirror image of the image that is to be presented to the student after embossing. A non-mirror-image version is shown in Figure 2 (p. 339) .


Height, velocity, and acceleration versus time
150
100
50
0
-50
-100
Height in feet
Velocity in feet per second
Time in seconds
Acceleration in feet per second per second
0
6
File: Phy1080a1.svg

Height, velocity, and acceleration versus time
150
100
Height in feet
50
Velocity in feet per second
Time in seconds
0
0
6
-50
Acceleration in feet per
second per second
-100
File: Phy1080a1.svg

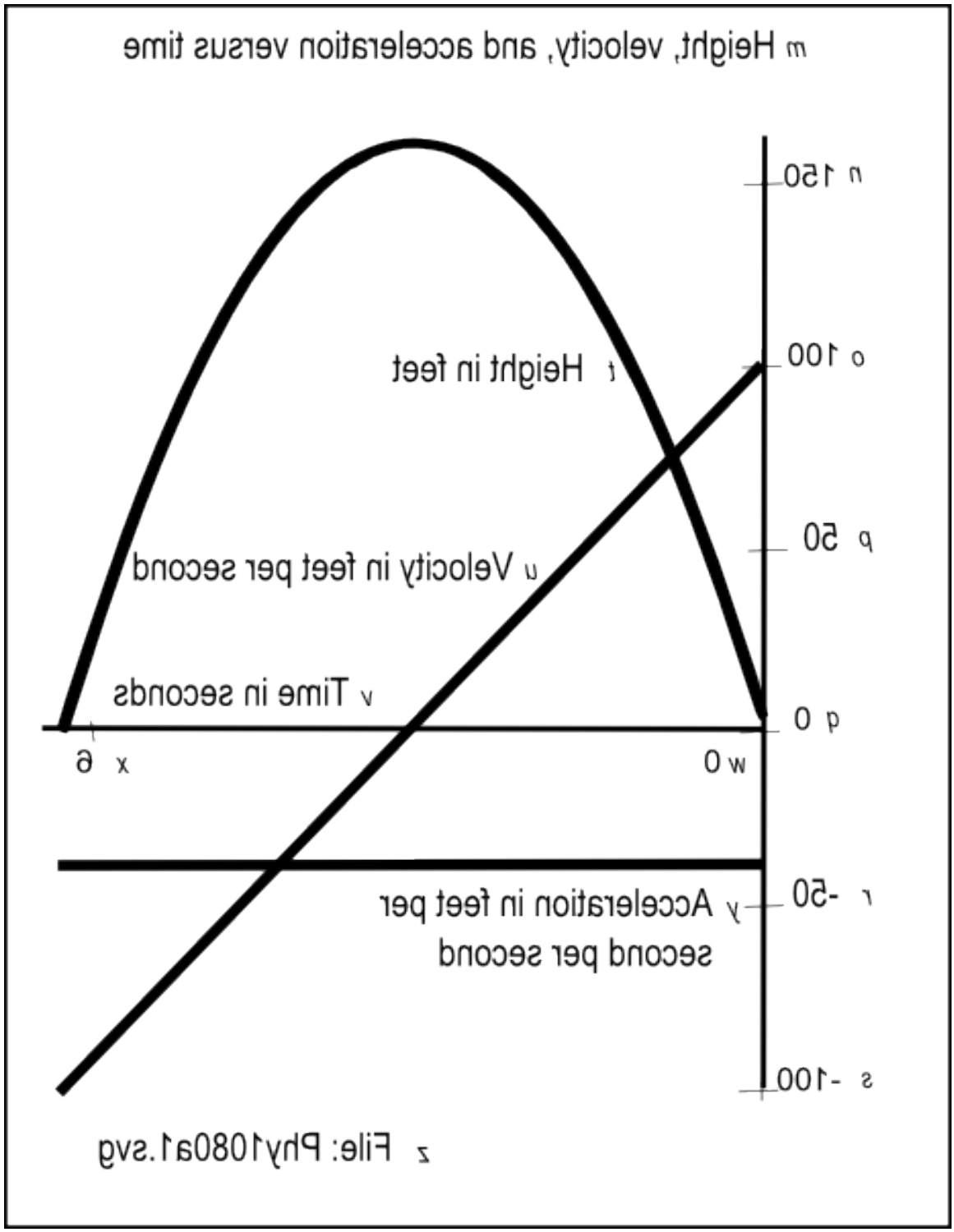

Height, velocity, and acceleration versus time
150
100
50
0
-50
-100
Height in feet
Velocity in feet per second
Time in seconds
0
6
Acceleration in feet per second per second
File: Phy1080a1.svg

Figure 1 . Mirror image contained in the file named Phy1080a1.svg.

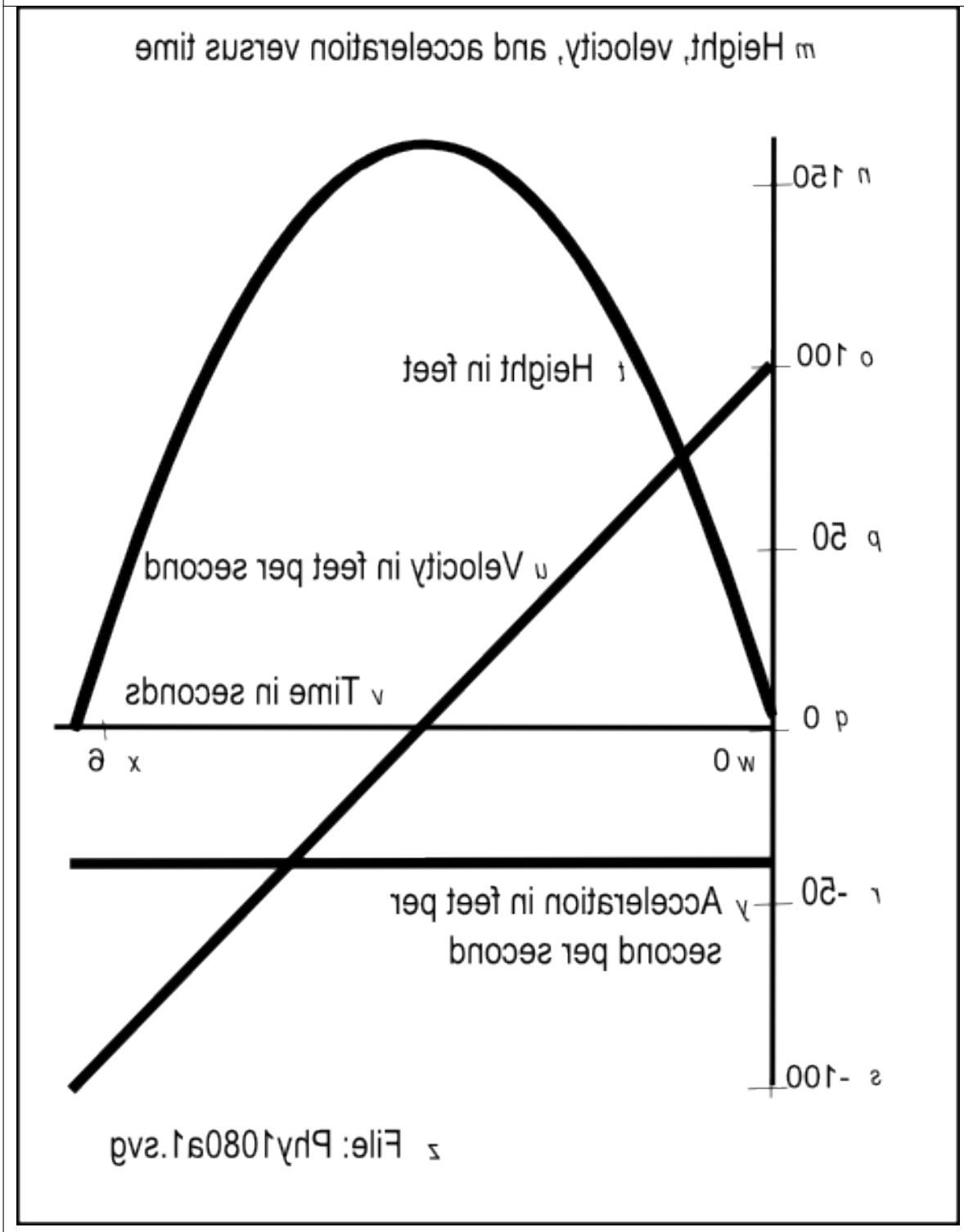
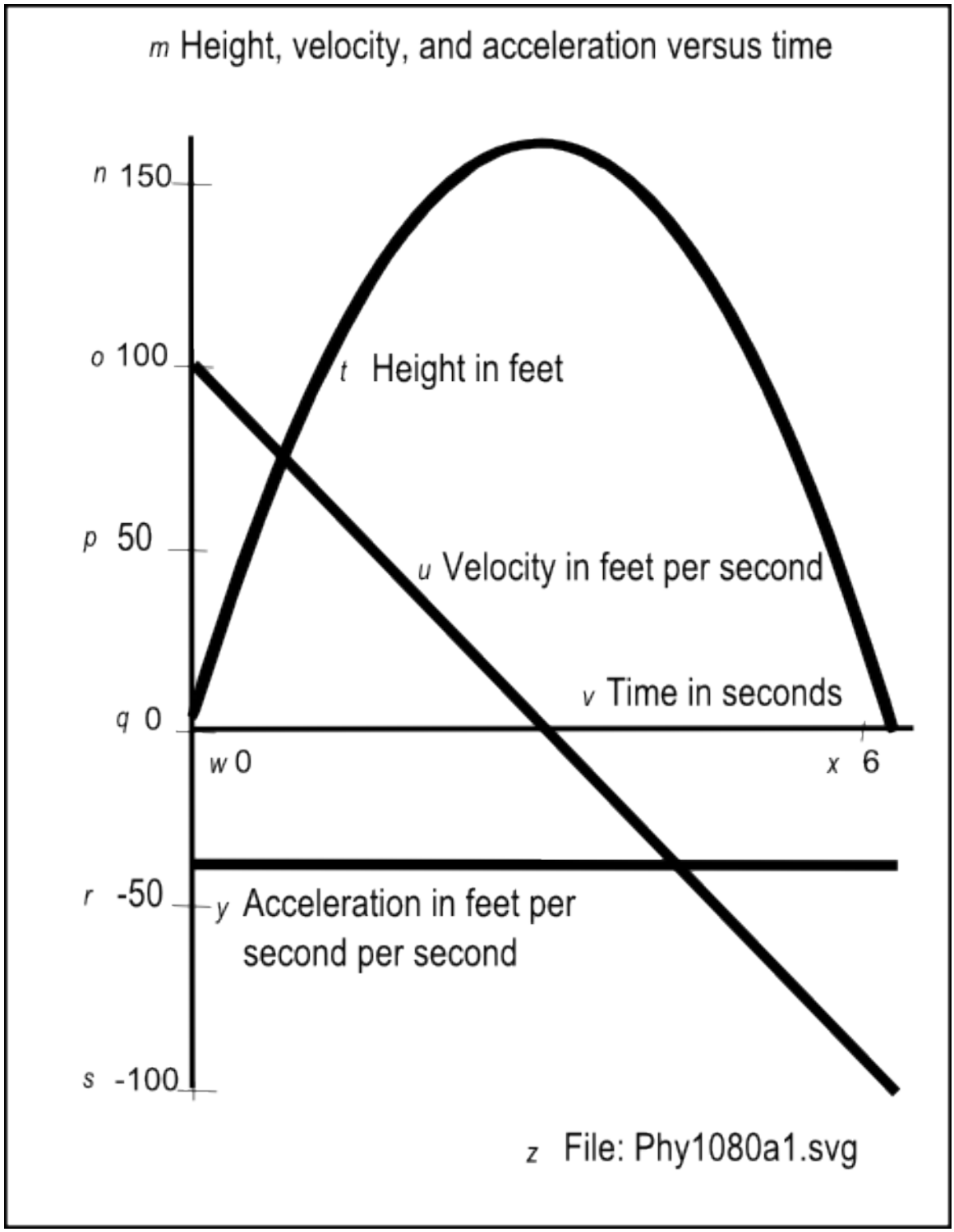
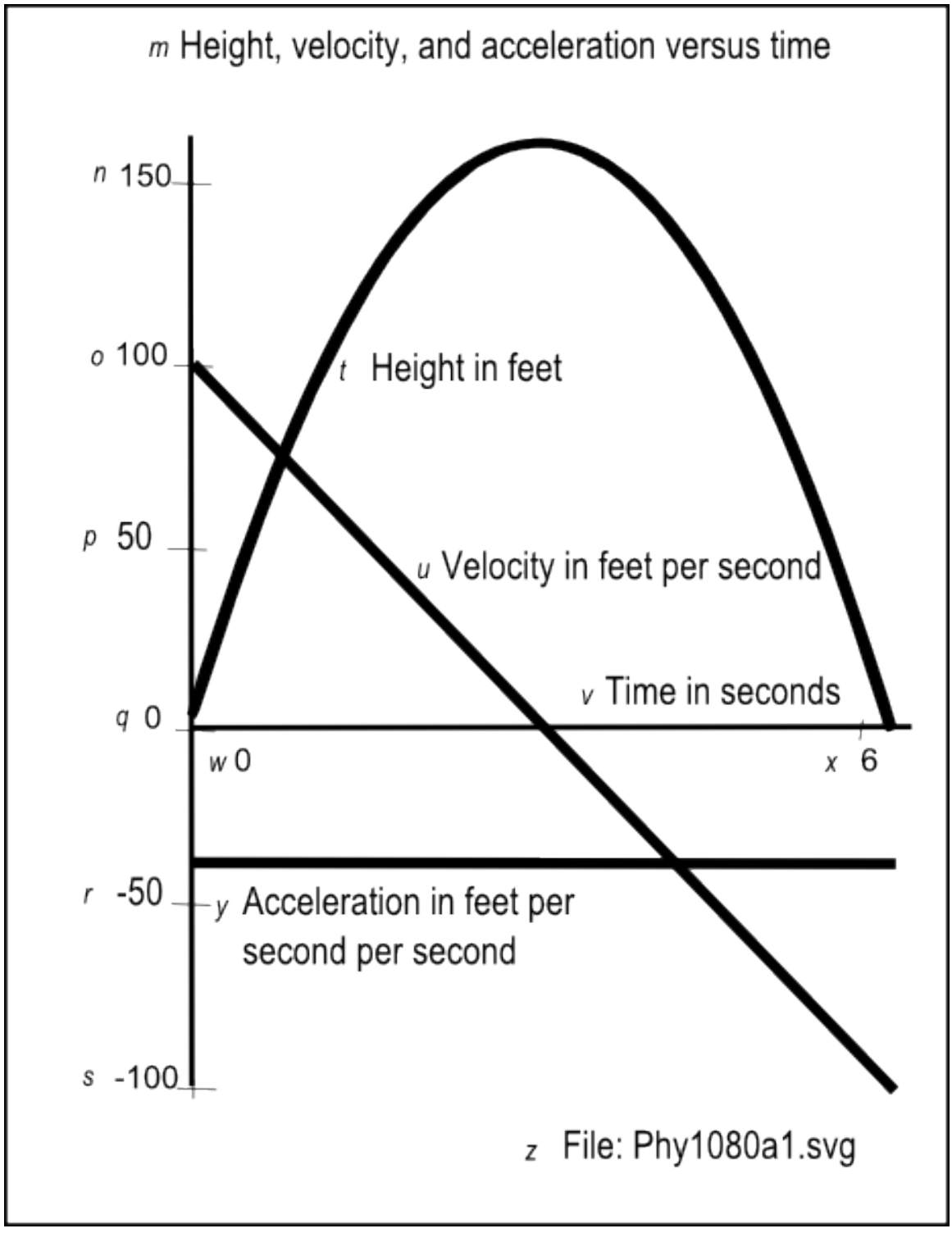


Table 12.1


m Height, velocity, and acceleration versus time
n 150
o 100
p 50
q 0
r -50
s -100
t Height in feet
u Velocity in feet per second
v Time in seconds
w 0
x 6
y Acceleration in feet per second per second
z File: Phy1080a1.svg


m Height, velocity, and acceleration versus time
n 150
o 100
p 50
q 0
r -50
s -100
t Height in feet
u Velocity in feet per second
v Time in seconds
w 0
x 6
y Acceleration in feet per second per second
z File: Phy1080a1.svg

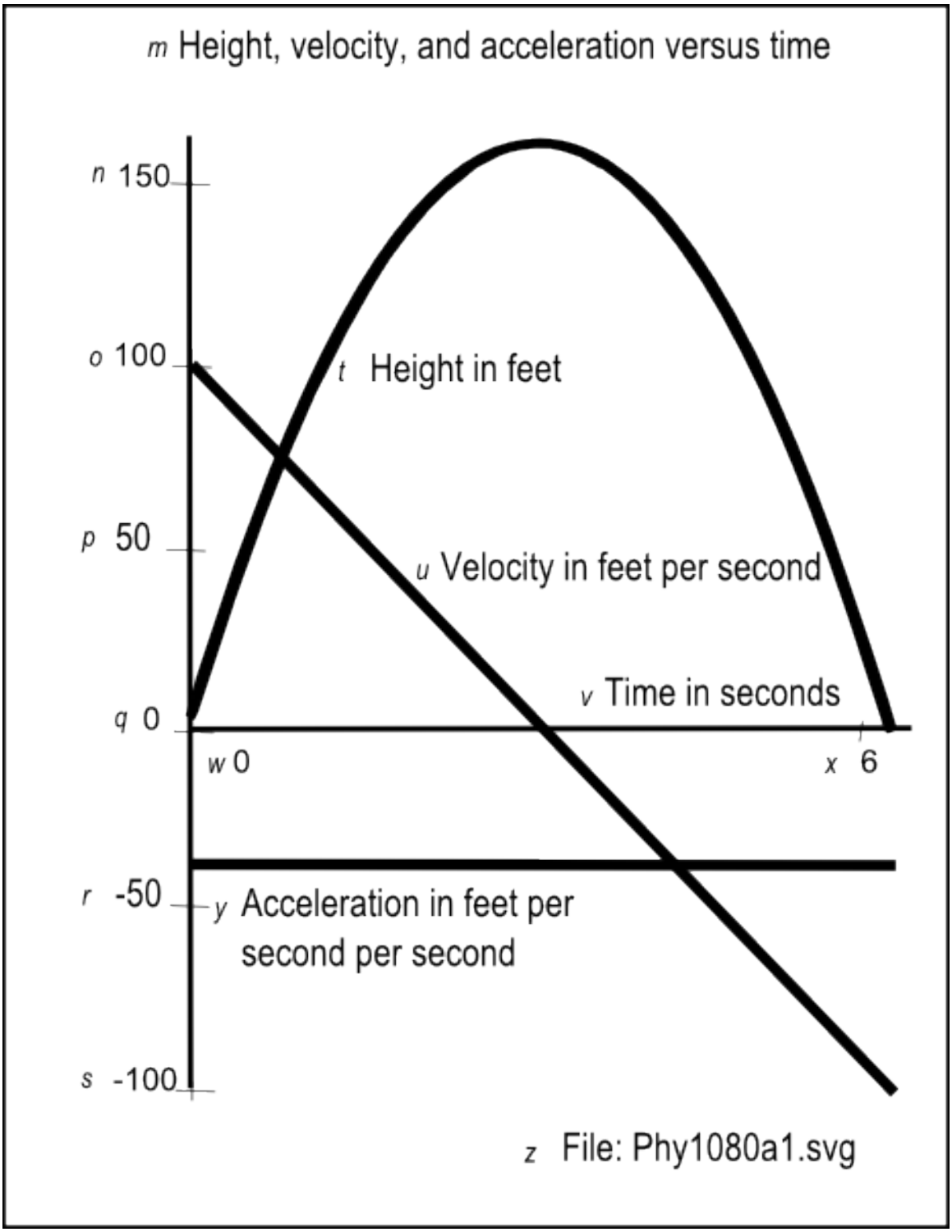

m Height, velocity, and acceleration versus time
n 150
o 100
p 50
q 0
r -50
s -100
t Height in feet
u Velocity in feet per second
v Time in seconds
w 0
x 6
y Acceleration in feet per second per second
z File: Phy1080a1.svg

Figure 2 . Non-mirror-image contained in the file named Phy1080a1.svg.

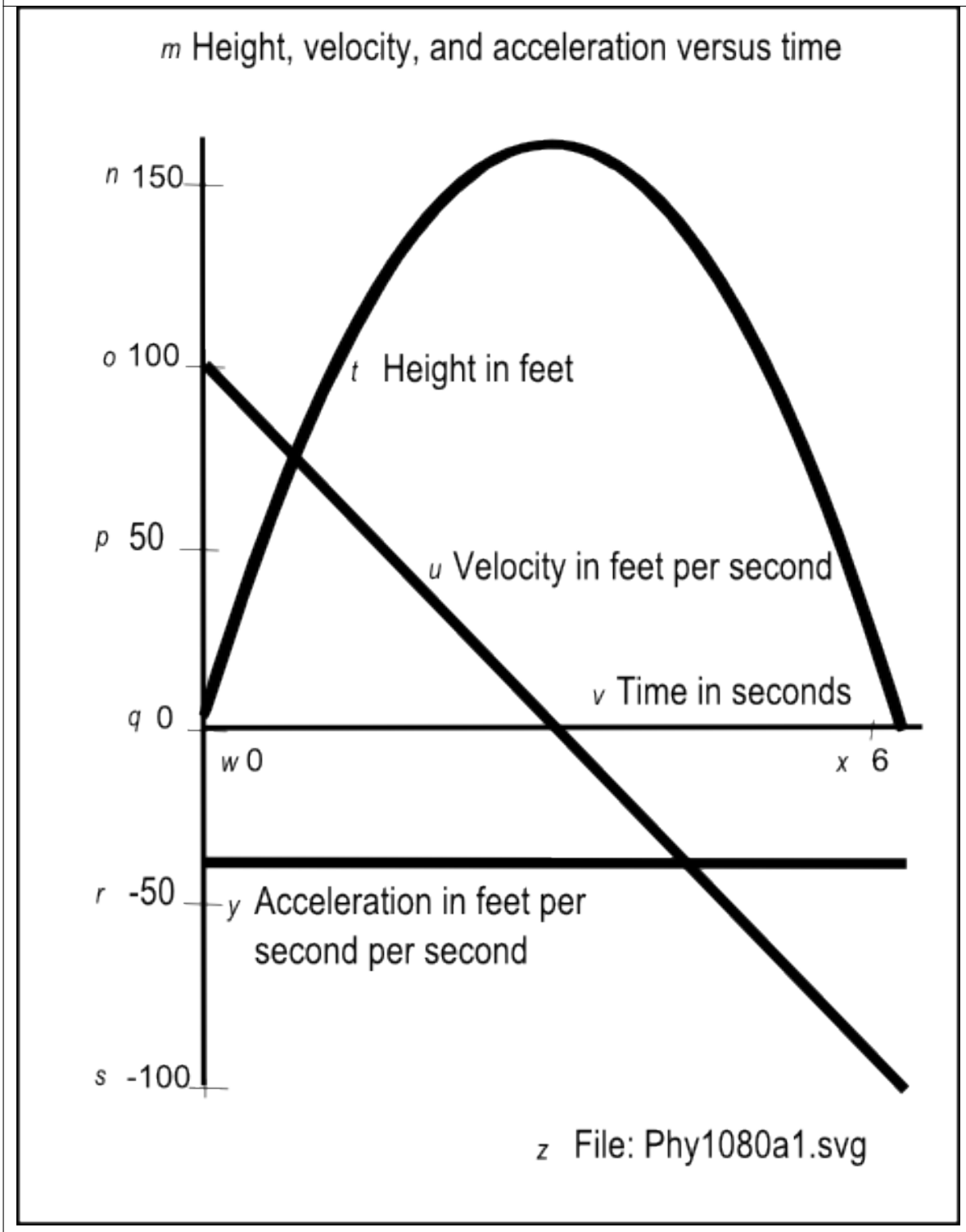


Table 12.2

Key-value pairs
The table of key-value pairs for this file is provided in Figure 3 (p. 340) .

Figure 3 . Key-value pairs for the image in Phy1080a1.svg.

```
m: Height, velocity, and acceleration versus time.
n: 150
o: 100
p: 50
q: 0
r: -50
s: -100
t: Height in feet
u: Velocity in feet per second
v: Time in seconds
w: 0
x: 6
y: Acceleration in feet per second per second
z: File: Phy1080a1.svg
```

Table 12.3

Create a script
Please copy the code shown in Listing 1 (p. 341) into an html file and open the file in your browser.

Listing 1 . Variable velocity exercise #1.

```
    <!---------------- File JavaScript01.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//Declare and initialize constants
var g = -32.2;//acceleration of gravity in ft/(sec*sec)
var v0 = 100;//initial velocity in ft/sec
var h0 = 6;//initial height in feet
var tInc = 0.25;//calculation interval in seconds

//Declare and initialize working variables
var t = .0001;//current time in seconds
var h = h0;//current height in feet

//The following variable is used to insert spaces in
// the output display.
var sp = "    "

//Compute and display various values for as long as the
// height of the projectile is above the ground.
while(h > 0){
  //This is the general equation for the height of a
  // projectile under the influence of gravity in a
  // vacuum.
  h = h0 + v0*t +0.5*g*t*t;

  //Display the information for this iteration
  document.write(
      "t = " + t.toFixed(2) + " seconds" + sp +
      " h = " + h.toFixed(1) + " feet" + sp +
      "</br>");

  //Increment the time for the next iteration.
  t = t + tInc;
}//end while loop

document.write("End Script");

</script>
</body></html>
```

Table 12.4

Screen output

The text shown in Figure 4 (p. 343) should appear in your browser window when you open the html file in your browser.

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Figure 4 . Screen output for Listing #1.

```
    Start Script
t = 0.00 seconds     h = 6.0 feet
t = 0.25 seconds     h = 30.0 feet
t = 0.50 seconds     h = 52.0 feet
t = 0.75 seconds     h = 72.0 feet
t = 1.00 seconds     h = 89.9 feet
t = 1.25 seconds     h = 105.8 feet
t = 1.50 seconds     h = 119.8 feet
t = 1.75 seconds     h = 131.7 feet
t = 2.00 seconds     h = 141.6 feet
t = 2.25 seconds     h = 149.5 feet
t = 2.50 seconds     h = 155.4 feet
t = 2.75 seconds     h = 159.2 feet
t = 3.00 seconds     h = 161.1 feet
t = 3.25 seconds     h = 160.9 feet
t = 3.50 seconds     h = 158.8 feet
t = 3.75 seconds     h = 154.6 feet
t = 4.00 seconds     h = 148.4 feet
t = 4.25 seconds     h = 140.2 feet
t = 4.50 seconds     h = 130.0 feet
t = 4.75 seconds     h = 117.7 feet
t = 5.00 seconds     h = 103.5 feet
t = 5.25 seconds     h = 87.2 feet
t = 5.50 seconds     h = 69.0 feet
t = 5.75 seconds     h = 48.7 feet
t = 6.00 seconds     h = 26.4 feet
t = 6.25 seconds     h = 2.1 feet
t = 6.50 seconds     h = -24.2 feet
End Script
```

Table 12.5

Analysis of the output

You should be able to tell that your curve begins at a height of 6 feet and increases on a point-by-point basis out to about 3 seconds. The maximum value should occur around 3 seconds and the plot should go to zero between 6.25 seconds and 6.5 seconds.

Not a trip to outer space

As you are probably aware, shooting an arrow upward does not cause the arrow to go into outer space, unless the arrow is self-propelled and manages to reach a velocity commonly known as the "escape velocity."

Instead, for any practical value of initial velocity (neglecting air resistance), gravitational attraction will eventually cause the arrow to slow down, reverse course, and fall back to earth with continually increasing velocity until it strikes the earth and stops. That is the message conveyed by the plot on your graph board.

The maximum height

At about 3 seconds and a height of about 161 feet, the kinetic energy provided by the initial velocity will have been dissipated by the gravitational attraction of the earth. At that point, the arrow won't go any higher. Instead, it will start falling back toward the earth. Somewhere around 6.25 seconds, it will strike the earth.

The shape of the curve

The shape of this curve is controlled by only two factors: the initial velocity of the arrow and the gravitational attraction of the earth. The archer has some degree of control over the initial velocity, but has no control over the gravitational attraction of the earth.

In theory, in the absence of an atmosphere, if the arrow is shot straight up, the arrow should land in the same place from which it was shot. In practice in the real world, wind and other factors would probably prevent that from happening.

Variable velocity

We learned in an earlier module that if velocity is uniform, the displacement should be the same during each successive equal interval of time. However, we can see from Figure 4 (p. 343) that is not true in this case. For example, Figure 5 (p. 344) shows the displacement versus time for the first five time intervals and we can see that it is anything but uniform.

Figure 5 . Displacement versus time for first five time intervals.

Interval	Displacement
1	24
2	22
3	20
4	17.9
5	15.9

Table 12.6

Successively decreasing or increasing displacement

Referring back to Figure 4 (p. 343) (along with Figure 5 (p. 344)), we see that the displacement during the first 0.25 second time interval was 24 feet, but the displacement between 2.75 seconds and 3.0 seconds was only 1.9 feet, Furthermore, the displacement was -0.2 feet during the interval between 3 seconds and 3.25 second indicating that the arrow had begun falling back to the earth.

From that point until contact with the ground, the displacement increased during each 0.25 second interval.

In this case, the velocity clearly wasn't uniform. Instead it was variable.

Measuring variable velocity in the real world

Measuring variable velocity in the real world can be difficult. What you need to do is to measure the displacement of an object in as short a time interval as possible and then divide the displacement by the value of the time interval. If you can do this at enough points in time, you can construct curves that represent the variable velocity to which the object is being subjected.

Estimating variable velocity graphically

Assuming that you are able to create a plot of displacement versus time at closely-spaced points, you can estimate the variable velocity curve by connecting each pair of points with a straight line and measuring the slope of each such straight line segment. An estimate of the velocity during the interval defined by a pair of points is the slope of the line that connects those points. The closer the points are, the better will be the estimate of the velocity at a point half way between those two points.

In the situation where you are able to collect enough data to draw a smooth curve of displacement versus time, the instantaneous velocity at any point on that curve is the slope of a line that is tangent to the curve at that point. Note, however, that the construction of such a tangent line is no small feat.

Units of velocity

If you recall your high school geometry course, you may remember that the slope of a line is given by the "rise over run." In other words, the slope of the line is the ratio of the height to the base of a right triangle for which the hypotenuse is on or parallel to the line.

Estimate some variable velocity values

At this point, I encourage you to use a straight edge, place it firmly against two pins that define a time interval on your graph board, estimate the slope of the edge, and record that slope as your estimate of the velocity at the center of that time interval.

Explanation of the code

The code in Listing 1 (p. 341) begins by declaring and initializing values to contain the parameters of the problem:

- The acceleration of gravity in feet/sec^2. Note that this value is negative indicating that the gravitational attraction is toward the center of the earth (down).
- The initial velocity of the arrow in feet/sec. This value is positive because the direction of velocity is away from the center of the earth (up).
- The height of the arrow when it is released, positive values meaning up from the ground.
- The time interval at which successive estimates of the height of the arrow will be computed.

Working variables

Following that, the code in Listing 1 (p. 341) declares and initializes two working variables for time and distance that will change as the program progresses.

Then Listing 1 (p. 341) declares and initializes a variable named **sp** that is used solely to insert spaces between the columns in the output display.

A while loop

A **while** loop is used to evaluate the equation given earlier (p. 328) , to compute and display the height of the arrow every 0.25 seconds for as long as the arrow is above the ground (height is positive). The loop continues to iterate and display time and height values in two separate columns (see Figure 4 (p. 343)) until the computation produces one negative height value.

When the test at the top of the **while** loop determines that the previously-computed value for height was negative,

- The body of the loop is skipped.
- The "End Script" message is displayed.
- The script terminates.

As you can see, this is not a complicated script in comparison with some of the scripts that were presented in earlier modules.

12.4.3 Variable velocity exercise #2

Estimating variable velocity with a computer model

Let's write a script that approximates the graphic procedure that I described earlier.

Please copy the code from Listing 2 (p. 346) into an html file and open the file in your browser.

Listing 2 . Variable velocity exercise #2.

```
<!---------------- File JavaScript2.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//Declare and initialize constants
var g = -32.2;//acceleration of gravity in ft/sec/sec
var v0 = 100;//initial velocity in ft/sec
var h0 = 6;//initial height in feet
var tInc = 0.25;//calculation interval in seconds

//Declare and initialize working variables
var t = .0001;//current time in seconds
var h = h0;//current height in feet
var v = 0;//current velocity in feet/sec

var oldT = 0;//previous time in seconds
var delT = 0;//change in time in seconds

var oldH = h0;//previous height in feet
var delH = 0;//change in height in feet

//The following variable is used to insert spaces in
// the output display.
var sp = "    "

//Compute and display various values for as long as the
// height of the projectile is above the ground.
while(h > 0){
  //This is the general equation for the height of a
  // projectile under the influence of gravity in a
  // vacuum.
  h = h0 + v0*t +0.5*g*t*t;

  //Compute and save the time interval since the
  // previous estimate of height, etc.
  delT = t-oldT;
  oldT = t;//Save the current time for the next iteration

  //Estimate the velocity based on the change in
  // height since the previous estimate.
  delH = h-oldH;
  v = delH/delT;//compute velocity
  oldH = h;//Save the current height for the next iteration.

  //Display the information for this iteration
  document.write(
      "t = " + t.toFixed(2) + " seconds" + sp +
      " h = " + h.toFixed(1) + "feet" + sp +
      " v = " + v.toFixed(2) + " feet/second" + sp +
      "</br>");

  //Increment the time for the next iteration.
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 12.7

Screen output

The text shown in Figure 6 (p. 347) should appear in your browser window when the html file opens in your browser.

Figure 6 . Screen output for Listing #2.

```
    Start Script
t = 0.00 seconds     h = 6.0 feet     v = 100.00 feet/second
t = 0.25 seconds     h = 30.0 feet     v = 95.97 feet/second
t = 0.50 seconds     h = 52.0 feet     v = 87.92 feet/second
t = 0.75 seconds     h = 72.0 feet     v = 79.87 feet/second
t = 1.00 seconds     h = 89.9 feet     v = 71.82 feet/second
t = 1.25 seconds     h = 105.8 feet     v = 63.77 feet/second
t = 1.50 seconds     h = 119.8 feet     v = 55.72 feet/second
t = 1.75 seconds     h = 131.7 feet     v = 47.67 feet/second
t = 2.00 seconds     h = 141.6 feet     v = 39.62 feet/second
t = 2.25 seconds     h = 149.5 feet     v = 31.57 feet/second
t = 2.50 seconds     h = 155.4 feet     v = 23.52 feet/second
t = 2.75 seconds     h = 159.2 feet     v = 15.47 feet/second
t = 3.00 seconds     h = 161.1 feet     v = 7.42 feet/second
t = 3.25 seconds     h = 160.9 feet     v = -0.63 feet/second
t = 3.50 seconds     h = 158.8 feet     v = -8.68 feet/second
t = 3.75 seconds     h = 154.6 feet     v = -16.73 feet/second
t = 4.00 seconds     h = 148.4 feet     v = -24.78 feet/second
t = 4.25 seconds     h = 140.2 feet     v = -32.83 feet/second
t = 4.50 seconds     h = 130.0 feet     v = -40.88 feet/second
t = 4.75 seconds     h = 117.7 feet     v = -48.93 feet/second
t = 5.00 seconds     h = 103.5 feet     v = -56.98 feet/second
t = 5.25 seconds     h = 87.2 feet     v = -65.03 feet/second
t = 5.50 seconds     h = 69.0 feet     v = -73.08 feet/second
t = 5.75 seconds     h = 48.7 feet     v = -81.13 feet/second
t = 6.00 seconds     h = 26.4 feet     v = -89.18 feet/second
t = 6.25 seconds     h = 2.1 feet     v = -97.23 feet/second
t = 6.50 seconds     h = -24.2 feet     v = -105.28 feet/second
End Script
```

Table 12.8

Tactile graphics

The velocity values shown in Figure 6 (p. 347) are plotted in the image that is contained in the file named Phy1080b1.svg, along with the height values and the acceleration values.

Analysis of the results

The difference between Figure 4 (p. 343) and Figure 6 (p. 347) is the addition of a column of velocity information on the right in Figure 6 (p. 347) . As you can see, the velocity begins with a positive value of 100 feet/second at the top of the velocity column.

The velocity goes to zero somewhere between 3 and 3.25 seconds, which is the point where the height of the arrow reaches its maximum.

At 3.25 seconds, the velocity switches from positive to negative meaning that the arrow begins falling back toward the earth. The velocity reaches -100 feet/second somewhere between 6.25 and 6.5 seconds, which is about the same time that the arrow hits the ground.

Plot the velocity curve

I believe you might find it useful to plot the velocity curve on the graph board without removing the height curve if you can. That will give you a good opportunity to use your hands to "see" the big picture insofar as the relationship between the height (displacement) and the velocity are concerned.

When you do that, hopefully you will recognize that the velocity curve for this particular situation is a straight line with a negative slope. That straight line crosses the horizontal axis going from positive territory into negative territory about half way between the original release of the arrow and the point where the arrow strikes the ground.

Analysis of the code

A comparison of Listing 2 (p. 346) with Listing 1 (p. 341) shows the addition of the following variables:

- var oldT = 0; //previous time in seconds
- var delT = 0; //change in time in seconds
- var oldH = h0; //previous height in feet
- var delH = 0; //change in height in feet

Save values for use in next iteration

As you will see when I get into an explanation of the **while** loop, the variables **oldT** and **oldH** are used to save the time value and the height value during each iteration to make those values available during the next iteration.

Compute changes in time and height

Inside the while loop, the current value of **t** and the value stored in **oldT** are used to compute the change in time since the previous iteration. The current value of **h** and the value stored in **oldH** are used to compute the change in height since the previous iteration. These change values are saved in the variables named **delT** and **delH** .

Velocity is the ratio of the changes

In this case, the value in **delH** represents the displacement of the arrow during the time interval and the value stored in **delT** represents the length of the time interval. As you already know, the velocity is the ratio of those two values. The value for velocity is computed along with the time and the height and then the script executes the next iteration of the **while** loop.

The while loop

The **while** loop in Listing 2 (p. 346) begins the same as in Listing 1 (p. 341) . After the new height value is computed, the script performs the computations described above to estimate and display the velocity for the time interval since the previous iteration.

In addition, the script saves the current values of **t** and **h** in **oldT** and **oldH** to be used to estimate the velocity during the next iteration of the **while** loop.

Finally, the time value is increased by the value stored in the variable named **tInc** , and control goes back to the top of the loop where the current value of height is tested to determine if it has become negative (meaning that the arrow has hit the ground).

12.4.4 Acceleration of gravity exercise #1

To repeat the earlier definition, acceleration is the rate of change of velocity.

You learned earlier that velocity is the rate of change of position. Then you learned that we can estimate the velocity of a moving object at a point in time by determining the slope of a curve that plots the position of the object versus time at that particular point in time.

Positive and negative velocity

Positive slopes indicate positive velocity and negative slopes indicate negative velocity. In the case of our arrow, the velocity went to zero and switched from positive to negative at the point where the arrow reached

its maximum height and began to fall back toward the earth. In a math class, that point would likely be referred to as an **inflection point** on the curve of position versus time.

Estimating acceleration

We can extend that procedure to estimate the acceleration of a moving object at a point in time by determining the slope of a curve that plots the velocity of the object versus time at that particular point in time. Positive slopes indicate positive acceleration and negative slopes indicate negative acceleration.

Estimate some velocity values graphically

If you have the velocity curve from Figure 6 (p. 347) plotted on your graph board, I recommend that you estimate and record the slope and hence the acceleration at a few points. That should be easy to do in this case, because the velocity curve should be a straight line with a negative slope.

A constant acceleration value

If the velocity curve is a straight line with a negative slope, and if the acceleration at any point in time is equal to the slope of the velocity curve, that means that the acceleration has a constant negative value throughout the time interval of interest. In fact, you should find that the value of the acceleration is approximately -32.2 feet/second^2, which is known as the **acceleration of gravity** .

Create a script

Please copy the code from Listing 3 (p. 350) into an html file and open that file in your browser.

Listing 3 . Acceleration of gravity exercise #1.

```
    <!---------------- File JavaScript99.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//Declare and initialize constants
var g = -32.2;//acceleration of gravity in ft/(sec*sec)
var v0 = 100;//initial velocity in ft/sec
var h0 = 6;//initial height in feet
var tInc = 0.25;//calculation interval in seconds

//Declare and initialize working variables
var t = .0001;//current time in seconds
var h = h0;//current height in feet
var v = 0;//current velocity in feet/sec
var a = 0;//current acceleration in feet/(sec*sec)

var oldT = 0;//previous time in seconds
var delT = 0;//change in time in seconds

var oldH = h0;//previous height in feet
var delH = 0;//change in height in feet

var oldV = 0;//previous velocity in feet/sec
var delV = 0;//change in velocity in feet/sec

//The following variable is used to insert spaces in
// the output display.
var sp = "  ";

//Compute and display various values for as long as the
// height of the projectile is above the ground.
while(h > 0){
  //This is the general equation for the height of a
  // projectile under the influence of gravity in a
  // vacuum.
  h = h0 + v0*t +0.5*g*t*t;

  //Compute and save the time interval since the
  // previous estimate of height, etc.
  delT = t-oldT;
  oldT = t;//Save the current time for the next iteration.

  //Estimate the velocity based on the change in
  // height since the previous estimate.
  delH = h-oldH;
  v = delH/delT;//compute velocity
  oldH = h;//Save the current height for the next iteration.

  //Estimate the acceleration based on the change in
  // velocity once the data pipeline is full.
  delV = v - oldV;
  oldV = v;//Save the current velocity for the next iteration.
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 12.9

Screen output

The text shown in Figure 7 (p. 351) should appear in your browser window when the html file is opened in your browser.

Figure 7 . Screen output for Listing #3.

```
    Start Script
t = 0.00 sec   h = 6 ft   v = 100.00 ft/sec   a = 0.00 ft/(sec*sec)
t = 0.25 sec   h = 30 ft   v = 95.97 ft/sec   a = 0.00 ft/(sec*sec)
t = 0.50 sec   h = 52 ft   v = 87.92 ft/sec   a = -32.20 ft/(sec*sec)
t = 0.75 sec   h = 72 ft   v = 79.87 ft/sec   a = -32.20 ft/(sec*sec)
t = 1.00 sec   h = 90 ft   v = 71.82 ft/sec   a = -32.20 ft/(sec*sec)
t = 1.25 sec   h = 106 ft   v = 63.77 ft/sec   a = -32.20 ft/(sec*sec)
t = 1.50 sec   h = 120 ft   v = 55.72 ft/sec   a = -32.20 ft/(sec*sec)
t = 1.75 sec   h = 132 ft   v = 47.67 ft/sec   a = -32.20 ft/(sec*sec)
t = 2.00 sec   h = 142 ft   v = 39.62 ft/sec   a = -32.20 ft/(sec*sec)
t = 2.25 sec   h = 149 ft   v = 31.57 ft/sec   a = -32.20 ft/(sec*sec)
t = 2.50 sec   h = 155 ft   v = 23.52 ft/sec   a = -32.20 ft/(sec*sec)
t = 2.75 sec   h = 159 ft   v = 15.47 ft/sec   a = -32.20 ft/(sec*sec)
t = 3.00 sec   h = 161 ft   v = 7.42 ft/sec   a = -32.20 ft/(sec*sec)
t = 3.25 sec   h = 161 ft   v = -0.63 ft/sec   a = -32.20 ft/(sec*sec)
t = 3.50 sec   h = 159 ft   v = -8.68 ft/sec   a = -32.20 ft/(sec*sec)
t = 3.75 sec   h = 155 ft   v = -16.73 ft/sec   a = -32.20 ft/(sec*sec)
t = 4.00 sec   h = 148 ft   v = -24.78 ft/sec   a = -32.20 ft/(sec*sec)
t = 4.25 sec   h = 140 ft   v = -32.83 ft/sec   a = -32.20 ft/(sec*sec)
t = 4.50 sec   h = 130 ft   v = -40.88 ft/sec   a = -32.20 ft/(sec*sec)
t = 4.75 sec   h = 118 ft   v = -48.93 ft/sec   a = -32.20 ft/(sec*sec)
t = 5.00 sec   h = 103 ft   v = -56.98 ft/sec   a = -32.20 ft/(sec*sec)
t = 5.25 sec   h = 87 ft   v = -65.03 ft/sec   a = -32.20 ft/(sec*sec)
t = 5.50 sec   h = 69 ft   v = -73.08 ft/sec   a = -32.20 ft/(sec*sec)
t = 5.75 sec   h = 49 ft   v = -81.13 ft/sec   a = -32.20 ft/(sec*sec)
t = 6.00 sec   h = 26 ft   v = -89.18 ft/sec   a = -32.20 ft/(sec*sec)
t = 6.25 sec   h = 2 ft   v = -97.23 ft/sec   a = -32.20 ft/(sec*sec)
t = 6.50 sec   h = -24 ft   v = -105.28 ft/sec   a = -32.20 ft/(sec*sec)
End Script
```

Table 12.10

Tactile graphics

The acceleration values shown in Figure 7 (p. 351) are plotted in the image that is contained in the file named Phy1080b1.svg, along with the height values and the velocity values.

Analysis of the results

The main difference between Figure 7 (p. 351) and Figure 6 (p. 347) is the addition of an acceleration column on the right in Figure 7 (p. 351) . (In addition, I abbreviated feet and seconds as ft and sec, and reduced the space between columns to make it all fit in the available space.)

Except for the first two values at the top of the acceleration column, every acceleration value is -32.2 ft/(sec*sec). This is what we predicted earlier when we observed that the velocity curve is a straight line

with a constant negative slope. (Note that the procedure used to estimate the acceleration did not allow for an accurate estimate of acceleration for the first two values in the acceleration column.)

Free fall

According to Wikipedia [20] , free fall is any motion of a body where gravity is the only force acting upon it.

Since this definition does not specify velocity, it also applies to objects initially moving upward (which is the case with our arrow that was shot upward).

Free fall was demonstrated on the moon by astronaut David Scott on August 2, 1971. He simultaneously released a hammer and a feather from the same height above the moon's surface. The hammer and the feather both fell at the same rate and hit the ground at the same time. This demonstrated Galileo's discovery that in the absence of air resistance, all objects experience the same acceleration due to gravity.

By the way, it is no accident that the value shown for acceleration in Figure 4 matches the value specified for the acceleration of gravity near the beginning of the code in Listing 3 (p. 350) .

Analysis of the code

The code in Listing 3 (p. 350) is very similar to the code in Listing 2 (p. 346) with the addition of the code necessary to estimate the acceleration values on the basis of the slope of the velocity curve.

The code that estimates the acceleration in Listing 3 (p. 350) is so similar to the code that estimates the velocity in Listing 2 (p. 346) that I won't bore you by explaining the code in detail.

However, there is one bit of new code that is worthy of note. Without going into detail as to the reasons why, this procedure is incapable of accurately estimating the slope of the velocity curve during the first two time intervals. Therefore, an **if** statement was written into the **while** loop to force the acceleration estimate to be zero for the first two time intervals.

12.4.5 Acceleration of gravity exercise #2

In the previous exercises, the arrow was shot straight up in the air. However, that is rarely the case. Normally, an arrow is shot in an attempt to strike a target some horizontal distance away.

A more realistic scenario

Let's modify our scenario such that the archer shoots the arrow with an initial velocity of 100 feet per second at an angle of 45 degrees relative to the horizontal axis. We will compute and plot the horizontal and vertical position of the arrow at uniform increments in time from the beginning to the end of its trajectory.

Motion of a projectile with uniform acceleration

The following equation describes the straight-line motion of a projectile **with uniform acceleration** .

d = v0*t +0.5*a*t^2

where

- d is distance in units of distance
- v0 is the initial velocity in units of distance/time
- t is time in units of time
- a is acceleration in units of distance/time^2

As usual, the units for distance, time, velocity, and acceleration must be consistent with one another.

Shooting the arrow straight up

If we shoot an arrow straight up, as was the case in the previous exercises, the initial velocity doesn't have a horizontal component. Instead, the initial velocity has only a vertical component and the corresponding motion of the arrow has only a vertical component. In other words, the arrow goes straight up and it comes down in the same spot.

Shooting the arrow other than straight up

However, if we shoot the arrow in a direction other than straight up or straight down, the initial velocity has both a vertical component and a horizontal component. Therefore, the resulting motion has both a vertical component and a horizontal component.

[20] http://en.wikipedia.org/wiki/Free_fall#Uniform_gravitational_field_without_air_resistance

To determine the position of the arrow at some time after it is released, we must determine both the vertical and the horizontal component of its motion.

Compute vertical and horizontal components independently

The equation given above (p. 352) can be used to compute the vertical and horizontal components of motion independently of one another. This approach can be used to give us the position as a function of time along each axis. We can then use that information to determine and plot the position of the arrow in x-y space at one or more points in time.

Vertical and horizontal motion components are independent

It is important to understand that the vertical and horizontal components of motion are independent of one another. In other words, changing the vertical component of motion doesn't effect the horizontal motion and changing the horizontal component of motion doesn't effect the vertical motion. The overall motion seen by an observer is the superposition of the two.

Create a script

Please copy the code shown in Listing 4 (p. 354) into an html file and open the file in your browser.

Listing 4 . Acceleration of gravity exercise #2.

```
    <!---------------- File JavaScript04.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//Define common parameters
var ang = 45;//firing angle degrees re the horizontal
var tInc = 0.25;//calculation interval in seconds
var v0 = 100;//initial velocity in ft/sec

//Define horizontal parameters
var ax = 0;//horizontal acceleration
var vx0 = v0*Math.cos(
  ang*Math.PI/180);//component of initial velocity in ft/sec
var x0 = 0;//initial x-offset in feet

//Define vertical parameters
var ay = -32.2;//vertical acceleration in ft/sec*sec
var vy0 = v0*Math.sin(
  ang*Math.PI/180);//component of initial velocity in ft/sec
var y0 = 6;//initial height in feet

//Declare and initialize working variables
var t = .0001;//current time in seconds
var x = x0;//current x in feet
var y = y0;//current height in feet

//The following variable is used to insert spaces in
// the output display.
var sp = "    "

//Compute and display the horizontal and vertical
// positions of the projectile at uniform increments
// of time
while(y > 0){
  //These are the general equations for the straight-
  // line motion of a projectile under the influence
  // of uniform acceleration in a vacuum.
  y = y0 + vy0*t + 0.5*ay*t*t;
  x = x0 + vx0*t + 0.5*ax*t*t;

  //Display the information for this iteration
  document.write(
      "t = " + t.toFixed(2) + " seconds" + sp +
      " x = " + x.toFixed(1) + " feet" + sp +
      " y = " + y.toFixed(1) + " feet" + sp +
      "</br>");

  //Increment the time for the next iteration.
  t = t + tInc;
}//end while loop

document.write("End Script");
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 12.11

Screen output

The text shown in Figure 8 (p. 355) should appear in your browser when you open the html file in your browser.

Figure 8 . Screen output for Listing #4 at 45 degrees.

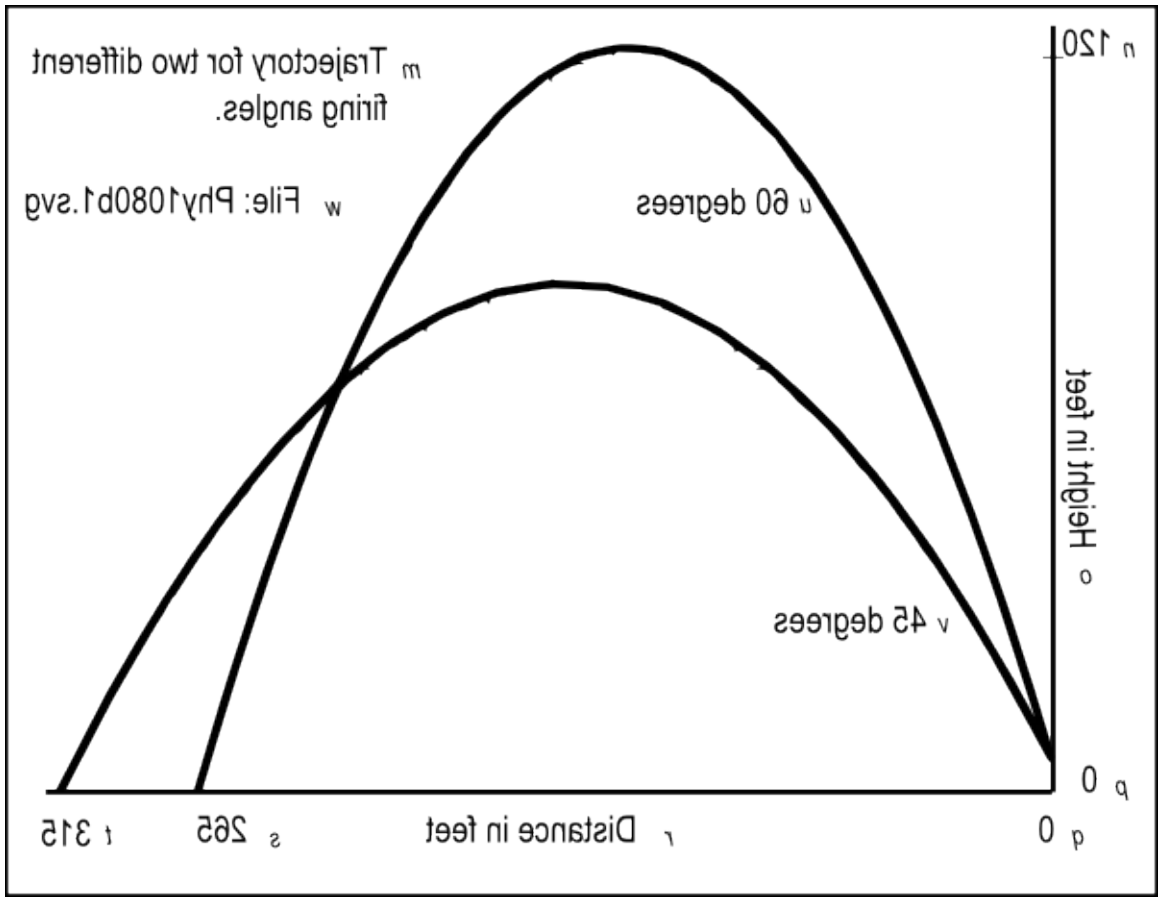
```
    Start Script
t = 0.00 seconds     x = 0.0 feet     y = 6.0 feet
t = 0.25 seconds     x = 17.7 feet     y = 22.7 feet
t = 0.50 seconds     x = 35.4 feet     y = 37.3 feet
t = 0.75 seconds     x = 53.0 feet     y = 50.0 feet
t = 1.00 seconds     x = 70.7 feet     y = 60.6 feet
t = 1.25 seconds     x = 88.4 feet     y = 69.2 feet
t = 1.50 seconds     x = 106.1 feet     y = 75.8 feet
t = 1.75 seconds     x = 123.8 feet     y = 80.4 feet
t = 2.00 seconds     x = 141.4 feet     y = 83.0 feet
t = 2.25 seconds     x = 159.1 feet     y = 83.6 feet
t = 2.50 seconds     x = 176.8 feet     y = 82.2 feet
t = 2.75 seconds     x = 194.5 feet     y = 78.7 feet
t = 3.00 seconds     x = 212.1 feet     y = 73.2 feet
t = 3.25 seconds     x = 229.8 feet     y = 65.8 feet
t = 3.50 seconds     x = 247.5 feet     y = 56.3 feet
t = 3.75 seconds     x = 265.2 feet     y = 44.8 feet
t = 4.00 seconds     x = 282.8 feet     y = 31.2 feet
t = 4.25 seconds     x = 300.5 feet     y = 15.7 feet
t = 4.50 seconds     x = 318.2 feet     y = -1.8 feet
End Script
```

Table 12.12

Tactile graphics

The svg file named Phy1080b1 contains a graph that shows the trajectory of the arrow for firing angles of both 45 degrees and 60 degrees. The vertical axis shows the height in feet and the horizontal axis shows the flight distance in feet.

The image contained in this file is shown in Figure 9 (p. 359) for the benefit of your assistant who will manually emboss the diagram. Note that as usual, this is a mirror image of the image that is to be presented to the student after embossing. A non-mirror-image version is shown in Figure 10 (p. 363) .


Trajectory for two different firing angles.
File: Phy1080b1.svg
60 degrees
45 degrees
120
Height in feet
0
0
Distance in feet
265
315

Trajectory for two different firing angles.
File: Phy1080b1.svg
60 degrees
45 degrees
120
Height in feet
0
0
Distance in feet
265
315

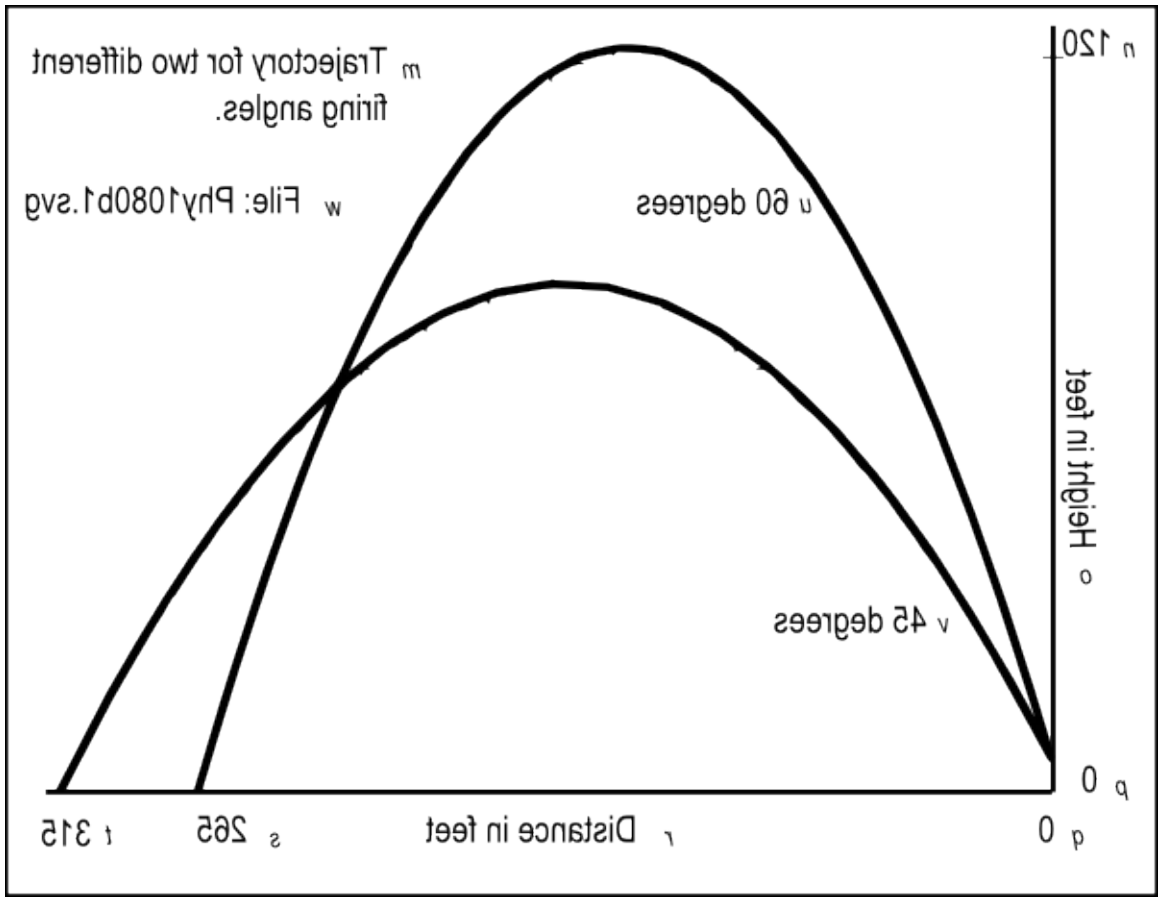

Trajectory for two different firing angles.
File: Phy1080b1.svg
60 degrees
45 degrees
120
Height in feet
0
0
Distance in feet
265
315

Figure 9 . Mirror image contained in the file named Phy1080b1.svg.

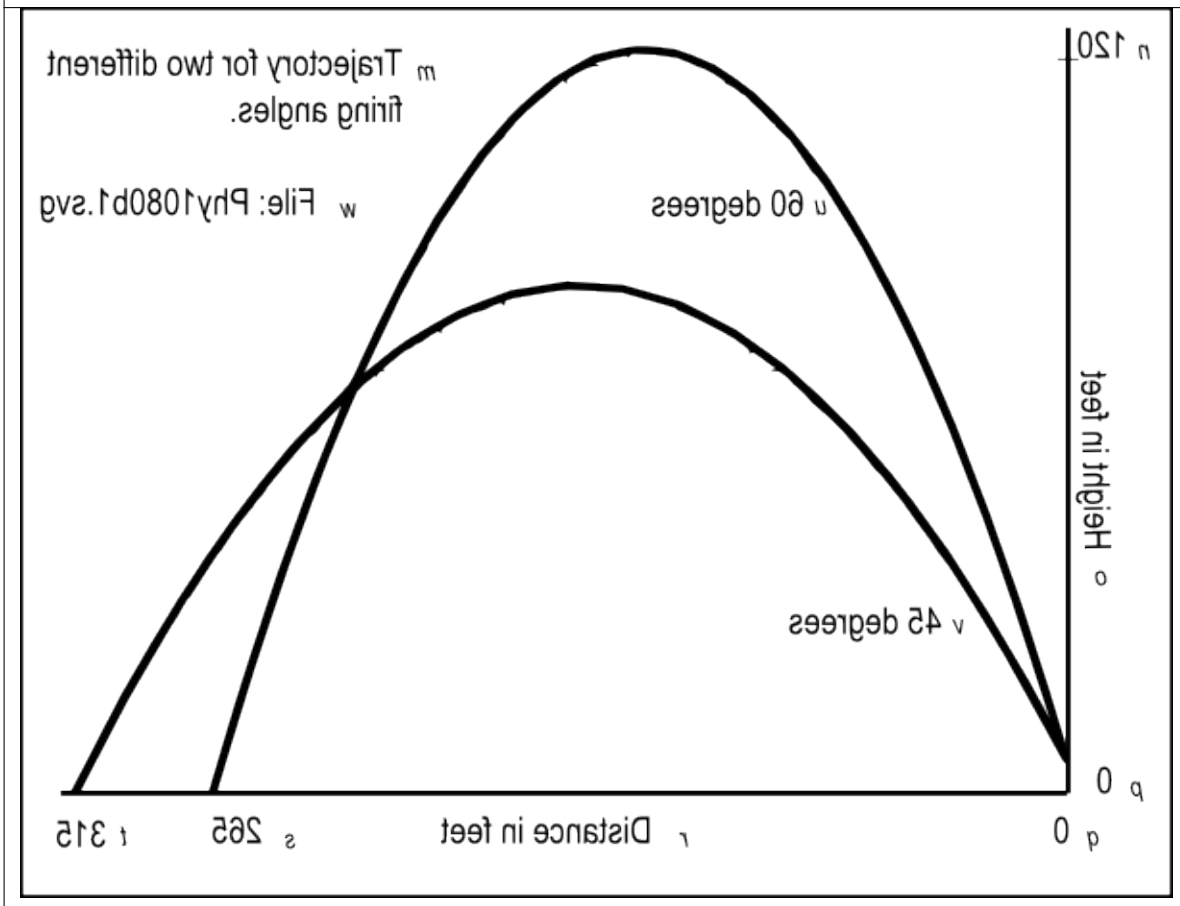
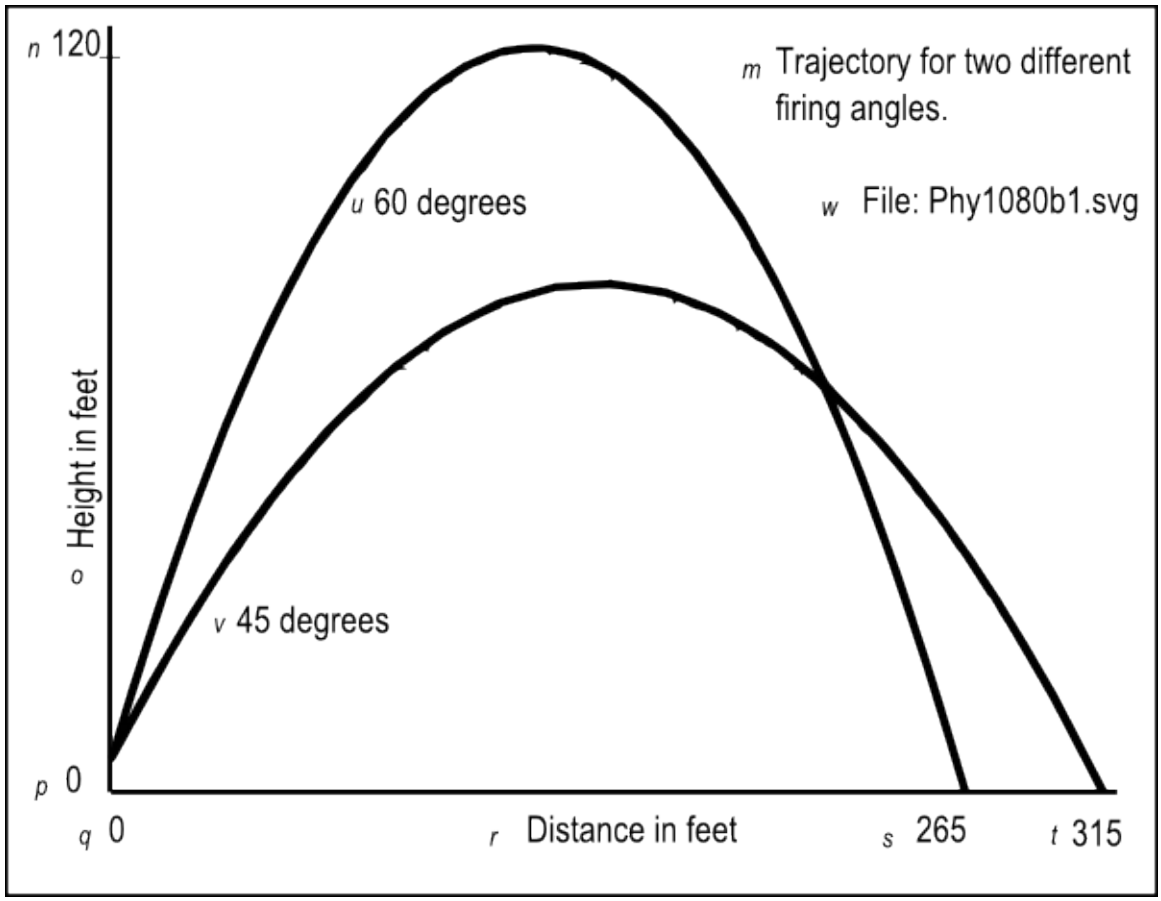
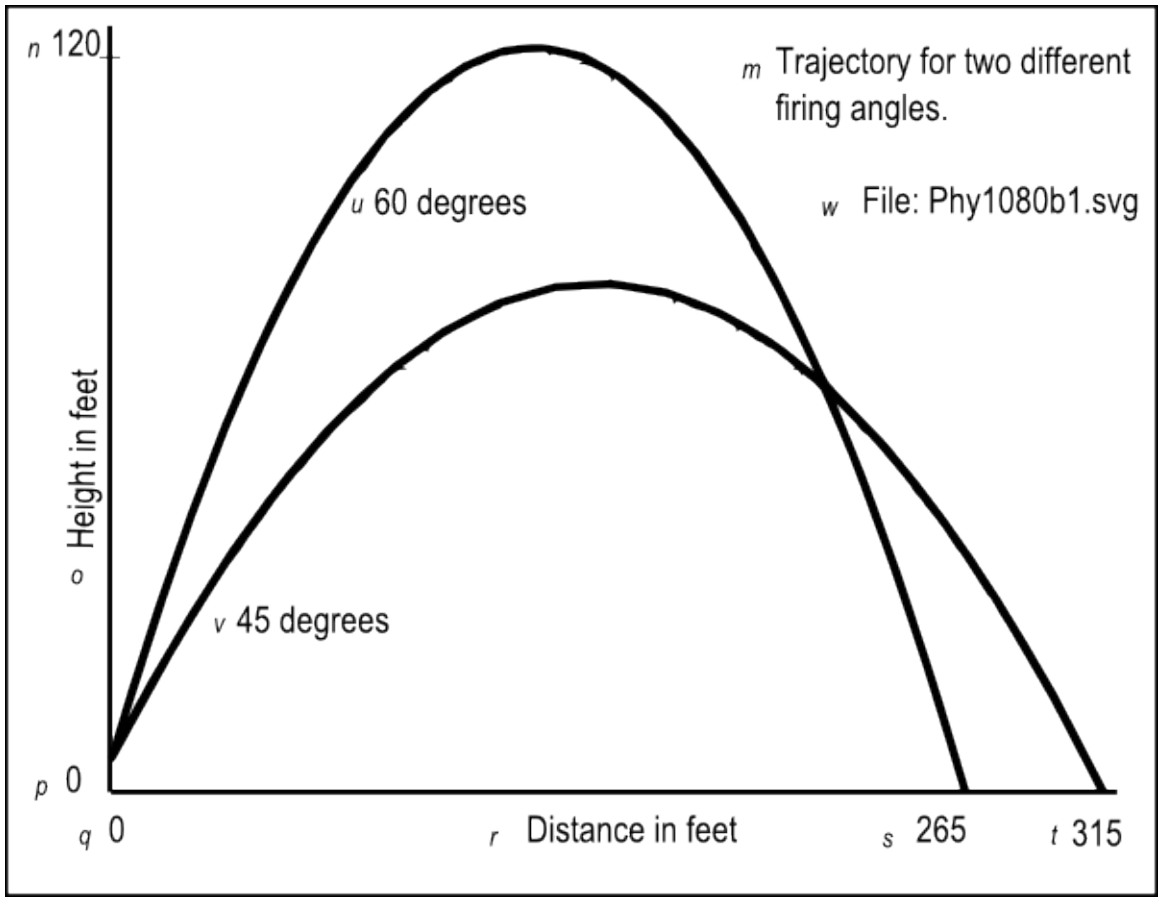


Table 12.13


n 120
m Trajectory for two different firing angles.
u 60 degrees
w File: Phy1080b1.svg
o Height in feet
v 45 degrees
p 0
q 0
r Distance in feet
s 265
t 315


n 120
m Trajectory for two different firing angles.
u 60 degrees
w File: Phy1080b1.svg
o Height in feet
v 45 degrees
p 0
q 0
r Distance in feet
s 265
t 315

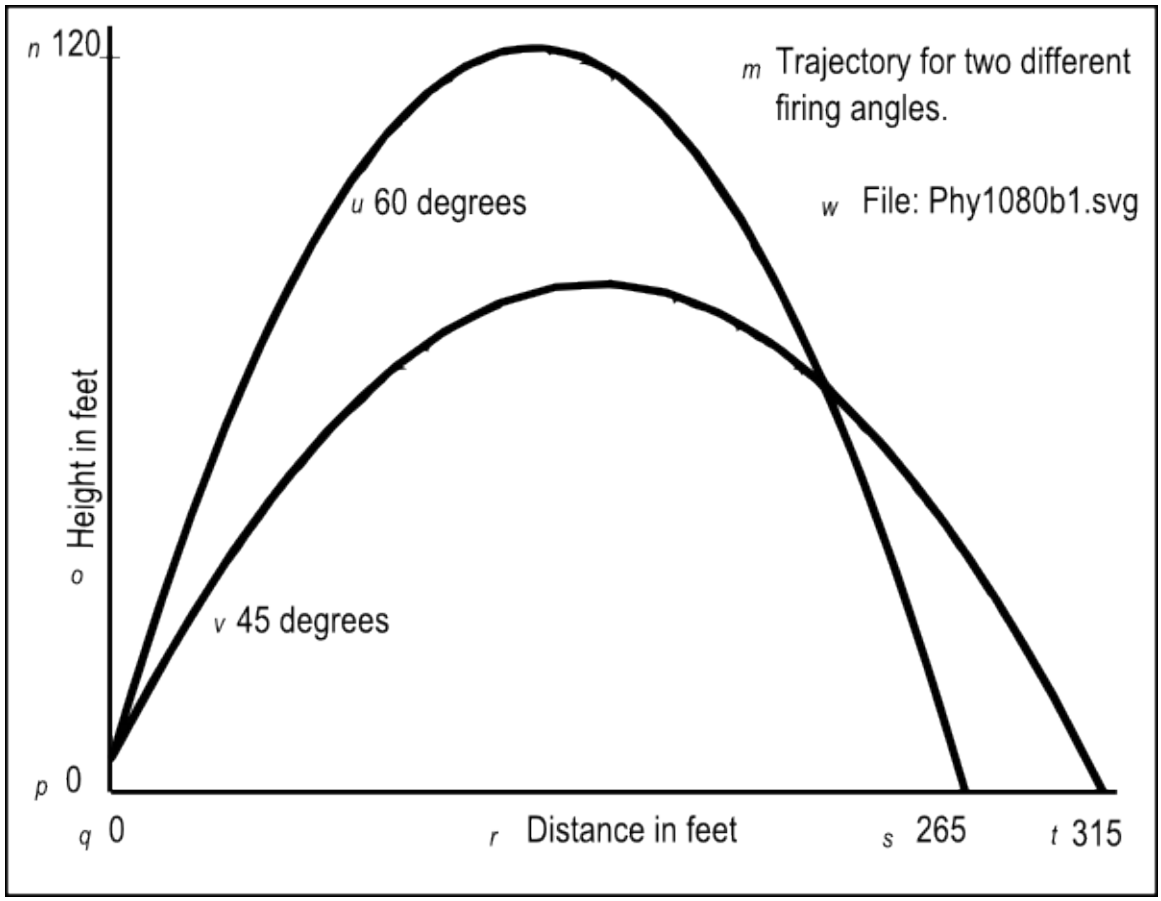

n 120
m Trajectory for two different firing angles.
u 60 degrees
w File: Phy1080b1.svg
o Height in feet
v 45 degrees
p 0
q 0
r Distance in feet
s 265
t 315

Figure 10 . Non-mirror-image contained in the file named Phy1080b1.svg.

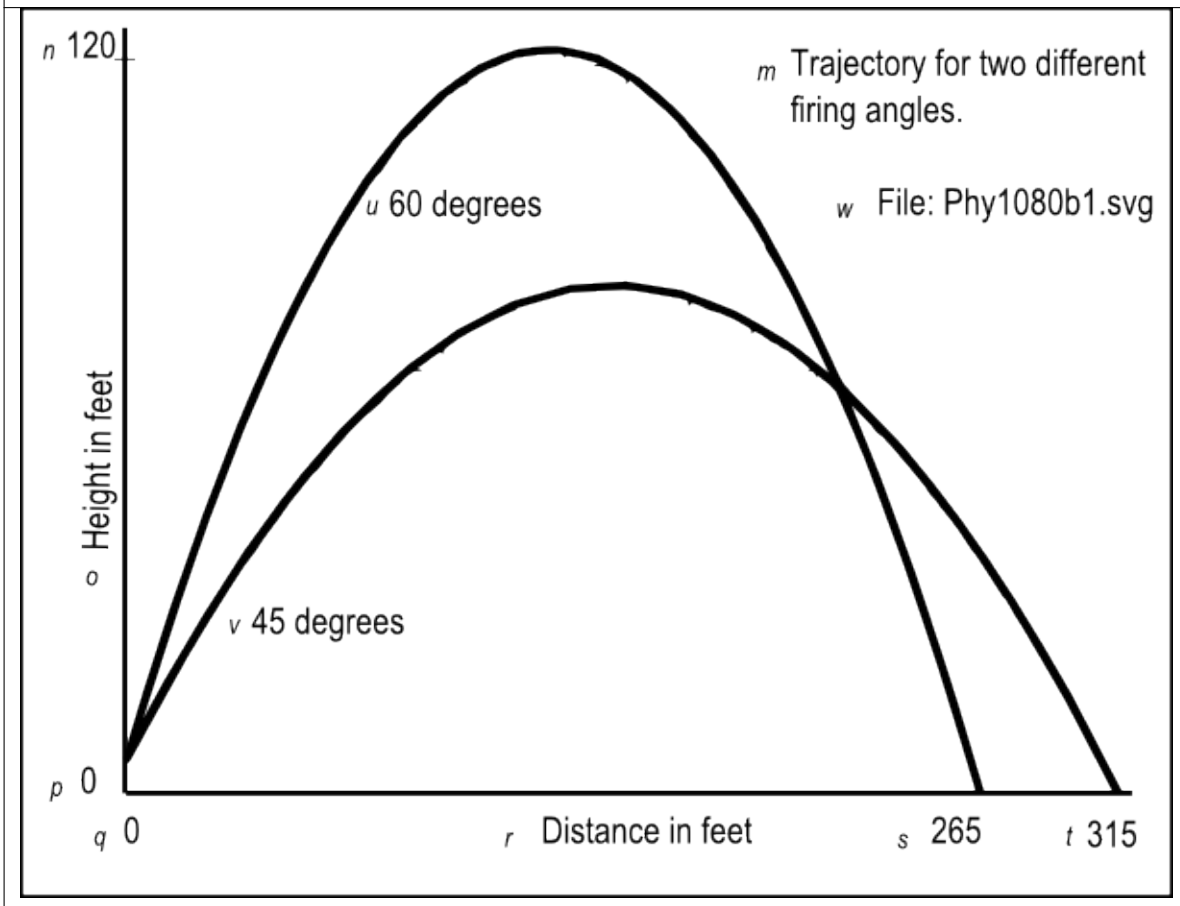


Table 12.14

Key-value pairs

The key-value pairs for the image in the file named Phy1080b1 are shown in Figure 11 (p. 363) .

Figure 11 . Key-value pairs for the image in Phy1080b1.svg.

```
m: Trajectory for two different firing angles
n: 120
o: Height in feet
p: 0
q: 0
r: Distance in feet
s: 265
t: 315
u: 60 degrees
v: 45 degrees
w: File: Phy1080b1.svg
```

Table 12.15

Analysis of the output

Figure 8 (p. 355) contains the x and y coordinates of the arrow's position every 0.25 seconds from the time that it is released until it strikes the ground a little less than 4.5 seconds later.

If you are unable to create tactile graphics based on the file named Phy1080b1.svg, I recommend that you plot the arrow's positions on your graph board in order to get a feel for the shape of the trajectory This is a classic trajectory curve that applies to many different types of projectiles.

Compute for a different angle

Next, I recommend that you modify the script by changing the value of the variable named **ang** from 45 degrees to 60 degrees and load the modified version of the html file into your browser. This should produce the output shown in Figure 12 (p. 364) . This output is also plotted in the image in the file named Phy1080b1.svg.

Figure 12 . Screen output for Listing #4 at 60 degrees.

```
    Start Script
t = 0.00 seconds     x = 0.0 feet     y = 6.0 feet
t = 0.25 seconds     x = 12.5 feet     y = 26.7 feet
t = 0.50 seconds     x = 25.0 feet     y = 45.3 feet
t = 0.75 seconds     x = 37.5 feet     y = 61.9 feet
t = 1.00 seconds     x = 50.0 feet     y = 76.5 feet
t = 1.25 seconds     x = 62.5 feet     y = 89.1 feet
t = 1.50 seconds     x = 75.0 feet     y = 99.7 feet
t = 1.75 seconds     x = 87.5 feet     y = 108.3 feet
t = 2.00 seconds     x = 100.0 feet     y = 114.8 feet
t = 2.25 seconds     x = 112.5 feet     y = 119.4 feet
t = 2.50 seconds     x = 125.0 feet     y = 121.9 feet
t = 2.75 seconds     x = 137.5 feet     y = 122.4 feet
t = 3.00 seconds     x = 150.0 feet     y = 120.9 feet
t = 3.25 seconds     x = 162.5 feet     y = 117.4 feet
t = 3.50 seconds     x = 175.0 feet     y = 111.9 feet
t = 3.75 seconds     x = 187.5 feet     y = 104.3 feet
t = 4.00 seconds     x = 200.0 feet     y = 94.8 feet
t = 4.25 seconds     x = 212.5 feet     y = 83.2 feet
t = 4.50 seconds     x = 225.0 feet     y = 69.7 feet
t = 4.75 seconds     x = 237.5 feet     y = 54.1 feet
t = 5.00 seconds     x = 250.0 feet     y = 36.5 feet
t = 5.25 seconds     x = 262.5 feet     y = 16.9 feet
t = 5.50 seconds     x = 275.0 feet     y = -4.7 feet
End Script
```

Table 12.16

Plot this curve also

Once again, if you are unable to create tactile graphics from the file named Phy1080b1.svg, I recommend that you plot this set of results on top of the previous results for 45 degrees and compare the two. Which arrow went higher? Which arrow went further along the horizontal axis?

Perhaps you could do the same thing for some other launch angles and determine the launch angle that provides the greatest horizontal range and which launch angle provides the greatest vertical height. How far does the arrow fly when launched at an angle of 0 degrees from a height of 6 feet?

Analysis of the code

The code in this script treats the horizontal and vertical components of motion independently and uses the equation given above (p. 352) to determine the position along that axis as a function of time every 0.25 seconds.

Common parameter values

The code in Listing 4 (p. 354) begins by defining common parameter values for the firing angle, the time increment at which coordinate values will be computed and displayed, and the initial velocity of the arrow in the firing direction.

Horizontal and vertical parameters

Following that, the code in Listing 4 (p. 354) defines horizontal parameters and vertical parameters that will be used to compute the horizontal and vertical components of motion respectively.

Horizontal acceleration

The horizontal acceleration is set to zero. In the absence of air resistance, there is nothing to cause the horizontal component of the arrow to speed up or slow down until it stops suddenly when it hits the ground.

Vertical acceleration

The vertical acceleration is set to the acceleration of gravity at the surface of the earth, which is the same value used for the previous exercises in this module.

Decompose the initial velocity

The cosine and sine functions are used to decompose the initial velocity into horizontal and vertical components of velocity.

Initial horizontal and vertical positions

Finally, the horizontal position when the arrow is released is set to 0 and the vertical position is set to 6 feet, approximately the height of the release point for an archer that is six feet tall.

Working variables

After defining the horizontal and vertical parameters, the code in Listing 4 (p. 354) declares working variables for time, horizontal position (x), and vertical position (y).

A while loop

A **while** loop is used to iterate for as long as the arrow is above the ground.

During each iteration, the equation given above (p. 352) is evaluated twice, once to determine the height of the arrow and once to determine the horizontal position of the arrow at the current time.

Time starts at 0, and increments by +0.25 seconds during each iteration.

The document.write method is called inside the while loop to display the results shown in Figure 8 (p. 355) .

12.4.6 Acceleration of gravity exercise #3

A quadratic equation

Now let's turn things around and approach the problem from a different viewpoint. If we subtract **d** from both sides of the motion equation given above (p. 352) , replace a by g to avoid confusion later, and then rearrange the terms, we get **the following equation** :

0.5*g*t^2+v0*t-d = 0

where

- d is distance in units of distance
- v0 is the initial velocity in units of distance/time
- t is time in units of time
- g is acceleration in units of distance/time^2

In this form, you may recognize this as a standard quadratic equation which is **often written as**

a*t^2 + b*t + c = 0

(Now you know why I replaced the acceleration term **a** by the acceleration term **g** earlier. I need to use the symbol **a** in the standard quadratic equation.)

Standard solution for a quadratic equation

Given values for a, b, and c, you should know that the solution for determining the values for t is to find the roots of the equation.

There are two roots (hence two values for t). You should also know that the two roots can be found by evaluating the **quadratic formula** in two forms.

t1 = (-b+Math.sqrt(b*b-4*a*c))/(2*a);

t2 = (-b-Math.sqrt(b*b-4*a*c))/(2*a);

Using the solution

Relating the coefficients in the standard motion equation (p. 365) to the coefficients in the standard quadratic equation (p. 366) gives us:

- a = 0.5*g
- b = v0
- c = -d

The scenario

Let's use the scenario posed in the first exercise (p. 331) in this module. In this scenario, an archer that is six feet tall shoots an arrow directly upward with a velocity of 100 feet per second. Assume that the arrow is at a height of 6 feet when it leaves the bow. Also ignore the effects of air resistance.

Referring back to Figure 1...

We learned from the table in Figure 4 (p. 343) that the arrow was at a height of 89.9 feet at 1 second on the way up from the surface of the earth, and was at a height of 89.9 feet again on the way down at approximately 5.2 seconds.

Let's pretend that we don't have the table in Figure 1 (p. 343) and write a script that uses the quadratic form of the standard motion equation (p. 365) along with the quadratic formula (p. 366) to compute the times that the arrow was at a height of 89.9 feet.

Do it also for the moon

To make the problem even more interesting, let's also cause the script to compute and display the times that the arrow was at a height of 89.9 feet assuming that the archer was standing on the surface of the moon instead of the earth.

An issue involving the initial height

The standard motion equation (p. 365) can be used to compute the time that the arrow has traveled to any specific height, relative to the height from which it was released. Therefore, since the arrow was released from a height of 6 feet and it initially traveled up, we need to determine the time at which it had traveled 83.9 feet to find the time that it was actually at 89.9 feet.

The JavaScript code

Please copy the code from Listing 5 (p. 367) into an html file and open the file in your browser.

Listing 5 . Acceleration of gravity exercise #3.

```
    <!---------------- File JavaScript05.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br></br>");

//The purpose of the getRoots function is to compute and
// return the roots of a quadratic equation expressed in
// the format
//   a*x^2 + b*x + c = 0
//The roots are returned in the elements of a two-element
// array. If the roots are imaginary, the function
// returns NaN for the value of each root.
function getRoots(a,b,c){
  var roots = new Array(2);
  roots[0] = (-b+Math.sqrt(b*b-4*a*c))/(2*a);
  roots[1] = (-b-Math.sqrt(b*b-4*a*c))/(2*a);
  return roots;
}//end getRoots

//Initialize the problem parameters.
var gE = -32.2;//gravity in ft/sec*sec on Earth
var gM = -32.2*0.167;//gravity in ft/sec*sec on the Moon
var v0 = 100;//initial velocity in ft/sec
var d0 = 6;//initial height in feet
var d = 89.9-d0;//target height corrected for initial height

//Equate problem parameters to coefficients in a
// standard quadratic equation.
var a = 0.5*gE;
var b = v0;
var c = -d;

//Solve the quadratic formula for the two times at which the
// height matches the target height corrected for the
// initial height.
var roots = getRoots(a,b,c);

//Extract the two time values from the array.
var t1 = roots[0];//time in seconds
var t2 = roots[1];//time in seconds

//Display the results for the earth.
document.write("On Earth </br>");
document.write("Arrow is at " + (d+d0) + " feet at " +
               t1.toFixed(2) + " seconds" + "</br>");
document.write("Arrow is at " + (d+d0) + " feet at " +
               t2.toFixed(2) + " seconds" + "</br></br>");

//Compute and display for the moon
//Adjust the value of a to represent the acceleration
// of gravity on the moon
a = 0.5*gM;
```

Table 12.17

Screen output

The text shown in Figure 13 (p. 368) should appear in the browser window when you open the html file in the browser.

Figure 13 . Screen output for Listing #5.

```
   Start Script

On Earth
Arrow is at 89.9 feet at 1.00 seconds
Arrow is at 89.9 feet at 5.21 seconds

On the Moon
Arrow is at 89.9 feet at 0.86 seconds
Arrow is at 89.9 feet at 36.33 seconds

End Script
```

Table 12.18

A function named getRoots

The quadratic formula isn't very complicated, but it is fairly tedious and easy to type incorrectly. Therefore, I decided to encapsulate it in a function that we can copy into future scripts saving us the need to type it correctly in the future.

Listing 5 (p. 367) begins with the definition of a function named **getRoots** that receives the parameters a, b, and c, and returns the roots of the quadratic equation in a two-element array.

Real or imaginary roots

The roots of a quadratic equation can be either real or imaginary. If the roots are imaginary, this function simply returns NaN (not a number) for each root.

The parameters of the problem

Following the definition of the getRoots function, Listing 5 (p. 367) declares and initializes several variables to establish the parameters of the problem, such as the acceleration of gravity on the earth and moon, the initial velocity of the arrow, etc.

The computed height versus the target height

The target height for the problem is 89.9 feet. Note that the variable named d contains that value less the initial height of 6 feet. Thus, the script will find the time at which the arrow has traveled 83.9 feet on the way up, and the time that it has traveled that same distance on the way back down.

Establish quadratic coefficients

The next three lines of code use the problem parameters to establish values for the standard coefficients of a quadratic equation, a, b, and c, as described above (p. 365) . Note that at this point in the script, the coefficient named **a** is based on the acceleration of gravity on earth. (Later, it will be changed to reflect the acceleration of gravity on the moon.)

Get the roots of the quadratic equation

Then the script calls the getRoots function, passing a, b, and c as parameters, and stores the returned array containing the roots in the variable named **roots** .

Following that, the script extracts the roots from the array and displays them as shown by the text in the upper half of Figure 13 (p. 368) .

Repeat the process for the moon

Then Listing 5 (p. 367) sets the value of the coefficient named **a** to reflect the acceleration of gravity on the moon, repeats the process, and displays the results in the lower half of Figure 13 (p. 368) .

Note that the arrow reaches the target height somewhat quicker on the moon due to the lower acceleration of gravity, and takes much longer to arrive at the same height on the way back down to the surface of the moon. Were we to create a chart similar to Figure 4 (p. 343) for the moon, we would see that the arrow goes much higher before turning around and falling back to the surface of the moon.

12.4.7 Other useful equations

You have learned how to use the following equation to solve various physics problems involving motion in a straight line with uniform acceleration so far in this module.

h = v0*t + 0.5*g*t^2

where

- h is the distance of the projectile above the surface of the earth in units of distance
- v0 is the initial velocity of the projectile in units of distance/time
- t is time in seconds
- g is the acceleration of gravity, approximately 9.8 meters per second squared, or approximately 32.2 feet per second squared.

Some physics textbooks also list the following equations **as being important** .

v = v0 + g*t

v^2 = v0^2 + 2*g*h

where v is the velocity of the object and the other terms are the same as described above (p. 369) .

12.4.7.1 Exercise to find the velocity

Let's do an exercise using the first of the two equations given above (p. 369) .

An individual on the surface of the earth shoots an arrow directly upward with a velocity of 100 feet per second. How many seconds elapse before the arrow turns and starts falling towards the surface of the earth. Ignore the effects of air resistance.

Create a script

Please copy the code from Listing 6 (p. 370) into an html file and open the file in your browser.

Listing 6 . Exercise to find the velocity.

```
    <!---------------- File JavaScript06.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br></br>");

//Initialize the problem parameters.
var g = -32.2;//gravity in ft/sec*sec on Earth
var v0 = 100;//initial velocity in ft/sec

//Given that v = v0 + g * t
//At what time does the velocity go to zero?

//Rearrange the terms in the equation.
var t = -v0/g;

//Display the results
document.write("Arrow has zero velocity at " +
               t.toFixed(2) + " seconds " + "</br>");

document.write("</br>End Script");

</script>
</body></html>
```

Table 12.19

Screen output

The text shown in Figure 14 (p. 370) should appear in your browser window when you open the html file in your browser.

Figure 14 . Screen output for Listing #6.

```
    Start Script

Arrow has zero velocity at 3.11 seconds

End Script
```

Table 12.20

Analysis of the code

Compared to working through the solutions to the previous exercises, this one seems almost trivial.

After establishing values for the acceleration of gravity and the initial velocity of the arrow, the code in Listing 6 (p. 370) rearranges the first equation given above (p. 369) and solves for the value of time at which

the velocity goes to zero. This is the point in time when the arrow turns from moving up and begins falling back toward the earth.

The results are shown in Figure 14 (p. 370) . You should compare this result with Figure 1 (p. 335) , which shows that the arrow reaches its maximum height at approximately 3 seconds, which agrees very well with the result shown in Figure 14 (p. 370) .

12.4.7.2 Exercise to find the height

Let's do an exercise using the second of the two equations given above (p. 369) .

An individual that is six feet tall standing on the surface of the earth shoots an arrow directly upward with a velocity of 100 feet per second. What is the maximum height achieved by the arrow before it turns and falls back towards the surface of the earth? Ignore the effects of air resistance.

Create a script

Please copy the code from Listing 7 (p. 371) into an html file and open the file in your browser.

Listing 7 . Exercise to find the height.

```
   <!---------------- File JavaScript07.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br></br>");

//Initialize the problem parameters.
var g = -32.2;//gravity in ft/sec*sec on Earth
var v0 = 100;//initial velocity in ft/sec
var h0 = 6;//initial height

//Given that v^2 = v0^2 + 2*g*h
//What is the maximum height reached by the arrow?
//Note that the maximum height is six feet more than
// the value given by the above equation because that
// equation is based on the point of release.

//The maximum height occurs when the velocity goes to zero.
//Setting the velocity to zero and rearranging the terms
// in the equation gives:
var h = h0 + (-(v0 * v0))/(2*g);

//Display the results
document.write("Arrow reaches maximum height of " +
               h.toFixed(2) + " feet " + "</br>");

document.write("</br>End Script");

</script>
</body></html>
```

Table 12.21

Screen output

The text shown in Figure 15 (p. 372) should appear in your browser window when the html file is opened in your browser.

Figure 15 . Screen output for Listing #7.

