

Importing Libraries

```
In [1]: 1 import numpy as np
        2 import pandas as pd
        3 import matplotlib.pyplot as plt
        4 import seaborn as sns
        5 from sklearn.cluster import KMeans
        6 from scipy.optimize import curve_fit
        7
        8 import warnings
        9 warnings.filterwarnings("ignore", category=FutureWarning)
```

Module Error


```

In [2]: 1  """
2  Module errors. Contains:
3  error_prop Calculates the error range caused by the uncertainty of the fit
4  parameters. Covariances are taken into account.
5  covar_to_corr: Converts covariance matrix into correlation matrix.
6  """
7
8
9  import numpy as np
10
11
12  def error_prop(x, func, parameter, covar):
13      """
14      Calculates 1 sigma error ranges for number or array. It uses error
15      propagation with variances and covariances taken from the covar matrix.
16      Derivatives are calculated numerically.
17
18      """
19
20      # initiate sigma the same shape as parameter
21
22      var = np.zeros_like(x)  # initialise variance vektor
23      # Nested loop over all combinations of the parameters
24      for i in range(len(parameter)):
25          # derivative with respect to the ith parameter
26          deriv1 = deriv(x, func, parameter, i)
27
28          for j in range(len(parameter)):
29              # derivative with respect to the jth parameter
30              deriv2 = deriv(x, func, parameter, j)
31
32
33              # multiplied with the i-jth covariance
34              # variance vektor
35              var = var + deriv1*deriv2*covar[i, j]
36
37
38      # Check for division by zero or invalid values
39      mask = np.isinf(var) | np.isnan(var)
40      var[mask] = 0 # Set invalid values to 0 or handle them appropriately
41
42      sigma = np.sqrt(var)
43      return sigma
44
45
46  def deriv(x, func, parameter, ip):
47      """
48      Calculates numerical derivatives from function
49      values at parameter +/- delta. Parameter is the vector with parameter
50      values. ip is the index of the parameter to derive the derivative.
51
52      """
53
54      # print("in", ip, parameter[ip])
55      # create vector with zeros and insert delta value for relevant parameter
56      # delta is calculated as a small fraction of the parameter value
57      scale = 1e-6 # scale factor to calculate the derivative
58      delta = np.zeros_like(parameter, dtype=float)
59      val = scale * np.abs(parameter[ip])
60      delta[ip] = val #scale * np.abs(parameter[ip])
61
62      diff = 0.5 * (func(x, *parameter+delta) - func(x, *parameter-delta))
63      dfdx = diff / val
64
65      return dfdx
66
67
68  def covar_to_corr(covar):
69      """ Converts the covariance matrix into a correlation matrix """
70
71      # extract variances from the diagonal and calculate std. dev.
72      sigma = np.sqrt(np.diag(covar))

```

```
73     # construct matrix containing the sigma values
74     matrix = np.outer(sigma, sigma)
75     # and divide by it
76     corr = covar/matrix
77
78     return corr
79
80
```

Tools for Support clusters


```

In [3]: 1 """ Tools to support clustering: correlation heatmap, normaliser and scale
2 (cluster centres) back to original scale, check for mismatching entries """
3
4
5 # def map_corr(df, size=6):
6 #     """Function creates heatmap of correlation matrix for each pair of
7 #     columns in the dataframe.
8
9 #     Input:
10 #         df: pandas DataFrame
11 #         size: vertical and horizontal size of the plot (in inch)
12
13 #     The function does not have a plt.show() at the end so that the user
14 #     can save the figure.
15 #     """
16
17 #     import matplotlib.pyplot as plt # ensure pyplot imported
18
19 #     corr = df.corr()
20 #     plt.figure(figsize=(size, size))
21 #     fig, ax = plt.subplots()
22 #     plt.matshow(corr, cmap='coolwarm', location="bottom")
23 #     # setting ticks to column names
24 #     plt.xticks(range(len(corr.columns)), corr.columns, rotation=90)
25 #     plt.yticks(range(len(corr.columns)), corr.columns)
26
27 #     plt.colorbar()
28 #     # no plt.show() at the end
29
30 def map_corr(df, size=15):
31     """Function creates a heatmap of the correlation matrix for each pair of
32     columns in the dataframe.
33
34     Input:
35         df: pandas DataFrame
36         size: vertical and horizontal size of the plot (in inches)
37
38     The function does not have a plt.show() at the end so that the user
39     can save the figure.
40     """
41     corr = df.corr(numeric_only=True)
42
43     plt.figure(figsize=(size, size))
44
45     cmap = sns.color_palette("coolwarm", as_cmap=True)
46
47     # Plot the heatmap without a mask
48     sns.heatmap(corr, cmap=cmap, annot=True, fmt=".2f", linewidths=.5)
49
50     plt.show()
51
52 def scaler(df):
53     """ Expects a dataframe and normalises all
54     columns to the 0-1 range. It also returns
55     dataframes with minimum and maximum for
56     transforming the cluster centres"""
57
58     # Uses the pandas methods
59     df_min = df.min()
60     df_max = df.max()
61
62     df = (df - df_min) / (df_max - df_min)
63
64     return df, df_min, df_max
65
66
67 def backscale(arr, df_min, df_max):
68     """ Expects an array of normalized cluster centres and scales
69     it back. Returns numpy array. """
70
71     # Convert df_min and df_max to numpy arrays for indexing
72     minima = df_min.to_numpy()

```

```
73     maxima = df_max.to_numpy()
74
75     # loop over the "columns" of the numpy array
76     for i in range(arr.shape[1]):
77         arr[:, i] = arr[:, i] * (maxima[i] - minima[i]) + minima[i]
78
79     return arr
80
81
82 def get_diff_entries(df1, df2, column):
83     """ Compares the values of column in df1 and the column with the same
84     name in df2. A list of mismatching entries is returned. The list will be
85     empty if all entries match. """
86
87     import pandas as pd # to be sure
88
89     # merge dataframes keeping all rows
90     df_out = pd.merge(df1, df2, on=column, how="outer")
91     print("total entries", len(df_out))
92     # merge keeping only rows in common
93     df_in = pd.merge(df1, df2, on=column, how="inner")
94     print("entries in common", len(df_in))
95     df_in["exists"] = "Y"
96
97     # merge again
98     df_merge = pd.merge(df_out, df_in, on=column, how="outer")
99
100    # extract columns without "Y" in exists
101    df_diff = df_merge[(df_merge["exists"] != "Y")]
102    diff_list = df_diff[column].to_list()
103
104    return diff_list
105
```

