

Assign3_naive_bayes_classifier

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1)

You are a meteorologist that places temperature sensors all of the world, and you set them up so that they automatically e-mail you, each day, the high temperature for that day. Unfortunately, you have forgotten whether you placed a certain sensor S in Portland or in the Sahara desert (but you are sure you placed it in one of those two places). The probability that you placed sensor S in Portland is 5%. The probability of getting a daily high temperature of 80 degrees or more is 20% in Portland and 90% in Sahara. Assume that probability of a daily high for any day is conditionally independent of the daily high for the previous day, given the location of the sensor.

Part a: If the first e-mail you got from sensor S indicates a daily high under 80 degrees, what is the probability that the sensor is placed in Portland?

Part b: If the first e-mail you got from sensor S indicates a daily high under 80 degrees, what is the probability that the second e-mail also indicates a daily high under 80 degrees?

Part c: What is the probability that the first three e-mails all indicate daily highs under 80 degrees?

$$a. P(<80 \text{ F} \mid \text{PDX}) = 1.00 - 0.20 = 0.80. P(<80 \text{ F} \mid \text{Sahara}) = 1.00 - 0.90 = 0.10$$

$$P(\text{PDX}) = 0.05, P(\text{Sahara}) = 0.95.$$

$$P(<80 \text{ F} \mid \text{PDX}) * P(\text{PDX}) = 0.80 * 0.05 = 0.04$$

$$P(<80 \text{ F} \mid \text{Sahara}) * P(\text{Sahara}) = 0.10 * 0.95 = 0.095$$

$$P(\text{PDX}) = P(<80 \text{ F} \mid \text{PDX}) * P(\text{PDX}) / (P(<80 \text{ F} \mid \text{PDX}) * P(\text{PDX}) + P(<80 \text{ F} \mid \text{Sahara}) * P(\text{Sahara})) = 0.04 / (0.04 + 0.095) = \mathbf{0.2963}.$$

$$b. P(<80 \text{ F}) = P(<80 \text{ F} \mid \text{PDX}) * P(\text{PDX}) + P(<80 \text{ F} \mid \text{Sahara}) * P(\text{Sahara}) = 0.04 + 0.095 = \mathbf{0.135}.$$

$$c. \text{ Assuming all 3 events are independent, } P(<80 \text{ F for 3 days}) = 0.135 * 0.135 * 0.135 = \mathbf{2.46 * 10^{-3}}.$$

2)

Function P is a function defined on a set of samples $S = \{A, B, C, D\}$. We do not know the value of P for all samples, but we know that $P(A) = 0.3$ and $P(B) = 0.6$. What can you say about whether P is a valid probability function? Is P definitely a probability function, possibly a probability function, or definitely not a probability function? Justify your answer.

P is possibly a probability function. The criteria for a probability function are: 1) each value for every event defined by the function must be between 0 and 1, and 2) the sum of the probabilities of all events must be equal to 1. $P(A) = 0.3$ and $P(B) = 0.6$, so if $P(C) + P(D) = 0.1$, then function P is a probability function.

3)

Function P is a function defined on the set of real numbers. We do not know the value of P for all cases, but we know that $P(x) = 0.3$ when $0 \leq x \leq 10$. What can you say about whether P is a valid probability density function? Is P definitely a probability density function, possibly a probability density function, or definitely not a probability density function? Justify your answer.

P is definitely not a probability function in this case. If $P(x)$ has a constant value of 0.3 across the range $0 \leq x \leq 10$, then the integral of $P(x)$ across that function is 3.0, which is greater than 1. A probability function has an integral across all real numbers that is equal to 1.

4) NOTE: the program output uses the \log_e approximation for $\arg\max_{\text{class} \in \{+1, -1\}} P(\text{class}) P(x_i | \tilde{\theta} | \text{class})$ in the Naïve Bayes classifier

Training data (yeast_training.txt)-

Class 1, attribute 1, mean = 0.52, std = 0.10
Class 1, attribute 2, mean = 0.54, std = 0.10
Class 1, attribute 3, mean = 0.52, std = 0.07
Class 1, attribute 4, mean = 0.41, std = 0.17
Class 1, attribute 5, mean = 0.50, std = 0.01
Class 1, attribute 6, mean = 0.00, std = 0.01
Class 1, attribute 7, mean = 0.50, std = 0.05
Class 1, attribute 8, mean = 0.24, std = 0.05