```
   Start Script

Arrow reaches maximum height of 161.28 feet

End Script
```

Table 12.22

Analysis of the code

Once again, compared to working through the previous exercises, this one also seems almost trivial.

After establishing values for the acceleration of gravity, the initial velocity of the arrow, and the height at which the arrow was released, the code in Listing 7 (p. 371) rearranges the second equation given above (p. 369) and solves for the value of the height (relative to the release point) at which the velocity goes to zero. This is the point in the trajectory where the arrow turns from moving up and begins falling back toward the earth.

Note that in order to get the actual height, it was necessary to add the initial height of 6 feet to the computed height.

Compare the results

The results are shown in Figure 15 (p. 372) . You should compare this result with Figure 4 (p. 343) , which shows that the arrow reaches its maximum height at approximately 161.1 feet, which agrees very well with the result shown in Figure 15 (p. 372) .

12.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

12.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

12.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Motion – Variable Velocity and Acceleration
- File: Phy1080.htm

- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · velocity
  - · instantaneous velocity
  - · variable velocity
  - · acceleration
  - · acceleration of gravity
  - · gravity
  - · tangent line
  - · slope of a line
  - · inflection point

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 13

Force – Introduction to Statics, Equilibrium, and Forces[1]

13.1 Table of Contents

- Preface (p. 375)
  - General (p. 375)
  - Prerequisites (p. 375)
  - Supplemental material (p. 376)
- General background information (p. 376)
  - A real-world example of force and equilibrium (p. 376)
  - Other examples of force (p. 379)
- Resources (p. 380)
- Miscellaneous (p. 380)

13.2 Preface

13.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

The purpose of this module is to provide an introduction to statics, equilibrium, and forces in a format that is accessible to blind students.

13.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

[1] This content is available online at <http://cnx.org/content/m38117/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

13.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

13.3 General background information

I will provide general information about forces and equilibrium in this module and will provide exercises involving those forces in a future module.

13.3.1 A real-world example of force and equilibrium

Assume that you are standing on the surface of the earth. As I explained in an earlier module, there is a gravitational attraction between your body mass and the mass of the earth that is directly proportional to the product of the two masses and inversely proportional to the square of the distance between the center of mass of your body and the center of mass of the earth.

What about a gas planet?

If the earth was a gas planet such as Jupiter, that gravitational attraction would probably pull your body into the inner portions of the planet and your life would probably come to an abrupt end. That would be similar to trying to stand on the surface of a lake.

Impenetrable material

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

Fortunately, the earth is made of rock and other relatively impenetrable material, which prevents you from being pulled into the inner portions of the earth by that gravitational attraction.

In order to prevent you from being pulled into the earth, the ground that you are standing on must push up on the bottom of your feet by an amount equal to the gravitational pull that is being exerted on your body.

Your skeleton is critical

In addition, the bones in your inner skeleton must be capable of withstanding the pull of gravity on your body to prevent your body from turning into a puddle on the surface of the earth. For example, a raw egg doesn't have an inner skeleton; it has an exoskeleton, the shell. When you break the shell over a frying pan and dump the egg out of the shell, the attraction of gravity causes it to become a puddle in the frying pan.

The push is a force

In simplified terms, a force is a push or a pull. The push that is being exerted upward on the bottom of your feet by the surface of the earth is commonly called a force.

The gravitational attraction that is trying to pull you into the center of the earth is also a force (which is often referred to as your weight).

Equilibrium

If you are standing in one spot without moving up or down, the entire system that constitutes your body and the earth is in equilibrium. This means that the upward force being exerted on the bottom of your feet by the surface of the earth is equal in magnitude and opposite in direction from the gravitational force that is trying to pull you into the earth.

An object is in equilibrium when it is not being accelerated. It may be at rest and remain at rest or it may be moving with a constant velocity and continue to move at the same velocity. If it is being accelerated, which includes changing its direction of motion, it is not in equilibrium.

Force is a vector quantity

In order for a body to be in equilibrium, the vector sum of all the forces acting on that body must be zero.

The bathroom scales

Many households in the U.S. have a device that is commonly known as the bathroom scale. We use it to determine "how much we weigh."

Assume that you walk into your bathroom and step onto the bathroom scale. To understand everything that is going on, we need to recognize that the house and the scale inside the house are also being pulled toward the center of the earth by gravitational attraction. Fortunately the surface of the earth normally exerts an upward force on the foundation of the house to prevent that from happening. The foundation exerts an upward force on the bathroom floor, and the floor exerts an upward force on the scale.

When the strength of the earth fails...

Sometimes, under some conditions, the surface of the earth fails to exert an upward force on the foundation of a house sufficient to prevent it from accelerating toward the center of the earth. I recently saw a video on the news of a house that was pulled into a sinkhole in Florida. I have seen several news videos where houses have been pulled toward the center of the earth during landslides in California.

How much do you weigh?

When you are standing on the scale, the gravitational force exerted on your body is being supported by the top surface of the scale. The top surface of the scale is pushing up with a force that is equal and opposite to the gravitational force that is trying to pull you toward the center of the earth.

The scale also has a weight

A gravitational force is also being exerted on the scale trying to pull it into the center of the earth. The floor underneath the scale is exerting an upward force on the scale equal to the combined gravitational force of you and the scale, causing you, the scale, and the floor to be in equilibrium.

Compression

These two forces, one pushing down on the top of the scale and the other pushing up on the bottom of the scale causes the scale to experience something commonly called compression. In other words, the opposing

forces are trying to squeeze the scale into a puddle but the material from which the scale was constructed won't allow that to happen.

Tension versus compression

As an aside, the state that we call tension is the opposite of the state that we call compression. If you sit with all of your weight on a stool, the legs of that stool will be in a state of compression. If you sit with all of your weight in a porch swing supported by chains attached to the ceiling, those chains will be in tension.

A readout in pounds

The bathroom scale contains a mechanism that is capable of measuring the amount of compression that it is experiencing. It also contains a display mechanism that displays the amount of compression. Typically in the U.S., that compression value is converted to force and displayed in units of pounds.

The objective of the bathroom scale

The objective of the bathroom scale, therefore, is to measure the gravitational force exerted on your body (your weight) and to display that value for your pleasure or displeasure as the case may be.

Units

In the so-called English system of units that is commonly used in the U.S., the word **pound** is used to indicate mass and the words **pound force** are used to indicate force. That can be confusing.

In the International System of Units, abbreviated SI, the standard unit of mass is the kilogram, abbreviated kg, and the standard unit of force is the newton, abbreviated N.

A derived unit

As you learned in an earlier module, the SI units consist of base units and units that are derived from base units. The newton is a derived unit, which when expressed in SI base units is:

1 N = 1 kg *m/s^2

where

- N is the symbol for newton
- kg is the symbol for mass in kilograms
- m is the symbol for length in meters
- s is the symbol for time in seconds
- ^2 means "raised to the second power" or squared

Knowing the relationship between the newton and its base units is important when you are evaluating an equation and need to cause the units to cancel out so as to end up with only the correct units for the result.

Conversion factor

The force that is trying to pull you into the center of the earth (your weight) can be expressed in any of the following units:

- pound force
- newtons
- kg*m/s^2

The following conversion factor can be used to convert from English units to SI units:

1 pound force = 4.44822162 newtons

To convert your weight from pounds to newtons, multiply your weight in pounds by 4.448.

To convert your weight from newtons to pounds, divide your weight in newtons by 4.448.

If your bathroom scale says that your weight is 100 pounds, a bathroom scale calibrated in newtons would say that your weight is 444.8 newtons.

The basis for your weight

You may be wondering why you weigh as much as you do (other than as a result of your appetite for sweets). As mentioned earlier, the basis for your weight is the gravitational attraction between your body mass and the mass of the earth.

The acceleration of gravity

In an earlier module, you learned that two bodies in free fall above the surface of the earth, in the absence of an atmosphere, will accelerate toward the center of the earth at the same rate, which is about 32.2 ft/sec^2 or 9.8 m/s^2.

Your weight, or the gravitational force exerted on your body, is the force necessary to cause your body to accelerate toward the center of the earth at 32.2 ft/sec^2.

Acceleration = force/mass

You also learned in an earlier module that the acceleration of an object due to an applied force is directly proportional to the force and inversely proportional to the mass of the object.

Therefore, a greater force must be applied to an object with greater mass to achieve the same acceleration. The greater your body mass, the greater must be the force that will cause your body to accelerate at 32.2 ft/sec^2 toward the center of the earth.

Your weight

That force is what we commonly refer to as your weight. The greater your body mass, the greater will be your weight.

Your body in equilibrium

If you climb out and sit on the limb of a tree, that limb must be capable of exerting an amount of upward force sufficient to cancel the gravitational force acting on your body. Otherwise, you will fall to the surface of the earth or to something between the limb and the surface of the earth that is capable of exerting that upward force..

If the limb doesn't break and you are able to sit there in comfort, the entire system including the tree, the limb, your body, and the earth will be in equilibrium.

Your body out of equilibrium

If the limb is incapable of sustaining that amount of upward force, it will break and allow you to accelerate towards the center of the earth. That means that the system is not in equilibrium.

13.3.2 Other examples of force

So far, I have limited the discussion to forces that are directly attributable to gravitational attraction. I began this explanation with that limitation because the effects of gravitational force are something that most of us understand pretty well.

However, there are many other examples of force, most which involve the conversion of energy into force.

A human who lifts weights

For example, a human who lifts weight eats a diet that contains energy, commonly measured in calories. The human body has the ability to convert that energy into the contraction of muscle tissue.

When the contraction of muscle tissue is applied in just the right way, a human is capable of generating forces that overcome the force of gravity exerted on an object and lift that object from the surface of the earth, or whatever is supporting it.

Conversion of chemical energy

A chemical explosion has the ability to convert chemical energy into force and cause large rocks to break their molecular bonds and shatter into small pieces, or to cause rockets to overcome the force of gravity and accelerate into outer space, leaving the effects of the earth's gravitational field behind.

Overcoming gravity

Much energy is expended in daily life to overcome the force of gravity. For example, chemical energy is converted to force to lift large steel beams high in the air to build skyscrapers. Human caloric energy is converted to various forces to tote roofing shingles up a ladder onto a rooftop to make a house water proof. Various types of energy are converted to various forces to lift water to irrigate crops. Chemical energy is converted to various forces to cause airplanes to fly. Thermal energy is converted to various forces to cause hot-air balloons to fly.

Converting kinetic energy to force

Various weather processes cause solar energy to be converted into kinetic energy in the form of moving air (wind). The blades on a windmill convert that kinetic energy to forces that are used to pump water or to make electrical power.

When water moves from a high altitude to a lower altitude, potential energy is converted to kinetic energy. The blades on a water wheel or a turbine convert that kinetic energy into forces that are used to grind grain or make electrical power.

Conversion of energy to force

With the possible exception of gravitational force, I can't think of any kind of force that doesn't result from the conversion of some form of energy. As an engineer, I strongly suspect that even gravitational force results from the conversion of energy at some level, although that conversion may take place at the molecular or atomic level.

13.4 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

13.5 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force – Introduction to Statics, Equilibrium, and Forces
- File: Phy1090.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · statics
  - · forces
  - · equilibrium

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 14

Force – Vector Solutions for Coplanar Forces Concurrent at a Point[1]

14.1 Table of Contents

- Preface (p. 383)
  - General (p. 383)
  - Prerequisites (p. 384)
  - Viewing tip (p. 384)
    - Figures (p. 385)
    - Listings (p. 385)
  - Supplemental material (p. 385)
- General background information (p. 385)
  - Creation of tactile graphics (p. 385)
  - The law of sines (p. 386)
  - The law of cosines (p. 396)
- Discussion and sample code (p. 397)
  - Using the graph board (p. 397)
  - Parallelogram of forces (p. 397)
  - Triangle of forces (p. 418)
  - Trigonometric solution to the triangle of forces (p. 428)
  - Using components to analyze for equilibrium (p. 428)
- Run the scripts (p. 442)
- Resources (p. 442)
- Miscellaneous (p. 442)

14.2 Preface

14.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38125/1.5/>.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

This module explains the law of sines and the law of cosines. It also explains vector solutions for coplanar forces that are concurrent at a point in a format that is accessible to blind students.

14.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

14.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

14.2.3.1 Figures

- Figure 1 (p. 390) . Mirror image from the file named Phy1100a1.svg.
- Figure 2 (p. 394) . Non-mirror-image version of the image from the file named Phy1100a1.svg.
- Figure 3 (p. 395) . Key-value pairs for the image in the file named Phy1100a1.svg.
- Figure 4 (p. 401) . Mirror image contained in the file named Phy1100b1.svg.
- Figure 5 (p. 405) . Non-mirror-image version of the image from the file named Phy1100b1.svg.
- Figure 6 (p. 406) . Key-value pairs for the image in the file named Phy1100b1.svg.
- Figure 7 (p. 411) . Mirror image from the file named Phy1100c1.svg.
- Figure 8 (p. 415) . Non-mirror-image version of the image from the file named Phy1100c1.svg.
- Figure 9 (p. 416) . Key-value pairs for the image in the file named Phy1100c1.svg.
- Figure 10 (p. 422) . Mirror image from the file named Phy1100d1.svg.
- Figure 11 (p. 426) . Non-mirror-image version of the image from the file named Phy1100d1.svg.
- Figure 12 (p. 427) . Key-value pairs for the image in the file named Phy1100d1.svg.
- Figure 13 (p. 433) . Mirror image from the file named Phy1100e1.svg.
- Figure 14 (p. 437) . Non-mirror-image version of the image from the file named Phy1100e1.svg.
- Figure 15 (p. 438) . Key-value pairs for the image in the file named Phy1100e1.svg.
- Figure 16 (p. 440) . Screen output for Listing #1.
- Figure 17 (p. 441) . Changes to Listing #1.
- Figure 18 (p. 441) . Screen output for modified Listing #1.

14.2.3.2 Listings

- Listing 1 (p. 439) . Tent pole exercise.

14.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

14.3 General background information

14.3.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explains how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1100.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

[14] http://www.dickbaldwin.com/toc.htm
[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[16] http://cnx.org/content/m38125/latest/Phy1100.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

14.3.2 The law of sines

Much of the background material needed for this module was presented in the earlier module titled Introduction to Statics, Equilibrium, and Forces [18] . However, beginning with this module, you will need some information on trigonometry that was not presented in the earlier module titled Brief Trigonometry Tutorial [19] . That item is known as the law of sines (see http://en.wikipedia.org/wiki/Law_of_sines [20]).

Proving identities

When I was a student in a trigonometry course many years ago, students were required to "prove identities" which was a process very similar to proving theorems in plane geometry. That activity included doing such things as proving that the law of sines is true.

I don't know if mathematics instructors still require students to prove identities or not. In any event, we will simply accept such mathematical truths in this collection of modules and won't worry about proving that they are true.

We will work through some exercises in this module to show how the law of sines can be used to advantage.

Beyond the right triangle

Previous modules using trigonometry have dealt almost exclusively with right triangles. Beginning in this module, we will start using trigonometry to solve problems that involve triangles that are not right triangles.

Tactile graphics

The svg file that is required to create tactile graphics for this exercise is named Phy1100a1.svg. You should have downloaded (p. 385) that file earlier.

Figure 1 (p. 390) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

[18] http://cnx.org/contents/52e8554d-a0e3-4a2f-b18f-6e876178c921/Phy1090-Force—-Introduction-
[19] http://cnx.org/contents/257998f1-068b-4c59-8924-78a29a68b610/Phy1020-Brief-Trigonometry-Tut
[20] http://en.wikipedia.org/wiki/Law_of_sines

Laws of sines and cosines
C = 40 degrees
b = 11.4
a = 10
A = 60 degrees
B = 80 degrees
c = 7.4
File: Phy110001.svg

Laws of sines and cosines
C = 40 degrees
b = 11.4
g = 10
A = 60 degrees
B = 80 degrees
c = 7.4
File: Phy1100a1.svg

Laws of sines and cosines
C = 40 degrees
b = 11.4
a = 10
A = 60 degrees
B = 80 degrees
c = 7.4
File: Phy1100a1.svg

Figure 1 . Mirror image from the file named Phy1100a1.svg.

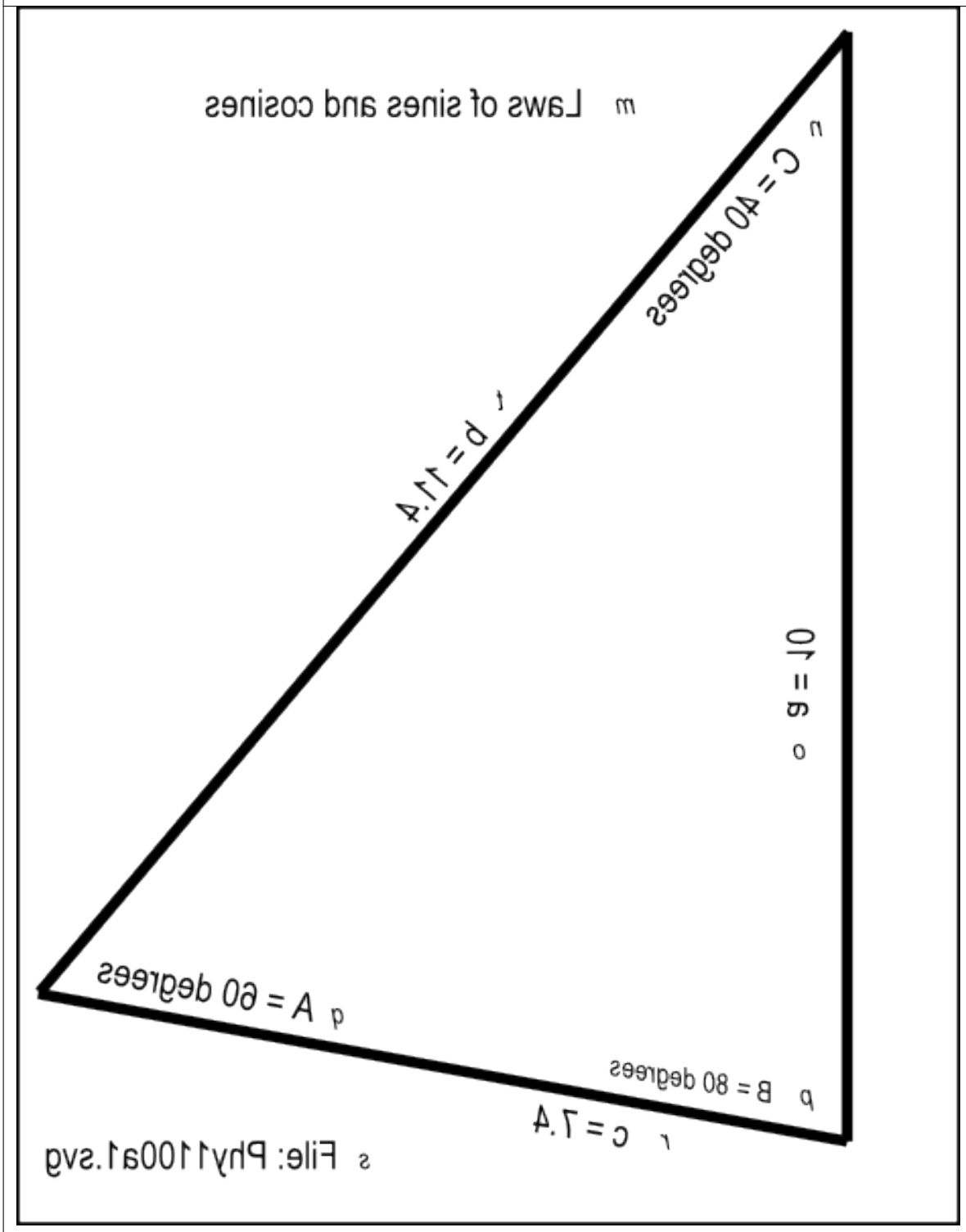
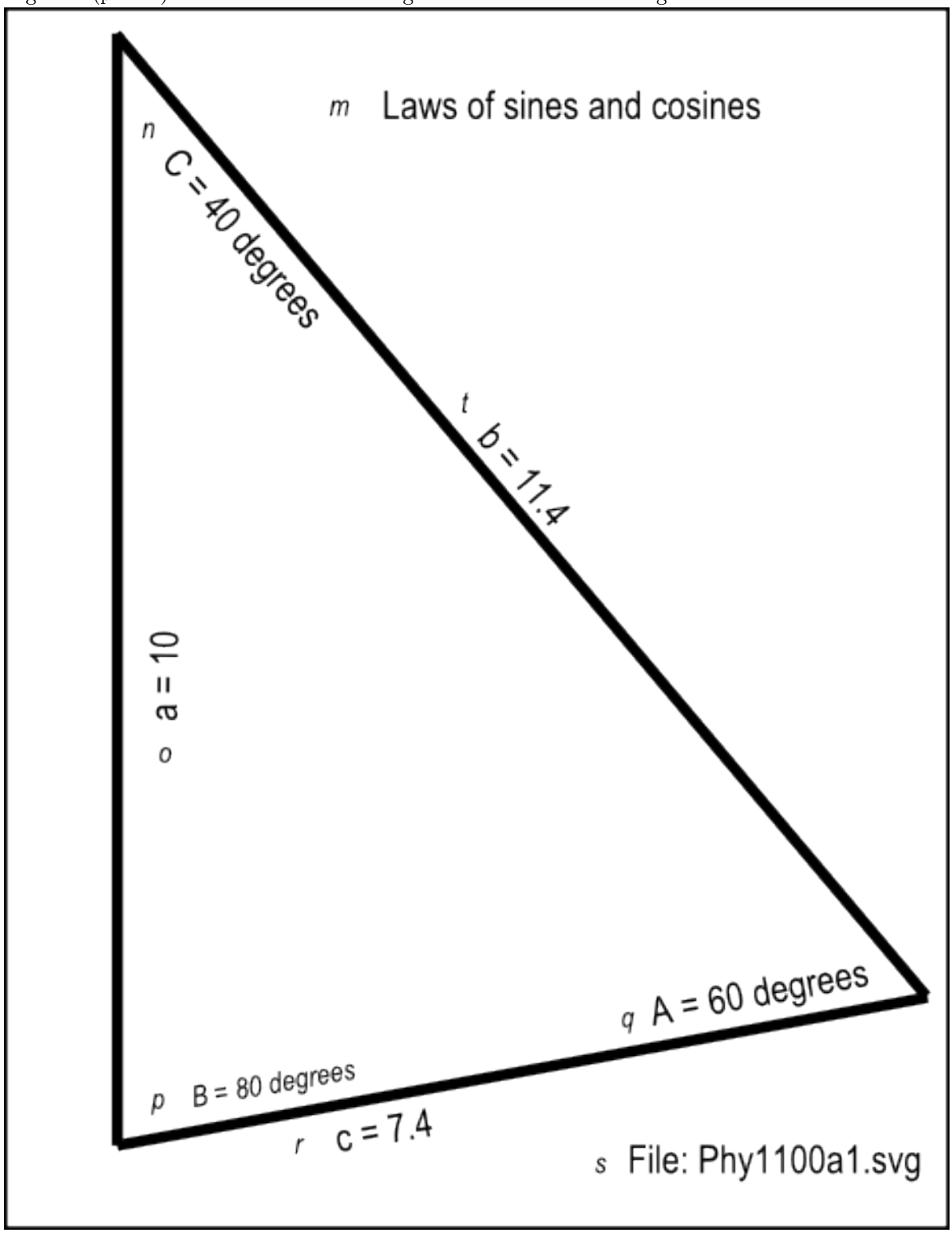
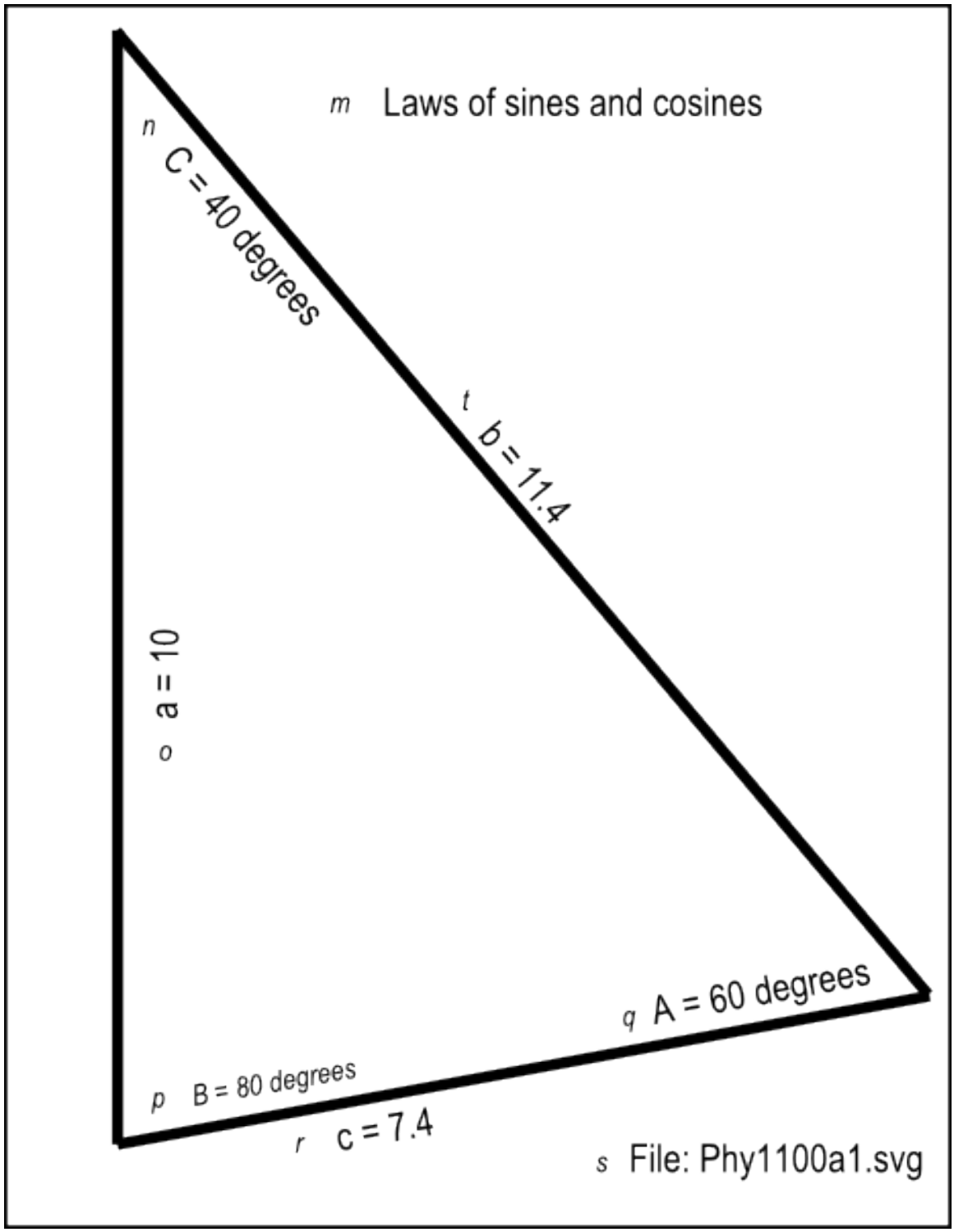


Table 14.1

Figure 2 (p. 394) shows a non-mirror-image version of the same image.




m Laws of sines and cosines
n C = 40 degrees
t b = 11.4
o a = 10
q A = 60 degrees
p B = 80 degrees
r c = 7.4
s File: Phy1100a1.svg

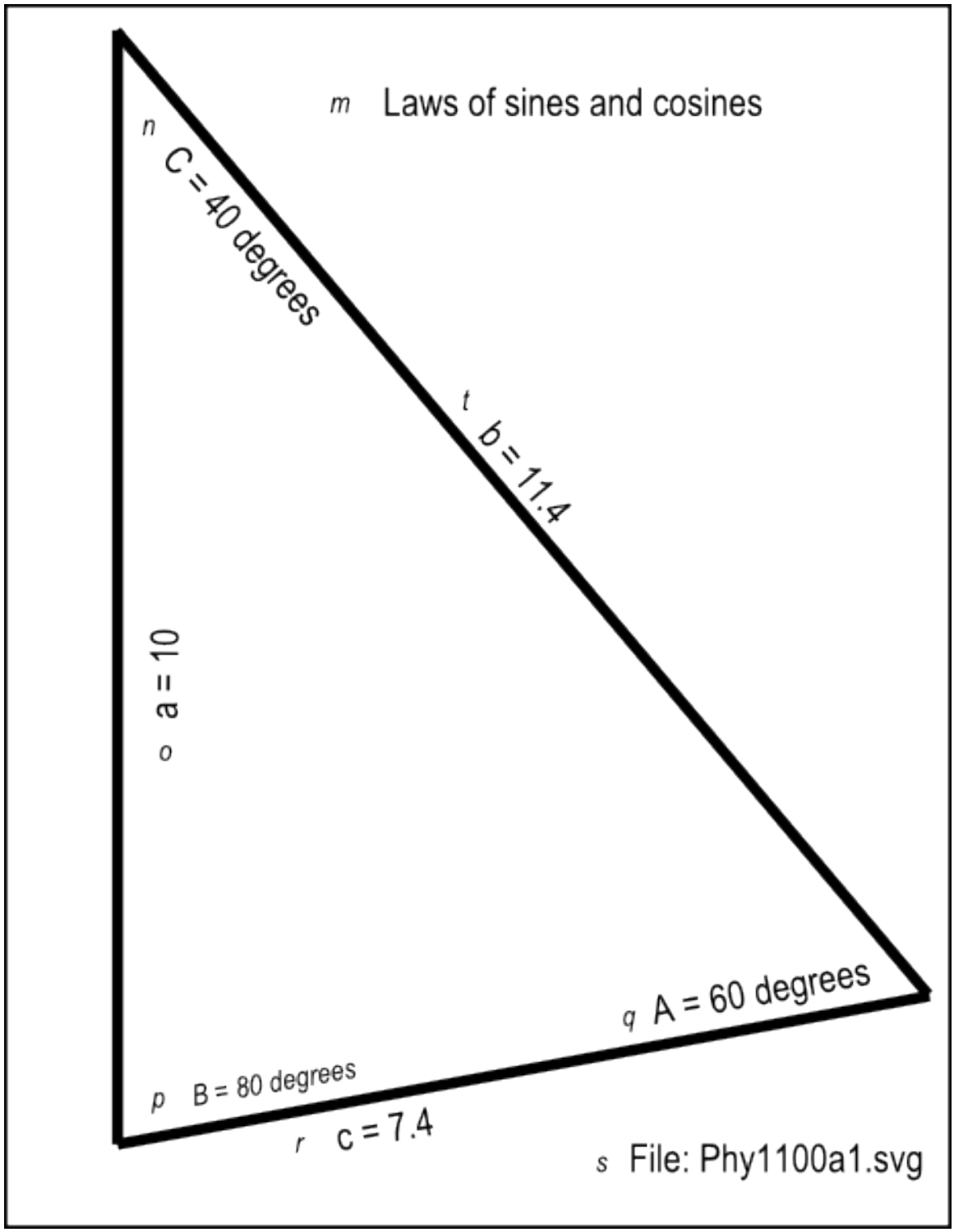

m Laws of sines and cosines
n C = 40 degrees
t b = 11.4
o a = 10
q A = 60 degrees
p B = 80 degrees
r c = 7.4
s File: Phy1100a1.svg

Figure 2 . Non-mirror-image version of the image from the file named Phy1100a1.svg.

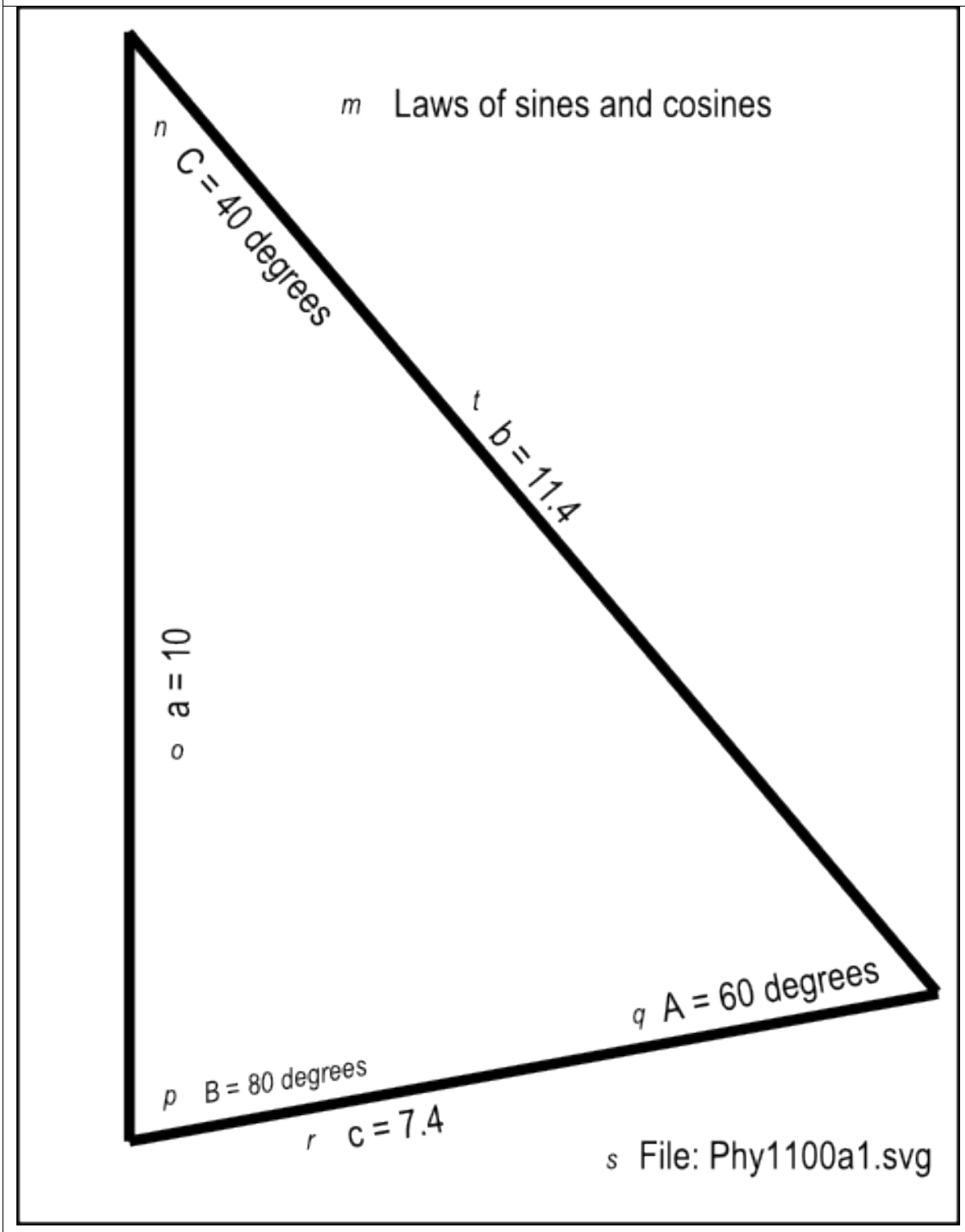


Table 14.2

Key-value pairs

Figure 3 (p. 395) shows the key-value pairs that go with the image in the file named Phy1100a1.svg.

Figure 3 . Key-value pairs for the image in the file named Phy1100a1.svg.

```
m: Laws of sines and cosines
n: C = 40 degrees
o: a = 10
p: B = 80 degrees
q: A = 60 degrees
r: c = 7.4
s: File: Phy1100a1.svg
t: b = 11.4
```

Table 14.3

Create a triangle

Please use your graph board to create a triangle with the dimensions, angles, and labels specified below.

Use your Braille labeler to label the sides of the triangle a, b, and c. Label the angle opposite from side a as A, the angle opposite side b as B, and the angle opposite side c as C.

Adjust your triangle to make side a equal to 10 units, angle B equal to 80 degrees, and angle C equal to 40 degrees. Using the above relationship, we also know that angle A is equal to 60 degrees because the angles in a triangle must sum to 180 degrees.

Using the law of sines

The law of sines can be used to compute the remaining sides of a triangle when two angles and a side are known. Of course, if you know two of the angles of a triangle, you also know the third angle because the sum of the angles in a triangle is always 180 degrees.

Use the Google calculator

All of the arithmetic and trigonometric requirements in this section can be satisfied using the Google calculator. Go to Google and enter the following in the search box:

sine 80 degrees

Google should produce a line of output text that reads:

sine(80 degrees) = 0.984807753

Record this value for future use.

Get and record two more values

Go to Google again and enter sine 40 degrees. The output should be:

sine(40 degrees) = 0.64278761

Doing the same thing again for 60 degrees produces:

sine(60 degrees) = 0.866025404

The law of sines formula

Given the names of the sides and angles that you have applied to your triangle, the law of sines says that the following is true:

(a/sin A) = (b/sin B) = (c/sin C)

In general, the law of sines states that the ratio of the length of any side to the sine of the opposite angle is equal to the ratio of any other side to the sine of its opposite angle.

Compute the length of side b

Substituting the known values in the above formula gives us:

(10/0.866) = (b/0.985) = (c/0.643)

Combining the first and second ratios and rearranging terms gives us:
b = 10 * 0.985/0.866 = 11.4
(You can also use the search box at Google to perform arithmetic such as 10 * 0.985/0.866.)

Compute the length of side c

Combining the first and third ratios and rearranging terms gives us:
c = 10*0.643/.866 = 7.42

Hopefully, you can confirm these values (approximately) by making measurements on the triangle on your graph board.

Another use for the law of sines

The law of sines can also be used when two sides and one of the non-enclosed angles is known to find the values of the other side and the values of the other two angles.

Let's give that scenario a test drive

Pretend that all we know about the triangle is the following:

- Side a = 10
- Side b = 11.3741339
- Angle B = 80 degrees
- sin B = 0.984807753

We need to solve for angles A and C as well as side c.

Substitution of known values

Recall that the general formula for the law of sines is:
(a/sin A) = (b/sin B) = (c/sin C)
Substitution of the known values into the formula gives:
(10/sin A) = (11.3741339/0.984807753) = (c/sin C)

Rearranging terms

Using the first two ratios and rearranging terms gives us:
sin A = 10*0.984807753/11.3741339 = 0.865830983
A = arcsine(0.865830983) = 1.04680884 radians = 60 degrees
(Note that I used an extreme number of decimal digits to avoid rounding errors and cause the final answer to come out almost exactly correct.)

Compute the value of angle c

Now that we know the values of angles A and B, we also know that
Angle C = 180 - 80 - 60 = 40 degrees.

Compute the length of side c

Using the second and third ratios and rearranging terms, we can solve for side c as:
c = sin C * (11.3741339/0.984807753)
= 0.64278761 * (11.3741339/0.984807753)
= 7.42393866
Thus, we have determined that:

- Angle A = 60 degrees
- Angle C = 40 degrees
- Side c = 7.42

These values all match what we know to be true from the first example.

14.3.3 The law of cosines

In addition to the law of sines, there is also a law of cosines. The law of cosines is useful for computing the third side of a triangle when two sides and their enclosed angle are known. The law is also useful in computing the angles of a triangle when all three sides are known.

Using the same names for the sides and the angles, there are three formulas for the law **of cosines** :

- c^2 = a^2 + b^2 - 2*a*b*cos C
- a^2 = b^2 + c^2 - 2*b*c*cos A
- b^2 = a^2 + c^2 - 2*a*c*cos B

Compute the length of side c

Using the known values from the earlier example, we can compute the length of side c by entering the following expression into the Google search box:

sqrt(10^2 + 11.3^2 - 2*10*11.3*cos(40*pi/180))

This produces an answer of 7.39, which is close enough to the known value for the length of side c.

Compute angle C in degrees

Similarly, using the known values from the earlier example, we can compute the angle C knowing the lengths of the three sides. Enter the following expression into the Google search box:

arccos((10^2 + 11.3^2 - 7.39^2)/(2*10*11.3))*180/pi

This produces an answer of 40.02 which is close enough to the known value for the angle C in degrees.

An exercise for the student

I will leave it as an exercise for the student to work through the algebra to determine how I used the known values and the first formula given above (p. 396) to construct the expressions that I entered into the Google search box.

14.4 Discussion and sample code

Now that we have expanded our knowledge of trigonometry, let's examine some solutions to problems involving force using vectors.

All of the exercises that we will work through in this module involve forces in a two-dimensional plane (coplanar) that meet at a point (concurrent at a point). This excludes forces that create moments or torques, which will be the topic of future modules, and also excludes forces that may come from any direction in a third dimension.

14.4.1 Using the graph board

You will be making heavy use of your graph board in this module. In order to make the instructions less awkward, I am going to begin using drawing syntax when talking about the graph board. For example, when I say draw a line from 3,5 to 10,15, that will mean for you to insert a pin at coordinates x=3 and y=5, to insert a second pin at x=10 and y=15, and to connect the two pins with a rubber band, a pipe cleaner, a wire, a piece of yarn, or whatever works for you.

14.4.2 Parallelogram of forces

Force is a vector. It has both magnitude and direction. You learned in an earlier module that you can resolve vectors two at a time using a method that involves parallelograms.

Stated more formally, any two forces meeting at a point can be replaced by a single resultant force, which is represented by the diagonal of a parallelogram whose two adjacent sides represent the two forces.

Tactile graphics

The file named Phy1100b1.svg contains an image from which your assistant can create a tactile graphic to illustrate the scenario described below (p. 406) .

Figure 4 (p. 401) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

m Schematic diagram of the scenario
n pipe
o cord under tension
p slack cord
q cord under tension
u small connecting ring
r cord under tension
s 1.02 kg mass
t File: Phy1100b1.svg

m Schematic diagram of the scenario
n pipe
o cord under tension
p slack cord
q cord under tension
u small connecting ring
r cord under tension
s 1.02 kg mass
t File: Phy110b1.svg

m Schematic diagram of the scenario
n pipe
o cord under tension
p slack cord
q cord under tension
u small connecting ring
r cord under tension
s 1.02 kg mass
t File: Phy1100b1.svg

Figure 4 . Mirror image contained in the file named Phy1100b1.svg.

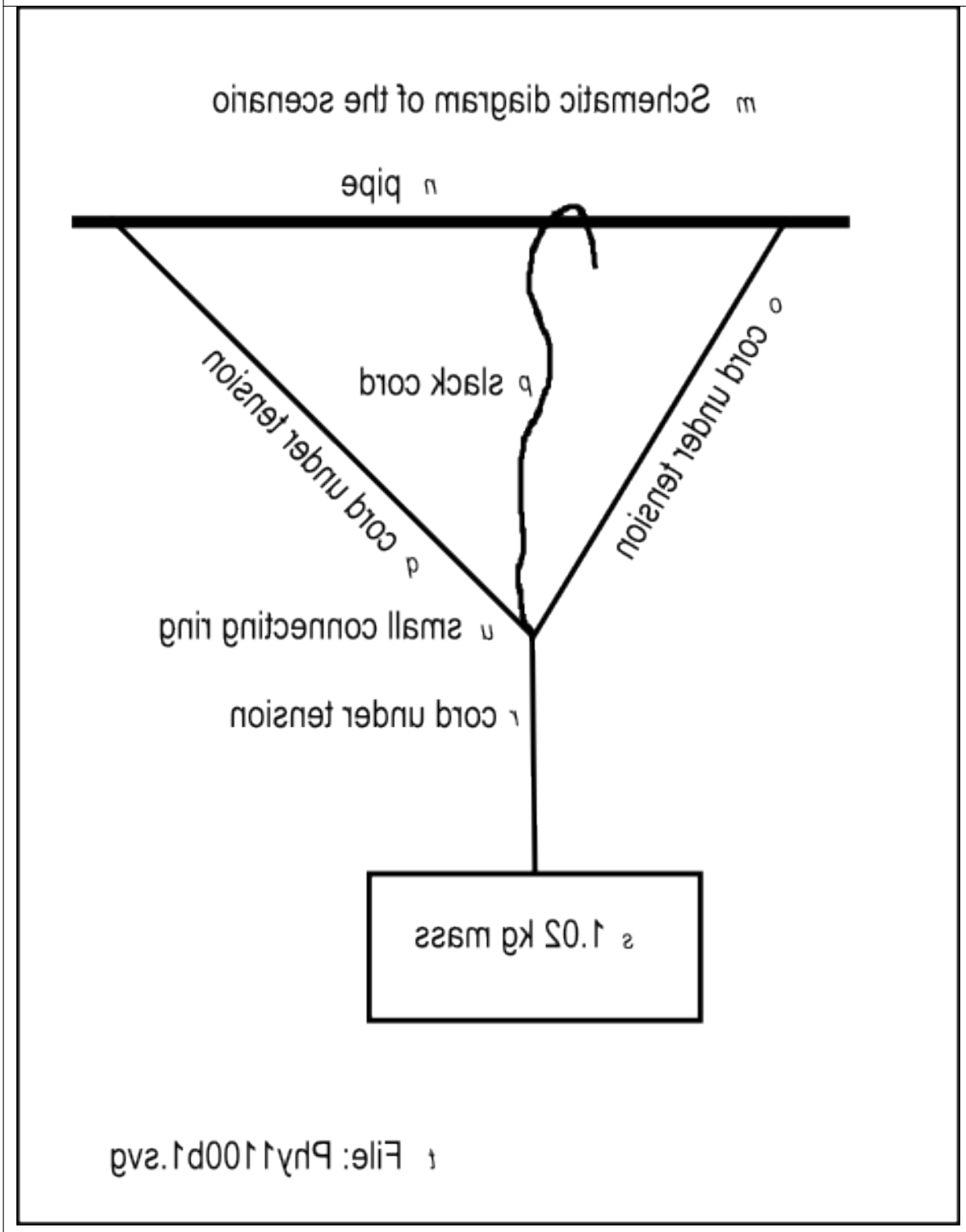
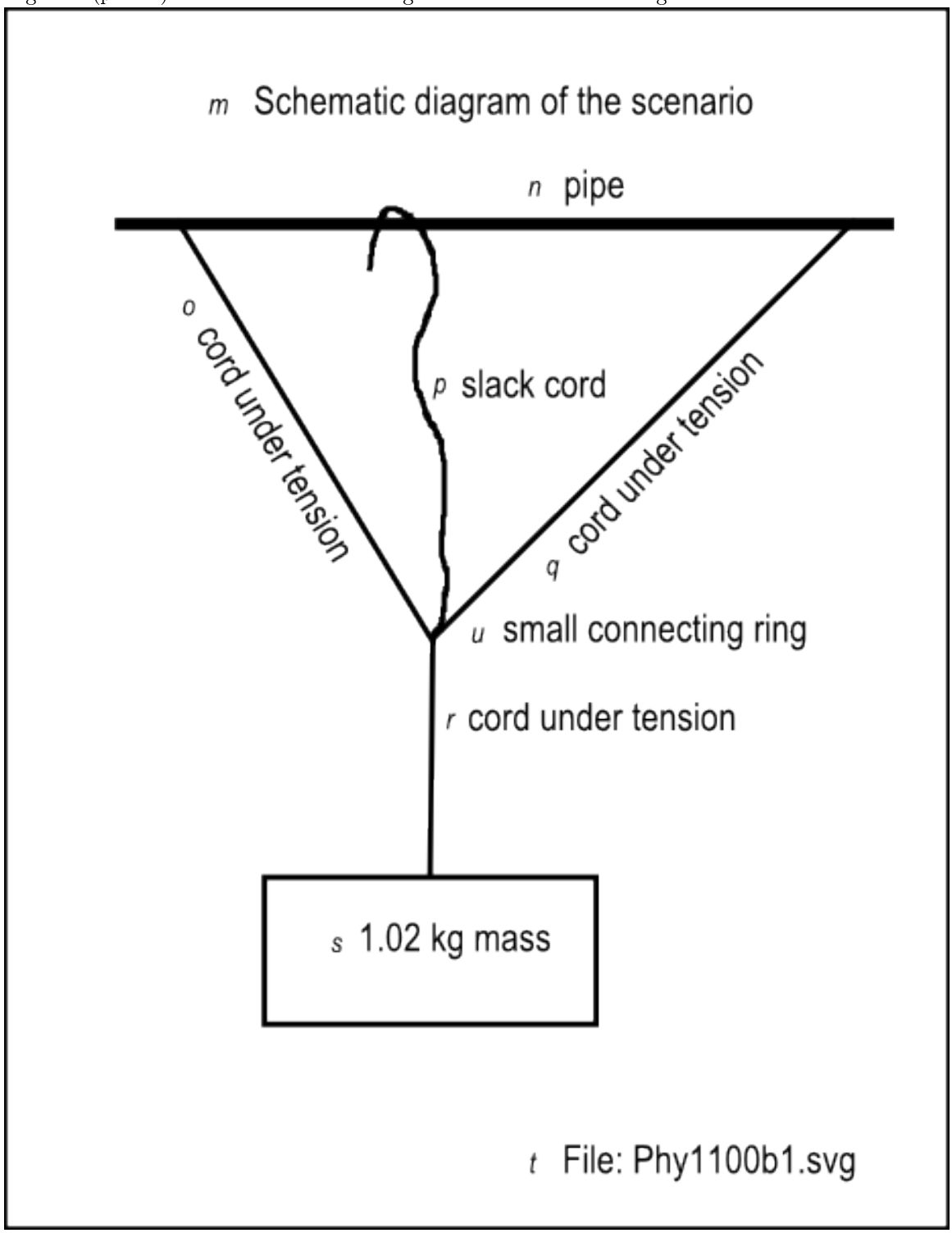
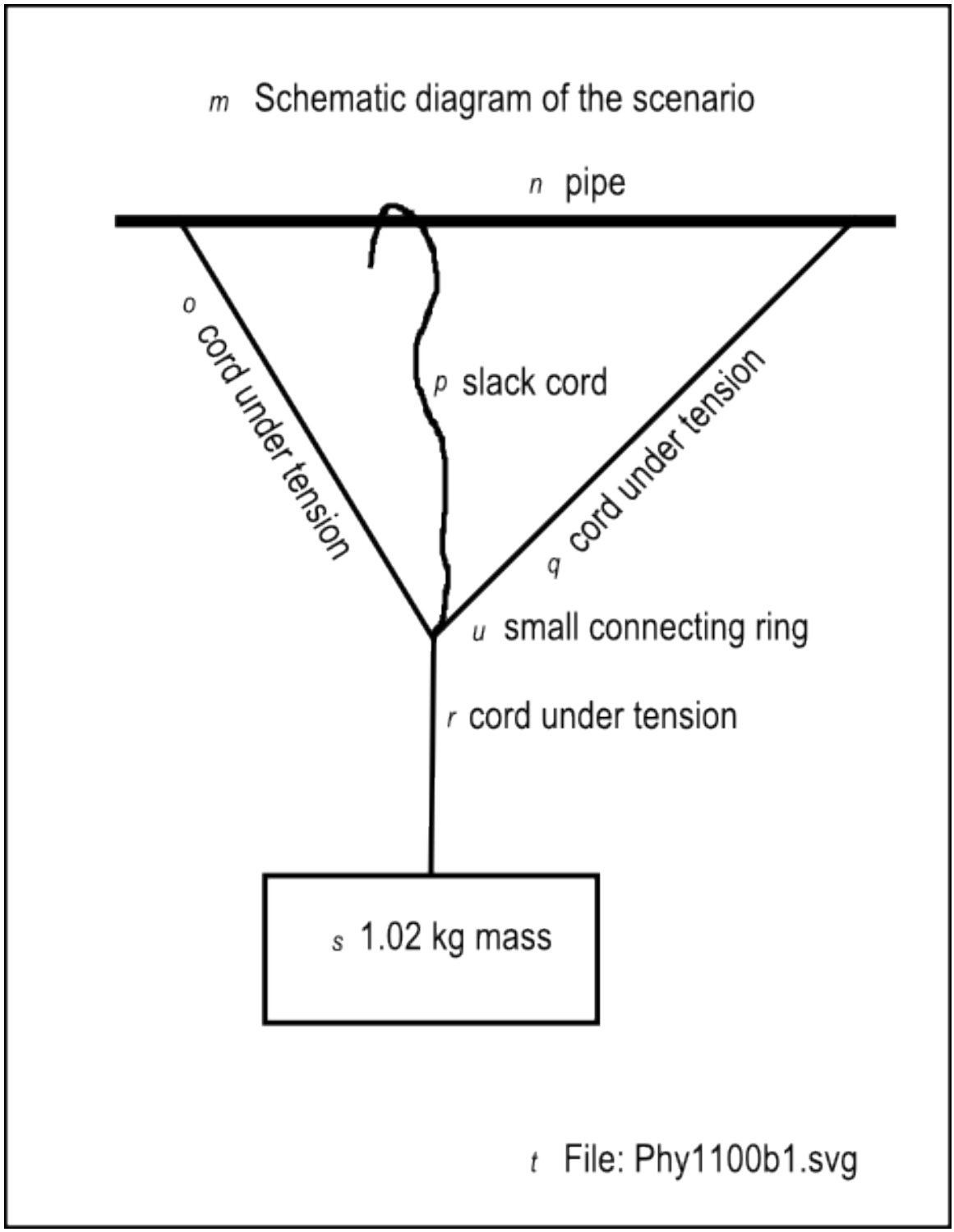


Table 14.4

Figure 5 (p. 405) shows a non-mirror-image version of the same image.




m Schematic diagram of the scenario
n pipe
o cord under tension
p slack cord
q cord under tension
u small connecting ring
r cord under tension
s 1.02 kg mass
t File: Phy1100b1.svg

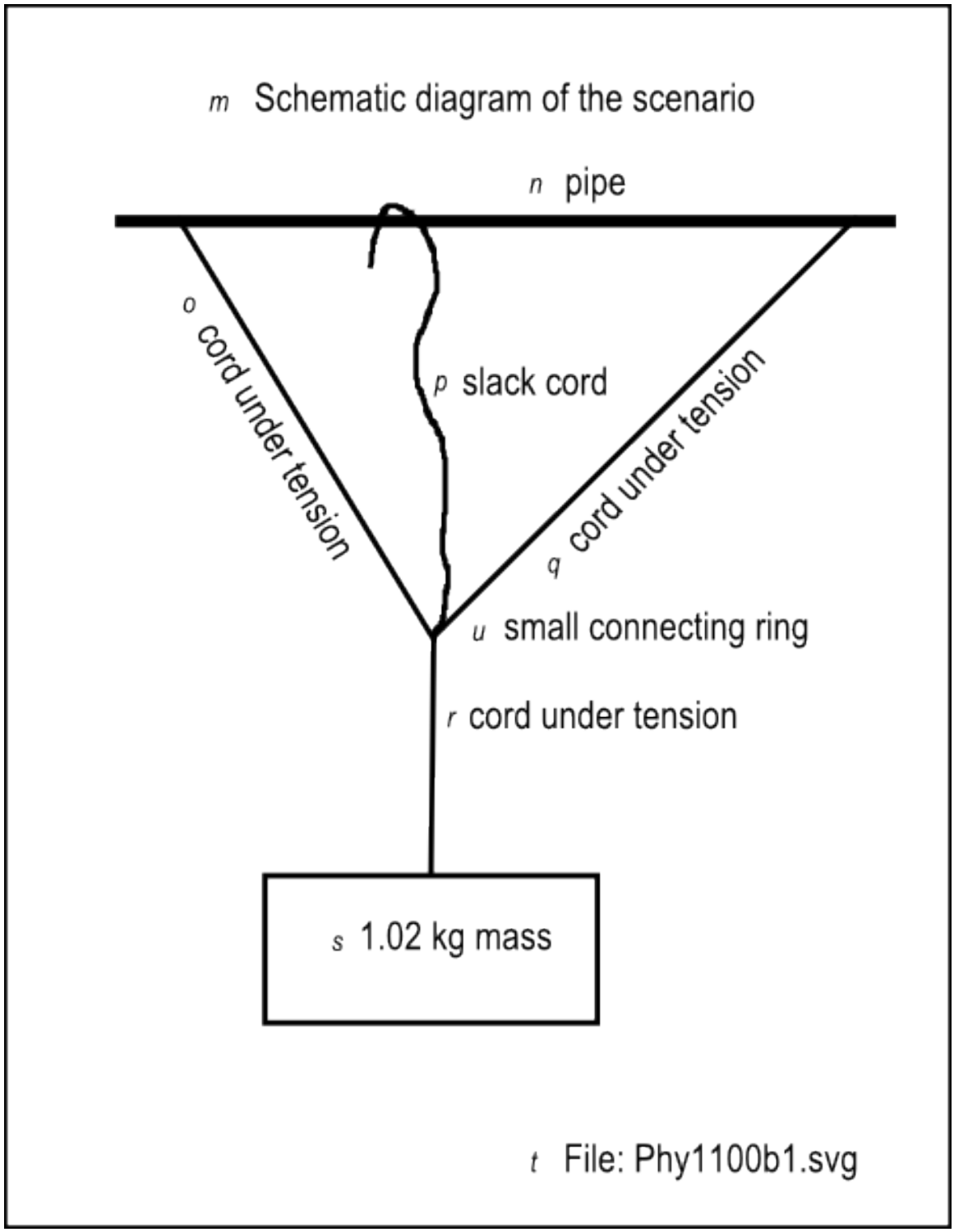

m Schematic diagram of the scenario
n pipe
o cord under tension
p slack cord
q cord under tension
u small connecting ring
r cord under tension
s 1.02 kg mass
t File: Phy1100b1.svg

Figure 5 . Non-mirror-image version of the image from the file named Phy1100b1.svg.

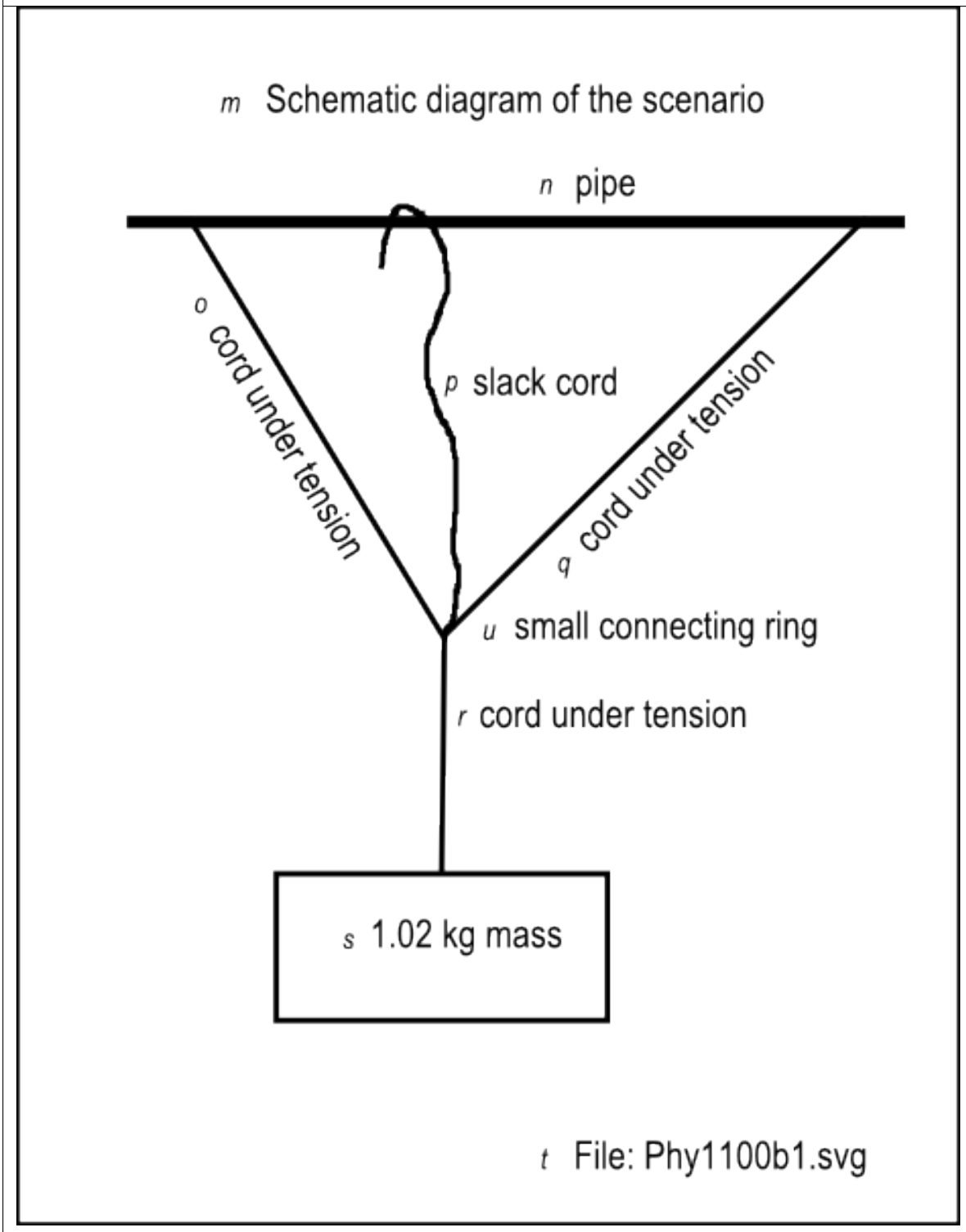


Table 14.5

Figure 6 (p. 406) shows the key-value pairs that go with the image in the file named Phy1100b1.svg.

Figure 6 . Key-value pairs for the image in the file named Phy1100b1.svg.

```
    m: Schematic diagram of the scenario
n: pipe
o: cord under tension
p: slack cord
q: cord under tension
r: cord under tension
s: 1.02 kg mass
t: File: Phy1100b1.svg
u: small connecting ring
```

Table 14.6

The scenario

Please draw a schematic diagram of this scenario on your graph board.

A strong pipe extends from left to right in the work area supported on both ends. You can get your hands around the pipe to attach things to it.

Light, strong, flexible cords

You tie two light, strong, flexible cords to the pipe approximately one meter apart. (the distance isn't critical). We will ignore the weight of the cords. You thread the ends of the two cords through a small ring that is just large enough for you to tie four cords onto the ring. (The only purpose of the ring is to provide a convenient way to attach cords together at a single point.)

Adjust the position of the ring

You adjust the position of the ring on each of the cords so that the cord on the left forms a 60-degree south of east angle with the pipe and the cord on the right forms a 45 degree south of west angle with the pipe. Then you tie both cords to the ring at that point.

A triangle

When you pull straight down on the ring, the two cords and the pipe form a triangle. The interior angle at the upper-left vertex is 60 degrees, the interior angle at the upper-right vertex is 45 degrees, and because the sum of the angles in a triangle is 180 degrees, the interior angle at the bottom vertex is 75 degrees.

Attach a mass to the ring

You tie a short cord to the bottom of the ring and tie a 1.02 kg mass to the cord. (A 1.02 kg mass has a weight of approximately 10 newtons.). Then you allow the two cords that are tied to the pipe to support the mass, which pulls straight down with a force of 10 newtons.

The structure of the system

Now, let's think about the structure of this system. It consists of three cords emanating out from a small ring with each cord exerting a force on the ring.

Compression or tension?

To begin with, a flexible cord cannot support a compressive force. If you push on one end of a cord in an attempt to push the object attached to the other end, the cord will simply bend. Therefore, the cords in this scenario cannot possibly exert a pushing force on the ring.

However, a cord can support a tension or pulling force. In other words, any or all of the cords can support a pulling force on the ring.

Nothing else is touching the ring

The ring is not in contact with anything other than the three cords at this point in time, and the ring is in equilibrium. By that I mean that the ring has zero velocity and no acceleration.

(We are also assuming that the ring is, for all practical purposes, a point and it is not experiencing a moment or a torque. Once again, we'll get into moments and torques in a future module.)

Thee pulling forces

So, we know that the ring is being subjected to three pulling forces. A pulling force of 10 newtons is pulling straight down toward the center of the earth. The forces associated with the two upper cords must be along the lengths of the cords. Otherwise, the cords and the ring would be moving.

Tactile graphics

The file named Phy1100c1.svg contains a force vector diagram for this scenario.

Figure 7 (p. 411) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

m Force vector diagram
n Parallelogram lines
10 newtons
q Resultant
Vector B
Vector C
7.3 newtons
5.3 newtons
u 60 degrees
v 45 degrees
w 10 newtons
Vector A
x
y File: Phy100c1.svg

m Force vector diagram
n Parallelogram lines
q Resultant
r 10 newtons
p Vector C
o 7.3 newtons
s Vector B
t 5.3 newtons
u 60 degrees
v 45 degrees
w 10 newtons
x Vector A
y File: Phy110Oc1.svg

m Force vector diagram
n Parallelogram lines
o 7.3 newtons
p Vector C
q Resultant
r 10 newtons
s Vector B
t 5.3 newtons
u 60 degrees
v 45 degrees
w 10 newtons
x Vector A
y File: Phy100c1.svg

Figure 7 . Mirror image from the file named Phy1100c1.svg.

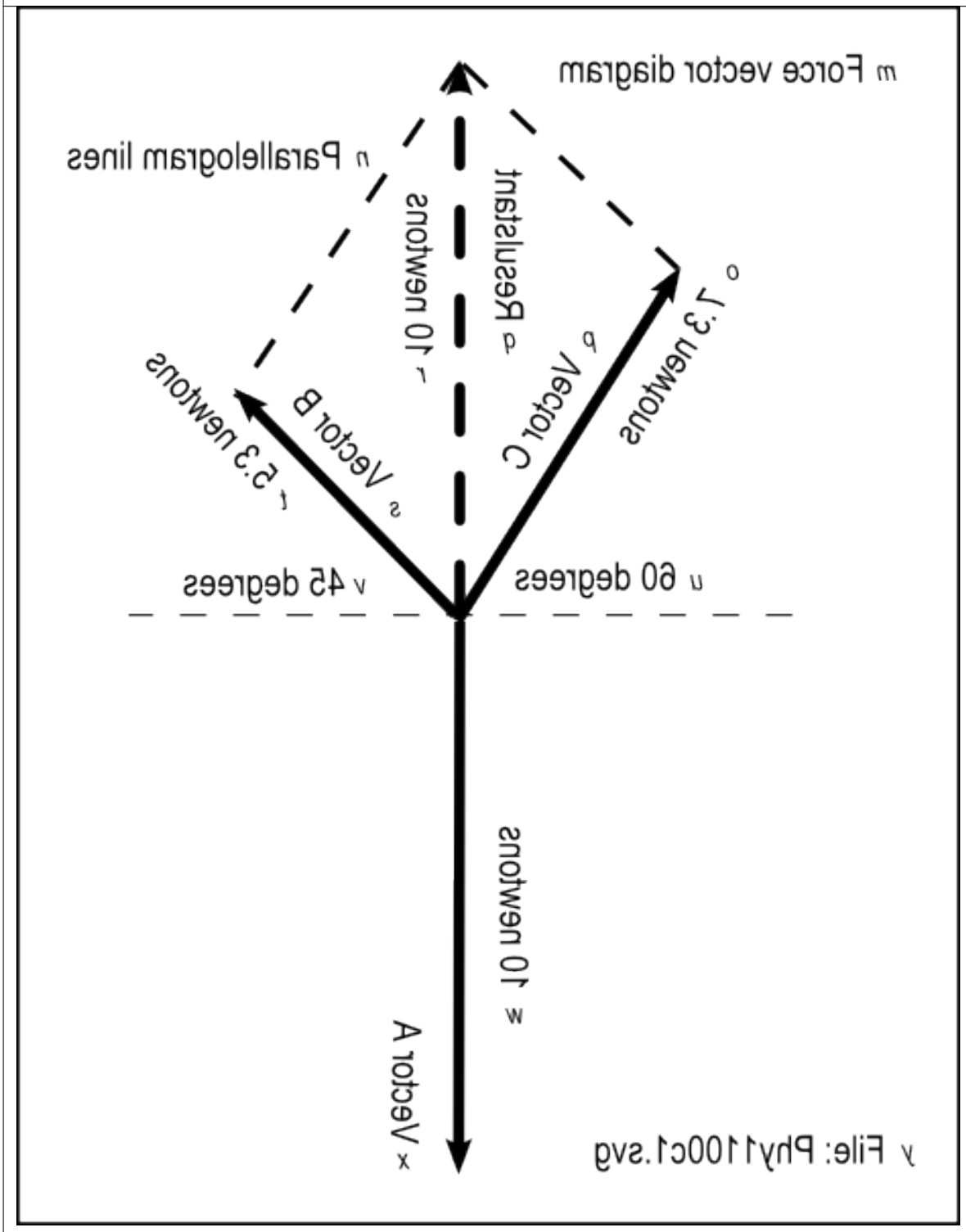
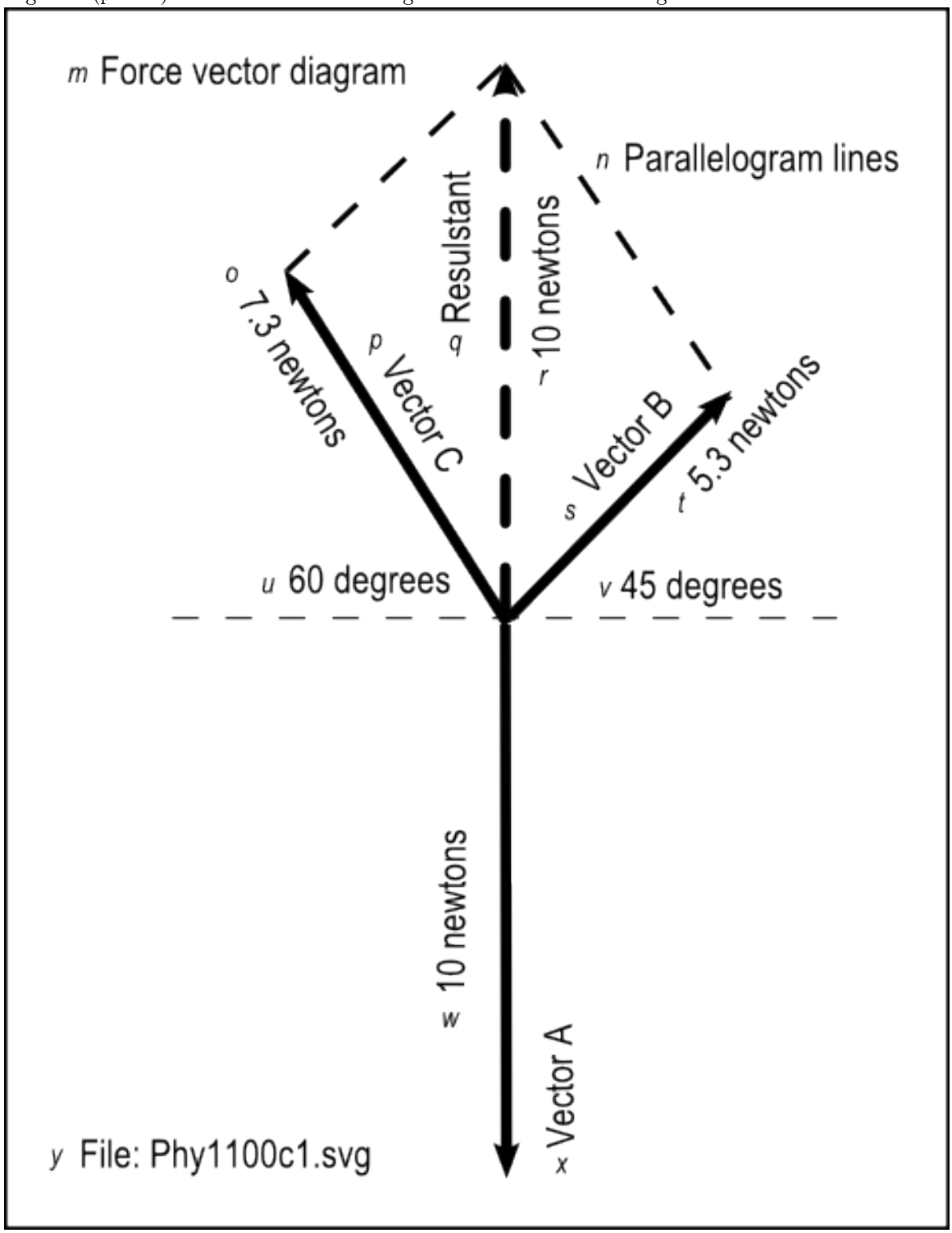
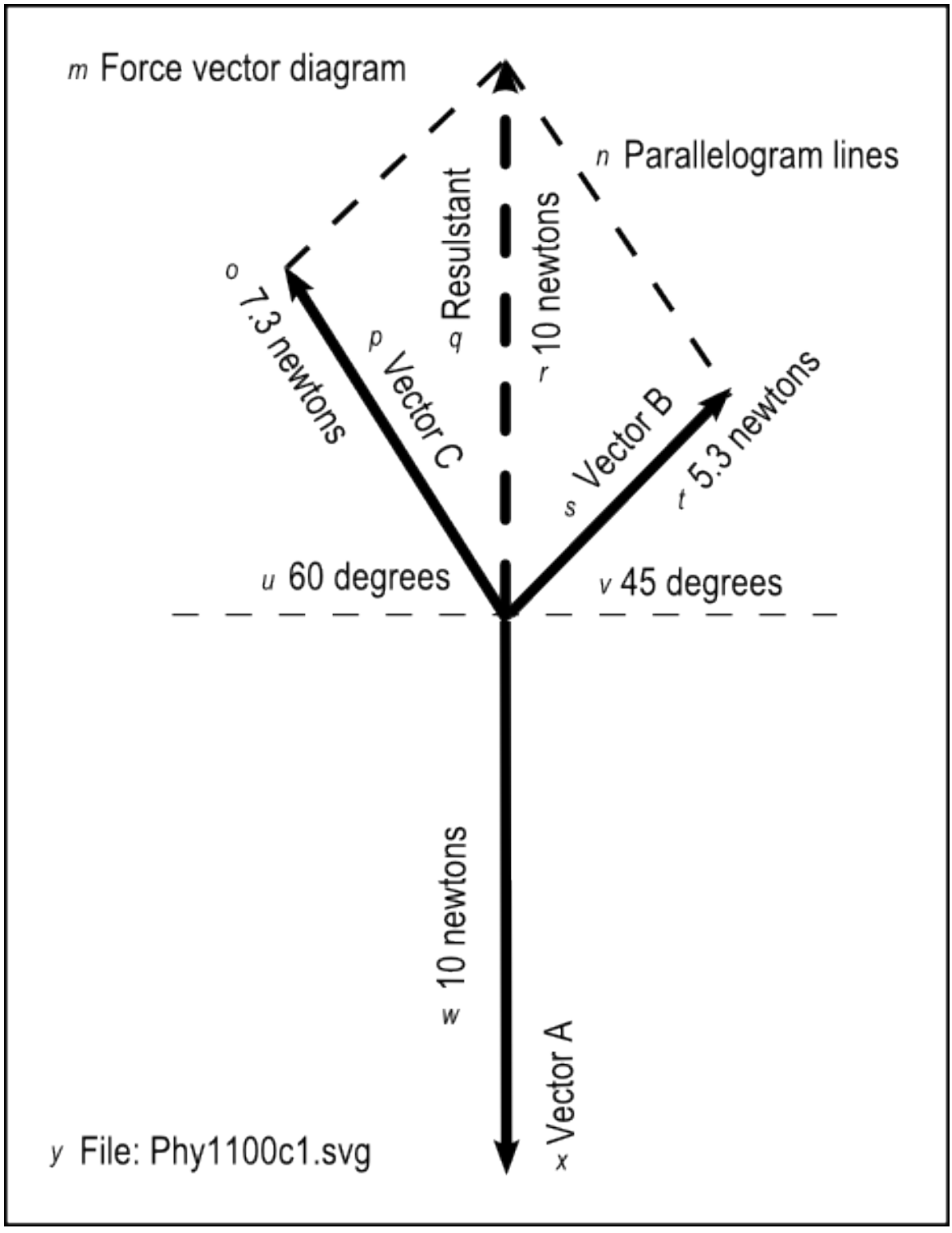


Table 14.7

Figure 8 (p. 415) shows a non-mirror-image version of the same image.




m Force vector diagram
n Parallelogram lines
o 7.3 newtons
p Vector C
q Resulstant
r 10 newtons
s Vector B
t 5.3 newtons
u 60 degrees
v 45 degrees
w 10 newtons
x Vector A
y File: Phy1100c1.svg

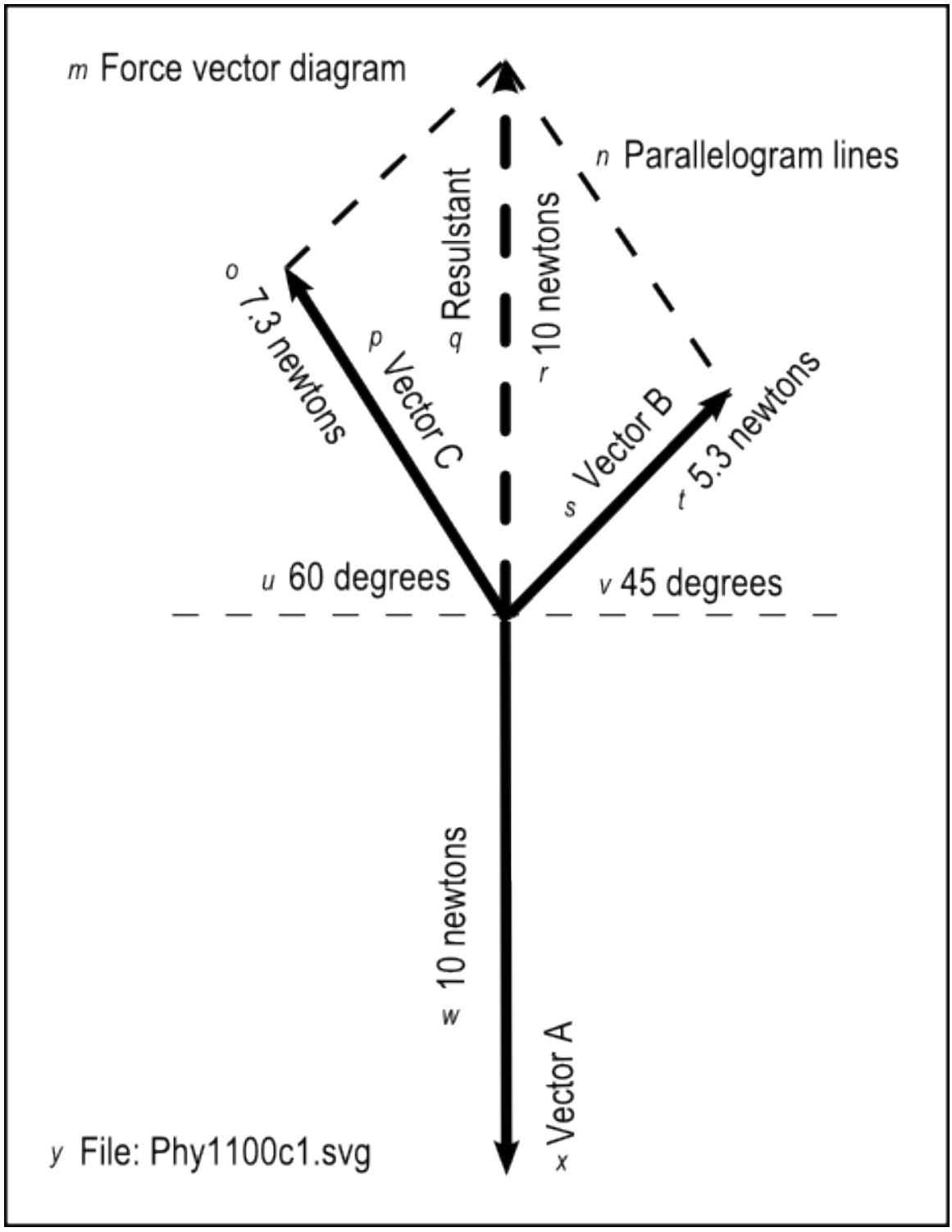

m Force vector diagram
n Parallelogram lines
o 7.3 newtons
p Vector C
q Resulstant
r 10 newtons
s Vector B
t 5.3 newtons
u 60 degrees
v 45 degrees
w 10 newtons
x Vector A
y File: Phy1100c1.svg

Figure 8 . Non-mirror-image version of the image from the file named Phy1100c1.svg.

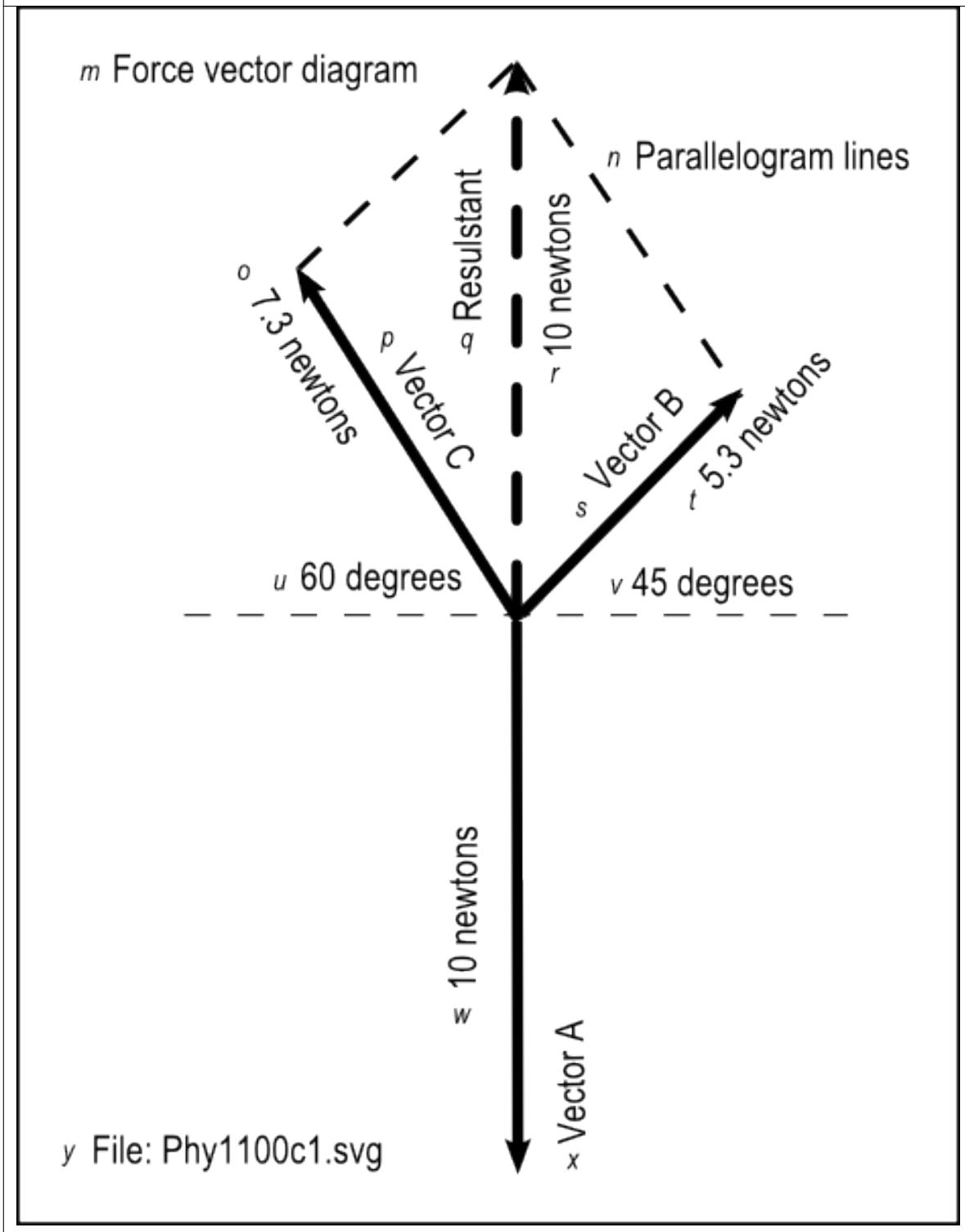


Table 14.8

Figure 9 (p. 416) shows the key-value pairs that go with the image in the file named Phy1100c1.svg.

Figure 9 . Key-value pairs for the image in the file named Phy1100c1.svg.

```
    m: Force vector diagram.
n: Parallelogram lines
o: 7.3 newtons
p: Vector C
q: Resultant
r: 10 newtons
s: Vector B
t: 5.3 newtons
u: 60 degrees
v: 45 degrees
w: 10 newtons
x: Vector A
y: File: Phy1100c1.svg
```

Table 14.9

Directions of the forces

The fact that the cords can only support tension tells us that one of the two forces pulling in an upward direction is actually pulling at an angle of 45 degrees north of east(the direction of the cord). The other force is pulling at angle of 60 degrees north of west.

What is the tension in each cord?

So, three interesting questions are:

- What is the tension in each of the cords.
- How much of the 10-newton load is being carried by the cord on the right.
- How much of the load is being carried by the cord on the left?

We can determine the answers to those questions using the parallelogram rule. At least we can find the tension in each of the cords and determine if those tensions are sufficient to carry the 10-newton load.

The parallelogram rule

As I told you earlier, **any two forces** meeting at a point (we're considering the ring to be a point) can be replaced by a single resultant force, which is represented by the diagonal of a parallelogram whose two adjacent sides represent the two forces.

If we can figure out what the resultant force is, we can figure out the answer to the above questions regarding the tensions and the load being borne by each of the upper cords.

We should be able to figure out what the resultant force is with a thought experiment based on our life experiences.

Tie a fourth cord to the ring

Pretend that you tie another cord to the top of the ring and loop it over the pipe somewhere between the tie points of the other two cords. (This cord is indicated by the object labeled "slack cord" in the image in the file named Phy1100b1.svg and also in the image in Figure 5 (p. 405) .)

Then pull very slowly on that fourth cord until both of the other cords go slack. At that point, the fourth cord, (which is somewhat analogous to the resultant of the forces in the other two cords) has assumed the entire 10-newton load.

Is the ring in equilibrium?

Did the ring and the mass move sideways when the other two cords went slack. If so, the equilibrium of the system was disrupted. Relax the tension on the fourth cord, move it slightly sideways, and repeat the experiment. Continue this process until you find a location where the ring doesn't move sideways when the fourth cord assumes the total load.

Only one location will suffice

You can probably agree, from prior experience on the swing set at the playground as a child, that the only location at which the fourth cord can assume the total load without the ring moving sideways is when the fourth cord passes over the pipe directly above the ring and the mass. Therefore, we know that the resultant of the forces being exerted by the original two upper cords must be on a vertical line directly above the ring and the mass.

The diagonal

We also know on the basis of the above rule (p. 416) that the length of the resultant force is proportional to the length of the diagonal of a parallelogram whose two adjacent sides represent the two forces that are being resolved.

The resultant force

The resultant force is indicated by a heavy dashed vector pointing upward in the image in Phy1100c1.svg. Two sides of the parallelogram are represented by lighter dashed lines. The other two sides of the parallelogram are represented by heavy solid vectors, one at 45 degrees relative to the horizontal and the other at 60 degrees relative to the horizontal.

The length of the resultant force vector

Let's think a bit about the required length of the resultant force vector. The above experiment resolves the force system into only two forces acting on the ring. (The original two upward force vectors are eliminated from the picture when those two cords go slack.) One force is directed up and the other force is directed down.

The vector sum must be zero for equilibrium

We also know that the vector sum of all the forces acting on a point must be zero in in order for the point to be in equilibrium. Since there are the only two forces acting on the ring, the only way that they can sum to zero is for them to be equal in magnitude and opposite in direction from one another.

Equal and opposite

The force vector pulling down on the ring has a magnitude of 10 newtons. We know because we tied a ten-newton load to the bottom of the ring. Therefore, the magnitude of the resultant vector pulling up on the ring must also be 10 newtons. That is the required length of the diagonal of the parallelogram discussed earlier.

The graphical solution

Draw the force vector pointing straight up from the ring with a length of ten units. (You can use whatever drawing scale is convenient for your drawing.) That is the diagonal of a parallelogram. Two force vectors lie along the lines of the two cords. Those two force vectors form the adjacent sides of a parallelogram, but we don't yet know their lengths.

Complete the parallelogram

The lengths of the force vectors that lie along the lines of the cords are equal to the sides of the parallelogram. That provides the answer to the original question regarding the tension in each of the cords. The tension in each cord is equal to the length of the side of the parallelogram using the same scale that was used to draw the 10-newton diagonal length.

Drawing the parallelogram might be difficult

Drawing the parallelogram is not particularly easy even for a sighted person without special drawing tools. Here is one way that I have found to do it.

Draw a horizontal line (parallel to the pipe) that just touches the tip of the force vector that represents the diagonal. Then use your protractor to draw lines that begin at the tip of the force vector and emanate downward on both sides of the diagonal at angles of 60 degrees south of east and 45 degrees south of west. Be sure to do it in the correct order so that you end up with a parallelogram.

Mark the spots

Mark the spots where those lines cross the two upper cords. Those spots are the tips of the force vectors in the cords.

My answer

When I did it graphically (using pencil, paper, protractor, and ruler), I got the length of the force vector pointing 45 degrees north of east to have a length of 5.3 newtons. I got the length of the force vector pointing north of west to be 7.3 newtons. These are the values on the upper force vectors in the image in Phy1100c1.svg.

Numeric sum exceeds 10 newtons

You may have noticed that the numeric sum of the magnitudes of these two force vectors exceeds the load of 10 newtons. How can that be?

The answer is that when force vectors that are supporting a vertical load go off in a direction other than vertical, only the vertical component of the force vector counts insofar as supporting the load is concerned.

The vertical components of the two force vectors are:

- 5.3*sin(45 degrees) = 3.74
- 7.3*sin(60 degrees) = 6.32

The sum of the vertical components is:

6.32 + 3.74 = 10.06

which is close enough to the downward force being exerted on the ring by the mass.

Your answer

Were you able to get reasonably close to my answers? If so, congratulations. If not, don't worry too much about it. The most important thing is for you to understand the process and not to develop expertise in constructing accurate drawings using pushpins and rubber bands. However, I do believe that constructing vector diagrams will help you gain a better understanding of the process.

Later I will show you how to solve the same .problem mathematically which is probably the better choice for a blind student.

14.4.3 Triangle of forces

Now let's take another look at the same problem from a somewhat different graphical viewpoint. To recap, we have a point in a plane (the ring) which is subjected to the following three vector forces and the ring is in equilibrium. (All angles are given relative to the positive horizontal axis).

- Vector A – A force of 10 newtons at an angle of -90 degrees.
- Vector B – A force of an unknown magnitude at an angle of 45 degrees.
- Vector C – A force of an unknown magnitude at an angle of 120 degrees.

We know that in order for the ring to be in equilibrium, the sum of the vectors for these three forces must add to zero. What are the magnitudes of the forces that vector B and vector C exert on the ring?

Drawing tail to tip

You learned in an earlier module that you can add vectors graphically by drawing them tail to tip. Once you have done that, the resultant vector is a vector that extends from the tail of the first vector to the tip of the last vector.

Tactile graphics

The file named Phy1100d1.svg contains the image that is required to create a tactile graphic for this scenario.

Figure 10 (p. 422) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

Tip-to-tail force vector diagram
Vector C
7.3 newtons
10 newtons
Vector A
120 degrees
Vector B
5.3 newtons
45 degrees
File: Phy1100d1.svg

Tip-to-tail force vector diagram
Vector C
7.3 newtons
10 newtons
Vector A
120 degrees
Vector B
5.3 newtons
45 degrees
File: Phy1100d1.svg

Tip-to-tail force vector diagram
Vector C
7.3 newtons
10 newtons
Vector A
120 degrees
Vector B
5.3 newtons
45 degrees
File: Phy1100d1.svg

Figure 10 . Mirror image from the file named Phy1100d1.svg.

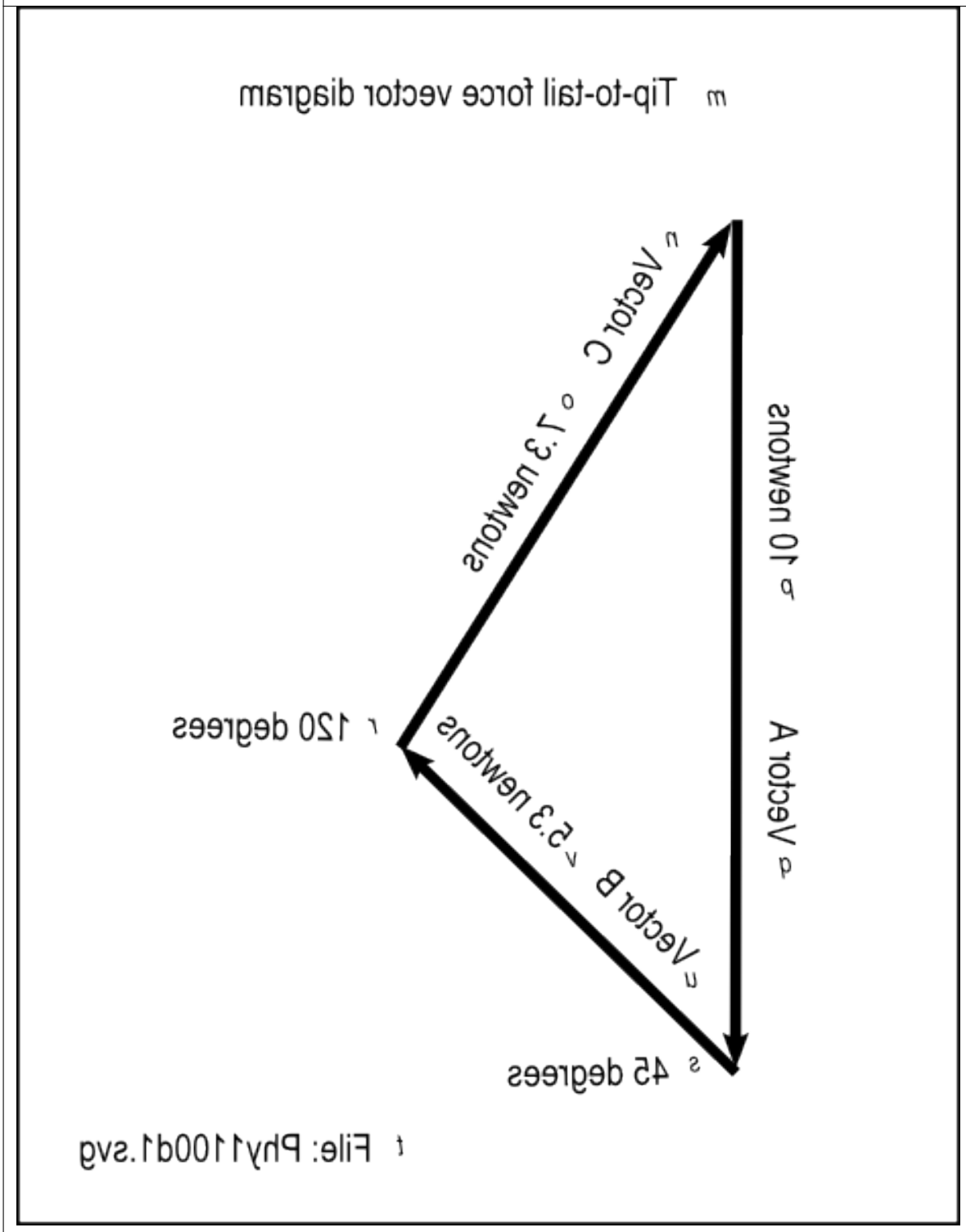
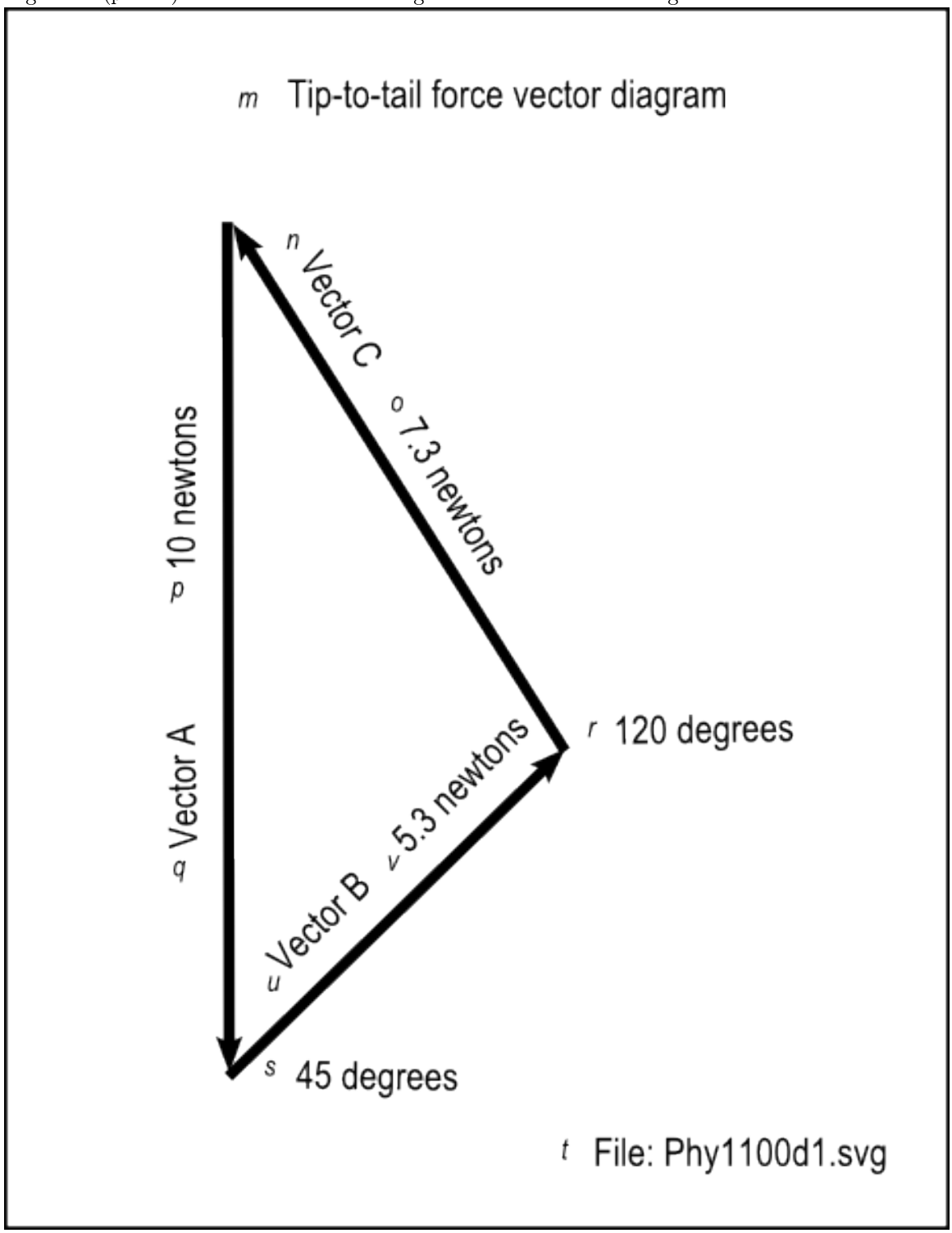
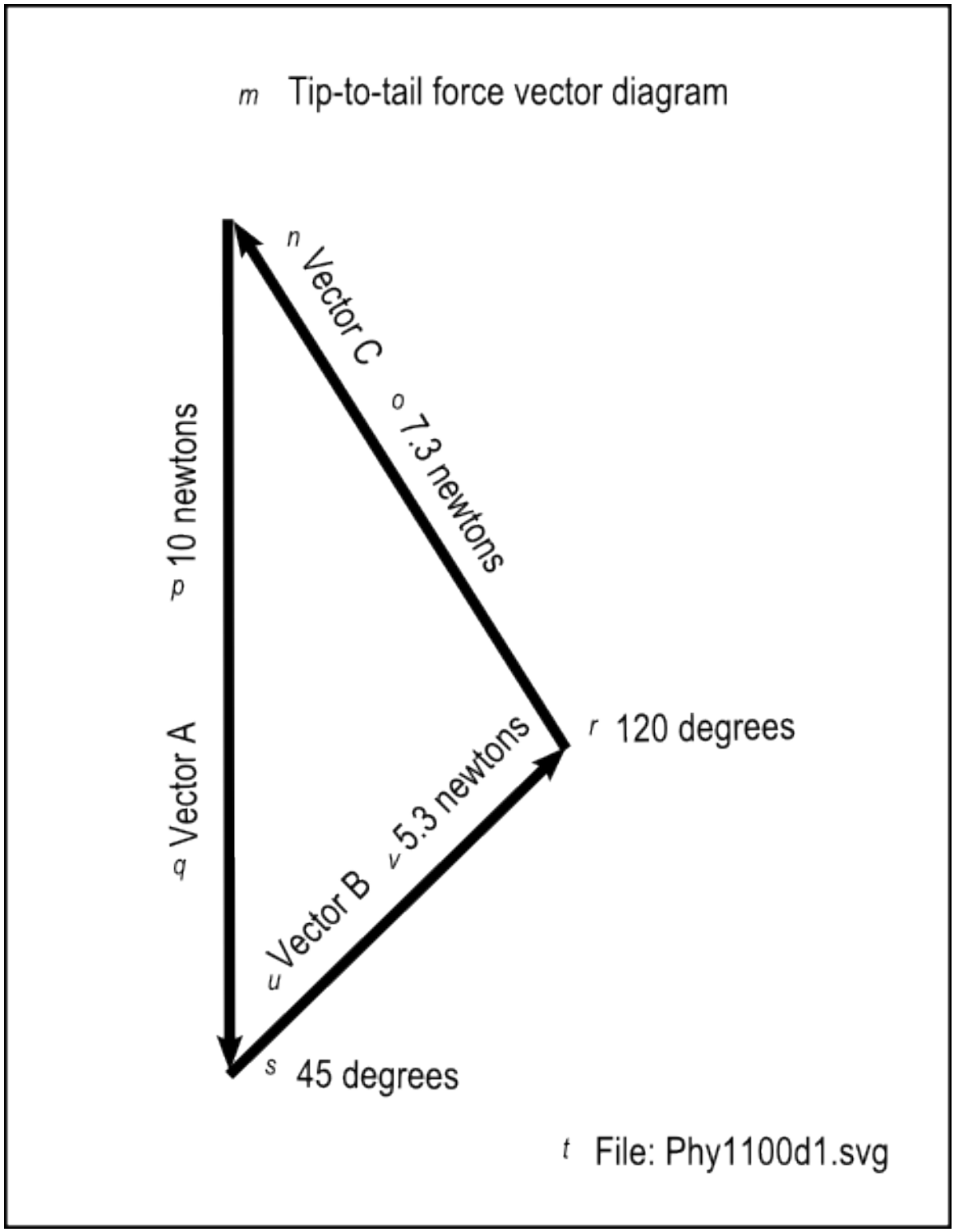


Table 14.10

Figure 11 (p. 426) shows a non-mirror-image version of the same image.




m Tip-to-tail force vector diagram
n Vector C
o 7.3 newtons
p 10 newtons
q Vector A
r 120 degrees
u Vector B
v 5.3 newtons
s 45 degrees
t File: Phy1100d1.svg

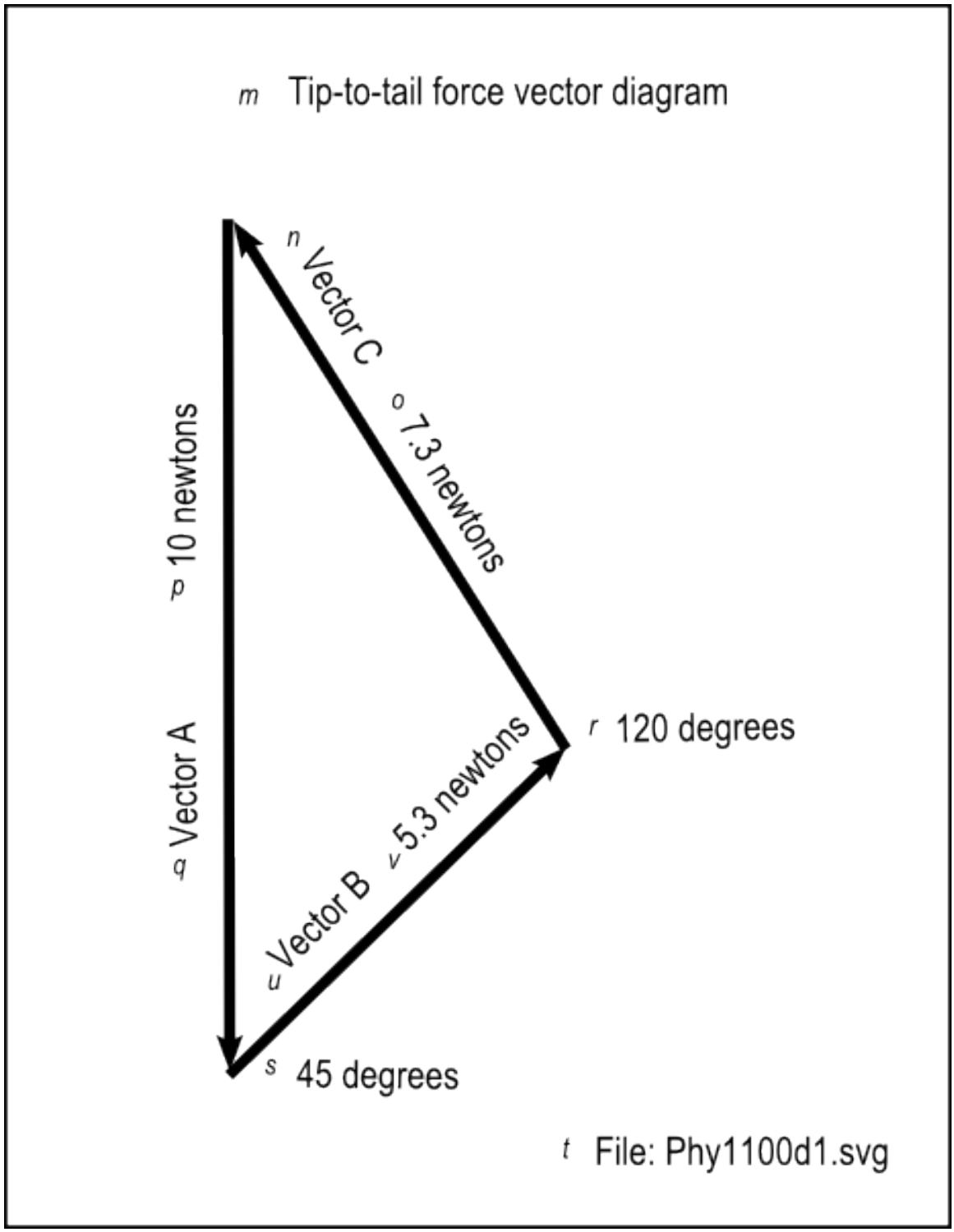

m Tip-to-tail force vector diagram
n Vector C
o 7.3 newtons
p 10 newtons
q Vector A
r 120 degrees
s 45 degrees
u Vector B
v 5.3 newtons
t File: Phy1100d1.svg

Figure 11 . Non-mirror-image version of the image from the file named Phy1100d1.svg.

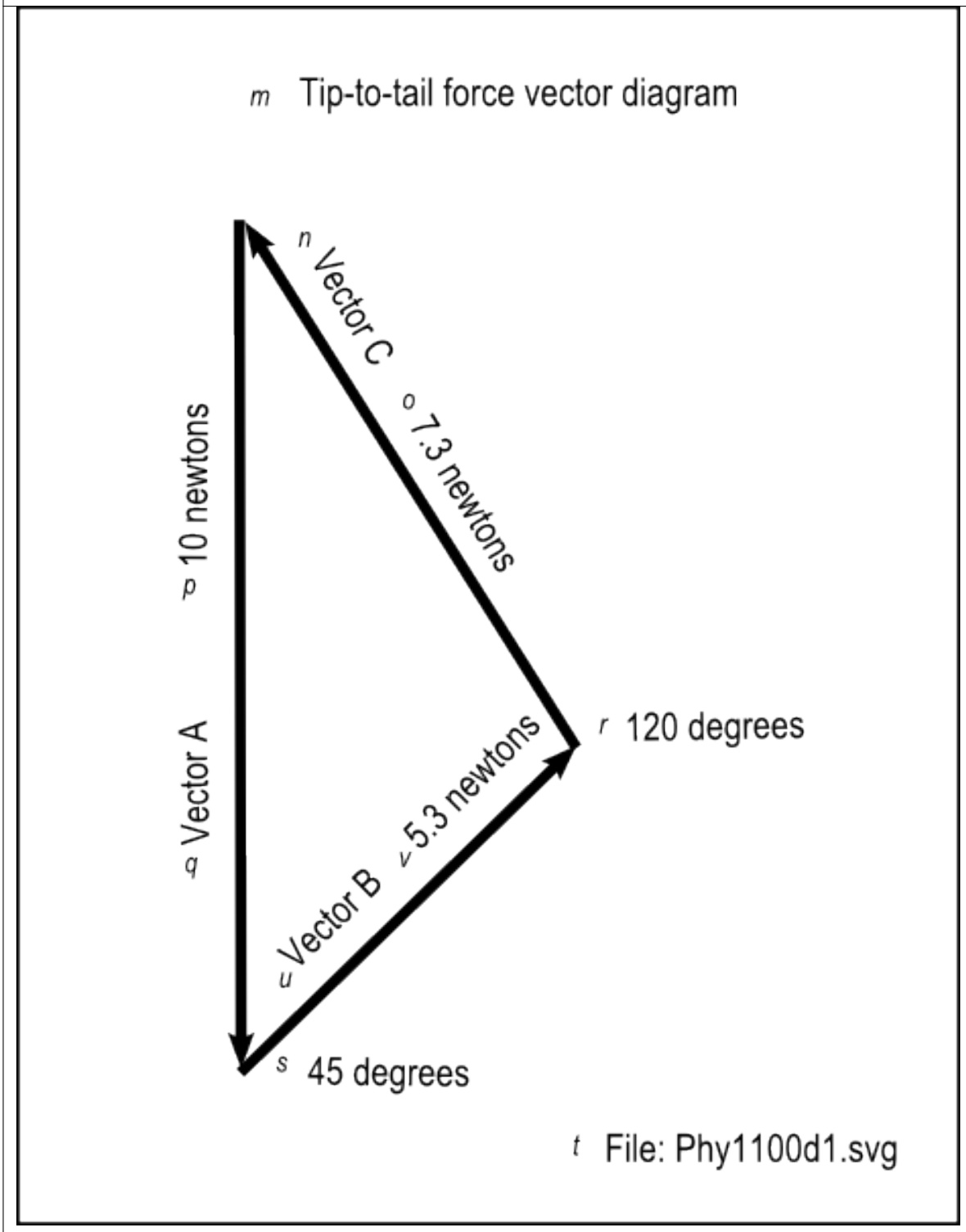


Table 14.11

Figure 12 (p. 427) shows the key-value pairs that go with the image in the file named Phy1100d1.svg.

Figure 12 . Key-value pairs for the image in the file named Phy1100d1.svg.