Loading Dataset

```
In [4]: 1 df = pd.read_csv('co2.csv')
        2 df
```

Out[4]:

	Country Name	Country Code	Indicator Name	Indicator Code	1990	1991	1992	1993	1994	1995	...	2013	
0	Africa Eastern and Southern	AFE	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.982975	0.942212	0.907936	0.909550	0.913413	0.933001	...	1.001154	1.
1	Afghanistan	AFG	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.191389	0.180674	0.126517	0.109106	0.096638	0.088781	...	0.298088	0.
2	Africa Western and Central	AFW	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.470111	0.521084	0.558013	0.513859	0.462384	0.492656	...	0.481623	0.
3	Angola	AGO	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.554941	0.545807	0.544413	0.710961	0.839266	0.914265	...	1.031044	1.
4	Albania	ALB	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	1.844035	1.261054	0.689644	0.644008	0.649938	0.612055	...	1.656390	1.
...
232	Samoa	WSM	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.529176	0.579131	0.606011	0.656505	0.597318	0.666659	...	0.983800	1.
233	Yemen, Rep.	YEM	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.496616	0.611585	0.632544	0.570608	0.600495	0.654007	...	1.031167	0.
234	South Africa	ZAF	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	6.209373	5.922276	5.717823	5.795258	5.826213	6.007616	...	8.116435	8
235	Zambia	ZMB	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	0.356578	0.364978	0.352722	0.304005	0.252979	0.245217	...	0.278215	0.
236	Zimbabwe	ZWE	CO2 emissions (metric tons per capita)	EN.ATM.CO2E.PC	1.634929	1.763473	1.735620	1.581818	1.469850	1.408363	...	0.901248	0.

237 rows × 37 columns

Cleaning & Preprocessing

```
In [5]: 1 df = df.dropna(axis=1)
        2 df.shape
```

Out[5]: (237, 34)


```
In [6]: 1 df = df.drop(['Country Code', 'Indicator Code'], axis=1)
```

```
In [7]: 1 df.head()
```

Out[7]:

	Country Name	Indicator Name	1991	1992	1993	1994	1995	1996	1997	1998	...	2011	2012	
0	Africa Eastern and Southern	CO2 emissions (metric tons per capita)	0.942212	0.907936	0.909550	0.913413	0.933001	0.943200	0.962203	0.963157	...	0.976840	0.989585	1
1	Afghanistan	CO2 emissions (metric tons per capita)	0.180674	0.126517	0.109106	0.096638	0.088781	0.082267	0.075559	0.071270	...	0.408965	0.335061	0
2	Africa Western and Central	CO2 emissions (metric tons per capita)	0.521084	0.558013	0.513859	0.462384	0.492656	0.554305	0.540062	0.506709	...	0.451578	0.452101	0
3	Angola	CO2 emissions (metric tons per capita)	0.545807	0.544413	0.710961	0.839266	0.914265	1.073630	1.086325	1.091173	...	0.983787	0.947583	1
4	Albania	CO2 emissions (metric tons per capita)	1.261054	0.689644	0.644008	0.649938	0.612055	0.621206	0.469831	0.576804	...	1.768109	1.565921	1

5 rows × 32 columns



```
In [34]: 1 df.to_csv('final.csv', index=False)
```

```
In [8]: 1 df1 = df.drop(['Country Name', 'Indicator Name'], axis=1)
2 df1.head()
```

Out[8]:

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	...	2011	2012	
0	0.942212	0.907936	0.909550	0.913413	0.933001	0.943200	0.962203	0.963157	0.902134	0.891352	...	0.976840	0.989585	1.00
1	0.180674	0.126517	0.109106	0.096638	0.088781	0.082267	0.075559	0.071270	0.058247	0.055167	...	0.408965	0.335061	0.29
2	0.521084	0.558013	0.513859	0.462384	0.492656	0.554305	0.540062	0.506709	0.502905	0.521689	...	0.451578	0.452101	0.48
3	0.545807	0.544413	0.710961	0.839266	0.914265	1.073630	1.086325	1.091173	1.109791	0.988416	...	0.983787	0.947583	1.03
4	1.261054	0.689644	0.644008	0.649938	0.612055	0.621206	0.469831	0.576804	0.960297	1.031568	...	1.768109	1.565921	1.65

5 rows × 30 columns



```
In [9]: 1 df1.columns
```

```
Out[9]: Index(['1991', '1992', '1993', '1994', '1995', '1996', '1997', '1998', '1999',
              '2000', '2001', '2002', '2003', '2004', '2005', '2006', '2007', '2008',
              '2009', '2010', '2011', '2012', '2013', '2014', '2015', '2016', '2017',
              '2018', '2019', '2020'],
              dtype='object')
```

```
In [10]: 1 scaled_df, df_min, df_max = scaler(df1)
```

```
In [11]: 1 scaled_df
```

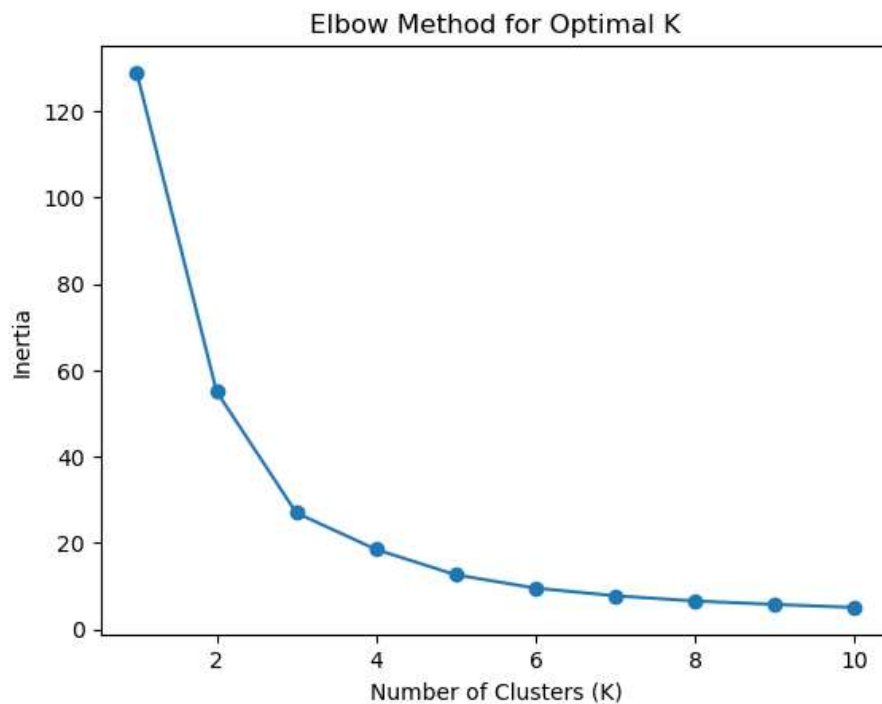
Out[11]:

