Model Based Inference for Political Science

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Your Training

0) Math Camp
1) Pol 450A: Regression
2) Pol 450B: Design-Based Inference
3) Pol 450C: Model-Based Inference
4) POL 450D: Topics in Quantitative Methods

   ⋮

Why Models?

Design-based inference:

- Randomness: comes from treatment assignment
- Examples: Experiments, Survey sampling, selection on observables
- Reliable, Clear, and Precise

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All require models

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5) Develop applied data analysis skills
6) Continue improving programming skills

How to Accomplish those Goals?

1) Introduce lots of tools $\rightsquigarrow$ common logic unifying them
2) Use proofs + simulation to establish properties of estimators
3) Applied data in both problem sets and a replication assignment
4) Lots of hard work!!

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Prerequisites:

1) Math Camp
2) POL 450A
3) POL 450B

OR My permission (if you haven't done 1-3, you shouldn't take the class)

Get some books!

Evaluation

Five components to evaluation:

1) Homework (30%): Weekly homework assignments to develop core intuition in class. Use R markdown to submit
2) Midterm Exam (15%): May 14th Closed Book, Pencil + Paper
3) Final Exam (25%): Closed Book, Computer Based
4) Replication Project (25%): (More on next slide)
5) Participation: ask lots of questions!

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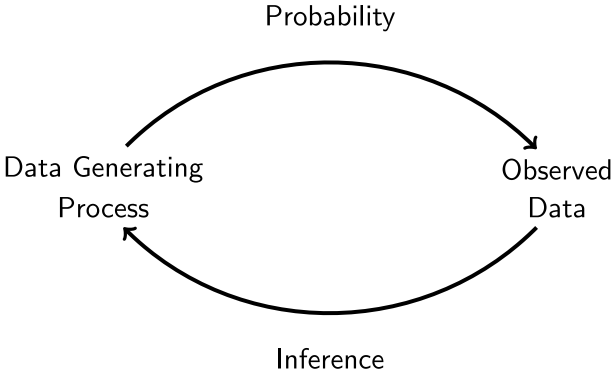
Probability

Data Generating Process

Observed Data

Data Generating Process $\xrightarrow{\text{Probability}}$ Observed Data


Probability
Data Generating Process
Observed Data
Inference

Probability Theory $\rightsquigarrow$ Refresher

1) Model of Probability, Axioms of Probability Function
2) Definition of Random Variable
3) Univariate ideas: Expectation, Variance
4) Multivariate ideas: Joint Distribution, Independence

We will review many random variables + properties as we develop data generating processes

Model of Probability

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$$\text{Var}(X) = E[(X - E[X])^2]$$

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Random variables $\rightsquigarrow$ Build Model

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Model individual i's voter turnout decision, Y

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$Y \sim \text{Bernoulli}(\pi)$

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$$P(Y = y) = p(y) \quad = \quad \pi^y (1 - \pi)^{1-y}$$

for $y \in \{0, 1\}$ (0 otherwise)

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$$\begin{aligned} E[Y] &= 1 \times P(Y = 1) + 0 \times P(Y = 0) \\ &= \pi + 0(1-\pi) = \pi \\ \text{var}(Y) &= E[Y^2] - E[Y]^2 \end{aligned}$$

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$$P(Y = y) = p(y) \quad = \quad \pi^y(1-\pi)^{1-y}$$

for $y \in \{0, 1\}$ (0 otherwise)

$$\begin{aligned} E[Y] &= 1 \times P(Y=1) + 0 \times P(Y=0) \\ &= \pi + 0(1-\pi) = \pi \\ \text{var}(Y) &= E[Y^2] - E[Y]^2 \\ E[Y^2] &= 1^2 P(Y=1) + 0^2 P(Y=0) \end{aligned}$$

Participation

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&= P(Y=1) = \pi \\
\text{var}(Y) &= \pi - \pi^2 \\
&= \pi(1-\pi)
\end{aligned}$$

Joint Random Variables: Many Turnout Decisions

- Interested in many individual's turnout decisions ($i = 1, \ldots, N$)
- Model joint decisions $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_N)$

Define:

$$p(Y_1 = y_1, Y_2 = y_2, \ldots, Y_N = y_N) \quad = \quad p(\boldsymbol{y})$$

This is very complicated $\rightsquigarrow$ Let's simplify

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\prod_{i=1}^{N} P(Y_i = y_i)
\end{aligned}$$

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$Y_i \sim \text{Bernoulli}(\pi)$
Independent, Identically, Distributed (IID)
Assumption$\rightsquigarrow$ similar to what assumption from design-based inference?

Joint Random Variables: Many Turnout Decisions

Suppose $Y_i \sim \text{Bernoulli}(\pi)$

$$\begin{aligned} p(\boldsymbol{y}) &= \prod_{i=1}^{N} P(Y_i = y_i) \\ &= \prod_{i=1}^{N} \pi^{y_i}(1-\pi)^{1-y_i} \\ &= \pi^{\sum_{i=1}^{N} y_i}(1-\pi)^{N-\sum_{i=1}^{N} y_i} \end{aligned}$$

Joint Random Variables: Many Turnout Decisions

Suppose $Y_i \sim \text{Bernoulli}(\pi_i)$
Independent

$$
\begin{aligned}
p(\boldsymbol{y}) &= \prod_{i=1}^{N} P(Y_i = y_i) \\
&= \prod_{i=1}^{N} \pi_i^{y_i}(1-\pi_i)^{1-y_i}
\end{aligned}
$$

Key insight:

- Given $\pi_i \rightsquigarrow$ probability of $\boldsymbol{y}$
- Given $\boldsymbol{y}$ we should be able to learn something about π_i

Modeling Incumbent Vote Share

Suppose we are interested in modeling an incumbent's vote share Y_i. Assume that Y_i is a Normal random variable. Specifically, we will assume:

$$Y_i \sim \text{Normal}(\mu, \sigma^2)$$

Equivalently, a normal random variable has pdf:

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-\mu)^2}{2\sigma^2}\right)$$

Properties of normal random variables:

- $E[Y_i] = \mu$
- $\text{var}(Y_i) = \sigma^2$

Modeling Many Incumbents' Vote Share

Suppose we are interested in many Congressional incumbents' vote shares
Assume $Y_i \sim \text{Normal}(\mu_i, \sigma^2)$
independent (non-identical) samples $\rightsquigarrow \boldsymbol{Y}$

$$
\begin{aligned}
f(\boldsymbol{y}) &= \prod_{i=1}^{N} f(y_i) \\
&= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y_i - \mu_i)^2}{2\sigma^2}\right) \\
&= \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(\sum_{i=1}^{N} \frac{-(y_i - \mu_i)^2}{2\sigma^2}\right)
\end{aligned}
$$

How do we make inferences using the model?

Modeling Many Incumbents' Vote Share

Define $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_N)$

$$f(\boldsymbol{y}, \underbrace{\boldsymbol{\mu}, \sigma^2}_{parameters}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left(\sum_{i=1}^{N} \frac{-(y_i - \mu_i)^2}{2\sigma^2}\right)$$

Writing down model of $\boldsymbol{y}$ creates link between data, parameters

All inferences depend initial modeling assumption (that won't be reflected in other measures of uncertainty)

Probability Term Refresher

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5) Bayes' Rule

Wednesday: Introduction to Likelihood Theory of Inference (inversion)