```
    m: Tip-to-tail force vector diagram
n: Vector C
o: 7.3 newtons
p: 10 newtons
q: Vector A
r: 120 degrees
s: 45 degrees
t: File: Phy1100d1.svg
u: Vector B
v: 5.3 newtons
```

Table 14.12

A closed path for equilibrium

For this case where the sum must be zero, the resultant vector must have a zero length. This means that the tip of the last vector must touch the tail of the first vector. In other words, the connected vectors must describe a closed path.

In the earlier modules, you were given the magnitude and direction of several vectors and you arranged them tail to tip in order to find the resultant vector. However, for this problem, you know the direction and magnitude of one vector but you only know the directions for the other two vectors. You need to find their magnitudes.

Knowing that the three vectors must form a closed path makes a graphical solution to this problem possible.

Draw a vector diagram

Begin by drawing vector A with a length of 10 newtons at an angle of -90 degrees, using whatever plotting scale works for you. The tail for this vector will be at the top of the vertical line and the tip will be at the bottom.

Add vector B to the diagram

Go to the tip of vector A and draw vector B with an unspecified length at an angle of 45 degrees. The tail of vector B will touch the tip of vector A and the tip of vector B will be somewhere up and to the right of the tip of vector A.

Add vector C to the diagram

Go to the tail of vector A and draw vector C with an unspecified length at an angle of 120 degrees. The tip of vector C will touch the tail of vector A and the tail of vector C will be somewhere down and to the right of the tail of vector A.

The lines should cross

If you did everything right, the lines that describe vector B and vector C should cross. Use a pushpin to mark the place where they cross. This point will be the tip of vector B and will be the tail of vector C.

Measure the lengths of vector B and vector C

At this point, you can measure the lengths of the lines that describe vector B and vector C. Using the same plotting scale that you used for the 10-newton vector A, you can determine the magnitudes of each of those vectors.

Since this is the same problem as before, the magnitudes for those two vectors should be the same as before. Vector B should be about 5.3 newtons and vector C should be about 7.3 newtons.

Was this easier than the parallelogram?

I suspect that you probably found the physical construction of the triangle solution to be easier than the physical construction of the parallelogram solution. I further suspect that the trigonometric solution, which I will explain in the next section, will be even easier for blind students.

14.4.4 Trigonometric solution to the triangle of forces

Let's pretend that you don't have a graphic solution for this problem. However, you do have a rough sketch of what the triangle looks like on your graph board. Use your Braille labeler to label the three vectors in the sketch A, B, and C. Then label the angles opposite each side as a, b, and c respectively.

Here are the facts as we know them at this point:

- Vector A = 10 newtons.
- Angle a = 105 degrees.
- Sine 105 degrees = 0.966
- Angle b = 30 degrees
- Sine 30 degrees = 0..5
- Angle c = 45 degrees
- Sine c = 0.707

The law of sines

We will solve this problem using the law of sines that I explained earlier. Given the names that we have applied to the sides and the angles for this triangle, the law of sines can be written as follows:

(A/sin a) = (B/sin b) = (C/sin c)

(Note that upper and lower-case letters were switched relative to the law of sines given near the beginning (p. 395) of this module.)

Solve for the length of side B

Using the first two ratios and rearranging terms gives

B = A*sin b/sin a = 5.18

Solve for the length of side C

Using the first and third ratio and rearranging terms gives

C = A*sin c/sin a = 7.32

The answers

These are the answers that we are looking for:

- B = 5.18 newtons
- C = 7.32 newtons

14.4.5 Using components to analyze for equilibrium

Tent pole exercise

Please sketch this scenario on your graph board. Then we will solve the problem using a script and components.

A bird's eye view of the top of a tent pole shows guy wires extending outward and exerting radial 10-newton forces on a tent pole at **the following angles** :

- 0 degrees
- 45 degrees
- 90 degrees
- 135 degrees
- 180 degrees
- 225 degrees
- 270 degrees

Tactile graphics

The image in the file named Phy1100e1.svg contains seven vectors of equal length emanating out from a common point at the angles given above.

Figure 13 (p. 433) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

m Tent pole exercise
n File: Phy1100e1.svg

m Tent pole exercise
n File: Phy110e1.svg

m Tent pole exercise
n File: Phy1100e1.svg

Figure 13 . Mirror image from the file named Phy1100e1.svg.

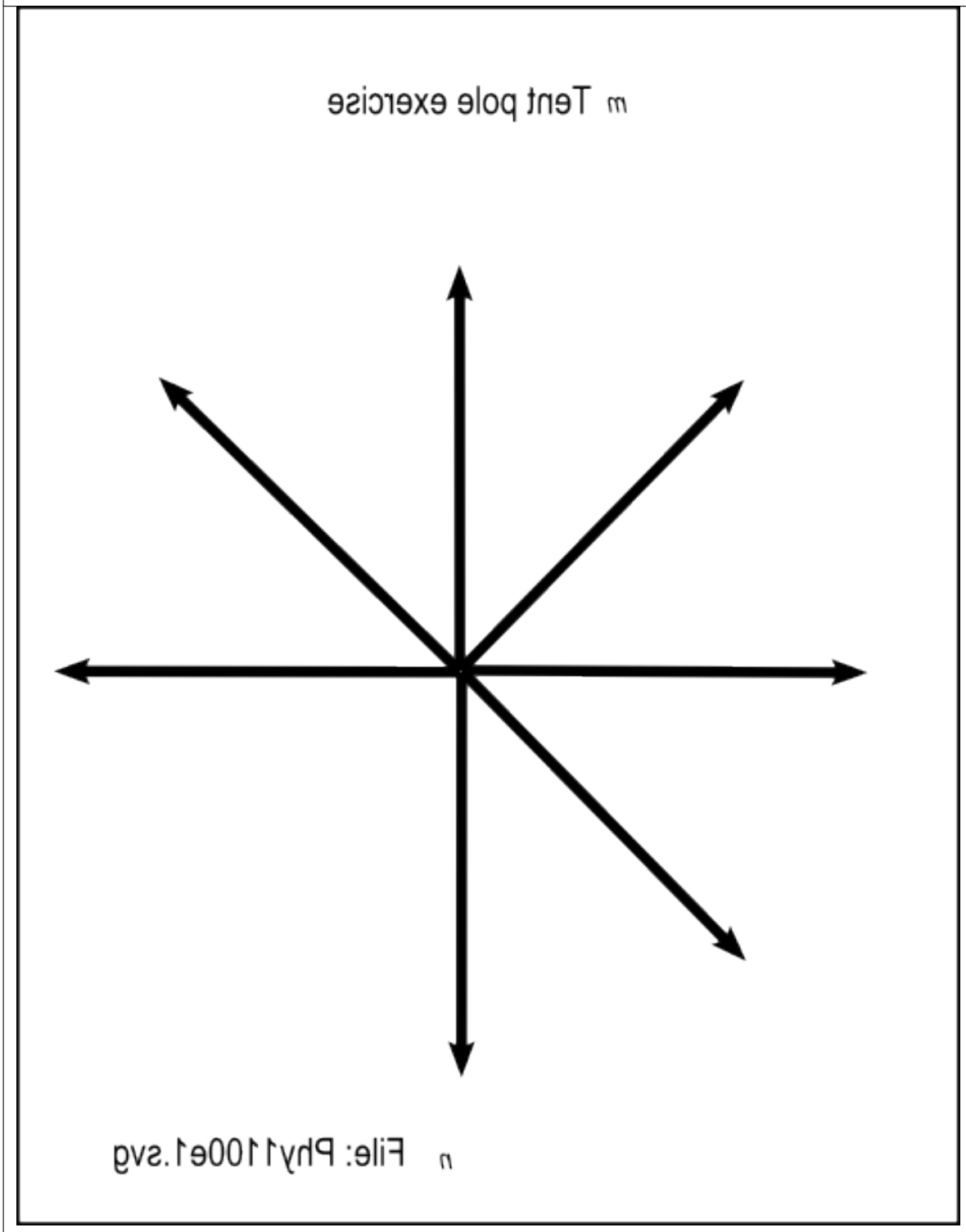
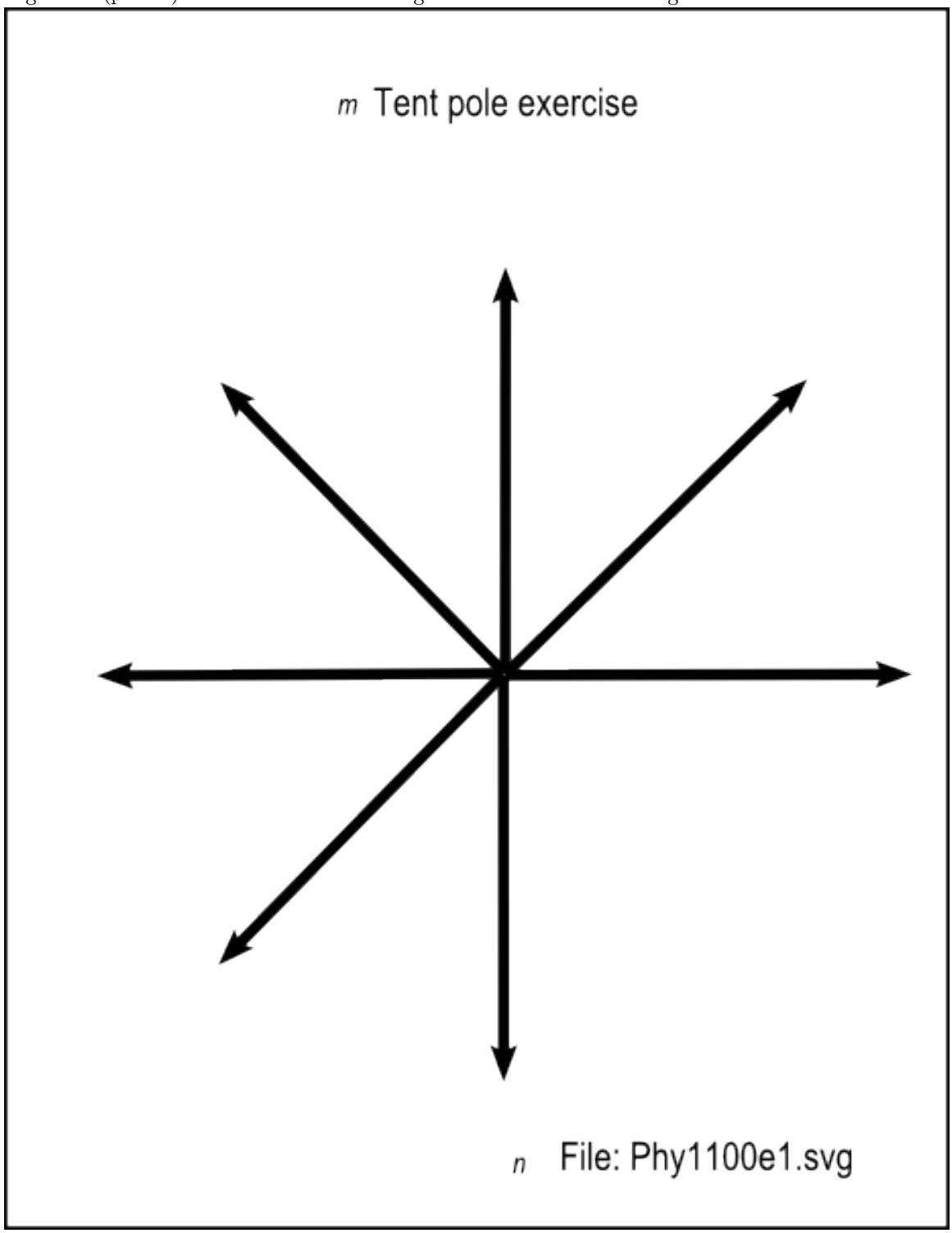
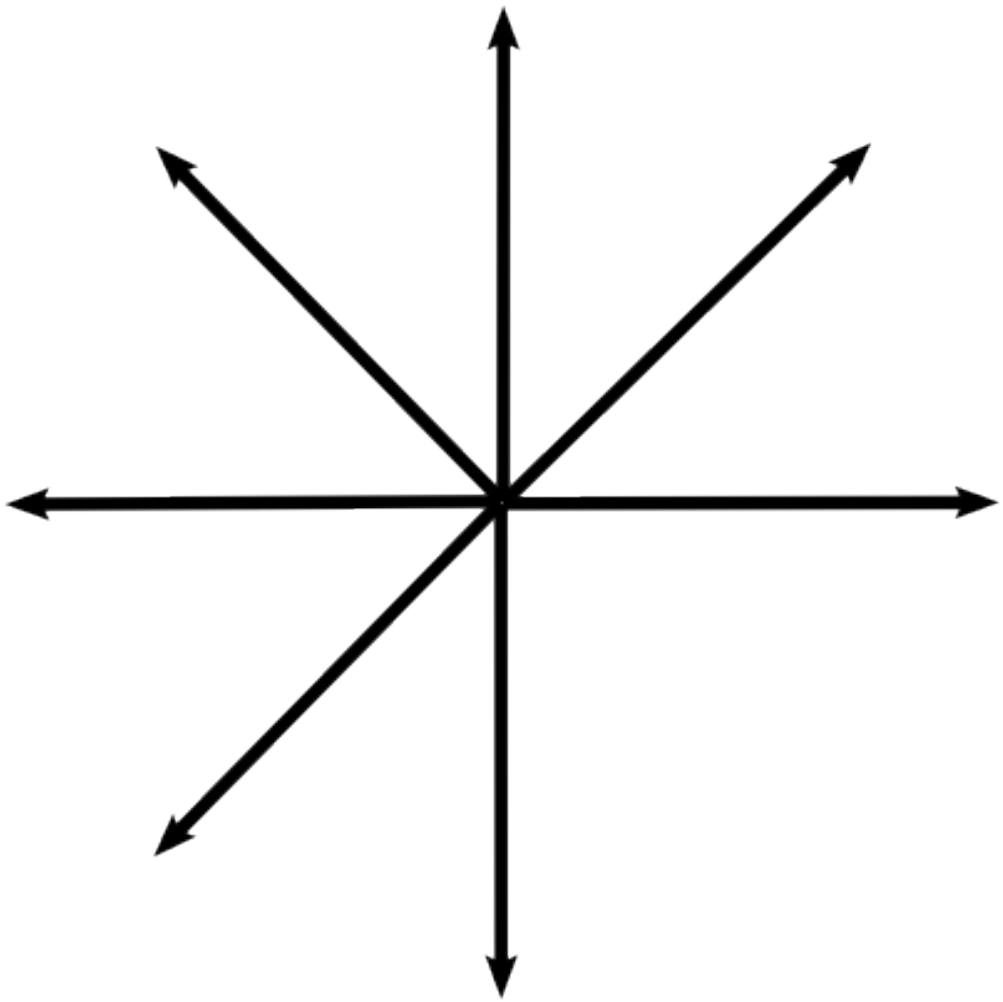


Table 14.13

Figure 14 (p. 437) shows a non-mirror-image version of the same image.



m Tent pole exercise



n File: Phy1100e1.svg

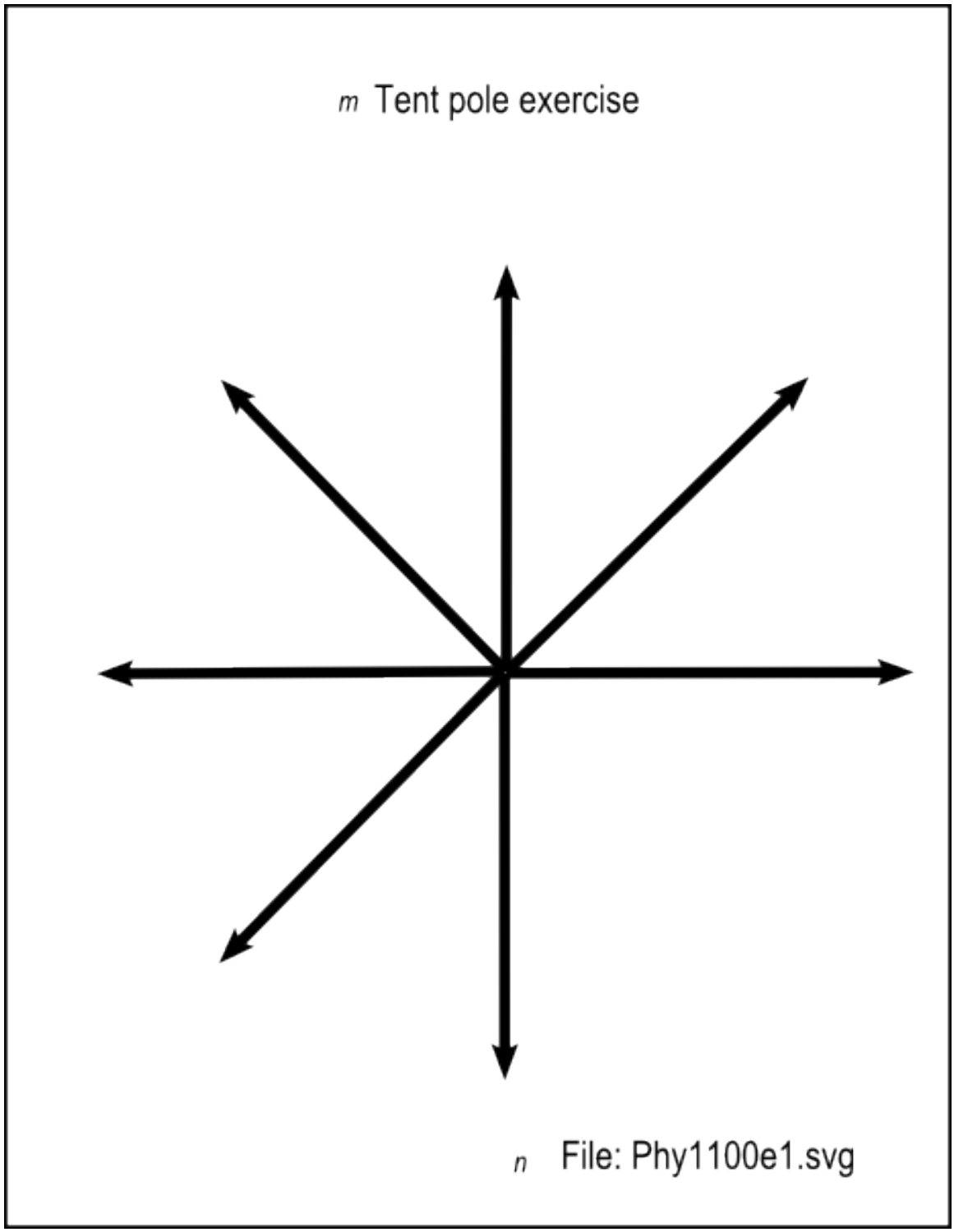

m Tent pole exercise
n File: Phy1100e1.svg

Figure 14 . Non-mirror-image version of the image from the file named Phy1100e1.svg.

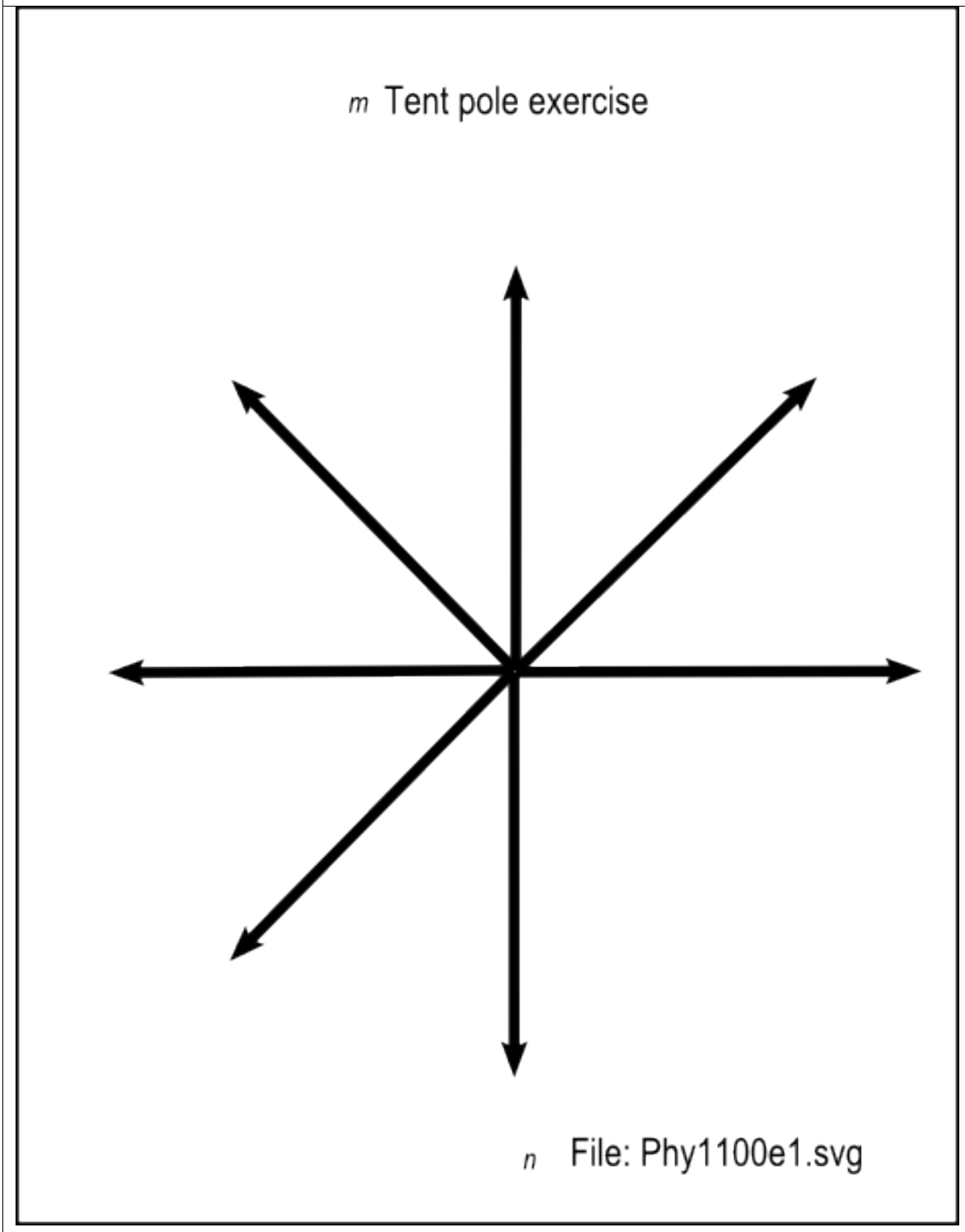


Table 14.14

Figure 15 (p. 438) shows the key-value pairs that go with the image in the file named Phy1100e1.svg.

Figure 15 . Key-value pairs for the image in the file named Phy1100e1.svg.

```
    m: Tent pole exercise
n: File: Phy1100e1.svg
```

Table 14.15

Equilibrium

Is the tent pole in equilibrium? If not, why not? What new force would need to be added to cause it to be in equilibrium?

You can probably solve this exercise in your head by inspection of the force vectors because they are all of the same length and they emanate outward at regularly spaced angles around the pole.

Could be more complex

Note, however, that the scenario could be much more complicated involving force vectors with different lengths arranged at seemingly random angles around the pole.

The important point is, the same general scheme would be used to solve the more complicated problem as is used to solve this simple exercise. However, the code in the script for the more complicated version would probably be somewhat more messy.

The scheme for the solution

The solution is to compute the sum of the force vectors to determine if the resultant vector is zero. If it is not zero, the tent pole would not be in equilibrium. It would be necessary to add a new force vector with the same magnitude as the resultant vector but in a direction that is the exact opposite of the direction of the resultant vector to cause the tent pole to be in equilibrium.

Create a script

Please copy the code from Listing 1 (p. 439) into an html file and open the file in your browser.

Listing 1 . Tent pole exercise.

```
    <!---------------- File JavaScript01.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br></br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle



var resultX = 0;//magnitude in newtons
var resultY = 0;//magnitude in newtons
var ang = 0;//degrees
var mag = 10;//newtons

while(ang <= 270){
  resultX = resultX + mag*Math.cos(ang*Math.PI/180);
  resultY = resultY + mag*Math.sin(ang*Math.PI/180);
  ang = ang + 45;
}//end while loop

var resultAng = getAngle(resultX,resultY);
var resultMag = Math.sqrt(resultX*resultX +
                          resultY*resultY);

document.write("The resultant is:" + "</br>");
document.write("Magnitude = " + resultMag.toFixed(2) +
               "</br>");
document.write("Angle = " + resultAng.toFixed(2) +
               "</br>");
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 14.16

Screen output

The text shown in Figure 16 (p. 440) should appear in your browser window when you open the html file in your browser.

Figure 16 . Screen output for Listing #1.

```
   Start Script

The resultant is:
Magnitude = 10.00
Angle = 135.00

End Script
```

Table 14.17

Analysis of the output

Figure 16 (p. 440) shows the resultant force when the vector forces shown earlier (p. 428) are added.

The requirement for equilibrium

Two or more coplanar forces concurrent at a point are in equilibrium if the sums of their components, taken along two mutually perpendicular axes are zero.

The vector sum is not zero

As you can see from Figure 16 (p. 440) , the vector sum of the forces shown earlier (p. 428) is not zero. The resultant vector has a magnitude of 10 newtons and a direction of 135 degrees.

Therefore, the tent pole is not in equilibrium and is subject to falling down. If it falls, it will probably fall in the direction of 135 degrees because there is a resultant force of 10 newtons pulling it in that direction.

Add a 10-newton force at 315 degrees

To put the tent pole in equilibrium, we would need to add one more force of 10 newtons in the opposite direction of the resultant shown in Figure 16 (p. 440) . In particular, one more guy wire is needed that adds a 10-newton pulling force in the direction of 315 degrees.

Analysis of the code

We will deal with causing the tent pole to be in equilibrium shortly. Right now, let's make certain that we understand the code shown in Listing 1 (p. 439) .

I doubt that you will find anything new in this script because I explained the use of components for computing vector sums in an earlier module.

A function named getAngle

Listing 1 (p. 439) begins with the definition of a function named getAngle that I explained in an earlier module. Therefore, I won't explain that code again in this module.

Declare working variables

Following the definition of the getAngle function, Listing 1 (p. 439) declares and initializes several working variables.

Enter a while loop

Then control goes into a while loop that computes the sum of the x-components and the sum of the y-components for each of the forces shown earlier (p. 428) .

Those forces are uniformly spaced around the tent pole with a 10-newton force vector every 45 degrees beginning at 0 and extending to 270 degrees inclusive. That makes it easy to compute and add the components using a while loop.

The code in the while loop begins by computing the components of the force vector at 0 degrees, and then computes and accumulates the x and y components for each of the remaining force vectors with a new force vector every 45 degrees up to and including the force vector at 270 degrees.

Storage locations for the vector sum

When the while loop exits, after computing the sum of the x-components and the sum of the y-components for the seven forces shown earlier (p. 428) , the sum of the x-components is stored in the variable named resultX and the sum of the y-components is stored in the variable named resultY.

Compute the magnitude and angle of the resultant vector

The magnitude of the resultant vector is computed as the square root of the sum of the squares of the x and y components. The getAngle method is called to get the angle of the resultant force vector.

Then those values are displayed in the browser window as shown in Figure 16 (p. 440) .

Now let's fix the problem

We said earlier that we can cause the tent pole to be in equilibrium by adding one additional 10-newton force at an angle of 315 degrees. We can easily accomplish this by making the changes shown in Figure 17 (p. 441) to the code in Listing 1 (p. 439) .

Figure 17 . Changes to Listing #1. .

```
Change the code that reads:

while(ang <= 270){

to read

while(ang <= 315){
```

Table 14.18

This change will cause one more 10-newton force vector at an angle of 315 degrees to be added to the sum of the vectors.

Make the change

Make that change and open the modified html file in your browser. When you do, the text shown in Figure 18 (p. 441) should appear in your browser window.

Figure 18 . Screen output for modified Listing #1.

```
    Start Script

The resultant is:
Magnitude = 0.00
Angle = 198.43

End Script
```

Table 14.19

Magnitude of resultant vector is zero

The significant thing about Figure 18 (p. 441) is that the magnitude of the resultant vector is now 0, which satisfies the requirement for equilibrium stated earlier (p. 440) .

Pay no attention to the angle shown in Figure 18 (p. 441) . It is meaningless. It is not possible to compute a valid angle for a vector that has a magnitude of zero.

14.5 Run the scripts

I encourage you to run the script and perform the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the code and the calculations, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

14.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

14.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force – Vector Solutions for Coplanar Forces Concurrent at a Point
- File: Phy1100.htm
- Revised: 10/02/15
- Keywords:
  - physics
  - accessible
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - coplanar forces
  - law of sines
  - law of cosines
  - parallelogram rule
  - triangle rule
  - resultant
  - equilibrium

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 15

Force – Moments, Torque, Couple, and Equilibrium[1]

15.1 Table of Contents

- Preface (p. 445)
  - General (p. 445)
  - Prerequisites (p. 446)
  - Viewing tip (p. 446)
    - Figures (p. 447)
  - Supplemental material (p. 447)
- General background information (p. 447)
- Discussion and sample code (p. 449)
  - Conditions for equilibrium (p. 449)
  - Equilibrium for a body with an arbitrary shape (p. 449)
  - A spring balance or spring scale (p. 460)
  - Exercise relating pound-force to kilogram-force (p. 461)
  - Exercise involving a trapeze bar (p. 463)
  - A more general case for the trapeze bar (p. 473)
  - Apply the weight at different locations on the trapeze (p. 474)
  - Hypothetical replacement by a single upward force (p. 474)
  - Exercise involving a bar supported at only one point (p. 475)
  - Couples (p. 484)
- Complete the exercises (p. 494)
- Resources (p. 494)
- Miscellaneous (p. 494)

15.2 Preface

15.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38167/1.4/>.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

This module explains moments, torque, and couples in a format that is accessible to blind students.

15.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

15.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following link to easily find and view the figure while you are reading about it.

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

15.2.3.1 Figures

- Figure 1 (p. 454) . Mirror image from the file named Phy1110a1.svg.
- Figure 2 (p. 458) . Non-mirror-image version of the image from the file named Phy1110a1.svg.
- Figure 3 (p. 458) . Key-value pairs for the image in the file named Phy1110a1.svg.
- Figure 4 (p. 462) . Relationship between pound-force and kilogram-force.
- Figure 5 (p. 467) . Mirror image from the file named Phy1110b1.svg.
- Figure 6 (p. 471) . Non-mirror-image version of the image from the file named Phy1110b1.svg.
- Figure 7 (p. 472) . Key-value pairs for the image in the file named Phy1110b1.svg.
- Figure 8 (p. 478) . Mirror image from the file named Phy1110c1.svg.
- Figure 9 (p. 482) . Non-mirror-image version of the image from the file named Phy1110c1.svg.
- Figure 10 (p. 482) . Key-value pairs for the image in the file named Phy1110c1.svg.
- Figure 11 (p. 488) . Mirror image from the file named Phy1110d1.svg.
- Figure 12 (p. 492) . Non-mirror-image version of the image from the file named Phy1110d1.svg.
- Figure 13 (p. 493) . Key-value pairs for the image in the file named Phy1110d1.svg.

15.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

15.3 General background information

Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explains how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1110.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

Conditions for equilibrium

You learned in earlier modules that the vector sum of all forces acting on a body must be zero in order for that body to be in equilibrium. There is a second, equally important condition for equilibrium:

The sum of all torques acting upon a body in equilibrium measured about any axis must be zero.

A see saw toy

[14] http://www.dickbaldwin.com/toc.htm
[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[16] http://cnx.org/content/m38167/latest/Phy1110.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

When I was a child, virtually every playground meant for children had one or more tows commonly called see saws. I never see them any more. They have probably been banned as a safety hazard along with swings and other toys.

Hopefully you have had the privilege of riding on a see saw, because riding on a see saw can provide several different physics lessons.

A simple device

A see saw is a simple device. It can be as simple as a long board balanced on a fulcrum near the center. A child sits on each end. As one child goes down, the other child goes up. Each child uses their feet to propel their end of the board upward. When one end goes up, the other end goes down.

It's the negative acceleration that hurts

Even under the worst of conditions a see saw can prove to be a good physics lesson for a child. The lesson begins when the child on one end jumps off while the child on the other end is high in the air. The remaining child suddenly begins a free fall toward the center of the earth with an acceleration of approximately 32.2 feet/sec^2.

After experiencing that once or twice, the child comes to learn that it is not the high downward velocity that hurts. Instead, it is the very high negative acceleration when the child makes contact with the ground that hurts.

Position relative to the fulcrum

Children also quickly learn that when two children of different body masses play on a see saw, the one with the greatest body mass must sit closer to the fulcrum. Otherwise, the see saw will only be in equilibrium when one end is touching the ground and the other end is high in the air.

Children learn that if each child sits just the right distance from the fulcrum with the more massive child closer to the fulcrum, they can cause the board to balance and be in equilibrium, at least for a short period of time. They also learn that is the seating position that provides the smoothest up and down motion.

Each child exerts a downward force

Although the children probably don't realize it, each child exerts a downward force on the board equal to the body mass of the child multiplied by the acceleration of gravity. Those forces are commonly referred to as the weights of the children.

Balance and equilibrium

When the see saw is in equilibrium (with both ends off the ground), the fulcrum exerts an upward force on the board that is equal to the combined weights of the children. Otherwise, the see saw and the children would either fly off into space, or sink into the earth.

In addition, when the see saw is in equilibrium with both ends off the ground, the product of each child's weight and the distance of the child from the fulcrum is equal to the product of the other child's weight and the distance of the other child from the fulcrum. Children usually figure out how that works but I doubt that they understand why it works.

The crank

I have an emergency light that has a crank on the side. When I turn the crank, an electrical generator inside the light turns, which generates electrical power that illuminates a light bulb in the end of the light.

There are many other examples of cranks in common use. For example, I have an automobile jack with a crank that is used to raise the automobile in order to replace a flat tire.

Sailboats have hand-operated windlasses with cranks that are used to shorten the length of ropes to raise the sails. Awnings have cranks that are used to raise or lower the awning to create more or less shade from the sun.

A toggle switch on the wall is a small crank with about a 30-degree angle of travel that is used to turn the lights on and off. Pushing the handle in one direction or the other causes an axel to turn inside the switch causing an electrical contact to be closed or open.

The turning effect of a crank

When analyzing a crank, there are at least two factors that must be considered. One factor is the force exerted on the handle. The second is the length of the crank arm. The turning effect depends on the product of the force and the length of the crank arm.

Moments or torques

Cranks are often used, (along with other mechanical devices such as screws) to provide mechanical advantage. The same turning effect (torque) can be achieved with a long crank arm and a small force, or a short crank arm and a large force. (Remember, the torque is the product of the two.) For example, much more torque is required to raise my automobile off the ground than is required to open or close the electrical contacts inside a toggle switch. Therefore, the crank arm on my automobile jack is much longer than the crank arm on a toggle switch.

By definition, (see *College Physics by Mendenhall, Keys, Eve, and Sutton)*

The torque or moment of a force about an axis through the point O is defined as the product of the force and the perpendicular distance from O to the line of action of the force.

Do torque and moment of a force mean the same thing?

This definition would have you believe that the torque of a force and the moment of a force mean the same thing. If you Google the difference between torque and moment, you will find many who agree that they mean the same thing and many who disagree. However, for practical purposes in this module, we will assume that they mean the same thing.

The turning action has a direction

Like force, the turning action of torque has a direction. Unlike force, however, for which the direction is a line through a point, the direction of the turning action of a torque is rotational about a point. The direction can be either clockwise or counter-clockwise. In this module, we will assume that a counter-clockwise direction is positive and a clockwise direction is negative.

(Although the turning action of a torque is rotational about a point, the true direction of a torque is said to be perpendicular to the plane containing the forces. That topic is beyond the scope of this module.)

A machine that converts force to torque

A crank is a machine that converts force along a line into torque about a point. When the crank arm on my automobile jack is horizontal with the crank handle to the right, and I push down on the crank handle, a clockwise torque is developed about the axle at the other end of the crank arm.

That axle is actually a large screw, and the torque causes the screw to turn. The other end of that screw is threaded through a mechanism that is often referred to as a "scissors jack." The rotational motion of the screw causes a small platform to raise underneath my car to lift it off the ground.

Oops, need to change direction

When I push down on the crank handle and the screw begins to turn, the crank arm rotates along with the screw. Therefore, the crank arm is no longer horizontal but is oriented at an angle slightly south of east. In order to keep the screw turning, I must adjust the direction of my force to being a little west of south instead of being straight down. As the process continues, I must continually adjust the direction of the force to be perpendicular to the length of the crank arm.

15.4 Discussion and sample code

Let's begin the discussion by expanding the conditions for equilibrium beyond the conditions that we have considered in earlier modules.

15.4.1 Conditions for equilibrium

In order for a body to be in equilibrium,

The vector sum of all forces acting on the body must be zero and the sum of all torques acting on the body measured about any axis must be zero.

15.4.2 Equilibrium for a body with an arbitrary shape

Achieving equilibrium may be easier said than done for a body with an arbitrary shape. Let's consider what first appears to be a simple example. Using your graph board, mark out a square that is four units on each

side. You should probably make it rather large using four or five divisions on the graph board to represent one unit. Let this square represent a square board that is free to move horizontally or vertically and is also free to rotate.

Mark four points with pins

Assume that the lower left corner of the square is the origin with coordinates of x=0 and y=0. Place pins at the following coordinate positions:

- A. x=1, y=1
- B.x=1, y=2
- C. x=3, y=2
- D. x=2, y=1

Draw vectors

Now use rubber bands or pipe cleaners to draw the following force vectors originating at the locations of the pins. (Positive angles are measured counter-clockwise relative to the positive x axis.)

- A. 5 units magnitude at 180 degrees
- B. 2.8 units magnitude at 45 degrees
- C. 3 units magnitude at 0 degrees
- D. 2 units magnitude at -90 or +270 degrees

Tactile graphics

The svg file that is required to create tactile graphics for this exercise is named Phy1110a1.svg. You should have downloaded (p. 447) that file earlier. This file contains a vector diagram that represents the instructions given above (p. 450) .

Figure 1 (p. 454) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

m Equilibrium for a body with an arbitrary shape
n B
o C
p D
q A
s The sum of the torques about point D is -8.936.
r File: Phy1110a1.svg

m Equilibrium for a body with an arbitrary shape
n B
o C
p D
q A
s The sum of the torques about point D is -8.936.
r File: Phy1110a1.svg

m Equilibrium for a body with an arbitrary shape
C o
B n
D p
A q
s The sum of the torques about point D is -8.936.
r File: Phy1110a1.svg

Figure 1 . Mirror image from the file named Phy1110a1.svg.

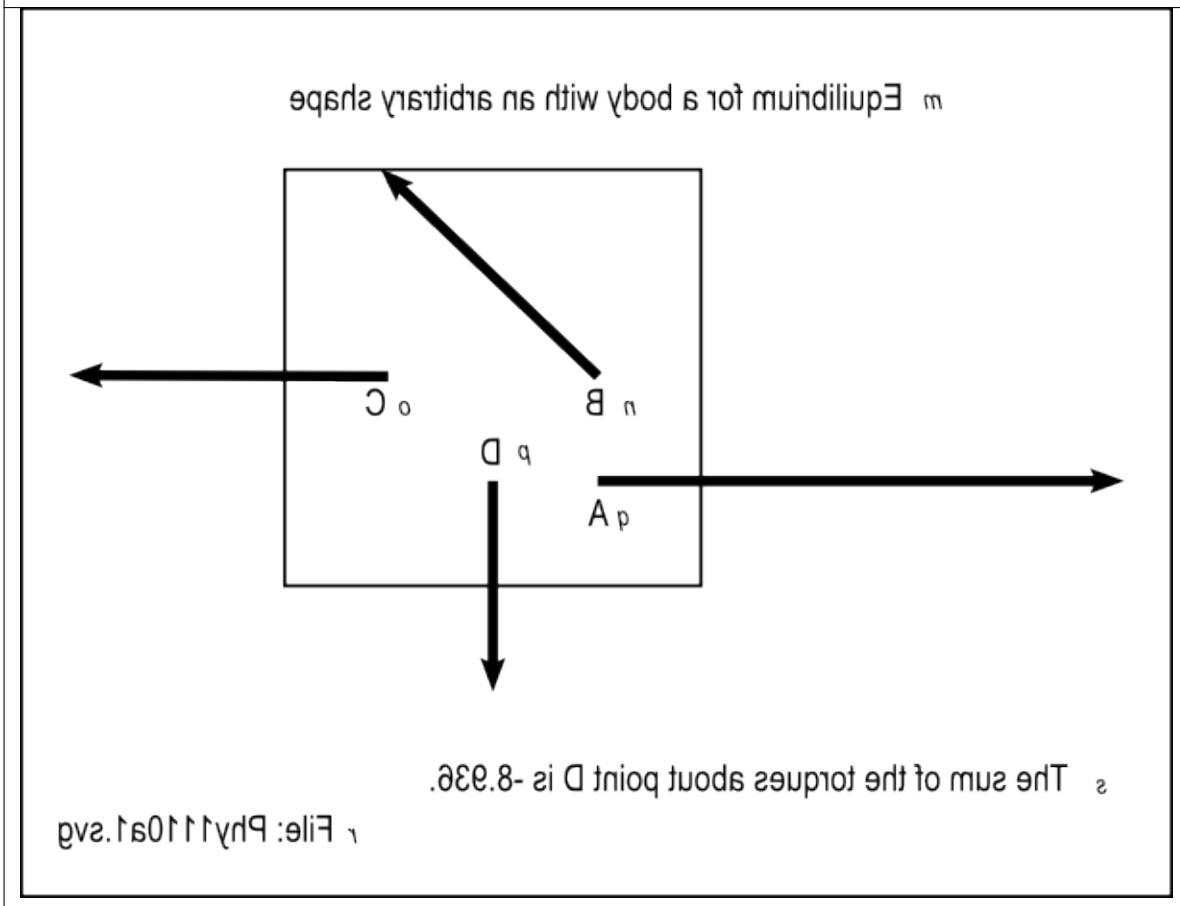
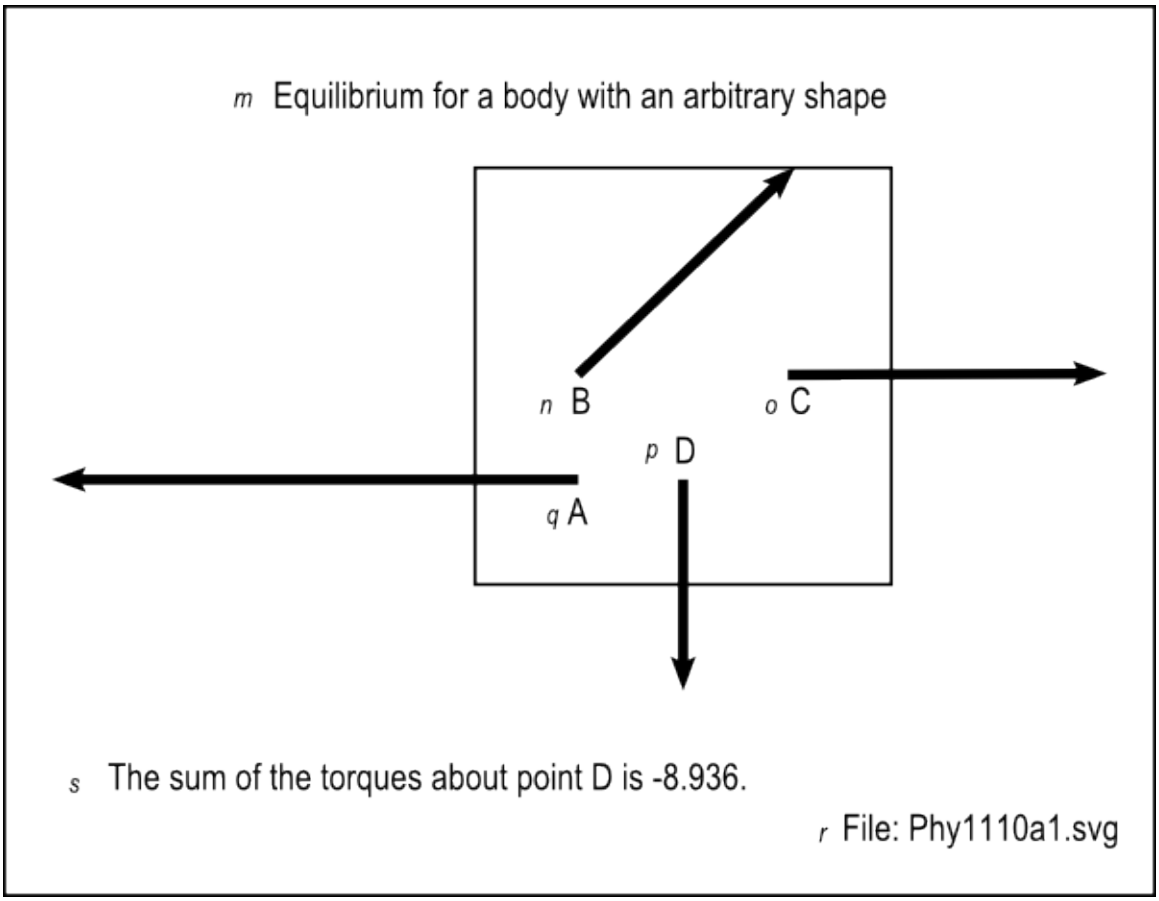
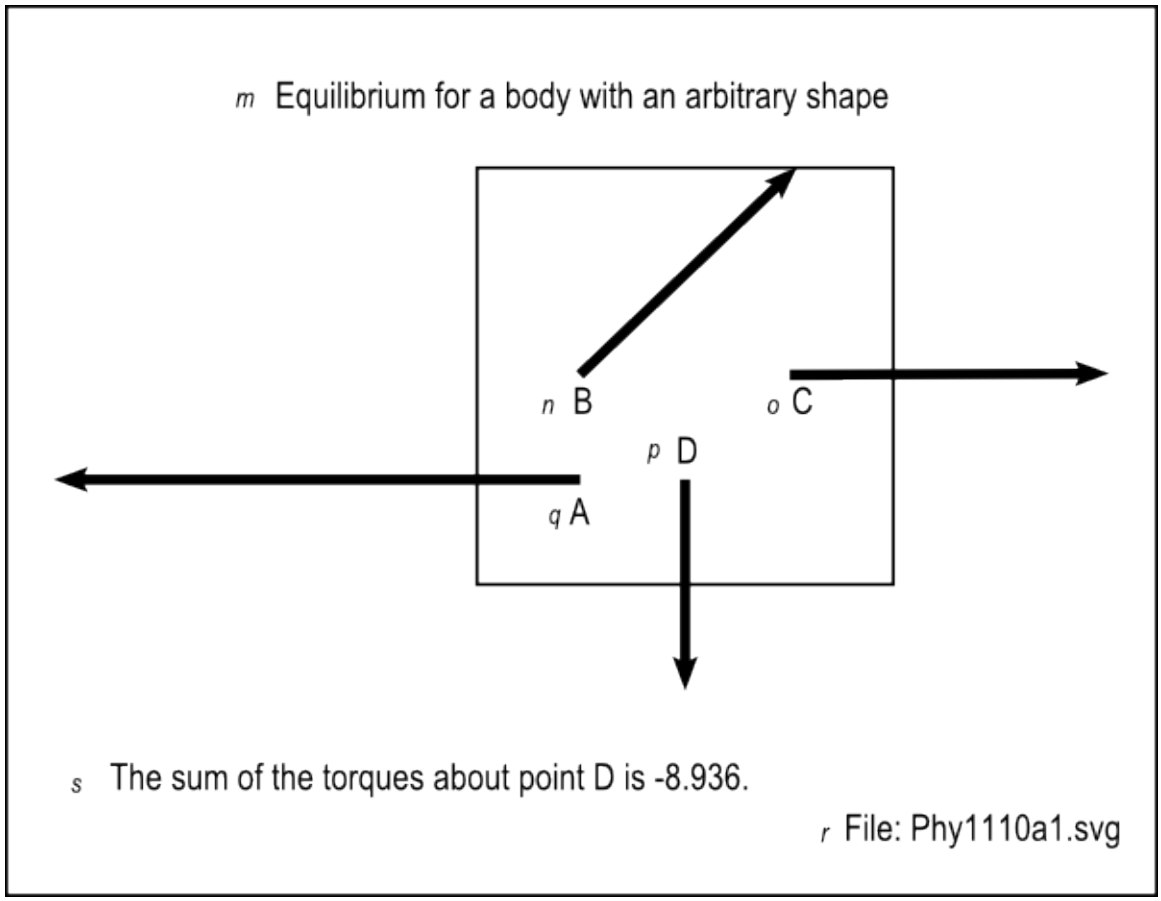


Table 15.1

Figure 2 (p. 458) shows a non-mirror-image version of the same image.


m Equilibrium for a body with an arbitrary shape
n B
o C
p D
q A
s The sum of the torques about point D is -8.936.
r File: Phy1110a1.svg


m Equilibrium for a body with an arbitrary shape
n B
o C
p D
q A
s The sum of the torques about point D is -8.936.
r File: Phy1110a1.svg

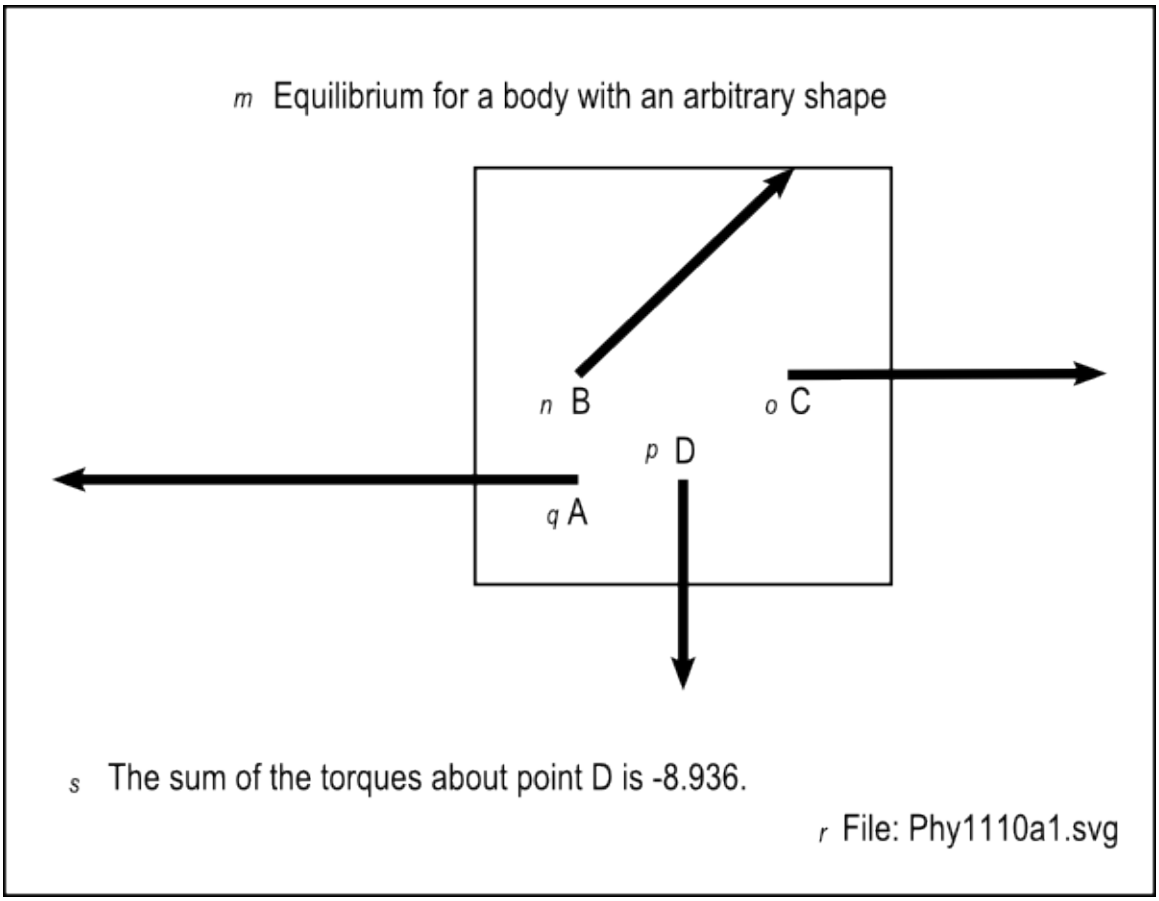

m Equilibrium for a body with an arbitrary shape
n B
o C
p D
q A
s The sum of the torques about point D is -8.936.
r File: Phy1110a1.svg

Figure 2 . Non-mirror-image version of the image from the file named Phy1110a1.svg.

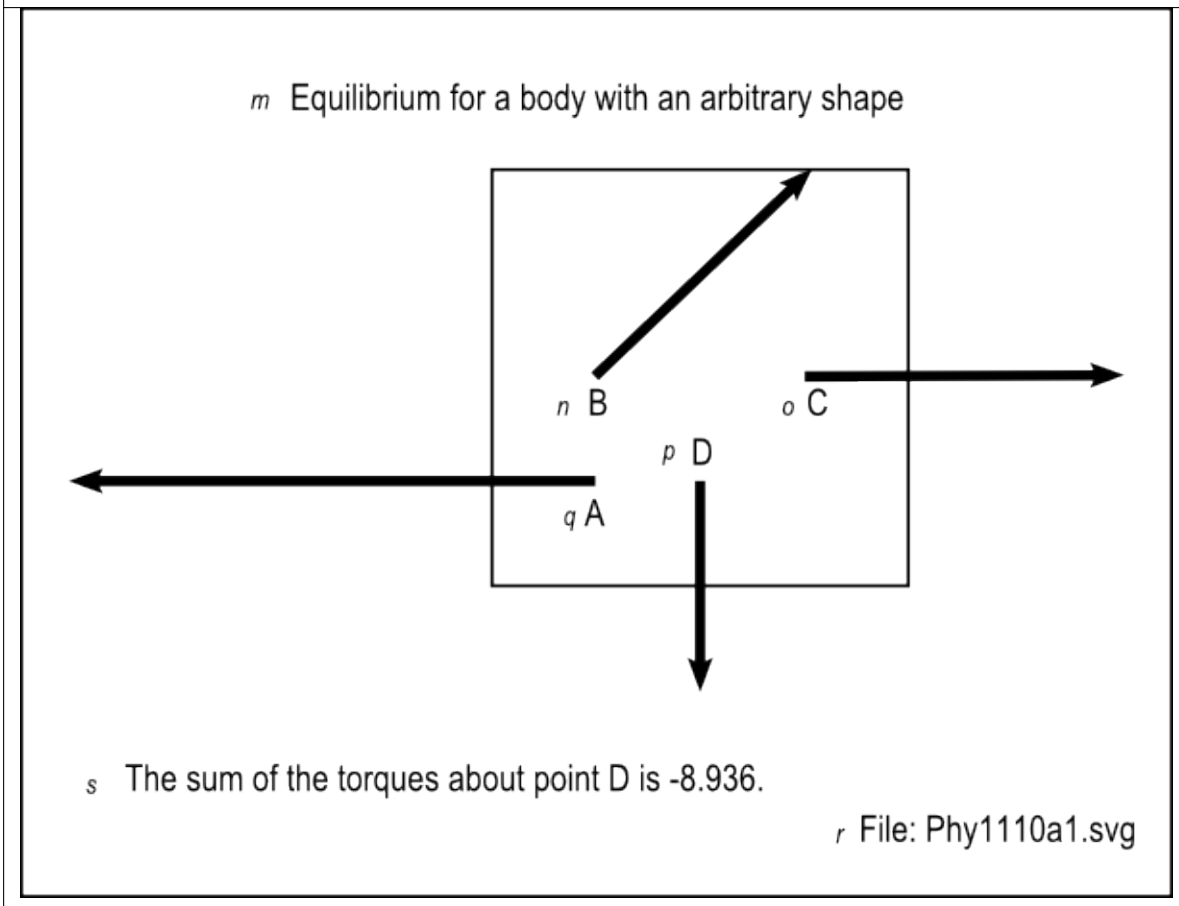


Table 15.2

Figure 3 (p. 458) shows the key-value pairs that go with the image in the file named Phy1110a1.svg.

Figure 3 . Key-value pairs for the image in the file named Phy1110a1.svg.

```
   m: Equilibrium for a body with an arbitrary shape
n: B
o: C
p: D
q: A
r: File: Phy1110a1.svg
s: The sum of the torques about point D is - 8.936.
```

Table 15.3

Compute the vector sum of the forces

Begin by computing the horizontal and vertical components of the force at B. As you know by now, the horizontal component is equal to the magnitude of the vector (2.8) multiplied by the cosine of 45 degrees and the vertical component is equal to the magnitude of the vector multiplied by the sine of 45 degrees. Both the sine and the cosine of 45 degrees is 0.707, so the horizontal and vertical components are both equal to 2. (Obviously, I planned it that way.)

The horizontal and vertical components

The remaining vectors are either horizontal or vertical so no trigonometry is required to compute their components.

The horizontal components of all the vectors consist of:

- A. 5 units at 180 degrees = -5
- Bx. 2 units at 0 degrees = +2
- C. 3 units at 0 degrees = +3

The sum of the horizontal components is 0.

The vertical components of all the vectors consist of:

- By. 2 units at 90 degrees = +2
- D. 2 units at -90 or +270 degrees = -2

The sum of the vertical components is also 0. Therefore, the system exhibits the first condition of equilibrium.

Compute the moment or torque

The second condition for equilibrium states that the sum of all torques acting on the body measured about any axis must be zero.

You can often reduce the complexity of the computation for computing torque by judicious selection of the point about which the torque will be computed.

Compute the torque about point D

In this case, we can simplify the arithmetic by choosing an axis that goes through the point at D.

If you examine your graph, you will see that the line of action for forces A and D both go through the point at D. Therefore, neither of those forces creates a moment about point D so we can exclude them from our computation.

Forces B and C create a torque about D

Further examination of your graph will reveal that forces B and C both create a torque about D, and they both create a clockwise or negative torque. Since there are no more forces being exerted on the board, the sum of the torques cannot be zero.

Thus, we can tell by inspection that this body is not in equilibrium.

For completeness...

Just for completeness, let's compute the torque about the point at D. If you extend the line of action for force B down and to the left, you will see that the distance from point D to that line, along a path that is perpendicular to the line, is 2.12 units. (You can compute that distance using either trigonometry or the Pythagorean theory.)

The components of the torque about the point at D are:

- B. 2.12 * (-2.8) = -5.936
- C. 1 * (-3) = -3

The sum of the torques about point D is not zero. Instead, it is -8.936. Therefore, the system does not satisfy the second condition for equilibrium.

Can equilibrium be achieved through the addition or modification of forces?

Obviously, if you add four more forces that are equal in magnitude and opposite in direction to the four existing forces, the system will be in equilibrium.

Also, if you allow the board to turn clockwise by several degrees, there is a point at which it will stop turning and will be in equilibrium.

Beyond those possibilities, I am unable, simply by inspection, to identify any forces that could be modified or added to bring the system into equilibrium. If it is required to add or modify forces to achieve equilibrium, I would need to construct and solve some simultaneous equations involving trigonometry. I have decided to opt out of that and will leave that as an exercise for the student.

The examples that follow will be less complicated than that.

15.4.3 A spring balance or spring scale

A fisherman's scale

What is a spring balance or spring scale? A spring scale is often called a fisherman's scale because a lot of fishermen carry one in their tackle box. They use it weigh the fish that they catch.

An old-fashioned spring scale consists of a stiff coil spring with a hook connected to each end. The spring is enclosed in a case that has a vertical slot with numbers along the side of the slot. There is a pointer attached to the spring that sticks out through the slot.

When you hold the top hook with one hand and hang something on the bottom hook, the spring extends causing the pointer to move downward. The distance that the pointer moves is an indication of the weight of the mass connected to the bottom hook.

A luggage scale

Because of the high fees that the airlines charge for overweight luggage, I carry a modern version of a spring scale when I travel so that I can weigh my luggage before check-in and make adjustments to the contents of each piece of luggage as appropriate. My spring scale is very similar to a fisherman's scale except that it is designed to weigh much heavier items than fish and is more modern in appearance.

My spring scale is designed to make it easy to measure the weight of a piece of luggage. (It also has a built-in tape measure, calibrated in both inches and centimeters that can be used to measure the dimensions of a piece of luggage, but that is not germane to this discussion.)

Construction of my spring scale

My spring scale has:

- A handle on the top
- A hook on the bottom
- A very stiff spring on the inside, and
- A circular dial with a red hand and a black hand on the front. The dial is calibrated both in lb (pounds-force) and in kg (kilograms-force).

The red and black hands

The red hand is normally in rotational equilibrium pointing to zero on the dial. The black hand can be rotated 360 degrees using a knob on the front of the scale.

Weighing a piece of luggage

You begin the process of weighing a piece of luggage by turning the black hand to point to zero. Then you hook the scale onto the handle on the luggage and pull up on the scale, causing the luggage to be lifted off the floor.

The red and black hands move

When you do that, the spring extends very slightly allowing the hook to move relative to the body of the scale, which in turn causes the entire scale to become slightly longer from top to bottom. The change in the state of the spring is proportional to the downward force (weight) exerted by the luggage on the hook.

The internal mechanism causes the red hand to rotate around the center of the dial. The position of the red hand indicates the weight of the luggage when compared to the printed calibrations on the face of the dial.

Equilibrium

When you are supporting the weight of the luggage with the spring scale, the luggage is being pulled down by the force of gravity and is being pulled up by the hook and your muscles. This causes the system to be in momentary equilibrium.

The black hand also moves

For convenience, the black hand moves along with the red hand, but unlike the red hand, the black hand doesn't return to zero when the luggage is allowed to settle back onto the floor. The position of the black hand, therefore, indicates the maximum angular excursion of the red hand, which indicates the weight of the luggage in either lb or kg.

Applying a force manually

When I hold the handle in one hand and pull on the hook with the other hand so that the overall length of the scale increases by about 1/4 inch, the red hand moves to a position indicating a force of approximately 20 lb or 9 kg.

When a spring balance is used in the following exercises, we will assume that the change in the overall length of the scale is negligible relative to the overall dimensions of the problem.

The difference between force and mass

The SI unit for mass is the kilogram with a symbol of kg.

An often-used non-SI unit for mass is the pound.

An area of potential confusion

Does the relationship between 20 lb and 9 kg as printed on the dial of my spring scale make sense? It only makes sense if you know what is meant by those abbreviations. (They do not indicate a 20-pound mass or a 9-kg mass as the abbreviations might lead you to believe.)

This is an area of physics that can be confusing – the difference between force and mass.

15.4.4 Exercise relating pound-force to kilogram-force

Let's work through an exercise using the following abbreviations and see if we can justify the relationship between lb and kg indicated by the dial on my spring scale:

- lbf means pounds-force
- pound means pounds mass
- kgf means kilograms-force
- kg means kilograms mass
- N means newtons
- m means meters
- s means seconds
- ft means feet

We will compute the relationship between pounds-force and kilograms-force using the force unit of newtons an intermediary.

We will assume that the acceleration of gravity is either 32.17 ft/s^2 or 9.81 m/s^2.

The calculations are shown in Figure 4 (p. 462) .

Figure 4 . Relationship between pound-force and kilogram-force.

```
Begin with known conversion factors:
1 lbf = 1 pound * 32.17 * ft/s^2
1 pound = 0.45 * kg

Substitute kg for pound.
1 lbf = 0.45 * kg * 32.17 * ft/s^2

Substitute meters for feet.
1 ft = 0.30 m
1 lbf = 0.45 * kg * 32.17 * 0.30 * m/s^2

Do the arithmetic and substitute (1 N)
for (1 kg*m/s^2)
1 lbf = 4.34 * kg*m/s^2 = 4.34 N



Now compute the relationship between kgf and N
Begin with some more known quantities.
1 kgf = 1 kg * 9.81 * m/s^2
1 N = 1 kg*m/s^2

Substitute (1 N) for (1 kg  * m/s^2)
1 kgf = 9.81 N

Now form a ratio between 1 lbf and 1 kgf
1 lbf/1 kgf = (4.34 N)/(9.81 N)

Cancel like terms, do the arithmetic, and
multiply both sides by kgf.
1 lbf = 0.44 kgf
```

Table 15.4

Is this the correct answer?

According to the online units converter [18] ,

1 pound-force (lbf) = 0.45359237 kilogram-force (kgf)

Considering that I rounded all computations to two decimal digits, my result is pretty close.

Apply conversion factor to the spring scale calibration

We now have a conversion factor that allows us to convert between lbf and kgf:

1 pound-force (lbf) = 0.45359237 kilogram-force (kgf)

If we multiply 20 lbf by0.45 to convert that value to force in units of kgf, we get 9.0 kgf. This agrees with the calibrations on my spring scale.

What do these terms really mean?

[18] http://www.translatorscafe.com/cafe/units-converter/force/calculator/kilogram-force-[kgf]-to-newton-[N]/

One lbf is the amount of force that is required to cause a one-pound mass to accelerate at 32.2 ft/s^2 due to the force of gravity.

One kgf is the amount of force required to cause a one-kg mass to accelerate at 9.81 m/s^2 due to the force of gravity.

15.4.5 Exercise involving a trapeze bar

Use your graph board to construct a picture that looks something like a very long trapeze bar. By this, I mean a horizontal bar with a rope tied to each end. The other end of each rope is firmly attached to the ceiling. Make the horizontal bar ten meters in length.

Tactile graphics

The file named Phy1110b1.svg contains an image that represents this scenario. The image shows the trapeze bar, the ropes, and the spring scales. The image also shows vectors that represent the forces acting on the trapeze bar.

Figure 5 (p. 467) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

Exercise involving a trapeze bar
Rope
Rope
Spring scale
Spring scale
Q = 2 N
P = 8 N
a
b
C = 2 m
A
B
0 m
Trapeze bar
10 m
W = 10 N
File: Phy1110b1.svg

Exercise involving a trapeze bar
Rope
Rope
Spring scale
Spring scale
Q = 2 N
P = 8 N
a
b
C = 2 m
A
B
0 m
Trapeze bar
10 m
W = 10 N
File: Phy1110b1.svg

Exercise involving a trapeze bar
Rope
Rope
Spring scale
Spring scale
Q = 2 N
P = 8 N
a
b
C = 2 m
A
B
0 m
Trapeze bar
10 m
W = 10 N
File: Phy1110b1.svg

Figure 5 . Mirror image from the file named Phy1110b1.svg.

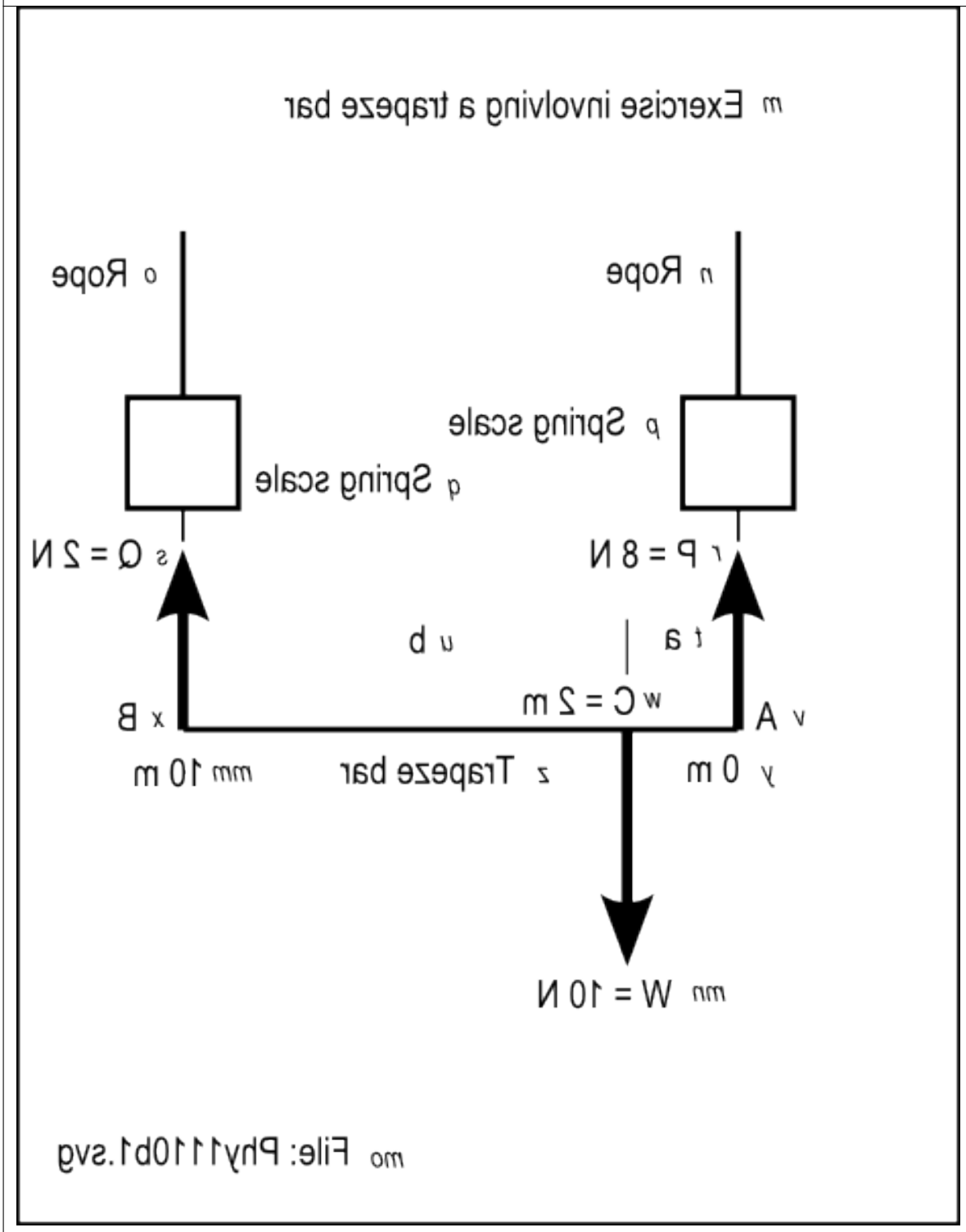
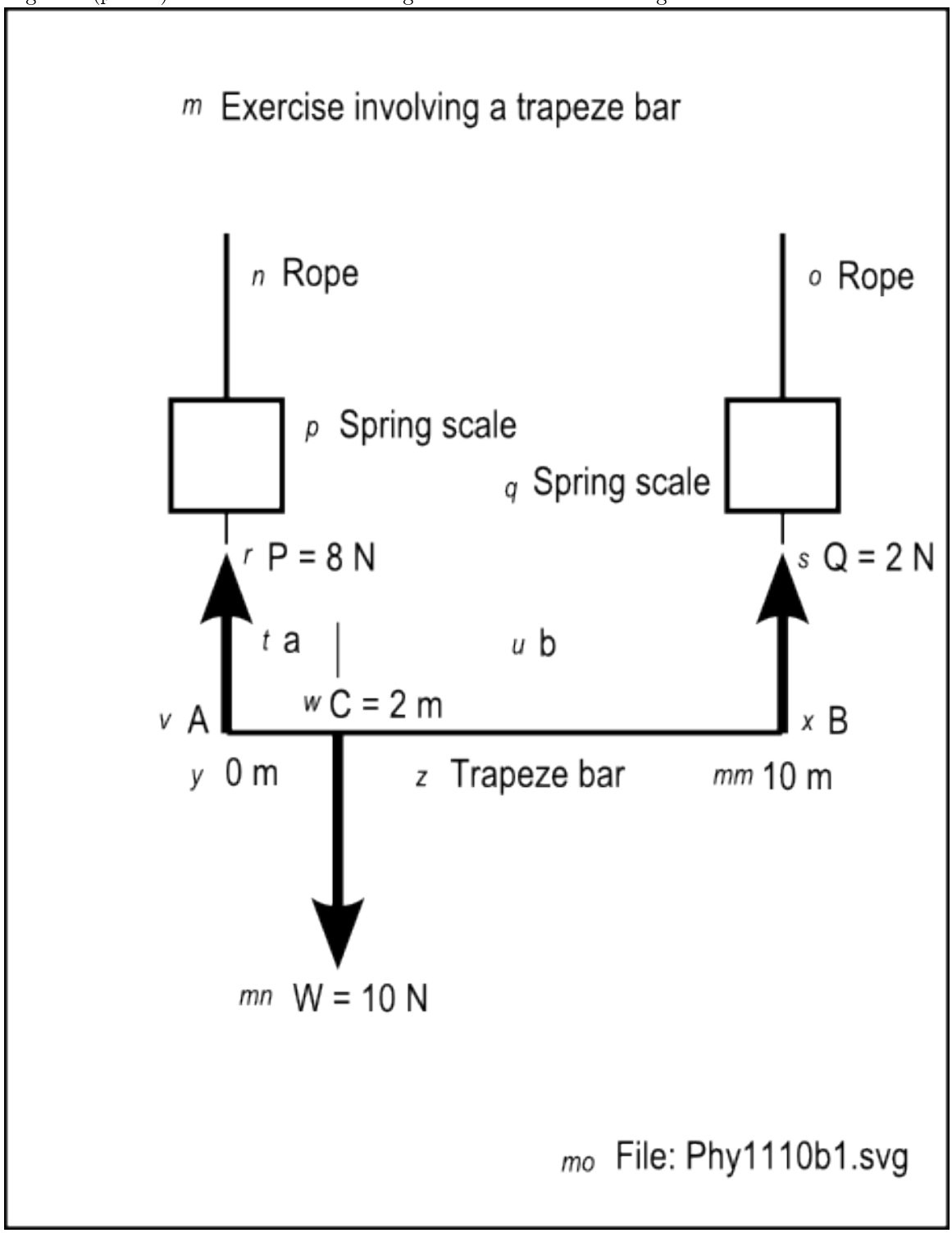
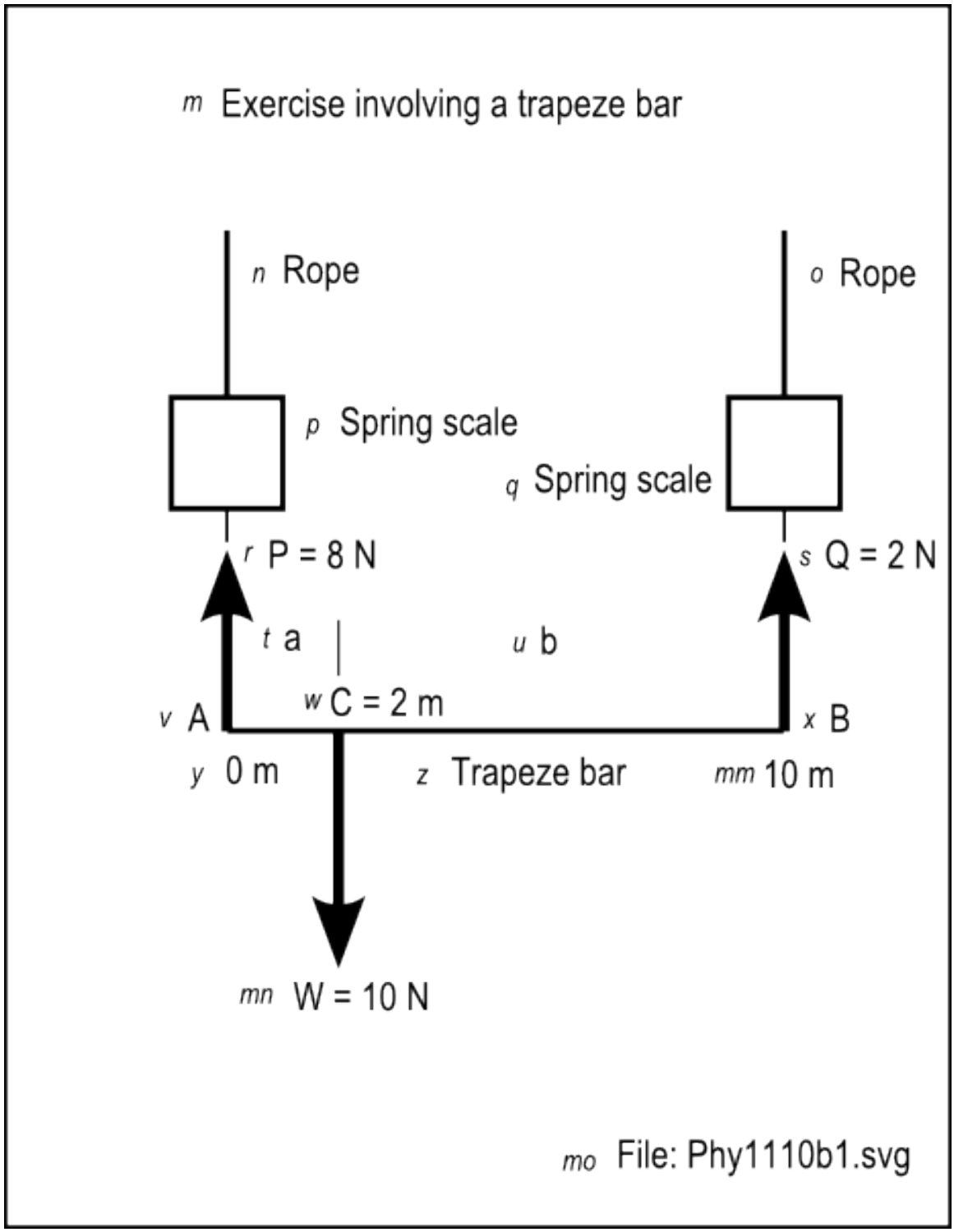


Table 15.5

Figure 6 (p. 471) shows a non-mirror-image version of the same image.




m Exercise involving a trapeze bar
n Rope
o Rope
p Spring scale
q Spring scale
r P = 8 N
s Q = 2 N
t a
u b
v A
w C = 2 m
x B
y 0 m
z Trapeze bar
mm 10 m
mn W = 10 N
mo File: Phy1110b1.svg

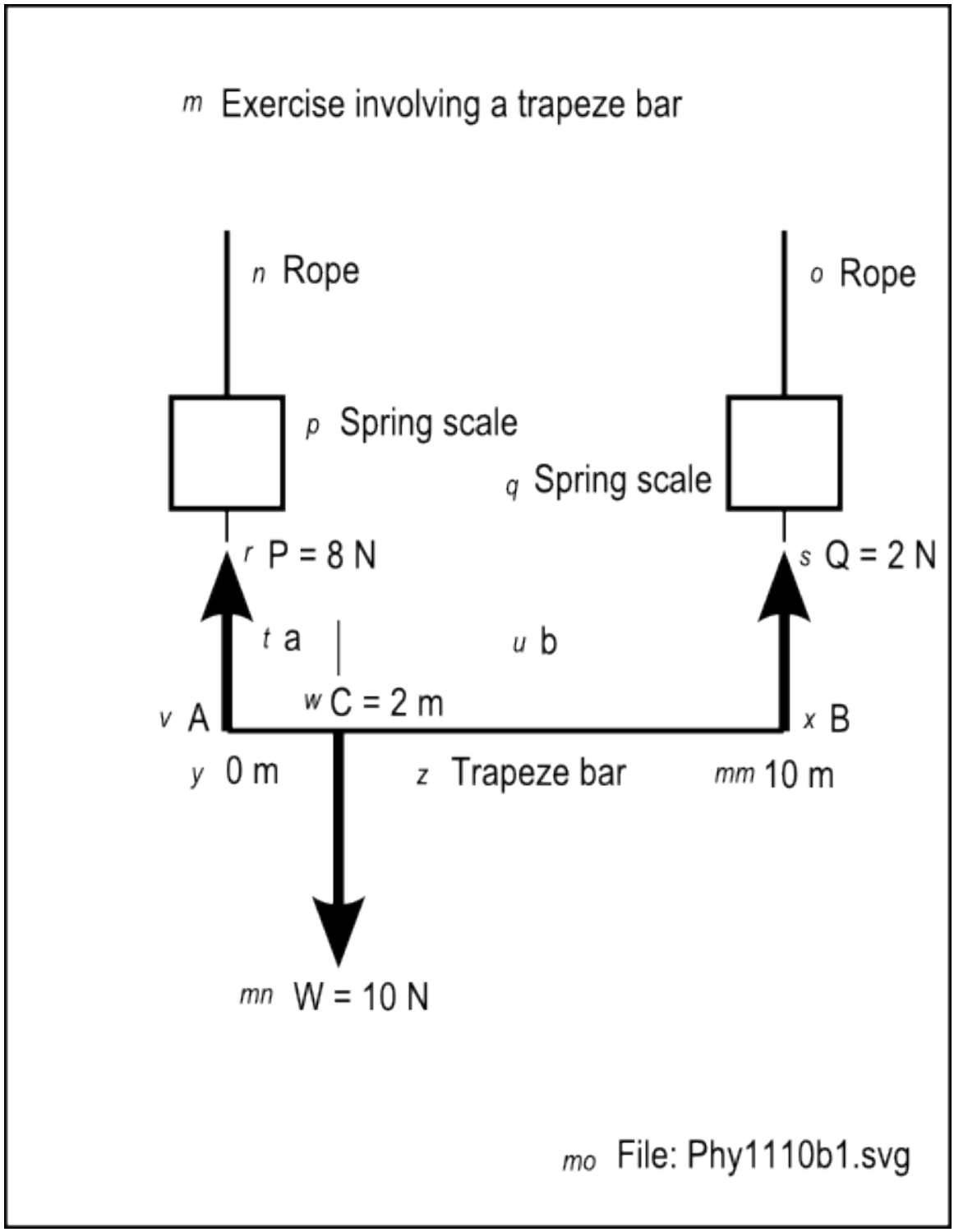

m Exercise involving a trapeze bar
n Rope
o Rope
p Spring scale
q Spring scale
r P = 8 N
s Q = 2 N
t a
u b
v A
w C = 2 m
x B
y 0 m
z Trapeze bar
mm 10 m
mn W = 10 N
mo File: Phy1110b1.svg

Figure 6 . Non-mirror-image version of the image from the file named Phy1110b1.svg.

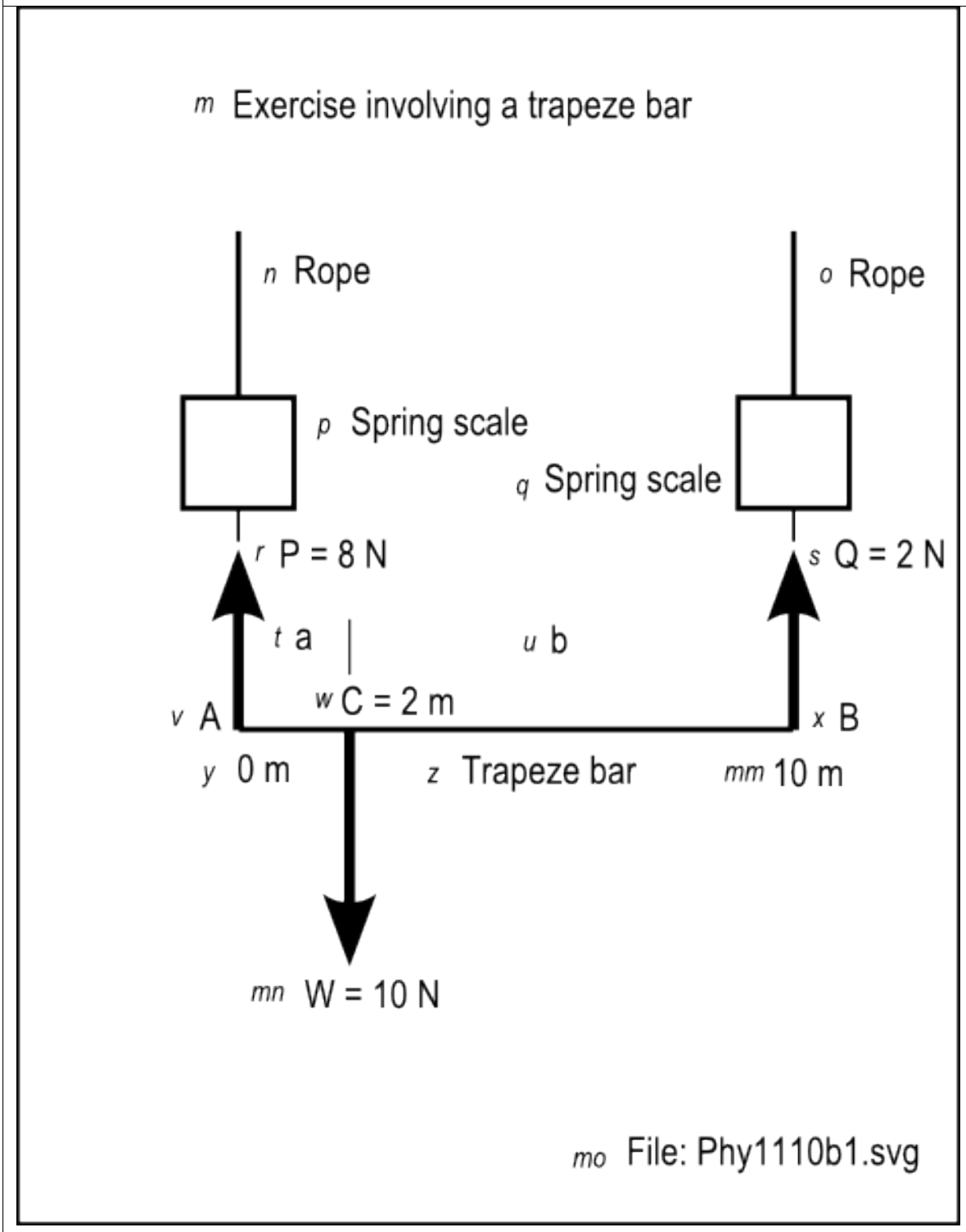


Table 15.6

Figure 7 (p. 472) shows the key-value pairs that go with the image in the file named Phy1110b1.svg.

Figure 7 . Key-value pairs for the image in the file named Phy1110b1.svg.

```
    m: Exercise involving a trapeze bar
n: Rope
o: Rope
p: Spring scale
q: Spring scale
r: P = 8 N
s: Q = 2 N
t: a
u: b
v: A
w: C = 2 m
x: B
y: 0 m
z: Trapeze bar
mm: 10 m
mn: W = 10 N
mo: File: Phy1110b1.svg
```

Table 15.7

Insert a hypothetical spring scale in each rope

Now cut each rope somewhere along its length and insert something that is meant to replicate a spring scale in each rope. Assume that the weights of the ropes, the spring scales, and the bar are negligible. Also assume that the extension of the spring scales caused by loading is negligible. (In other words, the bar will remain horizontal even when downward forces are applied.)

Each spring scale will exert an upward force on the bar and will, at the same time indicate the amount of force with which that end of the bar is pulling down on the rope that supports it.

The bar is in equilibrium

By default, the bar is in equilibrium. By that I mean that the bar doesn't accelerate in any direction, nor does it rotate about any axis. Furthermore, the weight being registered on each spring scale is negligible.

Label points and distances on your drawing

Use your Braille labeler to label the left end of the bar A and the right end of the bar B. Use pins to subdivide the bar into ten units.

Now assume that you will hang a 10 N weight somewhere on the bar between the ropes. Label that point C. Label the distance from C to A as a, and label the distance from C to B as b. Label the weight as W.

Add the weight to the bar

The situation changes when you add the 10 N weight to the bar. Each spring scale now exerts an upward force on its end of the bar that is no longer negligible. Label the upward force on the left end P and label the upward force on the right end Q.

Is the bar in equilibrium

I believe that most of you will know, based on experience with the trapeze bar on the playground, that the bar will still be in equilibrium. When you hung quietly on a trapeze bar as a child, it didn't accelerate off in some direction, nor did it rotate about any axis. So, we know from experience that the bar is still in equilibrium.

Three forces

There are now three forces being exerted on the bar: P, Q, and W. We know from earlier modules that the vector sum of those three forces must be zero in order for the bar to be in equilibrium. This system is simple enough that we can perform the vector sum in our heads. We don't need to draw a vector diagram. We see that

P + Q - W = 0

Adding W to both sides of the equation gives us:

P + Q = W (**eq. a1**)

The values for P and Q

What are the values for P and Q? We know that wherever we place the weight along the horizontal length of the bar, the bar will continue to be in equilibrium. Therefore, the sum of P and Q must be equal to W regardless of the position of W (so long as the position of W is between the ropes and the ropes don't break).

Compute the torque about C

Assume that positive coordinates are to the right with the origin at the left end of the bar. Computing the torque about the point C gives us:

(C-A)*(P) + (C-B)*(Q) = 0

Let's put some numbers in the problem now.

- Let C = 2
- Let A = 0
- Let B = 10
- Let W = 10

Substituting numbers in the above equation gives us:

(2)*(P) + (-8)*(Q) = 0

Simplify and rearrange terms

Simplifying, rearranging terms, and dividing both sides by 2 gives us:

P = 4*Q (**eq. a2**)

Two equations and two unknowns

Including eq. a1 (p. 473) , (repeated below) we now have two equations and two unknowns.

P + Q = W

Inserting the numeric value for W gives us

P + Q = 10, or

Q = 10 - P (**eq. a3**)

Substituting this value of Q into eq. a2 (p. 473) gives us

P = 4*(10 - P), or

P = 40 - 4*P, or

5*P = 40, or

P = 8

which is one of the answers that we are looking for.

Inserting the value for P into eq. a3 (p. 473) gives us

Q = 10 - 8, or

Q = 2

which is the other answer that we are looking for.

15.4.6 A more general case of the trapeze bar

Let's pick another point, label it X, and compute the moments about that point. Those moments must also sum to zero for the bar to be in equilibrium. (The moments computed about any point on the bar must sum to zero for the bar to be in equilibrium.)

The moments about X produced by the three forces are:

- P: (X-A)*(P)

- W: (X-C)*(W)
- Q: (X-B)*(Q)

Let X = 5

Substituting for X gives us:
(5)*(P) - (3)*(10) + (-5)*(Q) = 0
Simplifying, rearranging terms, and diving both sides by 5 gives us:
P - Q - 6 = 0, or
P = Q + 6 (**eq. a4**)
Now we can use this equation along with eq.a1 (p. 473) to solve for Q.
P + Q = 10, from eq.a1 (p. 473) , or
Q = 10 - P
Substituting this value for Q in eq. a4 (p. 474) gives us,
P = 10 - P + 6
Simplifying and dividing both sides by 2 gives us,
P = 8 , which is the same answer as before (which it should be)
Substitution of P back into eq. a1 gives us,
Q = 2

15.4.7 Apply the weight at different locations on the trapeze

If you solve for P and Q for any location of the weight between the ropes, you will find that the values for the upward forces at each end of the bar are inversely proportional to the distance from the weight to that end of the bar.

For example, for equilibrium, using the dimension symbols established for your drawing earlier:
a*P = b*Q
Dividing both sides by a gives:
P = (b/a)*Q
Weight is centered
If b/a = 1 (weight is centered), then:
P = Q meaning that both upward forces are equal.
Weight towards the left end
If b/a = 4 (weight at 2), then:
P = 4*Q meaning that most of the force is being exerted by the rope on the left end.
Weight toward the right
If b/a = 1/4 (weight at 8), then
P = Q/4 meaning that most of the force is being exerted by the rope on the right end.
Weight at the right end
If b/a = 0 (weight at 10, right end of the bar)
P = 0 meaning that all of the force is being exerted by the rope on the right end of the bar.
Because P + Q = W, we conclude that Q = W
The bar is essentially eliminated

The scenario where b/a = 0 essentially eliminates the bar from consideration. The weight is hanging directly on the rope on the right end of the bar and the weightless bar and the other rope are simply floating in the air.

15.4.8 Hypothetical replacement by a single upward force

If we were to draw an imaginary force labeled R pointing directly up from the point C where R is equal to P + Q, we could imagine the forces P and Q as being replaced by R. In that case, R would be the resultant of P and Q and would be equal in magnitude and opposite in direction to the downward force W.

In the absence of the forces P and Q, the force R would produce the same turning effect on the bar as do the forces P and Q jointly.