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	...	2011	2012	
0	0.028702	0.029154	0.026608	0.024733	0.025232	0.023837	0.020864	0.021114	0.019077	0.020085	...	0.024744	0.024084	0.
1	0.005504	0.004062	0.003192	0.002617	0.002401	0.002079	0.001638	0.001562	0.001232	0.001243	...	0.009777	0.007533	0.
2	0.015874	0.017918	0.015032	0.012520	0.013324	0.014009	0.011710	0.011108	0.010635	0.011755	...	0.010900	0.010492	0.
3	0.016627	0.017481	0.020798	0.022725	0.024726	0.027134	0.023555	0.023921	0.023468	0.022272	...	0.024928	0.023022	0.
4	0.038415	0.022144	0.018840	0.017598	0.016553	0.015700	0.010187	0.012645	0.020307	0.023244	...	0.045599	0.038658	0
...
232	0.017642	0.019459	0.019205	0.016174	0.018029	0.018807	0.015679	0.017756	0.016840	0.018197	...	0.025308	0.024344	0.
233	0.018631	0.020311	0.016692	0.016260	0.017687	0.016427	0.014753	0.015507	0.016412	0.018181	...	0.022742	0.019323	0.
234	0.180409	0.183597	0.169534	0.157757	0.162472	0.155074	0.138179	0.141613	0.126917	0.136923	...	0.204790	0.202237	0.
235	0.011118	0.011326	0.008893	0.006850	0.006632	0.005002	0.005426	0.005083	0.003877	0.004117	...	0.004635	0.005972	0.
236	0.053720	0.055730	0.046274	0.039799	0.038088	0.033602	0.026385	0.026715	0.028550	0.025854	...	0.021979	0.021849	0.

237 rows × 30 columns

Elbow Method

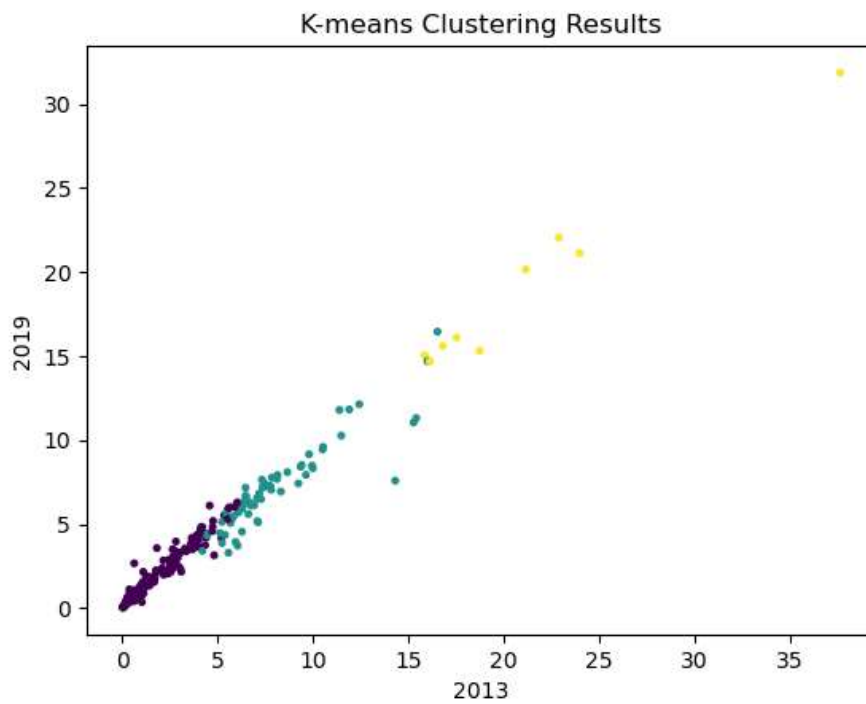
```
In [12]: 1 import warnings
2
3 # Suppress KMeans memory Leak warning on Windows
4 warnings.filterwarnings("ignore", category=UserWarning, module="sklearn.cluster._kmeans")
5
6 inertia = []
7
8 # Perform the Elbow Method for different values of K
9 for k in range(1, 11):
10     kmeans = KMeans(n_clusters=k, random_state=42)
11     kmeans.fit(scaled_df)
12     inertia.append(kmeans.inertia_)
13
14 # Plotting the Elbow Method
15 plt.plot(range(1, 11), inertia, marker='o')
16 plt.title('Elbow Method for Optimal K')
17 plt.xlabel('Number of Clusters (K)')
18 plt.ylabel('Inertia')
19 plt.show()
```