Class 2, attribute 1, mean = 0.45, std = 0.11

Class 2, attribute 2, mean = 0.45, std = 0.10
Class 2, attribute 3, mean = 0.53, std = 0.06
Class 2, attribute 4, mean = 0.23, std = 0.11
Class 2, attribute 5, mean = 0.50, std = 0.04
Class 2, attribute 6, mean = 0.00, std = 0.01
Class 2, attribute 7, mean = 0.49, std = 0.06
Class 2, attribute 8, mean = 0.33, std = 0.14

Class 3, attribute 1, mean = 0.43, std = 0.10
Class 3, attribute 2, mean = 0.48, std = 0.11
Class 3, attribute 3, mean = 0.36, std = 0.06
Class 3, attribute 4, mean = 0.22, std = 0.08
Class 3, attribute 5, mean = 0.51, std = 0.05
Class 3, attribute 6, mean = 0.00, std = 0.01
Class 3, attribute 7, mean = 0.51, std = 0.04
Class 3, attribute 8, mean = 0.27, std = 0.09

Class 4, attribute 1, mean = 0.79, std = 0.07
Class 4, attribute 2, mean = 0.76, std = 0.07
Class 4, attribute 3, mean = 0.38, std = 0.06
Class 4, attribute 4, mean = 0.32, std = 0.11
Class 4, attribute 5, mean = 0.50, std = 0.01
Class 4, attribute 6, mean = 0.00, std = 0.01
Class 4, attribute 7, mean = 0.51, std = 0.07
Class 4, attribute 8, mean = 0.27, std = 0.09

Class 5, attribute 1, mean = 0.74, std = 0.15
Class 5, attribute 2, mean = 0.62, std = 0.12
Class 5, attribute 3, mean = 0.42, std = 0.08
Class 5, attribute 4, mean = 0.30, std = 0.12
Class 5, attribute 5, mean = 0.50, std = 0.01
Class 5, attribute 6, mean = 0.00, std = 0.01
Class 5, attribute 7, mean = 0.51, std = 0.06
Class 5, attribute 8, mean = 0.24, std = 0.04

Class 6, attribute 1, mean = 0.54, std = 0.14
Class 6, attribute 2, mean = 0.50, std = 0.12
Class 6, attribute 3, mean = 0.51, std = 0.05
Class 6, attribute 4, mean = 0.24, std = 0.10
Class 6, attribute 5, mean = 0.50, std = 0.01
Class 6, attribute 6, mean = 0.49, std = 0.38
Class 6, attribute 7, mean = 0.51, std = 0.03
Class 6, attribute 8, mean = 0.24, std = 0.05

Class 7, attribute 1, mean = 0.48, std = 0.11
Class 7, attribute 2, mean = 0.47, std = 0.09
Class 7, attribute 3, mean = 0.54, std = 0.06
Class 7, attribute 4, mean = 0.22, std = 0.12
Class 7, attribute 5, mean = 0.50, std = 0.04
Class 7, attribute 6, mean = 0.00, std = 0.03
Class 7, attribute 7, mean = 0.50, std = 0.06
Class 7, attribute 8, mean = 0.26, std = 0.09

Class 8, attribute 1, mean = 0.74, std = 0.10
Class 8, attribute 2, mean = 0.73, std = 0.11
Class 8, attribute 3, mean = 0.49, std = 0.05
Class 8, attribute 4, mean = 0.29, std = 0.07
Class 8, attribute 5, mean = 0.50, std = 0.01
Class 8, attribute 6, mean = 0.00, std = 0.01
Class 8, attribute 7, mean = 0.46, std = 0.08
Class 8, attribute 8, mean = 0.23, std = 0.02

Class 9 , attribute 1 , mean = 0.55 , std = 0.14
Class 9 , attribute 2 , mean = 0.56 , std = 0.15
Class 9 , attribute 3 , mean = 0.51 , std = 0.06
Class 9 , attribute 4 , mean = 0.20 , std = 0.06
Class 9 , attribute 5 , mean = 0.50 , std = 0.01
Class 9 , attribute 6 , mean = 0.00 , std = 0.01
Class 9 , attribute 7 , mean = 0.53 , std = 0.05
Class 9 , attribute 8 , mean = 0.24 , std = 0.04

Class 10 , attribute 1 , mean = 0.78 , std = 0.05
Class 10 , attribute 2 , mean = 0.73 , std = 0.11
Class 10 , attribute 3 , mean = 0.48 , std = 0.09
Class 10 , attribute 4 , mean = 0.33 , std = 0.06
Class 10 , attribute 5 , mean = 1.00 , std = 0.01
Class 10 , attribute 6 , mean = 0.00 , std = 0.01
Class 10 , attribute 7 , mean = 0.55 , std = 0.02
Class 10 , attribute 8 , mean = 0.23 , std = 0.01

Classification (yeast_test.txt)-

classification accuracy = 0.3430