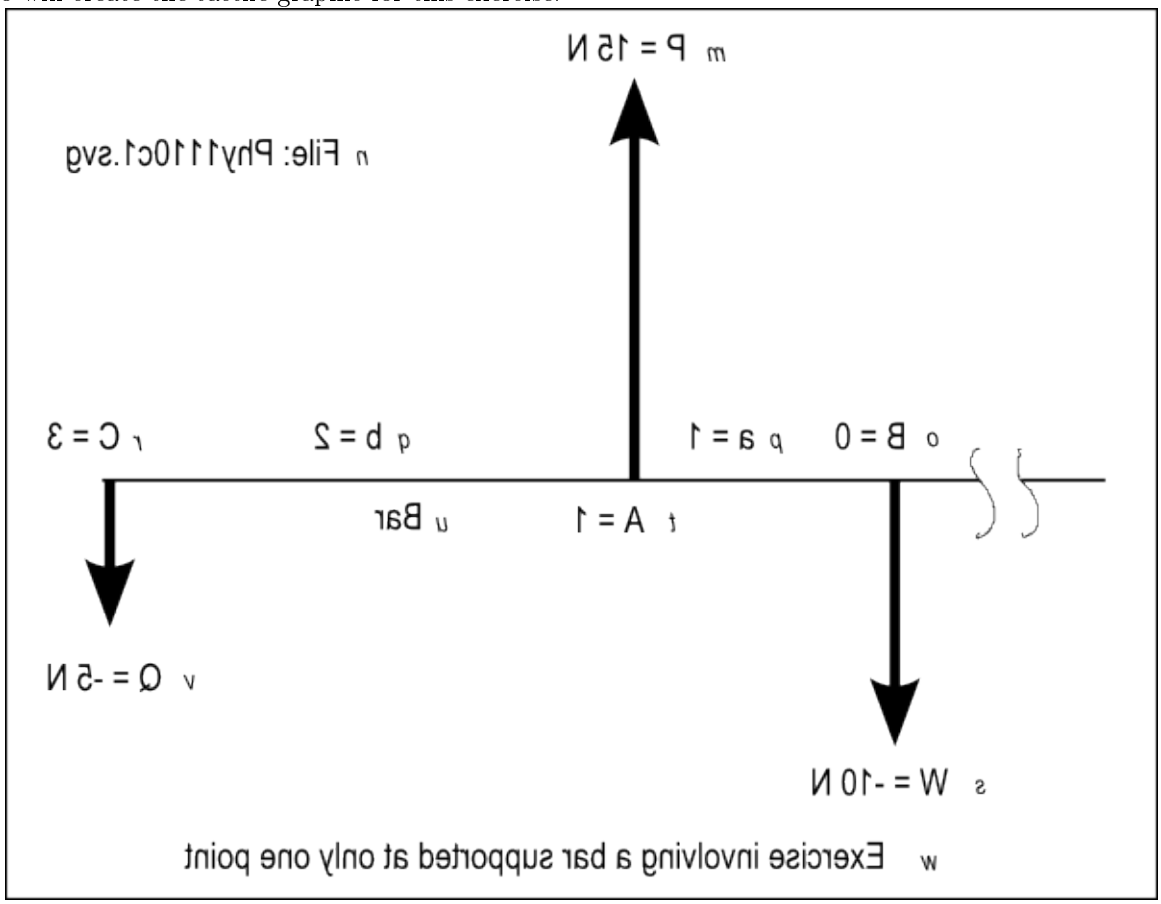
If the magnitude of R were not equal to the magnitude of W, the vector sum of the forces would not be zero, there would be a torque about any point on the bar other than at the point C, and the bar would not be in equilibrium.

15.4.9 Exercise involving a bar supported at only one point

Tactile graphics

The file named Phy1110c1.svg contains a vector diagram that represents this scenario. All of the action occurs near the right end of the bar, so only the right end is shown in the image.

Figure 8 (p. 478) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.



P = 15 N
File: Phy1110c1.svg
C = 3
b = 2
a = 1
B = 0
Bar
A = 1
Q = -5 N
W = -10 N
Exercise involving a bar supported at only one point

P = 15 N
File: Phy110c1.svg
C = 3
b = 2
a = 1
B = 0
Bar
A = 1
Q = -5 N
W = -10 N
Exercise involving a bar supported at only one point

Figure 8 . Mirror image from the file named Phy1110c1.svg.

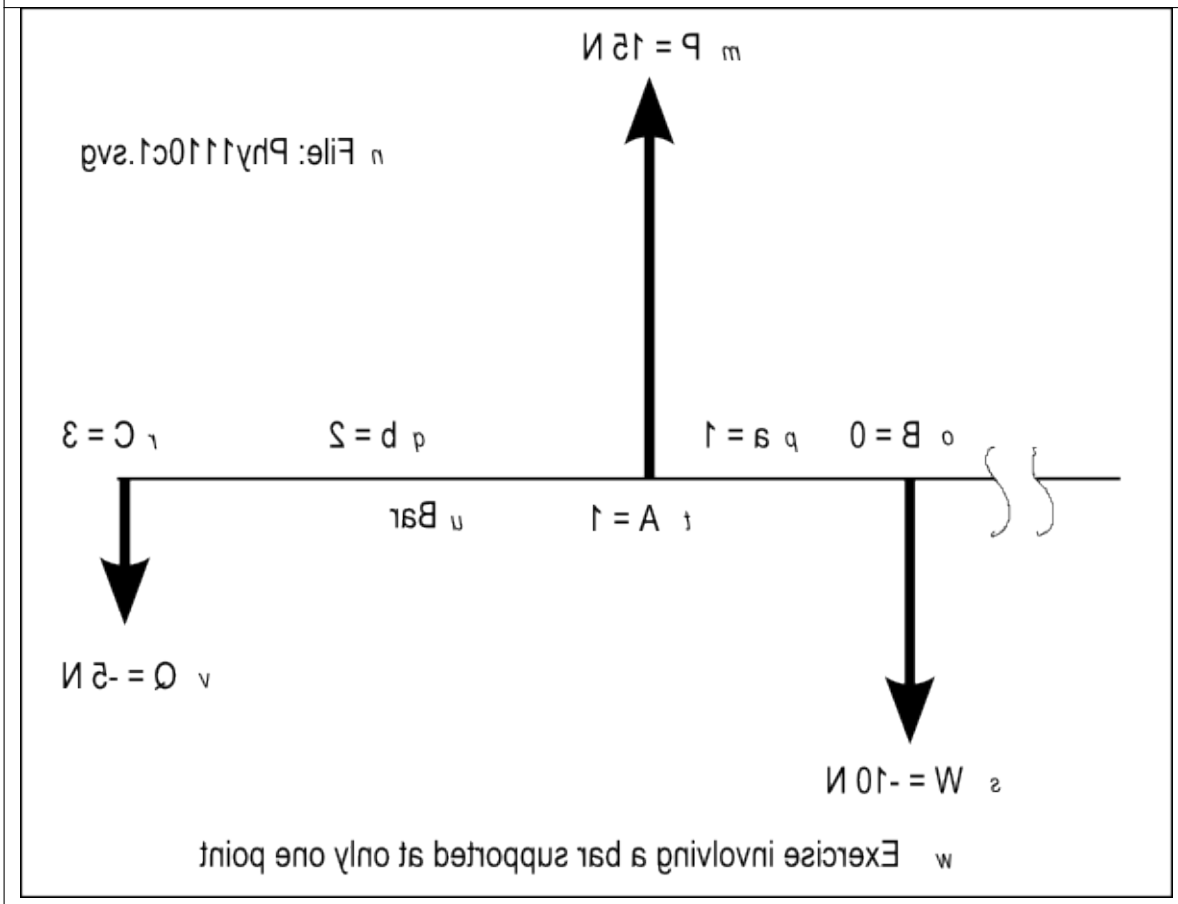
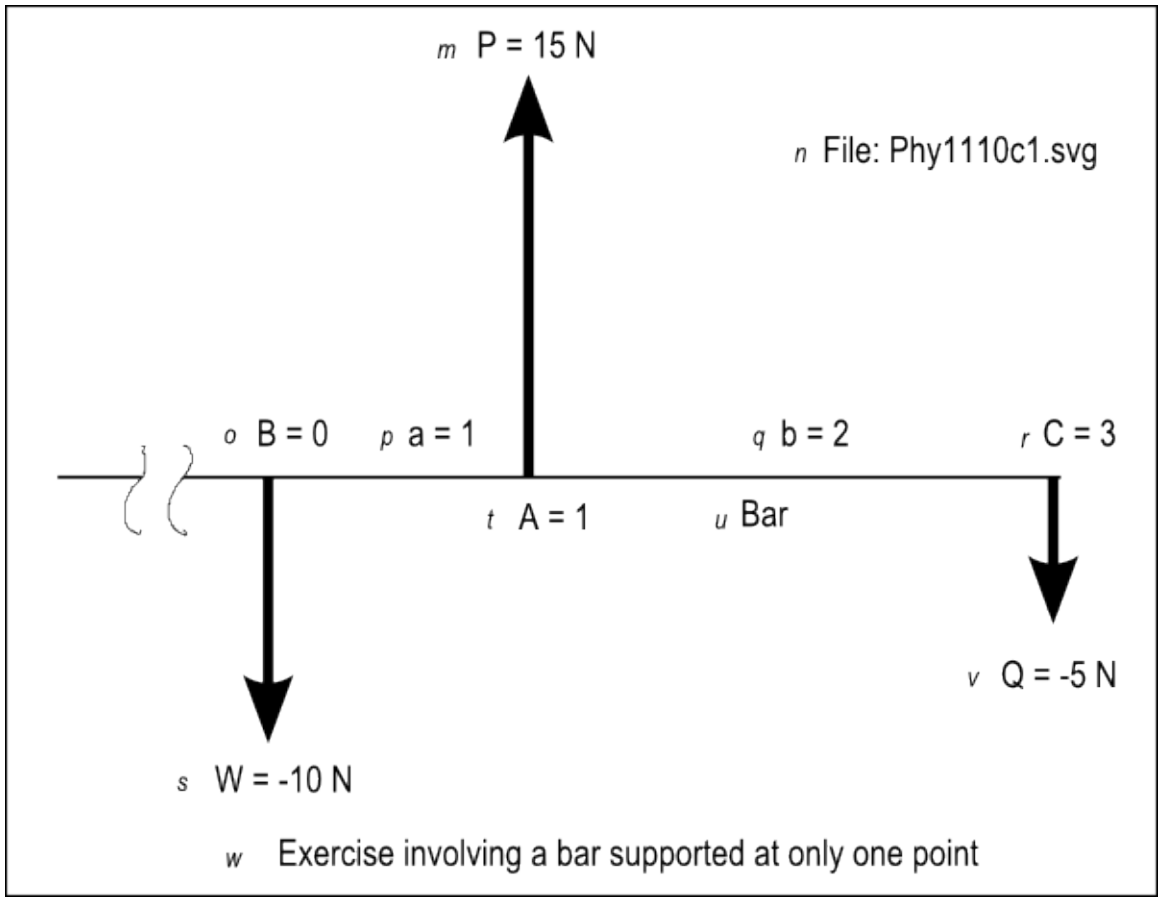
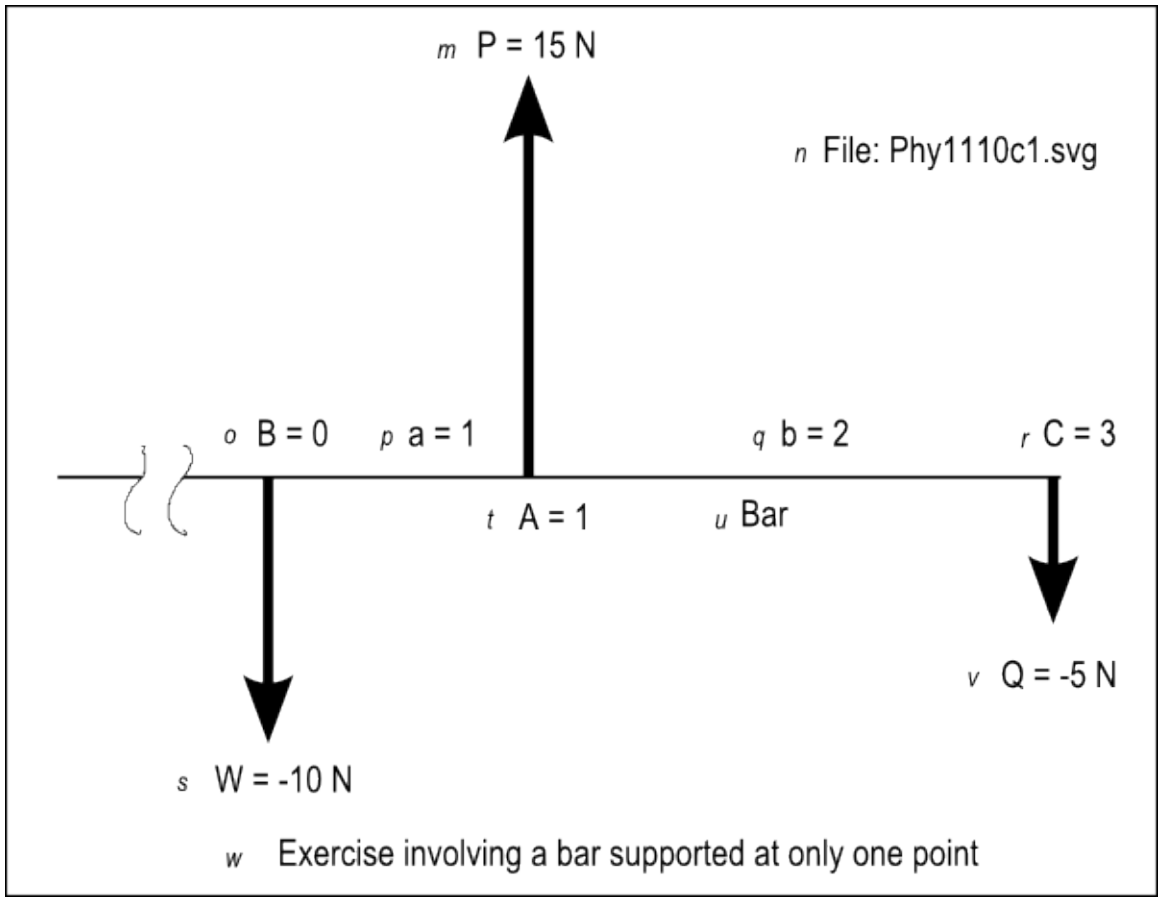


Table 15.8

Figure 9 (p. 482) shows a non-mirror-image version of the same image.


m P = 15 N
n File: Phy1110c1.svg
o B = 0
p a = 1
q b = 2
r C = 3
t A = 1
u Bar
v Q = -5 N
s W = -10 N
w Exercise involving a bar supported at only one point


m P = 15 N
n File: Phy1110c1.svg
o B = 0
p a = 1
q b = 2
r C = 3
t A = 1
u Bar
v Q = -5 N
s W = -10 N
w Exercise involving a bar supported at only one point

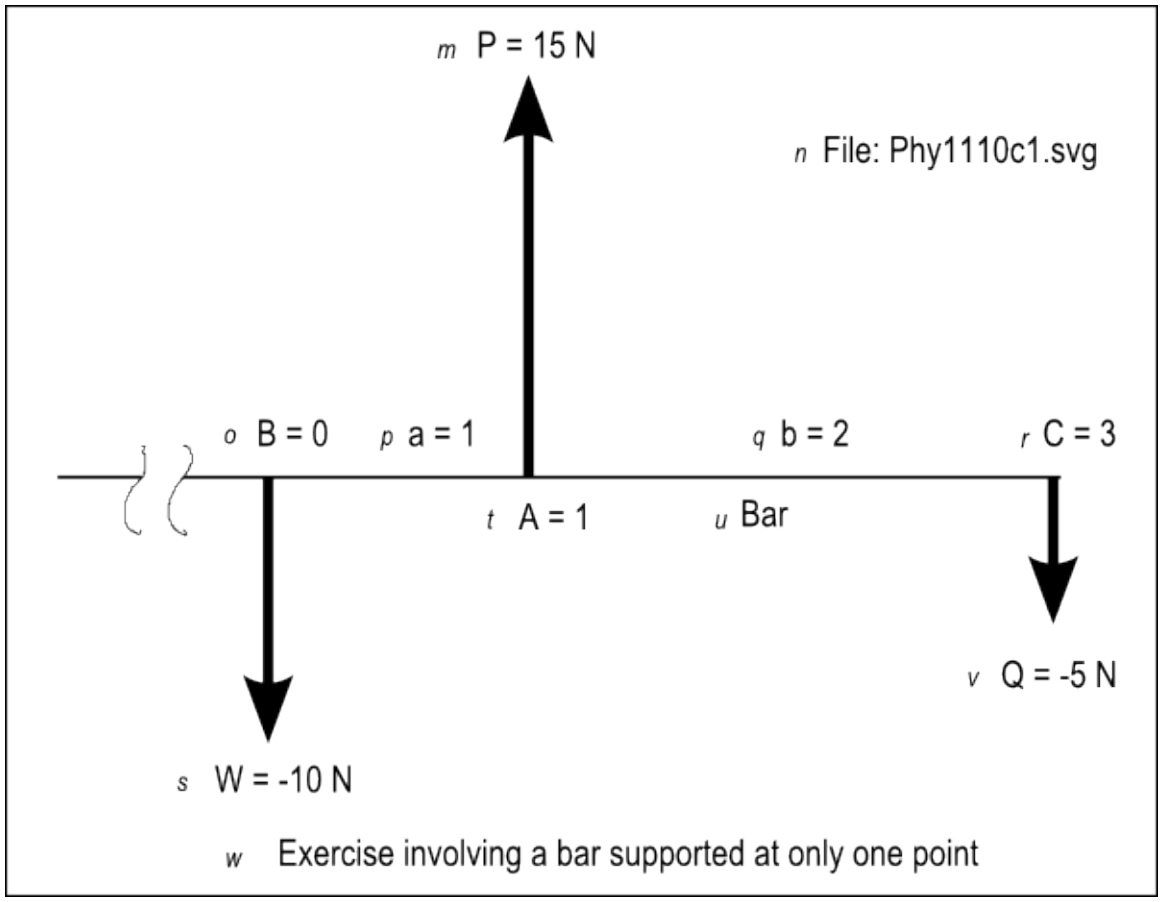

m P = 15 N
n File: Phy1110c1.svg
o B = 0
p a = 1
q b = 2
r C = 3
t A = 1
u Bar
v Q = -5 N
s W = -10 N
w Exercise involving a bar supported at only one point

Figure 9 . Non-mirror-image version of the image from the file named Phy1110c1.svg.

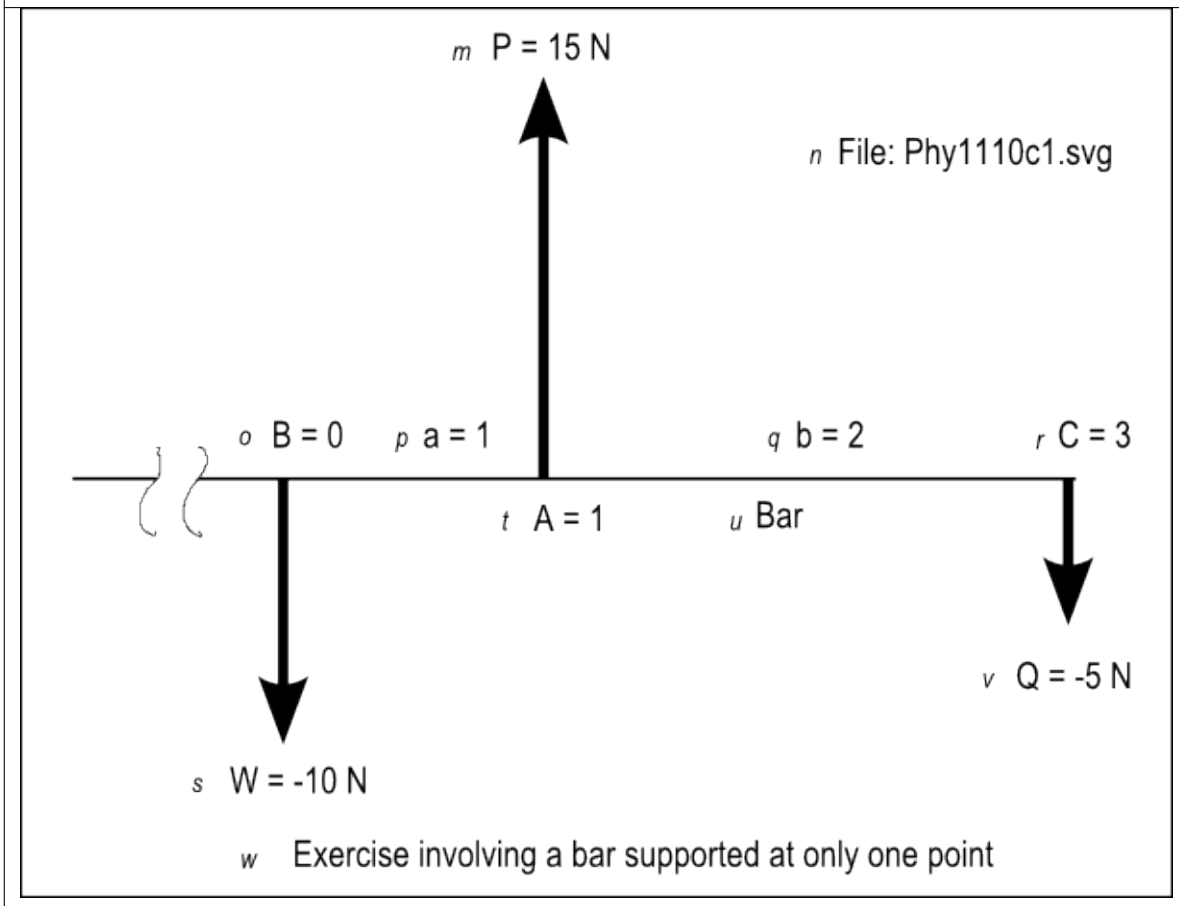


Table 15.9

Figure 10 (p. 482) shows the key-value pairs that go with the image in the file named Phy1110c1.svg.

Figure 10 . Key-value pairs for the image in the file named Phy1110c1.svg.

```
    m: P = 15 N
n: File: Phy1110c1.svg
o: B = 0
p: a = 1
q: b = 2
r: C = 3
s: W = -10 N
t: A = 1
u: Bar
v: Q = -5 N
w: Exercise involving a bar supported at only one point
```

Table 15.10

Unlike or **antiparallel**

It's time to modify your drawing to reflect a different scenario.

Make your bar 20 units in length. Draw a downward force labeled Q at the right end of the bar. Label this as point C on the bar.

Draw an upward force labeled P somewhere on the bar. Label the location of this point on the bar as A

In this case, we have two parallel forces, P and Q pulling in opposite directions but not in the same line of action. These forces are said to be **unlike** or **antiparallel** .

Add another downward force

Draw a downward force labeled W somewhere to the left of A. Label this point as B and consider it to be the horizontal origin. Directions to the right of B are positive directions.

Label the horizontal distance from B to A as a. Label the horizontal distance from A to C as b.

What is required for equilibrium?

We can look at this scenario and tell that the forces labeled W and P, both of which point down, must be on opposite sides of the force labeled P. Otherwise, the bar will rotate in a clockwise direction.

The following two conditions must be true for the bar to be in equilibrium.

P + W + Q = 0, or

P = - (W + Q) (vector sum must be zero)

a*P + (a+b) * Q = 0 (moments around point B)

The plan

We will set values for all the variables in the two conditions other than a and P and then solve for values of a and P that will result in the bar being **in equilibrium** .

- Let W = -10
- Let Q = -5
- Let b = 2

The computations

Substituting these values into the two conditions and rearranging terms gives us:

P = -(W + Q), or

P = 15 , this is one of the answers

a*P +(a+2)*(-5) = 0, or

15*a -5*a -10 = 0, or

10*a = 10, or

a = 1 , this is the other answer

The answers

Therefore, for the values of W, Q, and b given above (p. 483) , P must be equal to 15 and a must be equal to 1 for the bar to be in equilibrium.

Variation of a and P with changes in Q

Now let's see how P and a vary with respect to the value of Q

- Let W = -10
- Let Q = -20
- Let b = 2

P = -(W + Q) = 30

a*P +(a+2)*(-20) = 0, or

30*a -20*a -40 = 0, or

10*a = 40, or

a = 4

As you can see, keeping the location of P constant and increasing the value for Q requires the value of P to increase and the location of W to move further to the left.

15.4.10 Couples

Now draw a new scenario where antiparallel forces, each having a magnitude of 10 newtons are acting in opposite directions on a bar, with each force in the same plane and both forces perpendicular to the bar. Label one force P and the other force Q.

Tactile graphics

The file named Phy1110d1.svg contains a vector diagram that represents this scenario. It shows two 10 N forces labeled P and Q along with an unknown force labeled X.

Figure 11 (p. 488) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

An exercise on couples
P = 10 N
X = ?
Bar
Location of X = ?
Q = 10 N
File: Phy1110d1.svg

An exercise on couples
P = 10 N
X = ?
Bar
Location of X = ?
Q = 10 N
File: Phy1110d1.svg

An exercise on couples
P = 10 N
X = ?
Bar
Location of X = ?
Q = 10 N
File: Phy1110d1.svg

Figure 11 . Mirror image from the file named Phy1110d1.svg.

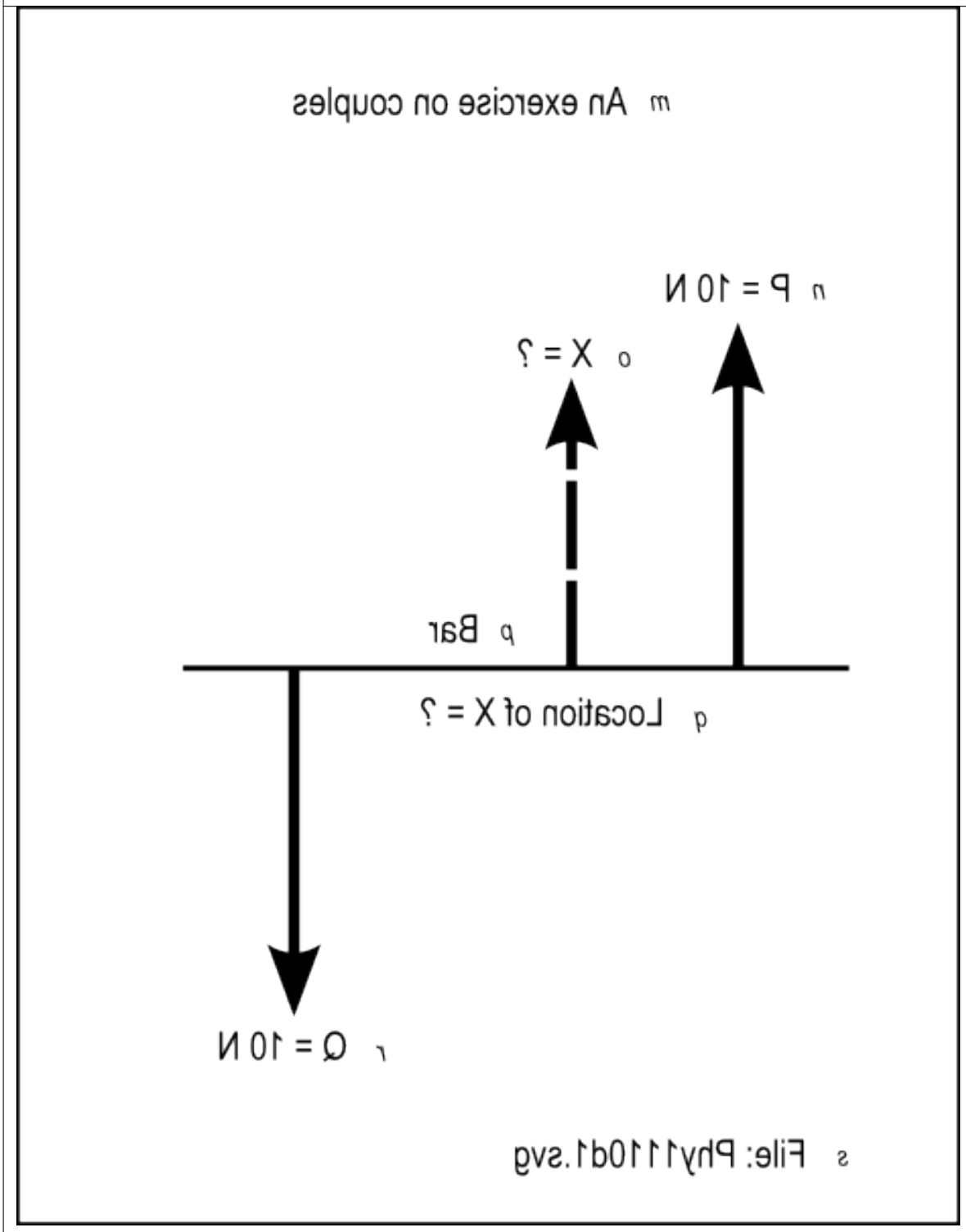
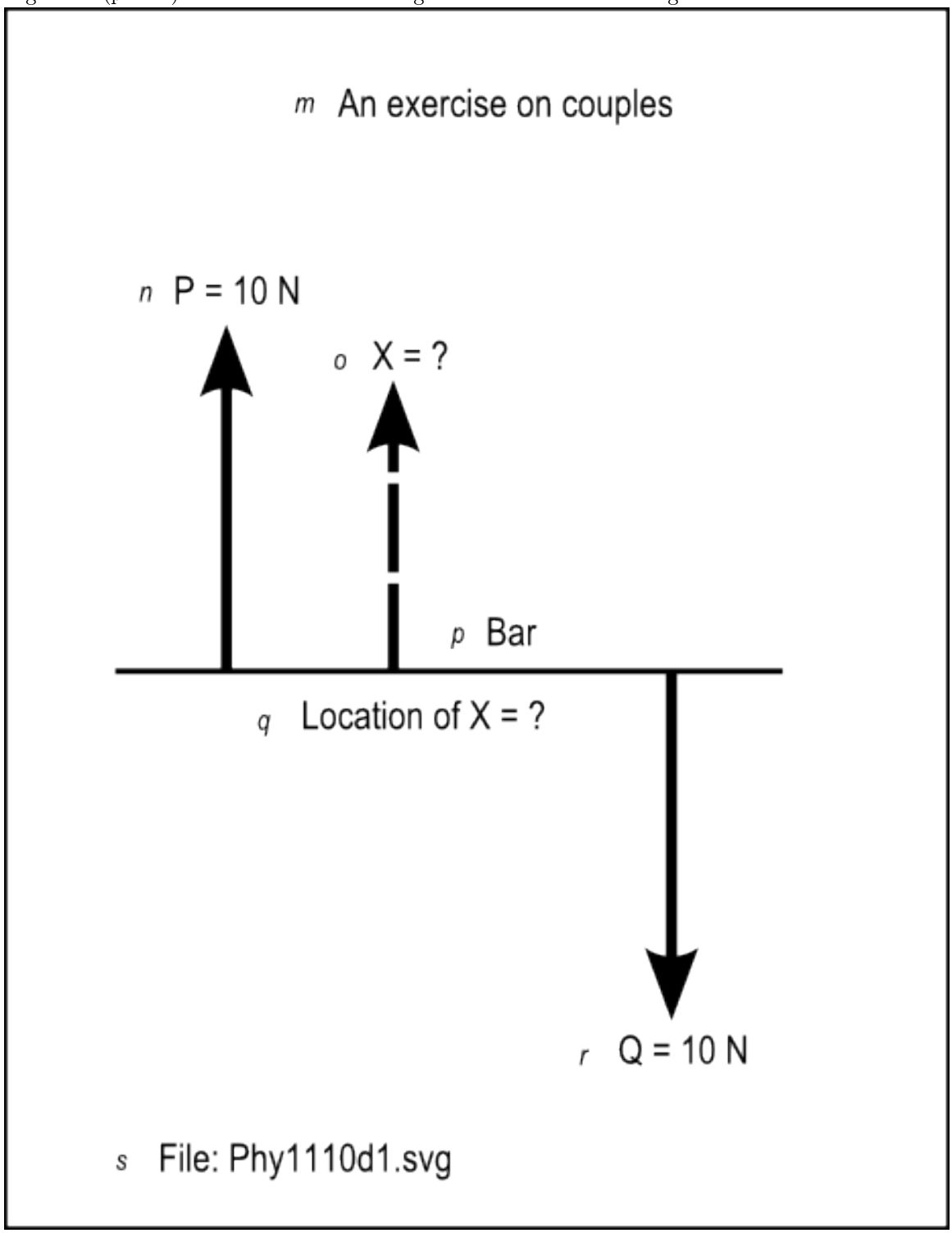
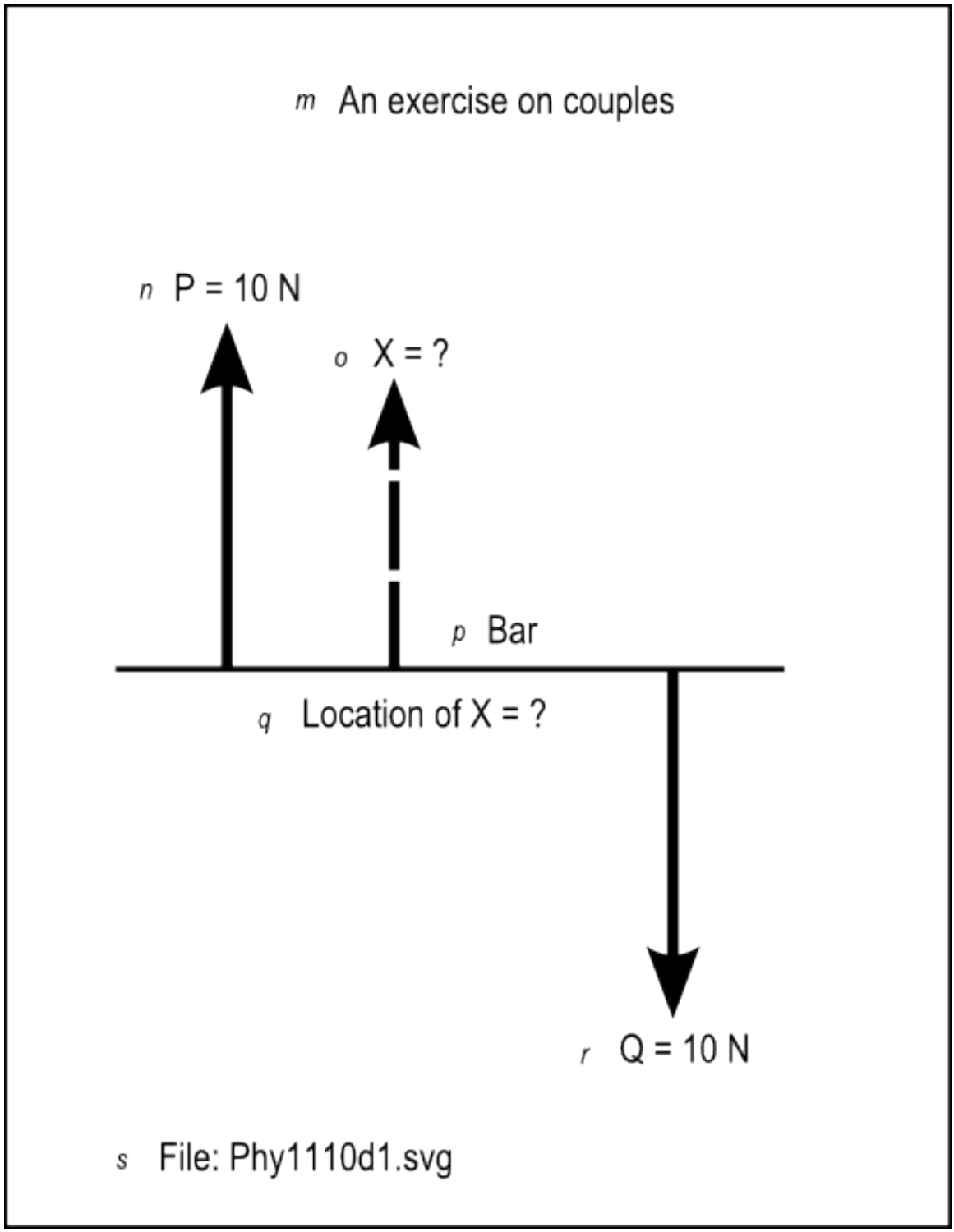


Table 15.11

Figure 12 (p. 492) shows a non-mirror-image version of the same image.




m An exercise on couples
n P = 10 N
o X = ?
p Bar
q Location of X = ?
r Q = 10 N
s File: Phy1110d1.svg

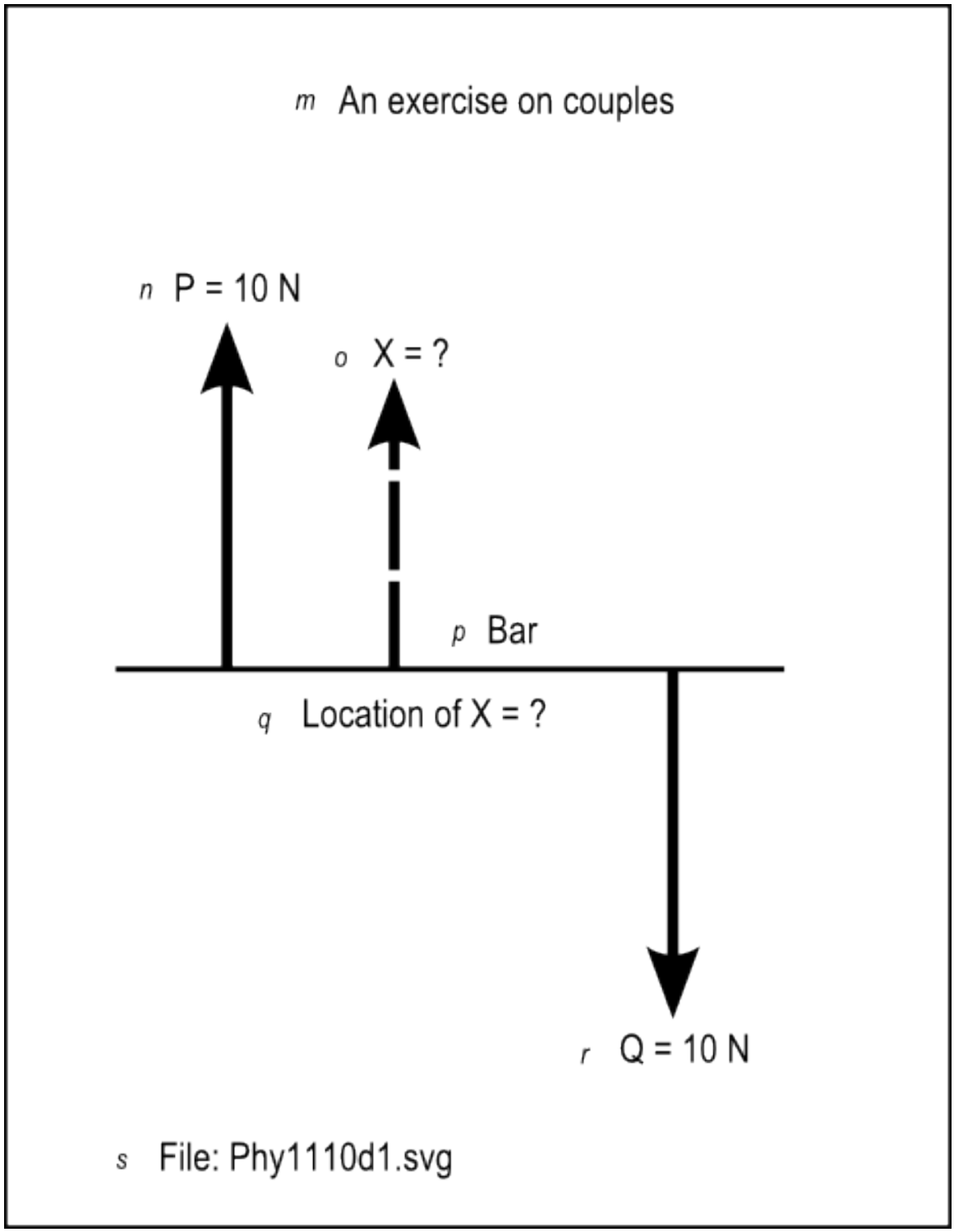

m An exercise on couples
n P = 10 N
o X = ?
p Bar
q Location of X = ?
r Q = 10 N
s File: Phy1110d1.svg

Figure 12 . Non-mirror-image version of the image from the file named Phy1110d1.svg.

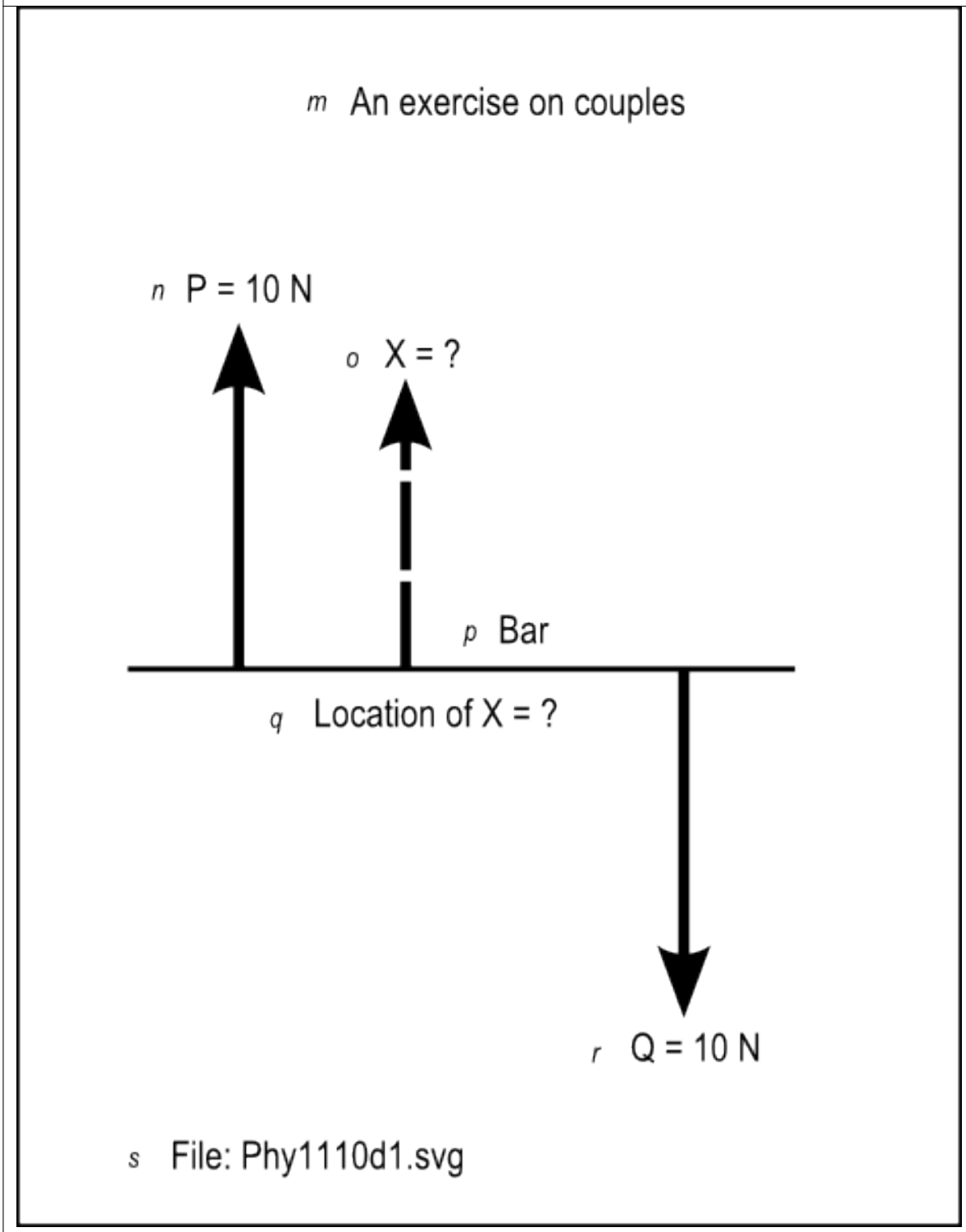


Table 15.12

Figure 13 (p. 493) shows the key-value pairs that go with the image in the file named Phy1110d1.svg.

Figure 13 . Key-value pairs for the image in the file named Phy1110d1.svg.

```
    m: An exercise on couples
n: P = 10 N
o: X = ?
p: Bar
q: Location of X = ?
r: Q = 10 N
s: File: Phy1110d1.svg
```

Table 15.13

The moments about a point

If you compute the moments about any point on this bar, they won't sum to zero. Therefore, the bar isn't in equilibrium. Can you find the location and magnitude of a single force that can be applied to the bar to cause it to be in equilibrium?

Consider the first condition for equilibrium

The vector sum of the forces must be zero.

In this case, the vector sum of the forces is

P - Q + X = 0

10 - 10 + X = 0

where X is an unknown force.

As you can see, there is no value for X that will satisfy this condition other than X=0. If X=0, there is no force, so there is no single force that can be applied to the bar to cause it to be in equilibrium.

A couple

Two forces equal in magnitude but opposite in direction and not in the same line constitute a ***couple*** .

Stated differently,

A couple is a system of forces with a non-zero moment but no resultant force. In other words, the vector sum of the forces is zero but the sum of the moments is not zero.

A simple couple

The simplest kind of couple consists of two equal and opposite forces whose lines of action do not coincide. This is often called a "simple couple."

The units for a couple

The SI unit for the torque is the newton meter, which when expanded into SI base units is kg*m^2/s^2.

The moment of force for a couple is unique

The moment of a force is only defined with respect to a certain point P (it is said to be the "moment about P"). In general when P is changed, the moment changes.

However, the moment (torque) of a couple is independent of the reference point P. Any point will give the same moment.

Let's prove that

The moment of a couple is independent of the point about which it is measured. Can we prove the truth of that statement?

Imagine a horizontal bar that is five units long. Imagine an upward force at the right end of the bar with a magnitude of P. Imagine a downward force with the same magnitude d units to the left of the first force. Imagine that the location of this force is labeled as X.

Imagine that the left end of the bar is labeled A and the distance from point A to point X is labeled as b. Imagine another point somewhere between A and X that is labeled as B. Imagine that the distance from A to B is labeled as c.

Compute the moments about the point A

Ma = (b+d)*P-b*P, or
Ma = b*P+d*P-b*p, or
Ma = d*p

Now compute the moments about the point B.

Mb = (b-c+d)*P-(b-c)*P, or
Mb = b*P-c*P+d*P-b*P+c*P, or
Mb = d*P

The moments are the same regardless of the point about which they are computed. The value d*P is called the moment or torque of a couple.

The torque of a couple

The torque of a couple is the product of either force and the perpendicular distance between the lines of actions of the forces.

15.5 Complete the exercises

I encourage you to complete the exercises that I have presented in this lesson to confirm that you get the same results. Experiment with the procedures, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

15.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

15.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force – Moments, Torque, Couple, and Equilibrium
- File: Phy1110.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · moment
  - · torque
  - · couple

· equilibrium

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 16

Force – Center of Gravity[1]

16.1 Table of Contents

- Preface (p. 497)
  - General (p. 497)
  - Prerequisites (p. 498)
  - Viewing tip (p. 498)
    - Figures (p. 498)
  - Supplemental material (p. 499)
- General background information (p. 499)
- Discussion and computations (p. 501)
  - Creation of tactile graphics (p. 501)
  - Equilibrium: stable, unstable, and neutral (p. 501)
  - An exercise involving the tipping point (p. 502)
  - A real-world example (p. 521)
- Do the computations (p. 522)
- Resources (p. 522)
- Miscellaneous (p. 522)

16.2 Preface

16.2.1 General

This module is part of a collection of modules designed to make physics concepts accessible to blind students.

If you opened this page in the context of the book, a Table of Contents for the book (or collection) should be available above and to the left of this paragraph. Otherwise, click here [2] to open the book at the beginning.

This book is intended to supplement but not to replace the textbook in an introductory high school or college physics course.

This module explains force and center of gravity in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38168/1.4/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

16.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

16.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

16.2.3.1 Figures

- Figure 1 (p. 506) . Mirror image from the file named Phy1120a1.svg.
- Figure 2 (p. 510) . Non-mirror-image version of the image from the file named Phy1120a1.svg.
- Figure 3 (p. 511) . Key-value pairs for the image in the file named Phy1120a1.svg.
- Figure 4 (p. 515) . Mirror image from the file named Phy1120b1.svg.
- Figure 5 (p. 519) . Non-mirror-image version of the image from the file named Phy1120b1.svg.
- Figure 6 (p. 520) . Key-value pairs for the image in the file named Phy1120b1.svg.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

16.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

16.3 General background information

The gravitational constant

The gravitational constant, G, is a constant that is used in the calculation of the gravitational attraction between objects with mass. The value of G is approximately:

G = 6.67 * 10^(-11) *m^3*kg^(-1)*s^(-2), or

G = 6.67 * 10^(-11) N*(m/kg)^2

According to the law of universal gravitation, the attractive force (F) between two bodies is proportional to the product of their masses (m1 and m2), and inversely proportional to the square of the distance (r) between them:

F = G*m1*m2/r^2

Newton's law of gravitation

Newton's law of gravitation proposes that every small element of matter, m, attracts every other small element of matter, m' with a force that is proportional to the product of the masses and inversely proportional to the square of the distance between them, so that at the small element level,

F = G*m*m'/r^2

As a result, we tend to fall toward the earth unless we are supported by upward forces, such as the floor that we are standing on.

An important simplification

It would be very difficult to deal with the attraction of gravity if we were required to deal with every object (including the earth) in terms of the many small elements of mass that make up the object.

Newton simplified that process by proving mathematically that the earth regarded as a sphere attracts a body as if the whole mass of the earth were concentrated at its center.

Almost parallel forces

The attraction of the earth on the elements of our body produces forces on those elements that are almost parallel. Those forces lie along lines that all end at a point that is almost 4000 miles beneath the surface of the earth.

At that distance, each pair of force lines forms the sides of a triangle with extremely long sides, an extremely small angle, and an extremely narrow base. As a practical matter, therefore, we can assume that those forces are parallel insofar as estimating the effect that the sum of all of those elemental forces have on our bodies.

The center of gravity C.G.

It can be proved that:

The resultant of the weights of all the elements of a rigid body passes through a certain fixed point commonly called its center of gravity, C.G., regardless of the orientation of that body relative to the earth.

For purposes of computations involving statics, the mass of a rigid body may be considered as if concentrated at it C.G., and its weight may be considered to center at this same point.

(Recall that the weight of an object is the force required to cause the mass of that object to fall toward the center of the earth with an acceleration of approximately 32.2 ft/s^2 or 9.81 m/s^2.)

A plumb-bob or plummet

A plumb-bob or a plummet is a weight, usually with a pointed tip on the bottom, that is suspended from a string and used as a vertical reference line, or plumb-line.

When the string is attached to an object that is fixed relative to the surface of the earth and the plummet is allowed to hang down, it will eventually (when it stops swinging) point directly at the center of mass of the earth.

[14] http://www.dickbaldwin.com/toc.htm

Common uses for a plummet

The plummet is most commonly used in construction projects to ensure that constructions are "plumb", or vertical.

It is also used in land surveying to set a surveying instrument exactly over a fixed survey marker, or to transcribe positions onto the ground for placing a marker.

Experimental verification of the C.G.

I'm aware that as a blind student, you probably can't perform this experiment, but hopefully you can imagine it.

Make a hanger

Use a plummet to draw a vertical line on a flat, smooth, vertical surface (such as a wall). Drive a finishing nail (a nail with a small head) into the surface on the vertical line allowing a small portion of the nail to stick out. Cause the nail to be as horizontal as practical.

Make an irregular object

Cut an irregular shape out of a piece of flat cardboard and punch several holes around the perimeter of the shape.

Perform the experiment

Using several holes in succession, including holes on all four sides of the cardboard, hang the cardboard on the nail in such a way that the cardboard is free to swing and settle into a stable orientation.

Once the cardboard stops swinging, mark the point on the bottom of the cardboard that coincides with the vertical line that you drew on the vertical surface.

After you have done this several times, draw lines from each hole to the corresponding mark that you made on the opposite edge of the cardboard. All of these lines should cross at a single point. That point is the C.G. for that particular irregular shape of cardboard.

Modify the experiment

Cut regular shapes of cardboard such as rectangles, triangles, hexagons, octagons, etc, and repeat the experiment with those shapes. Observe the location of the C.G. for those shapes.

Rectangular shapes

You should find that the C.G. for a rectangular piece of cardboard is at the center of the rectangle. This should be the case for all rectangles ranging from squares to long skinny rectangles (although it may difficult to measure for long skinny rectangles.)

Triangular shapes

The C.G. for a triangular shape should be at the intersection of its medians.

In case you have forgotten, the median of a triangle is a line segment from a vertex of the triangle to the midpoint of the side opposite that vertex. Because there are three vertices, there are three medians. No matter what shape the triangle, all three medians intersect at a single point, which is called the centroid of the triangle. That point would also be the C.G. for a triangular-shaped piece of cardboard.

The doughnut hole

Cut a piece of cardboard in the general shape of a doughnut or tire with a hole in the middle and perform the experiment using that shape. Observe that the C.G. can be located in the doughnut hole. The C.G. of an object doesn't have to be in a solid part of the object.

A void in a 3D object

It is also possible for the C.G. to be located in a cavity in a three-dimensional object. For example, the C.G. for a small section of cylindrical pipe that is cut square on both ends would be in the center of the void half way between the ends of the pipe.

16.4 Discussion and computations

16.4.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explains how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1120.zip [16] , which contains the svg files for this module. Extract the svg files from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

16.4.2 Equilibrium: stable, unstable, and neutral

When a vertical line through the C.G. of a body falls within the area covered by the supporting base, the body can rest in equilibrium. A body in equilibrium can be further qualified as being either stable, unstable, or neutral.

- Stable equilibrium: If a body at rest receives a small displacement and tends to return to its former position, that body is said to be in stable equilibrium.
- Unstable equilibrium: If that body tends to move further away, then that body is said to be in unstable equilibrium.
- Neutral equilibrium: If it does neither, the body is said to be in neutral equilibrium.

A practical example

Consider the case of a round pencil, sharpened on one end and flat on the other end. If you lay the pencil flat on the table and give it a very small push, it will move and then stop. That pencil is said to be in **neutral** equilibrium. It doesn't tend to return to its original position, and except for rolling a small distance as a result of inertia, it doesn't tend to move further away.

Balance it on the flat end

If you balance the pencil on its flat end and give it a very small push at the other end, it will tip slightly and then return to its original position. This assumes that you don't push it beyond its tipping point, which we will discuss in more detail later.

The pencil in this configuration is said to be in **stable** equilibrium. The key to success is that you don't tip it so far that a vertical line through the C.G. moves outside the circle that defines the area of the supporting base of the pencil.

Balance it on its point

Assume that you rub the pencil lead back and forth a few times to slightly flatten the pointed end. If you were able to balance the pencil on that very small flattened surface, and then blow on the pencil very lightly, it would probably tip over and fall flat on the table. The smaller the flattened surface supporting

[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[16] http://cnx.org/content/m38168/latest/Phy1120.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

the pencil, the easier it would be to cause it to tip over. The pencil in that state would be said to be in **unstable** equilibrium.

Stability is important

Stability is very important for many things such as boats and airplanes. Usually, the lower the C.G., the more stable will be the object.

A high-wire artist

For example, a high-wire artist at the circus may carry a long flexible pole, perpendicular to the wire held near the center of the pole at about thigh height. In this case, the supporting surface is the feet on the wire.

The flex in the pole allows the ends of the pole, and conceivably a large portion of the pole to hang lower than the soles of the feet. This has the effect of lowering the C.G. of the combination of the person and the pole considerably. If the C.G. is below the supporting surface, which in this case is the soles of the feet, the combination of the person and the pole will be very stable.

Cargo ships

The people that load cargo ships try to keep the C.G. of the loaded ship low in the hull of the ship. Sometimes they add extra weight called ballast very low in the hull to get the C.G. as low as possible.

Cargo aircraft

The people that load large military cargo aircraft are very careful how they load the cargo in order to control the location of the C.G. of the aircraft to maintain stability of the aircraft. The position of the C.G. relative to the axis of the wings is very important.

Children's toys

When my children were young, they had toys with a round bottom and something like a clown's head on the top. The toys had a heavy weight inside and very close to the bottom. This caused the C.G. to be very low in the toy. When the child pushed the toy over, it would return to an upright position. The toy was very stable.

16.4.3 An exercise involving the tipping point

Let's see if we can demonstrate stability or the lack thereof with a thought experiment. Begin by drawing two rectangles on your graph board side by side. Make each rectangle 2 units wide and five units tall.

Two boxes

Pretend that these rectangles represent side views of two cardboard boxes with dimensions of 2x2x5 ft. Assume that each box contains identical heavy material at one end and the remainder of the box is filled with very light packing material. Assume that the heavy contents are at the bottom of the left box and at the top of the right box. (The boxes and their contents are just alike, but one is upside down relative to the other.)

Tactile graphics

The file named Phy1120a1.svg contains an image that represents this scenario. The image shows two boxes, as described above (p. 502) , tipped over to their tipping points (p. 521) .

Figure 1 (p. 506) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

An exercise involving the tipping point
Tipping point = 14 degrees
Tipping point = 45 degrees
p
Box
Box
p
76 degrees
45 degrees
Floor
W
W
Note: Keys p and q refer to the center of gravity.
File: Phy1120a1.svg

m An exercise involving the tipping point
o Tipping point = 14 degrees
n Tipping point = 45 degrees
q
y Box
s 76 degrees
x Box
p
r 45 degrees
w Floor
u W
t W
Note: Keys p and q refer to the center of gravity.
v File: Phy1120a1.svg

An exercise involving the tipping point
Tipping point = 14 degrees
Tipping point = 45 degrees
q
Box
76 degrees
W
p
Box
45 degrees
Floor
W
Note: Keys p and q refer to the center of gravity.
File: Phy1120a1.svg

Figure 1 . Mirror image from the file named Phy1120a1.svg.

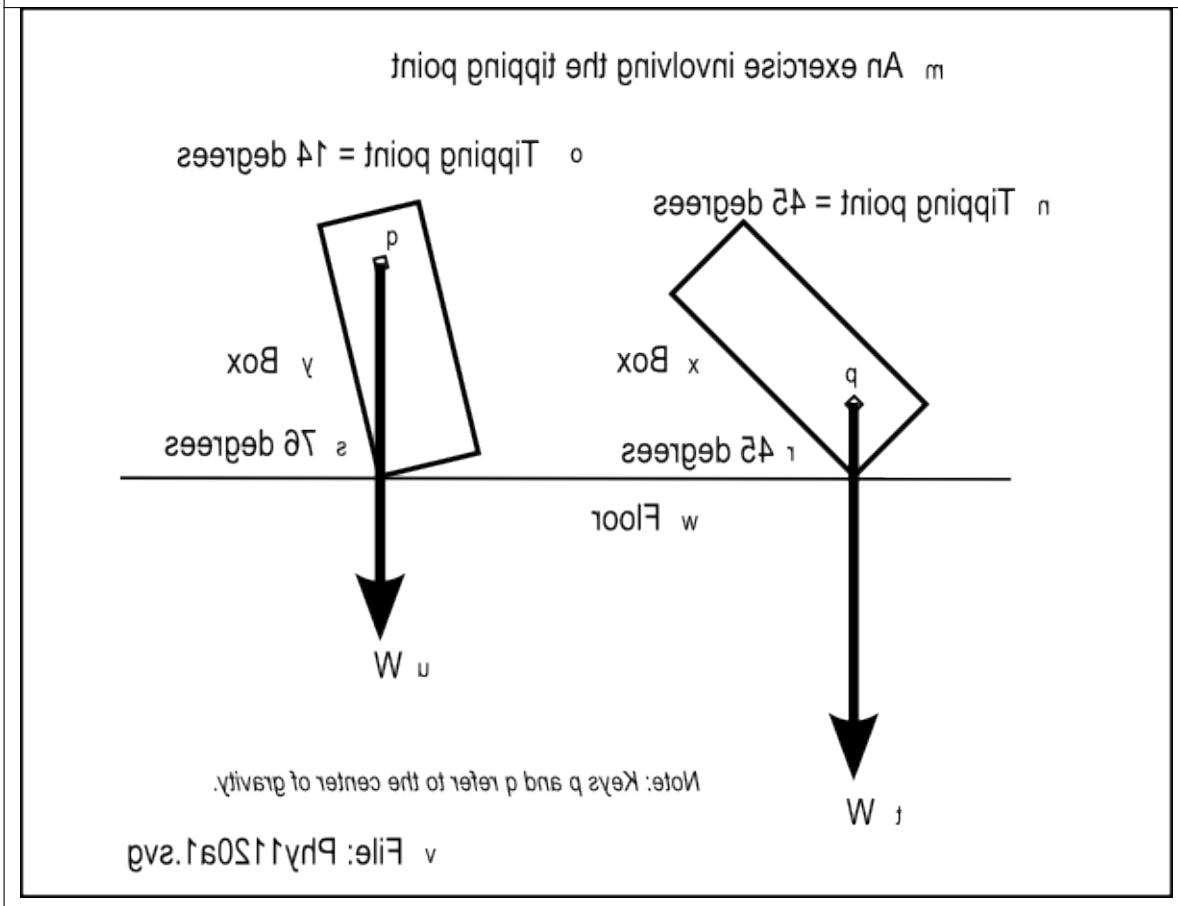
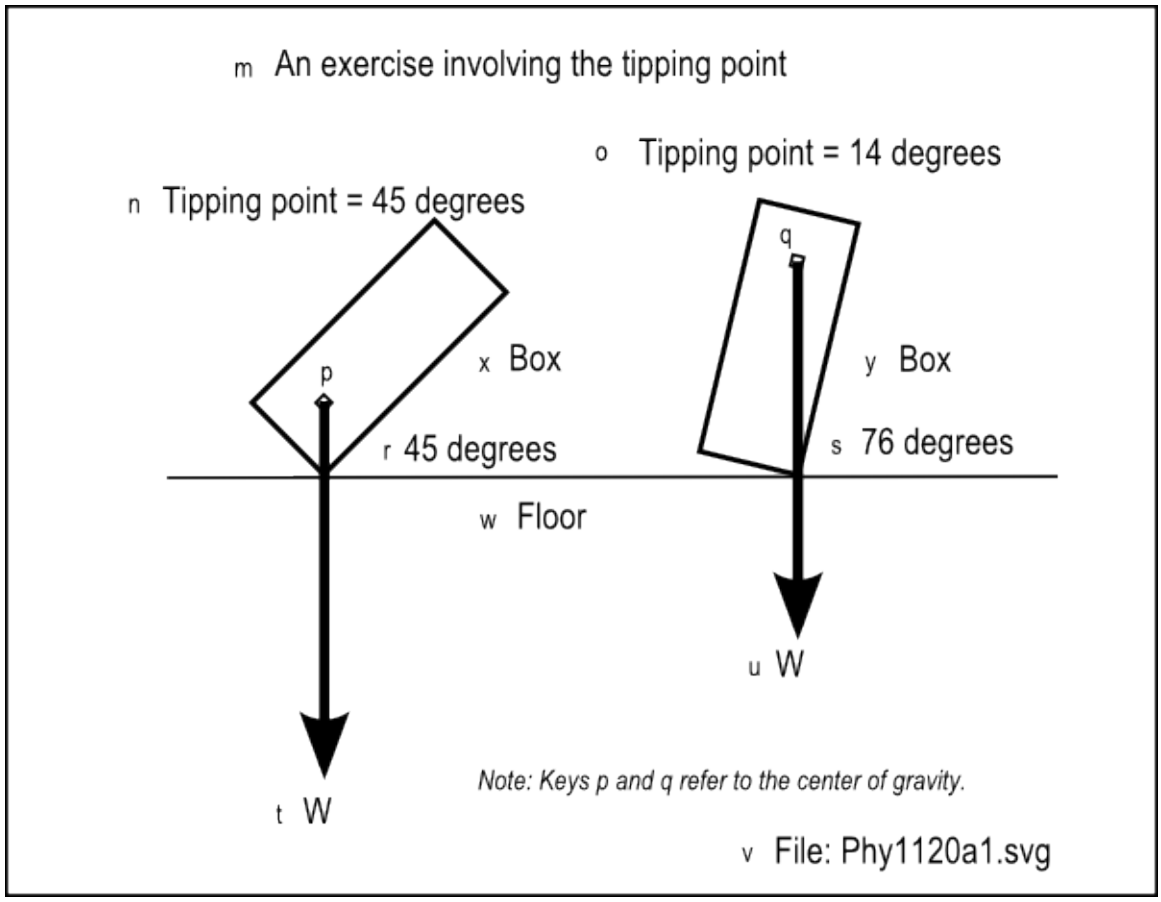
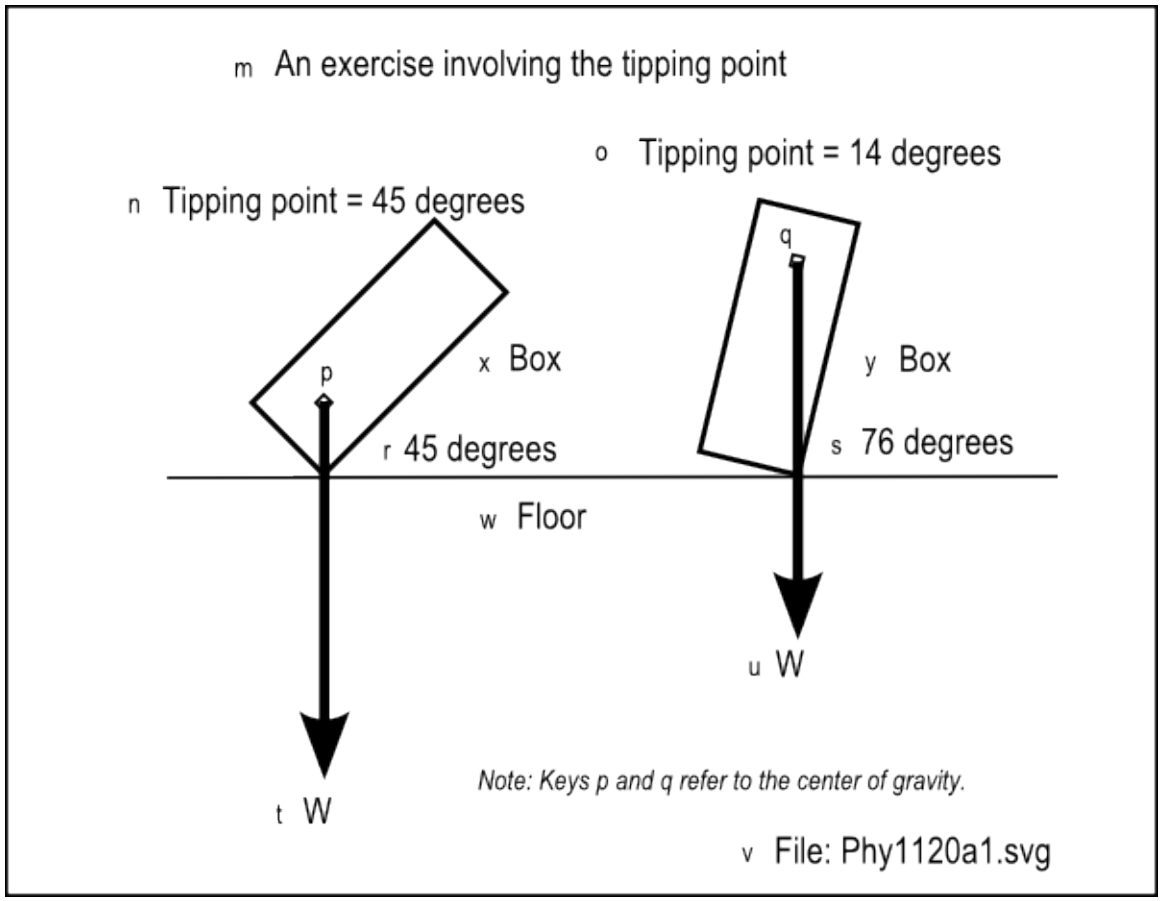


Table 16.1

Figure 2 (p. 510) shows a non-mirror-image version of the same image.


m An exercise involving the tipping point
o Tipping point = 14 degrees
n Tipping point = 45 degrees
q
p
x Box
y Box
r 45 degrees
s 76 degrees
w Floor
u W
t W
Note: Keys p and q refer to the center of gravity.
v File: Phy1120a1.svg


m An exercise involving the tipping point
o Tipping point = 14 degrees
n Tipping point = 45 degrees
q
p
x Box
y Box
r 45 degrees
s 76 degrees
w Floor
u W
t W
Note: Keys p and q refer to the center of gravity.
v File: Phy1120a1.svg

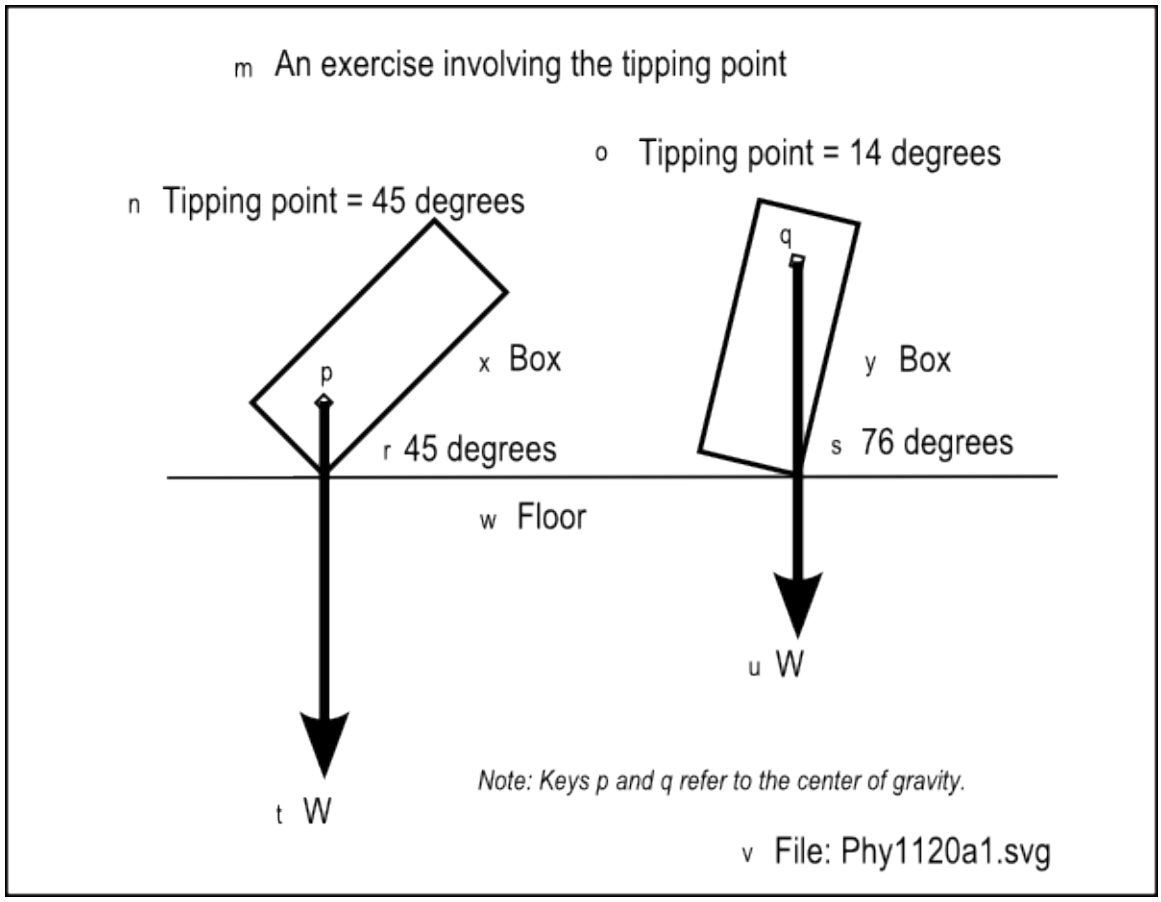

m An exercise involving the tipping point
o Tipping point = 14 degrees
n Tipping point = 45 degrees
q
p
x Box
y Box
r 45 degrees
s 76 degrees
w Floor
u W
t W
Note: Keys p and q refer to the center of gravity.
v File: Phy1120a1.svg

Figure 2 . Non-mirror-image version of the image from the file named Phy1120a1.svg.

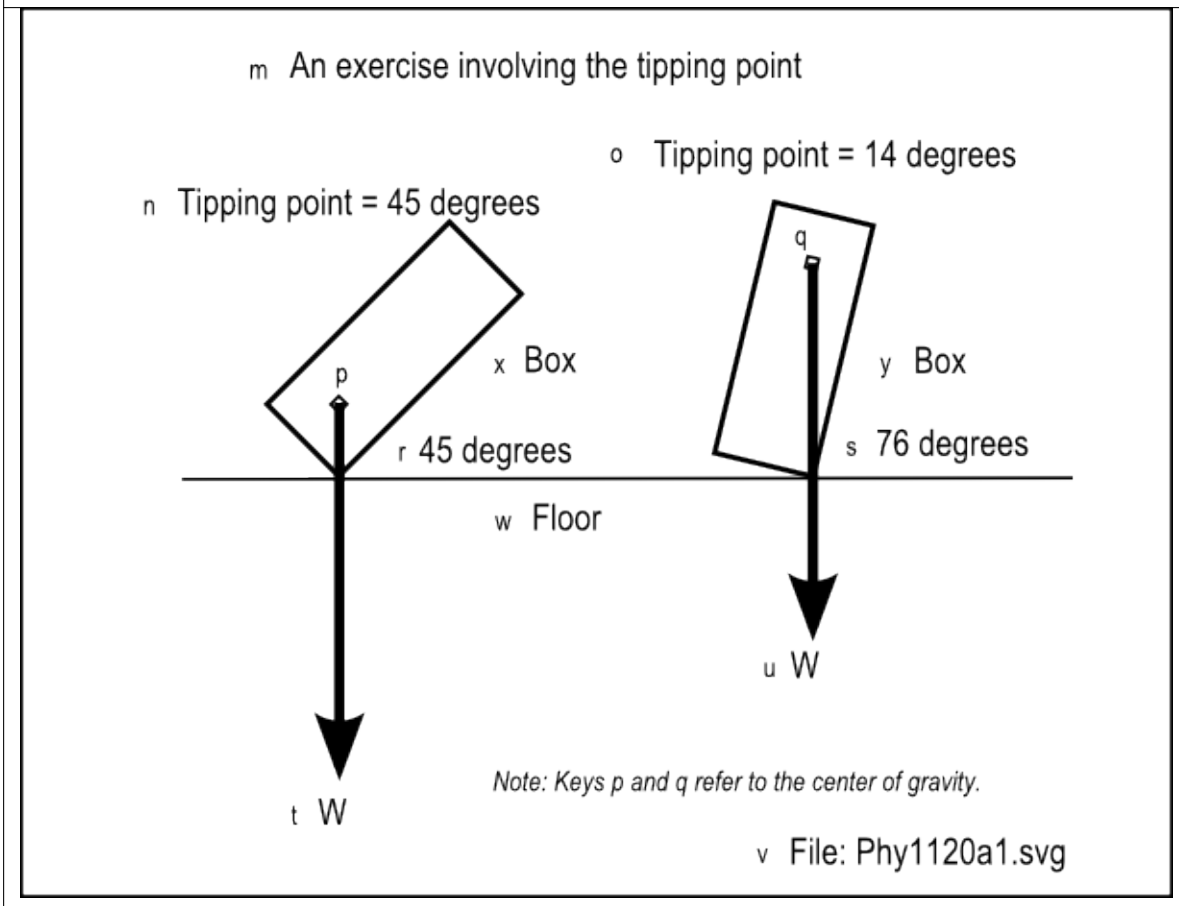


Table 16.2

Figure 3 (p. 511) shows the key-value pairs that go with the image in the file named Phy1120a1.svg.

Figure 3 . Key-value pairs for the image in the file named Phy1120a1.svg.

```
    m: An exercise involving the tipping point
n: Tipping point = 45 degrees
o: Tipping point = 14 degrees
p: Center of gravity
q: Center of gravity
r: 45 degrees
s: 76 degrees
t: W
u: W
v: File: Phy1120a1.svg
w: Floor
x: Box
y: Box
```

Table 16.3

Mark the C.G. of each box

Getting back to your drawing, assume that the C.G. of the left box is one foot up from the bottom and the C.G. of the right box is one foot down from the top (4 feet up from the bottom). In both cases, the C.G. is at the horizontal center of the box. Insert a pin at the location of the C.G. for both boxes and label it C.G. in both cases.

The weight of each box

The downward force that is the weight of each box is on a vertical line that goes through the C.G. and the total weight appears to be concentrated at the C.G. Therefore, when the box is setting flat on the floor, the weight vector for each box is on a line that is 1 ft. from either side of the box.

Vertical weight vectors

Begin at the C.G. for each box and draw a vertical line that extends 7 units down from the C.G. for each box. Label each line as W. These are weight vectors.

Both boxes are in equilibrium

We know from experience that both of these boxes are in equilibrium. We have also just learned that so long as the vertical weight vector that goes through the C.G. also goes through the two-foot wide supporting base of the box, the box will be in stable equilibrium.

What is the tipping point?

Label the bottom-right corner of each box as b. Pretend that you drive a nail immediately to the right of point b and allow it to protrude up from the floor so that when you push on the top-left corner of the box, it will rotate around point b instead of sliding to the right.

How far can we tip each box in a clockwise direction around the bottom right corner before we lose equilibrium?

Tactile graphics

The file named Phy1120b1.svg contains an image that shows the triangles discussed below (p. 520) .

Figure 4 (p. 515) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

An exercise involving the tipping point, part 2
w
o
q
u
v
n
p
t
y b
x b
Floor z
s
r
Key:
n: K1 = 1.414
o: K2 = 4.123
p: Z1 = 1
q: Z2 = 4
r: R = 1
s: R = 1
t: 45 degrees
u: 14 degrees
v: Center of gravity
w: Center of gravity
W mn
W mm
File: Phy1120b1.svg mo

An exercise involving the tipping point, part 2
w
o
q
u
v
n
p
t
y b
x b
Floor
z
s
r
Key:
n: K1 = 1.414
o: K2 = 4.123
p: Z1 = 1
q: Z2 = 4
r: R = 1
s: R = 1
t: 45 degrees
u: 14 degrees
v: Center of gravity
w: Center of gravity
W
mn
W
mm
File: Phy1120b1.svg
mo

An exercise involving the tipping point, part 2
w
o
q
u
v
n
p
t
y b
x b
Floor z
s
r
Key:
n: K1 = 1.414
o: K2 = 4.123
p: Z1 = 1
q: Z2 = 4
r: R = 1
s: R = 1
t: 45 degrees
u: 14 degrees
v: Center of gravity
w: Center of gravity
W mn
W mm
File: Phy1120b1.svg mo

Figure 4 . Mirror image from the file named Phy1120b1.svg.

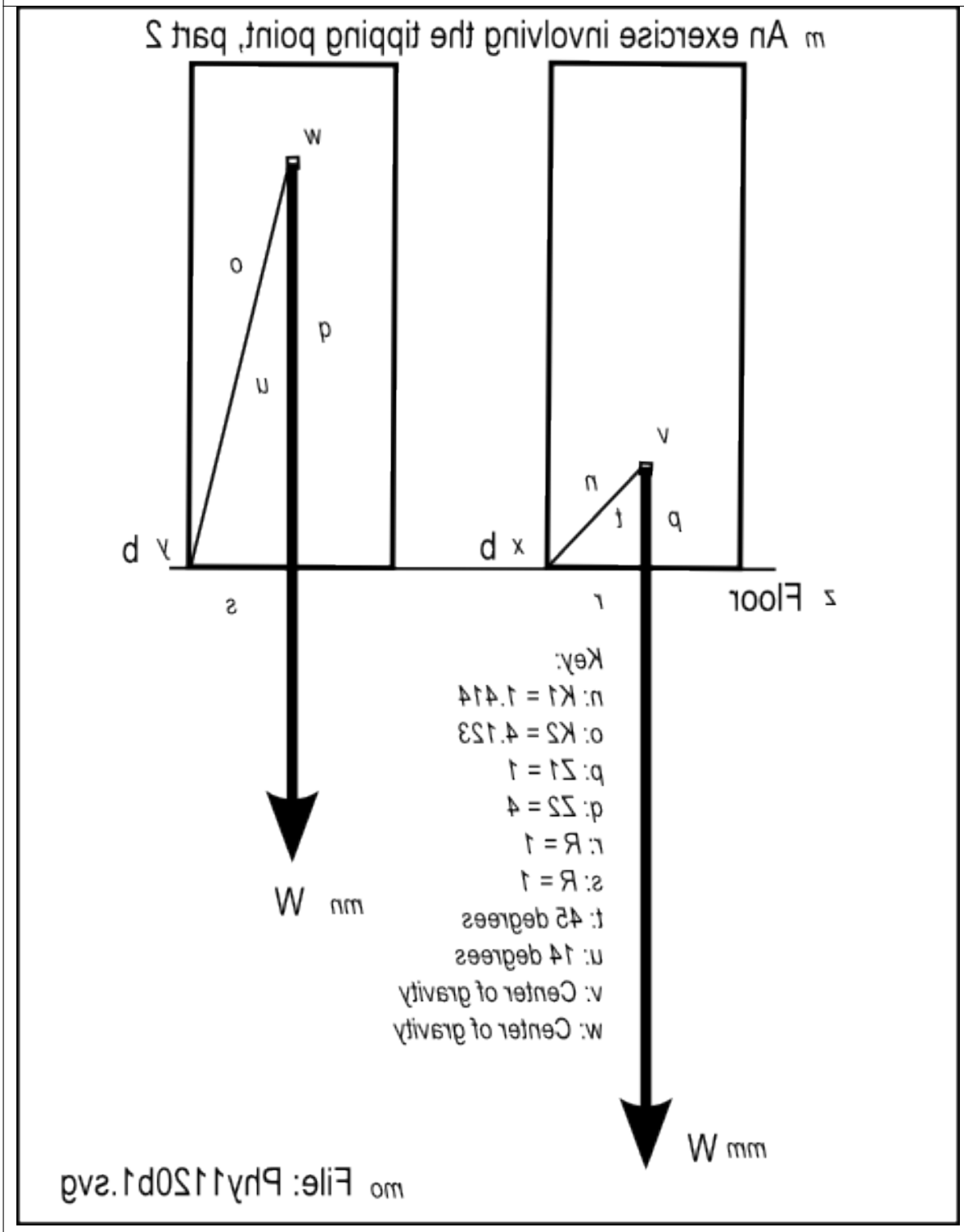
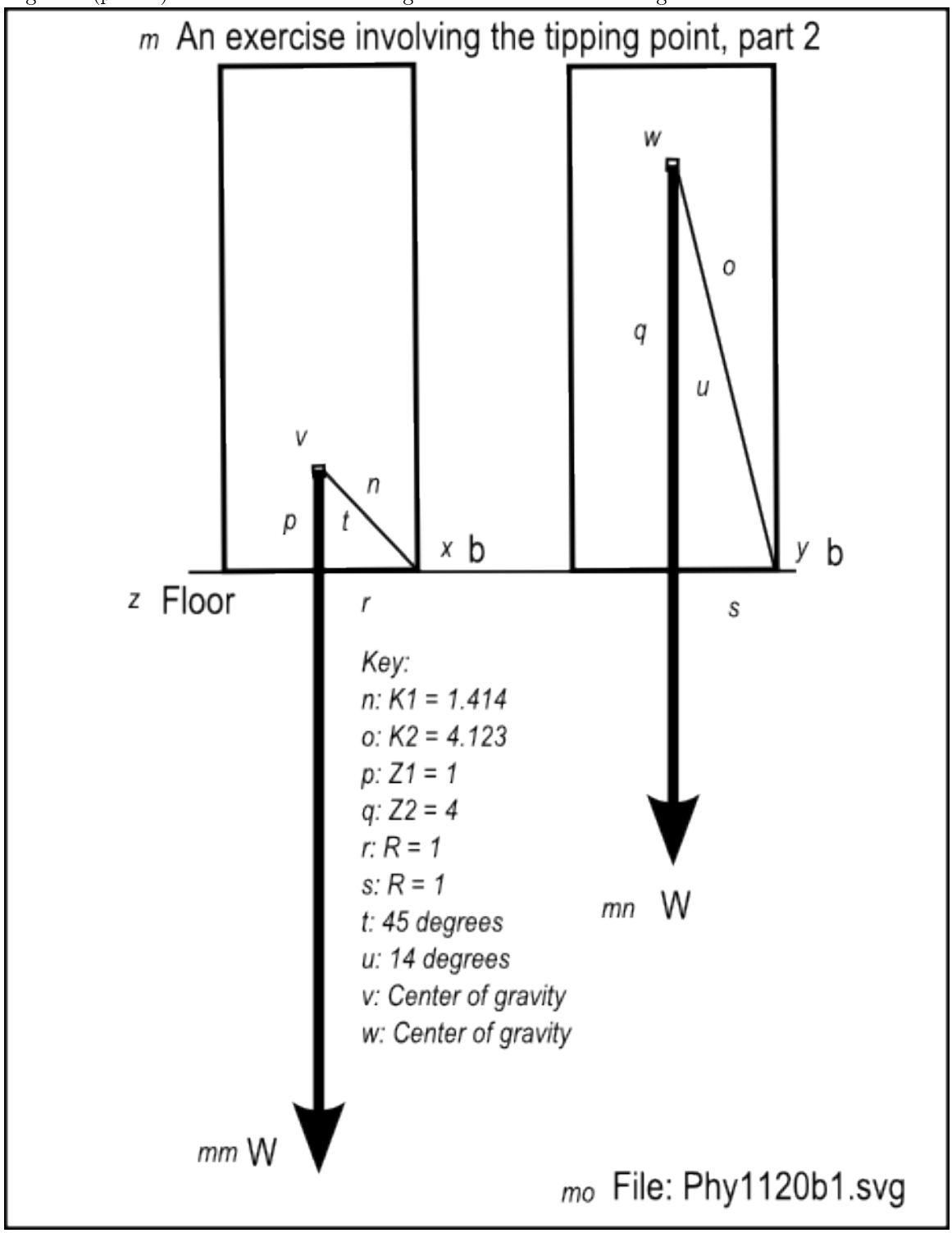
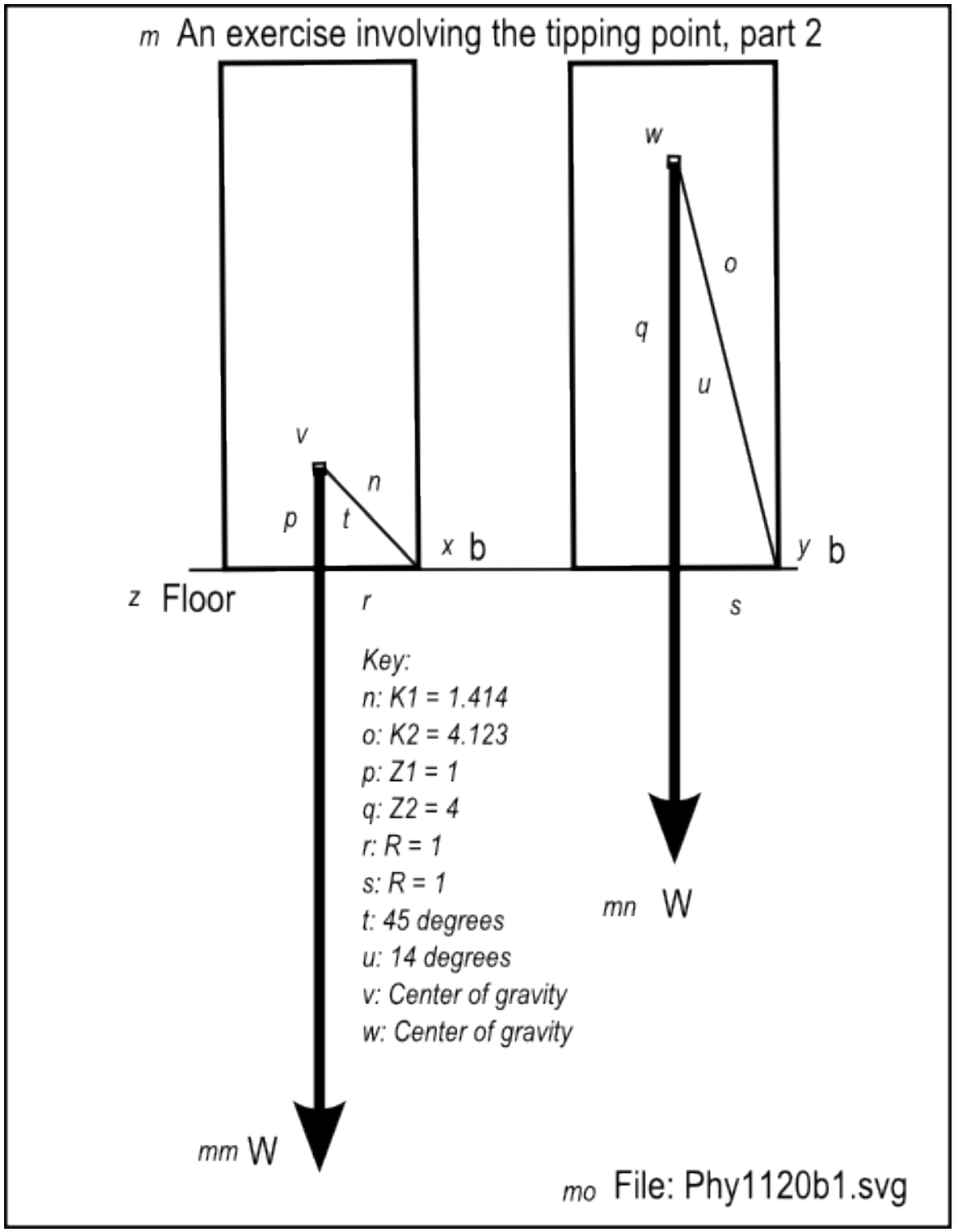


Table 16.4

Figure 5 (p. 519) shows a non-mirror-image version of the same image.




m An exercise involving the tipping point, part 2
w
o
q
u
v
n
p
t
x b
y b
z Floor
r
s
Key:
n: K1 = 1.414
o: K2 = 4.123
p: Z1 = 1
q: Z2 = 4
r: R = 1
s: R = 1
t: 45 degrees
u: 14 degrees
v: Center of gravity
w: Center of gravity
mn W
mm W
mo File: Phy1120b1.svg

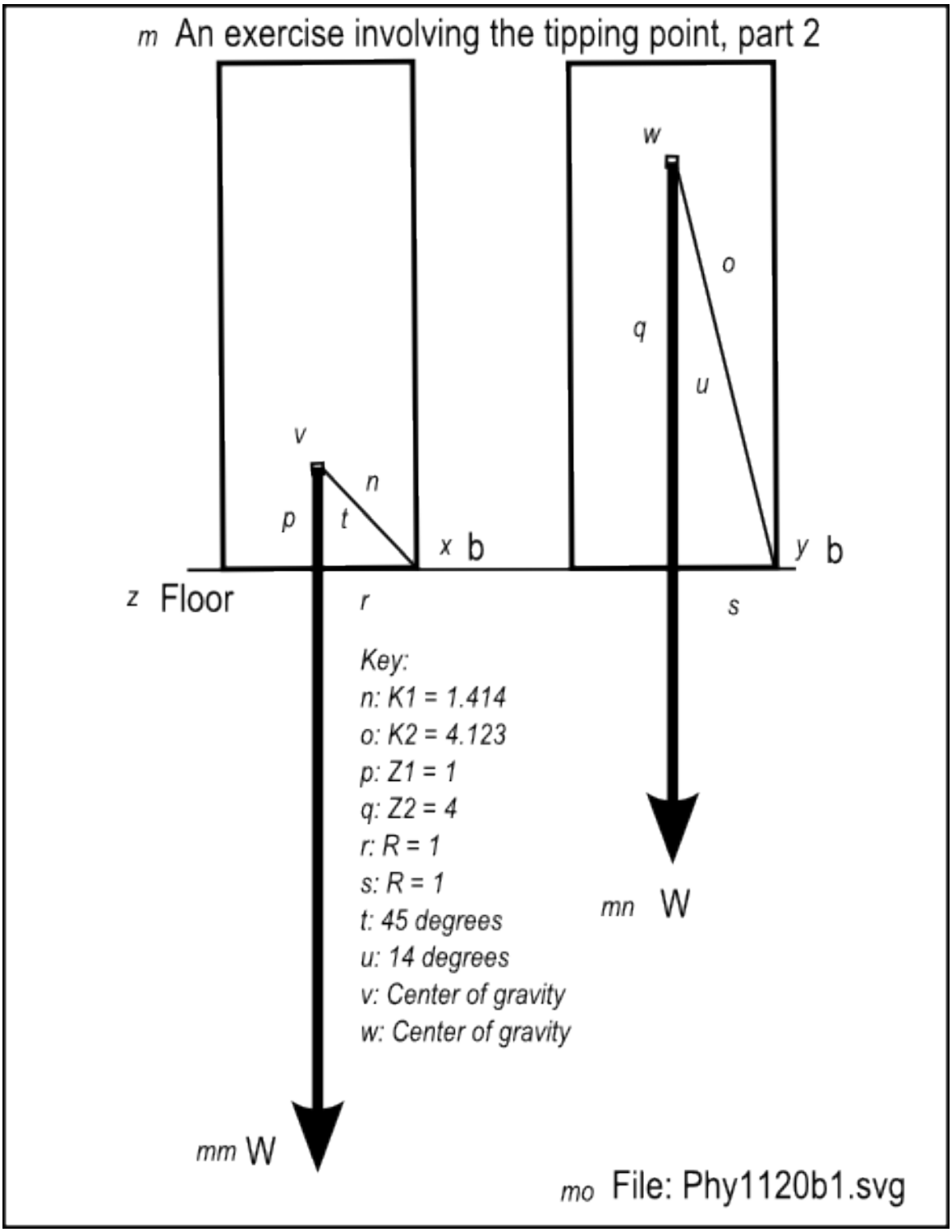

m An exercise involving the tipping point, part 2
w
o
q
u
v
n
p
t
x b
y b
z Floor
r
s
Key:
n: K1 = 1.414
o: K2 = 4.123
p: Z1 = 1
q: Z2 = 4
r: R = 1
s: R = 1
t: 45 degrees
u: 14 degrees
v: Center of gravity
w: Center of gravity
mn W
mm W
mo File: Phy1120b1.svg

Figure 5 . Non-mirror-image version of the image from the file named Phy1120b1.svg.

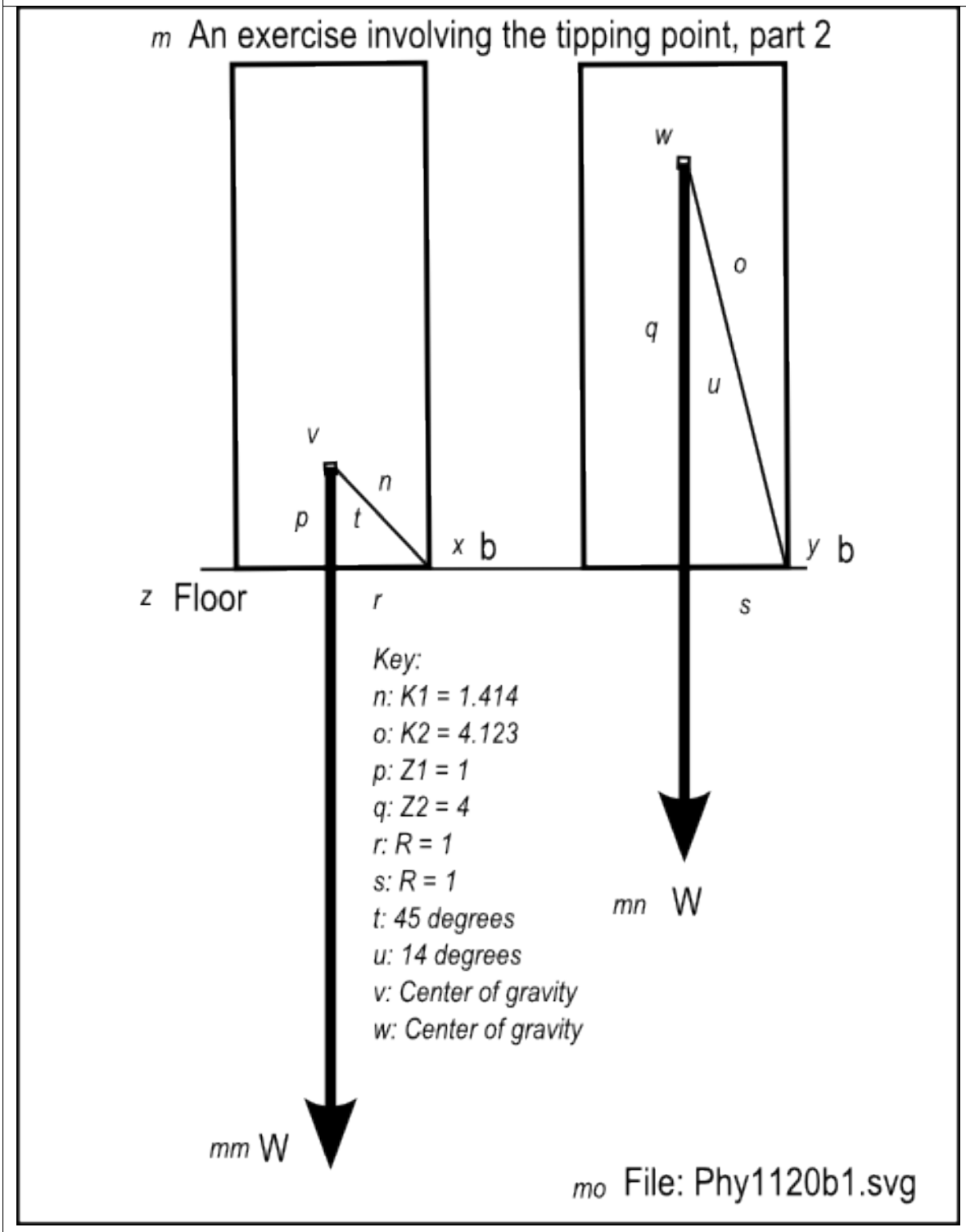


Table 16.5

Figure 6 (p. 520) shows the key-value pairs that go with the image in the file named Phy1120b1.svg.

Figure 6 . Key-value pairs for the image in the file named Phy1120b1.svg.

```
    m: An exercise involving the tipping point, part 2
n: K1 = 1.414
o: K2 = 4.123
p: Z1 = 1
q: Z2 = 4
r: R = 1
s: R = 1
t: 45 degrees
u: 14 degrees
v: Center of gravity
w: Center of gravity
x: b
y: b
z: Floor
mm: W
mn: W
mo: File: Phy1120b1.svg
```

Table 16.6

The lines K1 , K2, Z1, Z2, and R

To answer this question, begin by drawing a line from the C.G. to point b for each box. Label this line K1 for the left box and K2 for the right box.