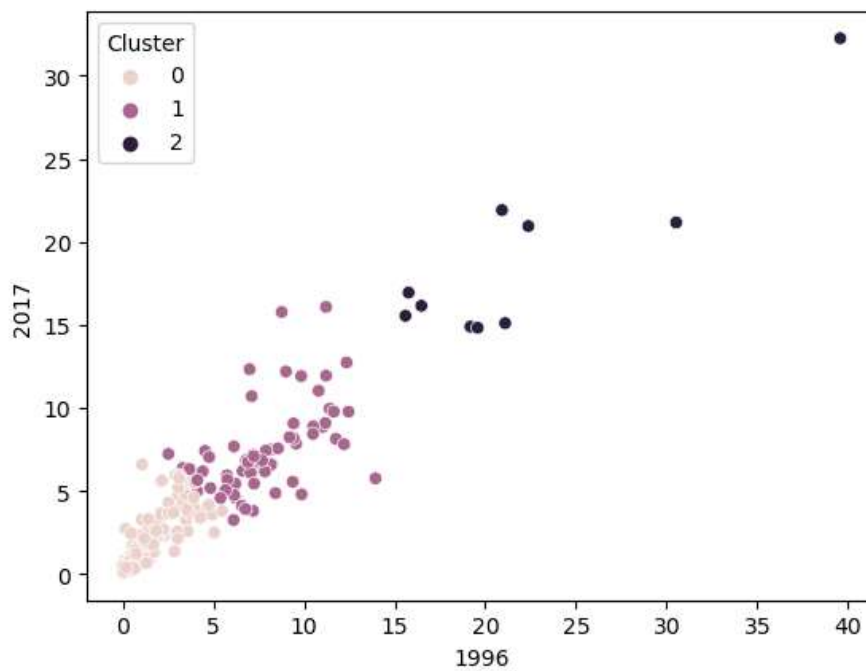
Applying K-means Clustering

```
In [13]: 1 optimal_k = 3
2
3 # Apply K-means clustering
4
5 kmeans = KMeans(n_clusters=optimal_k, random_state=42)
6 df['Cluster'] = kmeans.fit_predict(scaled_df)
```

```
In [14]: 1 dot_size = 7 # Adjust this value based on your preference
2
3 # Visualize the results using a scatter plot with smaller dots
4 plt.scatter(df['2013'], df['2019'], c=df['Cluster'], cmap='viridis', s=dot_size)
5 plt.title('K-means Clustering Results')
6 plt.xlabel('2013')
7 plt.ylabel('2019')
8 plt.show()
9
```



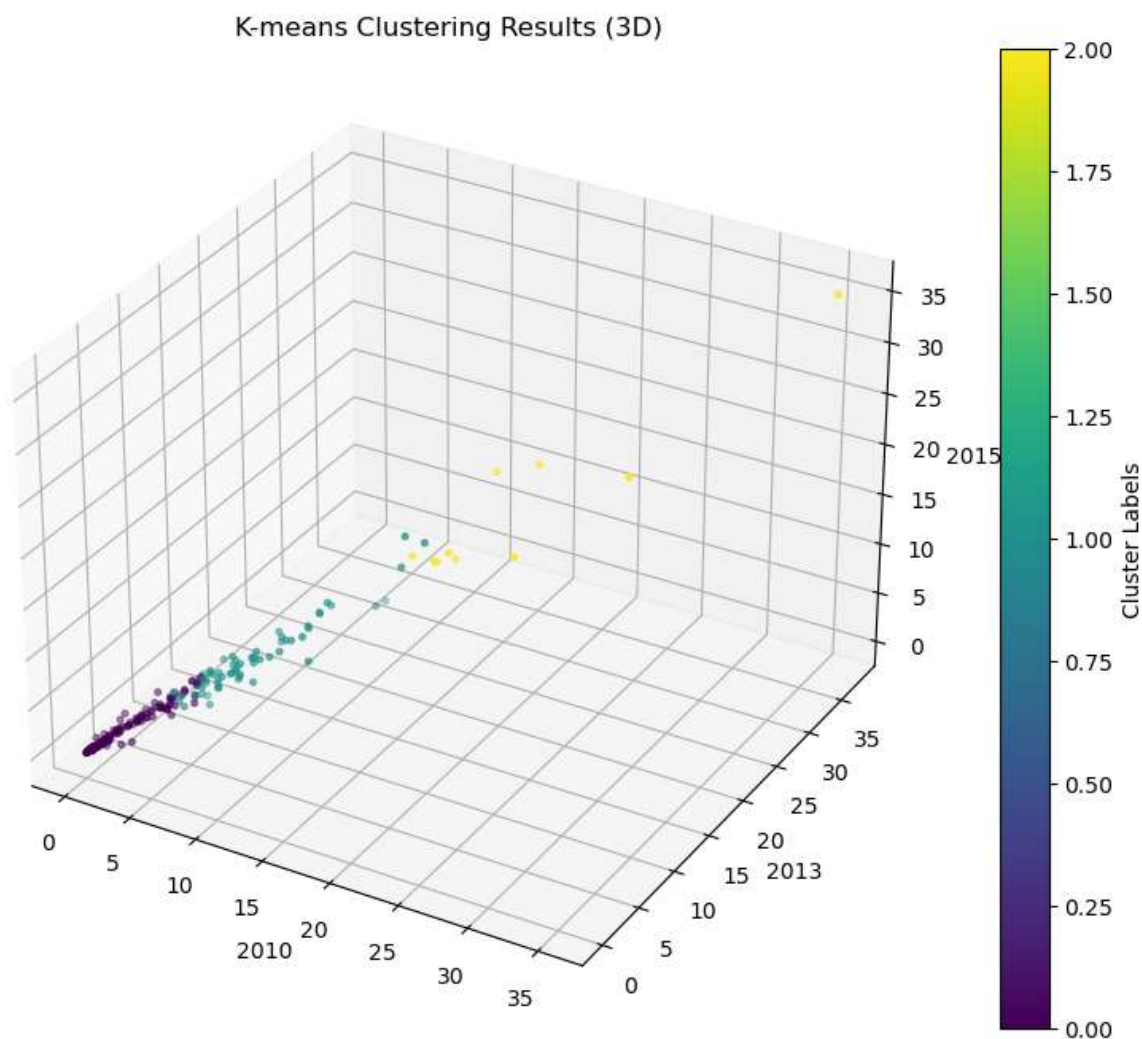
```
In [15]: 1 sns.scatterplot(x='1996', y='2017', hue='Cluster', data=df)
2 plt.show()
```



```

In [16]: 1 from mpl_toolkits.mplot3d import Axes3D
2
3 # Assuming 'original_dataset' contains your unscaled features and 'Cluster_Labels' column
4 fig = plt.figure(figsize=(10, 8))
5 ax = fig.add_subplot(111, projection='3d')
6
7 # Set a smaller size for the dots
8 dot_size = 7 # Adjust this value based on your preference
9
10 scatter = ax.scatter(
11     df['2010'],
12     df['2013'],
13     df['2015'],
14     c=df['Cluster'],
15     cmap='viridis',
16     s=dot_size # Set the size of the dots
17 )
18
19 ax.set_xlabel('2010')
20 ax.set_ylabel('2013')
21 ax.set_zlabel('2015')
22 ax.set_title('K-means Clustering Results (3D)')
23
24 # Add a colorbar to show cluster assignments
25 fig.colorbar(scatter, ax=ax, label='Cluster Labels')
26
27 plt.show()
28

```



```
In [17]: 1 cluster_summary = df.groupby('Cluster').mean()  
2 cluster_summary
```

Out[17]:

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	...	2010
Cluster												
0	1.427813	1.398819	1.361062	1.328024	1.353047	1.379618	1.423309	1.447565	1.439647	1.437016	...	1.78932
1	8.222173	8.080053	7.869553	7.793251	7.779457	7.931050	7.834609	7.778298	7.713726	7.765690	...	8.06386
2	19.919135	20.581825	21.543791	22.253110	21.786230	22.105216	22.824967	22.609215	22.740710	22.722836	...	21.14740

3 rows × 30 columns

```
In [18]: 1 from sklearn.model_selection import train_test_split  
2  
3 common_columns = ['1992', '1993', '1994', '1995', '1996', '1997', '1998', '1999', '2000',  
4                  '2001', '2002', '2003', '2004', '2005', '2006', '2007', '2008', '2009',  
5                  '2010', '2011', '2012', '2013', '2014', '2015', '2016', '2017', '2018',  
6                  '2019', '2020']  
7  
8 # Subset both data frames to include only common columns  
9 df1_common = df1[common_columns]  
10 df_common = df[common_columns + ['Country Name', 'Cluster']] # Add other relevant columns  
11
```

```
In [19]: 1 df_common
```

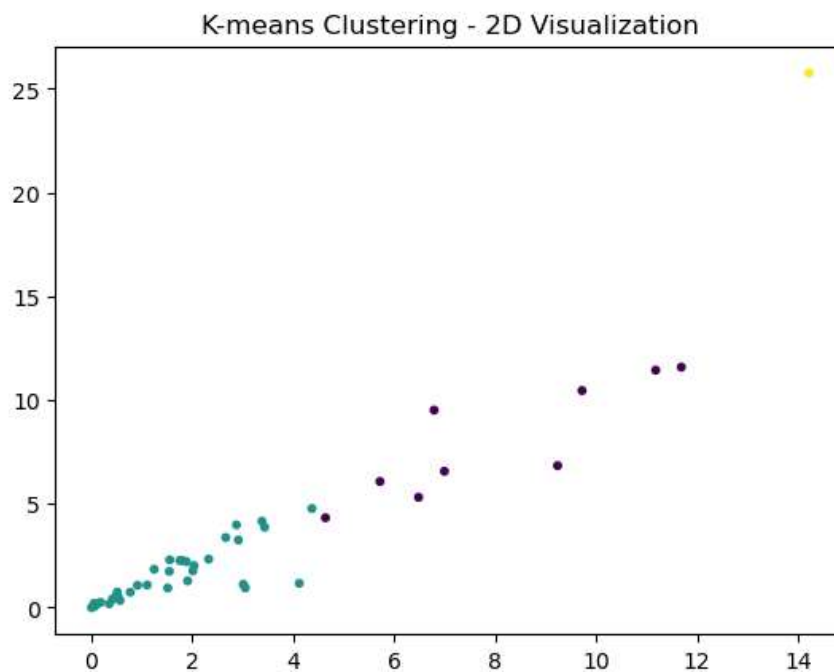
Out[19]:

	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	...	2013	2014	
0	0.907936	0.909550	0.913413	0.933001	0.943200	0.962203	0.963157	0.902134	0.891352	0.958883	...	1.001154	1.013758	0.
1	0.126517	0.109106	0.096638	0.088781	0.082267	0.075559	0.071270	0.058247	0.055167	0.055293	...	0.298088	0.283692	0.
2	0.558013	0.513859	0.462384	0.492656	0.554305	0.540062	0.506709	0.502905	0.521689	0.533552	...	0.481623	0.493505	0.
3	0.544413	0.710961	0.839266	0.914265	1.073630	1.086325	1.091173	1.109791	0.988416	0.941818	...	1.031044	1.091497	1.
4	0.689644	0.644008	0.649938	0.612055	0.621206	0.469831	0.576804	0.960297	1.031568	1.056868	...	1.656390	1.795712	1.
...
232	0.606011	0.656505	0.597318	0.666659	0.744144	0.723075	0.809934	0.796330	0.807574	0.876408	...	0.983800	1.027474	1.
233	0.632544	0.570608	0.600495	0.654007	0.649987	0.680397	0.707366	0.776116	0.806846	0.839206	...	1.031167	0.988347	0.
234	5.717823	5.795258	5.826213	6.007616	6.136002	6.372629	6.459824	6.001786	6.076553	6.783723	...	8.116435	8.191153	7.
235	0.352722	0.304005	0.252979	0.245217	0.197921	0.250242	0.231850	0.183344	0.182709	0.180071	...	0.278215	0.297755	0.
236	1.735620	1.581818	1.469850	1.408363	1.329556	1.216829	1.218623	1.350076	1.147382	1.137220	...	0.901248	0.866838	0.