Label the line segment that extends from the C.G. to the bottom of the left box as Z1 and label the line segment that extends from the C.G. to the bottom of the right box as Z2. Label the line segment that is the right half of the bottom of each box as R.

Right triangles

In the left box, the line segments K1, Z1, and R form a right triangle for which the lengths of two line segments are:

- Z1 = 1
- R = 1

In the right box, the line segments K2, Z2, and R form a right triangle for which the lengths of two line segments are:

- Z2 = 4
- R = 1

Compute the lengths of K1 and K2

Using the Pythagorean theorem and the Google calculator, we can compute the following lengths:

K1 = sqrt(1^2 + 1^2) = 1.414

K2 = sqrt(1^2 + 4^2) = 4.123

The sides of the right triangles

Next, we will compute the interior angle for each triangle at the C.G. vertex. Label the interior angle for the left box P1 and label the interior angle for the right box P2.

We know the lengths of all three sides of each triangle. There are a couple of ways that we can use trigonometry to find the interior angles. Considering the triangle from the viewpoint of the interior angle at the C.G. for the left triangle:

- hypotenuse = K1 = 1.414
- base = Z1 = 1
- opposite side = R = 1

For the right triangle,

- hypotenuse = K2 = 4.123
- base = Z2 = 4
- opposite side = R = 1

Compute the interior angles at each C.G. vertex

Using the Google calculator, we can compute the interior angle at the C.G. for each box as follows:

- P1 = arcsine(1/1.414)*180/PI = 45 degrees
- P2 = arcsine(1/4.123)*180/PI = 14 degrees

Tilt each box and observe the angles at the C.G. vertices

Now pretend that you exert a force on the top-left corner of each box causing it to rotate around the lower-right corner at point b. As you do that, the weight vector would rotate around the C.G. in a counter-clockwise direction (always pointing straight down) and the angle between the weight vector and the line K1 or K2 would decrease.

The tipping point

When the angle between the weight vector and the line K1 or K2 goes to zero degrees for either box, the tipping point for that box will have been reached. (See the image in the file named Phy1120a1.svg, or Figure 1 (p. 506) .)

At that exact point, the box will still be in equilibrium, but it will be in unstable equilibrium. Exerting a little more force to the right will cause the box to turn all the way over onto its side under its own weight. Relaxing the force a little when the box is in that position will allow it to rotate back towards its original position.

What are the tipping point angles?

The box on the left would need to rotate slightly more than 45 degrees clockwise around point b before the direction of the weight vector through the C.G. would move to the right of point b, causing the box to continue rotating under its own weight.

However, the box on the right would only need to rotate slightly more than 14 degrees clockwise around point b before the direction of the weight vector through the C.G. would move to the right of point b causing the box to continue rotating under its own weight.

Conclusion

Therefore, the box on the left is more stable than the box on the right. If you want to lessen the likelihood of an object tipping over, cause the C.G. of the object to be near the bottom of the object.

An exercise for the student

I will leave it as an exercise for the student to compute moments about point b to confirm the tipping angle for each box where the torque about point b changes from counter-clockwise to clockwise.

16.4.4 A real-world example

Let me illustrate this situation with a real-world example that may seem familiar to you. Assume that you put some flowers with long stems in a lightweight plastic vase and set the vase on a table. The C.G. of the vase and the flowers would be relatively high, and it wouldn't take much of a sideways push to cause the vase to turn over.

Now assume that you add water to the vase until it is about half full. This would cause the C.G. of the vase, the water, and the flowers to move down, probably into the bottom half of the vase. This would make it more difficult to tip the vase over. Of course, when it does tip over, it would make a bigger mess than would be the case without the water.

16.5 Do the computations

I encourage you to repeat the computations that I have performed in this module. Experiment with the computations, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

16.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

16.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force – Center of Gravity
- File: Phy1120.htm
- Revised: Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · force
  - · center of gravity
  - · gravitational constant

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection

that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 17

Force and Motion – Introduction[1]

17.1 Table of Contents

- Preface (p. 525)
  - General (p. 525)
  - Prerequisites (p. 525)
  - Supplemental material (p. 526)
- Discussion (p. 526)
  - Aristotle's contribution, or lack thereof (p. 527)
  - Galileo's contribution (p. 527)
  - Newton's contribution (p. 527)
    - Three laws of motion (p. 528)
      - Law 1 (p. 528)
      - Law 2 (p. 528)
      - Law 3 (p. 528)
  - Mass, momentum, force, and acceleration (p. 528)
- Resources (p. 529)
- Miscellaneous (p. 529)

17.2 Preface

17.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module introduces force and motion in a format that is accessible to blind students.

17.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

[1] This content is available online at <http://cnx.org/content/m38170/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

17.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

17.3 Discussion

We dealt with bodies in equilibrium in earlier modules. By equilibrium, I mean that we dealt with forces acting on bodies at rest or forces acting on bodies in uniform motion. A body in equilibrium does not accelerate as a result of the forces acting on it.

An example of a body experiencing forces while at rest

A highway bridge is an example of a body at rest being acted upon by a variety of forces. As vehicles enter and exit the bridge, those vehicles exert forces on the bridge. The impact of those forces is "felt" by all the structural members that make up the bridge. The tension and compression in the various members of the bridge change as vehicles enter and leave the bridge, but the bridge remains in the same fixed location. If properly designed, the bridge doesn't experience acceleration and crash into the river below as a result of those changing forces.

Unbalanced forces

In this module, we begin a study of the influence of unbalanced forces acting on bodies along with a study of the motion produced in bodies as a result of unbalanced forces.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

Statics versus dynamics

The earlier modules dealt with **statics** . We now begin a study of **dynamics** . The study of dynamics hinges largely on three important laws of motion, which were stated by Sir Issac Newton in 1686.

Making bodies move

We know from experience that we can cause small bodies to move by pulling or pushing them with our hands. In other words, we can cause a body to move by exerting a force on the body. We also know that larger bodies can be caused to move by pulling or pushing them with a machine or with a beast of burden.

We are also familiar with the idea of falling bodies that move independently of someone pushing or pulling. We have come to know this as the result of the gravitational attraction between masses.

17.3.1 Aristotle's contribution, or lack thereof

I have never forgotten my physics professor, Brother Rudolph, at St. Mary's University in San Antonio, Texas, telling the class that Aristotle was a hindrance to science.

Aristotle taught that heavier bodies fall faster than lighter bodies, but he was wrong. It has been proven that all bodies fall towards the earth at the same acceleration when the effects of air resistance are eliminated.

This is not too difficult to prove for yourself. A small piece of paper will fall to the ground much more slowly than a coin when the paper is in its normal state. However, if the paper is crumpled into a very tight ball, greatly reducing the effect of air resistance, it will fall to the ground almost as fast as the coin.

17.3.2 Galileo's contribution

Galileo and Newton clarified the ideas of motion through a series of experiments. For example, Galileo discovered the important relationship between force and acceleration. He concluded that in the absence of air resistance, freely falling objects have the same acceleration regardless of their differing weights.

Acceleration of gravity is constant

By rolling balls down inclined planes, Galileo discovered that the distance covered under a steady force is proportional to the square of the time of descent. By this, he concluded that acceleration is constant, at least near the surface of the earth.

Two cannon balls

The story goes that Galileo dropped two cannon balls of different weights off the Leaning Tower of Pisa and observed that they struck the ground below at exactly the same time. From this, he concluded that falling bodies are subject to the same acceleration regardless of their weight.

Inertia

By rolling a ball down one inclined plane and up a facing incline plane, Galileo observed that the ball tended to rise to (almost) the original height on the second inclined plane. This was true even if the second incline plane was less steep than the first.

This suggested that if the second inclined plane were perfectly flat, the ball would roll forever trying to regain its original height. From this, Galileo is said to have concluded that objects at rest tend to remain at rest, and objects in motion tend to remain in motion with the same velocity. (Recall that a change in direction is a change in velocity.)

This property of matter not to change its state of rest or of uniform motion at a constant velocity is called **inertia** .

Force is required to cause a change

Whenever a body changes from a state of rest to a state of motion, or changes its motion from one velocity to a different velocity, a force is required to cause that change.

17.3.3 Newton's contribution

Newton continued Galileo's study of motion and formulated his findings in three laws of motion. The first two laws were formal statements of Galileo's earlier conclusions. The third law formulated a finding that was original to Newton.

17.3.3.1 Three laws of motion

Unlike those before them, Galileo and Newton saw that in the absence of friction, force was required to change motion, not to maintain it.

The Principia

As mentioned earlier, Newton formulated the earlier findings of Galileo in two laws of motion and added a third law based on his own findings. He published those laws in a document which, when translated from the original Latin, was titled something like: "Mathematical Principles of Natural Philosophy." This document is often referred to simply as the Principia.

The three laws are paraphrased in the following sections.

17.3.3.1.1 Law 1

Every body remains in a state of rest or of uniform motion in a straight line unless compelled to change that state by external force acting upon it.

Another interpretation of this law reads as follows:

Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it. *(This suggests that a state of rest is motion with zero velocity.)*

This law is generally regarded as recognition of Galileo's concept of inertia and is often referred to as the *"Law of Inertia."*

The law explains what force does, but does not suggest how force should be measured.

17.3.3.1.2 Law 2

Rate of change of momentum is proportional to the impressed force and takes place in the direction of that force.

Another interpretation of the second law is:

The relationship between an object's mass m, its acceleration a, and the applied force F is F = m*a *(where the * indicates multiplication)* . Acceleration and force are vectors; in this law the direction of the force vector is the same as the direction of the acceleration vector.

This law, in conjunction with the third law, allows quantitative calculations of dynamics and a definition of mass. In particular, how do velocities change when forces are applied.

The relationships among momentum, force, mass, and accelerations will be explained later.

17.3.3.1.3 Law 3

Action and reaction are equal and opposite, and act on different bodies.

Another interpretation of this law is:

For every action there is an equal and opposite reaction.

This law means that forces act in equal and opposite pairs. Whenever two bodies act upon one another, equal and opposite changes of momentum occur in the two bodies.

An example of the effect of this law occurs when a person attempts to jump from a small boat to a pier. If the boat is free to move, it will tend to move away from the pier as the person attempts to launch himself in the direction of the pier. This may leave the person momentarily hanging in the air between the boat and the pier waiting to experience the acceleration of gravity and fall into the water.

In order to understand the full importance of these laws, it is necessary to understand the concepts of and relationships among mass, momentum, force, and acceleration.

17.3.4 Mass, momentum, force, and acceleration

Newton introduced the term mass as **being distinct from weight** . Mass is not easy to define in its own right. Mass can be thought of as being a "quantity of matter" or a "measure of inertia."

One of the properties of mass in a gravitational field is weight. Newton showed that weight is proportional to mass.

(In earlier modules, we have said that the weight of a body is a measure of the force required to cause that body to accelerate toward the surface of the earth at a rate of approximately 32.2 ft/s^2 or 9.81 m/s^2.)

Newton also worked with the concept of **momentum** , which is the product of the mass and the velocity of an object. Momentum is a vector quantity, having the same direction as the velocity to which it applies. Therefore, momenta are always added or subtracted as vectors.

The rate of change of momentum

As indicated in the second law (p. 528) , the force required to bring about a change in momentum is proportional to the rate of change of the momentum:

F $ (m*v2 - m*v1)/t

where the momentum is changed from a value of **m*v1** to a value of **m*v2** in a time interval **t** .

(Note that I elected to use the $ character to indicate "is proportional to." Textbooks typically use a Greek letter for this purpose, which is not compatible with many Braille displays and some audio screen readers.)

Given that the mass is constant (for the scenarios addressed in this collection of modules), and acceleration is given by the rate of change of velocity, factoring mass out of the above equation gives us

F $ m*(v2 - v1)/t, or

F $ m*a

where a is acceleration. Thus, force is proportional to the product of mass and acceleration. For a given mass, a specific force is required to cause that mass to accelerate by a given amount.

17.4 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

17.5 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force and Motion – Introduction
- File: Phy1140.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · Galileo
  - · Newton
  - · inertia
  - · mass

· momentum
· force
· acceleration

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 18

Force and Motion – Units of Force[1]

18.1 Table of Contents

- Preface (p. 532)
  - General (p. 532)
  - Prerequisites (p. 532)
  - Viewing tip (p. 532)
    - Figures (p. 533)
  - Supplemental material (p. 533)
- Background information (p. 533)
  - Creation of tactile graphics (p. 533)
  - Things can be confusing (p. 533)
  - SI units and the newton (p. 534)
  - A practical example using the Google calculator (p. 535)
  - Other units (p. 536)
    - poundal (p. 536)
    - dyne (p. 537)
  - Acceleration of gravity (p. 537)
    - pound-force (p. 537)
    - gram-force (p. 538)
    - kilogram-force (p. 538)
  - Weight is a force (p. 538)
- Sample problems (p. 539)
  - A static scenario (p. 539)
  - First dynamic scenario (p. 548)
  - Second dynamic scenario (p. 548)
  - Third dynamic scenario (p. 549)
- Do the calculations (p. 550)
- Resources (p. 550)
- Miscellaneous (p. 550)

[1] This content is available online at <http://cnx.org/content/m38211/1.3/>.

18.2 Preface

18.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains various units of force in a format that is accessible to blind students.

18.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.
- The ability to create tactile graphics as described here [7] .

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [8]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [9]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [10]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [11] and http://www.w3schools.com/js/default.asp [12]).
- An understanding of all of the material covered in the earlier modules in this collection.
- An understanding of the creation and use of tactile graphics as described here [13] .

18.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482
[8] http://www.youtube.com/watch?v=c8plj9UsJbg
[9] http://www.youtube.com/watch?v=v-F06HgiUpw
[10] http://www.clarku.edu/~djoyce/trig/
[11] http://www.dickbaldwin.com/tocjscript1.htm
[12] http://www.w3schools.com/js/default.asp
[13] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

18.2.3.1 Figures

- Figure 1 (p. 534) . SI base units.
- Figure 2 (p. 535) . Examples of SI derived units.
- Figure 3 (p. 543) . Mirror image of the image from the file named Phy1150a1.svg.
- Figure 4 (p. 547) . Non-mirror image of the image from the file named Phy1150a1.svg.
- Figure 5 (p. 547) . Key-value pairs for the image in the file named Phy1150a1.svg.

18.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [14] .

18.3 Background information

18.3.1 Creation of tactile graphics

The module titled Manual Creation of Tactile Graphics [15] explains how to create tactile graphics from svg files that I will provide.

If you are going to have an assistant create tactile graphics for this module, you will need to download the file named Phy1150.zip [16] , which contains the svg file for this module. Extract the svg file from the zip file and provide them to your assistant.

Also, if you are going to use tactile graphics, it probably won't be necessary for you to perform the graph board exercises. However, you should still walk through the graph board exercises in your mind because I will often embed important physics concepts in the instructions for doing the graph board exercises.

In each case where I am providing an svg file for the creation of tactile graphics, I will identify the name of the appropriate svg file and display an image of the contents of the file for the benefit of your assistant. As explained here [17] , those images will be mirror images of the actual images so that your assistant can emboss the image from the back of the paper and you can explore it from the front.

I will also display a non-mirror-image version of the image so that your assistant can easily read the text in the image.

Also in those cases, I will provide a table of key-value pairs that explain how the Braille keys in the image relate to text or objects in the image.

18.3.2 Things can be confusing

One of the more confusing things about physics textbooks can be their treatment of the units in which force is measured and reported. Several different units are often used including:

- newton
- poundal
- dyne
- pound-force
- gram-force
- kilogram-force

[14] http://www.dickbaldwin.com/toc.htm
[15] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482/Phy1004-Manual-Creation-of-Tac
[16] http://cnx.org/content/m38211/latest/Phy1150.zip
[17] http://cnx.org/contents/9fe07552-485f-43e7-9753-87133cf60482

The first three units in the above list are said to be **absolute units of force** because they are measured in fundamental units of mass, length, and time. The last three units are tied directly to the gravitational attraction between the earth and other objects.

The newton, which is an SI derived unit, is possibly the most universally accepted unit of force in the year 2011.

Force equals the product of mass and acceleration

You learned in an earlier module that the force that is required to cause a given mass to be accelerated by a given amount is proportional to the product of the mass and the acceleration. If we specify mass and acceleration in consistent units, we can write

f = m * a

where

- f is force
- m is mass
- a is acceleration

18.3.3 SI units and the newton

Many physics books use a system of units called **SI units** . SI is an abbreviation for a French name, which I am unable to pronounce, and which is probably also not compatible with your screen reader and your Braille display.

I won't attempt to explain much about SI units in this module. I provided some information in an earlier module titled Units and Dimensional Analysis [18] . Also, you can probably find a good explanation in your textbook, and if not, you can Google SI units and find hundreds of web pages that explain the system in varying levels of detail.

Tables of SI units

Most of those references will probably also provide tables for the units, but those tables may be partially incompatible with your screen reader and Braille display due to the extensive use of superscripts. Therefore, I will provide tables that should be accessible in Figure 1 (p. 534) and Figure 2 (p. 535) .

Base units and derived units

When reading about SI units, you will find that they are often divided into base units and derived units. I will put the base units in Figure 1 (p. 534) and some sample derived units in Figure 2 (p. 535) .

Figure 1 . SI base units.

Base Quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 18.1

[18] http://cnx.org/contents/c595fef6-42c1-4219-ac6c-0becf9651733/Phy1050-Units-and-Dimensional-

Note that the list of derived units in Figure 2 (p. 535) is only a sampling of different units that can be derived from the base units.

The exponentiation indicator

As is the case throughout these modules, the character "^" that you see used extensively in Figure 2 (p. 535) indicates that the character following the ^ is an exponent. Note also that when the exponent is negative, it is enclosed along with its minus sign in parentheses for clarity.

Figure 2 . Examples of SI derived units.

force	newton	kg*m/s^2
area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	m/s
acceleration	meter per second squared	m/s^2
wave number	reciprocal meter	m^(-1)
mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m^3/kg
current density	ampere per square meter	A/m^2

Table 18.2

The newton

The first derived unit listed in Figure 2 (p. 535) is the newton, which is the product of the base unit for mass (kg) and the base unit for acceleration (m/s^2). Thus, the units for the newton are

kg*m/s^2

A newton is a unit of force that causes a mass of one kilogram to be accelerated by one meter per second squared (1 m/s^2).

18.3.4 A practical example using the Google calculator

When making physics calculations, It is extremely important that you understand and keep track of the units that you are using. The Google search box can serve your needs as a scientific calculator if you are careful how you use it. For example, if you enter an expression such as 3+5 in the Google search box and press the Enter key, the result of evaluating that expression will be displayed immediately below the search box.

A physics problem

A typical problem in a physics textbook states that a 2-kg mass is moving in a circular path with a constant angular velocity of 5 radians per second and with a tangential velocity of 3 m/sec. The objective is to find the centripetal force on the mass.

You will learn all that you need to know to solve this problem in future modules. For now, just bear with me and concentrate on the use of the Google search box as a scientific calculator.

As you will learn in a future module, a radian is a dimensionless quantity, and the proper units for an angular velocity of 5 radians per second is simply 5/s. (It also works to spell radians out, but abbreviations for radians may not work in the Google calculator.)

The algebraic solution

Once you work through the algebra for this problem, you will have determined that the answer to the problem is given by the following expression:

centripetal force = 2kg * (5/s) *(3m/s)

Let Google do the work

Enter the following expression into the Google search box and press the Enter key:

2kg * (5/s) *(3m/s)

The following text should appear immediately below the search box.

2 kg * (5 / s) * (3 (m / s)) = 30 newtons

Note that in this case, I used the correct symbols for SI units.

Do it again with the units spelled differently

Enter the following expression into the search box and press the Enter key.

2 kilograms * (5 radians/second) * (3 meters/second)

The following text should appear immediately below the search box.

2 kilograms * ((5 radians) / second) * (3 (meters / second)) = 30 newtons

Do it one more time

Let's do it one more time, this time mixing metric and English units. Enter the following expression into the search box and press the Enter key.

2 kilograms * (5 radians/second) * (9.8425197 feet/second)

The following output should appear below the search box:

2 kilograms * ((5 radians) / second) * (9.8425197 (feet / second)) = 30 newtons

The conclusion

Given an input with units in the correct format, the Google calculator is not only able to deal with those units and perform the calculation correctly, it is also able to properly convert the result to the correct value in derived units (newtons in this case).

18.3.5 Other units

When analyzing a physics problem, the safest approach is probably to convert all of the given information into SI units and solve the problem in SI units, converting the result back to some other system of units if required. However, some textbooks and some physics professors may not allow that approach. Therefore, you need to know something about the other units of force that you may encounter.

18.3.5.1 poundal

As you learned in an earlier module, the **pound** is a unit of mass.

There is a very handy online mass unit converter at http://www.onlineconversion.com/weight_common.htm [19] . However, I don't know if it is accessible for screen readers and Braille displays. According to that converter,

1 pound = 0.45359237 kilograms

Can use Google for conversion

Even if that converter isn't accessible, you can use the Google calculator to make such conversions. Enter the following in the Google search box and press the Enter key:

conversion pound to kilogram

The following text should appear immediately below the search box:

1 pound = 0.45359237 kilograms

The foot

A foot is a unit of length commonly used for measurements in the United States.

1 foot = 0.3048 meters

The poundal unit of force

A poundal is a unit of force that causes a mass of one pound to accelerate at one foot per second squared (1 ft/s^2).

Because the poundal and the newton are both units of force, we can convert from one to the other as follows:

1 poundal = 0.138254954 newton

[19] http://www.onlineconversion.com/weight_common.htm

18.3.5.2 dyne

The gram is another unit of mass and the centimeter is another unit of length.

1 gram = 0.001 kilogram

1 centimeter = 0.01 meter

The dyne unit of force

A dyne is a unit for force that causes a mass of one gram to accelerate at one centimeter per second squared (1 cm/s^2). Once again, we can convert back and forth between dynes and newtons using the following relationship:

1 dyne = 0.00001 newton

18.3.6 Acceleration of gravity

The values for the next three units of force that I will explain depend on the gravitational attraction between the earth and other objects.

The universal law of gravitation tells us that objects having mass are attracted to one another by a force that is proportional to the product of their masses and inversely proportional to the distance between them.

attractive force = G*m1*m2/d^2

where

- G is the gravitational constant 6.673*10^(-11)*m^3*kg^(-1)*s^(-2)
- m1 and m2 are the masses involved
- d is the distance between the masses

It is also true that for large spherical masses like the earth, the effect is as if all of the mass is concentrated at a point located at the center of the sphere.

Objects are attracted to the earth

Therefore, all objects on or near the surface of the earth are attracted toward the center of the earth by an amount given by the above equation (p. 537) .

Neglecting friction such as air resistance, objects near the surface at different points on earth fall with an acceleration somewhere between about 9.78 m/s^2 and 9.82 m/s^2 depending on latitude.

18.3.6.1 pound-force

The unit of force named pound-force incorporates the gravitational pull of the earth in its definition. One pound-force is the force required to cause a mass of one standard pound to be accelerated by an amount equal to the standard acceleration of gravity that exists near the surface of the earth.

In accordance with the General Conference on Weights and Measures [20] , standard gravity is usually taken to be 9.80665 m/s^2 (32.174049 ft/s^2).

The acceleration of the standard gravitational field and the international avoirdupois pound [21] define the pound-force as:

- 1 pound-force = 1 pound * 32.174049 ft/s^2
- 1 pound-force = 0.45359237kg * 9.80665 m/s^2
- 1 pound-force = 4.44822162 newton

[20] http://en.wikipedia.org/wiki/General_Conference_on_Weights_and_Measures
[21] http://en.wikipedia.org/wiki/Pound_(mass)#International_pound

18.3.6.2 gram-force

As with pound-force, the unit of force named gram-force also incorporates the gravitational pull of the earth in its definition. One gram force is the force required to cause a one-gram mass to be accelerated by 980.665 cm/s^2.

We can convert force back and forth between units of gram-force and units of newton using the following relationship.

1 gram-force = 0.00980665 newton

18.3.6.3 kilogram-force

Finally, the unit of force named kilogram-force also incorporates the gravitational pull of the earth in its definition. One kilogram-force is the force required to cause a one-kilogram mass to be accelerated 9.80665 m/s^2.

Once again, we can convert between force units of kilogram-force and newton using the following relationship.

1 kilogram-force = 9.80665 newton

18.3.7 Weight is a force

Despite the fact that scales in the grocery stores in the United States have displays that read in pounds (which is a unit of mass), weight is not a measure of mass. Weight is a measure of force.

What exactly is the weight of an object?

The measurement that we normally think of as the weight of an object with a given mass is the force exerted on that object at the surface of the earth by the gravitational attraction between that object and the earth.

The weight of that same object on the surface of the moon would be the force exerted on that object by the gravitational attraction between that object and the moon.

Weighing a package of hamburger

Assume that you have a package that contains a one-pound mass of hamburger. You put it on a scale on the surface of the earth and the display reads 1 pound.

Now assume that you take the same scale and the same package of hamburger to the surface of the moon and place the package on the scale. The display would no longer read 1 pound.

The weight indicated by the scale would be different because the gravitational attraction between the mass of the hamburger and the mass of the moon would be less than the gravitational attraction between the mass of the hamburger and the mass of the earth.

According to the calculator at http://www.moonconnection.com/moon_gravity.phtml [22] , the scale would read 0.2 pounds on the moon.

What does the output of the scale really mean?

When you see a scale with a display that reads pound, it should read pound-force instead. If it reads kilogram, it should read kilogram-force instead. Pound-force, kilogram-force, and gram-force are units that are tied directly to the gravitational attraction between the earth and other objects.

Weightlessness

When astronauts go into space and speak of being weightless, they probably aren't completely weightless. However, their weight is probably so low that it seems to them to be zero.

The reduction in an astronaut's weight occurs because the distance between the astronaut and any large massive object (such as the earth or the moon) is so great that the gravitational attraction between them is very small.

[22] http://www.moonconnection.com/moon_gravity.phtml

18.4 Sample problems

Let's work through several sample problems involving forces. The first will be a statics problem and the last three will be dynamics problems.

18.4.1 A static scenario

It will probably help you to keep track of everything if you draw the scenario on your graph board.

Draw a side view of two cubes with different masses on the top of a flat level horizontal friction-free table. Label them Mass C and Mass B. Mass C is on the left and Mass B is on the right.

Mass B is close to the rightmost edge of the table and Mass C is to the left of Mass B.

Label Mass C as 3 kg and label Mass B as 2 kg.

A mass hanging on a cord

Draw a strong but lightweight cord, connected to the right side of Mass B and thread the cord over a very light frictionless pulley that changes the orientation of the cord from horizontal to vertical. The pulley is attached to the right edge of the table.

Draw a triangle-shaped mass connected to the cord that hangs down from the pulley. Label this Mass A and label it as 5 kg.

Two additional cords

Draw a strong but lightweight cord connecting Mass B to Mass C.

Draw another strong but lightweight cord connecting Mass C to a vertical wall on the left side of Mass C. That cord prevents any of the masses from moving and keeps the entire system in equilibrium with the 5-kg mass suspended from the cord that is threaded over the pulley.

Everything should be lined up

The pulley, both masses, and all three horizontal segments of cord above the tabletop should be in a straight line. The attachment heights of the cords and the top of the pulley should be such that all of the cords above the tabletop are horizontal. In other words, there should be no vertical components in any of the forces that act on the cords above the tabletop.

Label the tension in each cord

Label the tension in the cord between Mass B and Mass A as P. Label the tension in the cord between Mass C and Mass B as Q Label the tension in the cord between the wall and Mass C as R.

Tactile graphics

The svg file that is required to create tactile graphics for this exercise is named Phy1150a1.svg. You should have downloaded (p. 533) that file earlier. This file contains an image that represents the instructions given above (p. 539) .

Figure 3 (p. 543) shows the mirror image that is contained in that file for the benefit of your assistant who will create the tactile graphic for this exercise.

A static scenario
Mass C 3 kg
Mass B 2 kg
Friction free pulley
Q
R
P
Friction free table
Mass A 5 kg

A static scenario
File: Phy150a1.svg
Mass B 2 kg
Mass C 3 kg
Friction free pulley
Q
R
P
Friction free table
Mass A 5 kg

A static scenario
Mass C 3 kg
Mass B 2 kg
Friction free pulley
Q
R
P
Friction free table
Mass A 5 kg

Figure 3 . Mirror image of the image from the file named Phy1150a1.svg.

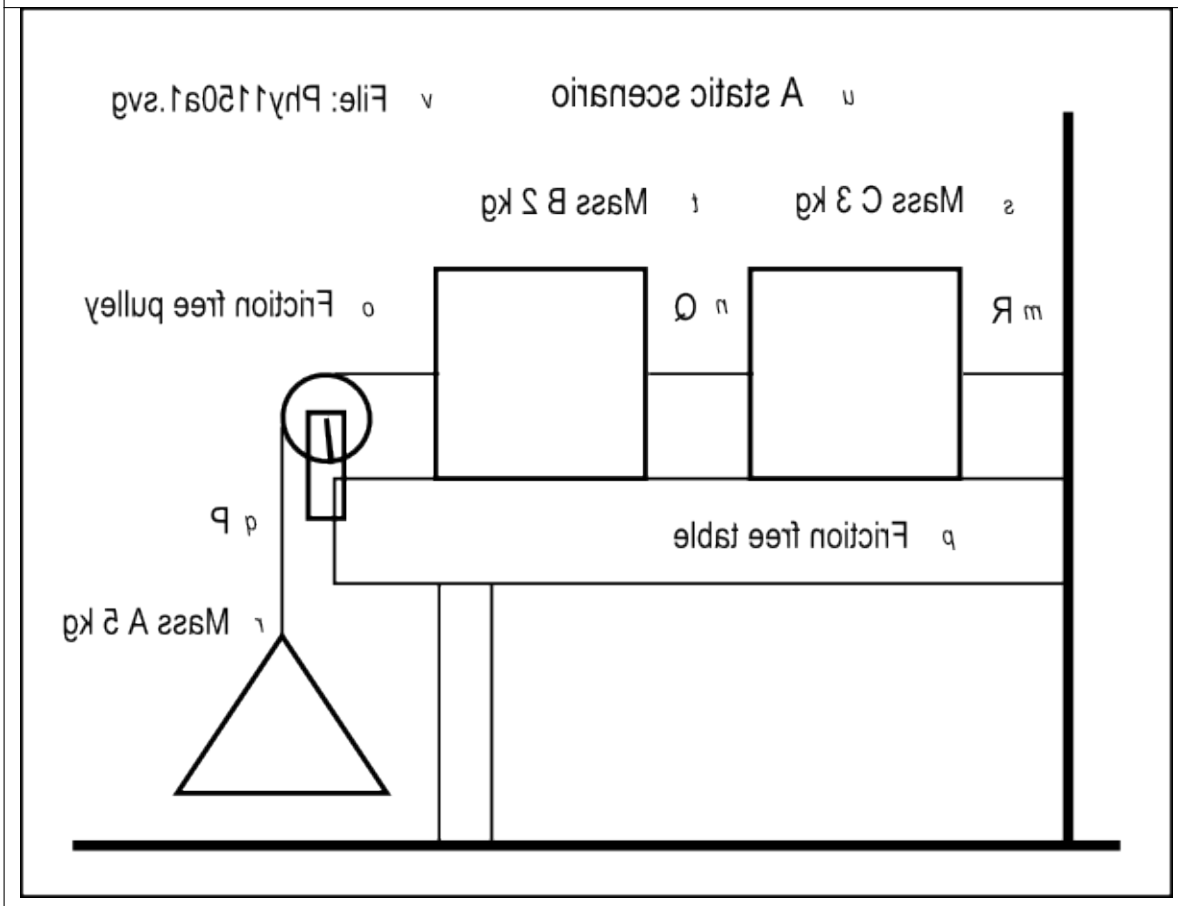
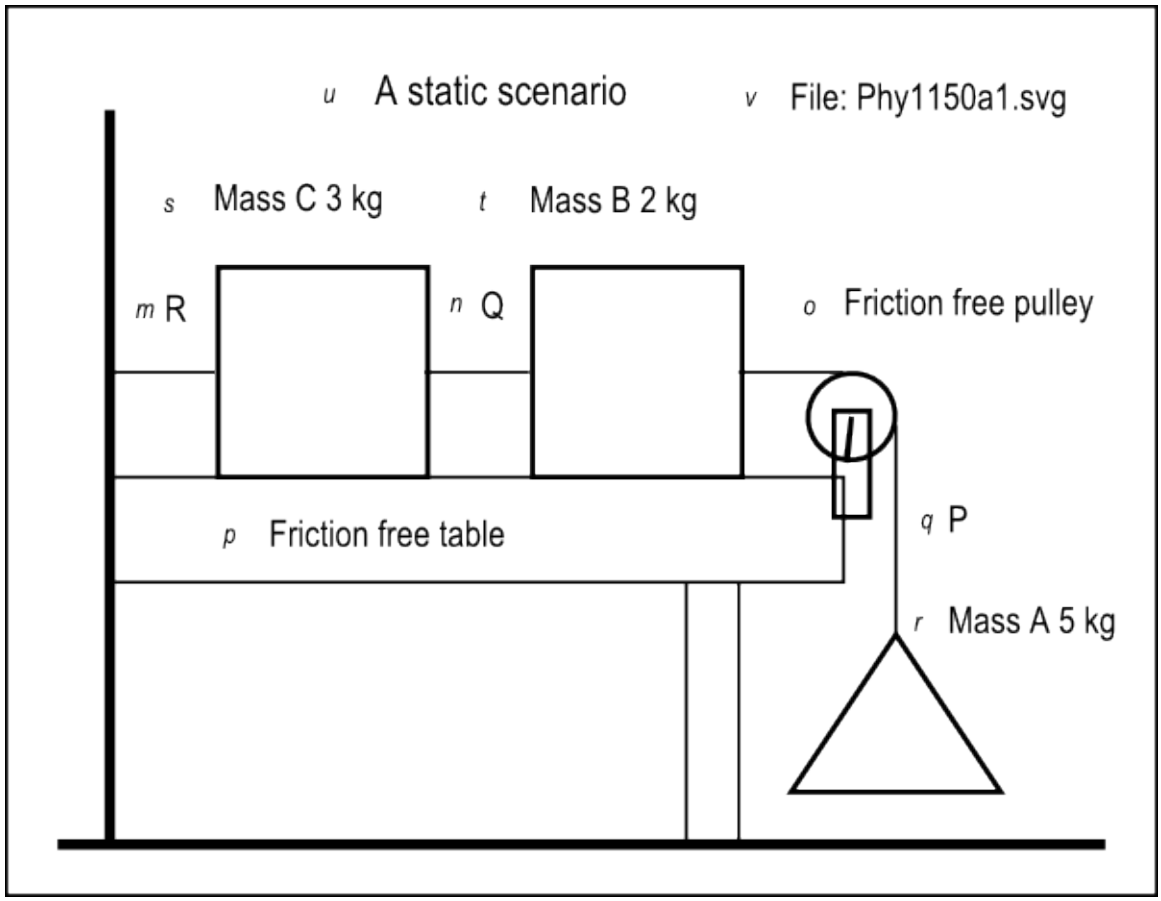
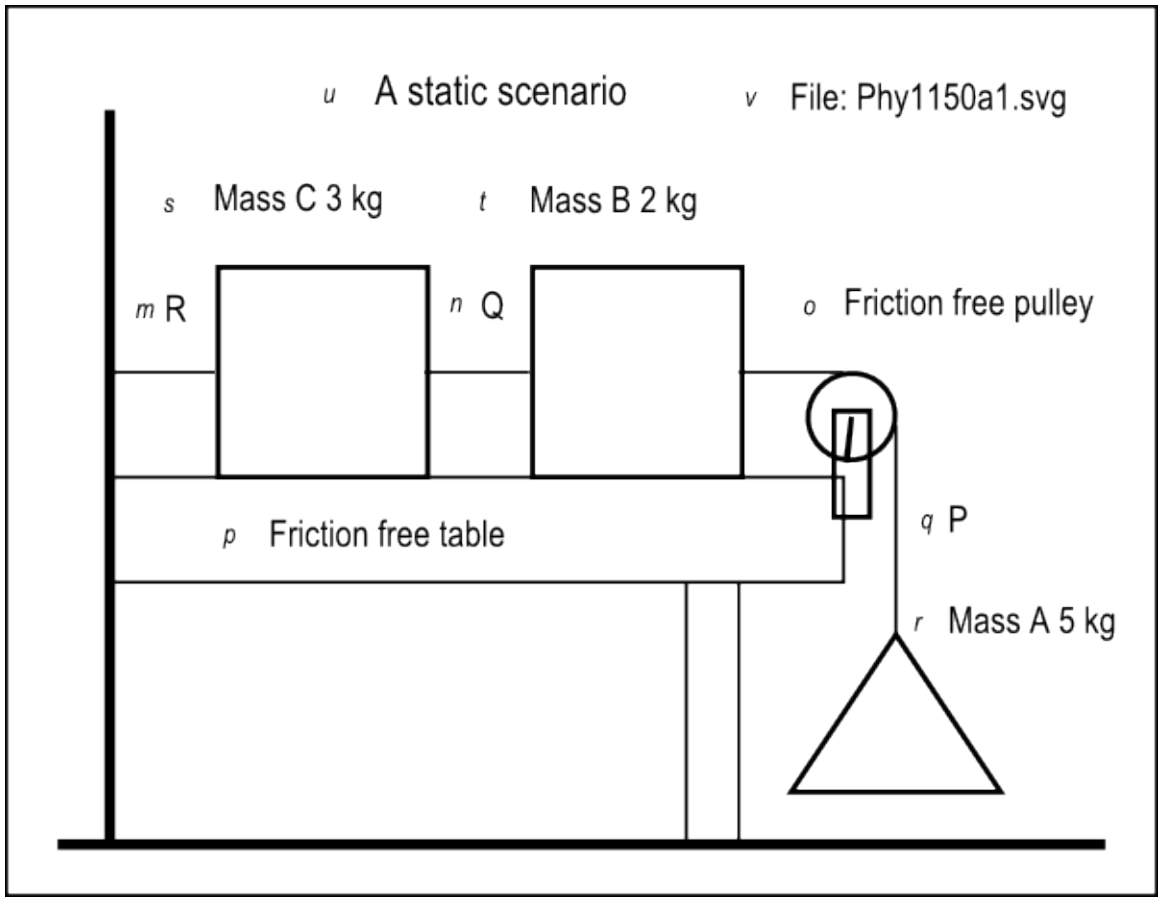


Table 18.3

Figure 4 (p. 547) shows a non-mirror-image version of the same image.


u A static scenario
v File: Phy1150a1.svg
s Mass C 3 kg
t Mass B 2 kg
m R
n Q
o Friction free pulley
p Friction free table
q P
r Mass A 5 kg


u A static scenario
v File: Phy1150a1.svg
s Mass C 3 kg
t Mass B 2 kg
m R
n Q
o Friction free pulley
p Friction free table
q P
r Mass A 5 kg

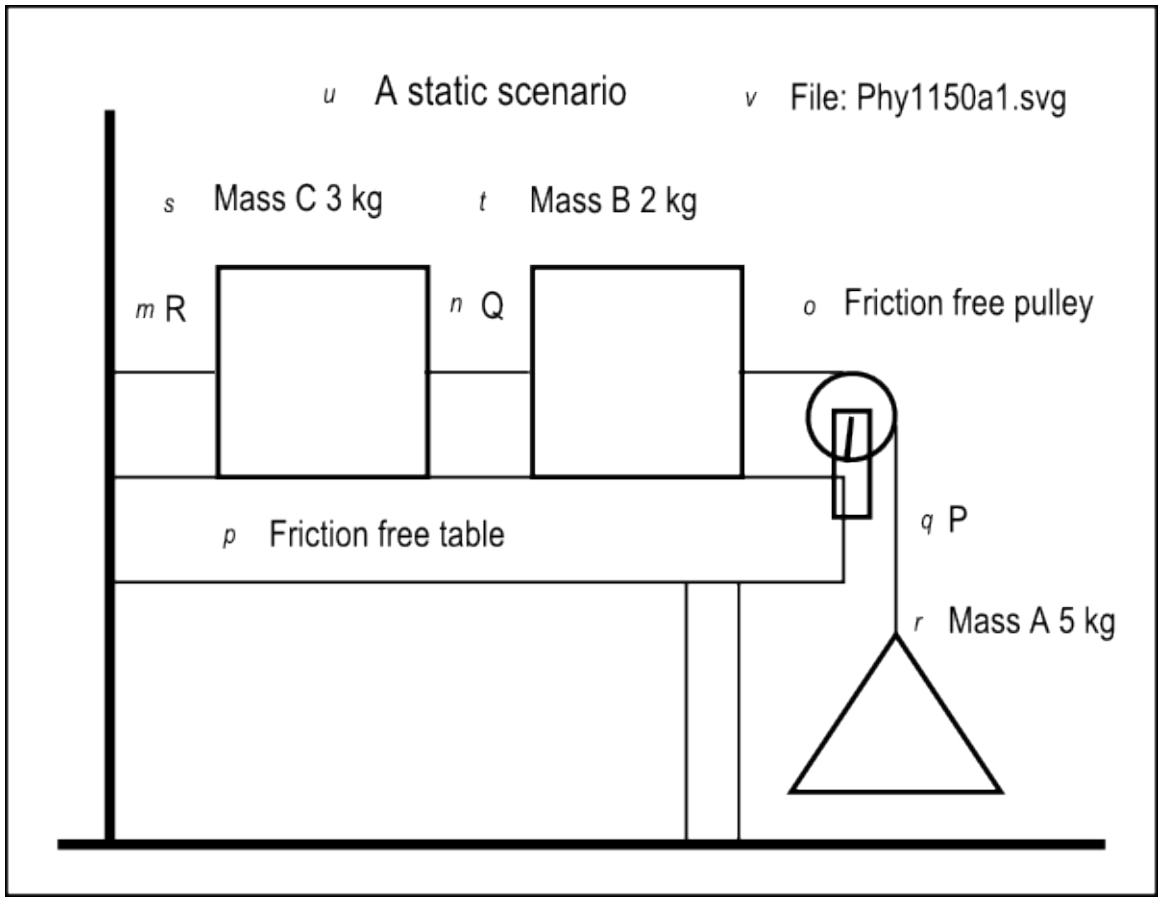

u A static scenario
v File: Phy1150a1.svg
s Mass C 3 kg
t Mass B 2 kg
m R
n Q
o Friction free pulley
p Friction free table
q P
r Mass A 5 kg

Figure 4 . Non-mirror image of the image from the file named Phy1150a1.svg.

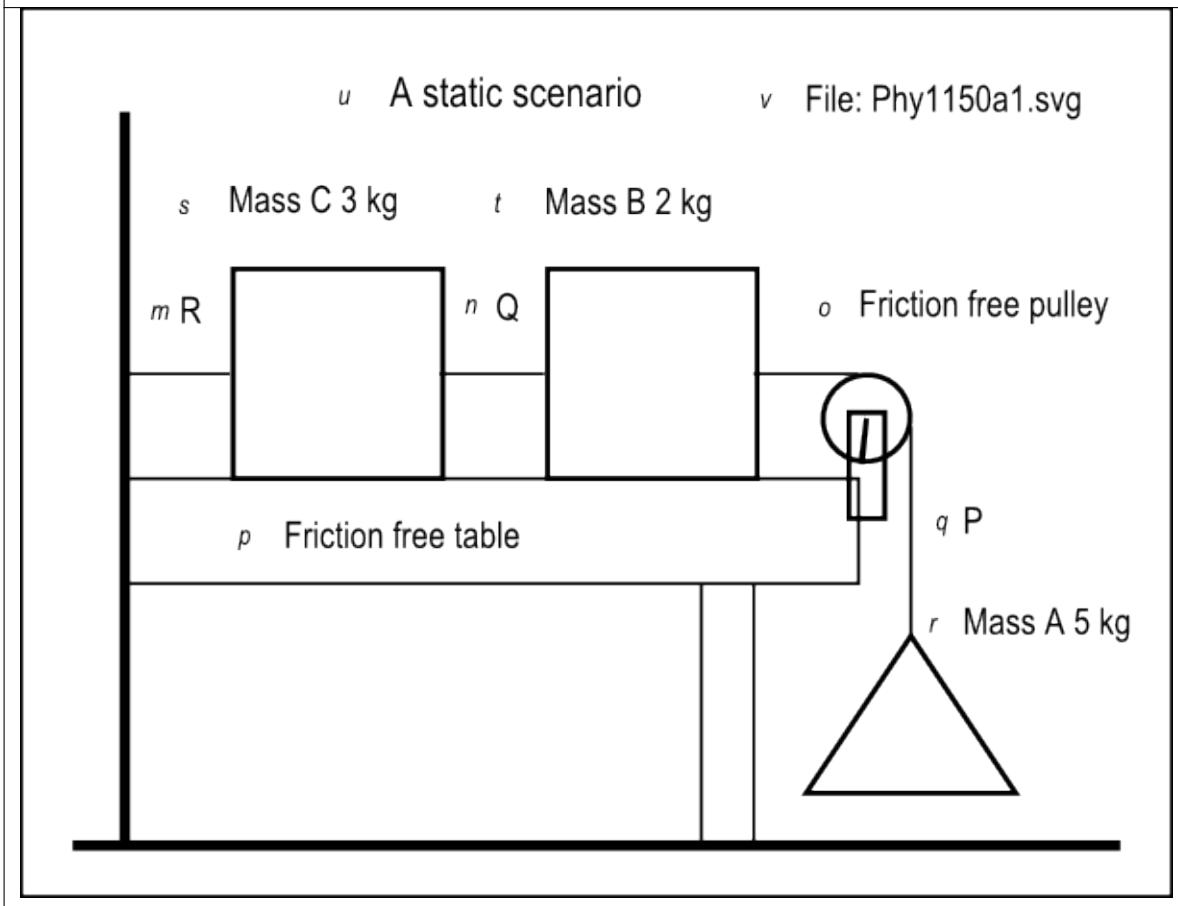


Table 18.4

Figure 5 (p. 547) shows the key-value pairs that go with the image in the file named Phy1150a1.svg

Figure 5 . Key-value pairs for the image in the file named Phy1150a1.svg.

```
    m: R
n: Q
o: Friction free pulley
p: Friction free table
q: P
r: Mass A 5 kg
s: Mass C 3 kg
t: Mass B 2 kg
u: A static scenario
v: File: Phy1150a1.svg
```

Table 18.5

The question

What are the tensions P, Q, and R in the individual cord segments when the system is in equilibrium and nothing is moving?

The answers

Mass A is in equilibrium because the upward force exerted by the cord attached to the top of Mass A is equal to the downward force of gravity that is exerted on Mass A.

Therefore, when the system is in equilibrium, the tension P is equal to the weight of Mass A, which is equal to the product of its mass and the acceleration of gravity.

P = 5*kg*9.81*m/s^2 = 49.05 newtons

The effect of the pulley

A single-wheel, frictionless, lightweight pulley as described here changes the direction of the cord, but does not change the tension P in the cord. The tension in the cord is the same on both sides of the pulley.

Support from the table

Mass B and Mass C are both in vertical equilibrium because their weight is being supported by the table.

Horizontal equilibrium

Mass B is in horizontal equilibrium because the force exerted on one side of the mass by tension Q is equal to the force exerted on the other side of the mass by tension P, which is 49.05 newtons.

Mass C is in horizontal equilibrium because the force exerted on one side of the mass by tension R is equal to the force exerted on the other side of the mass by tension Q, which as explained above, is 49.05 newtons.

The wall exerts a force that is equal in magnitude and opposite in direction to tension R in the cord that attaches Mass C to the wall.

Tensions are all the same

Therefore, the tensions in all three cords are the same and the force exerted by the wall supports the weight of Mass A hanging by the cord on the other end of the chain.

P = Q = R = 49.05 newtons

18.4.2 First dynamic scenario

Now assume that someone cuts the cord that attaches Mass A to Mass B. What happens to the tension in each cord? Which, if any of the masses move, and if they move, what is their acceleration?

We probably don't need to do any calculations to answer these questions. Life experience tells us that the tension in each cord immediately goes to zero when the cord holding up the weight is cut.

Movement

Mass B and Mass C remain in equilibrium with no horizontal forces acting on them and their weights being supported by the table. They don't move.

Mass A immediately begins a free fall toward the floor with an acceleration that is equal to the acceleration of gravity at 9.81 m/s^2. A short segment of cord trails out behind Mass A like an unopened parachute.

18.4.3 Second dynamic scenario

Now assume that we start over with the original static scenario and someone cuts the cord that attaches Mass A to Mass B. What happens to the tension in each cord? Which, if any of the masses move, and if they move, what is their acceleration?

This situation is a little more complicated and will probably require some calculations to sort out.

It seem obvious that the tensions labeled Q and R immediately go to zero, but the tension labeled P does not go to zero.

Movement

Mass C remains in equilibrium with no horizontal forces acting on it and its weight being supported by the table. It does not move.

However, Mass A starts falling toward the floor, dragging Mass B horizontally towards the pulley.

The driving force

The only force causing Mass A and Mass B to move is the weight of Mass A (49.05 newtons), which has not changed. However, that force is now trying to move a total of 7 kg instead of only 5 kg as in the first dynamic scenario above.

A force of 49.05 newtons is not sufficient to cause a mass of 7 kg to accelerate at 9.81 m/s^2. Instead, the acceleration of each mass is proportional to the force and inversely proportional to the total mass.

a = f/m = (49.05*newtons)/(7*kg)

Entering the rightmost expression into the Google search box and pressing Enter tells us that

a = 7.0 m / s^2

Thus, the acceleration of Mass A and Mass B is 7 m/s^2, a little less than the acceleration of gravity.

What is the value of tension P?

Tension P is exerting a horizontal force on the right side of Mass B that is causing that mass to accelerate at 7.0 m / s^2. The force required to achieve that acceleration on a mass of 2 kg is

P = m*a = 2 kg*7.0 m / s^2

Once again using the Google calculator, we learn that

P = 14 newtons

Thus, although the tension at P did not go to zero when the cord was cut at Q, the resulting tension in the cord at P was substantially reduced relative to the tension at P while the system was in equilibrium.

18.4.4 Third dynamic scenario

Now assume that we start over with the original static scenario and someone cuts the cord that attaches Mass C to the wall. What happens to the tension in each cord? Which, if any of the masses move, and if they move, what is their acceleration?

This situation is considerably more complicated and will definitely require some calculations. The tension labeled R goes to zero immediately, but the tensions labeled P and Q do not go to zero.

Movement

None of the masses remain in equilibrium. Mass A starts falling toward the floor, dragging Mass B and Mass C horizontally toward the pulley.

The driving force

Once again, the only force causing all three masses to move is the weight of Mass A (49.05 newtons), which has not changed. However, that force is now trying to move a total of 10 kg instead of only 5 kg or 7 kg as in the two scenarios described above.

A force of 49.05 newtons is not sufficient to cause a mass of 10 kg to accelerate at 9.81 m/s^2. Instead, the acceleration of each mass is proportional to the force and inversely proportional to the total mass.

a = f/m = (49.05*newtons)/(10*kg)

Entering the rightmost expression into the Google search box and pressing Enter tells us that

a = 4.9 m / s^2

Half the acceleration of gravity

Note that this is half the acceleration of gravity. This makes sense, because the force attributable to gravitational attraction acting on a mass of 5 kg is being applied to 10 kg of mass. It follows, therefore, that the acceleration that is achieved will be only half the acceleration of gravity.

What is the value of tension P?

Tension P is exerting a horizontal force on the right side of Mass B that is causing that mass and Mass C to accelerate at 4.9 m / s^2. The force required to achieve that acceleration on a mass of 5 kg is

P = m*a = 5*kg*4.9 m / s^2

Once again using the Google calculator, we learn that

P = 24.5 newtons

Mass B and Mass C together represent 50-percent of the total mass, and tension P is 50-percent of the force applied to the total mass.

What is the value of tension Q?

Tension Q is exerting a horizontal force on the right side of Mass C that is causing Mass C to accelerate at 4.9 m / s^2. The force required to achieve that acceleration on a mass of 3 kg is

Q = m*a = 3*kg*4.9 m / s^2
Once again using the Google calculator, we learn that
Q = 14.7 newtons
Mass C is 30-percent of the total mass, and tension Q is 30-percent of the force applied to the total mass.

18.5 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

18.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

18.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force and Motion – Units of Force
- File: Phy1150.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · newton
  - · poundal
  - · dyne
  - · weight
  - · weightlessness

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 19

Force and Motion – Momentum, Impulse, and Conservation of Momentum[1]

19.1 Table of Contents

- Preface (p. 554)
  - General (p. 554)
  - Prerequisites (p. 554)
  - Viewing tip (p. 555)
    - Figures (p. 555)
    - Listings (p. 555)
  - Supplemental material (p. 555)
- Discussion (p. 555)
  - Newton's cradle (p. 555)
  - Momentum (p. 556)
  - Impulse (p. 557)
  - The physics of collisions (p. 558)
  - Action and reaction (p. 560)
  - Conservation of momentum (p. 562)
  - Center of mass (p. 563)
    - Location of the center of mass (p. 563)
    - Motion of the center of mass (p. 564)
- Example scenarios (p. 564)
  - Momentum examples (p. 564)
    - A sprinter (p. 564)
    - A truck (p. 565)
    - Change in momentum due to change in speed and direction (p. 565)
    - Change in momentum due to change in speed only (p. 567)
    - Change in momentum due to change in direction only (p. 568)
  - Impulse examples (p. 569)

[1] This content is available online at <http://cnx.org/content/m38234/1.4/>.

    - Pushing a wagon part 1 (p. 569)
    - Pushing a wagon part 2 (p. 569)
    - Pushing a wagon part 3 (p. 570)
  - Action and reaction example (p. 570)
  - Conservation of momentum example (p. 571)
  - Center of mass examples (p. 571)
    - Two objects (p. 571)
    - Three objects (p. 574)
- Do the calculations (p. 575)
- Resources (p. 575)
- Miscellaneous (p. 575)

19.2 Preface

19.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains momentum, impulse, and the conservation of momentum in a format that is accessible to blind students.

19.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).

[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/

- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

19.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

19.2.3.1 Figures

- Figure 1 (p. 567) . Solution results.
- Figure 2 (p. 568) . Change in speed only.
- Figure 3 (p. 569) . Change in direction only.
- Figure 4 (p. 574) . Center of mass for two objects.
- Figure 5 (p. 575) . Center of mass for three objects.

19.2.3.2 Listings

- Listing 1 (p. 566) . Solution script.
- Listing 2 (p. 573) . Center of mass for two objects.
- Listing 3 (p. 574) . Change to add a third object.

19.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

19.3 Discussion

This module will describe and discuss some scenarios which, never having been seen, may be hard for a blind student to imagine. Some of those scenarios can be difficult to believe even when you can see them.

19.3.1 Newton's cradle

The behavior of Newton's cradle is somewhat difficult to believe even when you see it in operation. Newton's cradle is a gadget that is often found in novelty shops. It typically consists of an open wood or metal frame with about five steel balls suspended by strings on parallel beams that run from one end to the other along the top.

Several steel balls in a row

Each ball is suspended by two strings so that the ball forms the lower vertex of a triangle and the two equal-length strings form the sides of the triangle. Each string is attached at the upper end to a beam. The purpose of suspending each ball by two strings instead of suspending them on a single string is to cause all of the balls to swing back and forth along the same straight line.

A collision

[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

When the system is in equilibrium, the balls are lined up in a row and each ball barely touches the one next to it. If you pull one of the balls at the end back and then release it, allowing it to swing down, it will strike its neighbor on the downswing.

Only the ball on the other end appears to move

Surprisingly, the neighboring ball doesn't appear to move when struck, nor does its neighbor, nor does that neighbor's neighbor. The only ball that appears to move is the ball at the far end of the line. That ball will be sent off in an upswing.

The process is reversed

When that ball reaches the top of its upswing, it will reverse direction, swing back down, and collide with its neighbor. This causes the ball that was used to start the process to be sent off in an upswing.

The process continues

Left alone, this process will continue until all of the energy in the system has been dissipated, which can be many minutes or even hours later.

Newton's cradle illustrates the conservation of momentum. You will find an interesting article that explains some of the technical details at http://en.wikipedia.org/wiki/Newton%27s_cradle. [13]

19.3.2 Momentum

You learned in an earlier module that momentum is the product of the mass of an object and the velocity of the object. Because velocity is a vector quantity, momentum is also a vector quantity. The direction of the momentum vector is the same as the direction of the underlying velocity vector.

A common symbol for momentum is p

A common symbol for momentum is **p** . Momentum is a derived item in the SI system of units. In the SI system, momentum is defined as kg*m/s.

In equation form , therefore:

- momentum = mass * velocity, or
- p = m * v in units of kg*m/s

Facts worth remembering – Momentum

- momentum = mass * velocity, or
- p = m * v

The units of momentum are kg*m/s

Mass in action

Momentum can also be thought of as "mass in motion." Since all objects have mass, if an object is moving, its mass is in motion. Therefore, the object has momentum.

Proportional to both mass and velocity

From the above equation (p. 556) , it can be seen that an object can have a large momentum if either its mass or its velocity is large. Both variables are of equal importance in determining the momentum of an object.

A car and a tennis ball

Consider the case of a car and a tennis ball rolling down the street at the same speed. Because the car has greater mass, it has more momentum than the tennis ball. However, if the car stops and the tennis ball continues to roll, the tennis ball then has the greater momentum.

Momentum is zero at rest

The momentum of any object at rest is zero. Objects at rest do not have momentum because their mass is not in motion.

The quantity of momentum

The quantity of momentum possessed by an object depends on:

[13] http://en.wikipedia.org/wiki/Newton's_cradle

- How much mass is moving, and
- How fast the mass is moving.

For example, a small mass moving very fast can have the same momentum as a large mass moving slowly. You sometimes hear about the major damage that a very small piece of space junk moving at a very high speed could do if it were to strike the International Space Station.

A bullet shot from a firearm has a very small mass, but it has a very high velocity. Consequently, it probably has more momentum than a baseball pitched from second base to home plate, even though the baseball has much more mass.

What happened to the dinosaurs?

Similarly, you may have heard that an asteroid with a mass that was small relative to the mass of the earth but with an extremely high velocity led to the extinction of the dinosaurs about 160 million years ago when it collided with the earth in the Gulf of Mexico.

19.3.3 Impulse

Momentum can be changed by a force

An object with momentum can be stopped if a force is applied against it for a given amount of time. For example, when a car approaches a red traffic light, the driver applies the brakes. The friction of the tires on the pavement applies a force to the car, which eventually reduces the car's velocity to zero. When the velocity goes to zero, the momentum also goes to zero.

Therefore, the momentum of an object can be changed by applying a force to the object over a given period of time.

Unbalanced forces cause acceleration

As you learned in earlier modules, an unbalanced force always accelerates an object, either causing the object to speed up or causing the object to slow down. Either way, the application of an unbalanced force to an object will change the velocity of the object. When the velocity of the object is changed, the momentum of the object is changed as well.

The impulse

Let's use what we know from Newton's second law to derive a concept known as **impulse** .

The product of mass and acceleration

You learned in an earlier module that force is equal to the product of mass and acceleration:
F = m * a
where

- F represents Force
- m represents mass
- a represents acceleration

The rate of change of velocity

You also learned that acceleration is the time rate of change of velocity, or
a = (v2 - v1)/t
where

- v2 - v1 indicates a change in velocity during a time interval given by t.

Combine and rearrange equations

Therefore, by substitution we can write:
F = m * (v2 - v1)/t
Multiplying both sides by t gives
F * t = m * (v2 - v1), or
F*t = m*v2 - m*v1
where

- F*t represents impulse
- m*v2 and m*v1 each represent momentum
- m*v2 - m*v1 represents a change in momentum

A change in momentum

At this point, you should recognize that the product of mass and a change in velocity is a change in momentum.

In physics, the product of force and time is given the name **impulse** . It follows, therefore, that

impulse = F*t = change in momentum

This equation is often referred to as the *impulse-momentum change equation* .

Facts worth remembering – The *impulse-momentum change equation*

impulse = F*t = change in momentum

F*t = m*v2 - m*v1

where

- F*t represents impulse
- m*v2 and m*v1 each represent momentum
- m*v2 - m*v1 represents a change in momentum

19.3.4 The physics of collisions

One area of physics where momentum plays a large part is the physics of collisions. Momentum and possible changes in momentum are involved in the interactions among all moving objects, even when the changes in momentum are not visually obvious.

For example, the passage of a moving iron object through the magnetic field of a moving magnet will cause changes in the momentum of both the iron object and the magnet, but the result may not be obvious.

Collisions may be more obvious

The type of interaction that we call a collision may be the type of interaction that is the most familiar to us. Collisions between objects happen all the time. This module will discuss several examples that involve collisions.

Collisions in everyday life

I may be wrong, but I suspect that as a blind student, you may be more attuned to collisions in everyday life than your fellow sighted students. For example, each time the end of your cane touches an object, a collision has occurred. Regardless of how light the touch, you probably recognize that collision and take appropriate action.

The impulse-momentum change equation

A law associated with the impulse-momentum change equation (p. 558) may be expressed in the following way (see http://www.physicsclassroom.com/Class/momentum/U4l1b.cfm [14]):

In a collision, an object experiences a force for a specific amount of time that results in a change in momentum. The result of the force acting for the given amount of time is that the object's mass either speeds up or slows down (or changes direction). The impulse experienced by the object equals the change in momentum of the object.

In equation form,

F * t = m * (v2 - v1)

All objects in a collision experience an impulse

When a collision occurs, each object involved in the collision experiences an impulse. The impulse is equal to the change in momentum.

A hypothetical collision with a punching bag

[14] http://www.physicsclassroom.com/Class/momentum/U4l1b.cfm

In gyms where students practice boxing, there is usually a large object called a punching bag. Some punching bags are large cylindrical containers made of leather or some other pliable material filled with something like sand. Typically, they hang from the ceiling or from an overhead beam.

A void below the punching bag

Often, the punching bag is not attached to the floor and the bottom of the punching bag is often several feet above the floor. This leaves a void between the bottom of the punching bag and the floor.

A walk through the gym

Assume that you, as a blind student are walking through a gym that contains a punching bag of the type described above. Because of the void, you may not detect the presence of the punching bag with your cane and you might walk directly into the punching bag.

A force over a short period of time

Unlike a solid wall, the punching bag probably wouldn't stop your forward progress instantly. Instead, there would probably be a period of time during which the bag would exert a force on you and you would exert an equal but opposite force on the bag.

Initially, both you and the heavy punching bag would probably move in the direction that you are walking. Shortly thereafter, your forward velocity would probably go to zero.

Then the punching bag would probably push you backwards giving you a negative velocity as the bag swings like a pendulum. However the collision plays itself out, you would experience an impulse that would change your momentum and the momentum of the punching bag as well.

An ideal assumption

If we assume (ideally) that the force exerted on you by the bag is constant at ten newtons for a total of five seconds, the impulse would be equal to 50*N*s.

(In reality, the force would vary during the time interval and the computation of the impulse would be somewhat more complicated. For a time-varying force, the impulse is the area under a graph of force versus time. The average of the force over the given time interval cal also be used to compute the impulse when the force is not constant.)

The effect of the time interval

The impulse, or the change in momentum, is equal to the product of the force and the time. Therefore, a large force over a short period of time can produce the same change in momentum as is produced by a smaller force over a longer period of time.

The effect on the object experiencing the change in momentum can be quite different for the two cases. This is particularly true when the human body experiences a change in momentum. Among other things, air bags in cars are designed to lengthen the time over which the change in momentum occurs for a human body involved in a collision. In addition, the air bag can also spread the force over a larger portion of the body, which can reduce the damage to the body caused by the change in momentum.

Another representation of units

In any event, if we substitute the units for the newton in the above expression, we get

50*N*s = 50*(kg*m/s^2)*s = 50*kg*m/s

Two ways to represent the units of an impulse

Therefore, we can represent the units for an impulse as either N*s or as kg*m/s. You should recognize the latter as the product of mass and velocity, which are the same units as momentum.

In a collision, the impulse experienced by an object is always equal to the change in momentum experienced by the object.

A Super Ball

Consider the case of a *Super Ball* bouncing off of a solid concrete floor. *(Super Ball is a brand name and registered trademark of Wham-O Incorporated.)*

The main characteristic of a Super Ball is its ability to bounce almost as high as the height from which it was released when dropped onto a solid surface. As a result, when the Super Ball collides with the floor, it demonstrates a strong *rebound effect* . The greater the rebound effect, the greater will be the acceleration, momentum change, and impulse in a collision.

A rebound collision

A rebound collision involves a change in direction in addition to a change in speed. Because the direction changes, there is a large velocity change even if the magnitude of the velocity stays the same.

Elastic collision

Collisions in which objects rebound with the same speed (and thus, the same momentum and kinetic energy) as they had prior to the collision are known as *elastic collisions* .

Stated differently, an elastic collision is a collision between two bodies in which the total kinetic energy of the two bodies after the collision is equal to their total kinetic energy before the collision. Elastic collisions occur only if there is no net conversion of kinetic energy into other forms.

Facts worth remembering – Elastic collision

An elastic collision is a collision between two bodies in which the total kinetic energy of the two bodies after the collision is equal to their total kinetic energy before the collision.

Energy conversion

A future module will explain kinetic energy and other forms of energy, such as potential energy in detail. Briefly, kinetic energy is energy possessed by a moving object simply because it is moving. For example, it hurts more to be hit by a fast moving baseball than to be hit by a slow moving baseball, simply because the collision with a fast moving baseball imparts more energy into your body. In other words, the kinetic energy possessed by the fast-moving baseball is converted into pain in your body.

Characteristics of an elastic collision

An elastic collision is typically characterized by a large velocity change, a large momentum change, a large impulse, and a large force.

While the case of a Super Ball bouncing on a solid concrete floor isn't a perfect elastic collision, it comes very close. The amount of kinetic energy that is converted into other forms of energy during each bounce is very low, and the ball will continue bouncing for a very long time with the height of each bounce being almost as high as the height of the previous bounce.

19.3.5 Action and reaction

Two objects collide when they make contact while one or both are moving. As is the case with all interactions involving two or more moving objects, a collision results in a force being applied to all of the objects involved in the collision. The behavior of such collisions is governed by Newton's laws of motion.

Newton's third law

One paraphrased version of Newton's third law (see http://www.physicsclassroom.com/Class/momentum/u4l2a.cfm [15]) reads:

... in every interaction, there is a pair of forces acting on the two interacting objects. The size of the force on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs - equal and opposite action-reaction force pairs.

According to Newton's third law, when two objects are involved in a collision, the two objects experience forces that are equal in magnitude and opposite in direction.

In most cases, the collision will cause one object to gain momentum and the other object to lose momentum. This, in turn, will cause one object to speed up and the other object to slow down.

Railroad cars

As a child, I grew up living next to a very large railroad yard. I was accustomed to hearing the sounds of controlled collisions between railroad cars.

How railroad couples work

The devices that hold railroad cars together in a train are activated by a controlled collision. While one railroad car is either standing still, or moving at a slow speed, another railroad car purposely collides with the first car. When that happens, the two railroad cars become fastened together (coupled).

[15] http://www.physicsclassroom.com/Class/momentum/u4l2a.cfm

The distribution of momentum

Prior to the collision, each car possesses a given amount of momentum, which can be zero for a car at rest or non-zero for a car in motion. After the collision, the momentum of each car will have changed.

The conservation of momentum – a preview

As I will explain later, (except for the conversion of some kinetic energy into other forms, such as sound energy) the two cars coupled together will possess the total amount of momentum that was possessed by the individual cars prior to the collision. This typically means that one car speeds up and the other car slows down.

A change in momentum

During the time frame surrounding the instant of the collision, each car experiences a change in momentum. With sufficiently accurate measuring devices, you could measure and record the rate of change of momentum during that short time frame.

Accelerations are not necessarily equal

Although the forces experienced by the objects are equal in magnitude, the changes in velocity (accelerations) experienced by the two objects are not necessarily equal.

The acceleration is equal to...

As we learned in an earlier module, the acceleration experienced by an object is proportional to the applied force and inversely proportional to the mass of the object. Therefore, different masses experiencing the same magnitude of force will experience different magnitudes of acceleration.

Kicking a lightweight aluminum can

Consider what happens when someone leaves an empty lightweight aluminum drink can on the floor and you accidently kick it with your bare foot while walking briskly across the room. The can exerts a force on your foot, which fortunately doesn't hurt too much because of the small mass of the can. Your foot exerts an equal and opposite force on the can that probably sends it flying across the room.

The velocity of the can changes by a large amount, going from zero to a velocity that sends it flying. Thus, the acceleration of the can is large.

The velocity of your foot, on the other hand, changes very little at the moment of impact as a result of its large mass (although your muscular reaction might be such as to slow the foot down shortly thereafter). Thus, the magnitude of the acceleration of your foot due solely to the collision is small.

Kicking a heavy can

Now consider the same scenario except that this time, the can contains your brother's collection of rocks. In this case, the acceleration of the can due to the collision with your foot would probably be quite small, and the (negative) acceleration of your foot due to the collision might be much larger than with the lightweight can.

The can of rocks probably wouldn't go flying across the room, but might simply turn over, spilling the rocks on the floor.

The negative acceleration experienced by your foot might result in some broken toes.

Once again, the forces would be equal in magnitude and opposite in direction, but the acceleration of each object would be inversely proportional to the mass of the object.

Equal and opposite forces

When you kick the can, your foot would exert a force on the can in the direction of motion of your foot. The can would exert a force on your foot that is equal in magnitude but opposite in direction.

The forward force on the can would cause it to gain velocity in the direction that your foot is moving. The backward force on your foot would cause your foot to slow down.

Equal acceleration

In the unlikely event that the mass of the can is exactly equal to the mass of your foot, the negative acceleration experienced by your foot would be equal to the positive acceleration exerted on the can. That is the one case where not only the magnitudes of the forces, but also the magnitudes of the accelerations would be equal.

For collisions between equal-mass objects, each object experiences the same acceleration.

19.3.6 Conservation of momentum

When two objects interact in an *isolated system* , the total momentum of the two objects before the interaction is equal to the total momentum of the two objects after the interaction. The momentum lost by one object is gained by the other object.

Facts worth remembering – Isolated system

An isolated system is a system that is free from the influence of a net external force that alters the momentum of the system.

The total momentum of a collection of objects in a system is conserved. The total amount of momentum is constant.

Many forms of interaction are possible

There are many ways that two objects can interact. For example, when a car pulls away from a stoplight, it gains momentum by exerting frictional forces on the surface of the earth. When that happens, the earth loses an equal amount of momentum. (Fortunately, this represents a very small fraction of the earth's momentum, so the loss of momentum isn't noticeable.)

When the driver applies the brakes and stops the car at the next stop light (by exerting frictional forces on the surface of the earth), the car loses all of its momentum and the earth gains an equal amount of momentum. (Once again, this represents a very small fraction of the earth's momentum, so the gain of momentum isn't noticeable.)

Railroad cars and controlled collisions

Earlier in this module, I described a process where controlled collisions are used to couple railroad cars together.

While one railroad car is either standing still, or moving at a slow speed, another railroad car purposely collides with that car. When that happens, the two railroad cars become fastened together (coupled).

Distribution of momentum

Prior to the collision, each car possesses a given amount of momentum, which can be zero for a car at rest or non-zero for a car in motion. After the collision, the momentum of each car will have changed.

Except for the conversion of some kinetic energy into other forms, such as sound energy, the two cars coupled together will possess the total amount of momentum that was possessed by the individual cars prior to the collision.

This typically means that one car speeds up and the other car slows down. During the time frame surrounding the instant of the collision, each car experiences a change in momentum.

A logical proof of the conservation of momentum

We already know that when two objects interact, each object exerts a force on the other object. The two forces are equal in magnitude and opposite in direction.

We can probably agree that when two objects interact, the amount of time that object-A interacts with object-B is the same as the amount of time that object-B interacts with object-A.

Therefore, if the forces are equal and opposite, and the times are the same, we can write

Fa*t = -Fb*t

where

- Fa and Fb represent forces exerted on objects A and B respectively.
- t represents the time during which the objects interact.

Impulses acting on objects A and B

You should recognize these terms as the impulses acting on objects A and B. Therefore, the impulses acting on the two objects are equal and opposite.

Impulses are equal and object

You learned earlier that the impulse acting on an object is equal to the change in momentum of the object. If the impulses acting on the two objects are equal and opposite, then the change in momentum experienced by the two objects must be equal and opposite.

We can **express this in equation form** as
ma*(va2 - va1) = -mb*(vb2 - vb1)
where

- ma and mb are the masses of objects A and B respectively
- (va2 - va1) is the change in velocity of object-A
- (vb2 - vb1) is the change in velocity of object-B

The law of conservation of momentum

This equation is a statement of the law of conservation of momentum. The change in momentum experienced by object-A is equal to and opposite of the change in momentum experienced by object-B.

Stated differently, the momentum lost by object-A is gained by object-B, or vice-versa.

That being the case, the total momentum possessed by the system containing object-A and object-B remains unchanged by the interaction of the two objects. The total momentum of the system is conserved.

Facts worth remembering – The law of conservation of momentum

For a collision occurring between object-A and object-B in an isolated system, the total momentum of the two objects before the collision is equal to the total momentum of the two objects after the collision. (By total momentum we mean the vector sum of the individual momenta of the objects.)

Bowling ball and bowling pins

Bowling is a game where the players roll a heavy ball down a long smooth wooden platform in an attempt to knock down ten heavy wooden objects (bowling pins) arranged in a triangle at the end of the platform. The bowling pins are shaped something like a flower vase that is larger at the bottom than at the top. Thus, each bowling pin has a low center of gravity.

When the ball strikes the cluster of bowling pins, there are eleven objects involved in a conservation of momentum process. The momentum that is lost by the bowling ball is distributed among the ten bowling pins, but the momentum is probably not distributed evenly among the bowling pins. There is a lot of chaos at the time of impact with the bowling ball colliding with the pins, pins colliding with other pins, etc. Some of the energy is also converted to sound.

19.3.7 Center of mass

Interactions between parts of a system transfer momentum between the parts, but do not change the total momentum of the system. We can define a point called the center of mass that serves as an average location of a system of parts.

The center of mass need not necessarily be at a location that is either in or on one of the parts. For example, the center of mass of a pair of heavy rods connected at one end so as to form a "V" shape is somewhere in space between the two rods.

Having determined the center of mass for a system, we can treat the mass of the system as if it were all concentrated at the center of mass.

19.3.7.1 Location of the center of mass

For a system composed of two masses, the center of mass lies somewhere on a line between the two masses. The center of mass is a weighted average of the positions of the two masses.

Facts worth remembering – Center of mass for two objects

For a pair of masses located at two points along the x-axis, we can write

xcm = (m1*x1/M) + (m2*x2/M)

where

- xcm is the x-coordinate of the center of mass
- m1 and m2 are the values of the two masses
- x1 and x2 are the locations of the two masses
- M is the sum of m1 and m2

Multiple masses in three dimensions

When we have multiple masses in three dimensions, the definition of the center of mass is somewhat more complicated.

Facts worth remembering – Center of mass for many objects

Vector form:

rcm = sum over all i(mi*ri / M)

Component form:

xcm = sum over all i(mi*xi / M)

ycm = sum over all i(mi*yi / M)

zcm = sum over all i(mi*zi / M)

where

- **Vector form**
  - rcm is a position vector describing the location of the center of mass
  - ri are position vectors describing the locations of all the masses
  - mi are masses for i=1, i=2, etc.
- **Component form**
  - xcm, ycm, and zcm are the locations of the center of mass along 3-dimensional axes.
  - mi are masses for i=1, i=2, etc.
  - xi, yi, and zi are the locations of the masses along 3-dimensional axes for i=1, i=2, etc.
  - M is the sum of all of the masses

19.3.7.2 Motion of the center of mass

It can be shown that in an isolated system, the center of mass must move with constant velocity regardless of the motions of the individual particles.

It can be shown that in a non-isolated system, if a net external force acts on a system, the center of mass does not move with constant velocity. Instead, it moves as if all the mass were concentrated there into a fictitious point particle with all the external forces acting on that point.

19.4 Example scenarios

This section contains explanations and computations involving momentum, impulse, action and reaction, and the conservation of momentum.

19.4.1 Momentum examples

This section contains several examples involving momentum

19.4.1.1 A sprinter

Use the Google calculator to compute the momentum of a 70-kg sprinter running 30 m/s at 0 degrees.

Answer: 2100 kg*m/s at 0 degrees

19.4.1.2 A truck

Use the Google calculator to compute the momentum in kg*m/s of a 2205-lb truck traveling 33.6 miles per hour at 0 degrees when the changes listed below occur:

1. Initial momentum
2. Momentum when velocity is doubled
3. Momentum at initial velocity when mass is doubled
4. Momentum when both velocity and mass are doubled

Answers:

1. Enter the following into the Google calculator and press Enter to produce the results shown.

- convert 2205 lb to kg = 1000.17118 kilograms
- convert 33.6 mph to m/s = 15.020544 meters / second
- 1000 kg*15 m/s = 15000 m kg / s

Therefore, the initial momentum = 15000*kg*m/s at 0 degrees

2. 1000 kg*30 m/s = 30000 m kg / s at 0 degrees
3. 2000 kg*15 m/s = 30000 m kg / s at 0 degrees
4. 2000 kg*30 m/s = 60000 m kg / s at 0 degrees

19.4.1.3 Change in momentum due to change in speed and direction

A car with a weight of 10000 newtons is moving in a direction of 90 degrees at 40 m/s. After going around a curve in the road, the car is moving in a direction of 0 degrees at 20 m/s. What is the change in momentum of the car?

Solution:

While this problem could be solved using the Google calculator, because of the number of steps involved, JavaScript is probably a better approach.

The solution script for this problem is shown in Listing 1 (p. 566) .

Listing 1 . Solution script.

```
    <!---------------- File JavaScript01.html ---------------------->
<html><body>
<script language="JavaScript1.3">

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle

document.write("Start Script </br>");

var weight = 10000//N
var g = 9.8// m/s^2
//Find the mass of the car
var mass = weight/g;// kg


var ang1 = 90;//initial angle in degrees
var ang2 = 0; //final angle in degrees

var speed1 = 40;//initial speed in m/s
var speed2 = 20;//final speed in m/s

var ang1r = ang1*Math.PI/180;//initial angle in radians
var ang2r = ang2*Math.PI/180;//final angle in radians

//Remember, momentum is a vector quantity and momenta must
// be added and subtracted using vector arithmetic.

//Compute the components of the change in momentum.
var P1x = mass * speed1 * Math.cos(ang1r);
var P1y = mass * speed1 * Math.sin(ang1r);
var P2x = mass * speed2 * Math.cos(ang2r);
var P2y = mass * speed2 * Math.sin(ang2r);
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Table 19.1

The comments in Listing 1 (p. 566) explain the steps involved in finding the solution.

The output produced by Listing 1 (p. 566) is shown in Figure 1 (p. 567) with the magnitude and angle of the vector that describes the change of momentum at the end.

Figure 1 . Solution results.

```
   Start Script
The givens.
weight = 10000 kg
speed1 = 40 m/s
angle 1 = 90 degrees
speed2 = 20 m/s
angle 2 = 0 degrees
Computed mass.
mass = 1020 kg
Components of momentum vectors.
P1x = 0
P1y = 40816
P2x = 20408
P2y = 0
Components of momentum change vectors.
deltaPx = 20408
deltaPy = -40816
Magnitude and angle of change vector.
deltaPm = 45634 m kg/s
deltaPa = -63 degrees
End Script
```

Table 19.2

I find it interesting that the magnitude of the change in momentum is greater than the magnitude of either the initial or final momentum.

19.4.1.4 Change in momentum due to change in speed only

What happens if the car in the previous example changes speed but doesn't change direction.

Solution:

Change the given conditions in the script in Listing 1 to those shown at the beginning of Figure 2 (p. 568) .

Figure 2 . Change in speed only.

continued on next page

```
    Start Script
The givens.
weight = 10000 kg
speed1 = 40 m/s
angle 1 = 90 degrees
speed2 = 20 m/s
angle 2 = 90 degrees
Computed mass.
mass = 1020 kg
Components of momentum vectors.
P1x = 0
P1y = 40816
P2x = 0
P2y = 20408
Components of momentum change vectors.
deltaPx = -0
deltaPy = -20408
Magnitude and angle of change vector.
deltaPm = 20408 m kg/s
deltaPa = 270 degrees
End Script
```

Table 19.3

This change causes the car to slow down, but to continue in the same direction. As a result, the angle of the change in momentum is an angle that is opposite to the direction that the car is moving. The magnitude of the change in momentum depends entirely on the initial and final speeds.

19.4.1.5 Change in momentum due to change in direction only

What happens if the car in the previous example changes direction but doesn't change speed?

Solution:

Change the given conditions in the script in Listing 1 (p. 566) to those shown at the beginning of Figure 3 (p. 569) . This scenario simulates the car making a 10-degree turn to the right without changing speed.

Figure 3 . Change in direction only.

```
    Start Script
The givens.
weight = 10000 kg
speed1 = 40 m/s
angle 1 = 90 degrees
speed2 = 40 m/s
angle 2 = 80 degrees
Computed mass.
mass = 1020 kg
Components of momentum vectors.
P1x = 0
P1y = 40816
P2x = 7088
P2y = 40196
Components of momentum change vectors.
deltaPx = 7088
deltaPy = -620
Magnitude and angle of change vector.
deltaPm = 7115 m kg/s
deltaPa = -5 degrees
End Script
```

Table 19.4

19.4.2 Impulse examples

This section contains several examples involving the impulse.

19.4.2.1 Pushing a wagon part 1

1. What is the impulse experienced by pushing a 10-kg wagon that was initially at rest, with a constant force of 2 newtons for a period of 3 seconds?

Answer:

The impulse is given by the product of force and time. The mass of the wagon is superfluous for this question.

impulse = 2 N * 3 s = 6*N*s

19.4.2.2 Pushing a wagon part 2

2. What is the acceleration of the wagon in question 1 above?

Answer:

Now we do need to know the mass.

The most straightforward solution comes from the fact that we know the mass and that the force is uniform. Therefore,

F = m*a, or

a = F/m = 2N/10kg = 0.2 m/s^2

A more interesting solution comes from the fact that since

impulse = F*t, and
F = m*a, then
impulse = m*a*t, or
a = impulse/(m*t) = 6*N*s/(10*kg*3*s) = 0.2 m/s^2

19.4.2.3 Pushing a wagon part 3

3. What is the velocity at the end of the 3-second interval in question 1 above.
Answer:
The impulse is equal to the change in momentum, and the initial velocity is 0.
impulse = m*(v2 - v1) = m*v2, or
v2 = impulse/m = 6*N*s/(10*kg) = 0.6 m/s
We can check that answer by knowing that the acceleration is uniform at 0.3 m/s^2 for 3 s = 0.6 m/s.

19.4.3 Action and reaction example

A dip in the pool

You have a body mass of 70 kg. You are on your knees on an inflatable raft in a swimming pool. The raft has a mass of 1 kg. Your outstretched hands are about two meters from a safety rope that is strung across the pool.

You decide to launch yourself from the raft to catch the rope, exerting a force with a horizontal component of 5 newtons. (You assume that the vertical component of your launching force will take care of the downward pull of gravity, allowing you to fly in a parabolic arc to the rope.) Assuming uniform acceleration (which is unrealistic but we will assume that anyway), how long will it take you to fly through the air to reach the rope?

Answer:

The force that you exert on the raft will be equal and opposite to the force that the raft exerts on you. Therefore,

F = my*ay = -mr*ar
where

- my and mr are the mass of you and the raft respectively
- ay and ar are the accelerations experienced by you and the raft respectively

Therefore
ay = F/my = 5N/70kg = 0.07143 m/s^2 toward the rope
ar = -F/mr = -5N/1kg = -5.00000 m/s^2 away from the rope
We learned in an earlier module that given a constant acceleration, the distance traveled versus time is:
d = v0t + 0.5*a*t^2
In this case, v0 is zero, so
d = 0.5*ay*t^2, or
t = sqrt(d/(0.5*ay)), or
t = sqrt(2m/(0.5*0.07143m/s^2)) = 7.48324 seconds
I doubt that you will stay in the air long enough to reach the rope.

During that time period, the raft will travel the following distance in the opposite direction (assuming no resistance from the water).
d = 0.5*ar*t^2, or
d = 0.5*(-5m/s^2)*(7.48324s)^2 = -140 meters

19.4.4 Conservation of momentum example

Railroad cars

Getting back to my example of coupling railroad cars, when the collision has been completed, the two masses have effectively been joined into a single mass and they are moving at the same velocity.

In that case, we can write the above equation (p. 563) as

ma*(v2 - va1) = -mb*(v2 - vb1)

ma*v2 +mb*v2 = ma*va1 +mb*vb1

v2 = (ma*va1 +mb*vb1)/(ma + mb)

where

- ma an mb represent the masses for Car-A and Car-B respectively
- v2 is the velocity of the coupled cars after the collision
- va1 and vb1 represent the initial velocities for Car-A and Car-B respectively

Then for any set of assumed mass values for the railroad cars and assumed values for the initial velocities, we can calculate the final velocity of the coupled pair of railroad cars.

Scenario #1: Assume that the two railroad cars are just alike and empty giving them the same mass. Also assume that the initial velocity for Car-A is 10 m/s and the initial velocity for Car-b is 0.

Question: What would be the final velocity of the coupled railroad cars?

Answer: For this scenario, we have

v2 = (ma*va1 +mb*vb1)/(ma + mb), or

v2 = (ma*10)/2*ma = 5 m/s

The final velocity of the pair of coupled cars is half the initial velocity of the car that was moving.

Scenario #2: Now assume that due to loading, Car-A has twice the mass of Car-B, the initial velocity for Car-A is 10m/s and the initial velocity for Car-B is 0.

Question: What would be the final velocity of the coupled railroad cars?

Answer: For this scenario, we have

v2 = (ma*va1 +mb*vb1)/(ma + mb), or

v2 = (ma*va1)/(ma + 0.5ma), or

v2 = ma*10/1.5*ma = 6.67 m/s

When the car at rest is less massive than the car in motion, the final velocity is a little higher than when the two cars have the same mass.

Scenario #3: Finally, assume that due to loading, Car-A has twice the mass of Car-B, the initial velocity for Car-A is 10m/s and the initial velocity for Car-B is 5m/s.

Question: What would be the final velocity of each railroad car?

Answer: For this scenario, we have

v2 = (ma*va1 +mb*vb1)/(ma + mb), or

v2 = (ma*10 +0.5ma*5)/(1.5*ma), or

v2 = (10/1.5) + (0.5*5/1.5) = 8.33 m/s

When both cars are already moving in the same direction, the final velocity in that direction is greater than when one of the cars is stationary.

19.4.5 Center of mass examples

This section contains solutions to problems involving the center of mass.

19.4.5.1 Two objects

Two objects are located on a flat lawn with the following mass values and locations:

- obj1m = 15 kg
- obj1x = 1 m

- obj1y = 5 m
- obj2m = 3 kg
- obj2x = 4 m
- obj2y = 2 m

What are the coordinates of the center of mass?

Solution:

A JavaScript script that will solve this problem is shown in Listing 2 (p. 573) .

Listing 2 . Center of mass for two objects.

```
    <!---------------- File JavaScript02.html ---------------------->
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//Create arrays for mass,xCoor, and yCoor values;
var mass = new Array(15,3);
var xCoor = new Array(1,4);
var yCoor = new Array(5,2);

//Declare a counter variable.
var cnt = 0;

//Use a loop to compute total mass by summing the individual
// mass values.
var massTotal = 0;
for(cnt = 0;cnt < mass.length;cnt++){
  massTotal += mass[cnt];
}//end for loop

//Use a loop to compute x-coordinate of the center of mass
// by summing the normalized sum of products.
var cmX = 0;
for(cnt = 0;cnt < mass.length;cnt++){
  cmX += mass[cnt]*xCoor[cnt]/massTotal
}//end for loop

//Use a loop to compute y-coordinate of the center of mass
// by summing the normalized sum of products.
var cmY = 0;
for(cnt = 0;cnt < mass.length;cnt++){
  cmY += mass[cnt]*yCoor[cnt]/massTotal
}//end for loop

//Display the results
document.write("massTotal = " + massTotal + " kg</br>");
document.write("cmX = " + cmX + " meters</br>");
document.write("cmY = " + cmY + " meters</br>");

document.write("End Script");

</script>
</body></html>
```

Table 19.5

The comments describe how the problem is solved.

The output is shown in Figure 4 (p. 574) .

Figure 4 . Center of mass for two objects.

```
    Start Script
massTotal = 18 kg
cmX = 1.5 meters
cmY = 4.5 meters
End Script
```

Table 19.6

The x-coordinate for the center of mass is 1.5 meters, and the y-coordinate for the center of mass is 4.5 meters.

19.4.5.2 Three objects

Three objects are located on a flat lawn with the following mass values and locations:

- obj1m = 5 kg
- obj1x = 1 m
- obj1y = 1 m
- obj2m = 5 kg
- obj2x = 3 m
- obj2y = 1 m
- obj3m = 10 kg
- obj3x = 2 m
- obj3y = 3 m

What are the coordinates of the center of mass?

Use the code from Listing 2 (p. 573) but make the change shown in Listing 3 (p. 574) in order to populate the world with three objects having different mass values and different coordinates.

Listing 3 . Change to add a third object.

```
    //Create arrays for mass,xCoor, and yCoor values;
var mass = new Array(5,5,10);
var xCoor = new Array(1,3,2);
var yCoor = new Array(1,1,3);
```

Table 19.7

The output is shown in Figure 5 (p. 575) .

Figure 5 . Center of mass for three objects.