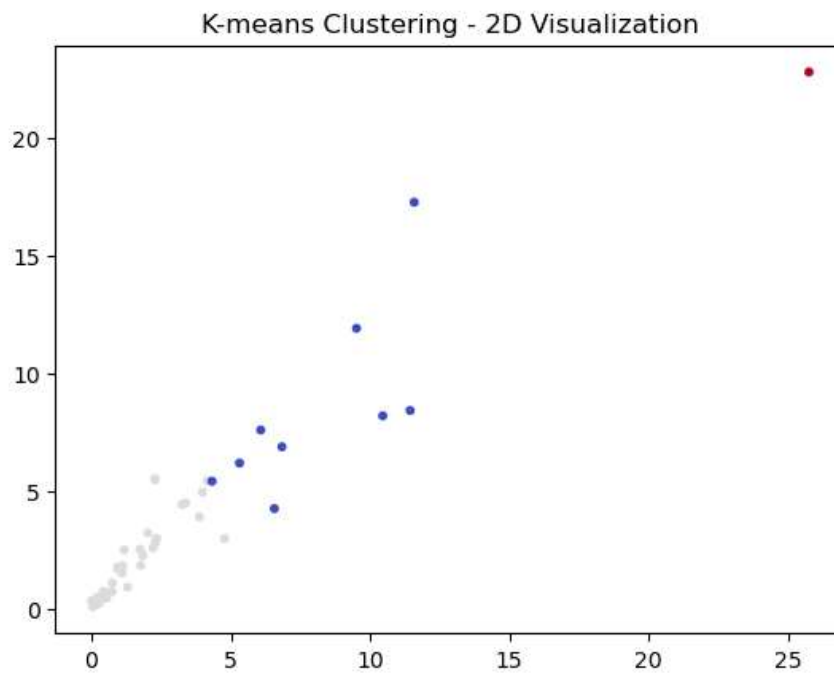
237 rows × 31 columns

```
In [20]: 1 features = df_common.drop(['Country Name', 'Cluster'], axis=1)
2 clusters = df_common['Cluster']
3
4 # Split the data
5 X_train, X_test, y_train, y_test = train_test_split(features, clusters, test_size=0.2, random_state=42)
6
7 # Train K-means on the training data
8 kmeans = KMeans(n_clusters=3, random_state=42)
9 kmeans.fit(X_train)
10
11 # Predict clusters on the test data
12 test_clusters = kmeans.predict(X_test)
13
```

```
In [37]: 1 # 2D Scatter Plot
2
3 plt.scatter(X_test['1992'], X_test['2000'], c=test_clusters, cmap='viridis', s=10) # Adjust 's' value
4 plt.title('K-means Clustering - 2D Visualization')
5 plt.show()
6
```

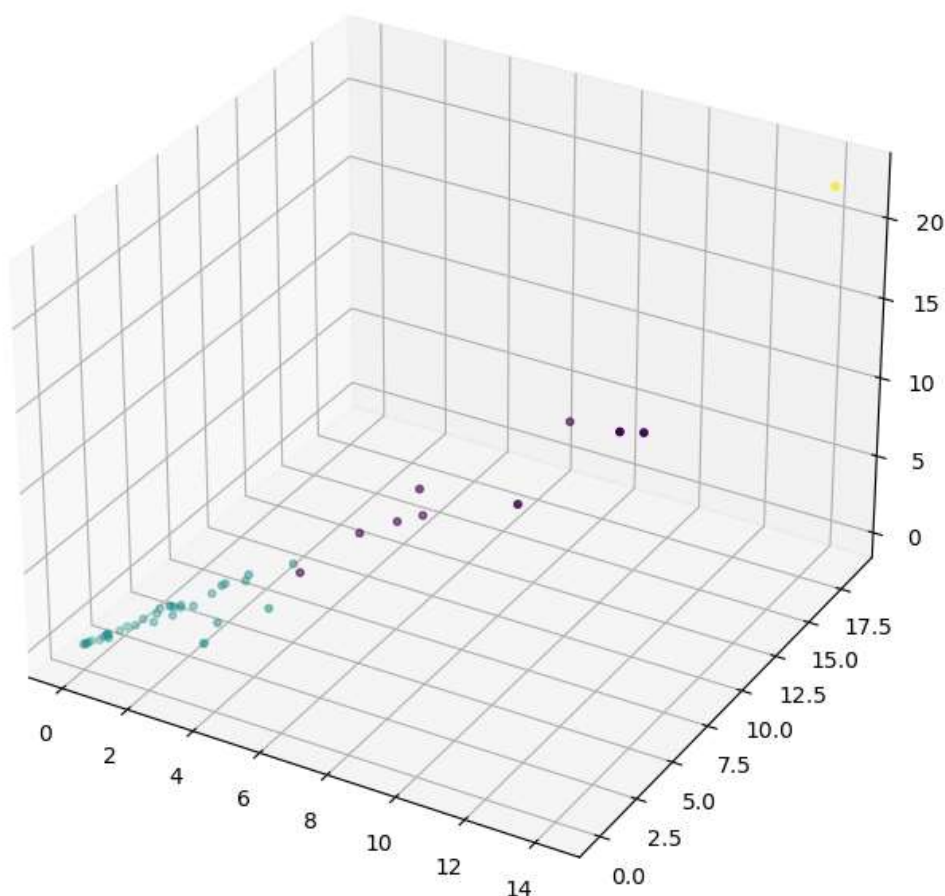


```
In [35]: 1 # 2D Scatter Plot -2
2
3 plt.scatter(X_test['2000'], X_test['2015'], c=test_clusters, cmap='coolwarm', s=10) # Adjust 's' value
4 plt.title('K-means Clustering - 2D Visualization')
5 plt.show()
6
```




```
In [22]: 1 from mpl_toolkits.mplot3d import Axes3D # for 3D plots
2
3 # 3D Scatter Plot
4
5 fig = plt.figure(figsize=(12, 8))
6 ax = fig.add_subplot(111, projection='3d')
7 ax.scatter(X_test['1992'], X_test['1993'], X_test['1994'], c=test_clusters, cmap='viridis', s=10) #
8 ax.set_title('K-means Clustering - 3D Visualization')
9 plt.show()
```

K-means Clustering - 3D Visualization



Evaluate Cohesion and Separation

```
In [23]: 1 from sklearn.metrics import silhouette_score
2
3 # kmeans is the trained K-means model
4 inertia = kmeans.inertia_
5 silhouette = silhouette_score(X_test, test_clusters)
6
7 print(f"Inertia: {inertia}")
8 print(f"Silhouette Score: {silhouette}")
9
```

Inertia: 31720.482657475437

Silhouette Score: 0.638077851285394

Inspect Cluster Sizes

```
In [24]: 1 cluster_sizes = X_test.groupby(test_clusters).size()
2 print("Cluster Sizes:")
3 print(cluster_sizes)
4
```

Cluster Sizes:

```
0    9
1   38
2    1
dtype: int64
```

```
In [25]: 1 new_data = df_common
```

```
In [26]: 1 df1 = df1.drop(['1991'], axis=1)
```

Make Predictions on New Data

```
In [27]: 1 new_data_clusters = kmeans.predict(df1)
```

Analyze Cluster Centers

```
In [28]: 1 cluster_centers = kmeans.cluster_centers_
2 print("Cluster Centers:")
3 print(cluster_centers)
```

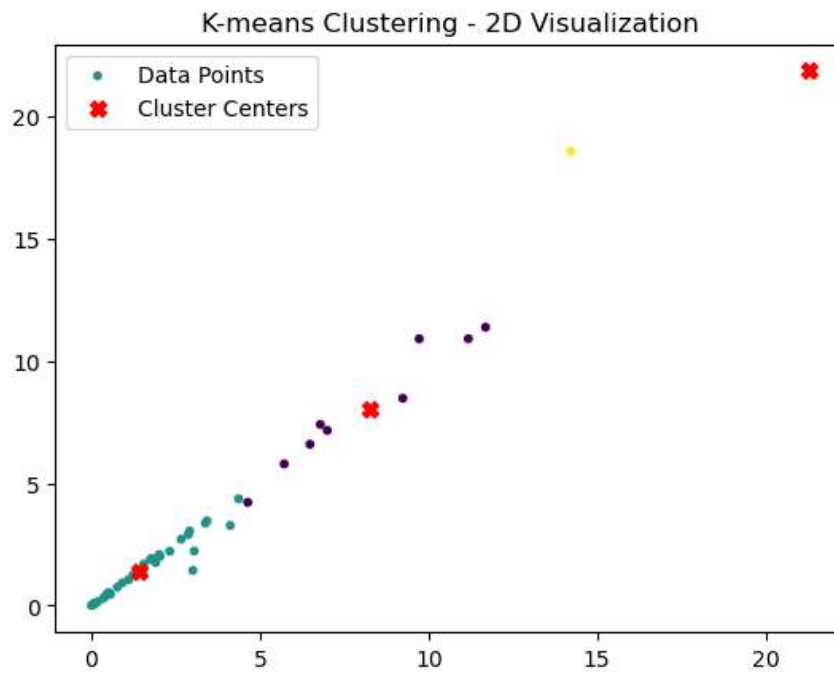
Cluster Centers:

```
[[ 8.28426524  8.01993061  7.94458669  7.92316716  8.0840896  7.97769568
  7.9330942  7.87665938  7.90147893  8.02057112  8.06742695  8.35174086
  8.35226625  8.32589279  8.46491597  8.5183357  8.37388878  7.83690559
  8.16436511  8.0285668  7.99279307  7.93501432  7.59839331  7.39217245
  7.31832485  7.34207293  7.29504353  6.99635447  6.37262892]
 [ 1.39321741  1.37022622  1.34656531  1.37564091  1.40225317  1.45007732
  1.47057223  1.47038827  1.45724774  1.50414889  1.49040285  1.52970369
  1.61564352  1.65816131  1.70811441  1.74345729  1.76607479  1.73851557
  1.79274071  1.83734154  1.86149989  1.84825408  1.85573135  1.85640443
  1.8901272  1.90979038  1.94234371  1.97078468  1.81887419]
 [21.28866745 21.87190048 22.24282654 21.77363312 22.23722059 23.00782918
 22.5484461 22.56034648 22.38565228 22.38657753 22.73653289 23.20446379
 23.37809936 23.19716465 23.19658392 22.32580692 21.75937132 20.29216765
 20.49594754 20.55339329 20.53951641 20.2962328 19.96881982 19.30893648
 18.99434427 18.64751924 18.44006864 18.39598242 18.06844054]]
```

```
In [29]: 1 df.columns
```

```
Out[29]: Index(['Country Name', 'Indicator Name', '1991', '1992', '1993', '1994',
  '1995', '1996', '1997', '1998', '1999', '2000', '2001', '2002', '2003',
  '2004', '2005', '2006', '2007', '2008', '2009', '2010', '2011', '2012',
  '2013', '2014', '2015', '2016', '2017', '2018', '2019', '2020',
  'Cluster'],
  dtype='object')
```

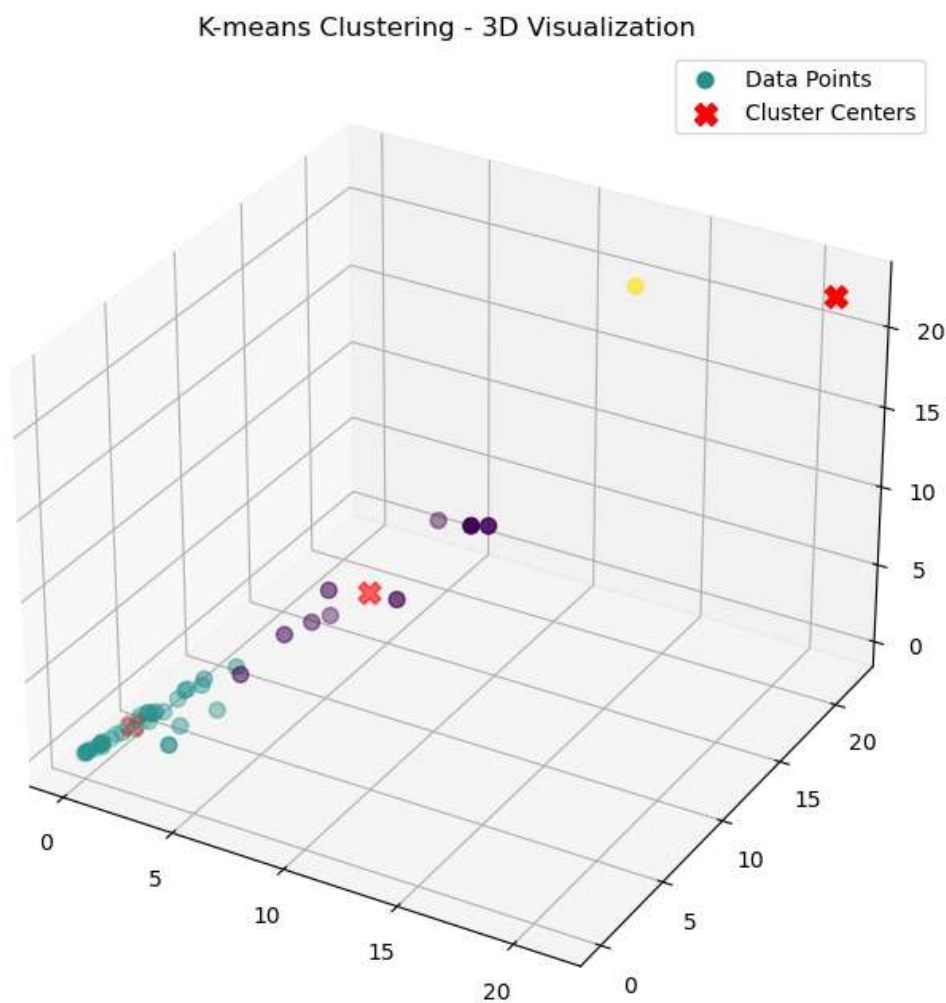
```
In [30]: 1 # 2D Scatter Plot with smaller dots and cross
2
3 plt.scatter(X_test['1992'], X_test['1993'], c=test_clusters, cmap='viridis', s=10, label='Data Points')
4 plt.scatter(cluster_centers[:, 0], cluster_centers[:, 1], marker='X', s=50, c='red', label='Cluster Centers')
5 plt.title('K-means Clustering - 2D Visualization')
6 plt.legend()
7 plt.show()
```



```

In [31]: 1 import matplotlib.pyplot as plt
2 from mpl_toolkits.mplot3d import Axes3D # Import this for 3D scatter plot
3
4 fig = plt.figure(figsize=(10, 8))
5 ax = fig.add_subplot(111, projection='3d')
6 ax.scatter(X_test['1992'], X_test['1993'], X_test['1994'], c=test_clusters, cmap='viridis', s=50, label='Data Points')
7 ax.scatter(cluster_centers[:, 0], cluster_centers[:, 1], cluster_centers[:, 2], marker='X', s=100, c='red', label='Cluster Centers')
8 ax.set_title('K-means Clustering - 3D Visualization')
9 ax.legend()
10 plt.show()
11