```
    Start Script
massTotal = 20 kg
cmX = 2 meters
cmY = 2 meters
End Script
```

Table 19.8

19.5 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

19.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

19.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Force and Motion – Momentum, Impulse, and Conservation of Momentum for Blind Students
- File: Phy1160.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · conservation of momentum
  - · momentum
  - · Newton's cradle
  - · impulse
  - · action and reaction

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 20

Energy – Work[1]

20.1 Table of Contents

- Preface (p. 577)
  - General (p. 577)
  - Prerequisites (p. 577)
  - Supplemental material (p. 578)
- General background information (p. 578)
  - An assumption of constant velocity (p. 578)
  - Use your graph board for sketches (p. 579)
  - Work in everyday life (p. 579)
  - Work in physics (p. 579)
- Sample calculations (p. 582)
  - Single force on a body (p. 582)
  - Multiple forces on a body (p. 583)
  - Force parallel to a ramp (p. 584)
- Do the calculations (p. 584)
- Resources (p. 585)
- Miscellaneous (p. 585)

20.2 Preface

20.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains energy and work in a format that is accessible to blind students.

20.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

[1] This content is available online at <http://cnx.org/content/m38267/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

20.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

20.3 General background information

I will explain some of the background material regarding work in this section and show how to work problems involving work later (p. 582) .

20.3.1 An assumption of constant velocity

In order to simplify the solutions to the problems, many of the example problems in this module will make the assumption that an object is moving with constant velocity. Let's review what that means in terms of what you have learned from earlier modules.

Unbalanced forces cause acceleration

We know that whenever unbalanced forces are applied to an object, that object will accelerate. What does it mean, therefore, when we make a statement like "a force of 1 newton to the right was applied to an object to cause that object to move 1 meter to the right at a constant velocity"?

Conditions for constant velocity

We know that the statement means at least the following things:

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

- At some point in the past, there was an unbalanced force applied to the object to cause it to begin moving to the right. That force is ignored by the statement of the problem.
- At the beginning of the time frame covered by the problem, the object was in equilibrium, moving at a constant velocity to the right.
- During the time frame of the problem, there must be a 1 newton balancing force being exerted to the left to prevent the 1 newton force being applied to the right from causing the object to accelerate towards the right. That force is also ignored by the statement of the problem.

20.3.2 Use your graph board for sketches

As a sighted person, I often find it useful to make sketches of the physical scenarios for problems of the types that I will discuss in this module.

You may also find it useful to use your graph board to create "sketches" of the physical scenarios in order to better understand the solutions to the problems.

20.3.3 Work in everyday life

You hear the word work in many situations in everyday life. For example, someone might say "My gasoline bill is high because I have to drive a long way to work every day."

Someone else might say "She works very hard at being popular."

Or, you might hear someone say "It's going to take a lot of work to lift that heavy sofa into the back of that truck."

The latter expression comes closest to describing what is mean by work in physics.

20.3.4 Work in physics

According to *College Physics* by Mendenhall et al

*If a body on which a force F is acting is displaced by a distance d, then the work done by the force is F*d*cos(theta), where theta is the angle between the path d and the line of action of F.*

Thus, the term **work** has a very precise meaning in physics. In equation form, **we can represent work as**

W = F * d * cos(theta)

where

- W represents work
- F represents a force that causes the displacement of an object
- d represents the magnitude of the displacement
- theta is the angle between the line of action of the force and the direction of the displacement

Three key ingredients

According to the Physics Classroom (see http://www.physicsclassroom.com/Class/energy/u5l1a.cfm [13]), there are three key ingredients to work: *force* , *displacement* , and *cause* .

In order for a force to qualify as having done work on an object, there must be a displacement and the force must cause the displacement.

Commonplace examples of work

Examples of work are commonplace in everyday life:

- A bulldozer pushing dirt. The bulldozer exerts a horizontal force to move the dirt.
- An elevator lifting people from the first to the tenth floor. The cable on the elevator exerts an upward force to move the people upward by approximately 110 feet.

[13] http://www.physicsclassroom.com/Class/energy/u5l1a.cfm

- You lifting your backpack full of books onto your back. Your muscles exert a force to raise several heavy books, possibly displacing them both vertically and horizontally at the same time.

However, some things that we might initially think of as work don't really qualify as work according to the physics definition of work.

Suzie and the heavy trunk

Your college dorm is equipped with a rope on a pulley that can be used to lift heavy objects and swing them into a window on the second floor. Suzie has used the pulley and the rope to lift her heavy trunk up to the level of the window. She asks you to hold the rope and maintain the trunk at that level while she goes up the stairs to pull it in the window.

Suzie likes to gossip

Suzie, however, really likes to gossip. On the way up to the window, she meets some friends and starts talking, leaving you holding onto the rope. After about 30 minutes, you become really tired and frustrated and you lower the trunk back to ground level.

Did you do any work on the trunk?

The answer is that you did not do any work on the trunk. Although you became very hot and tired standing there and holding the rope, you did not cause a displacement of the trunk. For 30 minutes, you simply held the rope causing the trunk to remain at the same location. At the end of that period, you carefully allowed the rope to slip through your hands while the force of gravity caused a downward displacement on the trunk.

Was any work done on the trunk?

Was any work done on the trunk from the time Suzie handed you the rope until the trunk was back on the ground?

Yes, the force of gravity caused the height of the trunk to be displaced from the level of the second floor window downward to the ground, so the force of gravity did work on the trunk.

Was any work done on the trunk from the time that Suzie started raising it until it was back on the ground?

The answer is no. The net displacement of the trunk over that time period was zero. If the displacement was zero, no work could have been done. In this case, the positive work done by Suzie in lifting the trunk to the second floor window was canceled out by the negative work done by gravity when you allowed the trunk to return to the ground.

Measurement of work can depend on the time interval

As you can see, therefore, the amount of work done in some cases can depend on the time interval over which the measurement is made.

Also, as you can see, certain activities (such as holding a rope to keep a trunk from falling) can make you very tired even though you are not doing any work.

Mathematical representation of work

As stated earlier, we can represent work as

W = F * d * cos(theta)

where

- F represents a force being applied to an object
- d represents the displacement of the object
- theta is the angle between the displacement path d and the line of action of the force.

Work is a scalar quantity

Although both force and displacement are vector quantities, the product of those two vectors with the cosine of an angle produces a scalar quantity. Work has a magnitude but no direction. The product is often called a scalar product or a dot product.

The scalar or dot product

The scalar product (or dot product) of two vectors is defined by the equation

vectorA * vectorB * cos(theta)

where theta is the angle between the two vectors when they are drawn tail to tail.

The special name and notation are used because this pattern occurs often in physics and mathematics.

Units of work

The SI unit of work (and energy) is the newton-meter (N*m), which is given the name joule (symbol: J), after the English physicist James Prescott Joule. One joule is equivalent to one newton of force causing a displacement of one meter.

A stubborn dog

Pretend that you have a dog that is very stubborn and just wants to sleep. You pull on the dog's leash and cause the dog to slide 2 meters across the floor. In this case, the displacement is horizontal and you are pulling at an angle of 30 degrees above the horizontal.

The work done on the dog would be the product of the horizontal component of the force that you exert on the leash (F*cos(theta)) and the displacement distance of 2 meters. If you were pulling with a constant horizontal force component of 3 newtons, then you would have done 2*3=6 joules of work.

A stubborn pig

A very large pig has escaped from its pen. You have a rope tied around the pig's neck and you are trying to pull the pig back into the pen. The rope is horizontal and you pull with a steady force of 3 newtons.

While you are pulling, however, the pig is backing up (actually pulling you forward away from the pen), and travels 2 meters in the direction opposite the direction that you are pulling.

In this case, the angle between the line of action of the force that you are exerting and the direction of the pig's displacement is 180 degrees. Since the cosine of 180 degrees is -1, the product of the force, displacement, and cosine of 180 degrees would be -6. Therefore, your efforts to move the pig toward the pen are unsuccessful and you have done -6 joules of work on the pig.

Therefore, work can be either positive or negative.

More on that stubborn pig

The very large pig has decided to lay down on a hydraulic lift on the back of a truck and someone accidentally flipped the switch causing the hydraulic lift to start moving up.

You still have a rope tied around the pig's neck and you attempt to pull the pig off the lift to prevent it from getting hurt. You pull as hard as you can on the rope but the pig doesn't budge. The lift goes up two meters to the height of the truck bed and turns itself off automatically with the pig still laying on the lift.

The angle is 90 degrees

In this case, the angle between the line of action of the force that you exert and the direction of displacement of the pig is 90 degrees. The cosine of 90 degrees is 0. Therefore, the product of the force, the displacement, and the cosine of 90 degrees would be 0.

While you might be ready for a rest after exerting all of that effort trying to pull the pig off the lift, you actually haven't done any work on the pig.

Therefore, work can even be zero if the line of action of the force is perpendicular to the displacement of the object. In this case, work did get done to raise the pig 2 meters off the ground, but you didn't contribute anything to that work. The work was done by the mechanical system that operates the hydraulic lift on the truck.

To do any work, a force must cause a displacement.

More on the stubborn dog

Let's go back and think a little more about the stubborn dog. Assume that you exert a force of 3.46 newtons on the leash and the leash makes an angle of 30 degrees with the horizontal floor. This causes the dog to slide 2 meters across the floor, but doesn't lift the dog off the floor. What is the work that you do on the dog?

The horizontal component Fx of the force F is given by

Fx = F*cos(30 degrees), or

Fx = 3.46*cos(30 degrees)

Enter the following into the Google calculator search box:

3.46*cos(30 degrees)

and with rounding, you will learn that the horizontal component of the force that you exerted on the leash is

3 newtons.

How much work was done?

The work done on the dog is the product of the horizontal component of the force that you exert on the leash and the horizontal displacement distance of 2 meters.

Assuming that you were pulling the leash with a constant force of 3.46 newtons, at an angle of 30 degrees relative to the horizontal floor, you would do 2*3=6 joules of work on the dog.

Be careful how you handle the angle theta

The angle theta is the angle between the line of action of the force and the direction of the displacement. It is only because I set the scenario up that way that the 30-degree angle in the previous stubborn-dog scenario is 30 degrees relative to the horizontal.

In that scenario, the line of action of the 3.46-newton force was at 30 degrees relative to the horizontal, but the displacement was horizontal. Therefore, it was necessary to take the angle into account when computing the work done on the dog. In other words, the work done on the dog was:

W = 3.46N * 2m * cos(30 degrees) = 6 joules

One more discussion about the stubborn dog

Assume that instead of pulling the dog across a horizontal floor, you are pulling the dog up a ramp that has an angle of 30 degrees relative to the horizontal.

In this case, you pull on the leash at an angle of 30 degrees relative to the horizontal, which is parallel to the ramp. Once again, you pull with a force of 3.46 newtons. This time, you cause the dog to slide 2 meters up the ramp.

In this case, the angle between the direction of the displacement and the line of action of the force is 0 degrees. The cosine of 0 degrees is +1. Therefore, the work done on the dog is

(3.46 N) * 2 m * cos(0 degrees) = 6.92 joules

(You may recognize the format of some of the mathematical expressions in the above paragraphs as being the format of the output of the Google calculator.)

20.4 Sample calculations

I will present and explain several scenarios in this section involving force and work.

In some cases, I will use the Google calculator to perform the arithmetic, and the format of the mathematical expressions will be the format of the output from the Google calculator (except that I will usually discard some of the digits to the right of the decimal point).

20.4.1 Single force on a body

A horizontal force

A horizontal force of 10 N towards the right is applied to a 15-kg body causing the body to move 5 m to the right at constant speed. How much work is done on the body?

Answer:

(10 newtons) * 5 meters = 50 joules

Note that the mass of the object is superfluous in this problem.

A force at 30 degrees

A force of 10 N is applied to a 15-kg body at an angle of 30 degrees relative to the horizontal causing the body to move 5 m to the right at constant speed. How much work is done on the body?

Answer:

(10 N) * 5 m * cos(30 degrees) = 43.3 joules

Once again, the mass of the object is superfluous.

A vertical force

The mass is not superfluous for this scenario.

A force is applied to a 10-kg body at an angle of 90 degrees relative to the horizontal causing the body to move 5 m straight up at constant velocity. How much work is done on the body?

Answer:

Weight = (10 kg) * 9.81 * (m / (s^2)) = 98.1 newtons

Under the constant velocity assumption explained earlier, an upward force equal to the weight of the object will cause the object to continue moving in an upward direction, and eventually move 5 m straight up. The angle between the action line of the force and the displacement is 0 degrees. Therefore, the work done to the object is:

(98.1 N) * 5 m * cos(0 degrees) = 490.5 joules

20.4.2 Multiple forces on a body

A block on a friction-free surface

A block on a friction-free surface is subjected to three forces:

1. Weight = 15 N
2. Upward force from the surface = 15 N
3. A 5 N force to the right that causes the block to move 5 m to the right at constant velocity.

How much work does each force do on the block.

Answer:

Forces 1 and 2 are perpendicular to the direction of displacement, so they cannot do work on the block.

Force 3 is in the same direction as the displacement and does the following work on the block:

(5 N) * 5 m * cos(0 degrees) = 25 joules

A moving block encounters friction

A block is sliding to the right on a friction free surface. Suddenly the block encounters an area on the surface where the friction is not zero. The friction causes the block to stop after moving 5 m to the right.

When the block encounters the frictional surface, it is subjected to the following three forces:

1. Weight = 15 N
2. Upward force from the surface = 15 N
3. A 5 N force to the left caused by the friction.

How much work does each force do on the block?

Answer:

Forces 1 and 2 are perpendicular to the direction of displacement, so they cannot do work on the block.

The angle between force 3 and the direction of displacement is 180 degrees. Force 3 does the following work on the block:

(5 N) * 5 m * cos(180 degrees) = -25 joules

Note that in this case, the work done is negative.

Another friction scenario

A block on a frictional surface is subjected to four forces:

1. Weight = 15 N
2. Upward force from the surface = 15 N
3. A 5 N force to the right causes the block to move 5 m to the right at constant speed.
4. A 5 N force to the left caused by friction.

How much work does each force do on the block?

Answer:

Forces 1 and 2 are perpendicular to the direction of displacement, so they cannot do work on the block.

The work done by force 3 is:

(5 N) * 5 m * cos(0 degrees) = 25 joules

The work done by force 4 is:

(5 N) * 5 m * cos(180 degrees) = -25 joules

20.4.3 Force parallel to a ramp

A man and a crate

A man needs to load a crate with friction-free wheels and a weight of 1000 N onto the bed of a truck that is 1 m off the ground. He has two different ramps that he can use to push the crate up the ramp and onto the truck.

One ramp is 2.5 m long and the other ramp is 5 m long.

He is able to push the crate up the ramp by pushing in a direction that is parallel to the ramp.

Questions

What is the force that he must exert parallel to each ramp to push the crate up the ramp.

Which ramp requires the least amount of work to push the crate up the ramp?

Required force

Trigonometry can be used to compute the amount of force required, parallel to the ramp, to push the crate up the ramp as a function of the angle that the ramp makes with the horizontal. That force is equal to the product of the weight of the crate (1000 N or 225 pound-force) and the sine of the angle that the ramp makes relative to the horizontal.

For example, other than the requirement to overcome inertia to get the crate moving, no force is required to push the crate if the angle is 0 degrees. At the other extreme, if the angle is 90 degrees, the man must lift the crate straight up, bearing the entire weight of the crate. For angles in between these two extremes, the required force is within the capability of one man to achieve.

Answers

Ramp #1

- Truck bed height = 1.0m
- Ramp length = 2.5m
- Ramp angle = arcsin(1 / 2.5) = 23.58 degrees
- Weight of crate = 1000N
- Component of crate's weight parallel to ramp = 1000 N * sin(23.58 degrees) = 400.03 newtons. This is the force that must be exerted to push the crate up the ramp.
- Work = (400.03 newtons) * 2.5 meters = 1000.08 joules

Ramp #2

- Truck bed height = 1.0m
- Ramp length = 5m
- Ramp angle = arcsin(1 / 5) = 11.54 degrees
- Weight of crate = 1000N
- Component of crate's weight parallel to ramp = 1000 N * sin(11.54 degrees) = 200.05 newtons. This is the force that must be exerted to push the crate up the ramp.
- Work = (200.05 newtons) * 5 meters = 1000.25 joules

The effect of the ramp

The man only has to exert half as much force to push the crate up Ramp #2 as is required for Ramp #1.

However, he has to push it twice as far with Ramp #2, so the amount of work done on the crate in both cases is 1000 joules.

Like many other simple machines, the use of a ramp (inclined plane) reduces the force required to do a job but it doesn't reduce the amount of work required to do the job.

20.5 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

20.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

20.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Energy – Work
- File: Phy1170.htm
- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · work
  - · energy
  - · power

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 21

Energy – Potential Energy[1]

21.1 Table of Contents

- Preface (p. 587)
  - General (p. 587)
  - Prerequisites (p. 587)
  - Supplemental material (p. 588)
- General background information (p. 588)
  - Graphic examples of potential energy (p. 588)
  - Potential energy (p. 589)
    - Gravitational potential energy (p. 590)
    - Elastic potential energy (p. 591)
  - Summary (p. 592)
- Sample calculations (p. 593)
- Do the calculations (p. 594)
- Resources (p. 594)
- Miscellaneous (p. 594)

21.2 Preface

21.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains potential energy in a format that is accessible to blind sudents.

21.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).

[1] This content is available online at <http://cnx.org/content/m38279/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg

- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

21.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

21.3 General background information

I will begin this explanation with a couple of graphic examples.

21.3.1 Graphic examples of potential energy

Gravitational potential energy

If you find a flat rock with a mass of 10 kg on the ground at a location on the surface of the earth where the ground is flat for miles around, that rock has little or no potential energy.

A change in potential energy

If you pick that rock up and balance it on a limb of a tree that is 2 meters off the ground, you have done at least two things:

- You have done work on the rock by exerting a force on the rock that displaced it upward by 2 meters. With some simplifying assumptions, we can calculate that you have done 196.2 joules of work on the rock.
- You have caused the rock to have gravitational potential energy that it did not have in its earlier position on the ground.

[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

An accident waiting to happen

Perhaps the change in potential energy can best be illustrated by stating that the rock now has the potential to crack someone's skull open if they happen to be standing under the limb when the rock falls towards the ground with an acceleration of 9.8 m/s^2.

When the rock was laying on the surface of the earth, it did not have that potential. You created that potential by imparting potential energy into the rock when you listed it from the ground and balanced it on the limb.

Elastic potential energy

Consider a common rubber band. Because of its small mass, a rubber band can never be expected to acquire much in the way of gravitational potential energy. However, it can acquire a considerable amount of elastic potential energy.

While I don't recommend that you try the experiment with the rock described earlier, this is an experiment that you can try with no permanent damage to your person.

Thread your arm through the rubber band

Find a strong rubber band and thread your left arm through the rubber band up to the wrist. Make certain that the rubber band fits loosely on your arm. At that point, the rubber band has little or no potential energy.

Change the potential energy

Now grasp the rubber band between the thumb and forefinger of your right hand and stretch it to the point where it is almost ready to break.

At that point, you have expended your energy in stretching the rubber band, and the rubber band has acquired elastic potential energy.

How much elastic potential energy?

How much elastic potential energy did the rubber band acquire? You can get a qualitative estimate of the amount of potential energy by releasing the rubber band and letting it strike your bare wrist.

When the rubber band contacts your skin, it will impart energy into the nerve ending in your skin as its potential energy goes back to zero. If it hurts a lot, there was a lot of elastic potential energy stored in the stretched rubber band. If it doesn't hurt very much, there probably wasn't much potential energy stored in the rubber band, or you have very tough skin with few nerve endings.

21.3.2 Potential energy

There are several ways that an object can acquire and store energy, which we will refer to as *potential energy* . Two of the most common ways are:

- Raising the object to a new height above the surface of the earth (gravitational potential energy)
- Deforming the object within its elastic limit (elastic potential energy)

Increasing the potential energy of the rock

For example, elevating the rock by 2 meters above the ground as described earlier imparted energy into the rock. (Work was done on the rock in order to increase its height above the ground.)

Storing the potential energy

Balancing the rock on the limb caused that energy to be (temporarily) stored in the rock as gravitational potential energy. The rock stores that energy for so long as it is held at that elevated position.

Releasing the potential energy

Tipping the rock off the limb so that it would fall back to the ground caused that potential energy to be released and caused the potential energy of the rock to go back to zero.

Potential energy and the rubber band

Similarly, stretching and then releasing the rubber band as described above first stored, and then released elastic potential energy in the rubber band.

Work is required to increase the potential energy of an object

In both cases, it was necessary for you to expend energy in order to increase the amount of energy stored in the rock and the rubber band.

21.3.2.1 Gravitational potential energy

Gravitational potential energy is the energy stored in an object as the result of the gravitational attraction between the earth and the object. In practical terms, the energy is stored as the result of the objects height above the surface of the earth.

Every object that has a position above the surface of the earth has stored gravitational potential energy. Some common examples are

- Books on a bookshelf
- An air conditioning unit in a window on the third story of an apartment building
- You, when you climb a tree and sit on a limb or walk up the stairs to your classroom.

Because all objects are attracted to a point in the center of the earth, all objects that don't rest on the surface of the earth have potential energy.

All objects have stored gravitational potential energy

Even those objects that do rest on the surface of the earth have potential energy, because they have a strong tendency to make their way to the center of the earth. In most cases, however, that desire cannot be fulfilled so we usually consider the surface of the earth to be the reference point for gravitational potential energy.

The deep

In the final analysis, however, only those objects that rest at the bottom of the deepest point in the ocean cannot be forced to give up stored gravitational potential energy. In theory, every other object could be transported to and dropped to that point on the ocean bed, thereby giving up stored potential energy in the process.

The height and the mass are critical

The magnitude of the potential energy possessed by the rock balanced on the tree limb in the earlier example depends on two things: the height of the rock above the ground and the mass of the rock.

Gravitational potential energy is proportional to mass

The gravitational potential energy of an object is proportional to the mass of the object. Objects with more mass are capable of having more potential energy than objects with less mass. Therefore, if the rubber band discussed in the earlier example were to be placed on the same limb as the rock, the rock would have more gravitational potential energy than the rubber band due to its greater mass.

Gravitational potential energy is proportional to height

Gravitational potential energy is also proportional to the distance of the object from the center of the earth. Thus, a 10 kg rock falling from 50 meters onto your head would do more damage than the same 10 kg rock falling onto your head from 2 meters. In both cases, the potential energy of the rock would be converted to kinetic energy, (which is a topic for a future module), and that kinetic energy would be converted to pain when the rock strikes your head.

The higher the starting point for the rock,

- the faster it would be going right before it strikes your head,
- the more kinetic energy it would possess at that point in time, and
- the more damage it would do to your head.

Gravitational potential energy is proportional to the product of mass, height, and gravity

Thus, the gravitational potential energy of an object that is elevated above the surface of the earth is equal to the product of the mass of the object, the height of the object, and the acceleration of gravity. Expressed as an equation, we can write:

PEg = mass*kg * gravity*m/s^2 * height*m

where

- PEg represents gravitational potential energy
- mass, gravity, and height represent their namesakes

- kg, m, and s represent kilogram, meter, and second

Units of gravitational potential energy

As you can see from above, the units of gravitational potential energy are

kg*m^2/s^2 = kg*(m/s^2)*(m) = N*m = joule

What is the zero height reference?

Because the determination of gravitational potential energy requires knowledge of the height of the object, you must determine the height of the object above a *zero-height* reference level. Therefore, to determine the gravitational potential energy of an object, you must first decide what level you are going to consider to be zero height.

The ultimate reference

The ultimate zero-height reference is the point in the center of the earth to which all objects are attracted. Using that point, however, would lead to a lot of arithmetic accuracy problems. The radius of the earth is something like 6378100 meters, and that depends on where on the earth you are standing – death valley, Mount Everest, or somewhere in between.

I will leave it as an exercise for the student to determine why the use of the center of the earth as the zero-height reference might lead to arithmetic accuracy problems when doing calculations involving gravitational potential energy.

Doubling the height doubles the gravitational potential energy

The gravitational potential energy of an object is directly proportional to its height above the zero position. Therefore, doubling or tripling the height of the object above the zero position will have a corresponding doubling or tripling effect on the gravitational potential energy.

21.3.2.2 Elastic potential energy

Elastic potential energy is the energy stored in an object as a result of deforming the object within its elastic limit, such as stretching a rubber band, stretching a coil spring in a fisherman's scale, or compressing a coil spring in the suspension of an automobile.

The elastic limit

Many materials are elastic up to a point, some more than others. This means that they can recover from a deformation up to a point.

However, most materials have an elastic limit. The elastic limit is the point beyond which the material cannot recover from a deformation.

A rubber band

For example, if you stretch a rubber band by an inch or two, it will usually recover when the load is removed. However, if you stretch it too far, it will break. In that case, the rubber band has clearly been deformed beyond its elastic limit.

A steel beam

A steel beam that supports a bridge can normally flex by a small amount when loaded by traffic on the bridge and return to its original shape once the load is removed. However, there is a point beyond which it cannot recover if flexed too far.

A coil spring

A coil spring in an old-fashioned fisherman's scale can stretch up to a certain limit when loaded with a fish, and then return to its original length when the fish is removed. However, if it is stretched beyond its elastic limit, it won't return to its original length when the load is removed. (The enclosure on a fisherman's scale is usually designed to prevent the spring from being stretched beyond its elastic limit.)

Many objects store elastic potential energy

Elastic potential energy can be stored in steel beams, rubber bands, golf balls, springs, automobile tires, etc. The amount of elastic potential energy stored in such a device is related to the amount by which the object is deformed (usually stretched or compressed). The more the object is deformed, up to a point, the more elastic potential energy is stored in the object.

Springs are a special case

Springs are a special case of an object that can store elastic potential energy either through stretching or compression. Some springs are probably manufactured with one or the other in mind while other springs may be manufactured to serve both purposes.

For example, the spring in a fisherman's scale is probably manufactured with only stretching in mind. The process of weighing a fish on a fisherman's scale is to hang the fish on a hook on the bottom end of the spring and measure how far the spring stretches.

On the other hand, I believe, but am not certain, that the coil springs in the suspension of a car serve their purpose by both stretching an compressing.

A force is required

A force is required to compress or stretch a spring. The more the spring is compressed or stretched (depending on its purpose), the more force will be required to compress or stretch it further.

The spring constant

For certain springs, the amount of force required to compress or stretch the spring (up to a limit) is directly proportional to the amount of stretch or compression. This can be expressed in equation form as

Fs = k * x

where

- Fs is the force applied to the spring
- k is the constant of proportionality
- x is the amount of stretch or compression of the spring

Hooke's law

The constant of proportionality in the above equation is known as the **spring constant** . Springs that behave this way are said to follow Hooke's law, named after the 17th century British physicist Robert Hooke.

The equilibrium state

If the spring is not compressed or stretched, it is in its equilibrium state, and there is no potential energy stored in it. The equilibrium state is the state that the spring naturally assumes when no forces are acting on it. This state could be called the zero-potential energy state.

(This is the state of the rubber band that hung loosely around your wrist in the experiment at the beginning of this module.)

Potential energy versus stretch or compression

There is an equation for springs that relates the amount of elastic potential energy to the amount of stretch (or compression) and the spring constant. The equation is

PEs = 0.5 * k * x^2

where

- PEs represents the elastic potential energy stored in the spring
- k is the spring constant for the material from which the spring is made
- x is the amount of compression or stretch relative to the equilibrium state.

21.3.3 Summary

Gravitational potential energy can be stored in an object by moving it further away from the center of the earth. As a practical matter, this usually means moving it to a higher position relative to the ground, the floor, or a tabletop.

Elastic potential energy can be stored in an object by deforming it within its elastic limit. Usually, but not always, this involves stretching or compressing the object, but it could also mean twisting it or deforming it in some other way. If the deformation doesn't exceed the elastic limit of the object, it will return to its original shape when the load is removed.

Some materials, such as the spring material in a fisherman's scale, can sustain considerable deformation before reaching the elastic limit. Other materials, such as toasted bread, not only reach their elastic limit, but also reach their structural limit and break at the slightest amount of deformation.

Other materials, such as fresh bread and modeling clay have a low elastic limit but a relatively high structural limit. In other words, they doesn't return to their original shape when deformed, but they also don't easily break when deformed.

21.4 Sample calculations

The rock and the tree

You pick a flat rock with a 10kg mass off the ground and balance it on a tree limb 2 meters above the ground.

What was the gravitational potential energy of the rock before your arrival and what change in gravitational potential energy did you impart to the rock by your actions?

Answer:

Assuming that the rock was flat and the ground was flat and there was essentially no chance the the rock could roll downhill, the gravitational potential energy of the rock before you picked it up was zero.

Using the Google calculator to do the arithmetic and handle the units, after you balanced the rock on the tree limb, the potential energy was:

(10 kg) * (9.8 (m / (s^2))) * (2 m) = 196 joules

Therefore, the change in gravitational potential energy was 196 joules.

A crate and a ramp

You push a 100 kg mass up a 5 meter-long ramp and onto a platform at the upper end of the ramp. The ramp makes an angle of 36.9 degrees relative to the ground on which it is setting.

What change in gravitational potential energy did you impart to the crate?

Answer:

First compute the height of the platform at the upper end of the ramp.

height = 5 * sin(36.9 degrees) = 3m

Now compute the change in gravitational potential energy.

(100 kg) * (9.8 (m / (s^2))) * (3 m) = 2940 joules

All that matters insofar as the change in gravitational potential energy is concerned is that the crate was lifted 3 meters higher than its initial position. How that lift was accomplished doesn't matter. It could be accomplished with a ramp, a pulley, or with brute strength and the change in gravitational potential energy would be the same.

More on the crate and the ramp

How much work was done on the crate in pushing it to the top of the ramp?

Answer:

The first task is to compute the force parallel to the ramp that is required to push the crate up the ramp, ignoring the extra push required to get it moving at the bottom of the ramp. That parallel force is equal to the component of the crate's weight that is pushing down parallel to the ramp.

A little trigonometry will reveal that the component of the crate's weight that is parallel to the ramp is equal to the product of the total weight of the crate and the sine of the ramp angle relative to the horizontal ground. That gives us a requirement for a force pushing up the ramp and parallel to the ramp of

(100 kg) * 9.8 (m / (s^2)) * sin(36.9 degrees) = 588 newtons

The angle between the line of action of the force and the displacement direction of the crate was 0 degrees. Therefore, the work done on the crate was:

(588 newtons) * 5 m * cos(0) = 2940 joules

Note that the work done on the crate to move it to its new position was equal to the change in gravitational potential energy for the crate. As you may have guessed, that is not a coincidence. You will learn more about that relationship in a future module.

21.5 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

21.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

21.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Energy – Potential Energy
- File: Phy1180.htm
- Revised: 10/02/15
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - potential energy
  - work
  - gravitational potential energy
  - elastic potential energy

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 22

Energy – Kinetic and Mechanical Energy[1]

22.1 Table of Contents

- Preface (p. 595)
  - General (p. 595)
  - Prerequisites (p. 595)
  - Supplemental material (p. 596)
- General background information (p. 596)
  - Kinetic energy (p. 596)
  - Mechanical energy (p. 599)
  - Total mechanical energy (p. 601)
- Sample calculations (p. 602)
  - Kinetic energy (p. 602)
  - Mechanical energy (p. 603)
- Do the calculations (p. 604)
- Resources (p. 604)
- Miscellaneous (p. 604)

22.2 Preface

22.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains kinetic and mechanical energy in a format that is accessible to blind students.

22.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

[1] This content is available online at <http://cnx.org/content/m38296/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

22.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

22.3 General background information

In an earlier module, you learned about two forms of potential energy:

- gravitational potential energy
- elastic potential energy

In this module, you will learn about kinetic energy and mechanical energy.

22.3.1 Kinetic energy

Review regarding gravitational potential energy

Recall that gravitational potential energy is energy possessed by an object due to its distance from the center of the earth. Gravitational potential energy is driven by the attraction between the earth and all objects on the earth.

Review regarding elastic potential energy

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

Elastic potential energy is energy possessed by objects that have been deformed within their elastic limits. Elastic potential energy is driven by a desire of those deformed objects to return to their non-deformed state of equilibrium.

Kinetic energy

Kinetic energy is energy possessed by an object because of its motion. Kinetic energy is driven by the tendency of a moving object to continue moving at the same velocity. When an object is in motion, something earlier did work on the object to cause it to attain its velocity and force will be required to cause it to change its velocity.

There's that little brother again

When your little brother throws a ball and scores a solid hit on your back (not a glancing blow), the velocity of the ball goes to zero upon contact and the kinetic energy possessed by the baseball is released into your body. This can sometimes cause pain, particularly if the ball possesses a lot of kinetic energy.

Forms of kinetic energy

Kinetic energy comes in a variety of forms, including:

- rotational, which is due to rotational motion such as a wheel on a bicycle
- translational, which is due to the movement of an object from one location to another location

I will concentrate on translational kinetic energy in this module and will deal with other forms of kinetic energy in future modules.

Two factors control the amount of kinetic energy

The amount of translational kinetic energy possessed by a moving object depends on two factors:

1. The mass of the object.
2. The velocity of the object.

How did the object attain its current velocity?

Note that it doesn't matter how the object attained its current velocity. The object may have attained its current velocity by accelerating very slowly over a period of years. The object may have fallen off a tall bookshelf and attained its current velocity fairly quickly as a result of the acceleration of gravity. Or, the object may have been shot out of a cannon due to an explosion of gun powder and acquired its current velocity very quickly.

With regard to the kinetic energy currently possessed by the object, the only things that are important are:

1. What is the mass of the object?
2. What is the current velocity of the object?

Calculating kinetic energy

We know that an object can have motion (and hence kinetic energy) at the current time only if work was done on the object earlier to put it in motion. We also know that the kinetic energy will be equal to the work that was performed on the object to put it in motion with the possible loss of kinetic energy due to friction or other factors over time.

Knowing those things, and assuming that the object gained its current velocity as a result of constant acceleration, we can derive an equation that represents the kinetic energy possessed by the object.

The definition of work – review

Recall the definition of work:

work = force * displacement * cos(theta)

where

- theta is the angle between the direction of displacement and the line of action of the force.

In this case, we might as well assume that theta is 0 degrees.

The equation for work

Therefore, W = f*d
where

- W represents work, and ultimately kinetic energy
- f represents force
- d represents distance or displacement

Acceleration is caused by force

We also know that a given force applied to an object will produce a given acceleration, as in
f = m*a
where

- f represents force
- m represents mass
- a represents acceleration

Distance traveled by an object under constant acceleration

In an earlier module, you learned that when an object is subjected to a constant acceleration, the distance traveled by the object in a given time is represented by
d = 0.5*a*t^2
where

- d represents the distance traveled
- a represents the constant acceleration
- t represents time

Substitution

Therefore, by substitution we can write
W = f*d = 0.5*f*a*t*t, or
W = 0.5*m*a*a*t*t

We're almost there

Recognizing that a = v/t where v represents velocity and t represents time, we can write
W = 0.5*m*(v/t)*(v/t)*t*t
Canceling terms in the numerator and denominator, we can write
W = 0.5*m*v*v = 0.5*m*v^2

And we're there

Since the kinetic energy possessed by the object equals the work that was done on the object to produce the current velocity (assuming no energy loss in the interval), we can write
KE = 0.5*m*v^2
where

- KE represents the kinetic energy possessed by the object
- m represents the mass of the object
- v represents the velocity of the object

Once again, even though I assumed constant acceleration to derive this equation, the equation is true regardless of the nature of the acceleration that caused the object to possess its current velocity.

KE varies as the square of the velocity

This is an extremely important equation, because it shows that the kinetic energy possessed by an object increases as the square of the velocity of the object. For example, if you are struck by a falling book with a velocity of 2 m/s, it will do four times as much work in damaging your body as a book with a velocity of only 1 m/s.

If the driver of a car increases the speed of the car from 55 mph to 70 mph, this represents about a 27-percent increase in speed but it also represents about a 62-percent increase in kinetic energy.

Miscellaneous facts about kinetic energy

Like work, kinetic energy is a scalar quantity. It has magnitude but not direction.

Also like work and potential energy, the standard unit of kinetic energy is the joule. However, there are many other units that are used to express energy including:

- 1 foot-pound = 1.36 joules
- 1 British thermal unit (Btu) = 1055.1 joules
- 1 kilowatt-hour = 3600000 joules
- 1 calorie = 4.18 joules
- 1 electron volt = 1.6*10^(-19) joules
- 1 hartree = 4.4*10^(-18) joules
- 1 ton of TNT = 4.18 *10^9 joules

Some sample calculations involving kinetic energy are presented later (p. 602) .

22.3.2 Mechanical energy

We know what it means to do work on an object. When a force is applied to an object causing a displacement of the object (such as lifting a small child and placing that child into a feeding chair), work has been done on the object.

Force is a two-way street

A force doesn't exist in isolation. From Newton's third law, we know that for each force that is applied to one object, an equal and opposite force is applied to the other object. When you lift the child and place the child into a feeding chair, you supply the force necessary to overcome the force of gravity and lift the child. (You also do the work necessary to increase the child's gravitational potential energy.)

Work is also a two-way street

In all cases where a force is applied to an object (work is done on the object), some other object must apply the force and do the work. That force may be applied by a person, a motor, a beast of burden, rushing water, wind, etc.

An exchange of energy

In all such cases, the object doing the work possesses either potential energy or kinetic energy or both that is exchanged with the object upon which the work is performed. (When you lift the child and place the child into a feeding chair, you are expending potential energy that you possess in order to increase the potential energy possessed by the child.)

The type of energy expended

For example, the energy being expended may be in the form of

- chemical energy such as the energy stored in food, fuel, or TNT
- atomic energy such as is used in a nuclear power plant
- radiant energy such as energy received from the sun
- gravitational potential energy and/or kinetic energy as in the case of water rushing downhill through a turbine,
- elastic potential energy such as the energy stored in the spring that closes the gate to my back yard after someone opens that gate and twists the spring

Work causes objects to gain energy

Whenever work is done on an object, that object gains energy (as in the case of lifting a child into a feeding chair, thereby increasing the child's gravitational potential energy).

Mechanical energy

According to The Physics Classroom (see http://www.physicsclassroom.com/class/energy/u5l1d.cfm [13]), the energy acquired by the objects upon which work is done is known as **mechanical energy** .

Mechanical energy is the energy that is possessed by an object due to its motion or due to its position (where position includes the deformation, stretching, compressing, etc., involved in elastic potential energy). Mechanical energy can be either *kinetic energy* resulting from motion or *potential energy* resulting from the position of the object.

A speeding car has kinetic energy. An airplane in flight has both kinetic energy and gravitational potential energy. A stretched or compressed or twisted spring has elastic potential energy. (The spring on my gate doesn't obviously stretch or compress when the gate is opened. Instead, it twists.)

The ability to do work

Mechanical energy is often defined as the ability to do work because an object that has mechanical energy can do work. For example, the book on the top bookshelf has the ability to do work on your head if it falls on your head.

When a falling book strikes your head

When at rest on the bookshelf, the book has mechanical energy in the form of gravitational potential energy. On the way down, some of that potential energy is converted to kinetic energy.

When the book strikes your head and causes your head, the bone in your head, or possibly both, to be displaced, it gives up some of its kinetic energy. The kinetic energy given up by the book is absorbed by your head.

At that point, the book still has mechanical energy in the form of potential energy, and possibly some kinetic energy as well. The remaining mechanical energy could do work on your toe if the book happens to land on your toe.

In summary...

The mechanical energy possessed by an object makes it possible for the object to apply a force to some other object to cause the other object to be displaced, thus doing work on the other object.

A moving hammer

For example, a moving hammer can cause a nail to be displaced and driven into a piece of wood. In the case of a hammer, simply laying the hammer on the head of the nail, which means applying the hammer's potential energy to the head of the nail, probably won't displace the nail. However, giving the hammer a good swing, thereby imparting kinetic energy into the hammer, gives the hammer the capability to displace the nail.

Potential energy being dissipated

If you swing a hammer repeatedly for the purpose of driving nails, after awhile you will probably begin to feel tired. This is because each time you swing the hammer and impart mechanical energy into the hammer, that energy is being expended by your body. After awhile, your available potential energy in the form of chemical energy will have been depleted (or at least reduced) and you will feel tired.

The suction- cup dart gun

When I was a child, a commonly available toy was a gun that was designed to shoot projectiles at a target. The projectiles (darts) were sticks with rubber suction cups on one end. (Note that darts also come in another form that has a sharp but sturdy pin on one end and a construction that is very aerodynamic. Those darts are meant to be thrown at a target instead of being shot from a gun.)

The suction-cup dart

For the suction-cup dart, the target was very smooth. If the dart struck the target straight on with very little angle, the suction cup would adhere to the target by squeezing air out from the suction cup and creating a weak vacuum.

Cocking the gun

To prepare the gun for shooting the dart, the stick would be inserted into the barrel of the gun and pushed against a spring contained inside the gun until some sort of catching mechanism would click into place. That mechanism would hold the spring in a compressed position until the child pulled the trigger.

[13] http://www.physicsclassroom.com/class/energy/u5l1d.cfm

With the spring compressed, the gun possessed mechanical energy due to the elastic potential energy of the spring.

Shooting the gun

When the trigger was pulled, the catch would release, the spring would expand, and the dart would shoot out of the gun at relatively high speed.

Be careful what you aim at

I still recall getting in trouble with my mother for lying on my bed and shooting darts at the ceiling, which made little round circles on the ceiling that she had to clean off. (I forgot to mention that you normally lick the suction cup before loading it into the gun so that it will make a good seal when it strikes the target.)

An exchange of mechanical energy

When the dart is loaded into the gun and the spring is compressed, the gun possesses mechanical energy. When the trigger is pulled, that mechanical energy does work on the dart causing the dart to be displaced very rapidly shooting out of the barrel of the gun. The elastic potential energy in the spring is exchanged for kinetic energy in the dart.

Another energy exchange

When the dart strikes the target, the kinetic energy in the dart is exchanged for elastic potential energy in the rubber suction cup on the front of the dart. When the striking angle is nearly perpendicular to the target, the air is forced out of the suction-cup causing the dart to adhere to the smooth surface.

Mechanical energy possessed by the dart

At that point in time, the dart has gravitational potential energy due to its height above the floor and the suction cup has elastic potential energy due to having been deformed within its elastic limit.

The slightest air leak will cause the suction cup to return to its original shape, thereby causing the dart to pop off the target. Then the gravitational potential energy possessed by the dart will cause it to fall to the floor.

22.3.3 Total mechanical energy

The total amount of mechanical energy possessed by an object is the sum of the potential and kinetic energy possessed by the object. Therefore,

TME = PE + KE

where

- TME represents the total mechanical energy
- PE represents the total potential energy
- KE represents the kinetic energy

Because the potential energy can be of two forms,

TME = PEs + PEg + KE

where

- TME represents the total mechanical energy
- PEs represents the potential energy due to deformation such as stretching, compressing, twisting, etc.
- PEg represents the potential energy due to gravity
- KE represents the kinetic energy

Energy is a scalar quantity

As mentioned earlier, energy is a scalar and not a vector quantity. Therefore, the total mechanical energy is simply the sum of the three types of energy possessed by the object. There are no angles or directions to contend with when computing the sum.

Conversion of energy between types

Over the course of time, a moving object can convert the types of energy between the different forms. A good example is the pendulum on a clock, or a child sitting motionless in a swing that is in motion.

When the pendulum reaches its highest point...

There is an instant in time where a swinging pendulum is at its highest point and it is not moving. At that instant, it has no kinetic energy but it has maximum gravitational potential energy.

When the pendulum reaches its lowest point...

As the pendulum goes through the lowest point in its swing, it has the greatest speed and therefore the greatest amount of kinetic energy. If you define the zero height reference as the height of the pendulum at that point, then it has no potential energy at that point in time.

Swapping kinetic energy for potential energy

Therefore, the total mechanical energy of the pendulum is swapped back and forth between kinetic energy and potential energy in a very smooth way.

If the pendulum were swinging in a friction-free environment, it would continue that process forever. The total amount of mechanical energy would stay the same but at any instant in time would be divided between potential and kinetic energy.

Of course, there is no such thing as a friction-free environment and a small amount of energy is dissipated by friction each time the pendulum swings through a cycle. Over time, therefore, the total mechanical energy in the pendulum would be depleted and the pendulum would stop swinging.

Pendulum clocks have springs

That is why pendulum clocks have springs or weights that store potential energy and impart a small amount of energy into the pendulum mechanism during each swing.

Where does the potential energy come from

That raises another issue. What happens to the clock when the spring or weights expend all of their stored energy. The answer is that eventually the pendulum will stop swinging and the clock will stop running unless a human imparts more energy into the spring or weight system that keeps the clock running.

Twenty-four hours of controlled energy transfers

I can still remember my grandfather winding the spring on his pendulum clock as the last thing he did before going to bed each night. He was converting potential energy derived from food into elastic potential energy in the clock's spring.

During the next 24 hours, that elastic potential energy would be converted to a combination of gravitational potential energy and kinetic energy in the pendulum, which would move the hands on the clock to indicate the correct time.

The next night, my grandfather would start the process all over again by imparting energy into the clock's spring.

22.4 Sample calculations

I will present and explain sample calculations involving both kinetic energy and mechanical energy in this section.

22.4.1 Kinetic energy

A roller coaster car

What is the kinetic energy possessed by a 1378 pound roller coaster car that is moving with a speed of 60 ft/sec?

Answer:

Enter the following expression into the Google search box:

0.5*1378lb * (60ft/s)^2

The resulting kinetic energy is 1.045 * 10^5 joules.

Similarly, you could enter the following into the Google search box:

0.5*625kg * (60ft/s)^2

and you would get the same answer of 1.045 * 10^5 joules.

As you can see, it doesn't matter whether you use English units or SI units for mass and velocity, as long as you do the calculation correctly. The answer is the same, because the kinetic energy of the object doesn't change just because you express those quantities in different units.

More on the roller coaster car

What would be the kinetic energy of another car with the same mass but with three times the speed?

Answer:

We don't even need to evaluate the equation to answer this question. We know that the kinetic energy varies as the square of the speed, so if we triple the speed, we will increase the kinetic energy by a factor of 3*3=9. Therefore,

KE = 9*1.045 * 10^5 joules = 9.405 * 10^5 joules

Dead Fred the daredevil

Dead Fred, the daredevil possessed a kinetic energy of 18000 joules just prior to hitting a solid brick wall on his motorcycle. If his mass was 176 pounds, what was his speed?

Answer:

KE = 0.5*m*v^2, or

v^2 = KE/(0.5*m), or

v = sqrt(KE/(0.5*m)), or

v = sqrt(18000joules/(0.5*176pound))

Enter the right-hand expression above into the Google search box and you should get a velocity of

v = 21.24 m/s

22.4.2 Mechanical energy

A dart gun

A dart with a mass of 0.1 kg leaves the muzzle of a dart gun with a velocity of 10 m/s. The child is holding the dart gun at a position that is 1 m above the ground. At that instant,

1. What is the potential energy of the dart?
2. What is the kinetic energy of the dart?
3. What is the total mechanical energy of the dart?
4. Assuming no losses of energy due to friction or other causes, what is the change in the total mechanical energy of the gun at the instant the dart exits the gun?

Answers:

1. Assuming that the dart was not deformed as it was forced out of the barrel of the gun, the potential energy of the dart is given by

mass*gravity*height

Enter the following into the Google search box:

(0.1 kg) * (9.8 (m / (s^2))) * (1 m)

The result should be:

Potential energy = 0.98 joules

2. The kinetic energy of the dart is given by

0.5*m*v^2

Enter the following into the Google search box:

0.5*0.1kg*(10m/s)^2

The result should be:

kinetic energy = 5 joules

3. The total mechanical energy of the dart is the simple sum of the potential energy and the kinetic energy, which is:

Total mechanical energy = 5.98 joules

4. When the dart exits the gun, the gravitational potential energy of the gun is reduced because the total mass of the gun and the dart is reduced by the mass of the dart.

In addition, the elastic potential energy stored in the spring is imparted into the dart in the form of kinetic energy.

Therefore, the loss in mechanical energy of the gun is equal to the total mechanical energy of the dart immediately upon exit from the gun barrel. Therefore, the total mechanical energy of the gun is reduced by 5.98 joules when the dart exits the gun.

A crate on a ski run

A crate containing soft drinks with a mass of 5 kg is accidentally released at the top of a ski run and slides down the ski run to the valley below. The height of the point where the crate is released is 100 m above the valley floor. The crate goes through numerous dips and over many small hills on the way down but never stops.

Assuming there is no friction, no air resistance, no deformation, and no loss of energy in any form during the trip, what is the magnitude of the crate's velocity when it reaches the valley floor?

Answer:

As presented, this is a simple case of the conversion of gravitational potential energy into kinetic energy. The fact that the crate slowed down and sped up several times during the trip while negotiating little dips and hills doesn't matter. All that really matters is the balance of energy between the starting point and point where the crate reached the valley floor. With no energy loss during the trip, the total mechanical energy at the end of the trip must equal the total mechanical energy at the beginning of the trip.

At the top of the hill, the crate's gravitational potential energy was equal to

m*g*h = 5kg*(9.8m/s^2)*100m = 4900 joules

Therefore, at the bottom of the hill, with no remaining potential energy, the crate's kinetic energy must be equal to

0.5*m*v^2 = 4900 joules

Rearranging terms gives us

v^2 = (4900 joules)/(0.5*m), or

v = sqrt((4900 joules)/(0.5*m)), or

v = sqrt((4900 joules)/(0.5*5kg))

Entering this expression into the Google calculator gives us the crate's velocity when it reached the valley floor as

v = 44.3 m/s

22.5 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

22.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

22.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Energy – Kinetic and Mechanical Energy
- File: Phy1190.htm

- Revised: 10/02/15
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · potential energy
  - · work
  - · gravitational potential energy
  - · elastic potential energy
  - · kinetic energy
  - · mechanical energy
  - · total mechanical energy

Disclaimers: **Financial** : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 23

Energy – Power[1]

23.1 Table of Contents

- Preface (p. 607)
  - General (p. 607)
  - Prerequisites (p. 607)
  - Supplemental material (p. 608)
- General background information (p. 608)
- Sample calculations (p. 610)
- Do the calculations (p. 611)
- Resources (p. 611)
- Miscellaneous (p. 611)

23.2 Preface

23.2.1 General

This module is part of a book [2] *(or collection)* designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains power in a format that is accessible to blind students.

23.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .

[1] This content is available online at <http://cnx.org/content/m38354/1.3/>.
[2] http://cnx.org/contents/a7842db2-93c6-4c06-a5e3-56c282358143
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/

- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

23.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

23.3 General background information

What is work?

You learned in an earlier module that work occurs when a force causes a mass to be displaced by some distance. You learned that the equation for the quantity of work done is equal to

W = (f*newton)*(d*meter) = f*d*N*m

You also learned that work is measured in joules, where one joule is equal to one newton multiplied by one meter.

1 joule = 1 N * 1 m, or

1 joule = (1 kg * m/s^2) * m, or

1 joule = 1 kg*m^2/s^2

Paste the right-hand expression into the Google search box and press Enter just to be sure.

What about time?

Note that the equation for work says nothing about time. The same amount of work is done if it takes one second or one month for the object to which the force is applied to move by the same distance.

That doesn't sound right!

This goes against our normal concept of work. If Joe spreads one cubic yard of topsoil on the lawn in one hour and Bill requires three hours to do the same job, we might say that Joe is working harder than Bill.

Power

To be correct from a physics viewpoint, we would need to say that Joe is delivering more **power** than Bill. In other words, **power** is a measure of the rate at which work is done. They both do the same amount of work, but Joe does it more quickly than Bill. Hence Joe delivers more power than Bill.

[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

Power in equation form

Power is the ratio of work to time. In equation form,

Power = Work/time

The SI unit for power

The SI unit for power is the **watt** . One watt of power is being delivered when one joule of work or energy is being delivered each second.

An electric heater

In other words, if you have an electric heater that is properly rated at 60 watts, it will deliver 60 joules of energy per second when it is turned on.

This means that somewhere in the world, someone or something must be doing work at a rate of 60 joules per second in order to insert the energy into the electrical grid that your heater will be taking out of the grid and turning into heat energy.

Horsepower

For historical reasons, particularly in the U.S., we also use the term *horsepower* to describe the power delivered by a machine. This is particularly true in the automotive industry, but it applies to other kinds of machines as well. We might speak of a car with a 300 horsepower engine, or a clothes washing machine with a quarter-horsepower motor.

One horsepower is equal to approximately 750 watts.

Most machines do work

Most machines are designed to consume electrical or chemical energy and do work on an object. Some machines, such as the treadmill at the health center, are designed to consume human energy in order to do work.

In order for a machine to do work, it must consume energy in some form.

Power ratings for machines

Machines are often described by a power rating. The power rating indicates the rate at which that machine can do work on objects.

When I was in the military many years ago, there were potato peeling machines in kitchens in the mess halls. Their purpose was to do work on potatoes by removing the peel. Presumably a machine with a high power rating could peel more potatoes per hour than one with a lower power rating.

Automobile engines and horsepower

Automobile engines are often rated in terms of power using horsepower as the units. At the drag-race track, contests are held to determine which vehicle can move from point A to point B in the shortest amount of time.

Since work is measured as force multiplied by distance, and power is measured as the work done per unit of time, everything else being equal, one would expect that the vehicle with the highest power rating would be the winner in moving a given distance in the shortest amount of time.

What about the units?

What are the units of power? We know that

Power = force * distance/seconds

We know that the units of force are

f = m*a = kg*m/s^2

We know that the units of time are seconds, and the units of distance are meters. Therefore,

Power = f*d/time = (kg*m/s^2)*(m/s) = (kg*m^2)/(s^3), or

Power = kg*(m^2)*(s^(-3))

Plug the right-hand expression into the Google search box and you will learn that

1 watt = 1 kg*(m^2)*(s^(-3)), or

1 watt = 1 N*m/s

Another viewpoint

As explained above,

Power = force*distance/time

We learned in earlier modules that velocity is equal to the ratio of displacement and time. Therefore,

Power = force * velocity

Therefore, power is proportional to both force and velocity. A truck in a load-pulling contest that moves rather slowly but with great force is powerful.

Similarly, a racing motorcycle that moves very fast with relatively little force is also powerful.

And the great granddaddy of them all, a huge boulder that plows through a house at great speed during a landslide is very powerful.

23.4 Sample calculations

A story of two cranes

One crane named A lifts a 1000 kg object to a height of 100 meters in 10 seconds. Another crane named B requires 100 seconds to do the same thing.

Which crane does the most work?

Which crane delivers the most power?

Solution:

Both cranes do the same amount of work by displacing the same object the same distance against the force of gravity. The work done is equal to

(1000kg*9.8m/s^2)*100m = 980000 joules

Crane A delivers 980000 joules in 10 seconds. Therefore, crane A delivers

(980000 joules) / (10 seconds) = 98000 watts

Crane B delivers

(980000 joules) / (100 seconds) = 9800 watts

Therefore, crane A delivers the most power.

Another story about cranes

One crane named A lifts a 1000 kg object to a height of 100 meters in 10 seconds. Another crane named B lifts a 500 kg object to a height of 100 meters in 5 seconds.

Which crane does the most work?

Which crane delivers the most power?

Solution:

The work done by crane A is

(1000 kg) * (9.8 (m / (s^2))) * (100 m) = 980000 joules

The work done by crane B is

(500 kg) * (9.8 (m / (s^2))) * (100 m) = 490000 joules

Therefore crane A does the most work.

Crane A delivers 980000 joules in 10 seconds. Therefore, crane A delivers

(980000 joules) / (10 seconds) = 98000 watts

Crane B delivers

(490000 joules) / (5 seconds) = 98000 watts

Therefore, both cranes deliver the same amount of power.

Your electric bill

An electric bill is often expressed in terms of kilowatt-hours (kwh). One kilowatt-hour represents a power expenditure of 1000 watts in one hour.

How many joules of energy are represented by 100 kwh?

Solution:

1 kwh = 1000 watt * 1 hour * 3600 s/hour, or

1 kwh = 3.6*10^6 watt*s

1 watt = 1 N*m/s, therefore

1 kwh = 3.6*10^6 *(N*m/s) * s = 3.6*10^6 N*m

1 joule = 1 N*m, therefore

1 kwh = 3.6*10^6 joules, and

100 kwh = 3.6*10^8 joules

23.5 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

23.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

23.7 Miscellaneous

This section contains a variety of miscellaneous information.

Housekeeping material

- Module name: Energy – Power
- File: Phy1200.htm
- Revised: 10/02/15
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - potential energy
  - work
  - gravitational potential energy
  - elastic potential energy
  - kinetic energy
  - mechanical energy
  - total mechanical energy
  - power
  - watt

Disclaimers: Financial : Although the **openstax CNX** site makes it possible for you to download a PDF file for the collection that contains this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

You also need to know that Prof. Baldwin receives no financial compensation from **openstax CNX** even if you purchase the PDF version of the collection.

In the past, unknown individuals have copied Prof. Baldwin's modules from cnx.org, converted them to Kindle books, and placed them for sale on Amazon.com showing Prof. Baldwin as the author. Prof. Baldwin neither receives compensation for those sales nor does he know who does

receive compensation. If you purchase such a book, please be aware that it is a copy of a collection that is freely available on **openstax CNX** and that it was made and published without the prior knowledge of Prof. Baldwin.

Affiliation : Prof. Baldwin is a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 24

Energy – Internal and External Forces[1]

24.1 Table of Contents

- Preface (p. 613)
  - General (p. 613)
  - Prerequisites (p. 613)
  - Supplemental material (p. 614)
- Discussion (p. 614)
- Resources (p. 617)
- Miscellaneous (p. 617)

24.2 Preface

24.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains internal and external forces in a format that is accessible to blind students.

24.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).

[1] This content is available online at <http://cnx.org/content/m38362/1.2/>.
[2] http://cnx.org/content/col11294/latest/
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php

- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

24.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

24.3 Discussion

Work

You learned in an earlier module that work is done on an object when that object is displaced by a force. You further learned that the amount of work done is proportional to the product of the force, the displacement distance, and the cosine of the angle between them.

Mechanical energy

You also learned in an earlier module that mechanical energy is the energy that is possessed by an object due to its motion or due to its position *(where position includes the deformation, stretching, compressing, etc., involved in elastic potential energy)* .

Mechanical energy can be either kinetic energy resulting from motion or potential energy resulting from the position of the object.

Two categories of force

When speaking of work and energy, we can categorize force into two categories:

- internal or conservative force
- external or non-conservative force

In theory, the distinction between the two categories is not complicated. Work done on an object solely by internal forces cannot change the total mechanical energy possessed by an object. Work done on an object by external forces can change the total mechanical energy possessed by an object.

Internal forces

Internal forces are forces that can act on an object without physically touching the object. Examples of internal forces include:

- the force of gravity

[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

- electrical forces
- magnetic forces
- spring forces

These forces cannot change the total mechanical energy possessed by an object, but can transform that energy between potential energy and kinetic energy.

External forces

Examples of external forces include:

- applied force
- normal force
- tension force
- friction force
- air resistance force

When work is done on an object by external forces, the total mechanical energy, consisting of kinetic energy plus potential energy, will change. The work can be positive, in which case the total mechanical energy will increase. The work can be negative, in which case the total mechanical energy will decrease. The change in mechanical energy will be equal to the net work that is done on the object.

A Super Ball example

Assume that a Super Ball *(a toy manufactured by Wham-O that bounces with great vigor)* is at rest on a table. The ball has a certain amount of potential energy due to its position relative to the floor. While at rest, however, it has no kinetic energy. The total mechanical energy possessed by the ball is the potential energy due to gravity.

The ball falls

Assume that the ball is dangerously close to the edge of the table and the family cat accidently tips the ball off the table.

As the ball falls towards the floor, it will be losing potential energy because its height above the floor will be decreasing. At the same time, it will be gaining kinetic energy because its speed will be increasing.

A transfer of energy

Assuming that we can neglect air resistance and that no forces other than the force of gravity are acting on the ball, during its fall, it will be subjected only to the *internal* force of gravity. During that time interval, its total mechanical energy cannot change. The loss in potential energy will be replaced by an increase in kinetic energy and the total mechanical energy possessed by the ball will remain constant.

An external force is applied

Suddenly the ball reaches the floor. At that instant, an *external* upward force will be applied to the ball by the normal force exerted by the floor. There will be a change in the total mechanical energy possessed by the ball, if for no reason other than the fact that some of the energy is converted into sound wave energy. Both the kinetic energy and the gravitational potential energy will go to zero for a very short period of time.

Negative work

The work done on the ball by the floor will be negative because the force will be in the opposite direction of the direction of the displacement. Thus, the total mechanical energy will be decreased.

Conversion into elastic potential energy

However, some of the mechanical energy will probably be converted into elastic potential energy as the ball is compressed.

Shortly thereafter, that elastic potential energy will be converted into kinetic energy as the ball expands causing it to bounce up.

Another trip involving only internal forces

On the way up from the bounce, the potential energy due to gravity will be increasing and the kinetic energy will be decreasing until the ball reaches the top of the bounce.

For an instant, the kinetic energy will be zero and the gravitational potential energy will be at its maximum. Then the ball will begin its trip back toward the floor, gaining kinetic energy and losing potential energy along the way.

Application of another external force

Then the ball will strike the floor again, and the process will repeat. Each time the ball strikes the floor, some of the total mechanical energy will be lost, having been converted into some other form of energy such as thermal energy, sound wave energy, etc.

All of the potential energy is expended

Eventually, an amount of energy equal to the original potential energy will have been converted into those other forms of energy and the ball will come to rest on floor. However, because of the design of the Super Ball, many cycles of the process may be required for the ball to come to rest.

An aside – a ball of Play Doh

Consider what would happen if instead of being a Super Ball, the toy were a ball of Play Doh *(a toy manufactured by Hasbro that doesn't bounce very well)* . The ball of Play Doh would probably only make one trip from the tabletop to the floor.

On the way down, the ball of Play Doh would exchange potential energy for kinetic energy. When it hits the floor, all of the kinetic energy would probably be expended by converting it to heat energy, sound energy, etc., and by deforming the ball of Play Doh beyond its elastic limit. (Play Doh has a very low elastic limit, if any.)

Not done with the Super Ball yet

Now let's get back to the case of the Super Ball. Keep in mind that the total mechanical energy possessed by the ball when it comes to rest on the floor is not zero, because it still feels an attraction due to the force of gravity. Thus, it still possesses gravitational potential energy.

If all of this has been happening in a second-story loft, and the cat comes along again and causes the ball to roll off the edge of the loft floor, a process similar to that described above will occur all over again involving the ball and the floor below the loft.

Another phenomena

There is another phenomena that would probably occur with the Super Ball, but which would be visible only through the use of high-speed photography or other sensitive technology.

Oscillations

The impact with the floor would probably cause the ball to go into spring-like oscillations. By this I mean that it would be compressed by the impact with the floor. When it expands, (in addition to causing it to bounce upward), the ball would probably over-expand or stretch and gain elastic potential energy in the stretched configuration.

Energy conversion

During this process, the potential energy due to compression would be converted to kinetic energy as the molecules in the ball fly away from one another, and then be converted into potential energy again as the molecules fly too far away from one another (stretch).

Compress again

Then the ball would compress again and the process of stretching and compressing would continue until the elastic potential energy stored by the compression of the impact with the floor is expended by transforming it into heat energy, sound energy, etc.

Between impacts with the floor, this process would be occurring in addition to the activity involving gravitational potential energy.

A Slinky toy

A Slinky is a toy that was patented by R.T. James in 1946 and is currently manufactured by James industries.

The original Slinky toy was a coil spring made of spring steel with a diameter of about two inches. I can still remember the first time I saw one "crawling" down a flight of stairs, and I can remember the student that brought it to elementary school to show it off. (Slinky toys are also made of plastic these days and are made with different diameters.)

If you hold one end of a Slinky toy and allow the other end to fall toward the floor, the type of oscillations described above can easily be observed by a sighted person. The bottom end of the spring goes up and down rather slowly, exchanging elastic potential energy for kinetic energy and back again during each cycle.

Probably discernable by a blind student

If a blind student were to do this with a Slinky toy, particularly with one of the older and heavier versions made of spring steel, the student would probably be able to feel the oscillations as the bottom end of the spring goes up and down.

The blind student could also probably experience the phenomena by gently touching one side of the spring and feeling the individual coils moving up and down as the spring stretches and compresses.

Conservative versus non-conservative forces

Internal forces are often referred to as *conservative* forces because they are incapable of changing the total mechanical energy possessed by an object. In other words, the mechanical energy possessed by an object is conserved when the object is acted upon by internal forces.

External forces, on the other hand, are often referred to as *non-conservative* forces for exactly the opposite reason. The mechanical energy possessed by an object is not conserved when the object is acted upon by external forces. In particular, the mechanical energy possessed by the object will either increase or decrease by the net positive or negative work done on the object by external forces.

External forces always occur

While discussing physics concepts, we often like to assume conditions such as a friction-free surface, etc. However, there is no such thing as a friction-free surface. If there were, we could build the ultimate perpetual-motion machine and solve the world's energy problems forever.

A skier on a friction-free surface

If we speak of a skier gliding down a hill on a friction free surface, we can talk about the skier losing potential energy and gaining kinetic energy on the way down. Neglecting air resistance and other external forces, we can say that there is no change in the mechanical energy possessed by the skier on the trip down the hill. The skier's potential energy is converted to kinetic energy.

Bad for the ski business

But what happens when the skier reaches the bottom of the hill? Some external force must be applied to the skier to cause the skier's velocity to change. (Remember, the velocity of an object can only be changed by the application of a force.)

Without the application of that external force, the skier would continue moving with the same velocity and disappear over the horizon. It would be bad for the ski business if every skier disappeared after only one trip down the ski slope.

A falling object

It has been said that a fall doesn't hurt you. It's the sudden stop (negative acceleration) at the end of the fall that hurts. We can discuss the concept of only internal forces acting on an object while it is falling by neglecting air resistance and other external forces.

Eventually, however, that falling object must reach the surface of the earth, the floor, the table top, or some other surface that represents the zero reference for gravitational potential energy. When that happens, an external force must occur to change the object's velocity and change the mechanical energy possessed by the object.

24.4 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

24.5 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Energy – Internal and External Forces for Blind Students

- File: Phy1210.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · potential energy
  - · work
  - · gravitational potential energy
  - · elastic potential energy
  - · kinetic energy
  - · mechanical energy
  - · total mechanical energy
  - · power
  - · watt
  - · internal force
  - · conservative force
  - · external force
  - · non-conservative force

NOTE: **Disclaimers: Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 25

Energy – Elastic and Inelastic Collisions in Two Dimensions[1]

25.1 Table of Contents

- Preface (p. 619)
  - General (p. 619)
  - Prerequisites (p. 620)
  - Viewing tip (p. 620)
    - Figures (p. 620)
    - Listings (p. 621)
  - Supplemental material (p. 621)
- Discussion (p. 621)
- Example scenarios (p. 624)
  - One-dimensional scenarios (p. 624)
    - The rear end car crash (p. 624)
    - A perfectly inelastic car crash (p. 628)
  - Two-dimensional scenarios (p. 631)
    - Elastic collision between two pucks on friction-free ice (p. 632)
    - Perfectly inelastic collision between objects with odd angles (p. 636)
    - Rotating the axes for simplification (p. 640)
- Run the scripts (p. 647)
- Resources (p. 647)
- Miscellaneous (p. 647)

25.2 Preface

25.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

[1] This content is available online at <http://cnx.org/content/m38431/1.2/>.
[2] http://cnx.org/content/col11294/latest/

This module explains elastic and inelastic collisions in two dimensions in a format that is accessible to blind students.

25.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

25.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

25.2.3.1 Figures

- Figure 1 (p. 622) . Equations for collisions of two objects in two-dimensional space.
- Figure 2 (p. 628) . Output for the rear end car crash.
- Figure 3 (p. 631) . Output for the perfectly inelastic car crash.
- Figure 4 (p. 636) . Output for an elastic collision between two pucks on friction-free ice.
- Figure 5 (p. 640) . Output for a perfectly inelastic collision between objects with odd angles.
- Figure 6 (p. 646) . Output for rotation of the axes for simplification.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp

25.2.3.2 Listings

- Listing 1 (p. 624) . The rear end car crash.
- Listing 2 (p. 628) . A perfectly inelastic car crash.
- Listing 3 (p. 632) . Elastic collision between two pucks on friction-free ice.
- Listing 4 (p. 636) . Perfectly inelastic collision between objects with odd angles.
- Listing 5 (p. 641) . Rotation of the axes for simplification.

25.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

25.3 Discussion

I have touched on collisions in one dimension in earlier modules. I will deal with collisions in a more rigorous manner in this module, and will also extend the analysis to two dimensions.

NOTE: **Facts worth remembering – Types of collisions**

An **elastic** collision is one in which the total kinetic energy is the same before and after the collision.

An **inelastic** collision is one in which the final kinetic energy is less than the initial kinetic energy.

A **perfectly inelastic** collision is one that results in the two objects sticking together. The decrease of kinetic energy in a perfectly inelastic collision is as large as possible (consistent with the conservation of momentum).

Momentum is conserved regardless of whether the collision is elastic or inelastic.

A general solution for elastic collisions

I will provide you with three equations that apply in general to elastic collisions in two dimensions. However, as you will see, there are more than three variables involved in such collisions. With only three equations, you can only solve for three unknowns. Therefore, in order to solve the general problem, the values of all the other variables must be known.

The two-dimensional solution can be applied to one-dimensional problems by constraining the directions of motion of the two objects to either be the same or to differ by 180 degrees. If possible, such problems should be structured to cause the directions to be along the x-axis. This will often simplify the solution.

A general solution for inelastic collisions

The case for inelastic collisions is more restrictive than the case for elastic collisions. Only two of the equations mentioned above apply to inelastic collisions. As a result, you can only solve for two unknown values for an inelastic collision. The values for all of the other variables must be known.

Collision equations

The equations for collisions of two objects in two-dimensional space are shown in Figure 1 (p. 622) . Note that it is assumed that the two objects constitute an isolated system – that is, a closed system that is not subject to external interactions. This requires that both the magnitude and the direction of the momentum for the system be the same at the beginning and the end of the collision.

[12] http://www.dickbaldwin.com/toc.htm

Equations for collisions of two objects in two-dimensional space.

```
    Using conservation of momentum alone, we have two
equations, allowing us to solve for two unknowns.

m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

Using conservation of kinetic energy for the elastic
case gives us one additional equation, allowing us
to solve for three unknowns.

0.5*m1*u1^2 + 0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

Velocities can be decomposed into their x and
y-components using the following equations:

u1x = u1*cos(a1)
u1y = u1*sin(a1)

u2x = u2*cos(a2)
u2y = u2*sin(a2)

v1x = v1*cos(b1)
v1y = v1*sin(b1)

v2x = v2*cos(b2)
v2y = v2*sin(b2)

Substitution yields the following for the two momentum
equations:

m1*u1*cos(a1) + m2*u2*cos(a2) = m1*v1*cos(b1) + m2*v2*cos(b2)
m1*u1*sin(a1) + m2*u2*sin(a2) = m1*v1*sin(b1) + m2*v2*sin(b2)

where:

m1 and m2 are the masses of the two objects in kg

u1 and u2 are the magnitudes of the initial
velocities of the two objects. Velocities are
measured in meters/second

v1 and v2 are the magnitudes of the final velocities
of the two objects

u1x, u1y, v1x, and v1y are the x and y-components
of the initial and final velocities in 2D space.

a1 and a2 are angles that describe the initial
directions of the two objects through 2D space.
Angles are measured counter-clockwise relative to
the positive x-axis
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

What do the equations imply?

The first two equations based on momentum in Figure 1 (p. 622) require that the combined momentum of the two objects be the same, along each axis, before and after the collision. Thus, one of the equations deals with momentum along the horizontal axis and the other equation deals with momentum along the vertical axis.

The terms in the two equations are:

- m1 and m2 represent the masses of object 1 and object 2.
- u1x and u2x represent the components of the velocities of the two objects along the horizontal or x axis before the collision.
- u1y and u2y represent the components of the velocities of the two objects along the vertical or y axis before the collision.
- v1x and v2x represent the components of the velocities of the two objects along the horizontal or x axis after the collision.
- v1y and v2y represent the components of the velocities of the two objects along the vertical or y axis after the collision.

The third or energy equation

As indicated in Figure 1 (p. 622) , this equation can only be used for the case of an elastic collision. This equation requires that the total kinetic energy of the two objects before the collision be equal to the total kinetic energy of the two objects after the collision.

In addition to the masses described above, this equation introduces the following terms:

- u1 and u2 represent the magnitudes of the velocities of the two objects before the collision.
- v1 and v2 represent the magnitudes of the velocities of the two objects after the collision.

Definition of the angles

In order to compute the horizontal and vertical components of the velocities before and after the equations, you must know the directions in which the objects are moving before and after the collision. Those directions appear as angles in Figure 1 (p. 622) where

- a1 and a2 are angles measured counter clockwise relative to the horizontal axis that represent the direction of travel of each of the objects respectively before the collision.
- b1 and b2 are angles measured counter clockwise relative to the horizontal axis that represent the direction of travel of each of the objects respectively after the collision.

Ten variables

As you can see in Figure 1 (p. 622) , these equations involve ten variables. That means that in order for a solution to be possible, the values for eight of the variables must be know for an inelastic collision, and the values for seven of the variables must be known for an elastic collision.

Not conceptually difficult

If you believe in the laws of conservation of momentum and conservation of energy on which these three equations are based, the solution to collision problems is not conceptually difficult.

However, depending on which variables are known, which are unknown, and whether the collision is elastic, inelastic, or perfectly inelastic, you can come up with equations that are difficult to solve from an algebra/trigonometry viewpoint.

Axis rotation

For the case where none of the given directions are along the x- axis or the y-axis, you can sometimes simplify the algebraic/trigonometric problem by rotating the axis so as to place one of those directions along the x-axis or the y-axis. This will often cause one or more terms in the set of equations to go to zero, thus simplifying the solution to the set of equations.

Having done that, you can rotate the axis by the same amount in the opposite direction at the end to cause the final solution to apply to the original axes. I will present an example of this in the next section.

25.4 Example scenarios

Several examples

I will use the information in Figure 1 (p. 622) to analyze several scenarios involving collisions in both one dimension and two dimensions in this section.

The use of JavaScript

All of these examples could be solved using the Google calculator. However, several steps are involved and I find it easier to keep things organized and perform the steps in the correct order by using JavaScript to compute and display the solution.

Note, however, that JavaScript will only do the arithmetic for you. You must still do the algebra/trigonometry yourself. In these examples, I will usually work through the algebra in comment sections and switch to actual code when it is time to compute and display one or more values.

25.4.1 One-dimensional scenarios

The first one-dimensional scenario involves an automobile accident.

25.4.1.1 The rear end car crash

The description as well as the solution to the problem are shown in Listing 1 (p. 624) .

Listing 25.1: The rear end car crash.

```
    <!---------------- File JavaScript01.html --------------------->
<html><body>
<script language="JavaScript1.3">

/*
This script simulates car #2 rear-ending car #1 in a
one-dimensional elastic collision while car #1 was stopped.
The script computes and displays the speed of car #2
immediately before the collision under the assumption that
car #1 was moving at 20 m/s just after the collision.


Using conservation of momentum alone, we have two
equations, allowing us to solve for two unknowns.

m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

Using conservation of kinetic energy for the elastic
case gives us one additional equation, allowing us
to solve for three unknowns.

0.5*m1*u1^2 + 0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

Variables:
m1, m2, u1, u2, v1, v2, a1, a2, b1, b2
*/

document.write("Start Script </br>");
```

```
//Solve for u2 and v2 for an elastic collision
var m1 = 1500;//kg
var m2 = 2000;//kg
//Velocities before the collision
var u1 = 0;//meters per second - standing still
var u2;//unknown value to be determined
//Velocities after the collision
var v1 = 20;//meters/second
var v2;//unknown value to be determined
//Angles
var a1 = 0;//car was not moving
var a2 = 0;//moving straight ahead
var b1 = 0;//moving straight ahead
var b2 = 0;//moving straight ahead

//Convert angles to radians
A1 = a1*Math.PI/180;
A2 = a2*Math.PI/180;
B1 = b1*Math.PI/180;
B2 = b2*Math.PI/180;

//Compute and print the x and y components of velocity
u1x = u1*Math.cos(A1)
u1y = u1*Math.sin(A1)

//u2x = u2*Math.cos(A2)//unknown
//u2y = u2*Math.sin(A2)//unknown

v1x = v1*Math.cos(B1)
v1y = v1*Math.sin(B1)

//v2x = v2*Math.cos(B2)//unknown
//v2y = v2*Math.sin(B2)//unknown

document.write("x and y components of velocity</br>");
document.write("u1x = " + u1x.toFixed(3) + "</br>");
document.write("u1y = " + u1y.toFixed(3) + "</br>");
document.write("v1x = " + v1x.toFixed(3) + "</br>");
document.write("v1y = " + v1y.toFixed(3) + "</br>");
document.write("================================"+ " </br>");

/*
Prepare the equations for use in solving the problem.

Given the following three equations
m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y
0.5*m1*u1^2 + 0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

Eliminate all of the components for which the above printout
```

```
shows zero or for which the given values show zero.

0 + m2*u2x = m1*v1x + m2*v2x
0 + m2*u2y = 0 + m2*v2y
0 + 0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

Although it isn't totally obvious from the equations, at this
point we need to recognize that because all velocities are
defined to occur along the x-axis, all of the terms in the
middle equation above that deals with the y-component of
velocity must be zero. Therefore, we can eliminate that
equation entirely.

We also need to recognize that because there are no velocity
components along the y-axis, the velocity components along
the x-axis are actually the magnitudes of those velocity
components. Thus, u2x = u2.

Now we will make the substitutions and eliminate terms with a
value of 0 in the process, yielding

m2*u2 = m1*v1 + m2*v2
0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

Substituting known values into the two equations yields

2000*u2 = 1500*20 + 2000*v2
2000*u2^2 = 1500*20*20 + 2000*v2^2

Simplifying the two equations yields

u2 = 15 + v2
u2*u2 = 300 + v2*v2

Now we need to eliminate one equation through substitution
v2 = u2 - 15

u2*u2 = 300 + (u2 - 15)*(u2 - 15)
u2*u2 = 300 + u2*u2 - 30*u2 +225
u2*u2 - 300 - u2*u2 + 30*u2 -225 = 0
u2*u2 - u2*u2 + 30*u2 - 300 -225 = 0
30*u2 - 525 = 0
30*u2 = 525
*/

//Compute and print the first speed value
document.write("Speed values</br>");
u2 = 525/30;
document.write("u2 = " + u2.toFixed(2) + " m/s</br>");

/*
```

```
Substituting this value back into an earlier energy equation
yields

v2*v2 = u2*u2 - 300;
*/

//Compute and display the second speed value
v2 = Math.sqrt(u2*u2 - 300);
document.write("v2 = " + v2.toFixed(2) + " m/s</br>");
document.write("================================"+ " </br>");

//Check the answers for conservation of momentum
document.write("Check for conservation of momentum</br>");
var mou = m1*u1 + m2*u2;
var mov = m1*v1 + m2*v2;

document.write("mou = " + mou.toFixed(0) + " Kg*m/s</br>");
document.write("mov = " + mov.toFixed(0) + " Kg*m/s</br>");

document.write("================================"+ " </br>");

//Check the answer for elastic collision
var keIn = 0.5*m1*u1*u1 + 0.5*m2*u2*u2;
var keOut = 0.5*m1*v1*v1 + 0.5*m2*v2*v2;

document.write("Check for conservation of energy</br>");
document.write("keIn = " + keIn.toFixed(0)
  + " Kg*m^2/s^2</br>");
document.write("keOut = " + keOut.toFixed(0)
  + " Kg*m^2/s^2</br>");

document.write("End Script");

</script>
</body></html>
```

The output

The output produced by this script is shown in Figure 2 (p. 628) .

The only comments that I will make in addition to the comments in Listing 1 (p. 624) are that the values displayed for mou, mov, keIn, and keOut near the end of Figure 2 (p. 628) show that both momentum and kinetic energy were conserved. Therefore, the requirements for an elastic collision were met.

The meaning of each of those terms is as follows:

- mou represents total momentum before the crash
- mov represents total momentum after the crash
- keIn represents total kinetic energy before the crash
- keOut represents total kinetic energy after the crash

Output for the rear end car crash.

```
    Start Script
x and y components of velocity
u1x = 0.000
u1y = 0.000
v1x = 20.000
v1y = 0.000
==============================
Speed values
u2 = 17.50 m/s
v2 = 2.50 m/s
==============================
Check for conservation of momentum
mou = 35000 Kg*m/s
mov = 35000 Kg*m/s
==============================
Check for conservation of energy
keIn = 306250 Kg*m^2/s^2
keOut = 306250 Kg*m^2/s^2
End Script
```

Figure 25.2: Output for the rear end car crash.

25.4.1.2 A perfectly inelastic car crash

The description and the solution to another rear end car crash are provided in Listing 2 (p. 628) . Whereas the previous car-crash scenario described an elastic collision, this scenario describes a perfectly inelastic collision.In this crash, the two cars become entangled and move forward as a single object following the collision. Therefore, this is an example of a perfectly inelastic collision.

Listing 25.2: A perfectly inelastic car crash.

```
    <!---------------- File JavaScript10.html ---------------------->
<html><body>
<script language="JavaScript1.3">

/*
This script simulates car #2 rear-ending car #1 in a
one-dimensional perfectly inelastic collision while car #1 was
stopped.
```

```
Assume that car #2 was moving at 17.5 m/s immediately before
the collision. The cars became entangled and moved as a single
object in a straight line following the collision. Find
the velocity of the two objects immediately following the
collision. Find the total momentum of the two objects
immediately before and immediately after the collision.


Using conservation of momentum alone, we have two
equations, allowing us to solve for two unknowns.

m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

However, because the cars are moving along the x-axis, all
terms in the second equation must be zero. This limits us to
only the first equation shown above. Note also that because
this is an inelastic collision, we can't use the equation
based on conservation of energy.

Variables:
m1, m2, u1, u2, v1, v2, a1, a2, b1, b2
*/

document.write("Start Script </br>");

var m1 = 1500;//kg
var m2 = 2000;//kg
//Velocities before the collision
var u1 = 0;//meters per second - standing still
var u2 = 17.5;//meters per second
//Velocities after the collision
var v1;//unknown but v2 = v1
var v2;//unknown value to be determined
//Angles
var a1 = 0;//car was not moving
var a2 = 0;//moving straight ahead
var b1;//unknown but not needed
var b2;//unknown but not needed

//Convert angles to radians
A1 = a1*Math.PI/180;
A2 = a2*Math.PI/180;
//B1 = b1*Math.PI/180;
//B2 = b2*Math.PI/180;

//Compute and print the initial x and y components of velocity.
// Because both cars are moving along the x-axis, all
// y-components are zero and all x components are equal to the
// magnitude.
u1x = u1
```

```
u1y = 0
u2x = u2
u2y = 0
//v1x = v1//unknown
v1y = 0
//v2x = v2//unknown
v2y = 0

document.write("Initial velocities</br>");
document.write("u1x = " + u1x.toFixed(2) + "</br>");
document.write("u2x = " + u2x.toFixed(2) + "</br>");
document.write("================================"+ " </br>");

/*
Prepare the equations for use in solving the problem.

Given the following three equations
m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y
0.5*m1*u1^2 + 0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

Only the first equation can be used for an inelastic collision
moving along the x-axis. This gives us the following equation
to work with.

m1*u1x + m2*u2x = m1*v1x + m2*v2x

Eliminate all of the components which are known to be zero.

m2*u2x = m1*v1x + m2*v2x

For a perfectly inelastic collision, v2=v1 yielding

m2*u2x = m1*v1x + m2*v1x, or
m2*u2x = (m1 + m2)*v1x

Rearranging terms gives the following:
*/

//Compute and print the final speed values
v1x = m2*u2x/(m1 + m2)
v2x = v1x;//required for a perfectly inelastic collision

document.write("Final speed values</br>");
document.write("v1x = " + v1x.toFixed(2) + " m/s</br>");
document.write("v2x = " + v2x.toFixed(2) + " m/s</br>");
document.write("================================"+ " </br>");

//Check the answer for conservation of momentum
document.write("Check for conservation of momentum</br>");
var mou = m1*u1x + m2*u2x;//momentum before the collision
```

```
var mov = m1*v1x + m2*v2x;//momentum after the collision

document.write("mou = " + mou.toFixed(0) + " Kg*m/s</br>");
document.write("mov = " + mov.toFixed(0) + " Kg*m/s</br>");

document.write("================================"+ " </br>");

document.write("End Script");

</script>
</body></html>
```

The output

Figure 3 (p. 631) shows the output for the car crash scenario portrayed by Listing 2.

Output for the perfectly inelastic car crash.

```
    Start Script
Initial velocities
u1x = 0.00
u2x = 17.50
================================
Final speed values
v1x = 10.00 m/s
v2x = 10.00 m/s
================================
Check for conservation of momentum
mou = 35000 Kg*m/s
mov = 35000 Kg*m/s
================================
End Script
```

Figure 25.3: Output for the perfectly inelastic car crash.

25.4.2 Two-dimensional scenarios

Now we will examine some two-dimensional scenarios. In these scenarios, the objects are free to move in a plane described by horizontal and vertical coordinates. Instead of the directions of motion being limited to only forward and backward, any object can move in any direction from 0 to 360 degrees.

25.4.2.1 Elastic collision between two pucks on friction-free ice

The description and the solution to a scenario involving an elastic collision between two pucks on friction-free ice is shown in Listing 3 (p. 632) . As you will see:

- The pucks are free to move in two dimensions.
- Even though this is an elastic collision, there were only two unknowns, so it wasn't necessary to use the energy-conservation equation to find a solution.

Three unknowns can be challenging

Elastic collisions involving three unknowns, particularly those where one or more angles are unknown, can be challenging from an algebraic/trigonometric viewpoint. Those solutions typically involve a quadratic equation containing trigonometric functions. This is not one of those especially challenging scenarios.

Listing 25.3: Elastic collision between two pucks on friction-free ice.

```
    <!---------------- File JavaScript02.html ---------------------->
<html><body>
<script language="JavaScript1.3">
/*
Obj1 and Obj2 are identical and are on friction-free ice. Obj1
has an initial velocity of 2.0 m/s in the direction
of 0 degrees. Obj2 is at rest. Obj1 collides elastically with
Obj2 and Obj1 moves off at 1.0 m/s at an angle of 60 degrees
north of east. What is the speed and direction of Obj2 after
the collision?


Using conservation of momentum alone, we have two
equations, allowing us to solve for two unknowns.

m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

Using conservation of kinetic energy for the elastic
case gives us one additional equation, allowing us
to solve for three unknowns.

0.5*m1*u1^2 + 0.5*m2*u2^2 = 0.5*m1*v1^2 + 0.5*m2*v2^2

The specifications tell us that this is an elastic collision,
so we are free to use any or all of the three equations to
solve the problem. In this case, we have three equations but
only two unknowns, v2 and b1, so we won't need all three
of the equations.

Note that the specifications don't specify the values of the
masses, but do specify that they are the same. Therefore, we
will be able to cancel them out of all three equations.

Variables:
m1, m2, u1, u2, v1, v2, a1, a2, b1, b2
```

```
*/

document.write("Start Script </br>");

//var m1 = unknown;//kg
//var m2 = m1;//kg
var u1 = 2;//meters per second
var u2 = 0;//meters per second -- at rest
var v1 = 1;//meter per second
var v2;//unknown -- to find
var a1 = 0;//degrees
var a2 = 0;//degrees -- at rest
var b1 = 60;//degrees
var b2;//unknown -- to find

//Convert angles to radians
A1 = a1*Math.PI/180;
A2 = a2*Math.PI/180;
B1 = b1*Math.PI/180;
//B2 = b2*Math.PI/180;//unknown

//Compute the initial x and y components of velocity
u1x = u1*Math.cos(A1)
u1y = u1*Math.sin(A1)

u2x = u2*Math.cos(A2)
u2y = u2*Math.sin(A2)

v1x = v1*Math.cos(B1)
v1y = v1*Math.sin(B1)

//v2x = v2*Math.cos(B2) //unknown
//v2y = v2*Math.sin(B2) //unknown

/*
For the special case of m2=m1 the three equations can be
written as follows (after canceling out the m-terms):
u1x + u2x = v1x + v2x
u1y + u2y = v1y + v2y
0.5*u1*u1 + 0.5*u2*u2 = 0.5*v1*v1 + 0.5*v2*v2

Identify all terms that are known to be zero
u1x + 0 = v1x + v2x
0 + 0 = v1y + v2y
0.5*u1*u1 + 0.5*0*0 = 0.5*v1*v1 + 0.5*v2*v2

Removing those terms yields
u1x = v1x + v2x
0 = v1y + v2y
0.5*u1*u1 = 0.5*v1*v1 + 0.5*v2*v2
```

```
Divide through the energy equation by 0.5 for simplification
u1x = v1x + v2x
0 = v1y + v2y
u1*u1 = v1*v1 + v2*v2

Substitute known values
2 = 0.5 + v2x
0 = 0.866 + v2y
u1*u1 + = v1*v1 + v2*v2

This problem can be solved without using the energy equation.
I will use the energy equation later to check the results.
*/

//Compute the components, magnitude, and direction of the
// final velocity of obj #2
v2x = 2 - 0.5;
v2y = -0.866;
v2 = Math.sqrt(v2x*v2x + v2y*v2y);
b2 = getAngle(1.5,-0.866);

//Display the known values along with the results.
document.write("u1x = " + u1x.toFixed(2) + " m/s</br>");
document.write("u1y = " + u1y.toFixed(2) + " m/s</br>");
document.write("u2x = " + u2x.toFixed(2) + " m/s</br>");
document.write("u2y = " + u2y.toFixed(2) + " m/s</br>");

document.write("v1x = " + v1x.toFixed(2) + " m/s</br>");
document.write("v1y = " + v1y.toFixed(2) + " m/s</br>");

document.write("v2x = " + v2x.toFixed(2) + " m/s</br>");
document.write("v2y = " + v2y.toFixed(3) + " m/s</br>");

document.write("u1 = " + u1.toFixed(2) + " m/s</br>");
document.write("u2 = " + u2.toFixed(2) + " m/s</br>");

document.write("v1 = " + v1.toFixed(2) + " m/s</br>");

document.write("================================"+ " </br>");
document.write("v2 = " + v2.toFixed(2) + " m/s</br>");
document.write("b2 = " + b2.toFixed(2) + " degrees</br>");
document.write("================================"+ " </br>");



//Check the answers assuming that the mass of the objects
// is one Kg each.
var moux = u1x + u2x;
var movx = v1x + v2x;
var mouy = u1y + u2y;
var movy = v1y + v2y;
```

```
document.write("moux = " + moux.toFixed(2) + " Kg*m/s</br>");
document.write("movx = " + movx.toFixed(2) + " Kg*m/s</br>");
document.write("mouy = " + mouy.toFixed(2) + " Kg*m/s</br>");
document.write("movy = " + movy.toFixed(2) + " Kg*m/s</br>");

//Check to confirm an elastic collision
var u1 = Math.sqrt(u1x*u1x + u1y*u1y);
var u2 = Math.sqrt(u2x*u2x + u2y*u2y);
var v1 = Math.sqrt(v1x*v1x + v1y*v1y);
var v2 = Math.sqrt(v2x*v2x + v2y*v2y);

var mou = 0.5*u1*u1 + 0.5*u2*u2;
var mov = 0.5*v1*v1 + 0.5*v2*v2;
document.write("mou = " + mou.toFixed(2) + " Kg*m/s</br>");
document.write("mov = " + mov.toFixed(2) + " Kg*m/s</br>");
document.write("================================"+ " </br>");


//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle

document.write("End Script");
</script>
</body></html>
```

The output

The output for this scenario is shown in Figure 4 (p. 636) .

Output for an elastic collision between two pucks on friction-free ice.

```
    Start Script
u1x = 2.00 m/s
u1y = 0.00 m/s
u2x = 0.00 m/s
u2y = 0.00 m/s
v1x = 0.50 m/s
v1y = 0.87 m/s
v2x = 1.50 m/s
v2y = -0.866 m/s
u1 = 2.00 m/s
u2 = 0.00 m/s
v1 = 1.00 m/s
==============================
v2 = 1.73 m/s
b2 = -30.00 degrees
==============================
moux = 2.00 Kg*m/s
movx = 2.00 Kg*m/s
mouy = 0.00 Kg*m/s
movy = 0.00 Kg*m/s
mou = 2.00 Kg*m/s
mov = 2.00 Kg*m/s
==============================
End Script
```

Figure 25.4: Output for an elastic collision between two pucks on friction-free ice.

Hopefully the comments in the script will be sufficient to explain the solution to this problem.

25.4.2.2 Perfectly inelastic collision between objects with odd angles

Listing 4 (p. 636) provides the description and the solution to a scenario involving a perfectly inelastic collision between two objects in a friction-free environment moving at odd angles. By odd angles, I mean that neither object is moving along either the x-axis or the y-axis, either before or after the collision.

Listing 25.4: Perfectly inelastic collision between objects with odd angles.

```
    <!---------------- File JavaScript03.html ---------------------->
<html><body>
<script language="JavaScript1.3">
/*
```

```
Obj1 with a mass of 1 Kg and an initial velocity of 3000 m/s
in a direction 67 degrees north of east collides in a perfectly
inelastic manner with Obj2, whose mass is also 1 Kg and whose
initial velocity is 2000 m/s in a direction 45 degrees north of east.

Calculate (1) the angle of motion of the combined bodies,
and (2) the magnitude of the momentum of the combined bodies
after the collision.

Using conservation of momentum alone, we have two
equations, allowing us to solve for two unknowns.

m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

Variables:
m1, m2, u1, u2, v1, v2, a1, a2, b1, b2
*/

document.write("Start Script </br>");

var m1 = 1;//kg
var m2 = 1;//kg
var u1 = 3000;//meters per second
var u2 = 2000;//meters per second
var v1;//unknown -- to be found
var v2;//unknown -- to be found
var a1 = 67;//degrees
var a2 = 45;//degrees
var b1;//unknown -- to be found
var b2;//unknown -- to be found

//Perfectly inelastic collision so v2=v1 and b2=b1

//Convert angles to radians
A1 = a1*Math.PI/180;
A2 = a2*Math.PI/180;
//B1 = b1*Math.PI/180;//unknown
//B2 = b2*Math.PI/180;//unknown

//Compute the x and y components of velocity
u1x = u1*Math.cos(A1)
u1y = u1*Math.sin(A1)

u2x = u2*Math.cos(A2)
u2y = u2*Math.sin(A2)

//v1x = v1*Math.cos(B1)//unknown
//v1y = v1*Math.sin(B1)//unknown

//v2x = v2*Math.cos(B2)//unknown
```

```
//v2y = v2*Math.sin(B2)//unknown

/*
For the special case of m2=m1=1 and v2=v1 (perfectly inelastic
collision) we can simplify the equations to the following:
u1x + u2x = 2*v1x
u1y + u2y = 2*v1y
*/

//Rearranging terms yields
v1x = (u1x + u2x)/2
v1y = (u1y + u2y)/2

//Knowing the x and y components of the final velocity, we can
// find the angle and magnitude as
b1 = getAngle(v1x,v1y);
v1 = Math.sqrt(v1x*v1x + v1y*v1y);


//Compute the momentum after the collision
var Px = v1x*(m1 + m2);
var Py = v1y*(m1 + m2);
var Pmag = Math.sqrt(Px*Px + Py*Py);

//Display the results
document.write("b1 = " + b1.toFixed(1) + " degrees</br>");
document.write("v1 = " + v1.toFixed(0) + " m/s</br>");
document.write("v1x = " + v1x.toFixed(0) + " m/s</br>");
document.write("v1y = " + v1y.toFixed(0) + " m/s</br>");
document.write("Px = " + Px.toFixed(0) + " Kg*m/s</br>");
document.write("Py = " + Py.toFixed(0) + " Kg*m/s</br>");
document.write("Pmag = " + Pmag.toFixed(0) + " Kg*m/s</br>");
document.write("================================"+ " </br>");

//Check the answer for perfect inelastic collision
v2x = v1x;
v2y = v1y;
v2 = v1;
var moux = u1x + u2x;
var movx = v1x + v2x;
var mouy = u1y + u2y;
var movy = v1y + v2y;
var mou = Math.sqrt(moux * moux + mouy * mouy);
var mov = Math.sqrt(movx * movx + movy * movy);

document.write("moux = " + moux.toFixed(0) + " Kg*m/s</br>");
document.write("movx = " + movx.toFixed(0) + " Kg*m/s</br>");
document.write("mouy = " + mouy.toFixed(0) + " Kg*m/s</br>");
document.write("movy = " + movy.toFixed(0) + " Kg*m/s</br>");
document.write("mou = " + mou.toFixed(0) + " Kg*m/s</br>");
document.write("mov = " + mov.toFixed(0) + " Kg*m/s</br>");
```

```
document.write("==============================="+ " </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle


document.write("End Script");

</script>
</body></html>
```

The output

The output for the script shown in Listing 4 (p. 636) is provided in Figure 5 (p. 640) .

Output for a perfectly inelastic collision between objects with odd angles.

```
    Start Script
b1 = 58.2 degrees
v1 = 2456 m/s
v1x = 1293 m/s
v1y = 2088 m/s
Px = 2586 Kg*m/s
Py = 4176 Kg*m/s
Pmag = 4912 Kg*m/s
==============================
moux = 2586 Kg*m/s
movx = 2586 Kg*m/s
mouy = 4176 Kg*m/s
movy = 4176 Kg*m/s
mou = 4912 Kg*m/s
mov = 4912 Kg*m/s
==============================
End Script
```

Figure 25.5: Output for a perfectly inelastic collision between objects with odd angles.

As before, I will allow the comments in Listing 4 (p. 636) serve as the explanation for the solution of this scenario.

25.4.2.3 Rotating the axes for simplification

The scenario shown in Listing 5 (p. 641) illustrates how, in some cases, you can simplify the algebra/trigonometry by rotating the axes. This situation occurs when none of the directions involving the two objects lies along either the x or y axis. In those cases, you may be able to simplify the three equations that you develop by rotating the axes such that one of the directions lies along either the x or the y axis. That will often cause some of the terms in the equations to go to zero and drop out of the equations.

A crash at an intersection

If you examine the scenario shown in Listing 5 (p. 641) carefully, you will see that it is the classic collision at the intersection of two streets that are perpendicular to one another. However, the streets don't run in north-south, east-west directions.

Rotating the axes simplified the problem

In this particular case, rotating the axes so that one street runs east and west while the other street runs north and south simplifies the equations considerably.

Don't forget the rotate the axes back at the end

Of course, if you rotate the axes to simplify the solution, you must remember to rotate it back again, and correct the solution accordingly, once you have a solution. The procedure for doing that is illustrated in the scenario shown in Listing 5 (p. 641) .

Listing 25.5: Rotation of the axes for simplification.

```
    <!---------------- File JavaScript04.html ---------------------->
<html><body>
<script language="JavaScript1.3">

/*
Two objects collide in a perfectly inelastic collision. Obj1
has a mass of 10000 kg and is traveling 30 degrees north of
east at 15 m/s. Obj2 has a mass of 1500 kg and is traveling
30 degrees west of north, or 120 degrees at 25 m/s. Find
the direction that the two object move and the speed of that
movement following the collision.

Note that this script illustrates rotating the axis to simplify
the problem.

Using conservation of momentum alone, we have two
equations, allowing us to solve for two unknowns.

m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

Variables:
m1, m2, u1, u2, v1, v2, a1, a2, b1, b2
*/

document.write("Start Script </br>");

var m1 = 10000;//kg
var m2 = 1500;//kg
var u1 = 15;//meters per second
var u2 = 25;//meters per second
var v1;//unknown -- to be found
var v2;//unknown -- to be found
var a1 = 30;//degrees
var a2 = 120;//degrees
var b1;//unknown -- to be found
var b2;//unknown -- to be found

//For a perfectly inelastic collision, v2=v1 and b2=b1

//Convert angles to radians
A1 = a1*Math.PI/180;
A2 = a2*Math.PI/180;
//B1 = b1*Math.PI/180;//unknown
//B2 = b2*Math.PI/180;//unknown

//Compute the x and y components of velocity
u1x = u1*Math.cos(A1)
u1y = u1*Math.sin(A1)
```

```
u2x = u2*Math.cos(A2)
u2y = u2*Math.sin(A2)

//v1x = v1*Math.cos(B1)//unknown
//v1y = v1*Math.sin(B1)//unknown

//v2x = v2*Math.cos(B2)//unknown
//v2y = v2*Math.sin(B2)//unknown

//Display the x and y components of initial velocity
document.write("x and y components of initial velocity</br>");
document.write("u1x = " + u1x.toFixed(2) + " m/s</br>");
document.write("u1y = " + u1y.toFixed(2) + " m/s</br>");
document.write("u2x = " + u2x.toFixed(2) + " m/s</br>");
document.write("u2y = " + u2y.toFixed(2) + " m/s</br>");
document.write("================================"+ " </br>");

/*
Given the following two equations
m1*u1x + m2*u2x = m1*v1x + m2*v2x
m1*u1y + m2*u2y = m1*v1y + m2*v2y

For the special case of v2=v1 (perfectly inelastic
collision), the equations can be simplified to the following:
m1*u1x + m2*u2x = (m1 + m2)*v1x
m1*u1y + m2*u2y = (m1 + m2)*v1y

Replace the terms with known x and y velocity component values.
m1*13 + m2*(-12.5) = (m1 + m2)*v1x
m1*7.5  + m2*21.6 = (m1 + m2)*v1y

A judicious examination of the problem reveals that if we were
to rotate the axis by 30 degrees clockwise, we could cause
some of the terms in these two equations to go to zero. We will
subtract 30 degrees from the given angles at this point and
add that 30 degrees back into the results at the end.
*/
a1 = a1 - 30;
a2 = a2 - 30;

//Convert the new angles to radians
A1 = a1*Math.PI/180;
A2 = a2*Math.PI/180;

//Recompute the x and y components of velocity
u1x = u1*Math.cos(A1)
u1y = u1*Math.sin(A1)

u2x = u2*Math.cos(A2)
u2y = u2*Math.sin(A2)
```

```
//Display the new x and y components of initial velocity
document.write(" New x and y components of velocity</br>");
document.write("u1x = " + u1x.toFixed(2) + " m/s</br>");
document.write("u1y = " + u1y.toFixed(2) + " m/s</br>");
document.write("u2x = " + u2x.toFixed(2) + " m/s</br>");
document.write("u2y = " + u2y.toFixed(2) + " m/s</br>");
document.write("================================"+ " </br>");

/*
Returning now to the special case of v2=v1 (perfectly inelastic
collision):
m1*u1x + m2*u2x = (m1 + m2)*v1x
m1*u1y + m2*u2y = (m1 + m2)*v1y

Replace the x and y components of velocity with the modified
values, two of which are now 0. This results in a simplification
of the equations.
m1*15 + m2*0 = (m1 + m2)*v1x
m1*0 + m2*25 = (m1 + m2)*v1y

Plugging in the values for mass and eliminating terms with
a zero value yields
10000*15 = (10000 + 1500)*v1x
1500*25 = (10000 + 1500)*v1y
*/

//Rearranging terms yields
v1x = (10000*15)/(10000 + 1500)
v1y = (1500*25)/(10000 + 1500)

//Compute the magnitude and the angle of the velocity of the
// two objects following the collision.
v1 = Math.sqrt(v1x*v1x + v1y*v1y);
b1 = getAngle(v1x,v1y);

//Because this is a perfectly inelastic collision, v2=v1
// and b2=b1
v2 = v1;
b2 = b1;

//Display results for the modified angles
document.write("Results for modified angles</br>");
document.write("v1x = " + v1x.toFixed(1) + " m/s</br>");
document.write("v1y = " + v1y.toFixed(1) + " m/s</br>");
document.write("v1 = " + v1.toFixed(1) + " m/s</br>");
document.write("v2 = " + v2.toFixed(1) + " m/s</br>");
document.write("b1 = " + b1.toFixed(1) + " degrees</br>");
document.write("b2 = " + b2.toFixed(1) + " degrees</br>");
document.write("================================"+ " </br>");
```

```
/*
Now we need to rotate the axis by 30 degrees counter-clockwise
to correct for the original rotation by 30 degrees clockwise.
The magnitude of the final velocity is the same regardless of
the orientation of the axes. Therefore, we will add 30 degrees
to the values of b1 and b2, and use the new angles along with
the magnitude of the final velocity to recompute the x and y
components of the final velocity.
*/

b1 = b1 + 30;
b2 = b1;
v1x = v1*Math.cos(b1*Math.PI/180);
v1y = v1*Math.sin(b1*Math.PI/180);

//Display results for the corrected angle
document.write("Results for corrected angle</br>");
document.write("v1x = " + v1x.toFixed(1) + " m/s</br>");
document.write("v1y = " + v1y.toFixed(1) + " m/s</br>");
document.write("v1 = " + v1.toFixed(1) + " m/s</br>");
document.write("v2 = " + v2.toFixed(1) + " m/s</br>");
document.write("b1 = " + b1.toFixed(1) + " degrees</br>");
document.write("b2 = " + b2.toFixed(1) + " degrees</br>");
document.write("================================"+ " </br>");

//Check the answer for a perfect inelastic collision. Must
// recognize that v2=v1 and correct the angles for a1 and a2.
v2x = v1x;
v2y = v1y;

u1x = u1*Math.cos((a1+30)*Math.PI/180);
u1y = u1*Math.sin((a1+30)*Math.PI/180);

u2x = u2*Math.cos((a2+30)*Math.PI/180);
u2y = u2*Math.sin((a2+30)*Math.PI/180);

var moux = m1*u1x + m2*u2x;
var movx = m1*v1x + m2*v2x;
var mouy = m1*u1y + m2*u2y;
var movy = m1*v1y + m2*v2y;
var mou = Math.sqrt(moux * moux + mouy * mouy);
var mov = Math.sqrt(movx * movx + movy * movy);

document.write("Check the answers.</br>");
document.write("moux = " + moux.toFixed(0) + " Kg*m/s</br>");
document.write("movx = " + movx.toFixed(0) + " Kg*m/s</br>");
document.write("mouy = " + mouy.toFixed(0) + " Kg*m/s</br>");
document.write("movy = " + movy.toFixed(0) + " Kg*m/s</br>");
document.write("mou = " + mou.toFixed(0) + " Kg*m/s</br>");
document.write("mov = " + mov.toFixed(0) + " Kg*m/s</br>");
document.write("================================"+ " </br>");
```

```
//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle


//The purpose of the getRoots function is to compute and
// return the roots of a quadratic equation expressed in
// the format
//   a*x^2 + b*x + c = 0
//The roots are returned in the elements of a two-element
// array. If the roots are imaginary, the function
// returns NaN for the value of each root.
function getRoots(a,b,c){
  var roots = new Array(2);
  roots[0] = (-b+Math.sqrt(b*b-4*a*c))/(2*a);
  roots[1] = (-b-Math.sqrt(b*b-4*a*c))/(2*a);
  return roots;
}//end getRoots

document.write("End Script");

</script>
</body></html>
```

The output

The output for this scenario is shown in Figure 6 (p. 646) .

Output for rotation of the axes for simplification.

```
    Start Script
x and y components of initial velocity
u1x = 12.99 m/s
u1y = 7.50 m/s
u2x = -12.50 m/s
u2y = 21.65 m/s
==============================
New x and y components of velocity
u1x = 15.00 m/s
u1y = 0.00 m/s
u2x = 0.00 m/s
u2y = 25.00 m/s
==============================
Results for modified angles
v1x = 13.0 m/s
v1y = 3.3 m/s
v1 = 13.4 m/s
v2 = 13.4 m/s
b1 = 14.0 degrees
b2 = 14.0 degrees
==============================
Results for corrected angle
v1x = 9.7 m/s
v1y = 9.3 m/s
v1 = 13.4 m/s
v2 = 13.4 m/s
b1 = 44.0 degrees
b2 = 44.0 degrees
==============================
Check the answers.
moux = 111154 Kg*m/s
movx = 111154 Kg*m/s
mouy = 107476 Kg*m/s
movy = 107476 Kg*m/s
mou = 154616 Kg*m/s
mov = 154616 Kg*m/s
==============================
End Script
```

Figure 25.6: Output for rotation of the axes for simplification.

Once more, I will allow the comments in Listing 4 (p. 636) to serve as the explanation for the solution of this scenario.

25.5 Run the scripts

I encourage you to run the scripts that I have presented in this lesson to confirm that you get the same results. Copy the code for each script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

25.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

25.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Energy – Elastic and Inelastic Collisions in Two Dimensions for Blind Students
- File: Phy1215.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · one-dimensional
  - · two-dimensional
  - · collision
  - · elastic collision
  - · inelastic collision
  - · perfectly inelastic collision
  - · kinetic energy
  - · conservation of momentum
  - · conservation of energy

NOTE: **Disclaimers: Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 26

Relationships Among Kinematics, Newton's Laws, Vectors, 2D Motion, 2D Forces, Momentum, Work, Energy, and Power[1]

26.1 Table of Contents

- Preface (p. 649)
  - General (p. 649)
  - Prerequisites (p. 650)
  - Supplemental material (p. 650)
- Discussion (p. 650)
  - An ideal rocket example (p. 651)
    - Leg A (p. 652)
    - Leg B (p. 652)
    - Leg C (p. 653)
    - Leg D (p. 654)
- Do the calculations (p. 655)
- Resources (p. 655)
- Miscellaneous (p. 655)

26.2 Preface

26.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module illustrates relationships among kinematics, Newton's laws, vectors, 2D linear motion, 2D forces, momentum, work, energy, and power in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38369/1.2/>.
[2] http://cnx.org/content/col11294/latest/

26.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

26.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

26.3 Discussion

A wrap-up module

The next module following this one will involve circular motion, which will be a major change in direction (no pun intended). Therefore, in this module, I will work through a major example involving a rocket that will tie together much of what you have learned in earlier modules.

Before we get to that example, however, let's do a quick review on external and internal forces.

A quick review of external forces

You learned in an earlier module that when work is done on an object by *external* forces, the total mechanical energy possessed by the object, consisting of kinetic energy plus potential energy, must change.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

The work done on the object by external forces can be positive, in which case the total mechanical energy will increase. The work can be negative, in which case the total mechanical energy will decrease. The change in mechanical energy will be equal to the net work that is done on the object.

In this case, the total mechanical energy is not *conserved* . Therefore, external forces are often referred to as *non-conservative* forces.

A quick review of internal forces

On the other hand, you also learned that if work is done on an object only by *internal* forces, the total mechanical energy possessed by the object cannot change. However, it can be transformed from potential energy to kinetic energy and vice versa.

In this case, the total mechanical energy is *conserved* . Therefore, internal forces are often referred to as *conservative* forces.

A quantitative relationship

The quantitative relationship between work and mechanical energy can be stated as follows:

MEf = MEi + We

where

- MEf and MEi represent the final and initial total mechanical energy possessed by the object respectively.
- We represents the work done on the object by external forces.

This equation states that the final amount of mechanical energy possessed by an object is equal to the initial mechanical energy plus the work done on the object by external forces.

Potential energy plus kinetic energy

The total mechanical energy at any point in time can be the sum of potential energy *(gravitational or elastic potential energy)* and kinetic energy due to motion.

Given that, we can rewrite the earlier equation as:

KEf + PEf = KEi + PEi + We

where

- KEf and PEf represent the final kinetic and potential energy respectively.
- KEi and PEi represent the initial kinetic and potential energy respectively.
- We represents the work done on the object by external forces.

As mentioned, the work done by external forces can be either positive or negative work. Whether the work is positive or negative depends on the cosine of the angle between the direction of the force and the direction of the displacement of the object.

26.3.1 An ideal rocket example

Consider the following scenario. The owners of an experimental rocket lift the rocket onto a platform above ground level and set it up for firing.

Later, when they fire the rocket, it goes straight up while the rocket engine is burning. When the rocket engine runs out of fuel and stops burning, the rocket coasts to its apex and stops climbing. Then it falls back to the surface of the earth in an unglamorous free fall.

Simplifying assumptions

We will make some simplifying assumptions:

- The mass of the fuel is insignificant relative to the combined mass of the rocket and its payload. Therefore, expenditure of fuel doesn't affect the mass of the rocket in a significant way.
- Air resistance is negligible. The rocket acts as if in a vacuum.

Initial conditions

Here are the initial conditions for the rocket experiment:

- Platform height = 15 meters.
- Mass of rocket and payload = 10kg.
- Thrust of rocket is constant at 150 newtons during burn.
- Burn time for the rocket = 10 seconds.

Legs of the trip

We will analyze the rocket's round trip from the ground, into the air, and back to the ground in several legs as described below:

- Leg A: Manually lifting the rocket from the ground to the platform.
- Leg B: Displacement of the rocket under rocket-engine power straight up.
- Leg C: Displacement of the rocket without power while coasting to the apex.
- Leg D: Displacement of the rocket in free fall from the apex back to the ground.

We will analyze several aspects of the state of the rocket at the end of each leg. We will also compare alternative ways of computing the state of the rocket.

26.3.1.1 Leg A

During this leg, the rocket is manually lifted from the ground to the platform. The rocket has no potential or kinetic energy while on the ground, so it begins with zero mechanical energy.

An external force must be provided to lift the rocket from the ground to the platform in order to overcome the internal force of gravity. As an external force, this force is capable of changing its mechanical energy, which it does.

When the rocket has been lifted onto the platform, the mechanical energy of the rocket consists of its gravitational potential energy, which is equal to the work done to lift it to the platform. The kinetic energy will be 0 at that point because the rocket isn't moving.

Weight of the rocket = m*g = 10kg*9.8m/s^2 = 98 newtons

Work = f * d = 98N * 15m = 1470 joules

State at the end of Leg A

Thus, the total mechanical energy possessed by the rocket at the end of Leg A is 1470 joules.

26.3.1.2 Leg B

During this leg, which begins when the rocket engine fires, the rocket flies straight up as a result of a constant upward force exerted by the rocket engine.

The net acceleration

For this leg, we need to determine the net acceleration that is applied to the rocket. The net acceleration consists of the upward or positive acceleration due to the force of the rocket engine and the downward or negative acceleration of gravity.

Aup = 150N/10kg = 15 m/s^2

Ag = -9.8 m/s^2

Anet = 15 m/s^2 - 9.8 m/s^2 = 5.2 m/s^2

How far will the rocket go?

The initial velocity of the rocket is zero. Given that, you learned in an earlier module that the distance that the rocket will travel during the burn is

d = 0.5*Anet*t^2 = 0.5 * (5.2 m/s^2) * (10s)^2, or

d = 260 meters

260 meters straight up

In other words, when the rocket runs out of fuel at the end of the 10-second burn, the rocket has traveled straight up by 260 meters. Given that it started 15 meters above the ground, it is at a height of 275 meters above the ground at that point in time.

Total mechanical energy

At that point in time, the total mechanical energy possessed by the rocket consists of its gravitational potential energy plus its kinetic energy.

The kinetic energy

To compute the kinetic energy, we need to know the velocity. You learned in an earlier module that we can compute the velocity as

v = Anet * t = (5.2 m/s^2) * 10s = 52 m/s

You also learned earlier that the kinetic energy is equal to

KE = 0.5 * m * v^2 = 0.5 * 10 kg * (52 m/s)^2 = 13520 joules

Gravitational potential energy

You learned in an earlier module that the gravitational potential energy of an object due to its height above the surface of the earth is equal to

PEg = m * g * h = 10 kg * (9.8m/s^2) * 275 m = 26950 joules

Note that this value is computed using the height above the ground and not the height above the platform, which is 260 meters.

The total mechanical energy

Thus, the total mechanical energy at this point in time is

ME = 13520 joules + 26950 joules = 40470 joules

Validation

Let's see if we can validate that result in some other way.

We know that the mechanical energy of the rocket at rest on the platform was equal to 1470 joules.

We can compute the work done in moving the rocket up by 260 meters by multiplying that distance by the upward force. Thus,

work = distance * thrust, or

work = 260 m * 150N = 39000 joules

Additional mechanical energy

This is the mechanical energy added to the rocket after it left the platform while the engine was burning. The total mechanical energy when the burn ends is the sum of that value and the mechanical energy that it had while at rest on the platform. Thus, the total mechanical energy at the end of the burn is:

ME = 1470 joules + 39000 joules = 40470 joules

A good match

This value matches the value computed earlier (p. 653) on the basis of the height of the rocket above the surface of the earth and the velocity of the rocket. Thus, the two approaches agree with one another up to this point.

State at the end of Leg B

Therefore, at the end of Leg B,

- **The total mechanical energy** possessed by the rocket is equal to 40470 joules.
- The gravitational potential energy is 26950 joules
- The kinetic energy is 13520 joules
- The rocket is out of fuel and is coasting upward with a velocity of 52 m/s.
- The only force acting on the rocket is an internal downward force due to gravity, which is equal to 10kg * 9.8m/s^2 = 98 newtons.

26.3.1.3 Leg C

This is the part of the trip where the rocket coasts from its height at the end of Leg B to the apex of its trip. The continued upward motion is due solely to its kinetic energy at the end of Leg B.

From the end of Leg B when the rocket engine stops burning, until the rocket crashes on the surface of the earth, the only forces acting on the rocket will be the internal force of gravity.

Total mechanical energy is conserved

Since internal forces cannot change the mechanical energy possessed by an object, the total mechanical energy for the rocket must remain at **40470 joules** for the remainder of the trip.

How long to reach the apex?

We can compute the time required for the rocket to reach the apex as

t = v/g = (52m/s)/(9.8m/s^2) = 5.31 seconds

How far will the rocket travel?

Knowing the time required to reach the apex, we can compute the distance to the apex (during this leg only) as

d = v0*t - 0.5*g*t^2, or

d = (52m/s) * (5.31s) - (0.5) * (9.8m/s^2)*(5.31s)^2, or

d = 138 meters

An additional 138 meters

In other words, the rocket travels an additional 138 meters straight up after the rocket-engine stops burning. This additional travel is due solely to the kinetic energy possessed by the rocket at the end of the burn.

The total height of the apex

Adding 138 more meters to the height at the end of the burn causes the height at the apex to be

height at apex = 275m + 138m = 413 meters

Mechanical energy equals potential energy alone

At that point, the total mechanical energy is equal to the gravitational potential energy because the rocket isn't moving and the kinetic energy has gone to zero.

Validation

We can compute the total mechanical energy at this point as

PEg = m*g*h = 10kg * (9.8m/s^2) * 413m = 40474 joules

This result is close enough to the total mechanical energy (p. 653) at the end of Leg B to validate the computations. In this case, we determined the height using time, velocity, and acceleration, and validated that height using work/energy concepts.

State at the end of Leg C

At the completion of Leg C:

- The rocket is at the apex at a height of 413 meters.
- The total mechanical energy is 40470 joules.
- The kinetic energy is 0 because for an instant, the rocket isn't moving.
- The mechanical energy consists totally of gravitational potential energy.

26.3.1.4 Leg D

Leg D of the trip is fairly simple. The rocket falls for a distance of 413 meters under the influence of the internal gravitational force.

No change in mechanical energy

Once again, because the force is an internal force, the total mechanical energy cannot be changed by the work done by the force. However, the mechanical energy can be transformed from potential energy to kinetic energy.

At the instant before the rocket strikes the ground, it must still have a total mechanical energy value of 40470 joules.

Kinetic energy: 40470, potential energy: 0

At the instant before the rocket strikes the ground, all of the mechanical energy has been transformed into kinetic energy. We can use that knowledge to compute the velocity of the rocket right before it strikes the ground.

KE = 0.5*m*v^2, or

v^2 = KE/(0.5*m), or

v = (KE/(0.5*m))^(1/2) = (40470 joules/(0.5*10kg))^(1/2), or
terminal velocity = v = 90 meters/sec
Thus, the terminal velocity of the rocket when it strikes the ground is 90 meters/sec straight down.
Validation
Let's see if we can validate that result using a different approach. Given the height of the apex and the acceleration of gravity, we can computer the transit time as
413m = 0.5*g*t^2, or
t^2 = 413m/(0.5*g), or
t = (413m/(0.5*g))^(1/2) = (413m/(0.5*9.8m/s^2))^(1/2), or
t = 9.18 seconds
Compute the terminal velocity
Knowing the time to make the trip to the ground along with the acceleration, we can compute the terminal velocity as
v = g * t = (9.8m/s^2)*9.18s = 90 m/s
which matches the terminal velocity (p. 655) arrived at on the basis of work and energy.
State at the end of Leg D
Therefore, at the end of Leg D, the rocket crashes into the ground. However, an instant before the crash,

- The total mechanical energy is 40470 joules.
- The gravitational potential energy is 0.
- The kinetic energy is 40470 joules.
- The velocity is 90 m/s straight down toward the center of the earth.

26.4 Do the calculations

I encourage you to repeat the calculations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

26.5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

26.6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Relationships Among Kinematics, Newton's Laws, Vectors, 2D Motion, 2D Forces, Momentum, Work, Energy, and Power for Blind Students
- File: Phy1220.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor

· screen reader
· refreshable Braille display
· JavaScript
· trigonometry
· potential energy
· work
· gravitational potential energy
· elastic potential energy
· kinetic energy
· mechanical energy
· total mechanical energy
· power
· watt
· internal force
· conservative force
· external force
· non-conservative force

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 27

Vector Subtraction[1]

27.1 Table of Contents

- Preface (p. 657)
  - General (p. 657)
  - Prerequisites (p. 658)
  - Viewing tip (p. 658)
    - Figures (p. 658)
    - Listings (p. 658)
  - Supplemental material (p. 659)
- Discussion (p. 659)
- Examples of vector subtraction (p. 660)
  - The parallelogram method (p. 660)
  - Angles, approaches, and vector naming convention (p. 660)
  - Trigonometric solutions (p. 661)
  - Orthogonal vectors (p. 663)
    - Graphical solution for Cs=B+A (p. 663)
    - Graphical solution for Dd=B-A (p. 663)
    - Trigonometric solution for Cs=B+A and Dd=B-A (p. 664)
  - Same magnitude, 45-degree angle (p. 664)
    - Graphical solutions for sum and difference vectors (p. 664)
  - Sum and difference for smaller and smaller angles (p. 665)
- Run the script (p. 667)
- Resources (p. 667)
- Miscellaneous (p. 667)

27.2 Preface

27.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

[1] This content is available online at <http://cnx.org/content/m38374/1.2/>.
[2] http://cnx.org/content/col11294/latest/

This module explains vector subtraction in a format that is accessible to blind students.

27.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

27.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures and listings while you are reading about them.

27.2.3.1 Figures

- Figure 1 (p. 664) . Program output for orthogonal vectors.
- Figure 2 (p. 666) . Program output for smaller and smaller angles.

27.2.3.2 Listings

- Listing 1 (p. 661) . Add and subtract vectors using trigonometry.

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp

27.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

27.3 Discussion

Earlier lessons have dealt quite a lot with vector addition, but I have had very little to say about vector subtraction. That is because we haven't had much need for vector subtraction until now. The next module will deal with circular motion and the need for understanding vector subtraction will be paramount in understanding the material in that lesson.

My objective in this lesson is to explain vector subtraction in sufficient depth that you can visualize what it means when the text says that vector A is subtracted from vector B.

A simple rule

If A and B are scalars and you are asked to subtract A from B, you should already know the rule that says change the sign of A and then add.

The same rule also applies to vector subtraction. If we need to subtract vector A from vector B, we need to change the sign on the vector named A and then add it to the vector named B.

Changing the sign of a vector

So the question is, how do you change the sign of a vector? I will answer the question in three ways that really mean the same thing:

1. Change the sign for the horizontal and vertical components of the vector.
2. Draw the vector so that it points in exactly the opposite direction.
3. Add 180 degrees to the angle that defines the vector.

If all else fails, just remember this simple rule and apply it using one of the three ways described above.

Three ways to add vectors

Let's review **three ways to add two vectors** :

1. Draw them sequentially tail-to-head. The sum will be a vector that extends from the tail of the first vector to the head of the last vector.
2. Draw them tail-to-tail. Then draw a parallelogram where the two vectors form two sides of the parallelogram. The sum will be a vector that extends from the point where the tails join to the opposite corner of the parallelogram.
3. Using trigonometry, decompose both vectors into horizontal and vertical components. Add the horizontal components and add the vertical components. The result will be the horizontal and vertical components of the sum vector. If needed, use trigonometry to convert the components of the sum vector into a magnitude and angle.

Creating vectors with a graph board is useful

I have stated in earlier modules that the third approach using trigonometry is probably the most practical for blind students.

However, I also believe that it is important to get a picture in the mind's eye as to what happens when we add or subtract vectors. Therefore, I believe it is also useful for blind students to

- Use a graph board, a protractor, pipe cleaners, and pushpins to "draw" vectors.
- Add them using one of the first two approaches listed above (p. 659) .
- Develop a picture of the result in the mind's eye.

(Being able to see the addition and subtraction of vectors in the mind's eye is very important. As a sighted person, I often close my eyes and draw vectors on the palm of my hand in order to get a better feel for what happens when vectors are added or subtracted.)

[12] http://www.dickbaldwin.com/toc.htm

27.4 Examp les of vector subtraction

With that as an introduction, I am going to discuss some examples that are intended to help you to get a good feel for what it means to add, and more importantly to subtract two vectors. I hope that you will not only follow along and work through the examples, but that you will also use your graph board and "draw" the examples as I describe them.

27.4.1 The parallelogram method

Before getting into the details of vector subtraction, let me make a few comments about the parallelogram method of vector addition.

Blind students should be able to do a pretty good job of adding two vectors with this method using a graph board, some pushpins, and five pipe cleaners. Try to keep the pipe cleaners as straight as practical. Consider making a loop at each end of four of the pipe cleaners that you can use to pin them down to the graph board.

Given two vectors...

Given two vectors A and B, which are specified in terms of the magnitude of each vector and the angle that each vector makes relative to the horizontal axis, follow these steps:

1. Cut or bend two pipe cleaners to the correct length for one of the vectors. Set one aside temporarily.
2. Cut or bent two pipe cleaners to the correct length for the other vector. Set one aside temporarily.
3. Position two of the pipe cleaners so as to represent the pair of vectors connected at their tails with the correct angle between them. (Make each vector form the correct angle relative to the horizontal axis.) Pin each end of each vector down to hold it at the correct location with the correct angle. Think of the point where their tails join as the origin of a Cartesian coordinate system.
4. Use the other two pipe cleaners to form a parallelogram. Pin the ends of those pipe cleaners down using the pushpins at the ends of the vectors plus one additional pushpin. If you pin the ends of those two pipe cleaners to the ends of the vectors first, you should be able to find the point where those two pipe cleaners meet and pin them down at that point. That will be the "opposite corner" of the parallelogram. If you end up with a four-sided geometric shape that is not a parallelogram, you have used the final two pipe cleaners in the wrong order. Switch them and draw the parallelogram again.
5. Place a fifth pipe cleaner from the origin to the opposite corner of the parallelogram. The length of that vector will be the magnitude of the sum of the other two vectors. The angle that vector forms with the horizontal axis will be the angle of the sum of the other two vectors.

27.4.2 Angles, approaches, and vector naming convention

Make one vector horizontal

In order to make the computations and the manipulations of the pipe cleaners in the following examples easier, I will cause one of the two vectors to lie on the horizontal axis with an angle of 0 degrees. That won't make the results any less general, but it will make it easier for you to get accurate results.

The parallelogram method

I may be wrong, but I believe that the parallelogram method for adding two vector is more practical for blind students than the tail-to-tip method. Therefore, I will walk you through a parallelogram solution and a trigonometry solution for several of the example scenarios.

Vector naming convention

I will define vectors in the following way:

- Bm = 10 units
- Ba = 45 degrees
- Cs = B + A
- Cd = B - A

where

- Bm is the magnitude of a vector named B.
- Ba is the angle that the vector named B makes with the horizontal axis.
- Cs and Cd are the sum and difference of two vectors respectively.
- Csm and Cdm are the magnitudes of the sum and difference of two vectors.
- Csa and Cda are the angles (relative to the horizontal axis) of the sum and difference of two vectors.

27.4.3 Trigonometric solutions

I'm going to write a JavaScript program such that we can simply plug numbers into the values of variables in order to solve for the sum and difference of two vectors. This will be easier and less error prone than doing lots of calculations using the Google calculator.

The code for the JavaScript program is shown in Listing 1 (p. 661) . There is nothing in Listing 1 (p. 661) that you haven't seen in earlier modules, so it shouldn't require an explanation.

Listing 27.1: Add and subtract vectors using trigonometry.

```
    <!------------ File JavaScript01.html ------------------>
<html><body>
<script language="JavaScript1.3">

document.write("Start Script </br>");

//The purpose of this function is to receive the adjacent
// and opposite side values for a right triangle and to
// return the angle in degrees in the correct quadrant.
function getAngle(x,y){
  if((x == 0) && (y == 0)){
    //Angle is indeterminate. Just return zero.
    return 0;
  }else if((x == 0) && (y > 0)){
    //Avoid divide by zero denominator.
    return 90;
  }else if((x == 0) && (y < 0)){
    //Avoid divide by zero denominator.
    return -90;
  }else if((x < 0) && (y >= 0)){
    //Correct to second quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else if((x < 0) && (y <= 0)){
    //Correct to third quadrant
    return Math.atan(y/x)*180/Math.PI + 180;
  }else{
    //First and fourth quadrants. No correction required.
    return Math.atan(y/x)*180/Math.PI;
  }//end else
}//end function getAngle


//Modify these values and run for different cases.
```

```
var Bm = 10;
var Ba = 90;
var Am = 10;
var Aa = 0;

//Do not modify any of the following code.
//Convert angles to radians
var bAng = Ba*Math.PI/180;
var aAng = Aa*Math.PI/180;

//Compute horizontal and vertical components
// for each vector.
var Bx = Bm*Math.cos(bAng);
var By = Bm*Math.sin(bAng);

var Ax = Am*Math.cos(aAng);
var Ay = Am*Math.sin(aAng);

//Compute sum of vectors
var Cx = Bx + Ax;
var Cy = By + Ay;
var Ca = getAngle(Cx,Cy);
var Cm = Math.sqrt(Cx*Cx + Cy*Cy);

//Compute difference between vectors
var Dx = Bx - Ax;
var Dy = By - Ay;
var Da = getAngle(Dx,Dy);
var Dm = Math.sqrt(Dx*Dx + Dy*Dy);

document.write("Bm = " + Bm.toFixed(2) + "</br>");
document.write("Ba = " + Ba.toFixed(2) + " deg</br>");
document.write("Am = " + Am.toFixed(2) + "</br>");
document.write("Aa = " + Aa.toFixed(2) + " deg</br>");

document.write("Bx = " + Bx.toFixed(2) + "</br>");
document.write("By = " + By.toFixed(2) + "</br>");
document.write("Ax = " + Ax.toFixed(2) + "</br>");
document.write("Ay = " + Ay.toFixed(2) + "</br>");
document.write("Cx = " + Cx.toFixed(2) + "</br>");
document.write("Cy = " + Cy.toFixed(2) + "</br>");
document.write("Ca = " + Ca.toFixed(2) + " deg</br>");
document.write("Cm = " + Cm.toFixed(2) + "</br>");
document.write("Da = " + Da.toFixed(2) + " deg</br>");
document.write("Dm = " + Dm.toFixed(2) + "</br>");

document.write("End Script");

</script>
</body></html>
```

The one unique thing

The only thing that is unique about Listing 1 (p. 661) is that we can plug values into the variables named Bm, Ba, Am, and Aa to specify the magnitudes and angles for the vectors named B and A. (The same naming scheme is used for the variables as was described earlier.) Then when we open the script in our browser, the sum and difference of the vectors B and A (plus some additional information) will be computed and displayed.

27.4.4 Orthogonal vectors

Orthogonal vectors are vectors that form a right angle when placed tail-to-tail. In other words, the angle between them is 90 degrees. Since this is a fairly easy case to work with, lets begin with a couple of examples of this sort.

Specification for two vectors

- Bm = 10 units
- Ba = 90 degrees
- Am = 10 units
- Aa = 0 degrees

Use the graphical method and the trigonometric method to find Cs=B+A and Dd=B-A.

27.4.4.1 Graphical solution for Cs=B+A

Draw the two vectors on your graph board and construct the parallelogram as described earlier.

In this case, the parallelogram simply becomes a square. You should find that the angle for the vector C is 45 degrees. Using the Pythagorean theorem, you should find that the magnitude of the vector C is 14.14. Thus,

- Cm = 14.14 units
- Ca = 45 degrees

27.4.4.2 Graphical solution for Dd=B-A

To subtract the vector A from the vector B, flip vector A over and draw it pointing in exactly the opposite direction. Stated differently, add 180 degrees to the angle for A and draw it. Then add the modified vector A to the original vector B.

Once again, the parallelogram is a square. Now you should find that the magnitude for vector D is 14.14 units, and the angle for vector D is 135 degrees. Thus

- Dm = 14.14 units
- Dz = 135 degrees

Sum and difference magnitudes are the same

Note that the magnitude of the difference vector is the same as the magnitude of the sum vector in this case. As you will see later, that is not the case in general.

Difference vector is perpendicular to the sum vector

Also, the angle of the difference vector is 90 degrees greater than the angle of the sum vector. In other words, the two vectors are perpendicular. As you will see later, that is the case in general.

27.4.4.3 Trigonometric solution for Cs=B+A and Dd=B-A

Figure 1 (p. 664) shows the program output for the sum and difference of a pair of orthogonal vectors. These are the same vectors for which you estimated the magnitudes and angles of the sum and difference vectors earlier.

Program output for orthogonal vectors.

```
Start Script
Bm = 10.00
Ba = 90.00 deg
Am = 10.00
Aa = 0.00 deg
Bx = 0.00
By = 10.00
Ax = 10.00
Ay = 0.00
Cx = 10.00
Cy = 10.00
Ca = 45.00 deg
Cm = 14.14
Da = 135.00 deg
Dm = 14.14
End Script
```

Figure 27.1: Program output for orthogonal vectors.

The last five lines of output text in Figure 1 (p. 664) show the same results that you got using graphical methods to add and subtract the vectors.

27.4.5 Same magnitude, 45-degree angle

Now let's modify the problem and reduce the angle between the two vectors to 45 degrees. Assume that

- Bm = 10 units
- Ba = 45 degrees
- Am = 10 units
- Aa = 0 degrees

Compute B+A and B-A as before.

27.4.5.1 Graphical solutions for sum and difference vectors

When you draw your parallelograms, you should find that:

- The angle for the sum vector is now 22.5 degrees (the sum vector falls half way between the two vectors being added).
- The angle for the difference vector is still equal to the angle for the sum vector plus 90 degrees, or 112.5 degrees. (The difference vector is perpendicular to the sum vector.)
- The magnitude of the sum vector is now longer than before.
- The magnitude of the difference vector is now shorter than before.

We could continue with graphical solutions

We could continue this process for smaller and smaller angles between two vectors with the same or different magnitudes, and we would find that

- The direction of the sum vector continues to be between the two vectors being added.
- The magnitude of the sum vector approaches the sum of the magnitudes of the two vectors as the angle between them approaches zero.
- The magnitude of the difference vector approaches the difference between the magnitudes of the two vectors as the angle approaches zero.

When the two vectors have the same magnitude

For the special case where the magnitudes of the two vectors being added and subtracted are the same,

- The magnitude of the sum vector approaches twice the magnitude of either vector as the angle between the two vectors approaches zero.
- The magnitude of the difference vector approaches zero as the angle between the two vectors approaches zero.
- The angle for the difference vector continues to be equal to the angle for the sum vector plus 90 degrees as the angle between the two vectors approaches zero. (The difference vector is perpendicular to the sum vector.)

This case will be very important in the modules on circular motion,

What happens to the angles?

Getting back to your graph board drawing of the most recent scenario, there is another very important characteristic that we might be able to recognize. When you add the negative of vector A to vector B in order to subtract vector A from vector B, the direction of the resulting vector points at an angle that bisects the angle made by vectors B and -A.

An almost straight line

As the angle between the vectors B and A approaches zero, the angle between the vectors B and - A approaches 180 degrees. Therefore, those two vectors tend to describe a straight line when joined at their tails as the angle between B and A approaches 0.

Perpendicularity

The magnitude of the vector that is the sum of B and -A approaches 0, while the direction of that vector approaches perpendicularity with the (almost) straight line. That means that the difference vector approaches perpendicularity with each of the original vectors, B and A, as the angle between them approaches 0.

Please remember this when we discuss centripetal force in a future module.

27.4.6 Sum and difference for smaller and smaller angles

It is difficult and time consuming for blind students to do vector addition and subtraction with the graph board. Therefore, I will make some minor modifications to the code in Listing 1 (p. 661) to cause the output to be more compact and then run the script for a series of decreasing angles between the vectors A and B while keeping the magnitudes of the two vectors the same. The results are shown in Figure 2 (p. 666) .

Program output for smaller and smaller angles.

```
    Start Script
Bm = 10.00 Ba = 90.00 deg
Am = 10.00 Aa = 0.00 deg
Cm = 14.14 Ca = 45.00 deg
Dm = 14.14 Da = 135.00 deg

Start Script
Bm = 10.00 Ba = 45.00 deg
Am = 10.00 Aa = 0.00 deg
Cm = 18.48 Ca = 22.50 deg
Dm = 7.65 Da = 112.50 deg

Start Script
Bm = 10.00 Ba = 22.50 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.62 Ca = 11.25 deg
Dm = 3.90 Da = 101.25 deg

Start Script
Bm = 10.00 Ba = 11.25 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.90 Ca = 5.62 deg
Dm = 1.96 Da = 95.63 deg

Start Script
Bm = 10.00 Ba = 5.63 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.98 Ca = 2.81 deg
Dm = 0.98 Da = 92.81 deg

Start Script
Bm = 10.00 Ba = 2.81 deg
Am = 10.00 Aa = 0.00 deg
Cm = 19.99 Ca = 1.41 deg
Dm = 0.49 Da = 91.41 deg

Start Script
Bm = 10.00 Ba = 1.41 deg
Am = 10.00 Aa = 0.00 deg
Cm = 20.00 Ca = 0.70 deg
Dm = 0.25 Da = 90.70 deg

Start Script
Bm = 10.00 Ba = 0.07 deg
Am = 10.00 Aa = 0.00 deg
Cm = 20.00 Ca = 0.04 deg
Dm = 0.01 Da = 90.04 deg
```

Available for free at Connexions <http://cnx.org/content/col11294/1.35>

Figure 27.2: Program output for smaller and smaller angles.

The results

Note in particular, the values of the magnitude of the difference vector (Dm) and the angle of the difference vector (Da) in Figure 2 (p. 666) as the angle between the vectors B and A approaches zero.

As you can see from Figure 2 (p. 666) , regardless of the angle between vectors B and A, the difference vector is always perpendicular to the sum vector.

As you also can also see from Figure 2 (p. 666) , for very small angles, the angle of the sum vector is very close to the angles of the other two vectors. (They almost overlay one another.) Therefore, for very small angles, the difference vector is very close to being perpendicular to each of the vectors being subtracted.

As I mentioned earlier, this conclusion will be very important in a future module dealing with circular motion.

27.5 Run the script

I encourage you to run the script that I presented in this lesson to confirm that you get the same results. Confirm some of those results with your graph board.

Copy the code for the script into a text file with an extension of html. Then open that file in your browser. Experiment with the code, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

27.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

27.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Vector Subtraction for Blind Students
- File: Phy1230.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - vector

NOTE: **Disclaimers: Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 28

Circular Motion – Speed and Velocity[1]

28.1 Table of Contents

- Preface (p. 669)
  - General (p. 669)
  - Prerequisites (p. 669)
  - Supplemental material (p. 670)
- Discussion (p. 670)
  - Uniform circular motion (p. 670)
  - Average speed (p. 671)
  - Velocity (p. 672)
- A thought experiment (p. 672)
- Resources (p. 673)
- Miscellaneous (p. 673)

28.2 Preface

28.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains speed and velocity for circular motion in a format that is accessible to blind students.

28.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).

[1] This content is available online at <http://cnx.org/content/m38399/1.2/>.
[2] http://cnx.org/content/col11294/latest/
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw

- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

28.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

28.3 Discussion

Much of what you learned in earlier modules pertaining to linear motion applies also to circular motion.

28.3.1 Uniform circular motion

A ride on a carousel

Suppose that you and three of your friends go to an amusement park and take a ride on the carousel. In case, you are unfamiliar with a carousel, it is usually a large disk containing models of horses positioned around concentric circles. Children sit on the horses while the disk spins. As the disk spins, music plays, and the horses go up and down.

Pay for a ride

Usually, you pay for a ride and when the carousel stops, you get on one the horses. After everyone is safely on a horse, the disk starts to spin. After getting up to speed, the disk spins at the same speed for a few minutes. Then it slows down and stops. Everyone gets off, and a new group of riders get on.

Ignore the up and down motion

For this discussion, we will ignore the music and the up and down motion of the horses and consider only the circular motion.

Four sets of horses

[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

Assume that it is a large carousel with horses on four equally spaced concentric circles. You take a seat on one of the horses on the outer circle and your three friends take seats on horses on the other three circles. The four of you are riding approximately side-by-side.

Constant speed

Once the carousel gets started and comes up to speed, it typically spins at a constant speed for several minutes, after which it slows down and stops.

During the time that the carousel is spinning at constant speed, you and each of your friends would be experiencing **uniform circular motion** .

28.3.2 Average speed

Let's assume that the radius of the circle on which your horse is positioned is 10 meters. In other words, you are sitting on a horse that is 10 meters from the center of the carousel.

The circumference of the circle

Then the circumference of the circle on which you are located would be equal to

c1 = 2*pi*r1 = 2*pi*10m = 62.83 meters

where

- c1 is the circumference of the outer circle
- r1 is the radius of the outer circle
- pi is the mathematical constant pi with a value of 3.14159
- m is distance in meters

Time required to complete one cycle

As the disk spins, you pass the same sighted observer standing close to the carousel again and again. Assume that the sighted observer determines that the time required for you to complete each cycle around the carousel is 31.41 seconds.

The period

The proper term for the time required for an object to complete one cycle with uniform circular motion is **period** . Thus, your period would be 31.41 seconds.

Periodic motion

The term **period** is also used to describe the motion of other objects, such as a pendulum, a rocking chair, etc., resulting in a related term: *periodic motion* . Periodic motion is motion that is repeated in equal intervals of time. Circular motion is only one of many forms of periodic motion.

Average speed

Getting back to your ride on the carousel, your average speed around the circumference of your circle would be equal to

Avg Speed = distance/time = circumference/time, or

Avg Speed = 2*pi*radius/time, or

Avg Speed = 2*pi*10m/31.41s = 2 m/s

where

- circumference is the circumference of the circle in meters
- Avg speed represents average speed
- time is the number of second required to complete one trip around the circumference; the period

Average speeds of your friends

Now consider the average speed that each of your friends are traveling. Each of you complete one cycle in 31.41 seconds, but your friends don't travel as far as you do in that amount of time. The circumference of the circles on which they are traveling is smaller than the circumference of the circle on which you are traveling. Therefore, your average speed is greater than your friends' average speeds.

Assume, for example, that the radii of the circles on which your friends are traveling are 9, 8, and 7 meters respectively. The average speed for you and each of your friends will be

- s1 = 2*pi*r1/time = 2*pi*10m/31.41s = 2 m/s
- s2 = 2*pi*r2/time = 2*pi*9m/31.41s = 1.8 m/s
- s3 = 2*pi*r3/time = 2*pi*8m/31.41s = 1.6 m/s
- s4 = 2*pi*r4/time = 2*pi*8m/31.41s = 1.4 m/s

where

- s1 through s4 represent the four different speeds
- r1 through r4 represent the radii of the four circles

Speed is proportional to radius

As you can see, the average speed for each rider is directly proportional to the radius of the circle on which that rider is traveling. Doubling the radius doubles the average speed. If there were a horse located 5 meters from the center of the carousel, the average speed for that horse would be only half of your average speed at 10 meters, or 1 m/s.

28.3.3 Velocity

Just because you are traveling at a constant speed (during a portion of the ride anyway), doesn't mean that you are traveling at a constant velocity. You learned in an earlier module that an object in motion tends to remain in motion in a straight line unless a force is applied to cause the object to change direction.

You are constantly changing direction

When you are riding on the carousel, you are constantly changing direction. Otherwise you would travel in a straight line instead of traveling in a circle.

A force is required to cause you to continually change direction and to travel in a circle. That force is exerted on your body by your grasp on the horse on which you are sitting.

Goodbye horse and rider

If the brackets that attach your horse to the carousel were to break, you would continue to travel in a straight line at that point, leaving the carousel behind.

A tangential velocity vector

At any instant in time, the direction of your velocity vector is a direction that is tangent to the circle on which you are traveling. Stated differently, the direction of your velocity vector is along a line that is perpendicular to a line that extends from your center of mass to the center of the circle. (A tangent line is a line that touches a circle at one point but does not intersect it.) If you sit very straight on the horse and face straight ahead, you will be facing the direction of your velocity vector.

Summary

An object moving in uniform circular motion is moving around the perimeter of the circle with a constant speed. Although the speed of the object is constant, the object's velocity is constantly changing.

The object's velocity vector has a constant magnitude but a changing direction. At any instant in time, the direction of the velocity vector is tangent to the circle. As the object travels along the circular path, during one full rotation (cycle) around the center, the tangent line is always pointing in a new direction. During each cycle, the velocity vector points in the same (infinite) set of directions.

A force must be exerted on an object to cause it to travel along a circular path instead of traveling in a straight line. As you will learn in a future module, this force is called the **centripetal** (center seeking) force.

28.4 A thought experiment

Assume that you attach one end of a garden hose to a faucet and then arrange the hose in a circle on the ground. When you turn the water on, will the water exit the hose and continue moving in a circular path, or will it move in a straight line (ignoring the effects of gravity, air resistance, etc.)?

You may already know from experience that when the water exits the hose, it moves in a straight line. While the water was inside the hose, it moved in a circular path due to the force exerted on the water molecules by the inside surface of the hose.

When the water exits the hose, that force will no longer be applied to the water molecules. According to Newton, the water will continue in motion at a constant velocity, meaning that the direction of the velocity vector for the water molecules will not change.

28.5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

28.6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Circular Motion – Speed and Velocity for Blind Students
- File: Phy1240.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · uniform circular motion
  - · average speed
  - · velocity
  - · velocity vector
  - · circumference
  - · radius
  - · period
  - · periodic motion
  - · carousel

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 29

Circular Motion – Acceleration and Centripetal Force[1]

29.1 Table of Contents

- Preface (p. 675)
  - General (p. 675)
  - Prerequisites (p. 676)
  - Supplemental material (p. 676)
- Discussion (p. 676)
  - Subtraction of vectors (p. 677)
  - An old-fashioned merry-go-round (p. 678)
  - A weight on a string (p. 679)
  - Water in a bucket (p. 679)
  - The moon and the Earth (p. 680)
  - The work-energy explanation (p. 680)
  - Centripetal, not centrifugal (p. 680)
  - Summary (p. 680)
- Example scenarios (p. 681)
  - Scenario #1 (p. 681)
  - Scenario #2 (p. 681)
  - Scenario #3 (p. 681)
- Resources (p. 682)
- Miscellaneous (p. 682)

29.2 Preface

29.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains acceleration and centripetal force in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38406/1.2/>.
[2] http://cnx.org/content/col11294/latest/

29.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

29.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

29.3 Discussion

I purposely published an earlier module titled Vector Subtraction for Blind Students [13] in preparation for this module.

An unbalanced net force is required

As you learned in an earlier module, in order for an object to travel in a circle under uniform circular motion, a force must act on that object to cause it to stay on the circular path. Otherwise, according to Newton, the object will fly off into space at a constant velocity (ignoring both air resistance and gravity).

Change of direction equals acceleration

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm
[13] http://cnx.org/content/m38374/latest/?collection=col11294/latest

Whenever the velocity vector that describes the motion of an object changes direction, that object has undergone an acceleration, even if the magnitude of the velocity vector hasn't changed.

Continual change in direction

The velocity vector for an object that is traveling on a circular path at a constant speed is constantly changing direction. Otherwise, the object would not stay on the circular path. As you are aware, velocity is a vector. That is, it has both magnitude and direction.

As you are also aware, acceleration (the time rate of change of velocity) is also a vector. That is to say, it also has magnitude and direction. The direction of the acceleration vector is the same as the direction of the force that caused the acceleration.

The direction of the acceleration vector

The direction of the acceleration vector for an object under uniform circular motion always points to the center of the circle.

That may be difficult for you to believe. I will try to convince you using several different approaches, some graphically satisfying, and some more anecdotal.

29.3.1 Subtraction of vectors

For this approach, I would like for you to use your graph board and follow along by creating and subtracting vectors. Use your graph board, some string, and a few pushpins to create an arc of a circle centered on the origin of a Cartesian coordinate system. All you will need is about one-half of the circle, on and to the right of the vertical axis.

Uniform circular motion

We will assume that on object is moving counter-clockwise around that circle with a uniform speed.

The initial velocity vector

Draw a vector with a length of ten units with it's tail at the intersection of the circle and the positive horizontal axis. This vector should point straight up in the direction of the positive y-axis. This vector represents the velocity of the object as it crosses the horizontal axis on its trip around the circle.

The final velocity vector

Now go up to a point on the circle about 30 degrees relative to the positive horizontal axis (the exact angle isn't critical). Draw a vector with a length of 10 units that is tangent to the circle at that point.

It isn't easy to draw a vector that is tangent to a circle. However, the direction of that vector should be perpendicular to a line that extends from the point on the circle to the center of the circle. That will cause the vector to be tangent to the circle at that point.

The initial and final velocity vectors

We now have two vectors that represent the velocity vectors for the object at two different points along its circular path separated by a time interval. (The exact time interval doesn't matter at this point in the discussion.) Label the vector at the horizontal axis as Vi (for initial vector). Label the vector at the 30-degree point as Vf (for final vector).

Although there is no way for you to label it on the graph, we will represent the time interval required for the object to move from the first point to the second point as dt (which is an abbreviation for delta-time, or change in time).

Average acceleration

We know that the average acceleration is the time rate of change of velocity. Therefore, we can write

Aavg = (Vf - Vi)/dt

where

- Aavg represents the average acceleration
- Vf represents the velocity at the second point
- Vi represents the velocity at the first point
- (Vf - Vi) represents the change in velocity
- dt represents the time interval over which that change took place

Subtracting vectors

In the earlier module titled Vector Subtraction for Blind Students [14] , you learned how to subtract the vector named Vi from the vector named Vf using the parallelogram method.

Let the time interval get smaller and smaller

What we want to do in this module is to estimate what happens to (Vf - Vi) as the time interval, dt, gets smaller and smaller.

Given that the speed of the object remains constant, the point on the circumference of the circle that represents the tail of Vf will move closer and closer to the horizontal axis as the time interval gets smaller.

The angle between the vectors gets smaller and smaller

That, in turn, will cause the angle that Vf forms with the horizontal axis to move closer and closer to 90 degrees. That means that the difference in the angles that the two vectors make with the horizontal axis will grow smaller and smaller, and the angle between the two vectors will grow smaller and smaller.

The difference vector

In the earlier module, you learned that for very small angles, the difference vector is very close to being perpendicular to both Vf and Vi. Being very close to perpendicular to both Vf and Vi means that the acceleration vector is very close to pointing directly at the center of the circle.

Using calculus, it can be shown that in the limit, as the time interval, dt, approaches zero, the direction of the acceleration vector points directly at the center of the circle.

The magnitude of the acceleration vector

In the earlier module, you also learned that the magnitude of the difference vector approaches zero as the angle between the two velocity vectors approaches zero. However, the acceleration is equal to the magnitude of the difference vector divided by the scalar value of the time interval, dt, which is also approaching zero. One very small value divided by another very small value is not necessarily a very small value. It can be shown using calculus that in the limit, the magnitude of the acceleration vector is not zero.

The conclusion

The conclusion is that an object undergoing uniform circular motion experiences an acceleration vector that points directly at the center of the circle and it has a non-zero magnitude.

29.3.2 An old-fashioned merry-go-round

That was the graphical explanation of the acceleration vector. Now I will cite a few anecdotal explanations.

A long wooden plank

When I was around ten or eleven years old, the child across the street from my house had an old-fashioned homemade merry-go-round in his back yard. This device consisted of a long wooden plank with a hole in the center. A large bolt was threaded through the hold and pushed vertically into a hole in the top of a post about two feet tall. The bolt formed a spindle and the plank was able to rotate around the spindle.

Two handles

Two short skinny boards were fastened to the plank about 18 inches from each end to form a sort of a cross in each end. These boards extended about 9 inches on either side of the plank and were intended to be handles.

A three-child operation

It took three children to operate this merry-go-round. One child sat on each end and the third child pushed it around and around as fast as possible.

Oops

We quickly learned that if we sat on the plank between the handle and the end of the plank and tried to hold onto the handle, we would slide off the end of the plank when the rotation of the plank reached a certain speed. Therefore, we learned to sit with our legs across the handle, with the handle pressing against the back insides of our knees. By doing this, we could survive without sliding off the plank no matter how fast the third child caused the plank to rotate.

Why am I telling you this?

[14] http://cnx.org/content/m38374/latest/?collection=col11294/latest

Although I didn't understand it at the time, when the plank was rotating around its spindle at a relatively high rate of speed, the handle exerted a force on the inside of both knees. In other words, the handle exerted a force on each leg in the direction of the center of the plank. This is a force that we will call a ***centripetal force*** .

NOTE: **Centripetal force**

According to The Physics Classroom [15] , any object moving in a circle (or along a circular path) experiences a centripetal force. That is, there is some physical force pushing or pulling the object towards the center of the circle. This is the centripetal force requirement.

The word centripetal is merely an adjective used to describe the direction of the force. We are not introducing a new type of force but rather describing the direction of the net force acting upon the object that moves in the circle. Whatever the object, if it moves in a circle, there is some force acting upon it to cause it to deviate from its straight-line path, accelerate inwards and move along a circular path.

A centripetal force on my body

The effect of this centripetal force was to cause my body to accelerate in the direction of the force, which was directly toward the spindle at the center of the plank. The spindle was at the center of the circle around which I was experiencing circular motion (although probably not uniform circular motion).

29.3.3 A weight on a string

Consider tying a weight, such as a full bottle of soda, to the end of a string. Then swing the bottle around and around above your head in an approximate circle. You will feel a force pulling your hand toward the periphery of the circle, so you will exert a force on the string to keep the bottle from flying away. The force that you exert on the string will be exerted on the bottle at the other end of the string and that force will be directed along the string towards the center of the circle. That is the force that we will refer to as centripetal (center seeking) force, and that force will act on the mass of the bottle to cause the bottle to accelerate in the direction of the center of the circle.

What happens if the string breaks?

If the string breaks, there will no longer be a centripetal force acting on the bottle and it will fly off into space along a line that is tangential to its location on the circle at the instant the string breaks. It won't continue moving in a circle. Instead, it will move in a straight line until the resistance of the air and force of gravity bring it to a sudden stop on the ground. (Actually, it will move along a parabolic arc under the force of gravity until it strikes the ground.)

29.3.4 Water in a bucket

If you don't mind taking a chance on getting wet, fill a small bucket about half full of water. Then swing the bucket rapidly in a circle in a vertical plane.

If you can swing the bucket fast enough, the water will stay in the bucket even when the bucket is upside down. Why is that? The water molecules want to move in a straight line. However, the inside surface of the bucket exerts a centripetal force on the water molecules causing them to accelerate toward the center of the circle. The centripetal force increases with the speed of the bucket. As long as the centripetal force is greater than the weight of the water, the water won't fall out of the bucket onto your head. However, if you allow the speed of the bucket to decrease, you will reach the point where gravity will overcome, and you will probably get wet.

[15] http://www.physicsclassroom.com/Class/circles/u6l1c.cfm

29.3.5 The moon and the Earth

Don't ask me how it got started in the first place, but somehow the moon got started circling the Earth.

The speed of the moon and the radius of its orbit is just exactly right so that the gravitational force that the Earth exerts on the moon causes the moon to accelerate towards the Earth. The amount of acceleration toward the earth, when combined with the speed of the moon, causes the moon to move in a uniform circular orbit around the Earth instead of either flying off into space or crashing into the Earth.

29.3.6 The work-energy explanation

It is probably time to start explaining this phenomena in a more technical and less anecdotal manner.

The application of a centripetal force for uniform circular motion causes the direction of the object to be changed without changing its speed. Let's see if we can explain this from a work-energy viewpoint.

Work

Recall from an earlier module that **work** is a force acting upon an object to cause a **displacement** . Also recall that the amount of work done on an object, expressed in Joules, is given by

Work = F * D * cosine(theta)

where

- F represents the force that causes the displacement expressed in Newtons.
- D represents the amount of the displacement expressed in meters.
- theta is the angle between the direction of the displacement and the line of action of the force.

Centripetal force points toward center of circle

As we showed (or claimed to show) earlier, the centripetal force for an object in uniform circular motion always points in the direction of the center of the circle. At the same time, the velocity vector, which represents the direction of the displacement is tangential to the circle. Therefore, the angle between the centripetal force and the direction of displacement is 90 degrees. This means that the centripetal force does no work on the object, because the cosine of 90 degrees is zero.

No work is done

When no work is done upon an object by external forces, the total mechanical energy, consisting of potential energy plus kinetic energy, of the object remains constant. If an object is moving in a circle in a plane that is parallel to the surface of the Earth, the effect of gravity may pull the entire plane toward the surface of the Earth, but it won't effect the circular motion unequally with respect to the position of the object in the circle.

Therefore, if an object is moving in a horizontal circle at constant speed, the centripetal force does no work on the object and cannot change the total mechanical energy of the object. The kinetic energy will remain constant and therefore, the speed of the object will remain constant. The centripetal force will accelerate the object by changing its direction but will not change its speed.

29.3.7 Centripetal, not centrifugal

Don't confuse centripetal force with the word *centrifugal* . For some reason, many people mistakenly use the word centrifugal (meaning outward) when they should use the word centripetal. In this case, centripetal is the correct word.

Maybe this is like the use of the words nuclear and nucular. Which is correct? I will leave that as an exercise for the student to determine.

29.3.8 Summary

An object in uniform circular motion must experience an unbalanced force pointing towards the center of the circle. This force is referred to as a centripetal force, where centripetal means *inward seeking* .

This force is required to cause the object to continually change its direction in order to move along a circular path without changing its speed.

Because the centripetal force is directed perpendicular to the tangential velocity vector, the centripetal force cannot change the total mechanical energy possessed by the object (the cosine of 90 degrees is 0).

Because the centripetal force has no impact on the potential energy possessed by the object, and because it cannot change the total mechanical energy, it cannot change the kinetic energy possessed by the object. Since it can't change the kinetic energy, it can't change the object's speed.

However, it can change the object's direction without changing its speed.

29.4 Example scenarios

29.4.1 Scenario #1

An object is moving with uniform circular motion in a counter-clockwise direction around a circle whose center is at the origin in a Cartesian coordinate system. When the object passes through the intersection of the circle with the positive horizontal axis, what is the direction of the velocity vector relative to the horizontal axis?

1. 0 degrees
2. 180 degrees
3. 90 degrees
4. 270 degrees
5. 30 degrees

The correct answer is #3, 90 degrees. At that point, the line tangent to the circle touches the circle where it intersects the positive horizontal axis and is perpendicular to the horizontal axis. Because the motion is counter-clockwise, the vector points up at 90 degrees (instead of down at 270 degrees) relative to the positive horizontal axis.

29.4.2 Scenario #2

An object is moving with uniform circular motion in a counter-clockwise direction around a circle whose center is at the origin in a Cartesian coordinate system. When the object passes through the intersection of the circle with the positive horizontal axis, what is the direction of the acceleration vector relative to the positive horizontal axis?

1. 0 degrees
2. 180 degrees
3. 90 degrees
4. 270 degrees
5. 30 degrees

The correct answer is #2, 180 degrees. The acceleration vector for an object under uniform circular motion always points in the direction of the center of the circle. At the intersection of the circle with the positive horizontal axis, the direction of the center of the circle is 180 degrees relative to the positive horizontal axis.

29.4.3 Scenario #3

A heavy object is being swung on a string in a counter-clockwise direction around a circle whose center is at the origin in a Cartesian coordinate system. The plane of the circle is parallel to the ground so that we can ignore the effects of gravity.

Just as the object passes through the intersection of the circle with the positive horizontal axis, the string breaks allowing the object to fly off into the nearby playground space. What is the direction of motion of the object relative to the positive horizontal axis?

1. 0 degrees
2. 180 degrees
3. 90 degrees
4. 270 degrees
5. 30 degrees
6. The object will continue to move in a circle.

The correct answer is #3, 90 degrees, which is the direction of the velocity vector at the instant that the string breaks. Because there will no longer be a centripetal force to cause the object to change direction and stay on the circular path, it will continue moving in the direction that it is moving when the string breaks.

29.5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

29.6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Circular Motion – Acceleration and Centripetal Force for Blind Students
- File: Phy1250.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · uniform circular motion
  - · tangential
  - · velocity vector
  - · centripetal force
  - · work-energy

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 30

Circular Motion – The Mathematics of Circular Motion[1]

30.1 Table of Contents

- Preface (p. 684) ;
  - General (p. 684)
  - Prerequisites (p. 684)
  - Supplemental material (p. 684)
- Discussion (p. 685)
  - Angular displacement and angular velocity (p. 685)
    - Angular displacement (p. 685)
    - Angular velocity (p. 686)
    - Radian measure (p. 686)
    - Tangential displacement versus angular displacement (p. 687)
    - Tangential speed versus angular velocity (p. 688)
  - The relationship between period and frequency (p. 689)
  - The relationship between angular velocity and frequency (p. 689)
  - Radial (centripetal) acceleration (p. 690)
- Example scenarios (p. 690)
  - Circumference of the Earth at the equator (p. 690)
  - Speed of a point on the equator (p. 690)
    - Solution A (p. 691)
    - Solution B (p. 691)
  - Period and frequency (p. 691)
  - Radial acceleration (p. 691)
- Work through the examples (p. 692)
- Resources (p. 692)
- Miscellaneous (p. 692)

[1] This content is available online at <http://cnx.org/content/m38411/1.4/>.

30.2 Preface

30.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains the mathematics of circular motion in a format that is accessible to blind students.

30.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

30.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

[2] http://cnx.org/content/col11294/latest/
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

30.3 Discussion

Now that you have an idea of how circular motion behaves from a physical viewpoint, let's take a look at the mathematics that describe circular motion.

30.3.1 Angular displacement and angular velocity

We begin this module with two new terms: *angular displacement* and *angular velocity* .

Dealing with points can be awkward

Up until now in this series of modules on circular motion, we have dealt mainly with the motion of points involved in uniform circular motion. However, in some situations, that is awkward. Consider a wheel on a car, for example. There are an infinite number of points on the wheel, and when the wheel is spinning, every point is moving with a different velocity and/or acceleration. It would be difficult for us to describe that motion in terms of the motions of all the points.

This causes us to seek a more comprehensive description that will encompass the motion of all of the points on the wheel. Two terms that accomplish that purpose are angular displacement and angular velocity.

30.3.1.1 Angular displacement

Approaching the situation from this viewpoint, we concentrate on angles instead of distances. If a wheel spins through one-fourth of a complete revolution, every point on the wheel moves through the same 90-degree angle. (However, as you learned in earlier modules, points at different radii move different distances.)

A set of new variables

We will define a set of variables involving angular motion that are analogous to displacement, velocity, and acceleration in the realm of linear motion. However, we will use angular measurements instead of linear distance measurements.

Angular displacement

Instead of linear displacement, for example, we will speak of angular displacement. **Angular displacement** is the angle through which a rotating body turns based on some starting and stopping criteria.

A pie-shaped wedge

As you learned in an earlier module, a point on a wheel moves along the circumference of a circle as the wheel rotates. Viewing the rotating wheel from a vantage point that is perpendicular to the wheel, during a given time interval, we see that the point sweeps out a pie-shaped wedge with its point at the center of the wheel.

An arc of a circle

The motion of the point describes an arc of a circle directly opposite the point at the center of the circle. This pie-shaped wedge describes an angle, which is the **angular displacement** during that episode of movement. (You should be able to simulate this on your graph board in order to get a better picture in your mind.)

NOTE: **Definition of angular displacement**

dA = Af - Ai

where

- dA represents the angular displacement
- Ai is the angle that a line through the point makes with the horizontal axis when the episode begins.
- Af is the angle that a line through the point makes with the horizontal axis when the episode ends.

Physics books typically use Greek letters such as phi and theta to represent angles. However, it is unlikely that your Braille display can handle Greek characters, so I will stick with standard qwerty keyboard characters.

Angular displacement is a signed quantity

The direction of rotation is indicated by the algebraic sign of the angular displacement. It is conventional to consider a counter clockwise rotation to result in a positive angular displacement.

Units of angular displacement

The units of angular displacement are typically degrees or radians.

30.3.1.2 Angular velocity

That brings us to angular velocity, The **average angular velocity** is the average rate of change of angular displacement.

NOTE: **Definition of angular velocity**

wAvg = dA/dT

where

- wAvg represents average angular velocity
- dA represents the angular displacement in a given time interval
- dT represents the time interval

It is customary in physics books to represent angular velocity with the Greek letter omega. In this module, I will use a lower-case "w" character to represent angular velocity where appropriate, simply because it looks a lot like a Greek omega character.

Reduce the time interval

If we allow the time interval dT to be come shorter and shorter, we are averaging over smaller and smaller time intervals. In the limit, as dT approaches zero, wAvg becomes w, which is the *instantaneous* angular velocity.

Angular velocity is a signed quantity

Angular velocity is also a signed quantity with the sign indicating the direction of rotation. By convention, counter clockwise rotation is viewed as positive rotation. The sign of angular velocity is the same as the sign of the angular displacement that forms the basis for the angular velocity.

Units of angular velocity

The units of angular velocity are typically degrees per second or radians per second. You will learn later that radians is a dimensionless quantity. Therefore, when angular velocity is measured in radians per second, it often appears simply as

w = 10/sec

30.3.1.3 Radian measure

The most familiar measurement of angles, in the U.S. is in degrees. However, in some situations, it is more convenient to measure angles in radians than in degrees.

This becomes most apparent when we need to relate angular displacement or angular velocity with the distance traveled or the tangential speed of a point on a rotating object.

NOTE: **Definition of a radian**

One radian is an angular measurement, which is equal to an angle at the center of a circle whose arc is equal in length to the radius of the circle.

Simulate with a graph board

I recommend that you use your graph board to simulate an angle of one radian.

Using your graph board along with some string and pushpins, draw a Cartesian coordinate system. Then draw a circle with a convenient radius with its center at the origin of your coordinate system.

Make the arc match the radius

Cut a piece of string to the length of the radius of the circle. Then, beginning at the intersection of the circle and the horizontal axis, lay the string along the circumference of the circle moving in a counter clockwise direction. Put a pushpin at the point where the string ends. Then stretch a rubber band from that point back to the center of the circle.

Measure the angle

Using your protractor, measure the angle that the rubber band makes with the horizontal axis. That angle should be about 57.3 degrees, which is one radian.

Measure the number of radians in 360 degrees

Now, using the string whose length is equal to the radius of the circle as a measuring tool, determine how many strings of that length you can lay end-to-end around the circumference of the circle.

You should find that about 6.28 (2*pi) such strings are required to go all the way around the circumference of the circle.

An angle in radians is a ratio of lengths

An angle measured in radians is a ratio of two values, each of which have units of length. Therefore, such an angle has no dimensions.

NOTE: **Measurement of an angle in radians**

angle = s/r

where

- s is the length of an arc along the circumference of a circle
- r is the radius of the circle
- angle is the angle subtended by the arc length, s

;

An example measurement

Assume that the arc length is equal to one-half the circumference of the circle. This arc represents a subtended angle of 180 degrees. Then,

s = circumference/2 = pi * radius

angle = (pi * radius)/radius = pi

From this we can see that pi radians is equal to 180 degrees.

Earlier we saw that 2*pi radians equal 360 degrees.

NOTE: **Facts worth remembering**

- One radian is equal to approximately 57.3 degrees
- Pi radians is equal to 180 degrees
- 2*Pi radians are equal to 360 degrees

30.3.1.4 Tangential displacement versus angular displacement

Consider the case of a 1.5-radian angular displacement of a wheel in a given time interval. What is the corresponding displacement of a point on the circumference of the wheel? Assume that the radius of the wheel is 0.5 meters.

angle = s/r, or

s = angle * r, or

s = 1.5 * 0.5m = 0.75m

A simple solution

Thus, we see that the tangential displacement of a point on the circumference of a wheel due to a given angular displacement of the wheel in radians is simply the product of the displacement and the radius of the wheel.

Solving for the same result using angular displacement in degrees would be somewhat more complicated.

30.3.1.5 Tangential speed versus angular velocity

A similar simplification occurs when dealing with the angular velocity of a wheel and the tangential speed of a point on the circumference of the wheel.

As in linear measurements, the average angular velocity of a wheel is equal to the angular displacement of the wheel divided by the time interval during which the displacement takes place.

NOTE: **Measurement of angular velocity in radians**

w = dA/dT

By substitution,

w = (s/r)/dT = s/(r*dT)

where

- s is the length of an arc along the circumference of a circle
- dA is an angular displacement in a given time interval
- dT is the time interval
- r is the radius of the circle
- w is the angular velocity

Another example

Consider the case of a 1.5-radian/second angular velocity of a wheel. What is the corresponding tangential speed of a point on the circumference of the wheel? Assume that the radius of the wheel is 0.5 meters.

The tangential speed is equal to the tangential displacement, s, divided by the time interval over which the displacement occurs. Given the above information, we can write:

w = s/(r*dT)

Given that

v = s/dT

Substitution yields

v; = w*r, or

v = (1.5/s)*(0.5m) = 0.75 m/s

Another simple relationship

Once again, if you keep your units straight, the tangential speed of a point on the circumference of the wheel is simply equal to the angular velocity in radians per second multiplied by the radius of the wheel.

NOTE: **Facts worth remembering**

tangential displacement = dA * r

tangential speed = w * r

where

- tangential displacement is the distance that a given point travels around the circumference of a circle as a function of an angular displacement in radians.
- tangential speed is the speed at which a point travels around the circumference of a circle as a function of an angular velocity in radians.
- w is the angular velocity in radians/second
- dA represents angular displacement
- r represents radius

;

30.3.2 The relationship between period and frequency

As you already know, when the speed of a point moving in a circle is constant, its motion is called uniform circular motion.

As you also already know, even though the speed of the point is constant, the velocity is not constant. The velocity is constantly changing because the direction of the velocity vector is constantly changing.

The period

The amount of time required for the point to travel completely around the circle is called the period of the motion.

The frequency

The frequency of the motion, which is the number of revolutions per unit time, is defined as the reciprocal of the period. That is,

frequency in rev per sec = 1/(period in sec per rev), or

f = 1/T

where

- f represents frequency in revolutions per second
- T represents period in seconds per revolution

30.3.3 The r elationship between angular velocity and frequency

The speed of a point moving completely around the circle is equal to the distance traveled divided by the time.

sT = 2*pi*r/T, or

sT = 2*pi*r*f

where

- sT is the tangential speed
- r is the radius
- T is the time required for the point to make one complete revolution
- f is the reciprocal of T

We know from before that

sT = w * r, or

w = sT/r

Therefore, by substitution from above,

w = 2*pi*r*f/r = 2*pi*f, or

the angular velocity in radians per second is the product of 2*pi and the frequency in revolutions per second.

where

- sT is tangential speed
- w is angular velocity in radians per second
- f is frequency in revolutions per second, or cycles per second, or hertz

The SI unit for frequency

The SI unit for frequency is hertz (Hz) where 1 Hz is equal to one revolution per second or one cycle per second.

NOTE: **Facts worth remembering**

w = 2*pi*f

where

- w is angular velocity in radians per second
- f is frequency in revolutions per second, or cycles per second, or hertz

The SI unit for frequency is hertz (Hz) where 1 Hz is equal to one revolution per second or one cycle per second

30.3.4 Radial (centripetal) acceleration

In an earlier module, you learned how to subtract vectors and; demonstrate that the acceleration vector of an object moving with uniform circular motion always points toward the center of the circle. However, in that lesson, we did not address the magnitude of the acceleration vector. We will do that here.

A very difficult derivation

Deriving the magnitude of the acceleration vector depends very heavily on the use of vector diagrams, complex assumptions, complicated equations. Unfortunately, this is one of those times that I won't be able to present that derivation in a format that is accessible for blind students. In this case, blind students will simply have to accept the final results in equation form and use those equations for the solution of problems in this area.

NOTE: **Facts worth remembering**

Ar = v^2/r, or

Ar = (w^2)*r

where

- Ar is the magnitude of the radial acceleration
- v is the magnitude of the tangential velocity of the object moving around the circle
- r is the radius of the circle
- w is the angular velocity of the object moving around the circle

30.4 Example scenarios

In this section, I will work through some examples that illustrate what you learned in the earlier section along with what you have learned in earlier modules.

30.4.1 Circumference of the Earth at the equator

The radius of the Earth at the equator is equal to approximately 6378km. What is the circumference of the earth at the equator.

Solution:

If you were to travel around the Earth at the equator, you would travel along a circular arc that subtends an angle of 2*pi radians. We know how to compute the length of the circular arc given the radius and the subtended angle:

arc length = (subtended angle) * radius, or

circumference = 2*pi*6378km = 40074 km

30.4.2 Speed of a point on the equator

The Earth rotates around its axis once each 24 hours. Therefore, a point on the equator makes one full trip around a circle with the circumference of the Earth each 24 hours.

Assume you are standing at a point on the equator. Ignoring all of the other motions of the universe, what is the speed with which you are traveling around that circle?

What is the angular velocity of the earth in radians per second.

Does the angular velocity of the Earth change when you drive North from the equator?

30.4.2.1 Solution A

Speed

Since we already know the circumference of the Earth, we know that you will travel 40074 km each 24 hours. Therefore,

speed = 40074km/24hr = 463.8 meters/second, or

speed = 1037 miles/hour

Did you know that you are constantly moving through space at a speed slightly greater than 1000 miles per hour?

Angular velocity

We also know that the earth rotates around its axis by 2*pi radians each 24 hours. Therefore, the angular velocity of the earth is

w = 2*pi radians/24 hours = 7.27 *10^(-5) radians / second

Differences in angular velocity

For purposes of this discussion, the Earth does not distort as it rotates. Therefore, the angular velocity of every point on the surface of the Earth rotates around the Earth's axis with the same angular velocity; namely 2*pi radians every 24 hours.

However I did see on TV the other day that the angular velocity of the Earth's core may be different from the angular velocity of the outer crust of the Earth. Apparently this can happen because the core is not firmly attached to the crust, but rather is suspended in a bath of molten rock.

30.4.2.2 Solution B

You learned earlier that the speed is equal to the product of the angular velocity and the radius. Therefore,

speed = w * r, or

speed = (7.27 *10^(-5) radians / second) * 6378km, or

speed = 463.7 m/s

which agrees with Solution A above.

Note that w represents angular velocity and r represents radius in the above calculations.

30.4.3 Period and frequency

A child's toy contains a round disk that rotates with a period of 0.628 seconds.

What is the frequency with which a spot on the edge of the disk passes a fixed mark on the body of the toy.

What is the angular velocity of the toy?

Solution:

f = 1/T = 1/(0.628) = 1.59 Hz

w = 2*pi*f = 10 radian/second

where

- f represents frequency
- T represents the period
- w represents angular velocity

30.4.4 Radial acceleration

A student ties a 10 kg mass onto a fishing line with a breaking strength of 5 newtons, and then starts swinging the mass around over his head. The student tries very hard to cause the path to be circular. The distance from the center of the circle to the mass is 3 meters.

As time goes on, the student swings the mass faster and faster until the fishing line breaks. What is the tangential velocity of the mass when the fishing line breaks.

Solution:
Centripetal force = mass * (centripetal acceleration), or
f = m * Ar, or
f = m * v^2/r, or
v^2 = f*r/m, or
v = (f*r/m)^(1/2), or
v = (5 newtons * 3 meters/10 kg)^(1/2), or
v = 1.22 m/s
The magnitude of the tangential velocity of the mass when the fishing line breaks is 1.22 meters/second
Check the solution using angular velocity, w
w = v / r, or
w = (1.22 m/s) / 3m = 0.407 radians/second
The angular velocity for a tangential velocity of 1.22 m/s is 0.407 radians/second
Ar = (w^2)*r, or
Ar = ((0.407 /second)^2)*3m, or
Ar = 0.5 m/s^2
The magnitude of the radial acceleration is 0.5 m/s^2
f = m * Ar, or
f = 10kg*(0.5 m/s^2), or
f = 5 newtons, or
the force equals 5 newtons, which matches the breaking strength of the fishing line.

30.5 Work through the examples

I encourage you to work through the examples that I have presented in this module to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

30.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

30.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Circular Motion – The Mathematics of Circular Motion for Blind Students
- File: Phy1260.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display

· JavaScript
· trigonometry
· angular displacement
· angular velocity
· radian measure
· tangential displacement
· tangential speed
· frequency
· period
· centripetal
· centripetal acceleration
· centripetal force

NOTE: **Disclaimers: Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 31

Angular Momentum – Rotational Kinetic Energy and Inertia[1]

31.1 Table of Contents

- Preface (p. 696)
  - General (p. 696)
  - Prerequisites (p. 696)
  - Viewing tip (p. 696)
    - Facts worth remembering (p. 697)
  - Supplemental material (p. 697)
- Discussion (p. 697)
  - Introduction (p. 697)
  - Calculating rotational kinetic energy (p. 698)
  - The rotational inertia (I) (p. 699)
  - Mass is no longer absolute (p. 700)
  - The parallel axis theorem (p. 701)
  - Examples of rotational inertia (p. 701)
- Example scenarios (p. 702)
  - The 2x4 scenario (p. 703)
  - The dumbbell scenario (p. 703)
  - A pulley and two objects, part 1 (p. 704)
  - A pulley and two objects, part 2 (p. 706)
  - A pulley and two objects, part 3 (p. 706)
  - A pulley and two objects, part 4 (p. 707)
  - A pulley and two objects, part 5 (p. 707)
  - An Atwood machine (p. 708)
  - A flywheel (p. 708)
- What have we learned? (p. 708)
- Work through the examples (p. 709)
- Resources (p. 709)
- Miscellaneous (p. 709)

[1] This content is available online at <http://cnx.org/content/m38440/1.2/>.

31.2 Preface

31.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains rotational kinetic energy and inertia in a format that is accessible to blind students.

31.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

31.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the material listed below while you are reading about it.

[2] http://cnx.org/content/col11294/latest/
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp

31.2.4 Facts worth remembering

- Rotational inertia (p. 699)
- Translational and Rotational Kinetic Energy (p. 700)
- Finding the rotational inertia (p. 700)
- The parallel axis theorem (p. 701)
- Examples of rotational inertia (p. 702)

31.2.5 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

31.3 Discussion

What do we mean by rotational kinetic energy and rotational inertia?

31.3.1 Introduction

In an earlier module, you learned of Newton's first law, which can be paraphrased something like the following:

Newton's first law

Every body (that has mass) in a state of rest tends to remain at rest. Similarly, every body (that has mass) in a state of motion tends to remain in motion in a straight line. In both cases, the body tends to remain in its current state unless compelled to change its state by external forces acting upon it.

This law is sometimes referred to as the "Law of Inertia" – objects that have mass don't like to change their velocity.

Newton's second law

Newton's second implies (once again paraphrasing) that the tendency to remain in the state of rest or motion depends on the amount of mass possessed by the body.

The greater the mass, the greater must be the external force required to cause the body to change its state by a given amount. You will recognize this as being characterized by the equation that tells us that acceleration (change of velocity) is proportional to the applied force and inversely proportional to the amount of mass.

Rotating rigid bodies

Similar considerations also apply to the rotation of rigid bodies. In the case of rigid-body rotation, however, it isn't simply the amount of mass that is important. The geometrical distribution of that mass also has an impact on the reluctance of a rotating body to change its state. We refer to this as *rotational inertia* , which is sometimes called the *moment of inertia* .

Rotational inertia or moment of inertia?

Rotational inertia and moment of inertia are simply two names that mean the same thing (some authors prefer one, other authors prefer the other)

I don't have a preference for either, but for consistency with the textbook currently being used for introductory physics courses at the college where I teach, I will use the term *rotational inertia* instead of *moment of inertia* .

An example

For example, the same amount of mass can be used to create

- a tall solid cylinder with a small radius or
- a thin disk with a large radius or
- a very thin disk with most of the mass concentrated around the outer circumference of the disk.

[12] http://www.dickbaldwin.com/toc.htm

When rotated about its central axis, either disk has a greater rotational inertia than the cylinder, and the disk with most of its mass concentrated around the outer circumference has a greater rotational inertia than the other two.

The rotational inertia increases as the mass is moved further from the axis of rotation, and the effect is proportional to square of that distance. As a result, given that the total mass is the same in all three cases, a greater external force is required to cause either disk to change its rotational state than is required to cause the cylinder to change its rotational state.

Work and kinetic energy

You also learned in an earlier module that work must be done on an object to cause it to have kinetic energy, and the kinetic energy possessed by an object is proportional to one-half of the product of the object's mass and the square of its velocity.

KE = (1/2)*m*v^2

where

- KE represents translational kinetic energy
- m represents mass
- v represents velocity

The rotational kinetic energy of a rotating body

When I was a youngster, I learned the hard way that a rotating rigid object has rotational kinetic energy. I had my bicycle turned upside down resting on the seat and the handle bars with the rear wheel turning very fast. I was using a long thin triangular file to chip mud off of the bicycle. I accidentally allowed one end of the file to come in contact with the tread on the spinning bicycle tire and ended up with a file sticking in the palm of my hand. Although I didn't know the technical term for rotational kinetic energy at the time, I did learn what rotational kinetic energy can do.

31.3.2 Calculating rotational kinetic energy

In principle, at least, we could calculate the rotational kinetic energy possessed by that spinning bicycle wheel by

- decomposing it into a very large number of small particles of mass,
- computing the kinetic energy of each particle of mass, and
- computing the sum of the kinetic energy values possessed by all of the particles of mass.

That would be a difficult computation. We need a simpler way to express the rotational kinetic energy of a rotating rigid body.

A simpler way

There is a simpler way that is based on the tangential speed of each particle of mass and the angular velocity of the rotating object.

You should recall that when the angular velocity is expressed in radians per second, the tangential speed of a point on the circumference of a circle is given by

v = r * w

where

- v represents the tangential speed of the point in meters per second
- r represents the radius of the circle in meters
- w represents the angular velocity in radians per second

Thus, the tangential speed of our hypothetical particle of mass is equal to the product of the distance of that particle from the center of rotation (the axle on my upturned bicycle) and the angular velocity of the wheel.

Terminology

The convention is to use the Greek letter omega to represent angular velocity, but I decided to use the "w" character because

- your Braille pad probably can't display the Greek letter omega
- the Greek letter omega looks a lot like a lower case "w"

Back to the bicycle wheel

Therefore, if we consider the bicycle wheel to be made up of an extremely large number of particles of mass, each located at a fixed distance from the axle, the kinetic energy of each particle would be given by

KEr = (1/2)*m*v^2, or
KEr = (1/2)*m*(r*w)^2, or
KEr = (1/2)*m*(r^2)*(w^2)
where all of the terms in this equation were defined earlier except for

- KEr, which represents rotational kinetic energy.

The total rotational kinetic energy of the bicycle wheel

Then the total rotational kinetic energy of the bicycle wheel would be

KErt = (1/2)*(sum from i=0 to i=N(mi*ri^2))*w^2
where

- KErt represents the total rotational kinetic energy of the wheel
- mi represents the ith mass particle in a set of N mass particles
- ri represents the distance of the ith mass particle from the axis of rotation
- w represents angular velocity in radians per second

Summation

It is conventional to use the Greek letter sigma to represent the sum with subscripts and superscripts providing the limits of the sum. However, since your Braille display probably won't display the Greek letter sigma with subscripts and superscripts, we will have to settle for something like *"sum from i=0 to i=N"* to mean the same thing.

Factoring out like terms

Note that then entire wheel rotates at the same angular velocity, so we can (and did) factor the (1/2) and the w^2 out of the summation equation given above.

Integral calculus is the key

Regardless of how difficult it may seem to you to perform the summation given above, when you complete a course in integral calculus, you will have learned how to do that sort of thing for a variety of geometric shapes such as solid cylinders, hollow cylinders, solid spheres, hollow spheres, squares, rectangles, rods, etc. As a result, engineering and physics handbooks contain tables with this sort of information for a variety of common geometrical shapes.

(There is also such a table at http://en.wikipedia.org/wiki/List_of_moments_of_inertia [13] , but if you are a blind student, your accessibility equipment may not be able to read it reliably.)

31.3.3 The rotational inertia (I)

Getting back to the earlier equation for rotational kinetic energy (p. 699) , the quantity in parentheses cannot change for a given geometric shape. The distance between each mass particle and the axis of rotation stays the same for a rigid body, and the mass of each mass particle doesn't change. It is conventional to give the quantity in parentheses the symbol I (upper-case "I") and refer to it as either the *rotational inertia* , or the *moment of inertia* .

Therefore, using the terminology from the earlier equation for rotational kinetic energy (p. 699) ,

NOTE: **Facts worth remembering – Rotational Inertia**

I = sum from i=0 to i=N(mi*ri^2)

where

[13] http://en.wikipedia.org/wiki/List_of_moments_of_inertia

- I represents rotational inertia
- SI units for rotational inertia are kg*m^2
- mi represents the ith mass particle in a set of N mass particles
- ri represents the distance of the ith mass particle from the axis of rotation

Translational versus rotational kinetic energy

Given that information, let's form an analogy between translational kinetic energy from an earlier module and rotational kinetic energy from this module.

NOTE: **Facts worth remembering – Translational and Rotational Kinetic Energy**

Kt = (1/2)*m*v^2

Kr = (1/2)*I*w^2

where

- Kt represents translational kinetic energy
- Kr represents rotational kinetic energy
- m represents mass in kg
- v represents translational velocity in m/s
- I represents rotational inertia in kg*m^2
- w^2 represents angular velocity in radians per second

Comparing the terms

When we compare the terms in the two expressions, we see that angular velocity in one case is analogous to translational velocity in the other case.

We also see that the rotational inertia in one expression is analogous to the mass in the other expression.

31.3.4 Mass is no longer absolute

As I explained earlier, mass is an absolute in translational terms. The translational inertia depends directly on the amount of mass.

However, mass is not an absolute in rotational terms. The rotational inertia for rotation depends not only upon the amount of mass involved, but also on how that mass is geometrically distributed within the object relative to the axis of rotation.

A measure of inertia

In translational terms, mass is a measure of the inertia of an object, or how difficult it is to cause the object to change its translational velocity.

In rotational terms, for a rigid rotating object, the rotational inertia (I), is a measure of how hard it is to cause the object to change its angular velocity.

Finding the rotational inertia

Here are a few tips on how you might go about finding the rotational inertia of an object.

NOTE: **Facts worth remembering – Finding the rotational inertia**

1. If the object consists of a small number of parts in an easily-handled geometric configuration, and you know the mass and position of each part, you may be able to estimate the rotational inertia by evaluating the expression given earlier for the rotational inertia (p. 699) .
2. For symmetrical objects with simple geometric shapes, you may be able to use calculus to perform the summation given earlier for the rotational inertia (p. 699) .
3. Because the rotational inertia is a sum, you may be able to decompose the object into several parts, find the rotational inertia for each part, and then add them together.
4. You may be able to apply the parallel axis theorem (p. 701) in conjunction with item 3.

The rotational axis is very important

The rotational inertia for an object depends heavily on the location of the axis of rotation. For example, some vehicles have doors on the back that are hinged on one side. Other vehicles have doors on the back that are hinged at the top. Given a door that has a rectangular shape, but which is not a square, the rotational inertia when the door is hinged on the side would be different from when the door is hinged at the top.

Assuming that both doors have the same mass, and are fastened to the vehicle with the same orientation, the center of mass for one arrangement would be further from the hinge than for the other arrangement. The arrangement for which the center of mass is further from the hinge would have the greater rotational inertia.

A simple experiment

Pick up an eight-foot piece of 2x4 lumber, grasp it near one end, and try swinging it like a baseball bat. You should find that to be relatively difficult because it has a large rotational inertia when rotated around its end. (It also has a lot torque due to gravity when supported only at the end. Torque will be the topic for a future module.)

Then grasp it in the center and rotate it as far as you can without hitting your body. You should find that to be somewhat easier because it has a smaller rotational inertia when rotated around its center than when rotated around its end.

I will have more to say about this later in this module.

31.3.5 The parallel axis theorem

It is possible to determine the rotational inertia of an object about any axis if we can determine the rotational inertia of that same object about a parallel axis that goes through the center of mass of the object.

I will explain this in much more detail in the dumbbell scenario later in this module.

NOTE: **Facts worth remembering – The parallel axis theorem**

The total rotational inertia of an object about a chosen axis is

- the rotational inertia about a parallel axis passing through the object's center of mass, plus
- the rotational inertia of the center of mass, treated as a point mass, about the chosen axis.

We can express this theorem in equation form as

Itotal = M*D^2 + Icm

where

- Itotal is the total rotational inertia of the object
- M is the mass of the object
- D is the distance from the center of mass of the object to the chosen axis
- Icm is the rotational inertia of the object through a parallel axis that passes through the object's center of mass

31.3.6 Examples of rotational inertia

In this section, I will attempt to describe some simple geometric shapes and provide the formula for the rotational inertia for each shape. This information is largely based on information gleaned from http://en.wikipedia.org/wiki/List_of_moments_of_inertia [14] .

Terminology

Unless I specify otherwise, the axis of rotation will be the axis of symmetry, such as at the center of a wheel. Also, unless I specify otherwise,

[14] http://en.wikipedia.org/wiki/List_of_moments_of_inertia

- r represents the radius
- m represents the mass

NOTE: **Facts worth remembering – Examples of rotational inertia**

Thin hollow cylindrical shape or hoop

Think of a can of beans without the beans and without the end caps.

I = m*r^2

Solid cylinder or disk

I = (1/2)*m*r^2

Thick-walled cylindrical tube with open ends, of inner radius r1, and outer radius r2

I = (1/2)*m*(r1^2 + r2^2)

Thin rectangular plate of height h and width w with axis of rotation in the center, perpendicular to the plate

I = (1/12)*m*(h^2 + w^2)

Solid sphere

I = (2/5)*m*r^2

Then hollow spherical shell

I = (2/3)*m*r^2

Thin rod of length L

Axis of rotation is perpendicular to the end of the rod.

I = (1/3)*m*L^2

Thin rectangular plate of width L and height H

Axis of rotation is along the edge of the plate parallel to the H dimension and perpendicular to the width L.

I = (1/3)*m*L^2

Thin rod of length L

Axis of rotation is through the center of the rod.

I = (1/12)*m*L^2

Thin rectangular plate of width L and height H

Axis of rotation is along the center of the plate parallel to the H dimension perpendicular to the width L.

I = (1/12)*m*L^2

31.4 Example scenarios

I will apply some of what we have learned to several different scenarios in this section.

31.4.1 The 2x4 scenario

Returning to the earlier example, pick up an eight-foot piece of 2x4 lumber, grasp it near one end, and try swinging it like a baseball bat.

Then grasp it in the center and rotate it as far as you can without hitting your body.

How does the rotational inertia with the axis at the end compare with the rotational inertia with the axis at the center? What is the ratio of the two?

Solution:

Although this may not be a good approximation, we will use the formulas for a thin rectangular plate of height h and width z from http://en.wikipedia.org/wiki/List_of_moments_of_inertia [15] . (A 2x4 isn't very thin so this may not be a good approximation.)

When rotated around the end,

Iend = (1/3)*(m*h^2) + (1/12)*(m*z^2)

When rotated around the center,

Icen = (1/12)*(m*h^2 + m*z^2)

where

- Iend is the rotational inertia with the axis of rotation at the end
- Icen is the rotational inertia with the axis of rotation at the center
- m is the mass
- h is the height
- z is the width (I avoided the use of w because I have been using that character for angular velocity.)

Define the numeric values

Let h = 96 inches and z = 3.75 inches (a 2x4 really isn't 2 inches thick and 4 inches wide)

Let mass = 1kg. We don't know what the mass of a 2.4 is. However, it will cancel out when we compute the ratio of the two cases. We can use any value so long as we don't ascribe any credibility to the absolute rotational inertia value.

Substitute numeric values for symbols

Iend = (1/3)*(1kg*(96inches)^2) + (1/12)*(1kg*(3.75 inches)^2)

Plugging this expression into the Google calculator gives us:

Iend = 1.98 m^2 kg

(Remember, however, that this isn't an accurate absolute value because we aren't using the actual mass of a piece of 2x4 lumber.)

Icen = (1/12)*(1kg*(96inches)^2 + 1kg*(3.75inches)^2)

The Google calculator gives us

Icen = 0.496 m^2 kg

And the ratio is...

If I formulated the problem correctly before plugging the expressions into the Google calculator, the ratio

Iend/Icen = (1.98 m^2 kg)/(0.496 m^2 kg) = 3.99

Therefore, it should have been about four times as difficult to swing the 2x4 like a baseball bat than to spin it at its center. (The downward torque caused by gravity probably made it seem even worse that that.)

31.4.2 The dumbbell scenario

Among other things, this scenario illustrates the parallel axis theorem (p. 701) .

Consider a dumbbell, or a barbell, whichever you choose to call it. This object consists of two identical solid spheres, each with mass M. The centers of mass of the spheres are separated by a distance L. The radius of each sphere is R.

The spheres are connected by a thin rod with mass m of length d. Thus, the length of the rod is L-2*R.

Find the rotational inertia of the dumbbell about an axis at the center of and perpendicular to the rod.

[15] http://en.wikipedia.org/wiki/List_of_moments_of_inertia

Solution:
The total rotational inertia of the dumbbell about the chosen axis consists of the sum of three parts:

1. The rotational inertia of one sphere about that axis.
2. The rotational inertia of the other sphere about that same axis.
3. The rotational inertia of the rod about that axis.

There are really five items in the sum
We learned from the parallel axis theorem (p. 701) that the first two items in the above list are each made up of the sum of two items:

1. The rotational inertia of a sphere about an axis passing through the sphere's center of mass
2. The moment of inertia of the center of mass of the sphere, treated as a point particle, about the chosen axis.

We learned from the earlier Examples of rotational inertia (p. 702) that the rotational inertia of a solid sphere about an axis through its center of mass is
Isphere = (2/5)*M*R^2
We learned in Rotational inertia (p. 699) that the rotational inertia of a point mass rotating about a chosen axis is
I = M*r^2, or in this case
I = M*(L/2)^2
Thus, the rotational inertia of each sphere about the chosen axis is the sum of those two, or
Isphere_axis = (2/5)*M*R^2 + M*(L/2)^2
The total rotational inertia of the dumbbell will be twice this value plus the rotational inertia of the rod about the chosen axis. We learned in Examples of rotational inertia (p. 702) that the rotational inertia about the chosen axis for the rod is
Irod = (1/12)*m*d^2
This, the total rotational inertia of the dumbbell about the chosen axis is
Itotal = Irod + 2*Isphere_axis, or
Itotal = (1/12)*m*d^2 + 2*((2/5)*M*R^2 + M*(L/2)^2)
Because the length of the rod, d is
d = L-2*R
We could substitute this expression for d giving us
Itotal = (1/12)*m*(L-2*R)^2 + 2*((2/5)*M*R^2 + M*(L/2)^2)
which reduces the expression down to include

- m represents the mass of the rod
- L represents the distance between the centers of mass of the two spheres
- R represents the radius of each sphere
- M represents the mass of each sphere

I will leave it as an exercise for the student to assign typical numeric values to each of those variables and to compute the rotational inertia of a dumbbell.

Another good exercise for the student would be to replace the spheres with disks having the same mass and compare the rotational inertia of the two configurations.

31.4.3 A pulley and two objects, part 1

In this scenario, a friction-free pulley with a mass M and a rotational inertia I is suspended from a beam. A cord is threaded around the pulley and two objects with masses of m1 and m2 are fastened to the ends of the cord.

The cord will not stretch and will not slip on the surface of the pulley.

When the two objects are held at the same level with the cord taut and released simultaneously, one may move up and the other may move down, depending of the relative mass values of the two objects. Because the cord cannot stretch, the magnitude of the velocity of each object must be the same.

Ignoring air resistance, write a general equation for the magnitude of the velocity of each object after each object has moved a distance h.

Solution:

With no friction and no air resistance, all forces acting on the system are conservative. Therefore, the mechanical energy of the system must be preserved.

deltaU + deltaK = 0

where

- deltaU is the change in potential energy
- deltaK is the change in kinetic energy

Since the pulley mechanism is in equilibrium, the only potential energy possessed by the system that can change is the gravitational potential energy of the two objects. Therefore, when the objects move, the gravitational potential energy of the two objects is converted into translational kinetic energy of the objects and rotational kinetic energy of the pulley.

Changes in potential energy of the system

In order to keep our algebraic signs straight, we will assume that m1 is greater than (or possibly equal to) m2. As a result, when the objects move, m1 will move down and m2 will move up.

This will result in the following changes in the potential energy of the system.

deltaU1 = -m1*g*h

deltaU2 = +m2*g*h

where

- deltaU1 and deltaU2 represent the changes in gravitational potential energy for m1 and m2 respectively.
- g represents the acceleration of gravity.
- h represents the magnitude of the change in height of each object.

The mechanical energy of the system

The mechanical energy of the system includes the translational kinetic energy of each of the two objects plus the rotational kinetic energy of the pulley.

deltaK = (1/2)*(m1+m2)*v^2 + (1/2)*I*w^2

where

- m1 and m2 are the mass values for each object
- v is the magnitude of the translational velocity of each object
- I is the rotational inertia of the pulley
- w is the angular velocity of the pulley

Translational speed versus tangential speed

Because the cord cannot slip on the pulley, the tangential speed of a point on the edge of the pulley must be equal to the translational speed of the objects.

This tangential speed at the edge of the pulley is related to the angular velocity of the pulley as follows

v = w*R, or

w = v/R

Where

- v is the tangential speed of a point on the edge of the pulley, which is also the speed of the cord, which is also the speed of each object.
- w is the angular velocity of the pulley.

- R is the radius of the pulley.

Substitute, combine terms, and simplify

Substitution of v/R for w yields
deltaK = (1/2)*(m1+m2)*v^2 + (1/2)*I*(v/R)^2
Combining potential and kinetic energy yields
deltaU + deltaK = -m1*g*h + m2*g*h
+ (1/2)*(m1+m2)*v^2 + (1/2)*I*(v/R)^2 = 0
Simplification yields
(1/2)*(m1+m2)*v^2 + (1/2)*I*(1/R^2)*v^2 = (m1 - m2)*g*h
Solving for v^2 yields
v^2 = (2*(m1 - m2)*g*h)/((m1+m2) + I*(1/R^2)), or
The **general equation** for velocity is
v = ((2*(m1 - m2)*g*h)/((m1+m2) + I*(1/R^2)))^(1/2)

31.4.4 A pulley and two objects, part 2

Make the following assumptions:

The pulley is a uniform disk with a mass, M, of 1 kg and a radius, R, of 1 meter.
m1 = 2 kg
m2 = 1 kg
h = 0.5 m
Find the velocity v.

Solution:

Referring back to Examples of rotational inertia (p. 702) , and changing the notation to match that being used in this scenario, we find that the rotational inertia for the pulley is
I = (1/2)*M*(R)^2

The rotational inertia

Let's begin by computing the rotational inertia of the pulley.
I = (1/2)*1kg*(1m)^2, or
I = 0.5*(m^2)*kg

Substitute values other than rotational inertia

Substituting values into the general equation (p. 706) yields
v = ((2*(2kg - 1kg)*(9.8m/s^2)*0.5m)/((2kg+1kg) + I*(1/(1m)^2)))^(1/2)
where we still have the rotational inertia as a variable.
We will use this equation again later in this module.

Substitute the value for rotational inertia

Substituting the value for rotational inertia computed above gives us
v = ((2*(2kg - 1kg)*(9.8m/s^2)*0.5m)/((2kg+1kg) + (0.5*(m^2)*kg)*(1/(1m)^2)))^(1/2)
We will also use this equation again later in this module.
Assuming that I managed to get all of that done without making an error, the velocity is
v = 1.67 m/s

31.4.5 A pulley and two objects, part 3

Assume the same conditions as in part 2 (p. 706) , except that m2 = m1 = 2 kg, what is the velocity?

Solution:

Starting with the equation from above (p. 706) and replacing each occurrence of 1kg with 2kg yields
v = ((2*(2kg - 2kg)*(9.8m/s^2)*0.5m)/((2kg+2kg) + (0.5*(m^2)*kg)*(1/(1m)^2)))^(1/2)
Using the Google calculator to solve this equation tells us that
v = 0 m/s.

This is what we should expect for the two objects having the same mass. Each object has an equal desire to fall toward the earth, so they balance one another, causing the system to be in equilibrium.

31.4.6 A pulley and two objects, part 4

Go back to the conditions for part 2 (p. 706) except instead of assuming that the pulley is a uniform disk, assume that the pulley approximates a *"thick-walled cylindrical tube with open ends, of inner radius r1, and outer radius r2"* .

For this configuration, we can approximate the rotational inertia as

I = (1/2)*m*(r1^2 + r2^2) (see Examples of rotational inertia (p. 702))

For example, think of a pulley that looks something like a bicycle wheel with very lightweight spokes connecting the outer rim to the axle.

Let r1 = 0.9*r2 and let the mass be unchanged.

Find the velocity.

Solution:

Given that the inner radius is 0.9 times the outer radius and the mass is unchanged, we can **approximate** the rotational inertia for this pulley with

I = (1/2)*M*((0.9*R)^2 + R^2)

The rotational inertia

Let's compute the rotational inertia for this pulley.

I = (1/2)*1kg*((0.9*1m)^2 + (1m)^2), or

I = 0.905 (m^2)*kg

As you can see, the rotational inertia for this configuration is almost double the rotational inertia for the uniform disk configuration.

Start with the original equation

Go back and get the original equation (p. 706) that contains numeric values but still has the rotational inertia as a variable.

v = ((2*(2kg - 1kg)*(9.8m/s^2)*0.5m)/((2kg+1kg) + I*(1/(1m)^2)))^(1/2)

Substitute the rotational inertia

Replace the rotational inertia, I, with the approximate (p. 707) rotational inertia for our new pulley.

v = ((2*(2kg - 1kg)*(9.8m/s^2)*0.5m)/((2kg+1kg) + (0.905 (m^2)*kg)*(1/(1m)^2)))^(1/2)

Once again, if I managed to get through all of that without making an error, the velocity is

v = 1.58 m/s

Analysis

Comparing this value with the velocity from part 2 (p. 706) , , we see that the velocity was reduced from 1.67 m/s to 1.58 m/s due to the increase in the rotational inertia of the pulley.

This is what we should expect. Increasing the rotational inertia of the pulley causes it to take longer to accelerate given the same external force that is causing it to change its angular velocity and acquire rotational kinetic energy.

31.4.7 A pulley and two objects, part 5

Let's make one more adjustment to the geometry of the pulley and observe the effect that it has on the system.

Keep all of the parameters the same as part 4 (p. 707) except let the radius of the pulley, R, be 10m.

Find the velocity.

Solution:

Begin by computing the rotational inertia of the new pulley.

I = (1/2)*M*((0.9*R)^2 + R^2), or

I = (1/2)*1kg*((0.9*10m)^2 + (10m)^2), or

I = 90.5 (m^2)*kg

Note that the rotational inertia is proportional to the square of the radius. Therefore, increasing the radius by a factor of 10 caused the rotational inertia to be increased by a factor of 100.

Generally speaking, moving the concentration of mass further from the axis of rotation will increase the rotational inertia.

Start with the original equation as before

Going back and getting the original equation (p. 706) that contains numeric values but still has the rotational inertia as a variable, we have.

v = ((2*(2kg - 1kg)*(9.8m/s^2)*0.5m)/((2kg+1kg) + I*(1/(1m)^2)))^(1/2)

Substitute the new rotational inertia

Replacing the rotational inertia, I, with the rotational inertia for our new pulley yields.

v = ((2*(2kg - 1kg)*(9.8m/s^2)*0.5m)/((2kg+1kg) + (90.5 (m^2)*kg)*(1/(1m)^2)))^(1/2)

Solving this equation with the Google calculator tells us that the new velocity value is

v = 0.323 m/s

which is much lower than the velocity for either part 2 (p. 706) or part 4 (p. 707) .

31.4.8 An Atwood machine

In case you want to learn more about this topic, the arrangement of the pulley and the objects that we have been discussing is commonly called an Atwood machine. The device was invented in 1784 by Rev. George Atwood as a laboratory experiment to verify the mechanical laws of motion.

31.4.9 A flywheel

A flywheel is a device that is commonly used to smooth out the irregularities of angular velocity in rotational motion. For example, in the age of steam engines, the rotational motion was powered only during a portion of each cycle, and was coasting during the remainder of the cycle. Unless corrected, therefore, a loaded steam engine output shaft would speed up and slow down once during each rotation of the shaft.

The correction

Most steam engines employed a large flywheel that was turned by the output shaft to cause the angular velocity to remain relatively constant despite the fact that rotational power was applied during only a portion of the cycle.

When rotational power was applied to the output shaft, the shaft turned the flywheel. When rotational power was not applied to the output shaft, the flywheel turned the shaft. Because the flywheel had a large rotational inertia, it preferred to turn at a near constant angular velocity.

Physically, the flywheels were usually large wheels with spokes and most of the mass distributed in a rim at the circumference of the wheel. This configuration produced a large rotational inertia for a given amount of mass and a given amount of available space.

31.5 What have we learned?

Perhaps the most important thing for you to take away from this module is that

- although the rotational inertia of a rotating object is influenced by the mass of the object,
- it is also very heavily influenced by the geometrical distribution of that mass.

Two rotating objects having exactly the same mass can have entirely different rotational inertia values (moments of inertia).

For example, a flywheel with the bulk of its mass concentrated a large distance from the axis of rotation is much more effective in smoothing out the angular velocity of the device than would be a flywheel with the same mass concentrated in a small radius near the axis of rotation.