```



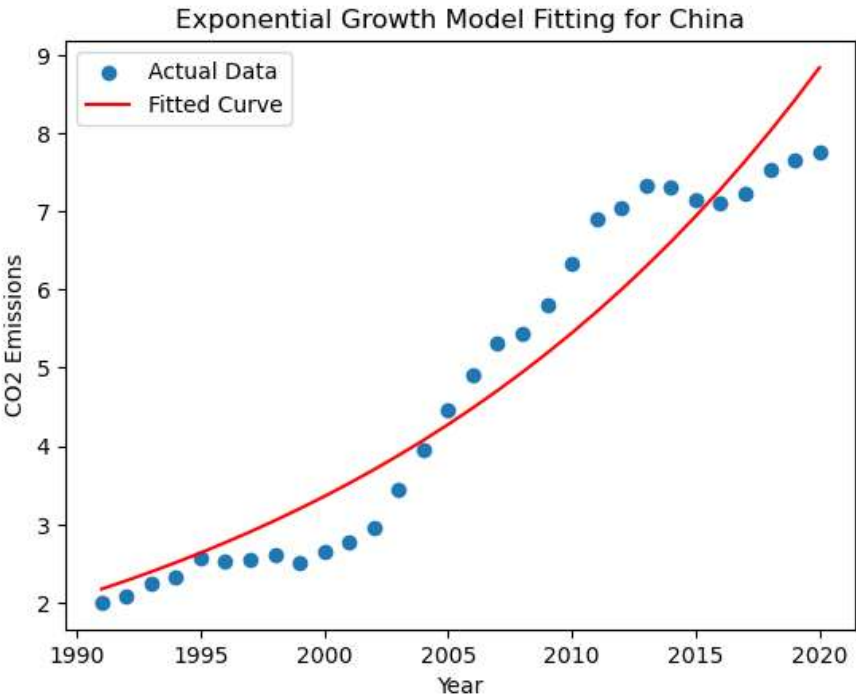
Model Fitting

```

In [32]: 1 selected_country = 'China'
2 co2_emissions = df[df['Country Name'] == selected_country].loc[:, '1991':'2000'].values.flatten()

```

```
In [33]: 1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.optimize import curve_fit
4
5 # Example: Replace 'United States' with the country for which you want to fit the model
6 selected_country = 'China'
7
8 # Extract relevant rows for the selected country
9 country_data = df[df['Country Name'] == selected_country]
10
11 # Extract the years and CO2 emissions values
12 years = country_data.columns[2:-1].astype(int) # assuming '1991' to '2020' columns
13 co2_emissions = country_data.iloc[:, 2:-1].values.flatten()
14
15 # Define the exponential growth model
16 def exponential_growth(t, a, b):
17     return a * np.exp(b * (t - 1991))
18
19 # Provide an initial guess for parameters 'a' and 'b'
20 initial_guess = [1.0, 0.01]
21
22 # Fit the model to the data
23 params, covariance = curve_fit(exponential_growth, years, co2_emissions)
24
25 # Predict using the fitted parameters
26 predicted_values = exponential_growth(years, *params)
27
28 # Plot the original data and the fitted curve
29 plt.scatter(years, co2_emissions, label='Actual Data')
30 plt.plot(years, predicted_values, color='red', label='Fitted Curve')
31 plt.xlabel('Year')
32 plt.ylabel('CO2 Emissions')
33 plt.title(f'Exponential Growth Model Fitting for {selected_country}')
34 plt.legend()
35 plt.show()
36
37 # Print the fitted parameters
38 print("Fitted Parameters (a, b):", params)
39
40 # Calculate confidence intervals
41 err_ranges = np.sqrt(np.diag(covariance))
42 lower_bound = params - 1.96 * err_ranges
43 upper_bound = params + 1.96 * err_ranges
44
45 # Print confidence intervals
46 print("Confidence Intervals (95%):")
47 print("Lower Bound:", lower_bound)
48 print("Upper Bound:", upper_bound)
49
```



Fitted Parameters (a, b): [2.17151711 0.04839454]
Confidence Intervals (95%):
Lower Bound: [1.88036033 0.04232672]
Upper Bound: [2.46267388 0.05446237]

```
In [ ]: 1
```