31.6 Work through the examples

I encourage you to work through the examples that I have presented in this lesson to confirm that you get the same results. Experiment with the examples, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

31.7 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

31.8 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Angular Momentum – Rotational Kinetic Energy and Inertia for Blind Students
- File: Phy1300.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · rotational kinetic energy
  - · rotational inertia
  - · moment of inertia
  - · Atwood machine
  - · flywheel

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 32

Vector Multiplication[1]

32.1 Table of Contents

- Preface (p. 712)
  - General (p. 712)
  - Prerequisites (p. 712)
  - Viewing tip (p. 712)
    - Figures (p. 712)
  - Supplemental material (p. 713)
- Discussion (p. 713)
  - Dot product, inner product, or scalar product (p. 713)
    - Background (p. 713)
    - Create a vector diagram on your graph board (p. 713)
    - The dot product (p. 714)
    - The projection of A onto B (p. 714)
    - Compute the projection (p. 714)
    - Compute the angle between the vectors (p. 715)
    - Perpendicular or parallel vectors (p. 715)
    - Summary for vector dot product (p. 715)
  - Cross product (p. 716)
    - Background (p. 716)
    - The right-hand rule (p. 717)
    - Create a vector diagram on your graph board (p. 717)
    - The cross product (p. 717)
    - The area of the parallelogram (p. 718)
    - The direction of the resultant vector (p. 718)
    - Perpendicular or parallel vectors (p. 718)
    - Summary for vector cross product (p. 719)
- Repeat the computations (p. 719)
- Resources (p. 720)
- Miscellaneous (p. 720)

[1] This content is available online at <http://cnx.org/content/m38455/1.2/>.

32.2 Preface

32.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains vector multiplication in a format that is accessible to blind students.

32.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

32.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

32.2.3.1 Figures

- Figure 1 (p. 716) . Facts worth remembering for the dot product.
- Figure 2 (p. 719) . Facts worth remembering for the cross product.

[2] http://cnx.org/content/col11294/latest/
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp

32.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

32.3 Discussion

You learned about vector addition and vector subtraction in earlier modules. It is also possible to multiply vectors. That capability will become important in future modules.

There is more than one way to multiply vectors (see http://mathworld.wolfram.com/VectorMultiplication.html [13]). I will explain two of those ways in this module:

- dot product
- cross product

32.3.1 Dot product, inner product, or scalar product

I will begin with some background information on the dot product of two vectors.

32.3.1.1 Background

The terms dot product, inner product, and scalar product all mean the same thing and are used in various context's by different authors.

The term *dot product* derives from the fact that a vector product of this sort is often written as the names of the two vectors separated by a dot. However, that special dot character is probably not compatible with your Braille display. Therefore, I will write the dot product of the vectors named A and B as

(A dot B)

The term scalar product derives from the fact that a vector product of this sort more closely resembles scalar arithmetic than vector arithmetic. In particular, unlike the cross product (that will be discussed later), the result of the dot product does not have a direction.

32.3.1.2 Create a vector diagram on your graph board

In order for you to better understand the nature of a vector dot product, I recommend that you create a Cartesian coordinate system on your graph board, and draw the following two vectors.

NOTE: **A vector diagram for your graph board**

```
    Draw the first vector from the origin to a point at
x = 1
y = 1.73
Label this vector A.

Draw a second vector from the origin to a point at
x = 2.9
y = 0.78
Label this vector B.
```

References to the vector coordinates

I will refer to the coordinates at the tip of vector A as ax and ay. Similarly, I will refer to the coordinates at the tip of vector B as bx and by.

[12] http://www.dickbaldwin.com/toc.htm
[13] http://mathworld.wolfram.com/VectorMultiplication.html

32.3.1.3 The dot product

Using this nomenclature , the dot product of any two vectors is given by
(A dot B) = (ax * bx) + (ay * by)
where

- (A dot B) represents the dot product of the vectors named A and B
- ax, ay, bx, and by are the coordinates of the tips of the vectors named A and B respectively

So what?
By now you are probably saying "So what? Why should I care?"
Although it isn't obvious from what you see above, the dot product of two vectors is also equal to the product of their magnitudes and the cosine of the angle between them. In other words,
(A dot B) = Amag*Bmag*cosine(angle between A and B)
where

- Amag is the magnitude of vector A
- Bmag is the magnitude of vector B

32.3.1.4 The projection of A onto B

If you divide the dot product of A and B by the magnitude of B, the result is equal to the projection of vector A onto vector B. In other words,
(A dot B)/(Bmag) = projection of A onto B
This sort of projection operation is an operation that occurs frequently in physics. For example, the horizontal component of a velocity vector is the projection of the velocity vector onto the horizontal axis. Similarly, the vertical component of a velocity vector is the projection of the velocity vector onto the vertical axis.

Let's work through some numbers
Substituting your coordinate values into the expression given above (p. 714) yields
(A dot B) = (ax * bx) + (ay * by), or
(A dot B) = (1.0*2.9) + (1.73*0.78), or
(A dot B) = 4.2494
We will make use of this value a little later in this module.

32.3.1.5 Compute the projection

Suppose your problem calls for the projection of vector A onto vector B. Let's compute the value of that projection. For this, we need to know the magnitude of vector B, which we can compute using the Pythagorean theorem:
Bmag = sqrt(2.9*2.9 + 0.78*0.78), or
Bmag = 3.0
The projection of A onto B is equal to
Projection = (A dot B)/Bmag, or
Projection = 4.294/3 = 1.43
We will also have more to say about this value a little later in this module.

What do we mean by the projection?
Using the vectors on your graph board, draw a line segment beginning at the tip of vector A. Make that line perpendicular to vector B and mark the point on vector B where that line intersects vector B.
The distance from the origin to that point is the value of the projection of vector A onto vector B.
According to the above arithmetic, that distance should be equal to 1.43 units. Hopefully when you measure it on your graph board, you will get approximately the same value.

32.3.1.6 Compute the angle between the vectors

Suppose instead that your problem calls for the angle between the two vectors. Given that the dot product is equal to the product of the magnitudes and the cosine of the angle between the vectors, the cosine of the angle between the vectors is equal to

cosine(angle) = (A dot B)/(Amag *Bmag)

You may or may not know this from your earlier experience with trigonometry, but the angle in the above expression is the arccosine of the cosine value.

To compute the angle, we first need to compute the magnitude of vector A. Once again using the Pythagorean theorem,

Amag = sqrt(1.0*1.0+1.73*1.73), or

Amag = 2

Now compute the angle between the vectors

We now have all of the information that we need to computer the angle between the vectors. Using Google calculator nomenclature,

Angle = arccos((A dot B)/(Amag *Bmag)) in degrees, or

angle = arccos(4.294/(2*3)) in degrees, or

angle = 44.30 degrees

(Actually, I chose the original values in hopes of causing this final answer to come out to 45 degrees, but the round off errors along the way threw things off a bit.)

What have we learned?

For these two vectors, we have learned that

- The angle between them is 44.3 degrees
- The projection of vector A onto vector B is equal to 1.43 units

32.3.1.7 Perpendicular or parallel vectors

Now consider what happens as the angle varies between 90 degrees (perpendicular vectors) and 0 degrees (parallel vectors) for a given pair of vectors.

As the angle approaches 90 degrees, the cosine of the angle approaches 0, and the dot product of the vectors approaches 0.

As the angle approaches 0 degrees, the cosine of the angle approaches 1.0 and the dot product approaches a value that is the product of the magnitudes of the two vectors.

For a given pair of vectors, the dot product can be thought of as a measure of the extent to which they are parallel. The closer they are to parallel, the greater will be the value of the dot product.

A check on the projection value

We can check our earlier projection value from a different viewpoint now that we know the angle between the vectors.

From the drawing on your graph board, you should see that vector A forms the hypotenuse of a right triangle and the projection of vector A onto vector B forms the base of that triangle. You should know from the earlier module on trigonometry that the length of the base is

base = Amag * cos(angle), or

base = 2 * cos(44.3 degrees), or

base = 1.43 units

which matches the length of the projection that we computed earlier.

32.3.1.8 Summary for vector dot product

Figure 1 (p. 716) contains some facts worth remembering about the vector dot product.

Facts worth remembering for the dot product.

```
Given two vectors, A and B with their tails at the origin and
their tips at ax, ay, bx, and by respectively

(A dot B) = (ax * bx) + (ay * by)

where
    (A dot B) represents the dot product of the vectors
    named A and B

also

(A dot B) = Amag*Bmag*cosine(angle between A and B)

where
    Amag is the magnitude of vector A
    Bmag is the magnitude of vector B

The projection of A onto B = (A dot B)/(Bmag)

The angle between A and B = arccos((A dot B)/(Amag *Bmag))

For a given pair of vectors, the dot product can be thought
of as a measure of the extent to which they are parallel.
The closer they are to parallel, the greater will be the
value of the dot product.
```

Figure 32.1: Facts worth remembering for the dot product.

32.3.2 Cross product

Let's begin our discussion of the vector cross product with some background information.

32.3.2.1 Background

The *cross product* , sometimes called a *vector product* , is an operation on two vectors in three-dimensional space. The operation results in a vector that is perpendicular to both of the vectors being multiplied.

The name of the operation

The name "cross product" derives from the fact that a special character that looks like an "x" is often used to indicate the nature of the operation.

I doubt that the special character will display properly on your Braille display. In this module, therefore, I will use an actual "x" character instead of the special character that is typically used. For example, I will indicate the cross product between vectors A and B as

AxB

A cross product with a zero result

If either of the vectors being multiplied has a magnitude of 0, the cross product will be zero. Also if the vectors being multiplied are parallel, their cross product will be zero.

The area of a parallelogram

Except for the case of perpendicular vectors, the magnitude of the cross product between two vectors equals the area of a parallelogram with the vectors forming two sides of the parallelogram. For the case of perpendicular vectors, the parallelogram becomes a rectangle and the magnitude of the product is the area of that rectangle.

The direction of the resultant vector

As mentioned earlier, the result of the cross product is a vector that is perpendicular to both of the vectors being multiplied. The resultant vector can satisfy that requirement and point in ether of two directions. The actual direction depends on certain orientation conventions as described by the *right-hand rule* .

32.3.2.2 The right-hand rule

For a "right-handed" coordinate system, the direction of the resultant vector for AxB can be determined as follows:

Point the forefinger of the right hand in the direction of A and point the second finger in the direction of B. The thumb will then point in the direction of the resultant vector.

The cross product is not commutative

If you think about this, you should realize that the cross product is not commutative. That is to say that AxB is not the same as BxA because the direction of the resultant vector would not be the same.

32.3.2.3 Create a vector diagram on your graph board

Once again, in order for you to better understand the nature of a vector cross product, I recommend that you create a Cartesian coordinate system on your graph board, and draw the following two vectors.

NOTE: **A vector diagram for your graph board**

```
    Draw the first vector from the origin to a point at
x = 1
y = 1.73
Label this vector A.

Draw a second vector from the origin to a point at
x = 2.9
y = 0.78
Label this vector B.
```

32.3.2.4 The cross product

The cross product, **AxB is defined as**

AxB = Amag*Bmag*sin(angle)

where

- Amag is the magnitude of the vector A
- Bmag is the magnitude of the vector B
- angle is the angle between the two vectors, which must be less than or equal to180 degrees

32.3.2.5 The area of the parallelogram

Use the vectors that you have drawn on your graph board to construct a parallelogram and see if you can estimate the area of that parallelogram.

Even if you were a sighted student having the parallelogram drawn on high-quality graph paper, it would be something of a chore to manually determine the area of the parallelogram.

Let's work through some numbers

Let's use the cross product to determine the area of the parallelogram.

Given the definition (p. 717) of the cross product, we see that there are three values that we need:

- Amag
- Bmag
- angle

Same vectors as before

If we were starting out with two new vectors, we could compute the magnitude of each vector using the Pythagorean theorem. We could also determine the angle by computing the vector dot product that I explained earlier in this module.

As you may have noticed, these are the same two vectors that we used earlier, and we computed those three values earlier. Going back and recovering those three values, we have

- Amag = 2.0
- Bmag = 3.0
- angle = 45 degrees (at least that is what I intended for it to be)

The area of the parallelogram

Using the earlier definition (p. 717) and the nomenclature for the Google calculator,
AxB = Amag*Bmag*sin(angle), or
AxB = 2.0*3.0*sin(45 degrees), or
AxB = 4.24 square units

32.3.2.6 The direction of the resultant vector

If you place the end of your thumb at the origin of your Cartesian coordinate system, you should be able, with reasonable comfort, to point your forefinger in the direction of A and your second finger in the direction of B.

According to the right-hand rule (p. 717) , this means that the direction of the resultant vector is the direction that your thumb is pointing, or straight down into the graph board.

32.3.2.7 Perpendicular or parallel vectors

Now consider what happens as the angle varies between 90 degrees (perpendicular vectors) and 0 degrees (parallel vectors) for a given pair of vectors.

As the angle approaches 0 degrees, the sine of the angle approaches 0, and the cross product of the vectors approaches 0.

As the angle approaches 90 degrees, the sine of the angle approaches 1.0 and the cross product approaches a value that is the product of the magnitudes of the two vectors.

Thus, for a given pair of vectors, the cross product can be thought of as a measure of the extent to which they are perpendicular to one another. The closer they are to perpendicular, the greater will be the value of the cross product.

Hence, the dot product is a measure of the extent to which two vectors are parallel to one another, while the cross product is a measure of the extent to which two vectors are perpendicular to one another.

32.3.2.8 Summary for vector cross product

Figure 2 (p. 719) contains some facts worth remembering about the vector cross product.

Facts worth remembering for the cross product.

```
Given two vectors, A and B with their tails at the origin

AxB = Amag*Bmag*sin(angle between A and B)

where
    AxB represents the cross product of the vectors
      named A and B
    Amag is the magnitude of vector A
    Bmag is the magnitude of vector B

The cross product is a vector operation. The resultant vector
is perpendicular to the two vectors being multiplied. The
direction of the resultant vector can be determined by the
right-hand rule.

The magnitude of the product equals the area of a
parallelogram with the vectors forming two sides of the
parallelogram.

The cross product is not commutative. In other words, AxB
is not equal to BxA.

For a given pair of vectors, the cross product can be
thought of as a measure of the extent to which they are
perpendicular to one another. The closer they are to
perpendicular, the greater will be the value of the
cross product.
```

Figure 32.2: Facts worth remembering for the cross product.

32.4 Repeat the computations

I encourage you to repeat the computations that I have presented in this lesson to confirm that you get the same results. Then do similar computations for pairs of different vectors. For example, swap the positions of vectors A and B and see what this does to the direction of the resultant vector for a cross product.

32.5 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

32.6 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Vector Multiplication for Blind Students
- File: Phy1310.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader
  - · refreshable Braille display
  - · JavaScript
  - · trigonometry
  - · dot product
  - · inner product
  - · scalar product
  - · cross product
  - · vector product
  - · projection

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 33

Angular Momentum – The Mathematics of Torque[1]

33.1 Table of Contents

- Preface (p. 721)
  - General (p. 721)
  - Prerequisites (p. 722)
  - Viewing tip (p. 722)
    - Figures (p. 722)
  - Supplemental material (p. 722)
- Discussion (p. 723)
  - A review of rotational kinematics (p. 723)
  - Torque from a mathematical viewpoint (p. 726)
  - Torque from an anecdotal viewpoint (p. 731)
  - A graph board exercise on torque (p. 734)
- Example scenarios (p. 736)
  - Net torque on the Lazy Susan turntable (p. 736)
  - A door-closing scenario (p. 736)
  - Torque and the moment of inertia (p. 737)
- Repeat the computations (p. 738)
- Resources (p. 738)
- Miscellaneous (p. 738)

33.2 Preface

33.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains the mathematics of torque in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38460/1.2/>.
[2] http://cnx.org/content/col11294/latest/

33.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

33.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

33.2.3.1 Figures

- Figure 1 (p. 727) . Tangential force, mass, radius, and angular acceleration.
- Figure 2 (p. 727) . Tangential force, radius, angular acceleration, and moment of inertia.
- Figure 3 (p. 728) . Torque, rotational inertia, and angular acceleration.
- Figure 4 (p. 730) . The torque vector.
- Figure 5 (p. 730) . A general equation for net torque.
- Figure 6 (p. 734) . The definition of torque.

33.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

33.3 Discussion

I will begin this discussion with a brief review of what we have learned about the rotation of rigid bodies and go from there into a discussion of torque.

33.3.1 A review of rotational kinematics

One of the objectives of this review is to summarize the angular kinematic variables that are used to describe rotational motion and relate them to the translational kinematic variables that we already know about.

Given a system of particles, we can describe motion as having two components:

- The motion of the center of mass.
- Motion relative to the center of mass.

Terminology

When an object rotates, it experiences an angular displacement, which I will refer to as theta in this review.

(Textbooks typically use the Greek letter theta for this purpose. However, your Braille display probably won't display the Greek letter theta. Therefore, I will spell it out when I use it in text, and will replace it by the character "Q" when I use it in an equation.)

The time rate of change of the angular displacement is the angular velocity, which I will refer to as omega in this review.

The time rate of change of the angular velocity is the angular acceleration, which I will refer to as alpha in this review.

Similarity to translational motion

These definitions are very similar to definitions from earlier modules having to do with translational motion. For example, when an object moves, it experiences a displacement. The time rate of change of the displacement is the velocity, and the time rate of change of the velocity is the acceleration.

This similarity derives from the fact that rotational motion can be described by a single angular displacement, theta, just as linear motion can be described by a single spatial displacement, x.

Constant angular acceleration

For constant angular acceleration, we can derive a set of equations that are analogous to corresponding equations for translational motion:

Q = Q0 + w0*t + (1/2)*a*t^2

w = w0 + a*t

w^2 = w0^2 + 2*a*(Q - Q0)

where

- Q represents the angular displacement theta
- Q0 represents the initial angular displacement
- w represents the angular velocity omega
- w0 represents the initial angular velocity
- a represents the angular acceleration alpha
- t represents time in seconds

Corresponding translational equations

For reference, here are the three translational equations that correspond to the equations given above (p. 723) . These equations were explained in an earlier module that involved the constant translational acceleration of gravity.

h = h0 v0*t + 0.5*g*t^2

v = v0 + g*t

v^2 = v0^2 + 2*g*(h-h0)

where

- h represents the translational displacement
- h0 represents the initial translational displacement
- v represents the translational velocity
- v0 represents the initial translational velocity
- g represents the acceleration of gravity
- t represents time in seconds

Motion of a point that is a fixed distance from the rotational axis

It is customary to define counter clockwise rotation as the positive direction of rotation. That will be the case in this review.

Consider a rotating disk. Since all points on the disk are rotating together, we can determine the linear displacement, speed and acceleration of any point on the disk in terms of the corresponding angular parameters.

Parameters for a point on a rotating disk

Consider a point on the disk that is a distance R from the axis of rotation. Assume that the disk has rotated through an angle theta. The point will have moved through a distance, s, along the circumference of a circle of radius R. The distance s is equal to the product of the radius and the angular displacement measured in radians:

s = R * Q

where

- s represents the distance traveled around the circumference of a circle of radius R
- R represents the radius of the circle
- Q represents the displacement angle theta measured in radians

Relationship between speed and angular velocity

With a little calculus, we can find a relationship between the tangential speed of the point along its path around the circle and the angular velocity of the disk:

v = R*w

where

- v represents the tangential speed of the point that is moving on the circumference of the circle
- R represents the radius of the circle
- w represents the angular velocity omega

Relationship between tangential acceleration and angular acceleration

By using a little more calculus, we can find a relationship between the tangential acceleration of the point along its circular path and the angular acceleration of the disk:

y = R*a

where

- y represents the tangential acceleration of the point along the circular path
- R represents the radius
- a represents the angular acceleration alpha

Kinetic energy in rotation

Consider the hypothetical case of a rigid object made up of a set of point particles connected by rods with zero mass. In other words, ignore the mass of the mechanism that holds the point particles in a rigid geometry.

Assume that this object rotates about a fixed axis with a constant angular velocity, omega. Also assume that you know the mass of each particle and that you know the distance of each particle from the axis of rotation.

Kinetic energy of each individual particle

We know how to compute the speed (p. 724) of each particle along its circular path.
The kinetic energy of each particle would be
Ki = (1/2)*mi*vi^2
where

- Ki is the kinetic energy of the ith point particle
- mi is the mass of the ith point particle
- vi is the speed of the ith point particle

Kinetic energy for translational motion

Hopefully you recognize the above equation (p. 725) as being the same as the following equation that we saw in an earlier module involving the kinetic energy of an object in translational motion.
KE = 0.5*m*v^2
where

- KE represents the kinetic energy possessed by the object
- m represents the mass of the object
- vv represents the velocity of the object

Kinetic energy for the object

The object described above (p. 724) is made up of a system of particles.
The total kinetic energy of the system is the sum of the kinetic energy values of each of its particles.
The kinetic energy for the system is
Ks = (1/2)*(sum from i=0 to i=N(mi*ri^2))*w^2
where

- Ks represents the kinetic energy of the system
- ((sum from i=0 to i=N(...)) represents the sum of the values resulting from evaluating the expression in parentheses for each one of N+1 particles
- mi represents the mass of the ith point particle
- ri represents the distance of the ith point particle from the axis of rotation
- w represents the angular velocity omega

Rotational inertia or moment of inertia

Define the rotational inertia, or the moment of inertia, whichever term you prefer as
I = sum from i=0 to i=N(mi*ri^2)
where

- I represents the rotational inertia or the moment of inertia
- mi represents the mass of the ith point particle
- ri represents the distance of the ith point particle from the axis of rotation

Rotational kinetic energy based on rotational inertia

From this, we can determine that
Ks = (1/2)*I*w^2
where

- Ks represents the kinetic energy for the system
- I represents the rotational inertia for the system
- w represents the angular velocity of the system

Similar to translational kinetic energy

Note the similarity between the kinetic energy of a rotating system and the kinetic energy an object undergoing translational motion:

Ke = (1/2)*I*w^2

Kt = (1/2)*m*v^2

where

- Kr represents the kinetic energy for the rotating system
- I represents the rotational inertia for the rotating system
- w represents the angular velocity of the rotating system
- Kt represents translational kinetic energy
- m represents the mass of an object undergoing translational motion
- v represents the velocity speed of the object undergoing translational motion

Similarities between rotational inertia and mass

The mass of an object tells us how its kinetic energy is related to the square of its velocity.

The rotational inertia of a rotating object tells us how its kinetic energy is related to the square of its angular velocity.

The rotational inertia plays the same role in rotational motion that mass plays in translational motion.

Differences between mass and rotational inertia

While mass and rotational inertia play similar roles, there are also major differences between the two including:

- Rotational inertial depends not just on the total mass of an object, but also on the geometric distribution of the mass within the object.
- Rotational inertia also depends on the choice of the rotation axis, because distances are measured relative to that axis.

33.3.2 Torque from a mathematical viewpoint

Consider a point mass that is constrained to move in a circle. Let the mass be acted upon by an arbitrary force F. We learned earlier that in order for the mass to be moving in a circle, there must be a component of force, (the centripetal force), that is directed toward the center of the circle.

Tangential force, mass, radius, and angular acceleration

If we assume that the speed of the mass is changing, there must also be a component of the force that is tangential to the circle at the location of the point mass acting on the mass. This force is required to produce acceleration. Therefore, we can write:

Ft = m*y

From above (p. 724) ,

y = r*a

Substitution yields

Ft = m*r*a

Tangential force, mass, radius, and angular acceleration.

Ft = m*r*a
where

- Ft represents the tangential force
- m represents the mass
- y represents the tangential acceleration of the point mass along the circular path
- r represents the radius
- a represents the angular acceleration alpha

Figure 33.1: Tangential force, mass, radius, and angular acceleration.

Tangential force, radius, angular acceleration, and moment of inertia

We confirmed earlier that the rotational analog of mass is the rotational inertia (p. 725) . In the case of a single mass,

I = m*r^2
Substitution yields
Ft = m*r*a, or
Ft = (I/r^2)*r*a, or
Ft = (I/r)*a
Multiplying both side by r yields
r*Ft = I*a

Tangential force, radius, angular acceleration, and moment of inertia .

r*Ft = I*a
where

- Ft represents the tangential force
- r represents the radius
- I represents the rotational inertia or the moment of inertia, whichever term you prefer
- a represents the angular acceleration alpha

Figure 33.2: Tangential force, radius, angular acceleration, and moment of inertia .

Similar to Newton's second law

The equation (p. 727) in Figure 2 (p. 727) looks a lot like Newton's second law for translational motion, which is often expressed as

Force = mass * acceleration

In this case, the rotational inertia, I, is analogous to mass and the rotational acceleration, a, is analogous to translational acceleration.

If this similarity holds, that must mean that the term on the left side (p. 727) is analogous to force in the tangential motion scenario.

Torque, rotational inertia, and angular acceleration

The term on the left (p. 727) , consisting of the product of the force and the distance from the point of application of the force to the axis of rotation is commonly known as the torque. The common symbol for torque is the Greek letter tau, which I will replace with the character T in equations in this module.

T = r*Ft **= I*a** , or

T = I*a

Torque, rotational inertia, and angular acceleration.

T = I*a
where

- T represents torque
- r represents radius
- Ft represents the tangential component of force
- I represents rotational inertia or moment of inertia
- a represents angular acceleration

Figure 33.3: Torque, rotational inertia, and angular acceleration.

Torque produces angular acceleration

Thus, we have determined that a torque, which is the product of a tangential force and the distance from the application point of this force to the axis of rotation, produces an angular acceleration.

This is beginning to look a lot like Newton's second law for rotation. We will refine it some more later to improve the analogy. To do that, we will need to define torque and angular acceleration as vector quantities. We will accomplish that using the cross product of two vectors that you learned about in an earlier module.

The convention for positive rotation

If you imagine a rotating object from a viewpoint in which the rotation axis is perpendicular to the page, it is conventional to define a counter clockwise rotation as the positive direction of rotation. That is the convention that I will use in this module.

You learned about the right-hand rule involving vectors in an earlier module. There is a similar right-hand rule that we use to describe rotating objects.

The right-hand rule for angular velocity

If you curl the fingers of your right hand in the direction of rotation of the object, your thumb will point in the direction of the angular velocity vector of the object.

In other words, if some object is spinning in the counter-clockwise direction in the x-y plane, curling the fingers of your right hand in this direction results in your thumb pointing in the +z direction which we define to be the direction of the angular velocity vector.

The rule for angular acceleration

If the magnitude of the angular velocity increases in time, then the angular acceleration vector has the same direction as that of the angular velocity. If the magnitude of the angular velocity decreases in time, then the angular acceleration vector has the opposite direction as that of the angular velocity.

Defining torque as a vector quantity

The magnitude of a torque is the product of two terms:

1. The length of a line that connects the axis of rotation to the point where the force acts. Refer to this line as r.
2. The component of the force, Ft, that is perpendicular to this line.

Define theta as the angle between the line and the force vector F (not the tangential component of the force vector but the force vector itself). Then the magnitude of the tangential force vector is given by

Ft = Fmag*sin(theta)

where

- Ft represents the magnitude of the tangential force vector that is perpendicular to the line r
- Fmag is the magnitude of the force vector
- theta is the angle between the line and the force vector

Define a vector R

Let R represent a vector that lies along the line from the axis of rotation to the point where the force acts with its tail at the axis of rotation. I will refer to the magnitude of this vector as Rmag

Doing a little algebra, we can write

T = r*Ft, or

Tvec = Rmag*Fmag*sin(theta)

where

- Tvec represents the tangential force vector
- Rmag is the magnitude of the vector R
- Fmag is the magnitude of the vector F

The cross product

Why do I refer to Tvec as a vector in the above (p. 729) equation?

You learned in an earlier module that the cross product of two vectors A and B is given by

AxB = Amag*Bmag*sin(angle between A and B)

where

- AxB represents the cross product of the vectors named A and B
- Amag is the magnitude of vector A
- Bmag is the magnitude of vector B

The torque vector

Comparing the torque vector (p. 729) with the cross product (p. 729) , we determine that

Tvec = RxF

The torque vector .

Tvec = RxF
where

- Tvec is a torque vector
- R is a vector that points from the axis of rotation to the point where the vector force F is applied
- F is a force vector
- RxF is the vector cross product between the vector R and the vector F

Figure 33.4: The torque vector .

The direction of the torque vector

Recall from the earlier module that the direction of Tvec is perpendicular to both R and F and obeys the right-hand rule in terms of its absolute direction.

The relationship of torque and rotational inertia

Combining this with what we learned earlier about the relationship among torque, rotational inertia, and rotational acceleration, we can write

Tvec = I*A

A general equation for net torque.

Tvec = I*A
where

- Tvec is the torque vector whose direction obeys the right-hand rule
- I is the rotational inertia or the moment of inertia of the object about the chosen axis of rotation
- A is the rotational acceleration treated as a vector whose direction obeys the right-hand rule

Figure 33.5: A general equation for net torque.

A general equation for net torque

The equation shown in Figure 5 (p. 730) is a general equation for net torque. The net torque about an axis of rotation is equal to the product of the rotational inertia about that axis and the angular acceleration.

Similar to Newton's second law

This equation is Newton's second law applied to a system of particles in rotation about a given axis. It makes no assumptions about constant rotational velocity.

33.3.3 Torque from an anecdotal viewpoint

One of the objectives of this module is to develop concepts involving rotational motion that are analogous to concepts from earlier modules that involve translational motion.

A Lazy Susan

On my dining room table, there is a device that is commonly called a Lazy Susan. In case you are unfamiliar with such devices, it is essentially a turntable. By a turntable, I mean a rather large disk mounted on bearings so that it is free to turn in a plane that is slightly above but parallel with the top of the dining room table.

The purpose of the Lazy Susan

The purpose of a Lazy Susan is to make it easier to serve food at the dining room table. Various dishes are placed on it . When someone wants a helping of carrots, for example, instead of saying "Please pass the carrots," they simply turn the Lazy Susan until they can reach the bowl of carrots and help themselves.

Not a module about carrots

However, we won't be discussing how to serve carrots in this module. In this module, I will use the Lazy Susan, in its empty state between meals, to discuss various aspects of rotating rigid objects.

To make it easier to type the material under discussion, I will refer to the Lazy Susan as a turntable. (For some reason, I can type turntable much more quickly than I can type Lazy Susan.)

Low angular acceleration when coasting

My turntable has pretty good bearings. It is also rather heavy for its size and therefore has a relatively large rotational inertia or moment of inertia, whichever term you prefer.

If you give it a good spin, it will spin for quite a while before all of its rotational energy is dissipated through friction in the bearings and air resistance. By default, therefore, its angular acceleration is low. In other words, the rate of change of its angular velocity is small.

A perpetual motion machine

If we could find a way to eliminate all of the frictional forces acting on the turntable, including air resistance, then it would spin forever. In that case, we would have invented what has been called a perpetual motion machine. The rate of change of angular velocity would be zero, meaning that its angular acceleration would also be zero.

Similar to Newton's first law

This reminds us of a moving body that satisfies Newton's first law, which can be paraphrased something like the following:

- a body in motion, being acted upon by no net forces, will continue moving forever in a straight line.

In other words, that law tells us that absent a force to the contrary, a moving body will continue to move with no change in velocity.

If we could eliminate all of the frictional forces acting on my turntable (which is a rotating body), we might like to say that

- absent any net forces acting on the body, a rotating body with a constant rotational inertia will continue rotating with the same angular velocity forever.

However, that would not be a true statement.

Houston, we have a problem

Assume that you do the following to our hypothetical frictionless turntable while it is spinning. Using one finger from each hand, press on opposite sides of the turntable, applying an equal force directed towards the center of the turntable on each side of the turntable.

No net force

In this case, the turntable would not experience any net forces. Assuming that you are facing north when you do this, and that you press on the east and west sides of the turntable, the frictional forces generated by your fingers would be directed in opposite directions.

If the turntable were spinning counter clockwise (when viewed from the top), the frictional force created by your finger on the east side would be directed toward the south. The frictional force created by your finger on the west side of the turntable would be directed toward the north.

From a translational viewpoint, at least, there would be no net force applied to the turntable. The force that points to the north would be cancelled by the force that points to the south. The force that points to the east would be cancelled by the force that points to the west. Therefore, the turntable would be in translational equilibrium.

Acceleration would no longer be zero

Despite that, the velocity would be decreased meaning that the acceleration would no longer be zero. From this we might conclude that just because a rotating object is in translational equilibrium, it is not necessarily in rotational equilibrium.

The world of torque

We have just entered the world of torque with this hypothetical experiment. Torque is a quantity that plays a role in rotation that is analogous to the role that force plays in translation.

However, torque is not separate from force. It is not possible to exert a torque without exerting a force.

Torque is a measure...

In anecdotal terms, torque is a measure of how effective a force is at changing the angular velocity of an object. Stated differently, torque is a measure of how effective a force is at causing an object to have a non-zero angular acceleration.

Angular acceleration can be either positive or negative

For an object that is rotating about a fixed axis (or is capable of rotating about a fixed axis), such as my turntable, torque can either increase or decrease the angular velocity of the object.

Net translational force was zero

When you pressed your fingers on my turntable as described earlier, the net translational force applied to the turntable was zero. Therefore, the turntable remained in translational equilibrium, meaning that it didn't go sliding towards the edge of the table.

Net torque was not zero

However, the net torque was not zero, so it was not in angular equilibrium. The kinetic frictional forces generated on each side of the turntable resulted in the same algebraic sign of non-zero angular acceleration.

Each force caused the angular velocity to decrease. The two torques created by the kinetic friction forces were not only equal, they had the same algebraic sign. Therefore, the turntable experienced a non-zero net torque.

How are force and torque related?

To begin with, torque is proportional to the magnitude of the applied force, but that is not the end of the story. The rest of the story involves exactly where and in what direction the force is applied.

Create a torque to close a door

For example, consider applying a force for the purpose of creating a torque to close an open door. Initially, the door has zero angular velocity.

You could apply a force in any direction at any point on either side or the edge of the door that you are tall enough to reach. However, with regard to the objective of closing the door, it would matter very much where and in what direction you applied the force.

A very intuitive topic

The interesting thing about this topic is that you already know all about it from a practical and intuitive viewpoint. You would already know intuitively where and in what direction to push on the door to cause it to close with a minimum expenditure of energy.

You probably wouldn't push on the edge of the door

If you pushed on the edge of door, directing your force directly at the hinges (the axis of rotation), the door wouldn't move. While this might prove to be a good form of isometric exercise, it would not be an

effective way to close the door.

A force in any other direction

A force that is applied to the door, (even on the edge of the door) acting in any direction other than directly toward the hinges could be decomposed into two components:

- A radial component acting directly towards the axis or rotation or the hinges.
- A perpendicular component acting perpendicular to the surface of the door.

The radial component is wasted effort

The radial component would make no contribution to the development of the torque required to change the angular velocity of the door and cause the door to close. Only the perpendicular component would contribute to the development of such a torque.

Could develop torque at any point on the door

So now you know that you could apply a force at any point on the door, and so long as that force has a component that is perpendicular to the surface of the door, the perpendicular component would contribute to the development of torque.

Location, location, and location

However, it is also important where you push on the door to apply the force. If you push on what we normally consider to be the inside surface of the door, it might create a torque, but that torque may have the wrong sign or direction to cause the door to close. In fact, that would cause the door to open even further.

Once you realize that you must push on what we would call the outside surface of the door, it would still be important where you push and apply the force. Suppose for example that you were to push at a point that is only one inch away from the hinge. You know intuitively that even for a lightweight door, you might have to apply a very strong force to cause the door to close by applying the force at that location.

Where would you push?

You would probably push on the door at a point somewhere between the center of the door and the outer edge of the door.

If the door happened to be a really heavy door, you would probably push on the door at a point as close to the outer edge as possible. This would make it possible for you to cause the door to close with the minimum effort on your part.

The magnitude of the torque

In an attempt to codify your intuition, your instinctive knowledge, or perhaps your acquired knowledge into something more mathematical, we will define the magnitude of the torque as

torque = r*F

The conventional symbol for torque is the Greek letter tau. However, your Braille display probably won't display that Greek letter, so in this module, I will represent torque with the letter T as shown in Figure 6 (p. 734) .

The sign convention for torque

In this module, I will use a sign convention such that a force whose perpendicular component, when acting alone, would cause the object to rotate in a counter clockwise direction as a positive torque.

If that torque is the only torque acting, it would cause a positive angular acceleration.

A definition of torque

You saw a mathematical definition of torque earlier (p. 730) . Figure 6 (p. 734) shows a somewhat less mathematical definition of torque.

The definition of torque.

T = r*F
where

- T represents torque
- r is the shortest distance from the axis of rotation to the point of application of the applied force
- F is the component of the applied force that is perpendicular to a line from the axis of rotation to the point of the applied force

The SI unit for torque is newton meters or N*m.

Figure 33.6: The definition of torque.

The units can be confusing

The units for torque can be confusing because the SI unit for work or energy in joules is also N*m. However, even though torque and energy have the same units, they have entirely different meanings. Torque is not a form of energy.

33.3.4 A graph board exercise on torque

Imagine a puck sliding in a circular groove that has been cut in the ice at an ice rink. A cross section of the grove is rectangular so that the puck just fits from side to side and sets level on the bottom of the groove. When a puck slides inside the groove, it will move in a large circle.

Apply a force to the puck

If you apply a force to the puck in (almost) any direction, a component of that force will directed toward or away from the center of the circle. For any case where the direction of the force doesn't lie on a line from the puck to the center of the circle, there will also be a component of the force that is perpendicular to that line, which will make it tangential to the circle.

Construct a graph board simulation

Use your graph board and create a Cartesian coordinate system with the origin near the lower-left corner of the graph board. Use pushpins and pipe cleaners to draw a quarter of a circle, with the center of the circle at the origin. Make the radius approximately one-half of the smallest dimension of the graph board. This circle should include the entire upper-right quadrant of your Cartesian coordinate system.

Identify the location of the puck

Now insert a pushpin at a point somewhere on the circle about mid way between the intersection of the circle and the x and y axes of the coordinate system.

Imagine that this is the puck mentioned above that is constrained to move in a circle. Label this point P.

A radial line from the center

Use a pipe cleaner or a rubber band to draw a line from the puck to the center of the circle. Label this line r.

Create a force vector

Make a little loop at one end of a pipe cleaner that is about half the radius of your circle and place the loop around the pushpin that represents the puck at P.

Leave it loose enough that it can be rotated around the pin. Imagine that this is a vector that describes a force being applied to the puck with the tail of the vector at the puck.

Point the force vector at the center

Begin by pointing the force vector directly at the center of the circle. You will probably be able to imagine that since the puck is not free to move directly to the center, a force in this direction will not cause the puck to move.

The technical reason that it won't cause the puck to move is because the force doesn't have a component that is tangent to the circle at the location of the puck.

Rotate the force vector

Now rotate the force vector clockwise by about 30 or 40 degrees and pin it down so that it won't move. Label the tip of the force vector F.

Draw the tangential component of the force vector

This may be the most difficult part of this exercise for a blind student. Use your protractor (or some other method that you know about) to find a point on the line labeled r such that a line through that point and perpendicular to r goes through the tip of the force vector. Mark that point with a pushpin and label it Q.

Draw a line from Q to F

Use a pipe cleaner to draw a line from Q to F. That line represents the component of the force vector that is tangent to the circle at the location of the puck. (Actually it is parallel to the tangential component of the force vector, but that is OK. It is still the correct length and points in the correct direction.) The direction of that tangential force component is from Q to F.

This is the component of the force vector that causes the puck to move. Label this vector Ft for tangential force.

The radial component of force

Use a pipe cleaner to draw a line from P to Q. This is the component of the force vector that points directly from the puck to the center of the circle. This component won't cause the puck to move.

A right triangle

If you examine your vector diagram at this point, you can determine that the points labeled P, Q, and F represent the vertices of a right triangle, with the right angle at the point Q.

The length of the tangential force vector

Label the interior angle at P with an A. Now you should be able to determine that the length of the tangential vector named Ft is equal to the product of the force F and the sine of the angle A.

Ft = F*sin(A)

where

- F is the force vector.
- Ft is the component of the force vector that is tangent to the circle at the location of the puck. This force is also perpendicular to the line from the puck to the center of the circle.
- A is the angle that the force vector makes with the line r.

The torque

Referring back to Figure 6 (p. 734) , we find that the torque produced by this force is equal to the product of the distance from the center to P and the tangential or perpendicular component of the force vector.

Therefore,

T = Ft*r, or

T = F*sin(A)*r

where

- T represents torque

- Ft represents the tangential component of the force vector
- r represents the distance from the puck to the center of the circle
- F represents the force vector
- A represents the angle between the force vector and the line from the puck to the center of the circle

33.4 Example scenarios

This section contains several example scenarios involving torque.

33.4.1 Net torque on the Lazy Susan turntable

The turntable discussed earlier, which has a radius of 24 cm, is spinning clockwise. You press your fingers on the east and west sides of the turntable with equal forces of 6.67 N. The coefficient of friction between the turntable and your fingers is 0.75. What is the net torque on the wheel?

Solution:

The 13.3 N force on each side of the table creates a tangential kinetic friction force on each side of the table equal to

Ft = 6.67 N * 0.75

Each force is in the opposite direction of the direction of rotation.

The net torque is equal to the sum of the torques.

Each torque is equal to the product of the force and the distance from the center of the turntable to the point at which the force is applied.

T = 2 * Ft * r, or

T = 2 * 6.67 newtons * 0.75 * 24 cm

Entering this expression into the Google calculator gives us

T = 2.4 joules

However, this is one case where the Google calculator gives us a misleading answer. We know that torque is not measured in joules. Instead, torque is measured in N*m. Therefore, the net torque on the turntable is

T = 2.4 N*m

33.4.2 A door-closing scenario

When viewed from above, the scenario is a door that is open. From above, the wall to which the door is attached can be represented by a horizontal line that runs from west to east. The door can be represented by a line segment at an angle of about 45 degrees south of east. The line segment (door) is attached to the wall at the upper-left end of the line segment. That is the point where the door is hinged, and that point is the axis of rotation for the door.

Assume that the axis of rotation extends out of the page towards you.

The door will need to rotate about 45 degrees counter clockwise to become flush with the wall and be closed.

A person is standing on the north side of the wall pulling on a rope that is attached to the door. The rope is attached 11.5 cm from the hinge and makes a 45 degree angle with the surface of the door. That person pulls on the rope with a force of 51 N.

Using the door hinges as the axis of rotation, find the magnitude of the torque that is exerted on the door. What is the sign of the torque.

Solution:

This solution is based on the cross product from Figure 4 (p. 730) .

The magnitude of the torque is given by

T = r * F * sin(angle), or

T = 11.5 cm * 51 newtons * sin(45 degrees)

Entering this expression into the Google calculator gives us

T = 4.15 N*m

The torque will cause the door to rotate in a counter clockwise direction. Therefore, the torque has a positive sign.

33.4.3 Torque and the moment of inertia

Three objects are rotating about their centers. All three objects have a mass of 10 kg. The three objects have the following shapes:

A. A solid disk with a moment of inertia given by
I = (1/2)*m*r^2
where

- r = 2m

B. A disk with a round hole in the center with a moment of inertia given by
I = (1/2)*m*(r1^2 + r2^2)
where

- r1 = 1m
- r2 = 2m

C. A square plate with a moment of inertia given by
I = (1/12)*m*(h^2 + w^2)
where

- h = w = 3.54m

Find the net torque required to cause each object to accelerate at a rate of 10 radians/sec^2.

Solution:

All three solutions are based on the general equation for torque given in Figure 5 (p. 730) .

A. T = I * A, or
T = (1/2)*m*r^2 * A, or
T = (1/2)*10kg*(2m)^2 * 10 radians/second^2
Entering this expression into the Google calculator gives us
T = 200 N*m

B. T = I * A, or
T = (1/2)*m*(r1^2 + r2^2) * A, or
T = (1/2)*10kg*((1m)^2 + (2m)^2) * 10 radians/second^2, or
T = 250 N*m

Note that because more of the mass is located close to the outer edge of the disk, the moment of inertia is higher and more torque is required to achieve the same acceleration for the same mass.

C. T = I * A, or
T = (1/12)*m*(h^2 + w^2) * A, or
T = (1/12)*10kg*((3.54m)^2 + (3.54m)^2) * 10 radians/second^2, or
T = 2.09 N*m

Note that the square in part C was designed to have the same surface area as the disk in part A. The mass in both cases was uniformly distributed throughout the entire surface. Under those conditions, a square has a slightly higher moment of inertia than a disk and thus requires a slightly greater torque to achieve the same acceleration.

33.5 Repeat the computations

I encourage you to repeat the computations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

33.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

33.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Angular Momentum – The Mathematics of Torque for Blind Students
- File: Phy1320.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - torque
  - force
  - mass
  - angular acceleration
  - moment of inertia
  - rotational inertia
  - vector

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 34

Angular Momentum – Torque, Work and Energy[1]

34.1 Table of Contents

- Preface (p. 739)
  - General (p. 739)
  - Prerequisites (p. 740)
  - Viewing tip (p. 740)
    - Figures (p. 740)
  - Supplemental material (p. 740)
- Discussion (p. 741)
  - Constant torque (p. 741)
  - Variable torque (p. 744)
- Example scenario (p. 744)
  - Part 1 (p. 744)
  - Part 2 (p. 745)
- Do the computations (p. 746)
- Resources (p. 746)
- Miscellaneous (p. 746)

34.2 Preface

34.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains torque, work and energy in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38461/1.2/>.
[2] http://cnx.org/content/col11294/latest/

34.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

34.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

34.2.3.1 Figures

- Figure 1 (p. 742) . Work done by perpendicular component of force.
- Figure 2 (p. 743) . Work done by constant torque.
- Figure 3 (p. 744) . Power generated or consumed by a constant torque.

34.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

34.3 Discussion

This section will begin by developing the equations from which you can compute the work done by a constant torque that causes a known displacement. Then it will provide a brief discussion of the situation where the torque is not constant.

34.3.1 Constant torque

One of the textbooks that I have read uses a very familiar example to illustrate that torque can do work. The example is that of a person pulling on the rope on a power mower or outboard engine to try to get it started.

If you are unfamiliar with that scenario, many small internal combustion engines use a rope wrapped around a pulley to start the engine. When the user pulls the rope, a torque is created on the pulley by the rope. The torque causes an angular displacement of the pulley, which in turn causes certain parts inside the engine to move. If you are lucky and everything is working properly, the engine starts.

The machine fights back

However, the machine fights back and the compression in the cylinders creates a resistive torque. If the user pulls hard enough, the torque created by the user overcomes the resistive torque, the pulley turns, the parts inside the engine move appropriately, and hopefully the engine starts running.

When the engine refuses to start...

Clearly when the engine refuses to start, it becomes apparent very quickly that torque can do work on a human. A few dozen pulls on the rope will cause even the most physically fit user to become exhausted.

The force does the work

The textbook point out that it is actually the force and not the torque that does the work. However, torque and force are related in a very definitive way, and the textbook points out that it is often easer to calculate the amount of work done on the basis of torque rather than making the calculation on the basis of force.

Review – what is work?

You learned in an earlier module on translational motion that the work done by a constant force is the product of the force and the displacement caused by that force. In other words,

Wt = Ft * d

where

- Wt represents translational work in joules or newton-meters
- Ft represents translational force in newtons
- d represents the displacement in meters

A rotational analogy

Similarly, work done by a constant torque can be calculated as the product of the constant torque and the displacement caused by that torque.

A constant torque

It is important to note that the entire remaining discussion in this section applies only to the application of a constant torque. I will have a few words about a variable torque in the next section (p. 744) .

Power

The power generated or consumed by the application of a constant torque can be calculated as the product of the constant torque and the angular velocity.

A wheel scenario

Imagine a force being applied to a point on the outer edge of a wheel to cause an angular displacement of the wheel. As you will recall from an earlier module, the torque produced by the force is equal to the product of

- the distance from the center of the wheel to the point where the force is applied and

- **the component of the force** that is perpendicular to a line drawn from that point to the center of the wheel.

The point moves through a circular arc

When the force causes an angular displacement of the wheel, the point at which the point is applied moves through a circular arc. The length of that circular, often referred to by s, can be measured. The work done is equal to the product of

- the length of the circular arc and
- the perpendicular component of applied force.

The work resulting from the application of the perpendicular force is given by the equation shown in Figure 1 (p. 742) .

Work done by perpendicular component of force.

W = Fp * s
where

- W represents the work done by the perpendicular force
- Fp is the perpendicular component of force described above (p. 742)
- s is the length of the circular arc through which the point moves

Figure 34.1: Work done by perpendicular component of force.

Work as a function of torque

Now that we have the work as a function of the perpendicular force and the length of the arc, let's rewrite it in terms of torque and displacement.

Torque

We know that torque is equal to
T = r * Fp
where

- T represents torque
- r represents the distance from the center of the wheel to the point where the perpendicular force is applied
- Fp represents the perpendicular force

Arc length

We also know that the arc length is given by
s = r * A
where

- s represents the arc length

- r represents the distance from the center of the wheel to the point where the perpendicular force is applied as before
- A represents the angle of displacement measured in radians

Through substitution
W = Fp * s, or
W = (T/r)*r*A, or
The work done by a constant torque is given by the equation shown in Figure 2.

Work done by constant torque.

W = T*A
where

- W represents the work done by a constant torque
- T represents the constant torque
- A represents the angle of displacement measured in radians resulting from the application of the constant torque

Work can be either positive or negative. If the torque and the angular displacement have the same sign, the work is positive. Otherwise, the work is negative.

Figure 34.2: Work done by constant torque.

Power

As in the translational case, power is a measure of the work done per unit of time. If we divide both sides of the above equation (p. 743) by time, we get
(W/t) = T*(A/t)
where

- W/t = work per second or power
- A is the angular displacement in radians
- t is time in seconds
- A/t is the displacement in radians per second, which we recognize as angular velocity

Thus, the power generated or consumed by applying a constant torque is given by the equation shown in Figure 3 (p. 744) .

Power generated or consumed by a constant torque.

P = T*w
where

- P represents power in watts (joules per second or newton-meters per second)
- T represents torque in newton meters
- w represents angular velocity in radians per second

Figure 34.3: Power generated or consumed by a constant torque.

34.3.2 Variable torque

A torque doesn't have to be constant to do work. In fact, the torque generated by the user with the starter rope on the power mower discussed in the previous section probably isn't constant.

However, if the torque is not constant, you cannot use the equations developed in the previous section (p. 743) to compute the work done by the torque.

Maybe you can use calculus

If the torque as a function of time can be described by a function that you can integrate using integral calculus, you can use calculus to compute the work done by the torque. However, in the real word, this is probably rarely the case.

Maybe you can use a computer

If you are in the business of computing work done by a variable torque, the most likely case is that you will have equipment that allows you to sample the torque and displacement values at uniform intervals of time and to save the values of the samples for digital processing. Then you can use any one of several digital methods to approximately integrate the product of the torque function and the displacement function.

34.4 Example scenario

I once visited a factory where mirrors were made. At one of the stations on the manufacturing line, a person used a large horizontal grinding wheel to grind a bevel on the edge of the mirror.

Assume that the grinding wheel is a uniform disk with:

- A moment of inertia, I, equal to (1/2)*M*R^2
- M = mass = 80 kg
- R = radius = 0.0.5 meters

34.4.1 Part 1

Find the amount of work that must be done to bring the wheel from rest to an angular velocity of 8.38 radians/sec

Solution:

Recall from a previous module that the rotational kinetic energy for a rotating object is given by
Ks = (1/2)*I*w^2

- where Ks represents the kinetic energy for the system
- I represents the rotational inertia for the system
- w represents the angular velocity of the system

We could rewrite this equation as
deltaKs = (1/2)*I*(w0 - wf)^2
where

- deltaKs represents the change in kinetic energy
- w0 represents the initial kinetic energy
- wf represents the final kinetic energy

However, since the initial kinetic energy value is zero, that would simply complicate the algebra. Therefore, we will stick with the original equation (p. 745) .

We either have, or can calculate values for all of the terms in this equation. Substituting the values given above gives us
Ks = (1/2)*I*w^2 , or
Ks = (1/2)*((1/2)*M*R^2)*w^2 , or
Ks = (1/2)*((1/2)*80kg*(0.5m)^2)*(8.38 radians/sec)^2
Entering this expression into the Google calculator gives us
Ks = 351 joules
This is the amount of work that must be done to bring the wheel from rest to an angular velocity of 8.38 radians/sec

34.4.2 Part 2

If the motor that drives the wheel delivers a constant torque of 10 N*m during this time, how many revolutions does the wheel turn in coming up to speed.

Solution:

We know how to relate the displacement angle and the work for a constant torque using the equation in Figure 2 (p. 743) .
W = T*A
where

- W represents the work done by a constant torque
- T represents the constant torque
- A represents the angle of displacement measured in radians resulting from the application of the constant torque

In this case, we know the amount of work and the value of the torque and need to find the angle. Therefore,
A = W*joules/T*n*m
However, this gives us the angular displacement in radians. We need to scale to convert it to revolutions.
A = (W*joules/T*n*m)/2*pi, or
A = (351joules/10newton meters)/(2*pi), or
A = 5.59 revolutions
This is the number of revolutions that the wheel turns in coming up to speed.

34.5 Do the computations

I encourage you to repeat the computations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

34.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

34.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Angular Momentum – Torque, Work and Energy for Blind Students
- File: Phy1330.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - force
  - torque
  - work
  - energy

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 35

Angular Momentum – Rotational Equilibrium[1]

35.1 Table of Contents

- Preface (p. 748)
  - General (p. 748)
  - Prerequisites (p. 748)
  - Supplemental material (p. 748)
- Discussion (p. 749)
- Example scenarios (p. 750)
  - Weights hanging on a beam (p. 750)
    - Part 1 (p. 750)
    - Part 2 (p. 750)
  - Beam supported by a diagonal cable (p. 750)
    - Part 1 (p. 751)
    - Part 2 (p. 751)
    - Part 3 (p. 751)
  - A crane scenario (p. 752)
    - Part 1 (p. 752)
    - Part 2 (p. 753)
    - Part 3 (p. 753)
  - Sliding a crate (p. 753)
    - Part 1 (p. 753)
    - Part 2 (p. 753)
  - Another crane scenario (p. 754)
    - Part 1 (p. 754)
    - Part 2 (p. 754)
    - Part 3 (p. 754)
  - A ladder scenario (p. 755)
    - Part 1 (p. 755)
    - Part 2 (p. 755)

[1] This content is available online at <http://cnx.org/content/m38465/1.3/>.

- Repeat the computations (p. 756)
- Resources (p. 756)
- Miscellaneous (p. 756)

35.2 Preface

35.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains rotational equilibrium in a format that is accessible to blind students.

35.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

35.2.3 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

[2] http://cnx.org/content/col11294/latest/
[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

35.3 Discussion

This module will consist of a short discussion followed by a lot of hands-on practical problem solving.

Translational and rotational equilibrium

You learned in an earlier module that an object is in translational equilibrium when the net force acting on it is zero. In other words, the vector sum of the forces acting on the object must be zero.

However, you also learned in an earlier module that it is possible for the net force to be zero, while the net torque is not zero. If the net torque is not zero, the object would have non-zero acceleration, and therefore would not be in rotational equilibrium.

Be careful when climbing a ladder

When you are climbing a ladder, for example, it would be highly undesirable for the system that includes you and the ladder to not be in rotational equilibrium.

Conditions for translational and rotational equilibrium

While zero net force is sufficient to ensure translational equilibrium, zero net torque is also required to ensure rotational equilibrium. When both translational and rotational equilibrium is achieved, the object is said to be in a state of static equilibrium.

In order to achieve static equilibrium, the sum of the forces acting on the object must be zero and the sum of the torques about all axes must also be zero.

Choosing an axis of rotation

If an object is not already rotating, it may not be obvious how to determine the axis about which the object is likely to rotate. You need to ensure that the sum of the torques about that axis are zero.

As it turns out, you can choose just about any axis you please when doing the calculations to confirm that the sum of the torques are zero. You touched on this in an earlier module involving the parallel axis theorem. It can be shown that if the net force acting on an object is zero and the net torque about one axis is zero, the net torque about every other parallel axis will also be zero.

The worst case scenario

In the worst case, therefore, you can ensure rotational equilibrium by ensuring that the net torque about three orthogonal axes are zero. In some cases, it will be obvious that you only need to worry about one axis. In some cases, you would need to worry about two or three axes.

A ladder

Before climbing a ladder, for example, you need to ensure rotational equilibrium about

1. One axis that is parallel to the rungs on the ladder
2. One axis that is perpendicular to a rectangle formed by two rungs and the side rails
3. One axis that is parallel to the side rails on the ladder

The lack of rotational equilibrium about any of these three axes can result in a nasty fall from a ladder.

Ensuring rotational equilibrium for a ladder

Ladder manufacturers install special "feet" with a high coefficient of friction on ladders to avoid accidents from the first item in the above list. These feet are designed to prevent the ladder from sliding away from the wall that it is leaning on.

They also install wide outriggers on the base of very tall ladders to avoid accidents from the second item in the list. The outriggers are designed to prevent top of the ladder from sliding sideways on the wall.

Tree trimmers that use ladders to trim trees take special precautions to prevent accidents from the third item in the list in those cases where both sides of the top of the ladder are not firmly supported.

Simplification through the choice of an axis

A judicious choice of axis can often greatly simplify the solution of an equilibrium problem by causing one or more forces to go through the axis. This eliminates the effect of those forces from the computation of rotational equilibrium.

35.4 Example scenarios

I will present several example scenarios involving rotational equilibrium in this section. In several cases, I will ask you to construct a replica of the scenario on your graph board.

All scenarios assume that counter clockwise rotation is the positive direction of rotation. All scenarios also assume that the positive horizontal axis is to the right and the positive vertical axis is up.

35.4.1 Weights hanging on a beam

Using your graph board, draw a picture of a beam of length L hanging from a rope at its center. Hang a mass M1 from the left end of the beam. Hang a mass M2 from the right end of the beam. Hang a mass M3 half way between M1 and the tie-point of the rope.

Assume that the system is in static equilibrium.

35.4.1.1 Part 1

Find the value of M3 given the following:

- M1 = 1kg
- M2 = 2kg

Solution:

Choose the tie-point of the rope as the axis of rotation.
The net torque about the axis of rotation is
Tnet = (L/2)*M1 + (L/4)*M3 - (L/2)*M2
For rotational equilibrium, the net torque must be equal to zero.
Rewriting and simplifying yields
L*(M1)/2 + L*(M3)/4 = L*(M2)/2
Dividing both sides by L yields
(M1)/2 + (M3)/4 = (M2)/2
Solving for M3 yields
M3 = 2*M2 - 2*M1
Substituting numeric values yields
M3 = 2*2kg - 2*1kg, or
M3 = 2kg

35.4.1.2 Part 2

If the mass of the beam is 2kg, what is the tension in the rope?

Solution:

In order for the system to be in translational equilibrium, the downward forces must be equal to the upward force exerted by the rope.
tension = (M1 + M2 + M3 + Mbeam)*g
Substituting numeric values yields
tension = (1kg + 2kg + 2kg + 2kg)*9.8m/s^2, or
tension = 68.6 newtons

35.4.2 Beam supported by a diagonal cable

Draw the following picture on your graph board. A vertical wall is on the left. A beam is attached to the wall with a hinge and extends outward horizontally to the right. (Without further support, the beam is free to rotate around the hinge causing the end of the beam to move up and down.)

To support the horizontal orientation of the beam, a cable is attached to the end of the beam and is attached to the wall above the attachment point of the beam. The angle between the beam and the cable at the end of the beam is 30 degrees.

Assume that the system is in static equilibrium.

Assume that

- the mass of the cable is negligible
- the mass of the beam is given by M = 20kg
- the length of the beam is L

35.4.2.1 Part 1

Draw vectors showing all of the forces that are exerted on the beam.

Solution:

The following forces are exerted on the beam:

- A horizontal force pushing outward on the left end of the beam. Label it H.
- A vertical force supporting the beam where it is connected to the wall by the hinge. Label it V.
- A tension force in the cable. Label it T.
- The weight of the beam acting downward at the center of the beam. Label it M*g.
- The horizontal component of the tension force pushing the beam towards the wall. Label it T*cos(30 degrees).
- The vertical component of the tension force pushing the end of the beam upward. Label it T*sin(30 degrees).

35.4.2.2 Part 2

What is the most judicious location for computing the sum of the torques to establish rotational equilibrium?

Solution:

This is probably a matter of opinion. However, if the torques are computed around the point where the beam attaches to the wall, three of the forces listed above pass through that point and can be ignored when computing the sum of the torques around that point. Therefore, I consider that to be the most judicious point.

35.4.2.3 Part 3

Given the above assumptions (p. 751) , what are the magnitudes of the forces labeled V, H, and T required to produce translational and rotational equilibrium?

Solution:

We begin by determining the required value of the tension in the cable, T, to produce rotational equilibrium.

Solve for the value of T

Choose the hinge as the axis of rotation for the reasons given above.

For rotational equilibrium,

L*T*sin(30 degrees) - (L/2)*M*g = 0

Dividing both sides by L yields

T*sin(30 degrees) - M*g/2 = 0

Note that the actual length of the beam is not important for this computation.

Solving for T yields

T = (20kg*(9.8m/s^2)/2)/sin(30 degrees), or

T = 196 newtons

Therefore, the tension in the cable is a force of 196 newtons directed from the end of the beam toward the wall at an angle of 30 degrees north of west.

Solve for the value of V

Now that we know the tension in the cable, we can compute the values for V and H.

For translational equilibrium in the vertical dimension,

V + T*sin(30 degrees) - M*g = 0, or

V = M*g - T*sin(30 degrees)

Inserting numeric values from above,

V = (20kg*9.8m/s^2) - 196newtons * sin(30 degrees), or

V = 98 newtons

Check the result for V

We can check this result by computing the sum of the torques about the center of the beam.

(L/2)*T*sin(30 degrees) - (L/2)*V = 0

Dividing through by L/2 yields

T*sin(30 degrees) - V = 0

Inserting numeric values yields

196*sin(30 degrees) - 98 = 0

and the values for T and V check.

Solve for the value of H

Knowing the value of T also makes it possible for us to compute the value of H.

For horizontal equilibrium,

H - T*cos(30 degrees) = 0, or

H = T*cos(30 degrees)

Inserting numeric values yields

H = 196newtons*cos(30 degrees), or

H = 169.7 newtons

35.4.3 A crane scenario

Draw the following schematic diagram of a crane on your graph board.

The crane consists of a boom attached to a horizontal surface with a pin. (The pin acts as a hinge and the boom can rotate around the pin.) The boom extends upwards and to the right at an angle of 30 degrees relative to the surface.

A weight of 6000 newtons is hanging from the end of the boom.

A vertical cable is attached to a winch on an overhead beam and is also attached to the boom 2 meters from the lower end.

Assumptions:

- The length of the boom = L = 8m
- The weight of the boom is negligible

35.4.3.1 Part 1

Draw a vector diagram showing the vertical forces being exerted on the boom.

Solution:

Three vertical forces are exerted on the boom:

1. A downward force at the lower end of the boom where the boom is attached to the pin. Label this force V.
2. An upward force at the point where the cable is attached to the boom. Label this force F.
3. A downward force at the upper end of the boom where the 6000 newton weight is attached. Label this force W.

35.4.3.2 Part 2

What is the downward force on the cable?

Let the lower end of the boom be the axis of rotation.

Compute the torques due to the cable and the weight. The sum of those torques must be zero for rotational stability.

2m * F*cos(30 degrees) - 8m*6000 newtons * cos(30 degrees) = 0

Divide both sides of the equation by cos(30 degrees)

2m*F = 8m*6000 newtons, or

F = (8M*6000newtons)/2m, or

F = 24000 newtons

Therefore, the force pulling down on the cable is 24000 newtons.

35.4.3.3 Part 3

What is the magnitude and direction of the force on the pin at the bottom of the boom?

For vertical equilibrium, the vertical forces must sum to zero.

V + 24000 newtons - 6000 newtons = 0, or

V = 6000 newtons - 24000 newtons, or

V = -18000 newtons

Therefore, the force on the pin is 18000 newtons down.

35.4.4 Sliding a crate

A crate is being slid to the right on a horizontal floor by pulling on a rope tied to the right side of the crate.

Width of the crate = w = 4 m

Height of the crate = h = 3 m

Coefficient of kinetic friction = u = 0..5

Attachment point of rope = d = ?

35.4.4.1 Part 1

Draw a picture of the crate showing the forces that are being exerted on the crate.

Solution:

The crate is a rectangle with the dimensions given above. There are three forces acting on the crate.

There is a downward force at the bottom center of the crate, which is the weight of the crate. Label it F1.

There is a horizontal force pointing to the left at the bottom of the crate. This is the force of friction. Label it F2.

There is a horizontal force pointing to the right on the right side of the crate that is d units above the floor. This is the force that is pulling the crate to the right. Label it F3.

35.4.4.2 Part 2

What is the maximum value for d that allows the crate to slide without tipping over?

Compute the sum of the horizontal forces.

F3 - F2 = 0, or

F3 - m*g*u = 0, or

F3 = m*g*u

Compute the torques about the bottom right corner.

(w/2)*F1 - d*F3 = 0, or

d*F3 = (w/2)*F1, or

d*F3 = (w/2)*m*g

Substitution yields
d*m*g*u = (w/2)*m*g
Simplification yields
d*u = (w/2), or
d = (w/2)/u
Inserting values yields
d = (4/2)/0.5, or
d = 4 m
Since the height of the crate is only 3 m, it is not possible to tip it over by pulling on a rope attached to the right side of the crate.

35.4.5 Another crane scenario

The boom of a crane is pinned at the intersection of a horizontal floor and a vertical wall. The boom slopes toward the upper right.

A mass hangs from the top end of the boom.

The top end of the boom is connected to the vertical wall by a cable that is stretched horizontally from the wall to the boom.

Length of cable = X = 4 m
Height of cable = Y = 3 m
Mass = M = 2000 kg
The weight of the boom is negligible.

35.4.5.1 Part 1

Draw the forces acting on the boom.

Solution:

There are four forces acting on the boom:

A downward force that is attributable to the mass hanging on the top end of the boom. Label this force F1.

A force pointing to the left from the top end of the boom due to the cable. Label this force F2.

A force pointing up at the bottom end of the boom. Label this force F3.

A force pointing to the right at the bottom of the boom. Label this force F4.

35.4.5.2 Part 2

Find the tension in the cable.

Solution:

For rotational equilibrium, the sum of the torques about the bottom of the boom must be zero.
F2*Y = F1*X, or
F2*Y = M*g*X, or
F2 = (M*g*X)*/Y
Inserting numeric values yields
F2 = (2000kg*(9.8m/s^2)*4m)/3m
F2 = 26133 newtons
Therefore, the tension in the cable is equal to 26133 newtons

35.4.5.3 Part 3

Find the force pushing against the bottom end of the boom along the length of the boom. Also find the angle between that force and the floor.

For horizontal equilibrium

F4 = F2 = 26133 newtons
For vertical equilibrium
F3 = F1 = M*g = 2000kg*9.8m/s^2, or
F3 = 19600 newtons
The magnitude of the force = sqrt(F4^2 + F3^2), or
Magnitude = sqrt(26133^2 + 19600^2), or
Magnitude = 32666 newtons
Therefore, the magnitude of the force is 32666 newtons.
The angle of the force is
angle = atan(19600 newtons/26133 newtons) in degrees
angle = 36.9 degrees north of east

35.4.6 A ladder scenario

A ladder is sitting on a horizontal floor and leaning against a smooth wall. A bucket of paint is hanging from a rung 80-percent up from the bottom of the ladder. The parameters are as follows:

- Length of ladder = L = 5 m
- Angle between ladder and floor = A = 50 degrees
- Mass of the bucket of paint = m = 2.5 kg
- Mass of the ladder = M = 12 kg
- Coefficient of static friction = u = ?

35.4.6.1 Part 1

Draw a vector diagram of the forces acting on the ladder.

Solution:

There are five forces acting on the ladder:

1. A force directed outward from the wall at the top of the ladder. Label this force F1.
2. A force pushing straight down due to the mass of the bucket of paint. Label this force F2.
3. A force pushing straight down due to the mass of the ladder. Label this force F3.
4. A force at the bottom of the ladder directed toward the wall due to static friction. Label this force F4.
5. A force at the bottom of the ladder directed straight up due to the weight of the ladder and the bucket of paint. Label this force F5.

35.4.6.2 Part 2

The ladder is in static equilibrium.

The coefficient of static friction between the ladder and the wall is zero.

What is the minimum possible value for the coefficient of static friction between the ladder and the floor.

F1 = ?
F2 = m*g = 2.5kg * 9.8m/s^2 = 24.5 newtons
F3 = M*g = 12kg*9.8m/s^2 = 117.6 newtons
F4 = F5*u
For vertical equilibrium,
F5 = F2 + F3 = (2.5kg + 12kg)*9.8m/s^2 = 142 newtons
For rotational equilibrium, the sum of the torques about the bottom of the ladder must be zero.
Compute the horizontal distance to the line of action of the weight of the ladder.
X1 = (L/2)*cos(50 degrees), or

X1 = ((5/2)m)*cos(50 degrees), or
X1 = 1.61m
Compute the horizontal distance to the line of action of the weight of the bucket of paint.
X2 = 0.8*5m*cos(50 degrees), or
X2 = 2.57m
Compute the vertical distance to the line of action of the horizontal force at the top of the ladder.
Y1 = 5m*sin(50 degrees), or
Y1 = 3.83m
Compute sum of the torques about the bottom of the ladder.
Y1*F1 - X2*F2 - X1*F3 = 0, or
F1 = (X2*F2 + X1*F3)/Y1, or
F1 = (2.57m*24.5 newtons+ 1.61m*117.6 newtons)/3.83m
F1 = 65.87 newtons
For horizontal equilibrium,
F4 = 65.87 newtons
By substitution,
F4 = F5*u = 65.87 newtons
u = (65.87 newtons)/F5, or
u = (65.87 newtons)/142 newtons, or
u = 0.46

While the coefficient of static friction could be higher than this and still achieve static equilibrium, if the coefficient were any lower, the ladder would slide away from the wall.

35.5 Repeat the computations

I encourage you to repeat the computations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

35.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

35.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Angular Momentum – Rotational equilibrium for Blind Students
- File: Phy1340.htm
- Keywords:
  - · physics
  - · accessible
  - · accessibility
  - · blind
  - · graph board
  - · protractor
  - · screen reader

· refreshable Braille display
· JavaScript
· trigonometry
· rotational equilibrium
· translational equilibrium
· static equilibrium
· beam
· boom
· crane
· crate
· ladder

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Chapter 36

Angular Momentum – The Physics of Rolling Objects[1]

36.1 Table of Contents

- Preface (p. 759)
  - General (p. 759)
  - Prerequisites (p. 760)
  - Viewing tip (p. 760)
    - Figures (p. 760)
  - Supplemental material (p. 760)
- Discussion (p. 761)
  - Rotational versus translational kinetic energy (p. 762)
  - Total kinetic energy (p. 763)
  - Acceleration of a rolling object (p. 764)
- Example scenarios (p. 764)
  - Rolling cylinders (p. 764)
    - Part 1 (p. 764)
    - Part 2 (p. 765)
  - Acceleration of a rolling cylinder (p. 765)
- Work through the computations (p. 767)
- Resources (p. 767)
- Miscellaneous (p. 767)

36.2 Preface

36.2.1 General

This module is part of a collection (see http://cnx.org/content/col11294/latest/ [2]) of modules designed to make physics concepts accessible to blind students. The collection is intended to supplement but not to replace the textbook in an introductory course in high school or college physics.

This module explains the physics of rolling objects in a format that is accessible to blind students.

[1] This content is available online at <http://cnx.org/content/m38498/1.2/>.
[2] http://cnx.org/content/col11294/latest/

36.2.2 Prerequisites

In addition to an Internet connection and a browser, you will need the following tools (as a minimum) to work through the exercises in these modules:

- A graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [3]).
- A protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [4]).
- An audio screen reader that is compatible with your operating system, such as the NonVisual Desktop Access program (NVDA), which is freely available at http://www.nvda-project.org/ [5] .
- A refreshable Braille display capable of providing a line by line tactile output of information displayed on the computer monitor (http://www.userite.com/ecampus/lesson1/tools.php [6]).
- A device to create Braille labels. Will be used to label graphs constructed on the graph board.

The minimum prerequisites for understanding the material in these modules include:

- A good understanding of algebra.
- An understanding of the use of a graph board for plotting graphs and vector diagrams (http://www.youtube.com/watch?v=c8plj9UsJbg [7]).
- An understanding of the use of a protractor for measuring angles (http://www.youtube.com/watch?v=v-F06HgiUpw [8]).
- A basic understanding of the use of sine, cosine, and tangent from trigonometry (http://www.clarku.edu/~djoyce/trig/ [9]).
- An introductory understanding of JavaScript programming (http://www.dickbaldwin.com/tocjscript1.htm [10] and http://www.w3schools.com/js/default.asp [11]).
- An understanding of all of the material covered in the earlier modules in this collection.

36.2.3 Viewing tip

I recommend that you open another copy of this document in a separate browser window and use the following links to easily find and view the figures while you are reading about them.

36.2.3.1 Figures

- Figure 1 (p. ??) . Examples of moment of inertia.
- Figure 2 (p. 763) . Relationship between rotational and translational kinetic energy.

36.2.4 Supplemental material

I recommend that you also study the other lessons in my extensive collection of online programming tutorials. You will find a consolidated index at www.DickBaldwin.com [12] .

[3] http://www.youtube.com/watch?v=c8plj9UsJbg
[4] http://www.youtube.com/watch?v=v-F06HgiUpw
[5] http://www.nvda-project.org/
[6] http://www.userite.com/ecampus/lesson1/tools.php
[7] http://www.youtube.com/watch?v=c8plj9UsJbg
[8] http://www.youtube.com/watch?v=v-F06HgiUpw
[9] http://www.clarku.edu/~djoyce/trig/
[10] http://www.dickbaldwin.com/tocjscript1.htm
[11] http://www.w3schools.com/js/default.asp
[12] http://www.dickbaldwin.com/toc.htm

36.3 Discussion

What happens when objects with different moments of inertia (rotational inertia) roll down a hill? Does the moment of inertia effect how the objects roll? Those are the kinds of questions that we will explore in this module.

A symmetrical object rolling down an incline

Imagine an object that is symmetrical about its center of mass (such as a sphere or a cylinder) rolling down an incline. The center of mass experiences translational motion as it rolls down the incline. In addition, the object is rotating about an axis that passes through the center of mass.

The translational velocity

If the object is rolling without slipping, the translational velocity of the center of mass is given by

Vcm = W*R

where

- Vcm represents the translational velocity of the center of mass
- W represents the angular velocity
- R represents the radius

A well-defined relationship

This means that there is a well-defined relationship between the rolling object's translational and rotational kinetic energies. The total kinetic energy of a rolling object is the sum of its translational and rotational kinetic energies.

A clue

This may give you a clue as to where we are heading with this. As a preview, when an object rolls down an incline it exchanges potential energy for kinetic energy. Some of that potential energy is transformed into translational kinetic energy and some is transformed into rotational kinetic energy. The manner in which that potential energy is distributed between the two forms of kinetic energy has an impact on how the object rolls.

Cylinders, disks, and spheres

Imagine a cylinder-like, disk-like, or sphere-like object that has a mass M and a radius R rolling down an incline. The moment of inertia for each of four different geometrical examples is shown in Figure 1 (p. **??**) .

I = M*R^2

Solid cylinder or disk

I = (1/2)*M*R^2

Solid sphere

I = (2/5)*M*R^2

Then hollow spherical shell

I = (2/3)*M*R^2

I = M*R^2

Solid cylinder or disk

I = (1/2)*M*R^2

Solid sphere

I = (2/5)*M*R^2

Then hollow spherical shell

I = (2/3)*M*R^2

Thin hollow cylindrical shape or hoop I = M*R^2	**Solid cylinder or disk** I = (1/2)*M*R^2	**Solid sphere** I

Figure 36.1: Examples of moment of inertia.

A very important constant

As you can see from Figure 1 (p. ??) , for these four shapes at least, the moment of inertia is equal to a constant multiplied by the product of the mass and the square of the radius. (As far as I know, this constant hasn't been given a widely-recognized name.)

In these four cases, we can write an expression for the moment of inertia for rotation about the center of mass as

Icm = Q*M*(R^2)

where

- Icm represents the moment of inertia through the center of mass
- Q represents a constant that applies for a particular shape
- M represents the mass
- R represents the radius

The purpose of the constant

The purpose of the constant, Q, is to specify how the mass is distributed relative to the axis of rotation. Larger values of Q generally indicate that the mass is distributed further from the axis of rotation. Similarly, larger values of Q also indicate larger moments of inertia.

36.3.1 Rotational versus translational kinetic energy

Given the relationships

Icm (p. 762) = Q*M*R^2, and

Vcm (p. 761) = W*R, and

Krot = (1/2)*Icm*W^2

where

- Icm represents the moment of inertia through the center of mass
- Q represents a constant that applies for a particular shape
- M represents the mass
- R represents the radius
- Vcm represents the translational velocity of the center of mass
- W represents the angular velocity
- R represents the radius
- Krot represents the rotational kinetic energy that you learned about in an earlier module [13]

We can substitute and write

Krot = (1/2)*(Q*M*R^2)*(Vcm/R)^2, or

Krot = (1/2)*(Q*M*R^2)*(Vcm^2/R^2), or

Krot = (1/2)*(Q*M)*(Vcm^2), or

Krot = Q*(1/2)*(M)*(Vcm^2)

Given that

Ktr = (1/2)*(M)*(Vcm^2)

where

- Ktr represents translational kinetic energy, which you also learned about in the earlier module [14]

[13] http://cnx.org/content/m38440/latest/#Translational_and_Rotational_Kinetic_Energy

[14] http://cnx.org/content/m38440/latest/#Translational_and_Rotational_Kinetic_Energy

A relationship between rotational and translational kinetic energy can be derived. That relationship is given by the expression shown in Figure 2 (p. 763) .

Relationship between rotational and translational kinetic energy.

Krot = Q*Ktr
where

- Krot represents rotational kinetic energy
- Ktr represents translational kinetic energy
- Q is a constant that depends on how the mass is geometrically distributed in the rolling object

Figure 36.2: Relationship between rotational and translational kinetic energy.

A very interesting equation

The equation shown in Figure 2 (p. 763) is very interesting. It shows that the relationship that determines the distribution of kinetic energy between translational kinetic energy and rotational kinetic depends solely on the constant Q, and is independent of the mass, the radius, etc.

The constant Q depends on how the mass is geometrically distributed in the rolling object. Generally speaking, the more the mass is distributed toward the outer edge of the object, the greater will be the value of Q.

36.3.2 Total kinetic energy

The total kinetic energy is given by

K = Ktr + Krot, or
K = (1/2)*M*(Vcm)^2 + (1/2)*Icm*W^2
where

- K represents total kinetic energy
- M represents the mass
- Vcm represents the translational velocity of the center of mass
- Icm represents the moment of inertia through the center of mass
- W represents the angular velocity

Through substitution we can write

K = Ktr + Krot, or
K = Ktr + Q*Ktr, or
K = (1 + Q)*Ktr, or
K = (1 + Q)*(1/2)*M*(Vcm)^2

Thus, an object with a given mass and a given translational velocity has a total kinetic energy that is proportional to (1+Q). The larger the value of Q, the greater will be the kinetic energy possessed by the object.

From Figure 1 (p. ??) , a rolling hollow cylinder with a Q value of 1 would have a greater total kinetic energy than a solid cylinder with the same mass and a Q value of 0.5 rolling with the same translational velocity. This is because the cylinder would have more rotational kinetic energy.

36.3.3 Acceleration of a rolling object

If there were no friction, an object would not roll down an incline. Instead it would simply slide down the incline. In the absence of torque, the rotational inertia of the object would prevent it from experiencing angular acceleration.

The frictional force parallel to the incline and pointing up the incline produces a torque that causes the object to experience angular acceleration.

36.4 Example scenarios

In this section, I will explain some example scenarios involving various aspects of rolling solid and hollow cylinders.

36.4.1 Rolling cylinders

A cylindrical shell and a solid cylinder, each of unknown mass and unknown radii roll down an incline of unknown height and unknown angle.

36.4.1.1 Part 1

Which object will have the greater translational velocity when they reach the bottom of the incline?

Solution:

From Figure 1 (p. ??) , the rotational inertia for a thin hollow cylinder is given by
I = M*R^2
The rotational inertia for a solid cylinder is given by
I = (1/2)*MR^2
Let the cylindrical shell be identified as M1 and the solid cylinder be identified as M2.

The total kinetic for each object at the bottom is equal to potential energy for that object at the top. Therefore,
M1*g*h = K1 = (1+Q1)*M1*(V1cm)^2, and
M2*g*h = K2 = (1+Q2)*M2*(V2cm)^2
Eliminating the mass from both sides of both equations yields
g*h = (1+Q1)*(V1cm)^2, and
g*h = (1+Q2)*(V2cm)^2
Solving each equation for the velocity yields
V1cm = ((g*h)/(1+Q1))^(1/2), and
V2cm = ((g*h)/(1+Q2))^(1/2)
Taking the ratios of the velocities yields
V1cm/V2cm = ((1+Q2)/(1+Q1))^(1/2)
Solving for V1cm in terms of V2cm yields
V1cm = V2cm * ((1+Q2)/(1+Q1))^(1/2)
From Figure 1 (p. ??) ,
For the cylindrical shell, Q1 = 1 and
For the solid cylinder, Q2 = (1/2)
Substituting values yields
V1cm = V2cm * ((1+(1/2))/(1+1))^(1/2)
Solving with the Google calculator yields

V1cm = V2cm * 0.866

Therefore, the translational velocity of the cylindrical shell is less than the translational velocity the solid cylinder by a factor of 0.866. This is because more of the potential energy is transformed into rotational kinetic energy in the cylindrical shell, resulting in less translational kinetic energy and less translational velocity.

Note that this result is independent of the mass of the objects, the radii of the objects, the height of the incline, and the angle of the incline.

36.4.1.2 Part 2

What percentage of the total kinetic energy is translational kinetic energy for each of the cylinders in Part 1 (p. 764) ?

Solution:

From above (p. 763) ,
K = (1 + Q)*(1/2)*M*(Vcm)^2
We know from earlier modules that
Ktr = (1/2)*M*(Vcm)^2
Taking the ratio of Ktr to K gives us
Ktr/K = ((1/2)*M*(Vcm)^2)/((1 + Q)*(1/2)*M*(Vcm)^2)
Simplification yields
Ktr/K = 1/(1+Q), or
Ktr = K * (1/(1+Q))
For the cylindrical shell
Ktr = K * (1/(1+1)) = K * (1/2)
Half or 50 percent of the total kinetic energy is translational kinetic energy for the cylindrical shell.
For the solid cylinder
Ktr = K * (1/(1+Q)) = K * (1/(1+(1/2))) = K * 0.67
Sixty-seven percent of the total kinetic energy is translational kinetic energy for the solid cylinder.

36.4.2 Acceleration of a rolling cylinder

A solid cylinder is rolling down an incline without slipping. What is the translational acceleration of the cylinder? How does that acceleration compare with the acceleration of the same object sliding down the incline?

Assume that

- The angle of incline is represented by U
- The value of Q for the cylinder is (1/2)
- The coefficient of friction is unknown
- The mass of the cylinder is unknown
- The radius of the cylinder is unknown

Solution:

The only forces acting on the cylinder are the force of gravity and an unknown frictional force parallel to the incline and pointing up the incline. The force of gravity does not produce a torque on the object acting about the center of mass. Therefore, the only torque is produced by the unknown force of friction.

Angular acceleration is commonly represented by the Greek letter alpha. However, your Braille display probably won't display the Greek letter alpha, so we will let angular acceleration be represented by "An" and will let translational acceleration be represented by "Atr".

We know that the relationship between torque, angular acceleration, and moment of inertia is
Sum of torques = I*An
We only have one torque. Therefore,

T = I*An
We also know that
T = R*f
where

- T is the torque
- R is the radius
- f is the unknown frictional force

Therefore,
T = I*An, or
An = T/I, or
An = R*f/I, or
f = An*I/R
where

- T represents the torque
- I represents the moment of inertia
- R represents the radius
- f represents the unknown frictional force

As we discussed earlier, when an object is rolling with a given angular velocity, the translational velocity of the axis of rotation is proportional to the radius of the object. The greater the radius, the greater will be the translational velocity. Thus, the translational velocity is proportional to the angular velocity with the radius being the proportionality constant. We can write

Vcm = W*R

If the object is not slipping, the rate of change of the translational velocity must also be proportional to the rate of change of the angular velocity through the same proportionality constant, which is the radius.

Therefore, we can write
Atr = R*An, or
An = Atr/R

The translational acceleration is also given by the net force divided by the mass. The net force is the component of the weight pointing down the incline minus the force of friction pointing up the incline. Therefore, we can write
M*g*sin(U) - f = M*Atr
Substitution from above yields
M*g*sin(U) - (An*I/R) = M*Atr
Further substitution from above yields
M*g*sin(U) - ((Atr/R)*I/R) = M*Atr, or
M*g*sin(U) - (Atr*I/(R^2)) = M*Atr
Solving for Atr yields
M*Atr + (Atr*I/(R^2)) = M*g*sin(U), or
Atr * (M + I/(R^2)) = M*g*sin(U), or
Atr = (M*g*sin(U))/(M + I/(R^2)), or
Atr = (g*sin(U))/(1 + I/(M*R^2))
For a solid cylinder,
I = (1/2)*M*R^2
By substitution
Atr = (g*sin(U))/(1 + ((1/2)*M*R^2)/(M*R^2)), or
Atr = (g*sin(U))/(1 + (1/2)), or
Atr = (g*sin(U))/(3/2), or
Therefore, the translational acceleration of the rolling solid cylinder is given by

Atr = (2/3)*(g*sin(U))

If the cylinder were sliding in the total absence of friction, from what you learned in earlier modules, the acceleration would simply be

Atr = g*sin(U)

Therefore, if there is sufficient friction to cause the cylinder to roll without slipping, the translational acceleration will be only (2/3) of the sliding acceleration. Once again, this is the result of a portion of the potential energy being transformed into rotational kinetic energy, resulting in less translational velocity, and less translational acceleration.

36.5 Work through the computations

I encourage you to work through the computations that I have presented in this lesson to confirm that you get the same results. Experiment with the scenarios, making changes, and observing the results of your changes. Make certain that you can explain why your changes behave as they do.

36.6 Resources

I will publish a module containing consolidated links to resources on my Connexions web page and will update and add to the list as additional modules in this collection are published.

36.7 Miscellaneous

This section contains a variety of miscellaneous information.

NOTE: **Housekeeping material**

- Module name: Angular Momentum – The Physics of Rolling Objects for Blind Students
- File: Phy1350.htm
- Keywords:
  - physics
  - accessible
  - accessibility
  - blind
  - graph board
  - protractor
  - screen reader
  - refreshable Braille display
  - JavaScript
  - trigonometry
  - rotational kinetic energy
  - translational kinetic energy

NOTE: **Disclaimers:** **Financial** : Although the Connexions site makes it possible for you to download a PDF file for this module at no charge, and also makes it possible for you to purchase a pre-printed version of the PDF file, you should be aware that some of the HTML elements in this module may not translate well into PDF.

I also want you to know that I receive no financial compensation from the Connexions website even if you purchase the PDF version of the module.

Affiliation : I am a professor of Computer Information Technology at Austin Community College in Austin, TX.

-end-

Index of Keywords and Terms

Keywords are listed by the section with that keyword (page numbers are in parentheses). Keywords do not necessarily appear in the text of the page. They are merely associated with that section. *Ex.* apples, § 1.1 (1) **Terms** are referenced by the page they appear on. *Ex.* apples, 1

A
acceleration, § 10(209), § 12(325), § 17(525)
acceleration of gravity, § 9(181), § 12(325)
accessibility, § 2(7), § 3(53), § 4(69), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
accessible, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 7(155), § 8(167), § 9(181), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
accuracy, § 8(167)
action and reaction, § 19(553)
angle, § 6(105)
angular acceleration, § 33(721)
angular displacement, § 30(683)
angular velocity, § 30(683)
arccosine, § 6(105)
arcsine, § 6(105)
arctangent, § 6(105)
argument, § 5(85)
arithmetic operators, § 5(85)
array object, § 11(273)
Atwood machine, § 31(695)
average speed, § 28(669)

B
beam, § 35(747)
blind, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 7(155), § 8(167), § 9(181), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
body, § 10(209)
boom, § 35(747)
Braille display, § 1(1), § 7(155), § 8(167), § 9(181)

C
carousel, § 28(669)
center of gravity, § 16(497)
centripetal, § 30(683)
centripetal acceleration, § 30(683)
centripetal force, § 29(675), § 30(683)
circumference, § 28(669)
collision, § 25(619)
conservation of energy, § 25(619)
conservation of momentum, § 19(553), § 25(619)
conservative force, § 24(613), § 26(649)
coplanar forces, § 14(383)
cosine, § 6(105)
couple, § 15(445)
crane, § 35(747)
crate, § 35(747)
cross product, § 32(711)

D
digitizer tablet, § 4(69)
dimensional analysis, § 9(181)
displacement, § 10(209), § 11(273)
dot product, § 32(711)
dyne, § 18(531)

E
elastic collision, § 25(619)
elastic potential energy, § 21(587), § 22(595), § 23(607), § 24(613), § 26(649)
embossing, § 3(53)
energy, § 20(577), § 34(739)
equilibrium, § 13(375), § 14(383), § 15(445)
external force, § 24(613), § 26(649)

F
factor, § 7(155)
flywheel, § 31(695)
force, § 16(497), § 17(525), § 33(721), § 34(739)
forces, § 13(375)
frequency, § 30(683)

function, § 5(85)

G Galileo, § 17(525)
Google calculator, § 7(155)
graph board, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 9(181), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
gravitational constant, § 16(497)
gravitational potential energy, § 21(587), § 22(595), § 23(607), § 24(613), § 26(649)
gravity, § 12(325)

I impulse, § 19(553)
inelastic collision, § 25(619)
inertia, § 17(525)
inflection point, § 12(325)
inner product, § 32(711)
instantaneous velocity, § 12(325)
internal force, § 24(613), § 26(649)
IVEO, § 3(53), § 4(69)
IVEO Hands-on-Learning System, § 4(69)
IVEO Learning System, § 4(69)

J JavaScript, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 7(155), § 8(167), § 9(181), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)

K kinematics, § 10(209)
kinetic energy, § 22(595), § 23(607), § 24(613), § 25(619), § 26(649)
kinetics, § 10(209)

L ladder, § 35(747)
law of cosines, § 14(383)
law of sines, § 14(383)

M mass, § 17(525), § 33(721)
mechanical energy, § 22(595), § 23(607), § 24(613), § 26(649)
minuend, § 10(209)
moment, § 15(445)
moment of inertia, § 31(695), § 33(721)
momentum, § 17(525), § 19(553)
motion, § 10(209), § 11(273)

N Newton, § 17(525), § 18(531)
Newton's cradle, § 19(553)
non-conservative force, § 24(613), § 26(649)

O one-dimensional, § 25(619)

P parallelogram rule, § 14(383)
parameter, § 5(85)
perfectly inelastic collision, § 25(619)
period, § 28(669), § 30(683)
periodic motion, § 28(669)
physics, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 7(155), § 8(167), § 9(181), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
potential energy, § 21(587), § 22(595), § 23(607), § 24(613), § 26(649)
poundal, § 18(531)
power, § 20(577), § 23(607), § 24(613), § 26(649)
powers of ten, § 9(181)
precision, § 8(167)
projection, § 32(711)
proportion, § 7(155)
protractor, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)

Q quadrant, § 6(105)

R radian measure, § 30(683)
radius, § 28(669)
ratio, § 7(155)
refreshable Braille display, § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607),

§ 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
relational operators, § 5(85)
relative velocity, § 11(273)
repetition, § 5(85)
rest, § 10(209)
resultant, § 10(209), § 11(273), § 14(383)
rotational equilibrium, § 35(747)
rotational inertia, § 31(695), § 33(721)
rotational kinetic energy, § 31(695), § 36(759)

S
scalable vector graphics, § 2(7)
scalar product, § 32(711)
scale, § 7(155)
scientific notation, § 8(167)
screen reader, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 7(155), § 8(167), § 9(181), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
selection, § 5(85)
sequence, § 5(85)
SI units, § 9(181)
significant figures, § 8(167)
sine, § 6(105)
slope of a line, § 12(325)
static equilibrium, § 35(747)
statics, § 13(375)
string, § 5(85)
string concatenation, § 5(85)
structured programming, § 5(85)
subtrahend, § 10(209), § 11(273)
SVG, § 2(7)

T
tactile graphics, § 3(53)
tangent, § 6(105)
tangent line, § 12(325)
tangential, § 29(675)
tangential displacement, § 30(683)
tangential speed, § 30(683)
torque, § 15(445), § 33(721), § 34(739)
total mechanical energy, § 22(595), § 23(607), § 24(613), § 26(649)
touchpad, § 4(69)
translational equilibrium, § 35(747)
translational kinetic energy, § 36(759)
triangle rule, § 14(383)
trigonometry, § 1(1), § 2(7), § 3(53), § 4(69), § 5(85), § 6(105), § 10(209), § 11(273), § 12(325), § 13(375), § 14(383), § 15(445), § 16(497), § 17(525), § 18(531), § 19(553), § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 25(619), § 26(649), § 27(657), § 28(669), § 29(675), § 30(683), § 31(695), § 32(711), § 33(721), § 34(739), § 35(747), § 36(759)
two-dimensional, § 25(619)

U
uniform circular motion, § 28(669), § 29(675)
uniform velocity, § 11(273)
units, § 9(181)

V
variable, § 5(85)
variable velocity, § 12(325)
vector, § 11(273), § 27(657), § 33(721)
vector product, § 32(711)
velocity, § 10(209), § 11(273), § 12(325), § 28(669)
velocity vector, § 28(669), § 29(675)

W
watt, § 23(607), § 24(613), § 26(649)
weight, § 18(531)
weightlessness, § 18(531)
work, § 20(577), § 21(587), § 22(595), § 23(607), § 24(613), § 26(649), § 34(739)
work-energy, § 29(675)

Attributions

Collection: *Accessible Physics Concepts for Blind Students*
Edited by: R.G. (Dick) Baldwin
URL: http://cnx.org/content/col11294/1.35/
License: http://creativecommons.org/licenses/by/3.0/

Module: "Phy1000: Introduction to Accessible Physics Concepts"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37432/1.5/
Pages: 1-6
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1002: If You Can Imagine It, You Can Draw It using SVG"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m39607/1.6/
Pages: 7-51
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1004: Manual Creation of Tactile Graphics"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38546/1.9/
Pages: 53-68
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1006: A small-scale demonstration of the IVEO Learning System"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m40516/1.2/
Pages: 69-83
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1010: JavaScript"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37433/1.4/
Pages: 85-104
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1020: Brief Trigonometry Tutorial"
Used here as: "Brief Trigonometry Tutorial"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37435/1.8/
Pages: 105-154
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1030: Scale Factors, Ratios, and Proportions"
Used here as: "Scale Factors, Ratios, and Proportions"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37436/1.3/
Pages: 155-165
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1040: Scientific Notation and Significant Figures"
Used here as: "Scientific Notation and Significant Figures"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37445/1.5/
Pages: 167-180
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1050: Units and Dimensional Analysis"
Used here as: "Units and Dimensional Analysis"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37480/1.6/
Pages: 181-207
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1060: Motion – Displacement and Vectors"
Used here as: "Motion – Displacement and Vectors"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m37660/1.4/
Pages: 209-271
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1070: Motion – Uniform and Relative Velocity"
Used here as: "Motion – Uniform and Relative Velocity"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38042/1.4/
Pages: 273-324
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1080: Motion – Variable Velocity and Acceleration"
Used here as: "Motion – Variable Velocity and Acceleration"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38074/1.4/
Pages: 325-373
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1090: Force – Introduction to Statics, Equilibrium, and Forces"
Used here as: "Force – Introduction to Statics, Equilibrium, and Forces"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38117/1.3/
Pages: 375-381
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1100: Force – Vector Solutions for Coplanar Forces Concurrent at a Point"
Used here as: "Force – Vector Solutions for Coplanar Forces Concurrent at a Point"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38125/1.5/
Pages: 383-443
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1110: Force – Moments, Torque, Couple, and Equilibrium"
Used here as: "Force – Moments, Torque, Couple, and Equilibrium"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38167/1.4/
Pages: 445-495
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1120: Force – Center of Gravity"
Used here as: "Force – Center of Gravity"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38168/1.4/
Pages: 497-523
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1140: Force and Motion – Introduction"
Used here as: "Force and Motion – Introduction"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38170/1.3/
Pages: 525-530
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1150: Force and Motion – Units of Force"
Used here as: "Force and Motion – Units of Force"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38211/1.3/
Pages: 531-551
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1160: Force and Motion – Momentum, Impulse, and Conservation of Momentum"
Used here as: "Force and Motion – Momentum, Impulse, and Conservation of Momentum"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38234/1.4/
Pages: 553-576
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1170: Energy – Work"
Used here as: "Energy – Work"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38267/1.3/
Pages: 577-585
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1180: Energy – Potential Energy"
Used here as: "Energy – Potential Energy"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38279/1.3/
Pages: 587-594
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1190: Energy – Kinetic and Mechanical Energy"
Used here as: "Energy – Kinetic and Mechanical Energy"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38296/1.3/
Pages: 595-605
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Phy1200: Energy – Power"
Used here as: "Energy – Power"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38354/1.3/
Pages: 607-612
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/4.0/

Module: "Energy – Internal and External Forces"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38362/1.2/
Pages: 613-618
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Energy – Elastic and Inelastic Collisions in Two Dimensions"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38431/1.2/
Pages: 619-648
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Relationships Among Kinematics, Newton's Laws, Vectors, 2D Motion, 2D Forces, Momentum, Work, Energy, and Power"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38369/1.2/
Pages: 649-656
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Vector Subtraction"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38374/1.2/
Pages: 657-668
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Circular Motion – Speed and Velocity"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38399/1.2/
Pages: 669-673
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Circular Motion – Acceleration and Centripetal Force"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38406/1.2/
Pages: 675-682
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Circular Motion – The Mathematics of Circular Motion"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38411/1.4/
Pages: 683-693
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Angular Momentum – Rotational Kinetic Energy and Inertia"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38440/1.2/
Pages: 695-709
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Vector Multiplication"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38455/1.2/
Pages: 711-720
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Angular Momentum – The Mathematics of Torque"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38460/1.2/
Pages: 721-738
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Angular Momentum – Torque, Work and Energy"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38461/1.2/
Pages: 739-746
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Angular Momentum – Rotational Equilibrium"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38465/1.3/
Pages: 747-757
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Module: "Angular Momentum – The Physics of Rolling Objects"
By: R.G. (Dick) Baldwin
URL: http://cnx.org/content/m38498/1.2/
Pages: 759-768
Copyright: R.G. (Dick) Baldwin
License: http://creativecommons.org/licenses/by/3.0/

Accessible Physics Concepts for Blind Students

Blind students should not be excluded from physics courses because of inaccessible textbooks. The modules in this collection present physics concepts in a format that blind students can read using accessibility tools such as an audio screen reader and an electronic line-by-line Braille display. These modules are intended to supplement and not to replace the physics textbook.

About OpenStax-CNX

Rhaptos is a web-based collaborative publishing system for educational material.